Funkcije:

(* Ortogonalna baza *)

In[•]:=
```
ortogonal[B_, v_] := Module[{B1}, B1 = B; Bo = Table[B1 = ReplacePart[
        B1, i → B1[[i]] - Sum[v[B1[[i]], B1[[k]]] / v[B1[[k]], B1[[k]]] * B1[[k]], {k, 1, i - 1}]],
    {i, 2, Length[B]}][[Length[B] - 1]]]
(* V katero vstavimo bazo in skalarni produkt v obliki "v[v1_,v2_]:= Dot[v1,v2]" *)
```

In[•]:=
```
(* Primer: *)
ClearAll[a, φ]
φ = {1, Sin[x], Cos[x]}
a[f_, g_] := Integrate[f g, {x, 0, π}]
ortogonal[φ, a]
```

Out[•]=
$$\{1, \text{Sin}[x], \text{Cos}[x]\}$$

Out[•]=
$$\left\{1, -\frac{2}{\pi} + \text{Sin}[x], \text{Cos}[x]\right\}$$

(* Ortonormirana baza; uporabimo funkcijo Orthogonalize[] v katero vstavimo bazo in skalarni produkt*)

In[•]:=
```
(* Primer: *)
ClearAll[X]
X = {{1, 2, 0, 0, 3, 3, 1, 1, 1}, {1, 1, 0, 0, 3, 3, 1, 6, 1}, {1, 2, 7, 0, 3, 0, 0, 1, 1}};
Orthogonalize[X]
```

Out[•]=
$$\left\{\left\{\frac{1}{\sqrt{26}}, \sqrt{\frac{2}{13}}, 0, 0, \frac{3}{\sqrt{26}}, \frac{3}{\sqrt{26}}, \frac{1}{\sqrt{26}}, \frac{1}{\sqrt{26}}, \frac{1}{\sqrt{26}}\right\},\right.$$
$$\left\{-\frac{3}{\sqrt{17\,342}}, -16\sqrt{\frac{2}{8671}}, 0, 0, -\frac{9}{\sqrt{17\,342}}, -\frac{9}{\sqrt{17\,342}}, -\frac{3}{\sqrt{17\,342}}, \frac{127}{\sqrt{17\,342}}, -\frac{3}{\sqrt{17\,342}}\right\},$$
$$\left\{\frac{260}{\sqrt{24\,514\,251}}, \frac{550}{\sqrt{24\,514\,251}}, 7\sqrt{\frac{667}{36\,753}}, 0, 260\sqrt{\frac{3}{8\,171\,417}},\right.$$
$$\left.\left.-407\sqrt{\frac{3}{8\,171\,417}}, -\frac{407}{\sqrt{24\,514\,251}}, \frac{110}{\sqrt{24\,514\,251}}, \frac{260}{\sqrt{24\,514\,251}}\right\}\right\}$$

(* Projekcija na podprostor s produktom v, kjer naj bo Ψ ortogonalna ali ortonormirana baza, l pa vektor, ki ga projeciramo na Ψ *)

In[63]:=
```
projUv[Ψ_, v_, l_] := Simplify[Sum[v[l, Ψ[[k]]] / v[Ψ[[k]], Ψ[[k]]] * Ψ[[k]], {k, 1, Length[Ψ]}]]
```

In[•]:=
```
(* Primer 1: *)(* Projeciramo x *)
ClearAll[a, φ, x]
a[f_, g_] := Integrate[f g, {x, 0, π}]
φ = {1, -2/π + Sin[x], Cos[x]};
projUv[φ, v, x]
```

Out[•]=

$$\frac{v[x,1]}{v[1,1]}+\frac{\text{Cos}[x]\,v[x,\text{Cos}[x]]}{v[\text{Cos}[x],\text{Cos}[x]]}+\frac{\left(-\frac{2}{\pi}+\text{Sin}[x]\right)v\left[x,-\frac{2}{\pi}+\text{Sin}[x]\right]}{v\left[-\frac{2}{\pi}+\text{Sin}[x],-\frac{2}{\pi}+\text{Sin}[x]\right]}$$

In[•]:=
```
(* Primer 2: *)
ClearAll[a, φ, x]
a[f_, g_] := Integrate[f g, {x, 0, π/2}]
φ = orotogonal[{1, Cos[x], Sin[x]}, a]
projUv[φ, v, x]
```

Out[•]=

$$\left\{1,-\frac{2}{\pi}+\text{Cos}[x],-\frac{2}{\pi}-\frac{\left(\frac{1}{2}-\frac{2}{\pi}\right)\left(-\frac{2}{\pi}+\text{Cos}[x]\right)}{-\frac{2}{\pi}+\frac{\pi}{4}}+\text{Sin}[x]\right\}$$

Out[•]=

$$\frac{v[x,1]}{v[1,1]}+\frac{\left(-\frac{2}{\pi}+\text{Cos}[x]\right)v\left[x,-\frac{2}{\pi}+\text{Cos}[x]\right]}{v\left[-\frac{2}{\pi}+\text{Cos}[x],-\frac{2}{\pi}+\text{Cos}[x]\right]}+\left(\left(4-2\pi-2\,(-4+\pi)\,\text{Cos}[x]+\left(-8+\pi^2\right)\text{Sin}[x]\right)v\left[x,\frac{4-2\pi-2\,(-4+\pi)\,\text{Cos}[x]+\left(-8+\pi^2\right)\text{Sin}[x]}{-8+\pi^2}\right]\right)\Big/\left(\left(-8+\pi^2\right)v\left[\frac{4-2\pi-2\,(-4+\pi)\,\text{Cos}[x]+\left(-8+\pi^2\right)\text{Sin}[x]}{-8+\pi^2},\frac{4-2\pi-2\,(-4+\pi)\,\text{Cos}[x]+\left(-8+\pi^2\right)\text{Sin}[x]}{-8+\pi^2}\right]\right)$$

(* Za lastne vrednosti uporabi Eigenvalues[], za lastne vektorje pa Eigenvectors[] *)

In[•]:=
```
(* Primer: *)
Eigenvalues[{{1, -2, 0}, {0, 3, -1}, {1, 1, 1}}]
Eigenvectors[{{1, -2, 0}, {0, 3, -1}, {1, 1, 1}}]
```

Out[•]=
```
{3, 1 + i, 1 - i}
```

Out[•]=
```
{{-1, 1, 0}, {-2 + 4 i, 2 + i, 5}, {-2 - 4 i, 2 - i, 5}}
```

(* Matrika linearne preslikave, kjer je ϕ baza in ψ predpis preslikave*)

In[•]:=
```
matrikapreslikave[ϕ_, ψ_] :=
 (Transpose[Table[ψ[ϕ[[k]]], {k, 1, Length[ϕ]}]].Inverse[Transpose[ϕ]])
```

In[•]:=
```
(* Primer: *)
ClearAll[h, s1]
s1 = {{1, 0, 0}, {0, 1, 0}, {0, 1, 1}};
h[f_] := Dot[f, a] * a + Cross[f, a] - 2 f
matrikapreslikave[s1, h] // MatrixForm
(* poglejmo, še kaj je v jedru (NullSpace[]) *)
NullSpace[matrikapreslikave[s1, h]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -2+\{1,0,0\}\times a+a\{1,0,0\}.a & \{0,1,0\}\times a+a\{0,1,0\}.a & -\{0,1,0\}\times a+\{0,1,1\}\times a \\ \{1,0,0\}\times a+a\{1,0,0\}.a & -2+\{0,1,0\}\times a+a\{0,1,0\}.a & -\{0,1,0\}\times a+\{0,1,1\}\times a \\ \{1,0,0\}\times a+a\{1,0,0\}.a & \{0,1,0\}\times a+a\{0,1,0\}.a & -2-\{0,1,0\}\times a+\{0,1,1\}\times \end{pmatrix}$$

Out[•]=

$\{\}$

Primeri:

Projekcija:

In[•]:=
```
ClearAll[a, φ, x, b]
(* bazo podprostora φ ortagonaliziramo glede na skalarni produkt
 a in poiščemo projekcijo Sin[x] na podprostor, nato jo narišemo *)
φ = {1, Cos[x], x};
a[f_, g_] := Integrate[f g, {x, 0, π/2}]
ortogonal[φ, a]
b = projUv[ortogonal[φ, a], a, Sin[x]]
Plot[{Sin[x], b}, {x, -5, 5}, PlotStyle → {Dashed, Black}]
```

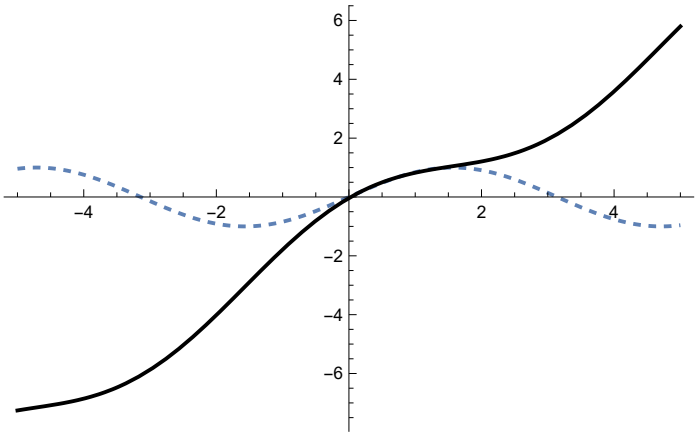
Out[•]=

$$\left\{1, -\frac{2}{\pi} + \text{Cos}[x], -\frac{\pi}{4} + x - \frac{(-4+\pi)\left(-\frac{2}{\pi} + \text{Cos}[x]\right)}{4\left(-\frac{2}{\pi} + \frac{\pi}{4}\right)}\right\}$$

Out[•]=

$$\left(8\left(\pi^4 + 6\pi^2(-5+x) - 192(1+x) + 72\pi(2+x) - \pi^3(2+3x)\right) + 2\pi\left(192 - 96\pi + 8\pi^2 + \pi^3\right)\text{Cos}[x]\right) \Big/ \left(\pi\left(-384 + 192\pi - 32\pi^2 + \pi^4\right)\right)$$

Out[•]=



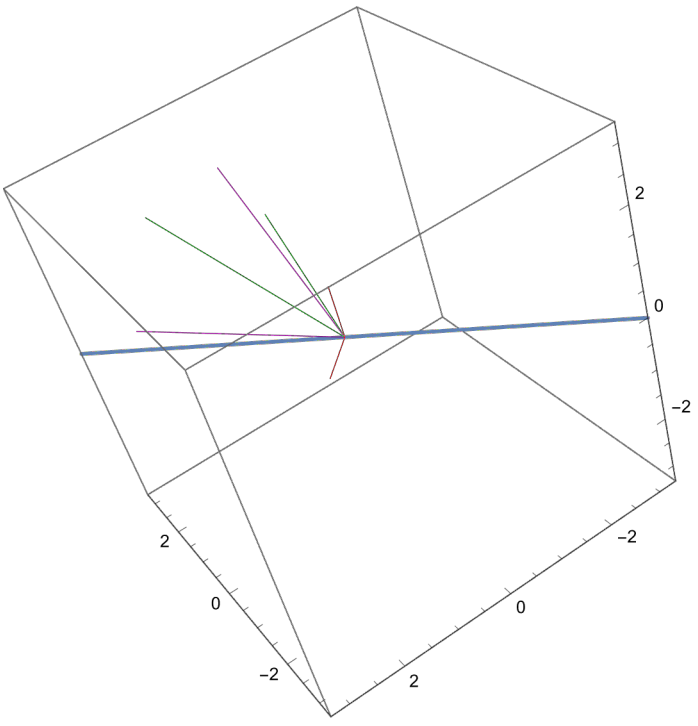
Rotacija za kot $\frac{\pi}{2}$ okoli premice $x = z$, $y = 0$:

In[•]:=
```
(* Razberemo smerni vektor s=
 {1,0,1} poiščemo še dva pravokotna vektorja in ju ortonormirajmo
   v1={1/Sqrt[2],0,-1/Sqrt[2]} in v2={-1/Sqrt[2],0,1/Sqrt[2]}. Imamo ortonormirano bazo...
    ker predpis za preslikavo ni podan jo izračunamo sami. *)
ClearAll[A, v1, v2, v3]
A = {{1/2, -1/Sqrt[2], 1/2}, {1/Sqrt[2], 0, -1/Sqrt[2]}, {1/2, 1/Sqrt[2], 1/2}};
A // MatrixForm
v1 = {1/Sqrt[2], 0, -1/Sqrt[2]};
v3 = {3, 2, 0};
v2 = {2, 3, 1};
Show[ParametricPlot3D[{t, 0, t}, {t, -10, 10}],
 Graphics3D[
  {Red, Line[{{0, 0, 0}, v1}], Line[{{0, 0, 0}, A.v1}], Green, Line[{{0, 0, 0}, v2}],
   Line[{{0, 0, 0}, A.v2}], Purple, Line[{{0, 0, 0}, v3}], Line[{{0, 0, 0}, A.v3}]}],
 BoxRatios → {1, 1, 1}, PlotRange → {{-3, 3}, {-3, 3}, {-3, 3}}]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \end{pmatrix}$$

