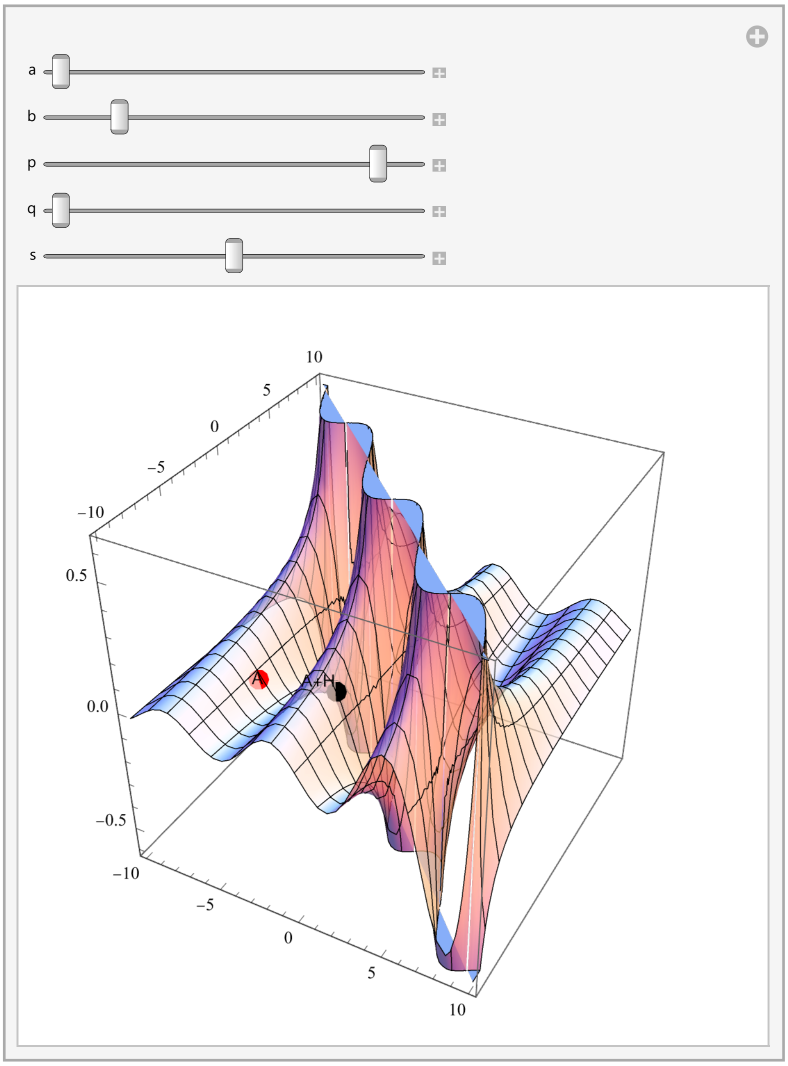
Taylorjev razvoj
(f(a+sh) je krivulja med A in A+H)

In[1280]:=

```
ClearAll[a, b, p, q, s, f]
f[x_, y_] = If[x + y == 0, 0, Sin[x] / (x + y)];
graf = Plot3D[f[x, y], {x, -10, 10}, {y, -10, 10},
   PlotTheme → "Classic",
   BoxRatios → {1, 1, 1},
   PlotPoints → 40,
   ImageSize → Medium,
   PlotStyle → Opacity[0.7]];

Manipulate[
 Show[graf,
  Graphics3D[
   {PointSize[0.03], Red, Point[{a, b, f[a, b]}],
    Black, Point[{a + s (p - a), b + s (q - b), f[a + s (p - a), b + s (q - b)]}],
    Text["A", {a, b, f[a, b]}],
    Text["A+H", {a + s (p - a), b + s (q - b), f[a + s (p - a), b + s (q - b)]}, {1, -1}]
   }
  ]
 ],
 {{a, -6}, -6, 6, 0}, {{b, -4}, -6, 6, 0},
 {{p, 5}, -6, 6, 0}, {q, -6, 6, 0}, {{s, 0.5}, 0, 1}]
(* Točka→ (a,b), Vektor spremembe→ h=(u,v), u=(p-a), v=(q-b), F[s]=f[Točka+sh] *)
```

