Sheet 1

Exercise 0 (Warmup):

Compute the following objects:

1. $(\{1,2\} \cap \{2,3\}) \cup \{2,4\}$ $= \{2,4\}$
2. $(\{1,2\} \cup \{2,3\}) \setminus \{2,4\}$ $= \{1,3\}$
3. $(\{1,2\} \cap \{2,3\}) \,\Delta\, \{2,4\}$ $= \{4\}$
4. $(\{1,2\} \cup \{2,3\}) \times \{2,4\}$ $= \{1,2,3\} \times \{2,4\} = \{\{1,2\}, \{1,4\}, \{2,2\}, \{2,4\}, \{3,2\}, \{3,4\}\}$
5. $\mathcal{P}(\{1,3,4\})$ $= \{1, \{1,3\}, \{1,4\}, 3, \{3,4\}, 4, \{1,3,4\}\}$

Exercise 1 (Sets):

(a) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

(b) Prove that $A \cap (A \cup B) = A$.

(c) Prove that if $B \subseteq A$, then $A \cap B = B$ and $A \cup B = A$.

Exercise 2 (De Morgan's law):

Prove that $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$ using a written proof in the style of Exercise 1(a).

Exercise 3 (Distributivity of Cartesian Product):

Let A, B, C, D be sets.

1. Show that $(A \cup B) \times C = (A \times C) \cup (B \times C)$
2. Show that $(A \cap B) \times C = (A \times C) \cap (B \times C)$
3. Show that $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$
4. Do we have $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$? If not, is one of both inclusions true?
5. Illustrate the results of this exercise using figures. You can use for example $A = [0,2]$, $B = [1,3]$, $C = [0,3]$, $D = [1,4]$.

Exercise 4 (Symmetric Difference):

This is a supplementary exercise that you should do at home, to get more practice after the tutorials.

(a) Let $A \Delta B = C$. What is $A \Delta C$?

(b) Prove that $A \Delta B = (A \setminus B) \cup (B \setminus A)$.

(c) Show that the symmetric difference is associative, i.e., that $A\Delta(B\Delta C) = (A\Delta B)\Delta C$ is true for all sets A, B, C. Can you find a natural description of the elements that are in $A\Delta B\Delta C$?

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notes

0: $(A \cup B) \times C \overset{!}{=} (A \times C) \cup (B \times C)$ | (Thesis)

Hyp. 1: A, B, C are Sets

Hyp. 2: $x \in (A \cup B) \times C$

Hyp. 3: using $S = (A \cup B) \to S \times C$

Hyp. 4: $A \subseteq S$

Hyp. 5: $B \subseteq S$

Hyp. 6: $A \cup B \subseteq S$

Hyp. 6: $x \in (A \cap C) \times (B \cap C)$

proof

Def. 0: Thesis means „Every tuple from $(A \cup B) \times C$ must correspond to a tuple from $(A \times C) \cup (B \times C)$ (and vice versa)

Def. 1: For $S = (A \cup B)$ and $S \times C$, $S \times C$ can be written as a union between all of S's subsets $\times C$

Def. 2: A and B are subsets of, and make up S. Any operation on S can be written as a union between that operation performed on all its subsets

Def. 3: Therefore: $(A \cup B) \times C = (A \times C) \cup (B \times C)$

q.e.d.