Out[•]=



Projekcija:

In[•]:=
```
ClearAll[a, φ, x, b]
(* bazo podprostora φ ortagonaliziramo glede na skalarni produkt
 a in poiščemo projekcijo Sin[x] na podprostor, nato jo narišemo *)
φ = {1, Cos[x], Sin[x]};
a[f_, g_] := Integrate[f g, {x, 0, π}]
ortogonal[φ, a]
b = projUv[ortogonal[φ, a], a, x^2]
Plot[{x^2, b}, {x, -5, 5}, PlotStyle → {Dashed, Black}]
```

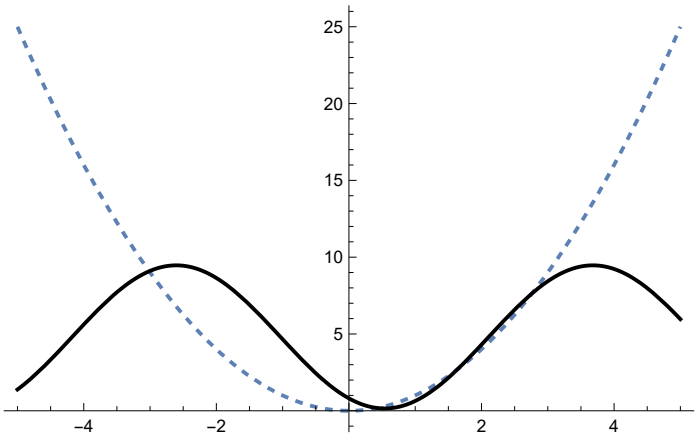
Out[•]=

$$\left\{1, \text{Cos}[x], -\frac{2}{\pi} + \text{Sin}[x]\right\}$$

Out[•]=

$$\frac{1}{3}\left(\pi^2 - 12\,\text{Cos}[x] + \frac{2\left(-12+\pi^2\right)\left(-2+\pi\,\text{Sin}[x]\right)}{-8+\pi^2}\right)$$

Out[•]=



Projekcija:

In[•]:=
```
ClearAll[a, φ, x, b]
(* bazo podprostora φ ortagonaliziramo glede na skalarni produkt
 a in poiščemo projekcijo Sin[x] na podprostor, nato jo narišemo *)
φ = {1, x, x^2};
a[f_, g_] := Integrate[f g, {x, 0, π}]
ortogonal[φ, a]
b = projUv[ortogonal[φ, a], a, Sin[x]]
Plot[{Sin[x], b}, {x, -5, 5}, PlotStyle → {Dashed, Black}]
```

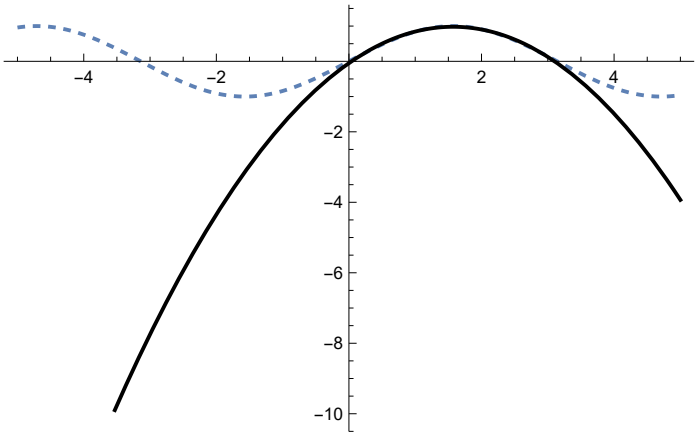
Out[•]=

$$\left\{1, -\frac{\pi}{2}+x, -\frac{\pi^2}{3}+x^2-\pi\left(-\frac{\pi}{2}+x\right)\right\}$$

Out[•]=

$$\frac{12\left(\pi^4+60\,\pi\,x-5\,\pi^3\,x-60\,x^2+5\,\pi^2\left(-2+x^2\right)\right)}{\pi^5}$$

Out[•]=



In[•]:=
```
ClearAll[a, φ, x, b]
(* bazo podprostora φ ortagonaliziramo glede na skalarni produkt
 a in poiščemo projekcijo Sin[x] na podprostor, nato jo narišemo *)
φ = {1, x, Sin[x]};
a[f_, g_] := (f /. x → 0) * (g /. x → 0) +
  (f /. x → 1) * (g /. x → 1) + (D[f, x] /. x → 0) * (D[g, x] /. x → 0)
ortogonal[φ, a]
b = projUv[ortogonal[φ, a], a, Cos[x]]
Plot[{Cos[x], b}, {x, -5, 5}, PlotStyle → {Dashed, Black}]
```

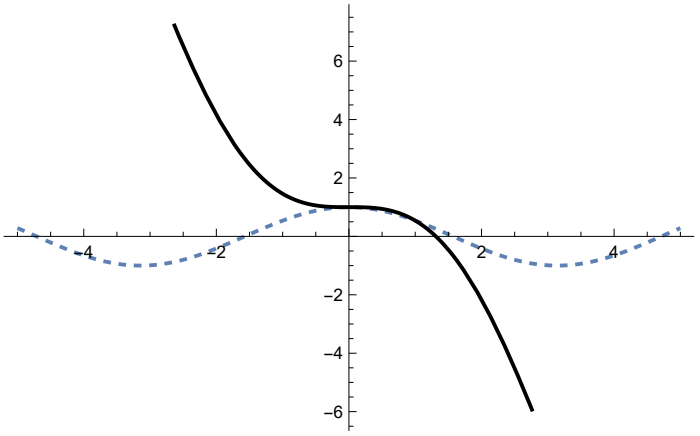
Out[•]=

$$\left\{1, -\frac{1}{2}+x, -\frac{2}{3}\left(-\frac{1}{2}+x\right)\left(1+\frac{\text{Sin}[1]}{2}\right)-\frac{\text{Sin}[1]}{2}+\text{Sin}[x]\right\}$$

Out[•]=

$$\frac{-1+x-x\,\text{Cos}[1]+\text{Sin}[1]+(-1+\text{Cos}[1])\,\text{Sin}[x]}{-1+\text{Sin}[1]}$$

Out[•]=



Aproksimacija:

Iskanje najboljše aproksimacije za f v podprostoru (zdi se kot, da je aproksimacija najboljša v sredini intervala na katerem integriramo)

```
ClearAll[a, φ, x, b]
φ = {1, x, x^2};
a[f_, g_] := Integrate[f g, {x, -π, π}]
ortogonal[φ, a]
b = projUv[ortogonal[φ, a], a, Sin[x]]
Plot[{Sin[x], b}, {x, -5, 5}, PlotStyle → {Dashed, Black}]
```