Out[1283]=

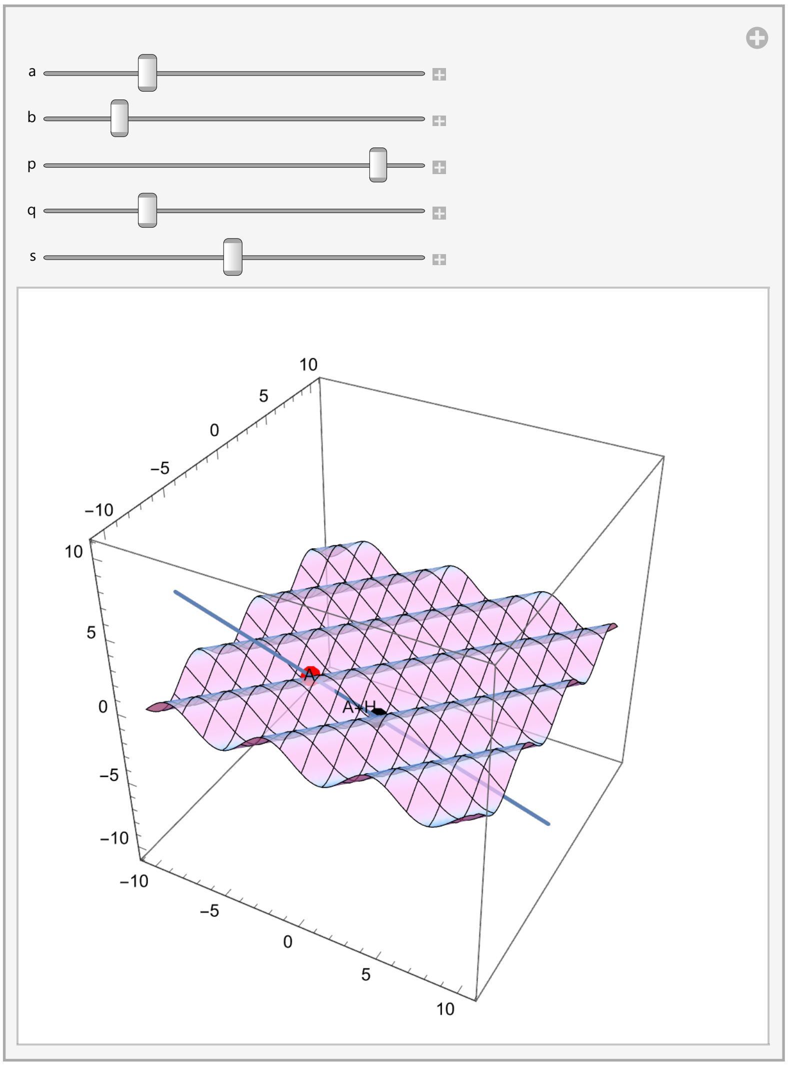


Taylorjev razvoj (Odvod f(a+sh) in njegova tangenta)

