

2. let us assume $A = \{a^{2^n}b^n | n \geq 0\}$ is regular.

let pumping length = p $A = \{a^{2^p}b^p\}$ divide this into xy^iz

lets say $p=2$ $S = a^{2*2}b^2$ which is $aaaabb$ where $x = ay = aaaa$ and $z = bb$

case 1 'y' is in a part ($a + aaa + bb$)

so xy^iz is xy^2z for $i = 2$

therefore we have $aaaaaaabb$ which is a^7b^2 does not come in $a^{2^n}b^n$

case 2: y lies in b part so $aaaabb$ where $x = aaaaay = bz = b$

for $i = 2$ xy^2z equals $aaaa bbb$ (a^4b^3) does not come with $a^{2^n}b^n$

case 3: y lies in a and b part so we have $aaaabb$ where $x = aaaaay = abz = b$

for $i = 2$ xy^2z equals $aaa abab b$ which does not come under $a^{2^n}b^n$

hence our assumption was wrong. so according to pumping lemma we can say that $\{a^{2^n}b^n | n \geq 0\}$ is not regular

1. Consider set $X = \{n \mid n \neq f(n)\}$

we can say that x is not a value of f . lets suppose that $X = f(k)$. does k belong to $f(k)$?

if yes then no. for if $k \in f(k)$ then $k \notin X$ but X is $f(k)$

if no then yes. if $k \notin f(k)$ then $k \in X$ and $P(X)$

so there is no such f between X and $P(X)$