Problem 1 *Sequences*

For each of the following sequences $(x_n)_{n\in\mathbb{N}}$, determine all accumulation points. Afterwards, indicate whether the respective sequence converges (and if so, to which limit) or diverges.

(a) $x_n = (1 + (-1)^n)^n$

(b) $x_n = 2\sin\left(\frac{n\pi}{2}\right)$

Problem 2 *Series*

(a) Use the geometric series to calculate

$$\sum_{n=4}^{\infty} \frac{1}{2^n}.$$

(b) Apply the Leibniz test to prove convergence of

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n} + \sqrt[3]{n}}.$$

Problem 3 *Series*

Determine whether the following series converge or diverge. In case of convergence, also check for absolute convergence.

(a) $\sum_{k=1}^{\infty} \frac{1}{2^k}$

(b) $\sum_{k=1}^{\infty} \frac{(-1)^k}{3k+1}$

(c) $\sum_{k=1}^{\infty} \frac{k}{k+1}$

(d) $\sum_{k=1}^{\infty} \frac{\mathrm{e}^k}{(2k)!}$

(e) $\sum_{k=1}^{\infty} \frac{k2^k}{3^k}$

(f) $\sum_{k=1}^{\infty} \frac{4k}{2k^3+100}$

(g) $\sum_{k=1}^{\infty} \frac{k+1}{3k^2-1}$