1. **A)** $[(p \vee (q \wedge r)]$

$Columns$	1	2	3	4	5
$Rows$	p	q	r	$q \wedge r$	$[(p \vee (q \wedge r)]$
1	0	0	0	0	0
2	0	0	1	0	0
3	0	1	0	0	0
4	0	1	1	0	1
5	1	0	0	0	1
6	1	0	1	1	1
7	1	1	0	0	1
8	1	1	1	1	1

B) $[(p \vee q) \wedge (p \vee r)]$

$Columns$	1	2	3	4	5	6
$Rows$	p	q	r	$(p \vee q)$	$(p \vee r)$	$[(p \vee q) \wedge (p \vee r)]$
1	0	0	0	0	0	0
2	0	0	1	0	1	0
3	0	1	0	1	0	0
4	0	1	1	1	1	1
5	1	0	0	1	1	1
6	1	0	1	1	1	1
7	1	1	0	1	1	1
8	1	1	1	1	1	1

As we see in out of following tables result in 5Th raw and 6Th raw are same therefore they are equal

2. **we should consider a graph which has at least one degree without any loops or cycles in. if we think maximum number of degrees that a node can have equals $n-1$. because, if $n =$ number of nodes, it can connect to all other nodes but not itself. below i will demonstrate my example with a table**

$Number of Nodes$	$Degree$
1	4
2	3
3	2
4	1
5	0 or 5

As you can see in the table $1st$ node has 4 degree. This is because it can connect to all other nodes but not on itself. Then on the $2nd$ node it can't have 5 degrees with the same reason as $1st$ node. it also can have 4 nodes however, that will prove given task that two nodes must repeat number of degrees. so, since we still have nodes to discuss $2nd$ node will have 3 degrees. Then $3rd$ node can't have $5, 4, or$ 3. 5 because of mentioned above, 4 and 3 because it is already used, therefore $3rd$ node will have 2 degrees. Then $4th$ node can't

have $5, 4, 3, or2$ 5 because of above mentioned reason. and 432 because it is already used. Therefore it only can have 1 degree. As we continue we left $5th$ node. as we can see there is only three number to choose from. First is 5 but, we can's use it. second number is 0 however, we cant use 0 as well because it means there are no degrees therefore it will not be part of graph. and third choice is to use other number of degrees which are $4, 3, 2, or1$ this means that 2 or mode nodes will must have same number of degrees

3. **A) the sum of the first n natural numbers ($S_n = 1 + 2 + 3... + n$) is given by $S_n = \frac{1}{2}n(n+1)$ Now lets calculate for 1 and 2 so we make sure that base case and formula is good.**

 when $n = 1$ then $\frac{1}{2}1(1+1)$

 when $n = 2$ then $1 + 2 = \frac{1}{2}2(2+1)$

 Now we can have formula for all n

$$\sum_{i=1}^{n} \frac{1}{2} 2(2+1).$$

 Then by rule of induction we have to add $n + 1$ which gives us $S_{n+1} = \frac{(n+1)(n+2)}{2}$

 above equality proves that given equality for the sum of n natural $S_n = \frac{1}{2}n(n+1)$ is also true for $(n + 1)$. therefore give equality is correct.

 B)

 Sum of the cube of first n natural numbers is given by $C_n = \frac{1}{4}(n^4 + 2n^3 + n^2) = \frac{1}{4}n^2(n+1)^2$

 Now lets calculate for $n = 1$

 $1^3 = \frac{1}{4}1^2(1+1)^2$

 base case is true. Now we can write formula which will be true for n

$$\sum_{i=1}^{k=n} i^3 = 1^3 + 2^3 + ... + n^3 = \frac{1}{4}n^2(n+1)^2.$$

 so for $(n + 1)$ it would be $C_n = 1^3 + 2^3 + ... + n^3 + (n+1)^3 = \frac{(n+1)^2(n+2)^2}{4}$

 which means $C_{n+1} = \frac{(n+1)^2(n+2)^2}{4}$

 So above equality proves that give equality for the sum of the cube of natural n numbers is true for $n + 1$ therefore equality is correct

 From above explanation we can see that $C_n = \frac{1}{4}(n^4 + 2n^3 + n^2) = \frac{1}{4}n^2(n+1)^2 = [\frac{n(n+1)}{2}]^2 = (S_n)^2$

 $C_n = (S_n)^2$

 it is concluded that sum of the cube of the first n natural number is equal to sum of the square of n natural numbers.

4. **A) Lets say N= set of natural numbers. R(x,y)= relation such that for any (x,y) x¡=y**

   Reflexive: x=x. so R(x,x) holds true. therefore above relation R(y.x) does not holds true

   Symmetric:For any two natural numbers x and y such that R(x,y) does not hold true. for example lets say $(2, 3)$ belongs to R. so $2 \leq 3$ holds true. however $(3, 2)$ is not true because $2 \leq 3$ is false

   Transitive: for (x,y) belongs to R and (y,z) belongs to R then (x,z) also belongs R. in this case $(2, 3)$ belongs to r and $(3, 4)$ also belongs are. since $2 \leq 3$ and $3 \leq 4$ then $(2, 4)$ also belongs R. also $2 \leq 4$ so relation is transitive

   Missing property is SYMMETRIC

   B)

   Lets say N = the set of natural numbers that are larger than 2) R(x,y)= a relation such that there is a prime number which divides both x and y.

   Reflexive: For natural number that belongs N and R(x,x) holds true. For example, suppose 4 belongs to N, then $(4, 4)$ holds true as 4 is divisible by 2. Therefore, the relation is reflexive.

   Symmetric: For two natural number (x,y) that belongs N so R(x,y) holds true, R(y,x) also holds true. For example, suppose $(2, 4)$ belongs to N, then $(2, 4)$ also belongs R as 2 and 4 are divisible by 2. Moreover, $(4, 2)$ also belongs to R as 4 and 2 are divisible by 2. Therefore, the relation is symmetric.

   Transitive: For (x, y) that belongs to R and (y, z) also belongs to R, (x, z) may not belongs to R. For instance, suppose $(3, 6)$ belongs to R as 3 and 6 are divisible by 3 and $(6, 8)$ belongs to R as 6 and 8 are divisible by 2, then $(3, 8)$ does not belongs to R as there is no prime number which divides both 3 and 8. Therefore, the relation is not transitive.

   Missing property is TRANSITIVE