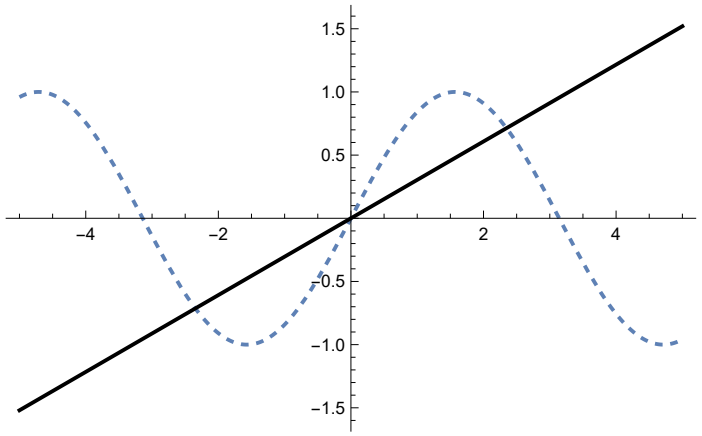
Out[•]=

$$\left\{1,\ x,\ -\frac{\pi^2}{3}+x^2\right\}$$

Out[•]=

$$\frac{3\,x}{\pi^2}$$

Out[•]=



```
ClearAll[a, φ, x, b]
φ = {1, Cos[x], Sin[x]};
a[f_, g_] := Integrate[f g, {x, -π, π}]
ortogonal[φ, a]
b = projUv[ortogonal[φ, a], a, x^2]
Plot[{x^2, b}, {x, -5, 5}, PlotStyle → {Dashed, Black}]
```

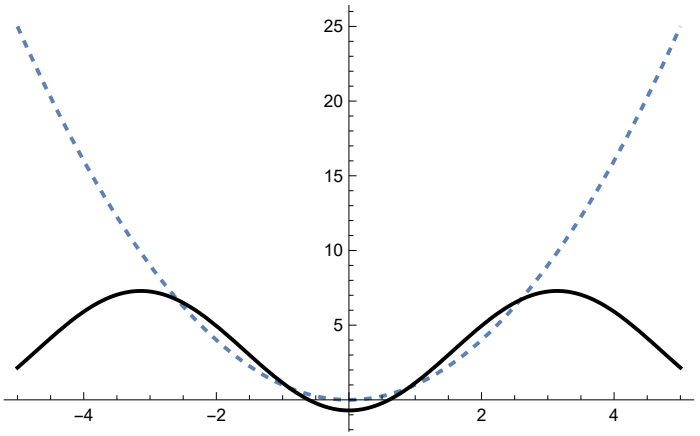
Out[•]=

```
{1, Cos[x], Sin[x]}
```

Out[•]=

$$\frac{\pi^2}{3} - 4\,\text{Cos}[x]$$

Out[•]=




```
ClearAll[a, φ, x, b]
φ = {1, Cos[x], Sin[x], x};
a[f_, g_] := Integrate[f g, {x, -π, π}]
ortogonal[φ, a]
b = projUv[ortogonal[φ, a], a, x^3]
Plot[{x^3, b}, {x, -5, 5}, PlotStyle → {Dashed, Black}]
```

```
ClearAll[a, φ, x, b]
φ = {1, Cos[x], Sin[x], x, x^2};
a[f_, g_] := Integrate[f g, {x, -π, π}]
ortogonal[φ, a]
b = projUv[ortogonal[φ, a], a, E^x]
Plot[{E^x, b}, {x, -5, 5}, PlotStyle → {Dashed, Black}]
```

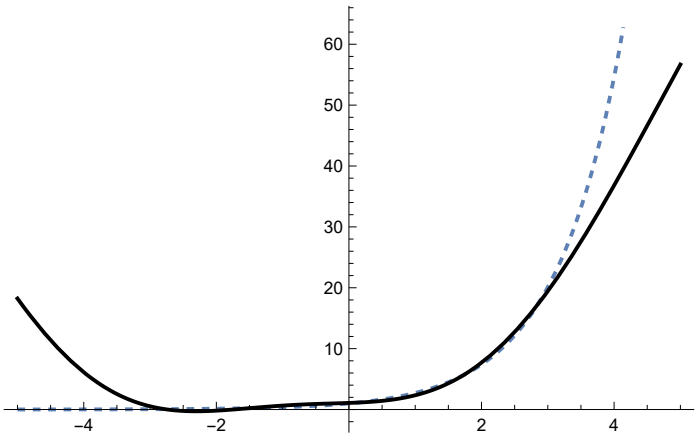
Out[•]=

$$\left\{1, \text{Cos}[x], \text{Sin}[x], x - 2\,\text{Sin}[x], -\frac{\pi^2}{3} + x^2 + 4\,\text{Cos}[x]\right\}$$

Out[•]=

$$\frac{1}{2}\left(\frac{-e^{-\pi} + e^{\pi}}{\pi} + \frac{6\,(x - 2\,\text{Sin}[x])\,(\pi\,\text{Cosh}[\pi] - 2\,\text{Sinh}[\pi])}{\pi\,\left(-6 + \pi^2\right)} - \frac{2\,\text{Cos}[x]\,\text{Sinh}[\pi]}{\pi} + \frac{2\,\text{Sin}[x]\,\text{Sinh}[\pi]}{\pi} - \frac{5\,\left(\pi^2 - 3\,x^2 - 12\,\text{Cos}[x]\right)\,(-3\,\text{Cosh}[\pi] + \pi\,\text{Sinh}[\pi])}{-90 + \pi^4}\right)$$

Out[•]=



```
ClearAll[a, φ, x, b]
φ = {1, Cos[x], Sin[x], x, x^2};
a[f_, g_] := Integrate[f g, {x, 0, 2 π}]
ortogonal[φ, a]
b = projUv[ortogonal[φ, a], a, E^x]
Plot[{E^x, b}, {x, -5, 5}, PlotStyle → {Dashed, Black}]
```

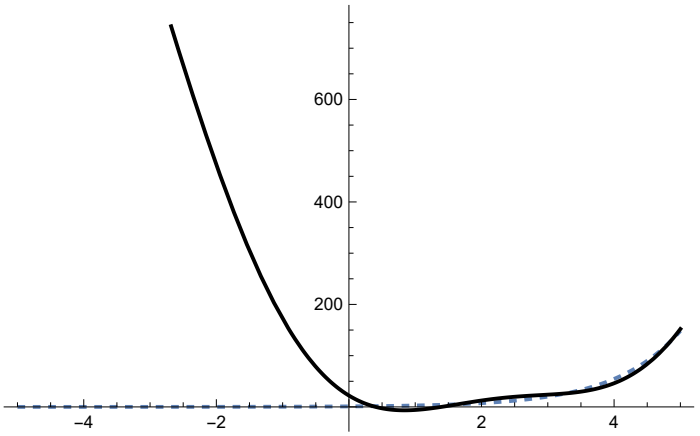
Out[•]=

$$\left\{1,\ \text{Cos}[x],\ \text{Sin}[x],\ -\pi + x + 2\,\text{Sin}[x],\ -\frac{4\pi^2}{3} + x^2 - 4\,\text{Cos}[x] + 4\pi\,\text{Sin}[x] - 2\pi\,(-\pi + x + 2\,\text{Sin}[x])\right\}$$

Out[•]=

$$\frac{1}{4}\left(\frac{2\left(-1 + e^{2\pi}\right)}{\pi} + \frac{5\left(-3 + e^{2\pi}\,(-3+\pi) - \pi\right)\left(2\pi^2 - 6\pi x + 3x^2 - 12\,\text{Cos}[x]\right)}{-90 + \pi^4} + \frac{6\left(2 + e^{2\pi}\,(-2+\pi) + \pi\right)(-\pi + x + 2\,\text{Sin}[x])}{\pi\left(-6 + \pi^2\right)} + \frac{4\,e^{\pi}\,\text{Cos}[x]\,\text{Sinh}[\pi]}{\pi} - \frac{4\,e^{\pi}\,\text{Sin}[x]\,\text{Sinh}[\pi]}{\pi}\right)$$

Out[•]=



```
ClearAll[a, φ, x, b]
φ = {1, Cos[x], Sin[x], x, x^2};
a[f_, g_] := Integrate[f g, {x, 2 π, 4 π}]
ortogonal[φ, a]
b = projUv[ortogonal[φ, a], a, E^x]
Plot[{E^x, b}, {x, 3, 15}, PlotStyle → {Dashed, Black}]
```

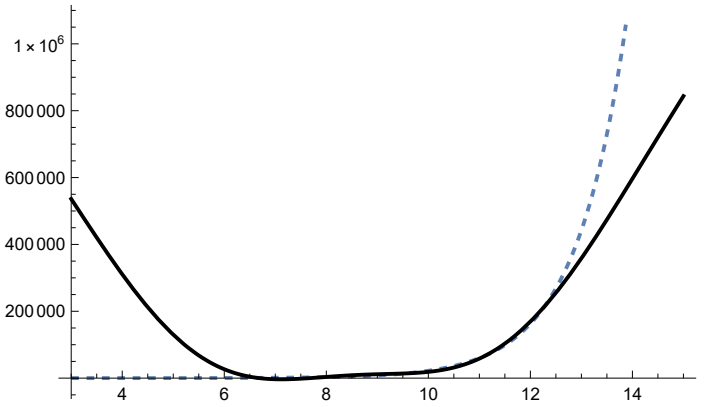
Out[•]=

$$\left\{1,\ \text{Cos}[x],\ \text{Sin}[x],\ -3\,\pi + x + 2\,\text{Sin}[x],\ -\frac{28\,\pi^2}{3} + x^2 - 4\,\text{Cos}[x] + 12\,\pi\,\text{Sin}[x] - 6\,\pi\,(-3\,\pi + x + 2\,\text{Sin}[x])\right\}$$

