Topological Quantum Field Theories

Proposition/Preview for a Master Course

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What are Topological Quantum Field Theories? And what are they for?

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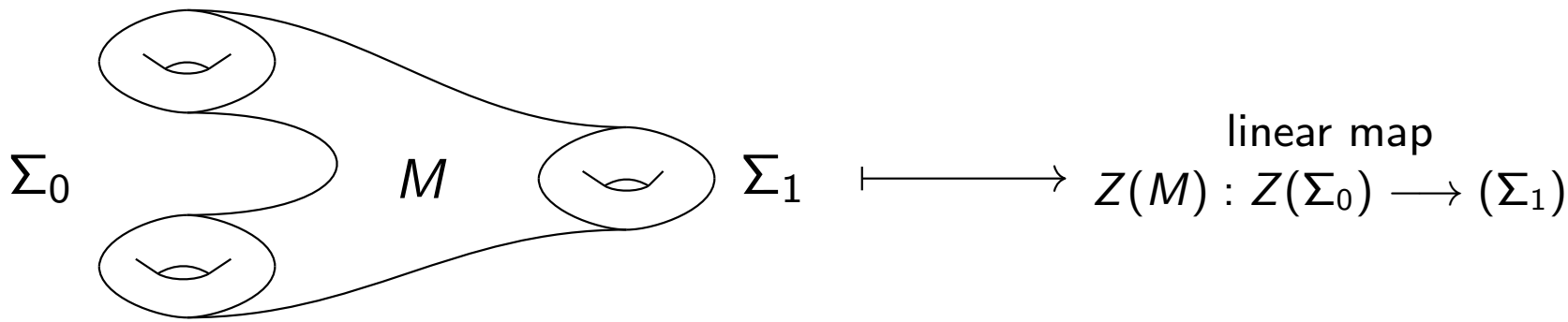
- Topological quantum field theories are an active research area and an innovative force within mathematics. For example, they have been, through their appearance in the cobordism hypothesis, at least one of the main driving force behind the development of the homotopy theory of higher (symmetric monoidal) categories.

How does a topological quantum field theory look like?

An (oriented) topological quantum field theory, here illustrated in dimension three, is a 'consistent assignment' of the following form:

compact oriented surface



equivalence class of
compact oriented
three-dimensional bordisms

We may interpret $Z(\Sigma)$ as a *state space*.

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3. *You will be introduced to the theory of three-dimensional topological field theory.* In dimension three, topological quantum field theories behave completely differently from dimension two, and the connections to representation theory become even richer. We will discuss a relatively manageable class of three-dimensional theories, the so-called Dijkgraaf-Witten theories, a type of discrete gauge theory.