Waves and Pulses in Cables

Zeyu Ye and Luka Filipovic

***Abstract*— This is an abstract. It will be filled at a later time. I'm just using this opportunity to perform basic tests, such as formatting concerns, and, well, other stuff. At this point, I'm actually running out of things to write down, and honestly the real abstract should not be significantly longer than this piece, so I'll just stop writing bullshit around, well, now.**

I. Introduction

As opposed to the simple DC (direct current) transmission through a conducting wire, AC (alternating current) is usually transmited using transmission cables consisting of coaxial conductors separated by a dielectric. The outer cable is grounded. (figure) Due to electrons changing directions in the wire, current and voltage exhibit wave-like properties, and hence measuring some of these properties is important when considering practical applications.

In our experiment, we examine how voltage and current behave with respect to varying AC frequencies and varying terminating conditions at the end of the cable (as explained below), and analyze this data to find resonant frequencies, speed of propagation in the cable, and dielectric constant of the insulator - which determines the dielectric material.

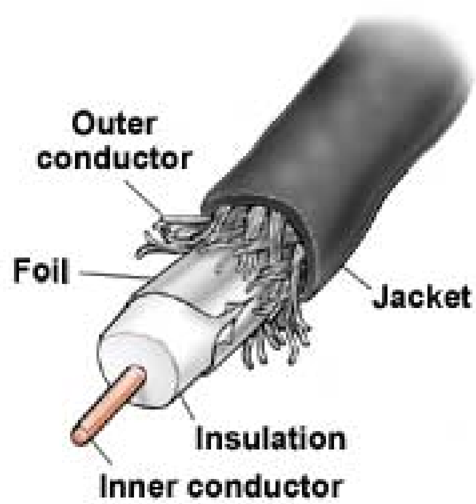

Fig. 1: Cross-section of the transmission cable

II. Theory

If we denote by L inductance per unit lenght, we have, by definition:

$$\frac{\partial V}{\partial x} = -L\frac{\partial I}{\partial t} \tag{II.1}$$

Similarily, if C is capacitance per unit length:

$$\frac{\partial I}{\partial x} = -C\frac{\partial V}{\partial t} \tag{II.2}$$

By combining these equations we have:

$$\frac{\partial^2 V}{\partial x^2} = \frac{\partial \frac{\partial V}{\partial x}}{\partial x} = \frac{-L\partial \frac{\partial I}{\partial t}}{\partial x} = -L\frac{\partial \frac{\partial I}{\partial x}}{\partial t} = -L\frac{-C\partial \frac{\partial V}{\partial t}}{\partial t}$$

or, written compactly:

$$\frac{\partial^2 V}{\partial x^2} = LC\frac{\partial^2 V}{\partial t^2} \tag{II.3}$$

Similarily

$$\frac{\partial^2 I}{\partial x^2} = LC\frac{\partial^2 I}{\partial t^2} \tag{II.4}$$

For an infinite cable where there are no reflection from the end, we can assume that the solution for voltage and current pulses is a superposition of waves in two directions, i.e.

$$V = V_1 e^{j(wt-kx)} + V_2 e^{j(wt+kx)} \tag{II.5}$$

and

$$I = I_1 e^{j(wt-kx)} + \phi_1 + I_2 e^{j(wt+kx)} + \phi_2 \tag{II.6}$$

where ϕ_1 and ϕ_2 are relative phases between the voltage and the current, k is the wavenumber, and ω the angular frequency. Now, if we put II.5 into II.3 we get

$$\frac{\partial^2 V}{\partial x^2} = \frac{\partial^2 (V_1 e^{j(wt-kx)} + V_2 e^{j(wt+kx)}}{\partial x} =$$

$$= (-k)^2 V_1 e^{j(wt-kx)} + (-k)^2 V_2 e^{j(wt+kx)} =$$

$$= k^2 (V_1 e^{j(wt-kx)} + V_2 e^{j(wt+kx)} = k^2 V$$

and similarily

$$LC\frac{\partial^2 V}{\partial t^2} = LC\omega^2 V$$

so from II.3 we have:

$$k^2 V = LC\omega^2 V$$

or

$$(k/\omega)^2 = LC \tag{II.7}$$

If the end of the cable, at position $x = l$, is open circuited, the current there is zero, and thus we have a reflected wave which is 180° out of the incident wave. This can be written as:

$$I = I_0(e^{j(\omega t - k(x-l))} - e^{j(\omega t + k(x-l))}) \tag{II.8}$$

The voltage is maximum at the end, so two waves are in phase at the end, which we can write as:

$$V = V_0(e^{j(\omega t - k(x-l))} + e^{j(\omega t + k(x-l))}) \tag{II.9}$$

For input impedance at arbitrary t and x we have:

$$Z_{in} = \frac{V}{I} = \frac{V_0}{I_0} \frac{e^{j(\omega t - k(x-l))} + e^{j(\omega t + k(x-l))}}{e^{j(\omega t - k(x-l))} - e^{j(\omega t + k(x-l))}}$$