Out[•]=

$$\frac{1}{2}\,e^{2\pi}\left(\frac{3\left(2 + e^{2\pi}\,(-2+\pi) + \pi\right)\,(-3\,\pi + x + 2\,\text{Sin}[x])}{\pi\left(-6+\pi^2\right)} + \frac{2\,e^{\pi}\,\text{Sinh}[\pi]}{\pi} + \frac{2\,e^{\pi}\,\text{Cos}[x]\,\text{Sinh}[\pi]}{\pi} - \frac{2\,e^{\pi}\,\text{Sin}[x]\,\text{Sinh}[\pi]}{\pi} + \frac{5\,e^{\pi}\left(26\,\pi^2 - 18\,\pi\,x + 3\,x^2 - 12\,\text{Cos}[x]\right)\,(-3\,\text{Cosh}[\pi] + \pi\,\text{Sinh}[\pi])}{-90 + \pi^4}\right)$$

Out[•]=



```
ClearAll[a, φ, x, b]
φ = {1, Cos[x], x, x^2, Log[x]};
a[f_, g_] := Integrate[f g, {x, -π, π}]
ortogonal[φ, a]
b = projUv[ortogonal[φ, a], a, √(x^3)]
Plot[{√(x^3), Re[b]}, {x, -5, 5}, PlotStyle → {Dashed, Black}]
```

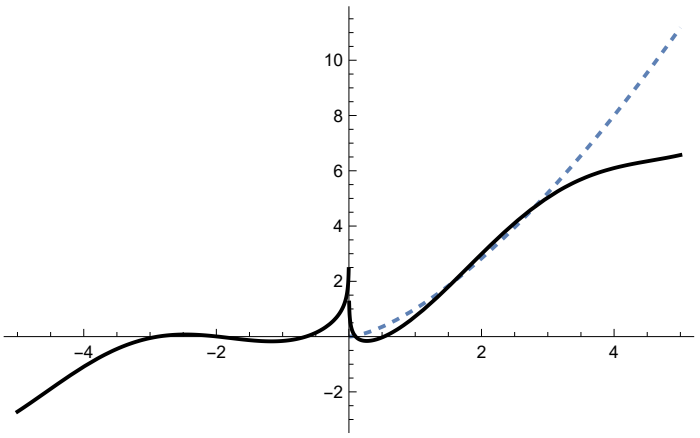
Out[•]=

$$\left\{1,\ \text{Cos}[x],\ x,\ -\frac{\pi^2}{3}+x^2+4\,\text{Cos}[x],\ \frac{3\,i\,x}{4}+\frac{1}{2}\,(2-i\,\pi-2\,\text{Log}[\pi])+\text{Log}[x]-\right.$$
$$\left.\frac{45\left(-\frac{\pi^2}{3}+x^2+4\,\text{Cos}[x]\right)\left(\frac{4\pi^3}{9}-8\,\text{SinIntegral}[\pi]\right)}{8\,\pi\left(-90+\pi^4\right)}+\frac{2\,\text{Cos}[x]\,\text{SinIntegral}[\pi]}{\pi}\right\}$$

Out[•]=

$$\frac{1}{\pi}\left(\frac{1}{2800}+\frac{i}{2800}\right)\left(560\,\pi^{5/2}-1200\,i\,\pi^{3/2}\,x-\right.$$
$$350\,\text{Cos}[x]\left(3\,\sqrt{2\,\pi}+4\,\pi^{5/2}\left(\text{ExpIntegralE}\left[-\frac{3}{2},-i\,\pi\right]+\text{ExpIntegralE}\left[-\frac{3}{2},i\,\pi\right]\right)\right)+$$
$$\frac{1}{-90+\pi^4}175\left(-\frac{\pi^2}{3}+x^2+4\,\text{Cos}[x]\right)$$
$$\left(8\,\pi^{9/2}-45\left(3\,\sqrt{2\,\pi}+4\,\pi^{5/2}\left(\text{ExpIntegralE}\left[-\frac{3}{2},-i\,\pi\right]+\text{ExpIntegralE}\left[-\frac{3}{2},i\,\pi\right]\right)\right)\right)+$$
$$\left(64\,\pi^{5/2}\left(1-\frac{i\,\pi}{2}+\frac{3\,i\,x}{4}-\text{Log}[\pi]+\text{Log}[x]+\right.\right.$$
$$\left.\frac{5\left(\pi^2-3\,x^2-12\,\text{Cos}[x]\right)\left(\pi^3-18\,\text{SinIntegral}[\pi]\right)}{6\,\pi\left(-90+\pi^4\right)}+\frac{2\,\text{Cos}[x]\,\text{SinIntegral}[\pi]}{\pi}\right)$$
$$\left(\pi^4\,(56+45\,\pi)+945\left(-22+25\,\sqrt{2}\,\text{FresnelC}\left[\sqrt{2}\right]\right)-\right.$$
$$\left.\left.225\,\pi\left(18+7\left(-2+3\,\sqrt{2}\,\text{FresnelC}\left[\sqrt{2}\right]\right)\text{SinIntegral}[\pi]\right)\right)\Big/$$
$$\left.\left(-12\,960+64\,\pi^4-9\,\pi^6+2880\,\pi\,\text{SinIntegral}[\pi]+\pi^2\left(810-288\,\text{SinIntegral}[\pi]^2\right)\right)\right)$$

Out[•]=



```
ClearAll[a, φ, x, b]
φ = {1, Cos[x], Sin[x]};
a[f_, g_] := (f /. x → 0) * (g /. x → 0) +
  (f /. x → 1) * (g /. x → 1) + (D[f, x] /. x → 0) * (D[g, x] /. x → 0)
ortogonal[φ, a]
t = Tanh[x]
b = projUv[ortogonal[φ, a], a, t]
Plot[{t, b}, {x, -5, 5}, PlotStyle → {Dashed, Black}]
```

Out[•]=

$$\left\{1, \frac{1}{2}\,(-1-\text{Cos}[1])+\text{Cos}[x],\right.$$
$$\left.-\frac{\text{Sin}[1]}{2}-\frac{\left(\frac{1}{2}\,(-1-\text{Cos}[1])+\text{Cos}[1]\right)\left(\frac{1}{2}\,(-1-\text{Cos}[1])+\text{Cos}[x]\right)\text{Sin}[1]}{\left(1+\frac{1}{2}\,(-1-\text{Cos}[1])\right)^2+\left(\frac{1}{2}\,(-1-\text{Cos}[1])+\text{Cos}[1]\right)^2}+\text{Sin}[x]\right\}$$

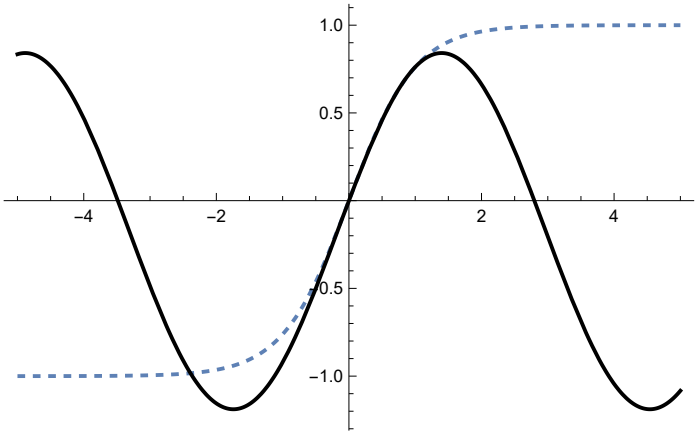
Out[•]=

$$\text{Tanh}[x]$$

Out[•]=

$$\frac{\text{Sin}[1]+(-1+\text{Cos}[1])\,\text{Sin}[x]-\text{Tanh}[1]+\text{Cos}[x]\,(-\text{Sin}[1]+\text{Tanh}[1])}{-1+\text{Cos}[1]}$$

Out[•]=



In[•]:=
```
ClearAll[a, φ, x, b]
φ = {1, Cos[x], Sin[x]};
a[f_, g_] := (f /. x → 0) * (g /. x → 0) +
  (f /. x → 1) * (g /. x → 1) + (D[f, x] /. x → 0) * (D[g, x] /. x → 0)
ortogonal[φ, a]
t = Cosh[x]
b = projUv[ortogonal[φ, a], a, t]
Plot[{t, b}, {x, -5, 5}, PlotStyle → {Dashed, Black}]
```

Out[•]=

$$\left\{1, \frac{1}{2}\left(-1-\text{Cos}[1]\right)+\text{Cos}[x],\right.$$
$$\left.-\frac{\text{Sin}[1]}{2}-\frac{\left(\frac{1}{2}\left(-1-\text{Cos}[1]\right)+\text{Cos}[1]\right)\left(\frac{1}{2}\left(-1-\text{Cos}[1]\right)+\text{Cos}[x]\right)\text{Sin}[1]}{\left(1+\frac{1}{2}\left(-1-\text{Cos}[1]\right)\right)^2+\left(\frac{1}{2}\left(-1-\text{Cos}[1]\right)+\text{Cos}[1]\right)^2}+\text{Sin}[x]\right\}$$

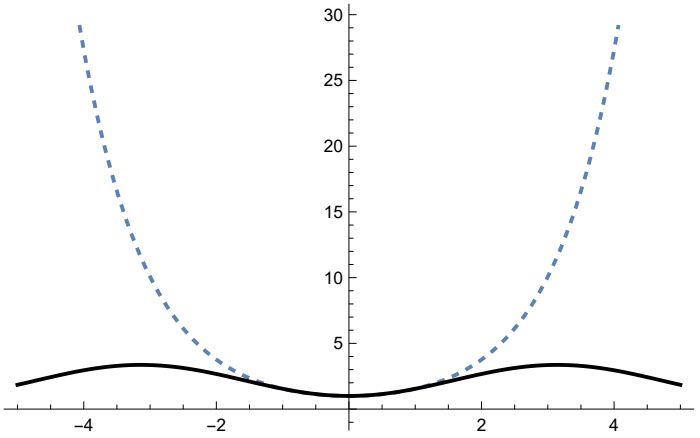
Out[•]=

$$\text{Cosh}[x]$$

Out[•]=

$$\frac{\text{Cos}[1]+\text{Cos}[x]\left(-1+\text{Cosh}[1]\right)-\text{Cosh}[1]}{-1+\text{Cos}[1]}$$

