1. Forward pass

Saturday, 2 March 2024 14:28

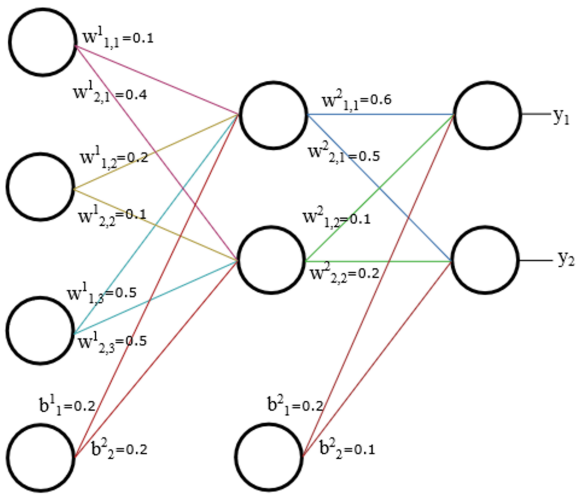

Figure 1: Architecture of a two-layer neural network.

$$x = [0.5, 0.5, 0.1]$$

$$\hat{y} = [0.9, 0.1]$$

$$\eta = 0.1$$

$$L = \frac{1}{2} \sum_{i=1}^{n_{out}} (y_i - \hat{y}_i)^2$$

$$\sigma(z) = \frac{1}{1+e^{-x}}$$

I. Forward pass

Layer 2:

$$z^1_1 = w^1_{11} \cdot x_1 + w^1_{12} \cdot x_1 + w^1_{13} \cdot x_1 + b_1 =$$

$$= 0,1 \cdot 0,5 + 0,2 \cdot 0,5 + 0,5 \cdot 0,5 + 0,2 = 0.6$$

$$a^1_1 = \sigma(z^1_1) = \sigma(0,6) = \frac{1}{1+e^{-0,6}} = 0,646$$

$$z^1_2 = w^1_{21} \cdot x_2 + w^1_{22} \cdot x_2 + w^1_{23} \cdot x_2 + b_2 =$$

$$= 0,4 \cdot 0,5 + 0,1 \cdot 0,5 + 0,5 \cdot 0,5 + 0,2 = 0,7$$

$$a^1_2 = \sigma(z^1_2) = \sigma(0,7) = \frac{1}{1+e^{-0,7}} = 0.668$$

Output layer:

$$z^2_1 = w^2_{11} \cdot a^1_1 + w^2_{12} \cdot a^1_2 + b^2_1 =$$

$$= 0,6 \cdot 0,646 + 0,1 \cdot 0,668 + 0,2 = 0,6544$$

$$a^2_1 = \frac{1}{1+e^{-0,6544}} = 0,658$$

$$a_1^2 = \frac{1}{1+e^{-0,16544}} = 0,658$$

$$z_2^2 = w_{21}^2 \cdot a_1^1 + w_{22}^2 \cdot a_2^1 + b_2^2 =$$

$$= 0,\overline{5} \cdot 0,1646 + 0,2 \cdot 0,1668 + 0,1 = 0,\overline{5566}$$

$$a_2^2 = \frac{1}{1+e^{-0,\overline{5566}}} = 0,1636$$

$$y = [0,658 \; ; \; 0,1636]$$

2. Backward pass

Saturday, 2 March 2024 15:32

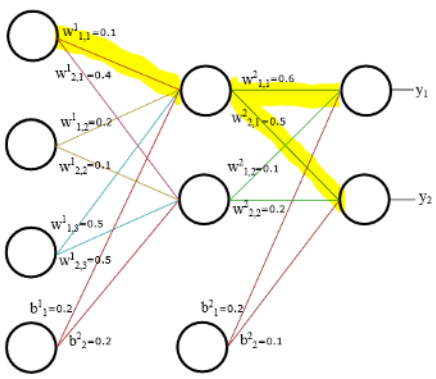

Figure 1: Architecture of a two-layer neural network.