To make calculations more compact, let us substitute $A := \omega t$ and $B := k(x-l)$. The above then becomes:

$$Z_{in} = Z_0 \frac{e^{j(A-B)} + e^{j(A+B)}}{e^{j(A-B)} - e^{j(A+B)}} =$$

$$Z_0 \frac{\cos(A-B) + i\sin(A-B) + \cos(A+B) + i\sin(A+B)}{\cos(A-B) + i\sin(A-B) - \cos(A+B) - i\sin(A-B)}$$

$$= Z_0 \frac{2\cos(A)\cos(B) + 2i\sin(A)\cos(B)}{-2\sin(A)\sin(B) + 2i\cos(A)\sin(B)}$$

Where last line was derived by using basic trigonometric identities, such as

$$\cos(A-B) + \cos(A+B) = 2\cos(A)\cos(B)$$

etc. Continuing the chain we obtain:

$$= Z_0 \frac{\cos(B)}{\sin(B)} \frac{\cos(A) + i\sin(A)}{-\sin(A) + i\cos(A)}$$

$$= Z_0 \cot(B) \frac{e^{jA}}{-je^{jA}} = jZ_0 \cot(B)$$

.

Hence by substituting back $B = k(x-l)$ we get:

$$Z_{in} = jZ_0 \cot(k(x-l)) \tag{II.10}$$

at the generator side of the cable, where $x = 0$, this gives us:

$$Z_{in} = jZ_0 \cot(-kl) = -jZ_0 \cot(kl) \tag{II.11}$$

If the end of the cable is shorted, we have that voltage at $x = l$ is zero, and current is maximum. Hence, the equations are similar to II.8 and II.9, with inverted signs:

$$I = I_0(e^{j(\omega t - k(x-l))} + e^{j(\omega t + k(x-l))}) \tag{II.12}$$

$$V = V_0(e^{j(\omega t - k(x-l))} - e^{j(\omega t + k(x-l))}) \tag{II.13}$$

Calculations are the same, only with numerator and denominator switched. This gets us:

$$Z_{in} = Z_0 \frac{\sin(B)}{\cos(B)} \frac{-jejA}{e^{jA}} = -jZ_0 \tan(B)$$

or, when substituting B back:

$$Z_{in} = -jZ_0 \tan(k(x-l))$$

At the generator side, $x = 0$, this means

$$Z_{in} = -jZ_0 \tan(-kl) = jZ_0 \tan(kl) \tag{II.14}$$

III. Experimental Design and Procedure

Equipment used in the experiment:

- Tektronix function generator
- CW & Pulse Mode Buffer
- one variable resistance terminal box 0-110Ω
- one current probe tap
- BNC connectors
- one BNC shorting termination
- one 60m cable
- two 9m cables
- Agilent Oscilloscope Model DSO6012A
- Voltage Probe
- Oscilloscope camera
- Vernier caliper
- cross-sectioned cable

Using the Vernier caliper, we begin by measuring diameter of the iner conducting wire, and the insulating dielectric, of the cross-sectioned cable (cut like shown in figure 1).

60 meter cable is connected to the function generator, and oscilloscope is used to display the input and reflected voltages. Current probe is clipped over the conductor. Buffer source impedance is set to 75 ohms and its output mode to CW.

We vary the frequency from 1MHz to 10MHz, recording the voltage and current observed. Near frequency values where voltage appers to be the lowest, and current highest, we take about 5 measurements separated by 0.05-0.1MHz. Then we take some additional measurements in between those frequency tresholds. After finishing measurements, we attach the BNC shorting termination at the end of our cable, and repeat the same procedure, this time doing measurements for the shorted circuit termination instead of the open circuit termination.

Next, buffer is set to pulse mode, and controls are set so that the pulses are 22ns in duration and have the frequency of 100MHz. We store the recorded reflection of pulses in the case of open circuit termination, shorted circuit termination (both as described above), and matched load, which is achieved by connecting the end of the 60m cable to the variable resistance terminal box. We repeat this procedure for a 9m cable, and an 18m cable (which we get by connecting two 9m cables).

Finally, we record the phase relationship near frequencies where $Z_{in} = Z_{min}$, i.e. lowest voltage, highest current, for all three cases mentioned above - open termination, shorted termination, matched load. We repeat the procedure for $Z_{in} = Z_{max}$, i.e. highest voltage, lowest current.

IV. Analysis

Hello.

V. Conclusion

Hello.