

1. We define C as:

$$C = A \cap B$$

We can map all the elements in A to those in $A \cap B$ as $A \cap B \subseteq A$.

Also we can map all elements in B to those in $A \cap B$ as $A \cap B \subseteq B$.

Hence we have that

$$A \leq_m C, \text{ and}$$

$$B \leq_m C$$

Also C is minimal as for any other set D such that $A \leq_m D$ & $B \leq_m D$, $C \subseteq D$.

$$\text{Hence } C \leq_m D$$

4. No. For example, the languages $A = \{0^n 1^n \mid n \geq 0\}$ and $B = \{1\}$,

both over the alphabet $\Sigma = \{0, 1\}$

$f(w) = 1$ if w belongs to A ;

0 if w does not belong to A :

Observe that A is a context-free language, so it is also Turing-decidable. Thus, f is a

computable function. Also, w belongs to A if and only if $f(w) = 1$, which is true if and only

if $f(w)$ belongs to B . Hence, $A \leq_m B$. Language A is non regular, but B is regular since it is finite.