In[1284]:=

```
ClearAll[a, b, p, q, s, f]
f[x_, y_] = Sin[x - y];
graf = Plot3D[f[x, y], {x, -10, 10}, {y, -10, 10},
   PlotTheme → "Classic",
   PlotRange → {-10, 10},
   PlotPoints → 40,
   ImageSize → Medium,
   PlotStyle → Opacity[0.7]];

Manipulate[
 Show[
  ParametricPlot3D[
   {a + t ((a + s (p - a)) - a), b + t ((b + s (q - b)) - b),
    f[a, b] + t (f[a + s (p - a), b + s (q - b)] - f[a, b])}, {t, -3 * 1/s, 3 * 1/s}],

  graf,

  Graphics3D[
   {PointSize[0.03], Red, Point[{a, b, f[a, b]}],
    Black, Point[{a + s (p - a), b + s (q - b), f[a + s (p - a), b + s (q - b)]}],
    Text["A", {a, b, f[a, b]}],
    Text["A+H", {a + s (p - a), b + s (q - b), f[a + s (p - a), b + s (q - b)]}, {1, -1}]}],

  BoxRatios → {1, 1, 1},
  PlotRange → {{-10, 10}, {-10, 10}, {-10, 10}}
 ],

 {{a, -6}, -6, 6, 0}, {{b, -4}, -6, 6, 0},
 {{p, 5}, -6, 6, 0}, {q, -6, 6, 0}, {{s, 0.5}, 0.01, 1}
]
(* Točka→ (a,b), Vektor spremembe→ h=(u,v), u=(p-a),v=(q-b) *)
```

