2.9 A language can be split into two languages which are defined as follows,

$A_1 = a^ib^jc^k|i, j, k \geq 0, i = j$ and $A_2 = a^ib^jc^k|i, j, k \geq 0, j = k$

using the language A1 and A2 the users can construct a CFG for A_1 and A_2 the grammar for language A is the union of grammar of two languages which is defined as follows $S \to S_1|S_2$

in the language A_1 the values of i and j are equal so there must be equal number of a's and b's in the language A^1 CFG for language A^1 and A^2 are given below

$S^2 \to aS^2|F| \in$

$F \to bFc| \in$

since the generating string $w = abc$ using the language a, either s_1 or s_2 can be user. therefore the context free grammar for the language A is ambiguous.

2.10 As we know from previous example we have $A_1 and A_2$ $A_1 = a^ib^jc^k|i, j, k \geq 0, i = j$ and $A_2 = a^ib^jc^k|i, j, k \geq 0, j = k$

push down automation follows as

Read and push $a's$

read b' while popping $a's$

if $b's$ finish when stack is empty skip $c's$ on input and accept.

2.14 lets add a new start variable S_0 and rule $S_0 \to A$. so grammar is

$S_0 \to A$

A $S_0BAB|B|\epsilon$

B $\to 00|\epsilon$

now we remove rules that contain ϵ $S_0 \to A|\epsilon$

$A \to BAB|BA|AB|A|B|BB$

$B \to 00$

the rule $S_0 \to \epsilon$ is accepted since S_0 is the start variable and that is allowed in Chomsky normal form.

now remove the unit rules

$S_0 \to A|\epsilon$

$A \to BAB|BA|AB|00|BB$

$B \to 00$

$S_0A\epsilon$

$A \to BAB|BA|AB|00|BB$

$B \to 00$

$S_0 \to BAB|BA|AB|00|BB|\epsilon$

$a \to BAB|BA|AB|00|BB$

$B \to 00$

now we replace third placed terminals 0 by variable U with new.

$S_0 \to BAB|BA|AB|UU|BB|\epsilon$

$A \to BAB|BA|AB|UU|BB$

$B \to UU$

$U \to 0$

$A_1 \to AB$

This is the final CFG in Chomsky normal form equivalent to the given CFG.

2.26 Given that G is a CFG in Chomsky normal form. the length of the string w ∈ L(G) is n≥ 1 for the string w. it is required to show that exactly $2n - 1$ steps are required for the derivation of string w. it can be proved applying the induction method by on the string w of length n.

for $n = 1$ consider string " a " of length 1 in Chomsky normal form the valid derivation for this will be $s \to a$. the number of steps can be obtained as follows

the number of steps can be obtained as follows $2n - 1 = 2(1) - 1 = 2 - 1 = 1$

now $n = k + 1$ is in chomsky normal form.

since $n > 1$ consider a language in CNF where derivation starts with start symbol S.

$S \to BC$

$B \to *x$

$C \to *y$

using the inductive hypothesis, for the above language in CNF the length of any derivation of string w must be.

$1 + 2(|x| - 1) + (2|y| - 1) = 2|x| + 2|y| + 1 - 1 - 1 = 2(|x| + |y|) - 1$

here $n = |x| + |Y|$

since $B \to *x$ has a length of —x— and $C \to *y$ has a length of —y—.

hence it is proved that it requires 2n-1 steps required for the derivation of string $w \in L(G)$ in Chomsky normal form.