Out[•]=

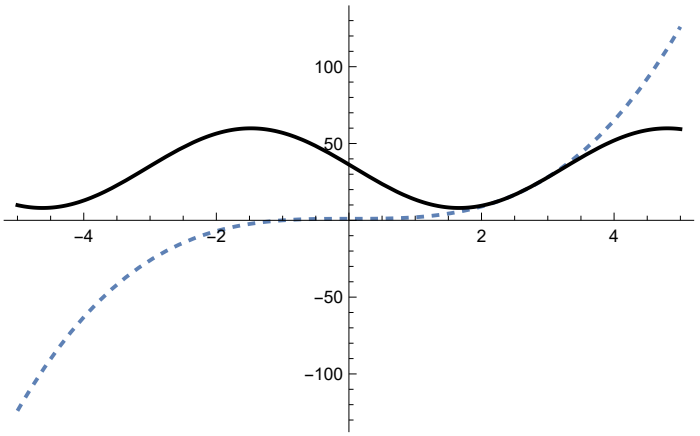


In[•]:=
```
ClearAll[a, φ, x, b]
φ = {1, Cos[x], Sin[x]};
tocke = {1/4, 1/3, 1/2, 1} + 2
a[f_, g_] := Sum[(f /. x → tocke[[i]]) * (g /. x → tocke[[i]]), {i, 1, Length[tocke]}]
a[x, Cos[x]]
ortogonal[φ, a];
t = x^3 + 1;
b = projUv[ortogonal[φ, a], a, t];
Plot[{t, b}, {x, -5, 5}, PlotStyle → {Dashed, Black}]
```

Out[•]=

$$\frac{9}{4}\cos\left[\frac{9}{4}\right] + \frac{7}{3}\cos\left[\frac{7}{3}\right] + \frac{5}{2}\cos\left[\frac{5}{2}\right] + 3\cos[3]$$

Out[•]=

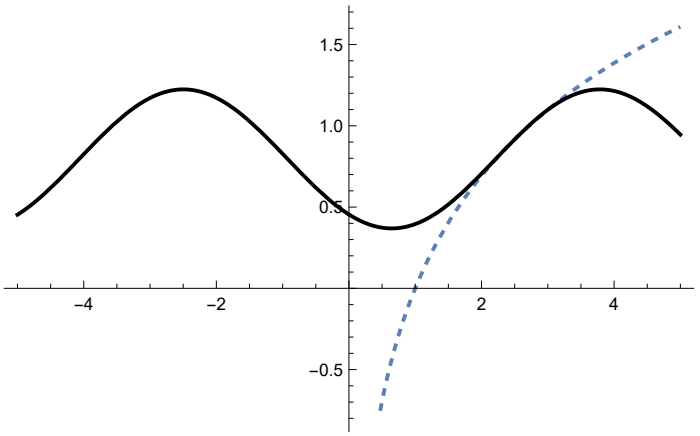


In[•]:=
```
ClearAll[a, φ, x, b]
φ = {1, Cos[x], Sin[x]};
tocke = {1/4, 1/3, 1/2, 1} + 2
a[f_, g_] := Sum[(f /. x → tocke[[i]]) * (g /. x → tocke[[i]]), {i, 1, Length[tocke]}]
a[x, Cos[x]]
ortogonal[φ, a];
t = Log[x];
b = projUv[ortogonal[φ, a], a, t];
Plot[{t, b}, {x, -5, 5}, PlotStyle → {Dashed, Black}]
```

Out[•]=

$$\frac{9}{4}\cos\left[\frac{9}{4}\right] + \frac{7}{3}\cos\left[\frac{7}{3}\right] + \frac{5}{2}\cos\left[\frac{5}{2}\right] + 3\cos[3]$$

Out[•]=

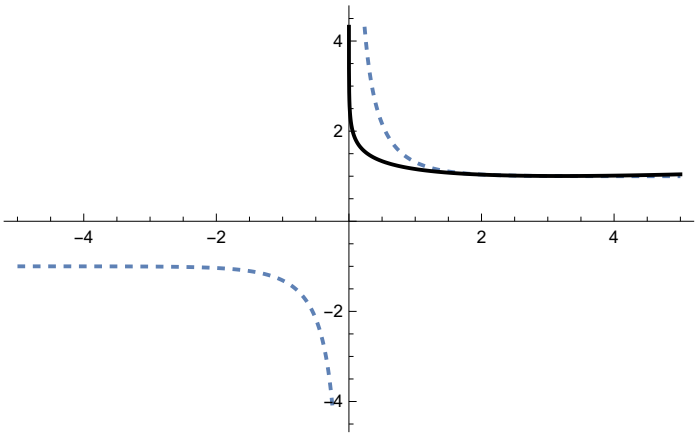


In[•]:=
```
ClearAll[a, φ, x, b]
φ = {1, x, Log[x]};
tocke = {1/4, 1/3, 1/2, 1} + 2
a[f_, g_] := Sum[(f /. x → tocke[[i]]) * (g /. x → tocke[[i]]), {i, 1, Length[tocke]}]
a[x, Cos[x]]
ortogonal[φ, a];
t = Coth[x];
b = projUv[ortogonal[φ, a], a, t];
Plot[{t, b}, {x, -5, 5}, PlotStyle → {Dashed, Black}]
```

Out[•]=

$$\frac{9}{4}\cos\left[\frac{9}{4}\right]+\frac{7}{3}\cos\left[\frac{7}{3}\right]+\frac{5}{2}\cos\left[\frac{5}{2}\right]+3\cos[3]$$

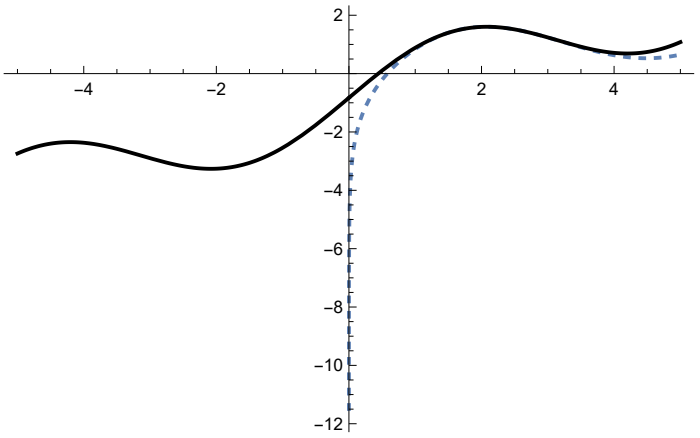
Out[•]=



In[•]:=
```
ClearAll[a, φ, x, b]
φ = {1, x, Sin[x]};
tocke = {1/4, 1/3, 1/2, 1} + 2
a[f_, g_] := Sum[(f /. x → tocke[[i]]) * (g /. x → tocke[[i]]), {i, 1, Length[tocke]}]
a[x, Cos[x]]
ortogonal[φ, a];
t = Log[x] + Sin[x];
b = projUv[ortogonal[φ, a], a, t];
Plot[{t, b}, {x, -5, 5}, PlotStyle → {Dashed, Black}]
```

Out[•]=
$$\frac{9}{4}\cos\left[\frac{9}{4}\right] + \frac{7}{3}\cos\left[\frac{7}{3}\right] + \frac{5}{2}\cos\left[\frac{5}{2}\right] + 3\cos[3]$$

Out[•]=

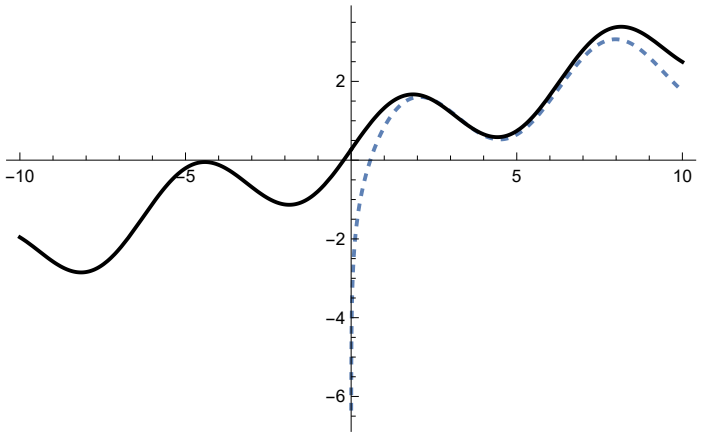


In[•]:=
```
ClearAll[a, φ, x, b]
φ = {1, x, Sin[x]};
tocke = {1/4, 1/3, 1/2, 1, 1 + 1/4, 1 + 1/3, 1 + 1/2, 1 + 1} + 2
a[f_, g_] := Sum[(f /. x → tocke[[i]]) * (g /. x → tocke[[i]]), {i, 1, Length[tocke]}]
a[x, Cos[x]]
ortogonal[φ, a];
t = Log[x] + Sin[x];
b = projUv[ortogonal[φ, a], a, t];
Plot[{t, b}, {x, -10, 10}, PlotStyle → {Dashed, Black}]
```

Out[•]=

$$\frac{9}{4}\cos\left[\frac{9}{4}\right]+\frac{7}{3}\cos\left[\frac{7}{3}\right]+\frac{5}{2}\cos\left[\frac{5}{2}\right]+3\cos[3]+\frac{13}{4}\cos\left[\frac{13}{4}\right]+\frac{10}{3}\cos\left[\frac{10}{3}\right]+\frac{7}{2}\cos\left[\frac{7}{2}\right]+4\cos[4]$$