Out[1287]=



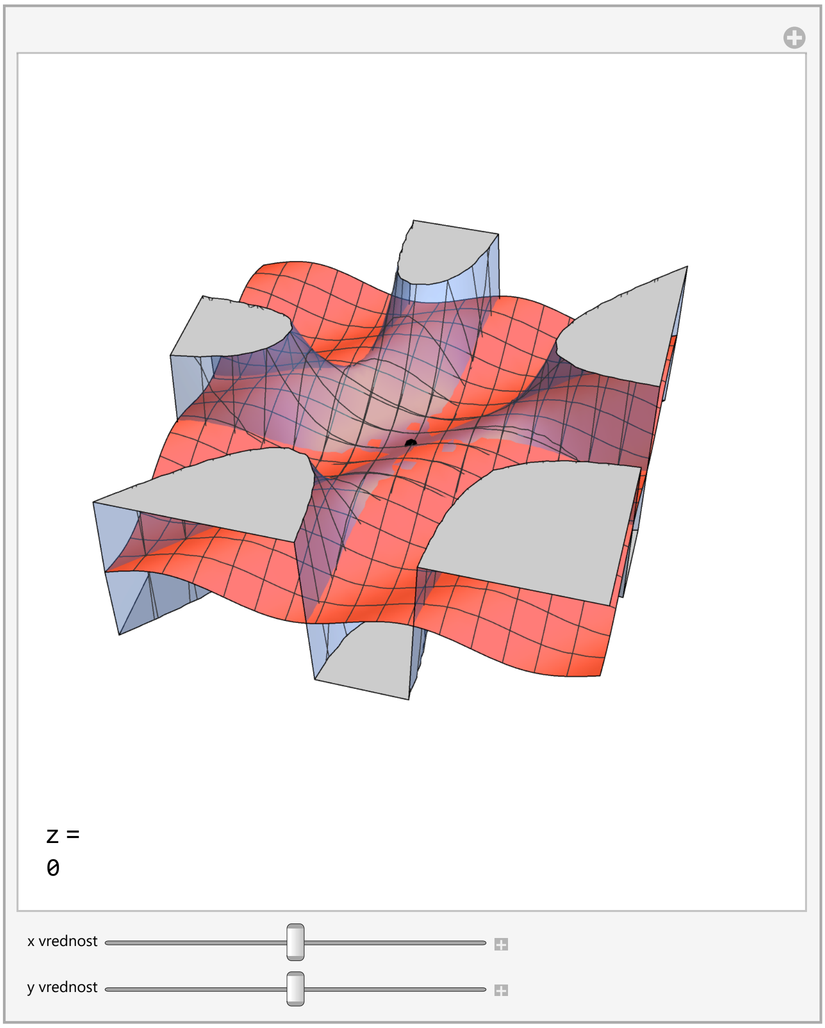
Taylor

Taylorjev razvoj

In[1288]:=

```
ClearAll[x, y, x0, y0, z]
g[x_, y_] = Sin[x - y];
m[x0_, y0_] = Normal[Series[g[x, y], {x, x0, 3}, {y, y0, 3}]];
Manipulate[
 Column[{
   Show[
    Plot3D[
     {g[x, y], m[x0, y0]},
     {x, -2 π, 2 π}, {y, -2 π, 2 π},
     PlotStyle → {Pink, Opacity[0.4], Gray},
     Boxed → False,
     Axes → False,
     AspectRatio → Automatic,
     PlotRange → {-2 π, 2 π},
     ImageSize → 380],

    Graphics3D[
     {PointSize[Large], Point[{x0, y0, g[x0, y0]}]}
    ]
   ],

   Text["z ="], z /. Flatten[Solve[g[x0, y0] == z, z]]
  }],

 {{x0, 0, "x vrednost"}, -E, E}, {{y0, 0, "y vrednost"}, -E, E},
 ControlPlacement → Bottom
]
```

Out[1291]=



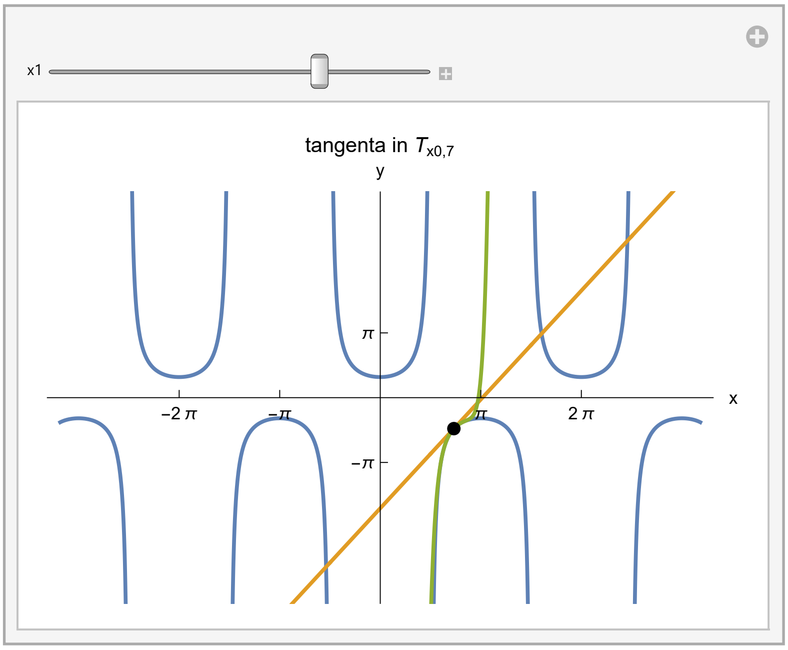
Taylor

Taylorjev razvoj

In[1292]:=

```
ClearAll[f, v, q, b, x1, x]
f[x_] = Sec[x];
v[x1_] := f'[x1];
q[x1_] := f[x1] - v[x1] x1;
b[x_, x1_] = Normal[Series[f[x], {x, x1, 7}]];

Manipulate[
 Plot[{f[x], x v[x1] + q[x1], b[x, x1]}, {x, -10, 10},
  PlotRange → {-10, 10},
  PlotLabel → "tangenta in \!\(\*SubscriptBox[\(T\), \(x0, 7\)]\)",
  AxesLabel → {"x", "y"},
  Ticks → {{-Pi, -2 Pi, Pi, 2 Pi}, {-Pi, Pi}},
  Epilog → {PointSize[Large], Point[{x1, f[x1]}]}],
 {{x1, 0}, -5, 5}
 ]
```

