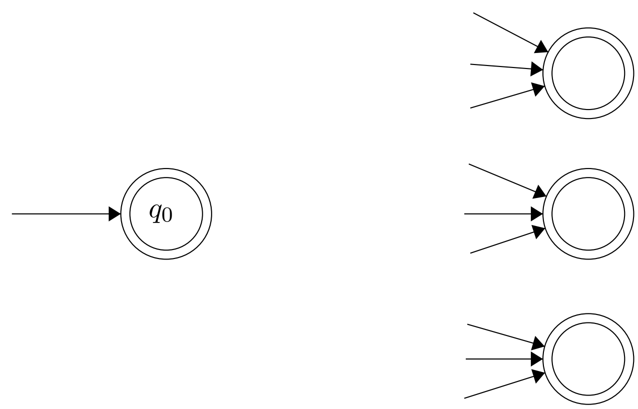
1. First let define M $(Q, \sum, \delta, q_0, F)$ is a DFA of A. Lets build NFA M' for A^R with the following steps:

   Reverse all arrows

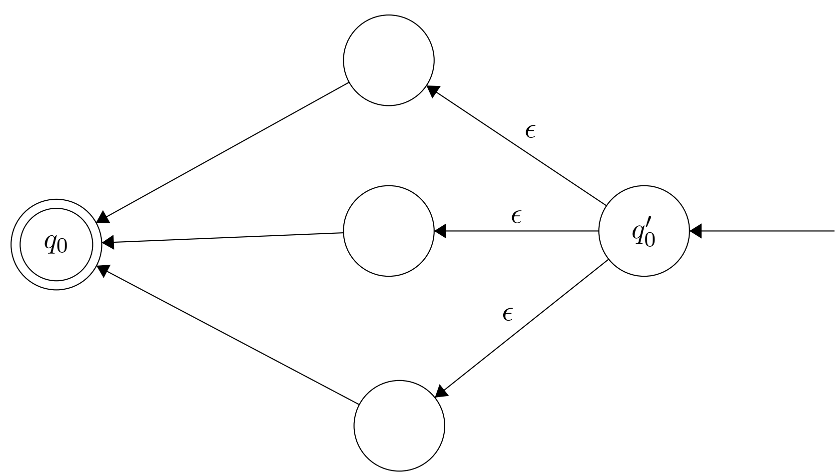
   Convert start state for M as the only accept state for M'

   Add new start state for M' and for q_0'

   Graph M



   Graph M'



   here $q_0' = q_{accept}'$. for any w$\epsilon \sum *$. there is a path following w from the start state to an accept state in M if there is a path following w$^R from$q$_0'$ to q_{accept}' in M'

2. **Consider that B_n is a^k — k is a multiple of n in order to prove that give expression is regular, the value of n is chosen as greater than or equal to 1**

   lets say that $k = ni$, where i is positive integer. lets say that when $i = 1$ and $n = 1$

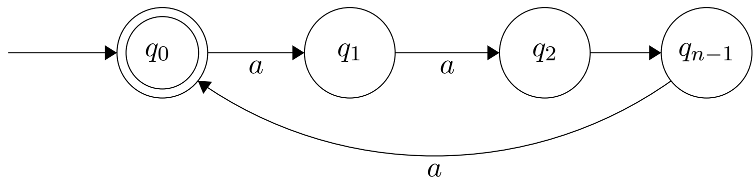
   $B_1 = a^k$ $B_2 = a^k$ $B_3 = a^k$

   $B_1 = a^{ni}$ $B_2 = a^{n1}$ $B_3 = a^{n1}$

   $B_1 = a^{1*1}$ $B_2 = a^{2*1}$ $B_3 = a^{3*1}$

   $B_1 = a$ $B_2 = aa$ $B_2 = aaa$

   now lets build finite automation for the expression



Union of B_1 and B_2 results in the third string and it is also a regular expression similarly, if we apply any property of close then the result is regular expression. Hence we proved that the above expression is a regular expression

3.A Let say that N= $(Q, \sum, \delta, q_0, F)$ be an NFA with k states that recognizes some language A and lets suppose A is non empty.

Then there must be an accept state that cab be reached from the start state q_o

Then let w be the string that can be accepted by N when traveling along the shortest path q_0 to q

Then let n be the length of 2

the sequence of state in the shortest path from q_0 to q has length $n+1$

since there are only J states in N that are n distinct states in the shortest path from q_0 to q we have $n < k$

so W is accepted by N because q is an accept state

so a contains a string of length at most K