Out[•]=

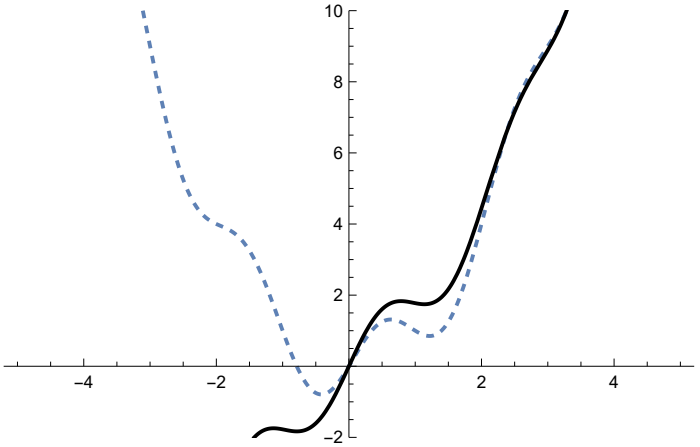


In[•]:=
```
ClearAll[a, φ, x, b]
φ = {x, Sin[π x], x^3};
tocke = {1/4, 1/3, 1/2, 1, 1 + 1/4, 1 + 1/3, 1 + 1/2, 1 + 1} + 2;
a[f_, g_] := Sum[(f /. x → tocke[[i]]) * (g /. x → tocke[[i]]), {i, 1, Length[tocke]}]
ortogonal[φ, a];
t = Sin[π x] + x^2;
b = projUv[ortogonal[φ, a], a, t]
Plot[{t, b}, {x, -5, 5}, PlotStyle → {Dashed, Black}, PlotRange → {-2, 10}]
```

Out[•]=

$$\left(x \left(-31\,500\,920\,230\,386 + 1\,560\,392\,751\,534\,\sqrt{2} + 1\,768\,885\,001\,680\,\sqrt{3} + 627\,071\,948\,627\,\sqrt{6} + 144 \left(-21\,802\,528\,956 + 1\,407\,660\,562\,\sqrt{2} + 1\,531\,964\,100\,\sqrt{3} + 603\,541\,245\,\sqrt{6}\right) x^2\right) + \left(-19\,337\,293\,477\,204 + 2\,045\,008\,509\,475\,\sqrt{2} + 2\,033\,893\,867\,858\,\sqrt{3} + 480\,027\,105\,744\,\sqrt{6}\right) \text{Sin}[\pi x]\right) \Big/ \left(24 \left(-841\,993\,994\,403 + 48\,010\,326\,572\,\sqrt{2} + 53\,203\,058\,258\,\sqrt{3} + 20\,001\,129\,406\,\sqrt{6}\right)\right)$$

Out[•]=

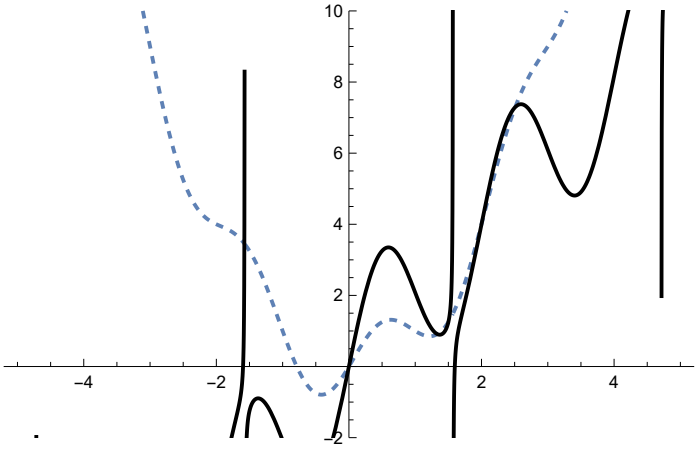


In[•]:=
```
ClearAll[a, φ, x, b]
φ = {x, Sin[π x], Tan[x]};
tocke = {1 + 1/3, 1 + 1/2, 1 + 1};
a[f_, g_] := Sum[(f /. x → tocke[[i]]) * (g /. x → tocke[[i]]), {i, 1, Length[tocke]}]
ortogonal[φ, a];
t = Sin[π x] + x^2;
b = projUv[ortogonal[φ, a], a, t]
Plot[{t, b}, {x, -5, 5}, PlotStyle → {Dashed, Black}, PlotRange → {-2, 10}]
```

Out[•]=

$$\left(2 \sin[\pi x] \left(126 \tan\left[\frac{4}{3}\right] - 4 \left(16 + 9\sqrt{3}\right) \tan\left[\frac{3}{2}\right] + 9 \left(-4 + 3\sqrt{3}\right) \tan[2]\right) + x \left(288 \tan\left[\frac{4}{3}\right] - 144\sqrt{3} \tan\left[\frac{3}{2}\right] + \left(-128 + 81\sqrt{3}\right) \tan[2]\right) + 2 \left(-64 + 27\sqrt{3}\right) \tan[x]\right) \Big/ \left(6 \left(24 \tan\left[\frac{4}{3}\right] - 12\sqrt{3} \tan\left[\frac{3}{2}\right] + \left(-16 + 9\sqrt{3}\right) \tan[2]\right)\right)$$

Out[•]=

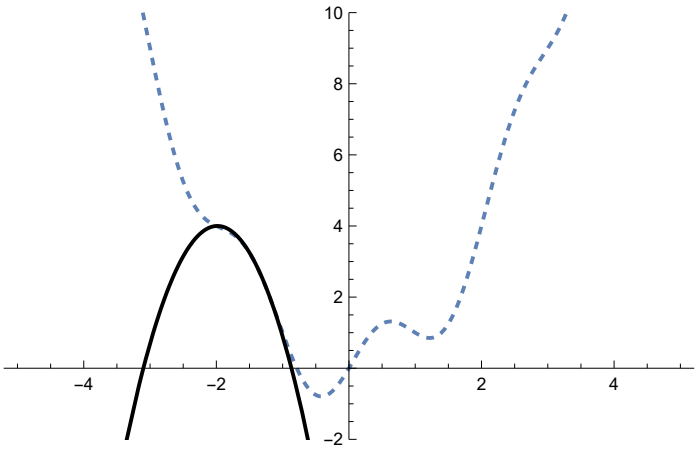


In[•]:=
```
ClearAll[a, φ, x, b]
φ = {1, x, x^2};
tocke = -{1 + 1/3, 1 + 1/2, 1 + 1};
a[f_, g_] := Sum[(f /. x → tocke[[i]]) * (g /. x → tocke[[i]]), {i, 1, Length[tocke]}]
ortogonal[φ, a];
t = Sin[π x] + x^2;
b = projUv[ortogonal[φ, a], a, t]
Plot[{t, b}, {x, -5, 5}, PlotStyle → {Dashed, Black}, PlotRange → {-2, 10}]
```

Out[•]=

$$-32+\frac{27\sqrt{3}}{2}+\left(-40+\frac{63\sqrt{3}}{4}\right)x+\left(-11+\frac{9\sqrt{3}}{2}\right)x^2$$

Out[•]=



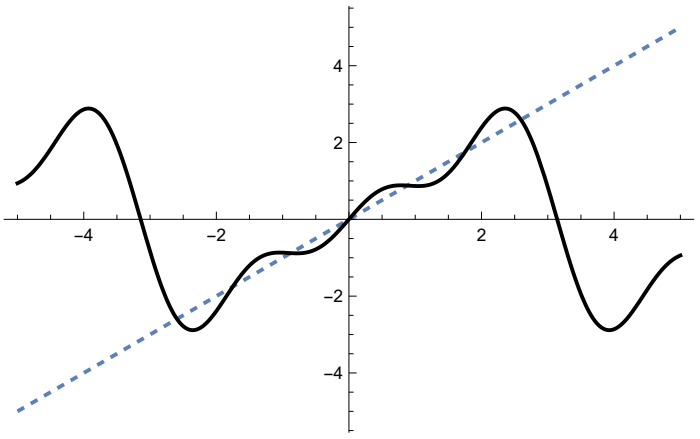
Fourierjev razvoj

In[•]:=
```
ClearAll[a, φ, x, b]
φ = {Sin[x], Cos[x], Sin[2 x], Cos[2 x], Sin[3 x], Cos[3 x]};
a[f_, g_] := Integrate[f g, {x, -π, π}]
ortogonal[φ, a]
b = projUv[ortogonal[φ, a], a, x]
Plot[{x, Re[b]}, {x, -5, 5}, PlotStyle → {Dashed, Black}]
```

Out[•]=

$$2 \sin[x] - \sin[2x] + \frac{2}{3} \sin[3x]$$

Out[•]=



In[•]:=
```
ClearAll[a, φ, x, b]
φ = {Sin[x], Cos[x], Sin[2 x], Cos[2 x], Sin[3 x], Cos[3 x]};
a[f_, g_] := Integrate[f g, {x, -π, π}]
ortogonal[φ, a]
b = projUv[ortogonal[φ, a], a, x^2]
Plot[{x^2, Re[b]}, {x, -5, 5}, PlotStyle → {Dashed, Black}]
```

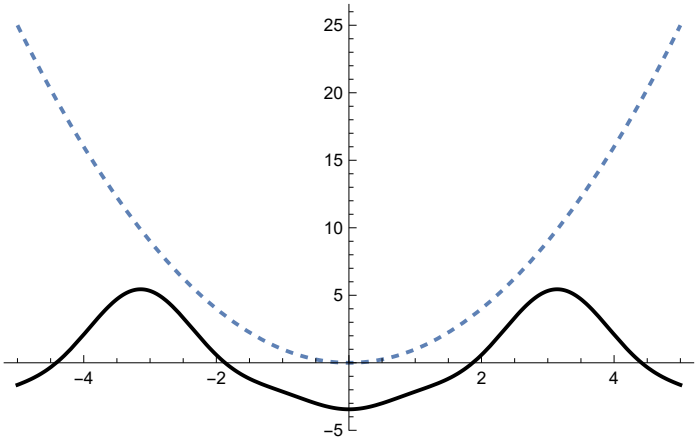
Out[•]=

$$\{\sin[x], \cos[x], \sin[2x], \cos[2x], \sin[3x], \cos[3x]\}$$

Out[•]=

$$-4 \cos[x] + \cos[2x] - \frac{4}{9} \cos[3x]$$

Out[•]=

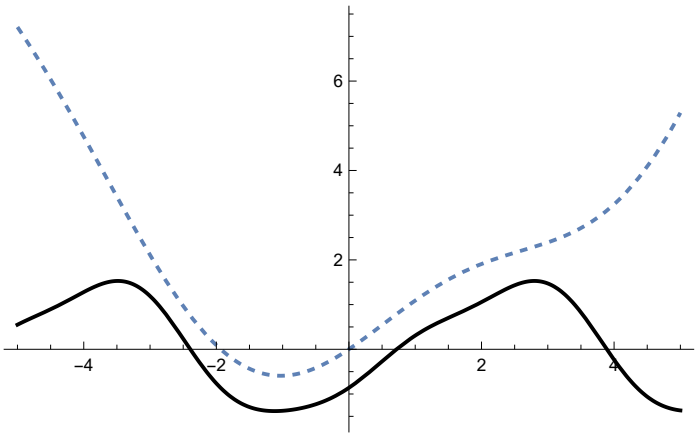


In[•]:=
```
ClearAll[a, φ, x, b]
φ = {Sin[x], Cos[x], Sin[2 x], Cos[2 x], Sin[3 x], Cos[3 x]};
a[f_, g_] := Integrate[f g, {x, -π, π}]
ortogonal[φ, a]
b = projUv[ortogonal[φ, a], a, 1/4 x^2 + Sin[x]]
Plot[{1/4 x^2 + Sin[x], Re[b]}, {x, -5, 5}, PlotStyle → {Dashed, Black}]
```