Out[1297]=



Zveznost

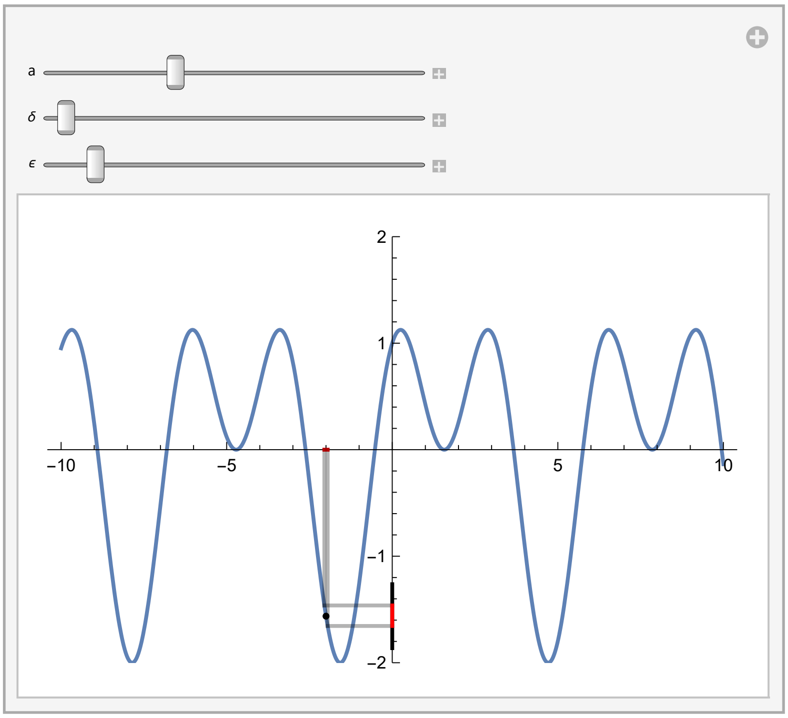
Zveznost

```
(* f je zvezna v a če za vsak epsilon obstaja delta da iz
 x element Df, |x-a|<delta sledi, da je |f(x)-f(a)|<epsilon *)
```

In[1298]:=

```
ClearAll[f, a, ϵ, δ]
f[x_] = Sin[x] + Cos[2 x];

Manipulate[
 Plot[f[x], {x, -10, 10}, PlotRange → {-2, 2},
  Epilog → {
    PointSize[0.01], Point[{a, f[a]}],
    Thick, Red, Line[{{a - δ, 0}, {a + δ, 0}}],

    Black, Line[{{0, f[a] - ϵ}, {0, f[a] + ϵ}}],
    Black, Opacity[0.3], Line[{{a - δ, 0}, {a - δ, f[a - δ]}}],
    Line[{{a + δ, 0}, {a + δ, f[a + δ]}}],

    Line[{{a + δ, f[a + δ]}, {0, f[a + δ]}}],
    Line[{{a - δ, f[a - δ]}, {0, f[a - δ]}}],

    Red, Opacity[1], Line[{{0, f[a - δ]}, {0, f[a + δ]}}]
   }],
 {{a, -2}, -6, 6}, {{δ, 0.05}, 0, 3}, {{ϵ, 0.3}, 0, 3}
]

(* Če obstaja delta dober za vse epsilone je funkcija enakomerno zvezna *)
```