$$x=[0.5,\ 0.5,\ 0.1]$$
$$\hat{y}=[0.9,\ 0.1]$$
$$L=\frac{1}{2}\sum_{i=1}^{n_{out}}(y_i-\hat{y}_i)^2$$
$$\sigma(x)=\frac{1}{1+e^{-x}}$$
$$\eta=0.1$$

$$w_{jk}^{l\,\prime}=w_{jk}^{l}-\eta\frac{\partial C}{\partial w_{jk}^{l}}$$

$$\frac{\partial C}{\partial w_{11}^2}=\frac{\partial C}{\partial a_1^2}\cdot\frac{\partial a_1^2}{\partial z_1^2}\cdot\frac{\partial z_1^2}{\partial w_{11}^2}$$

$$w_{11}^{2\,\prime}=w_{11}^2-\eta(a_1^2-\hat{y}_1)\sigma(z_1^2)(1-\sigma(z_1^2))\cdot a_1^1=$$
$$=0.6-0.1\,(0.658-0.9)\,0.658\,(1-0.658)\cdot 0.646=$$
$$=\underline{\underline{0.604}}$$

$$\frac{\partial C}{\partial a_1^2}=(a_1^2-\hat{y}_1)$$
$$\frac{\partial a_1^2}{\partial z_1^2}=\sigma(z_1^2)(1-\sigma(z_1^2))$$
$$\frac{\partial z_1^2}{\partial w_{11}^2}=a_1^1$$

$$\frac{\partial C}{\partial w_{21}^2}=\frac{\partial C}{\partial a_2^2}\cdot\frac{\partial a_2^2}{\partial z_2^2}\cdot\frac{\partial z_2^2}{\partial w_{21}^2}$$

$$w_{21}^{2\,\prime}=w_{21}^2-\eta(a_2^2-\hat{y}_2)\sigma(z_2^2)(1-\sigma(z_2^2))\cdot a_2^1=$$
$$=0.5-0.1\,(0.636-0.1)\,0.636\,(1-0.636))\cdot 0.646$$
$$=\underline{\underline{0.491}}$$

$$\frac{\partial C}{\partial a_2^2}=(a_2^2-\hat{y}_2)$$
$$\frac{\partial a_2^2}{\partial z_2^2}=\sigma(z_2^2)(1-\sigma(z_2^2))$$
$$\frac{\partial z_2^2}{\partial w_{21}^2}=a_2^1$$

$$\frac{\partial C}{\partial w_{11}^1}=\frac{\partial C}{\partial a_1^1}\cdot\frac{\partial a_1^1}{\partial z_1^1}\cdot\frac{\partial z_1^1}{\partial w_{11}^1}=\frac{\partial a_1^1}{\partial z_1^1}\cdot\frac{\partial z_1^1}{\partial w_{11}^1}\cdot\left(\frac{\partial z_1^2}{\partial a_1^1}\cdot\frac{\partial a_1^2}{\partial z_1^2}\cdot\frac{\partial C}{\partial a_1^2}+\frac{\partial z_2^2}{\partial a_1^1}\cdot\frac{\partial a_2^2}{\partial z_2^2}\cdot\frac{\partial C}{\partial a_2^2}\right)$$

$$\frac{\partial z_1^2}{\partial a_1^1}=\frac{\partial}{\partial a_1^1}\left(0.604\cdot a_1^1+0.1\cdot a_2^1+b_2^1\right)=\frac{\partial}{\partial a_1^1}(0.604\,a_1^1)+\frac{\partial}{\partial a_1^1}(0.1\cdot a_2^1)+\frac{\partial}{\partial a_1^1}(b_2^1)=$$

$$=0.604$$

$$\frac{\partial z_1^1}{\partial w_{11}^1}=a_1^0=x_1=0.5$$

$$\frac{\partial a_1^1}{\partial z_1^1}=\sigma(z_1^1)(1-\sigma(z_1^1))$$

$= 0.604$

$\frac{\partial z_2^2}{\partial a_1^1} = 0.491$ $\quad \frac{\partial z_1^1}{\partial w_{11}^1} = a_1^0 = x_1 = 0.5$ $\quad \frac{\partial a_1^1}{\partial z_1^1} = \sigma(z_1^1)(1-\sigma(z_1^1))$

$$\frac{\partial C}{\partial w_{11}^1} = 0.5 \cdot \sigma(z_1^1)(1-\sigma(z_1^1)) \cdot \left(0.604 \cdot \sigma(z_1^2)(1-\sigma(z_1^2)) \cdot (a_1^2 - \hat{y}_1) + 0.491 \cdot \sigma(z_2^2)(1-\sigma(z_2^2))(a_2^2 - \hat{y}_2)\right) =$$

$$= 0.5 \cdot 0.646(1-0.646)(0.604 \cdot 0.658 \cdot (1-0.658)(0.658-0.9) + 0.491 \cdot 0.636(1-0.636)(0.636-0.1)) =$$

$$= 0.003$$

$$w_{11}^{1)} = w_{11}^1 - \eta \cdot \frac{\partial C}{\partial w_{11}^1} = 0.1 - 0.1 \cdot 0.003 = \underline{\underline{0.0997}}$$