Out[•]=

$$-\text{Cos}[x] + \frac{1}{4}\text{Cos}[2x] - \frac{1}{9}\text{Cos}[3x] + \text{Sin}[x]$$

Out[•]=

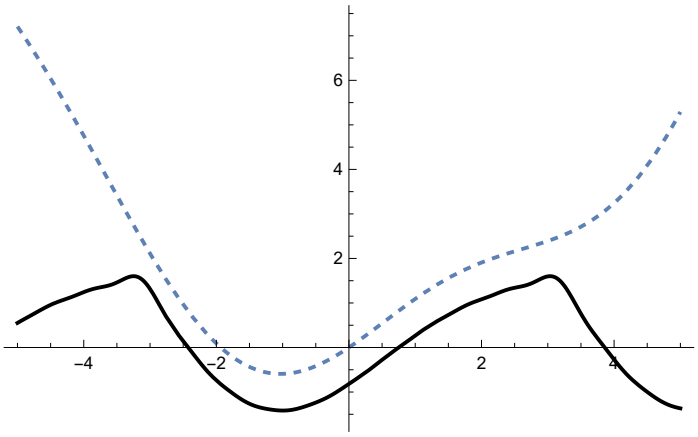


In[•]:=
```
ClearAll[a, φ, x, b]
φ = Join[Table[Sin[i x], {i, 1, 10}], Table[Cos[i x], {i, 1, 10}]];
a[f_, g_] := Integrate[f g, {x, -π, π}]
ortogonal[φ, a]
b = projUv[ortogonal[φ, a], a, 1/4 x^2 + Sin[x]]
Plot[{1/4 x^2 + Sin[x], Re[b]}, {x, -5, 5}, PlotStyle → {Dashed, Black}]
```

Out[•]=

$$-\text{Cos}[x]+\frac{1}{4}\text{Cos}[2x]-\frac{1}{9}\text{Cos}[3x]+\frac{1}{16}\text{Cos}[4x]-\frac{1}{25}\text{Cos}[5x]+\frac{1}{36}\text{Cos}[6x]-\frac{1}{49}\text{Cos}[7x]+\frac{1}{64}\text{Cos}[8x]-\frac{1}{81}\text{Cos}[9x]+\frac{1}{100}\text{Cos}[10x]+\text{Sin}[x]$$

Out[•]=

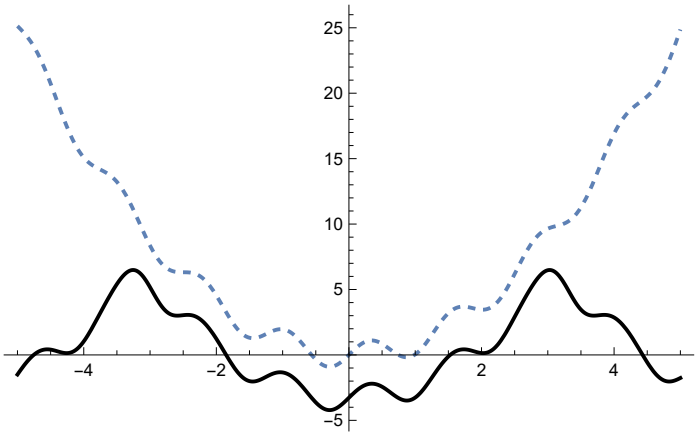


In[•]:=
```
ClearAll[a, φ, x, b]
φ = Join[Table[Sin[i x], {i, 1, 10}], Table[Cos[i x], {i, 1, 10}]];
a[f_, g_] := Integrate[f g, {x, -π, π}]
ortogonal[φ, a]
b = projUv[ortogonal[φ, a], a, x^2 + Sin[5 x]]
Plot[{x^2 + Sin[5 x], b}, {x, -5, 5}, PlotStyle → {Dashed, Black}]
```

Out[•]=

$$-4\,\text{Cos}[x] + \text{Cos}[2x] - \frac{4}{9}\,\text{Cos}[3x] + \frac{1}{4}\,\text{Cos}[4x] - \frac{4}{25}\,\text{Cos}[5x] + \frac{1}{9}\,\text{Cos}[6x] - \frac{4}{49}\,\text{Cos}[7x] + \frac{1}{16}\,\text{Cos}[8x] - \frac{4}{81}\,\text{Cos}[9x] + \frac{1}{25}\,\text{Cos}[10x] + \text{Sin}[5x]$$

Out[•]=



```
ClearAll[a, φ, x, b]
φ = Join[Table[Sin[i x], {i, 1, 10}], Table[Cos[i x], {i, 1, 10}]];
a[f_, g_] := Integrate[f g, {x, -π, π}]
ortogonal[φ, a];
b = projUv[ortogonal[φ, a], a, x^5]
Plot[{x^5, b}, {x, -5, 5}, PlotStyle → {Dashed, Black}]
```

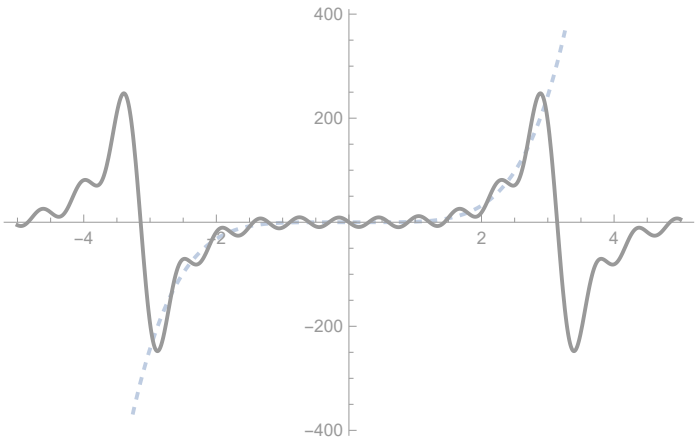
Out[•]=

```
{Sin[x], Sin[2 x], Sin[3 x], Sin[4 x], Sin[5 x], Sin[6 x],
 Sin[7 x], Sin[8 x], Sin[9 x], Sin[10 x], Cos[x], Cos[2 x], Cos[3 x],
 Cos[4 x], Cos[5 x], Cos[6 x], Cos[7 x], Cos[8 x], Cos[9 x], Cos[10 x]}
```

Out[•]=

$$2\left(120-20\pi^2+\pi^4\right)\text{Sin}[x]-\left(15-10\pi^2+2\pi^4\right)\text{Cos}[x]\,\text{Sin}[x]+\frac{2}{81}\left(40-60\pi^2+27\pi^4\right)\text{Sin}[3x]-\frac{1}{64}\left(15-40\pi^2+32\pi^4\right)\text{Sin}[4x]+\frac{2}{625}\left(24+25\pi^2\left(-4+5\pi^2\right)\right)\text{Sin}[5x]-\frac{1}{162}\left(5-30\pi^2+54\pi^4\right)\text{Sin}[6x]+\frac{2\left(120-980\pi^2+2401\pi^4\right)\text{Sin}[7x]}{16\,807}-\frac{\left(15-160\pi^2+512\pi^4\right)\text{Sin}[8x]}{2048}+\frac{2\left(40-540\pi^2+2187\pi^4\right)\text{Sin}[9x]}{19\,683}-\frac{\left(3-50\pi^2+250\pi^4\right)\text{Sin}[10x]}{1250}$$

Out[•]=

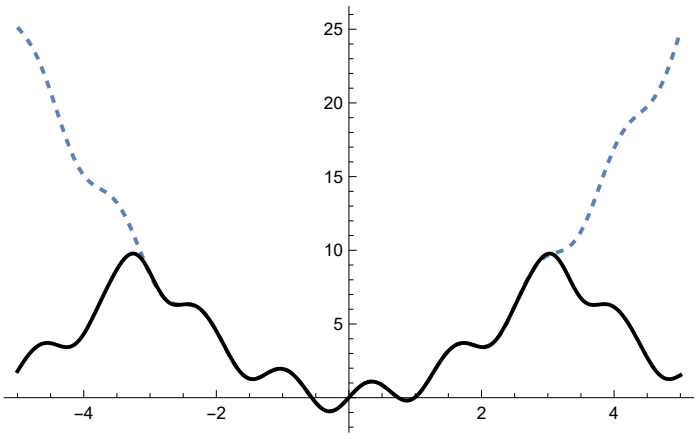


In[•]:=
```
ClearAll[a, φ, x, b]
φ = Join[Table[Sin[i x], {i, 1, 10}], Table[Cos[i x], {i, 1, 10}], {1}];
a[f_, g_] := Integrate[f g, {x, -π, π}]
ortogonal[φ, a]
b = projUv[ortogonal[φ, a], a, x^2 + Sin[5 x]]
Plot[{x^2 + Sin[5 x], b}, {x, -5, 5}, PlotStyle → {Dashed, Black}]
```