Out[1300]=



Taylor

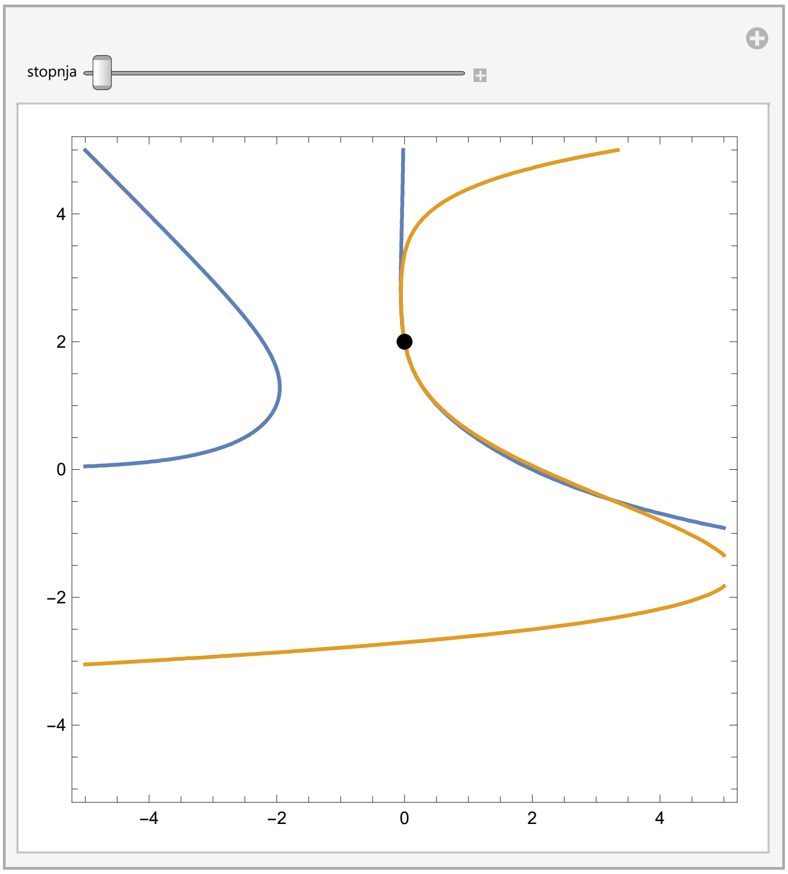
Taylor implicitne funkcije

In[1301]:=

```
ClearAll[g, o1, o1s, o1ss, o2, o2s, o2ss,
  o3, o3s, o3ss, o4, o4s, o4ss, o5, o5s, o5ss, t, r, k, i];
g[x_, y_] = x E^y + y E^-x - 2;
D[g[x, y], y];
(*točka (x,y)=(t,r) naj leži na krivulji*)
t = 0;
r = 2;
(*izrazil sem x=h[y],torej t=h[r]*)

o1 = D[g[h[y], y], y];
o1s = o1 /. {y → r, h[y] → t};
o1ss = Solve[o1s == 0, h'[r]];

o2 = D[o1, y];
o2s = o2 /. {y → r, h[y] → t} /. Flatten[o1ss];
o2ss = Solve[o2s == 0, h''[r]];

o3 = D[o2, y];
o3s = o3 /. {y → r, h[y] → t} /. Flatten[o1ss] /. Flatten[o2ss];
o3ss = Solve[o3s == 0, h'''[r]];

o4 = D[o3, y];
o4s = o4 /. {y → r, h[y] → t} /. Flatten[o1ss] /. Flatten[o2ss] /. Flatten[o3ss];
o4ss = Solve[o4s == 0, h''''[r]];

o5 = D[o4, y];
o5s = o5 /. {y → r, h[y] → t} /. Flatten[o1ss] /. Flatten[o2ss] /. Flatten[o3ss] /.
   Flatten[o4ss];
o5ss = Solve[o5s == 0, h'''''[r]];

(*taylor*)
kontrola[x_] := If[x < 0, 0, 1];
(*taylor=h[r]+h'[r] (y-r) (kontrola[5-k])+1/2h''[r] (y-r)^2 (kontrola[4-k])+
        1/6h'''[r] (y-r)^3 (kontrola[3-k])  +1/24h''''[r] (y-r)^4 (kontrola[2-k])+
        1/120h'''''[r] (y-r)^5 (kontrola[1-k])/.h[r]→t /.Flatten[o1ss]/.
     Flatten[o2ss]/.Flatten[o3ss]/.Flatten[o4ss]/.Flatten[o5ss]*)

Manipulate[
 Module[{k = i},
  taylor = h[r] + h'[r] (y - r) (kontrola[5 - k]) + 1/2 h''[r] (y - r)^2 (kontrola[4 - k]) +
```

```
    1/6 h'''[r] (y - r)^3 (kontrola[3 - k]) + 1/24 h''''[r] (y - r)^4 (kontrola[2 - k]) +
    1/120 h'''''[r] (y - r)^5 (kontrola[1 - k]) /. h[r] → t /. Flatten[o1ss] /.
  Flatten[o2ss] /. Flatten[o3ss] /. Flatten[o4ss] /. Flatten[o5ss];];
ContourPlot[{g[x, y] == 0, x == taylor}, {x, -5, 5},
 {y, -5, 5}, Epilog → {PointSize[0.02], Point[{t, r}]}],
{{i, 1, "stopnja"}, 1, 5, 1}
]
```

