

2.3 A) the variable of G are R,S,T and X. variable are the non terminal symbols that appear in the rules of the grammar.

B) the terminals of g are a,b. terminals are the terminal symbols that appear in the rules of the grammar.

c) the start variable G is R.  $R \rightarrow XRX$ —S. start variable is a variable usually occurs on the left hand side of the topmost rule.

D) the three strings not in  $\epsilon(G)$  are aba, b and  $\epsilon$ . since these strings cannot be derived from the given grammar G.

E) the string cannot be derived using G.

2.4 D) The context free grammar that generates the language  $\{w \mid \text{the length of the } w \text{ is odd and its middle symbol is a } 0\}$  is given

$$S \rightarrow 0|0S0|0S1|1S0|1S1$$

E) CFG that generates the language  $\{w \mid w = W^R \text{ that is } w \text{ is a palindrome}\}$  is

$$S \rightarrow 0|1|0S0|1S1|\epsilon$$

2.16 to show that CFG is closed under union operation considers two starts  $S_1$  and  $S_2$  for the two different languages  $L_1$  and  $L_2$

grammar for union operation is given

$$S \rightarrow S_1|S_2$$

if booth the language belongs to the CFG then union of both the language should belong to CFG. so if user generates  $S_1$  and  $S_2$  string or both then in that case union of the language is generated. so this implies that CFG is closed under union.

to show that CFg is closed under star operation. we should consider one start variable  $S_1$  for language  $L_1$

grammar of union is shown

$S \rightarrow S_1S|\epsilon$  if the language belongs to the CFG then star of the language should belong to CFG. so shown below if user generates zero or many strings which is definition of the star.

2.26 we can prove is by applying the induction method on the string w of length n.

for  $n=1$  : consider a string a of length 1 in Chomsky normal form. so the valid derivation for this will be  $S \rightarrow a$ ,

the number of step can be obtained as follows:

$$2n - 1 = 2(1) - 1$$

$$= 2 - 1$$

$$= 1$$

for  $n=k$

$$2n - 1 = 2(k) - 1$$

$$= 2k - 1$$

assuming a string of length at most k  $\leq 1$  terminal symbols and it has a string of length.

$n = k + 1$  in Chomsky normal form. since  $n \geq 1$  consider a language as follows in cnf where derivation starts with start symbol s.

$$S \longrightarrow BC$$

$$B \longrightarrow *X$$

$$C \longrightarrow *Y$$

since  $B \longrightarrow *X$  has length of  $x$  and  $C \longrightarrow *Y$  had a length of  $y$ . so it proved that it requires  $2n-1$  step to derivation the string  $w \in L(G)$  in Chomsky normal form.