Out[•]=
$$\{\text{Sin}[x], \text{Sin}[2x], \text{Sin}[3x], \text{Sin}[4x], \text{Sin}[5x], \text{Sin}[6x], \text{Sin}[7x], \text{Sin}[8x], \text{Sin}[9x], \text{Sin}[10x], \text{Cos}[x], \text{Cos}[2x], \text{Cos}[3x], \text{Cos}[4x], \text{Cos}[5x], \text{Cos}[6x], \text{Cos}[7x], \text{Cos}[8x], \text{Cos}[9x], \text{Cos}[10x], 1\}$$

Out[•]=
$$\frac{\pi^2}{3} - 4\,\text{Cos}[x] + \text{Cos}[2x] - \frac{4}{9}\,\text{Cos}[3x] + \frac{1}{4}\,\text{Cos}[4x] - \frac{4}{25}\,\text{Cos}[5x] + \frac{1}{9}\,\text{Cos}[6x] - \frac{4}{49}\,\text{Cos}[7x] + \frac{1}{16}\,\text{Cos}[8x] - \frac{4}{81}\,\text{Cos}[9x] + \frac{1}{25}\,\text{Cos}[10x] + \text{Sin}[5x]$$

Out[•]=



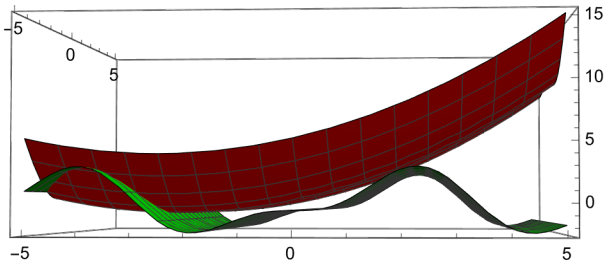
In[207]:=

```
ClearAll[a, φ, x, b, t]
φ = {Cos[x] Cos[y], Sin[x] Sin[y], Cos[2 x] Cos[2 y], Sin[2 x] Sin[2 y]};
a[f_, g_] := Integrate[f g, {x, -π, π}]
t = 1/5 (x^2 + y^2) + Cos[y] + x;
b = projekcija = projUv[φ, a, t]
Plot3D[{t, b}, {x, -5, 5}, {y, -5, 5}, PlotStyle → {Red, Green}, PlotPoints → 15]
```

Out[211]=

$$\frac{1}{5}\left(-4\cos[x]+\cos[2x]+10\sin[x]-5\sin[2x]\right)$$

Out[212]=



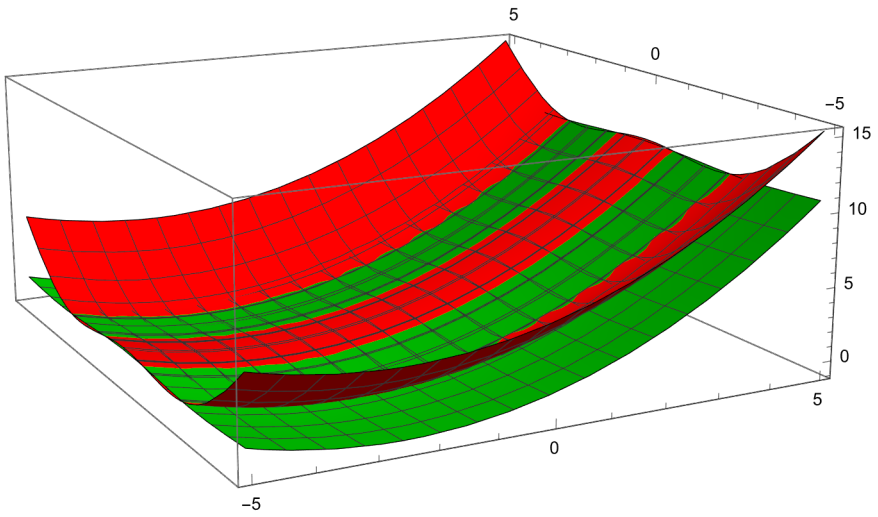
In[189]:=

```
ClearAll[a, φ, x, b, t]
φ = {Cos[x] Cos[y], Sin[x] Sin[y], x, y};
a[f_, g_] := Integrate[f g, {y, -π, π}]
t = 1/5 (x^2 + y^2) + Cos[y] + x;
b = projekcija = projUv[φ, a, t]
Plot3D[{t, b}, {x, -5, 5}, {y, -5, 5}, PlotStyle → {Red, Green}, PlotPoints → 15]
```

Out[193]=

$$\frac{1}{15}\left(\pi^2+3x\,(5+x)+3\cos[y]\right)$$

Out[194]=



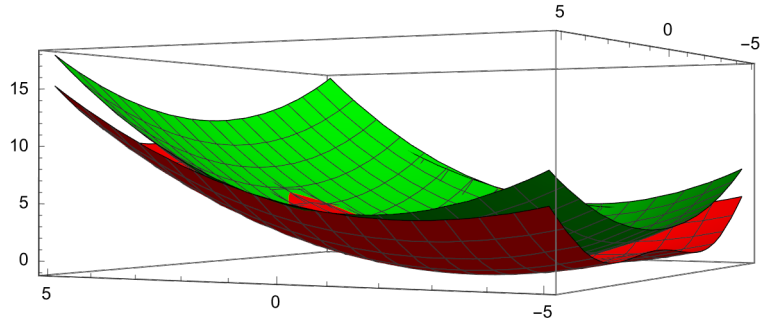
In[219]:=

```
ClearAll[a, φ, x, b, t]
φ = {x, x^2, y, y^2};
a[f_, g_] := Integrate[f g, {x, -π, π}, {y, -π, π}]
t = 1/5 (x^2 + y^2) + Cos[y] + x;
b = projekcija = projUv[φ, a, t]
Plot3D[{t, b}, {x, -5, 5}, {y, -5, 5}, PlotStyle → {Red, Green}, PlotPoints → 15]
```

Out[223]=

$$x + \frac{14\,x^2}{45} + \frac{2}{45}\left(7 - \frac{225}{\pi^4}\right) y^2$$

Out[224]=



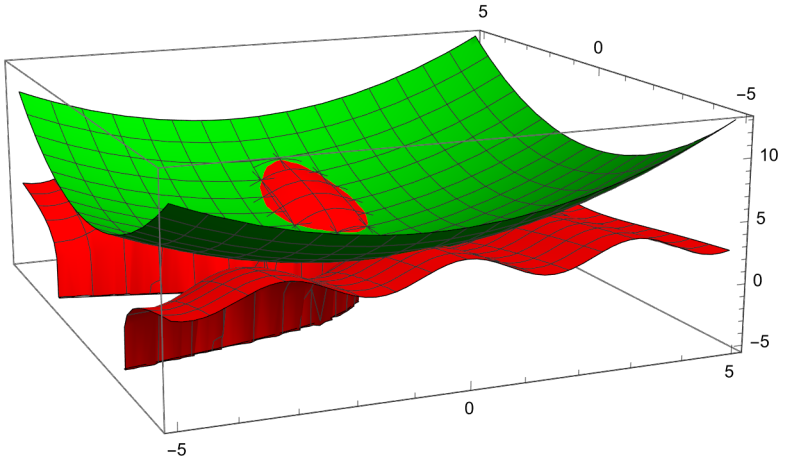
In[242]:=

```
ClearAll[a, φ, x, b, t]
φ = {x, x^2, y, y^2};
a[f_, g_] := Integrate[f g, {x, -π, π}, {y, -π, π}]
t = Log[x + y^2] + Cos[2 x];
b = projekcija = projUv[φ, a, t]
Plot3D[{t, Re[b]}, {x, -5, 5}, {y, -5, 5}, PlotStyle → {Red, Green}, PlotPoints → 15]
```

Out[246]=

$$\frac{1}{1260\,\pi^6}\Big(189\,\pi^2\,x\,\Big(2\,\pi^4+10\,\pi^3\,\text{ArcCoth}[\pi]-2\,\pi^5\,\text{ArcCoth}[\pi]+8\,\pi^{5/2}\,\Big(\text{ArcTan}\Big[\sqrt{\pi}\,\Big]-\text{ArcTanh}\Big[\sqrt{\pi}\,\Big]\Big)\Big)+70\,y^2\,\Big(18\,\pi^5\,\text{ArcCoth}[\pi]-12\,\pi^{5/2}\,\Big(\text{ArcTan}\Big[\sqrt{\pi}\,\Big]-\text{ArcTanh}\Big[\sqrt{\pi}\,\Big]\Big)+\pi^4\,\Big(-38+30\,\text{Log}[\pi]+15\,\text{Log}\Big[-1+\pi^2\Big]\Big)\Big)+50\,\pi^2\,x^2\,\Big(63-6\,\pi^4+6\,\pi^5\,\text{ArcCoth}[\pi]+36\,\pi^{3/2}\,\Big(\text{ArcTan}\Big[\sqrt{\pi}\,\Big]+\text{ArcTanh}\Big[\sqrt{\pi}\,\Big]\Big)+\pi^2\,\Big(-86+42\,\text{Log}[\pi]+21\,\text{Log}\Big[-1+\pi^2\Big]\Big)\Big)\Big)\Big)$$

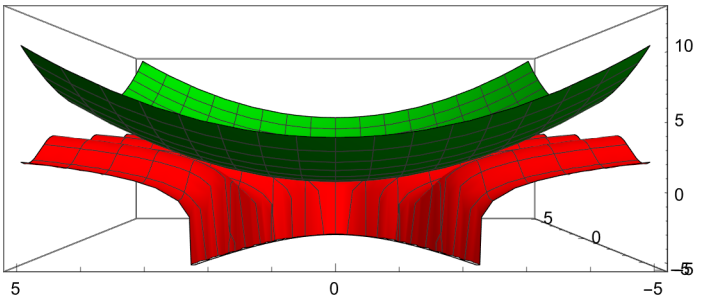
Out[247]=



In[249]:=

```
Show[%247, ViewPoint → Left]
```

Out[249]=



In[248]:=

```
Show[%247, ViewPoint → Front]
```

Out[248]=