Out[1322]=



Lagrange

F[x,y,λ]

In[187]:=

```
ClearAll[f, g1, g2, L1, L2, ℛ, lagrangePoints1,
 lagrangePoints2, extremePoints1, extremePoints2]
f[x_, y_] := Sin[x] Cos[y];
g1[x_, y_] := x^2 + y^2 - 1;
g2[x_, y_] = x^2 + y^2 - 1/2;
ℛ = ImplicitRegion[g1[x, y] ≤ 0 && g2[x, y] ≥ 0, {x, y}];
Print["Tocke kjer ima f lokalne extreme"]
tockef = {x, y, f[x, y]} /. # & /@ Solve[D[f[x, y], x] == D[f[x, y], y] == 0, {x, y} ∈ ℛ]

(*Lagrangeva funkcija*)
L1[x_, y_, λ_] := f[x, y] + λ g1[x, y];
L2[x_, y_, λ_] := f[x, y] + λ g2[x, y];

(*Lagrangeve enačbe*)
lagrangePoints1 = Solve[{D[L1[x, y, λ], x] == 0,
    D[L1[x, y, λ], y] == 0, D[L1[x, y, λ], λ] == 0, g1[x, y] == 0}, {x, y, λ}, Reals];
lagrangePoints2 = Solve[{D[L2[x, y, λ], x] == 0,
    D[L2[x, y, λ], y] == 0, D[L2[x, y, λ], λ] == 0, g2[x, y] == 0}, {x, y, λ}, Reals];


(*koordinate lagrangevih funkcij*)
Print["Tocke lagrangeve funkcije za g1"]
extremePoints1 = {x, y, f[x, y]} /. lagrangePoints1
Print["Tocke lagrangeve funkcije za g2"]
extremePoints2 = {x, y, f[x, y]} /. lagrangePoints2

(*graf*)
Manipulate[
 Show[Graphics3D[{Red, PointSize[0.02],
    Point[extremePoints1], Point[extremePoints2], Point[tockef]}],
  Plot3D[f[x, y], {x, y} ∈ ℛ, BoxRatios → {1, 1, 1}, PlotRange → All],
  Plot3D[f[x, y], {x, -10, 10}, {y, -10, 10}, PlotRange → All],

  PlotRange → {{-v, v}, {-v, v}, {-v, v}}
 ],
 {{v, 1, "velikost"}, 1/3, 10, 1/2}
]
```

Tocke kjer ima f lokalne extreme

Out[193]=

{}

Tocke lagrangeve funkcije za g1

Out[199]=

$\{\{-1, 0, -\mathrm{Sin}[1]\}, \{1, 0, \mathrm{Sin}[1]\}\}$

Tocke lagrangeve funkcije za g2

Out[201]=

$$\left\{\left\{-\frac{1}{\sqrt{2}}, 0, -\mathrm{Sin}\left[\frac{1}{\sqrt{2}}\right]\right\}, \left\{\frac{1}{\sqrt{2}}, 0, \mathrm{Sin}\left[\frac{1}{\sqrt{2}}\right]\right\}\right\}$$

Out[202]=