Example of written examination

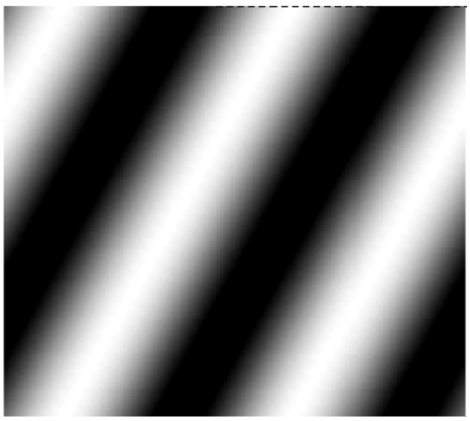
(Questions will be provided also in German)

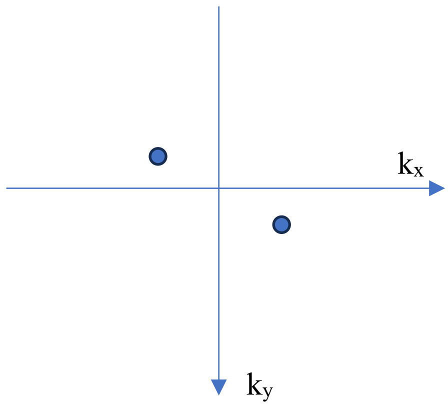
Question	1	2	3	4	5	6	7	8	9	10
Max points	4	6	8	4	9	6	6	6	6	5

X-ray based imaging

Question 1 - Image characteristics (4 points)

1) Sketch the Fourier representation of the following image, explaining the plot (**4 points**)





Premise (extra explanation for educational purposes, not necessary to get full score at the exam)
The image represents a gradual and periodic variation of grey levels (between black and white) in space. The rate of change of grey intensity in space is called 'spatial frequency' and is denoted by 'k', and the modulus of its amplitude is shown in the Fourier plane (frequency amplitude spectrum, we did not consider the phase spectrum in this course). In image processing, because of the shift applied to the FFT (Fast Fourier Transform), positive components of k are directed as in the figure.

In this image, only one spatial frequency can be recognized by eye, with two components k_x and ky. (both positive because the slope is ≤90°). Along the x-direction of the Fourier plane, the variation of grey levels is faster (about twice as fast) than in the y-direction. This means that the k_x component is about 2 times k_y component. Furthermore, since an image is a function of real values, it holds that $F(k_x, k_y) = F(-k_x, -k_y)$, then we also represent the symmetrical value.

Question 2 - Image characteristics (6 points)

Define

1) what is the contrast in an image (**2 points**),
2) how it can be improved or deteriorated (**2 points**)
3) and how it is measured in a CT scanner (**2 points**)

1) The contrast in an image is due to the difference of gray level (or color) intensity between regions (tissues in medical imaging) within the image. If we want to measure the contrast of an object A with respect to the background B, we compute

$$contrast = \frac{mean\ value\ (A) - mean\ value\ (B)}{mean\ value\ (B)}$$

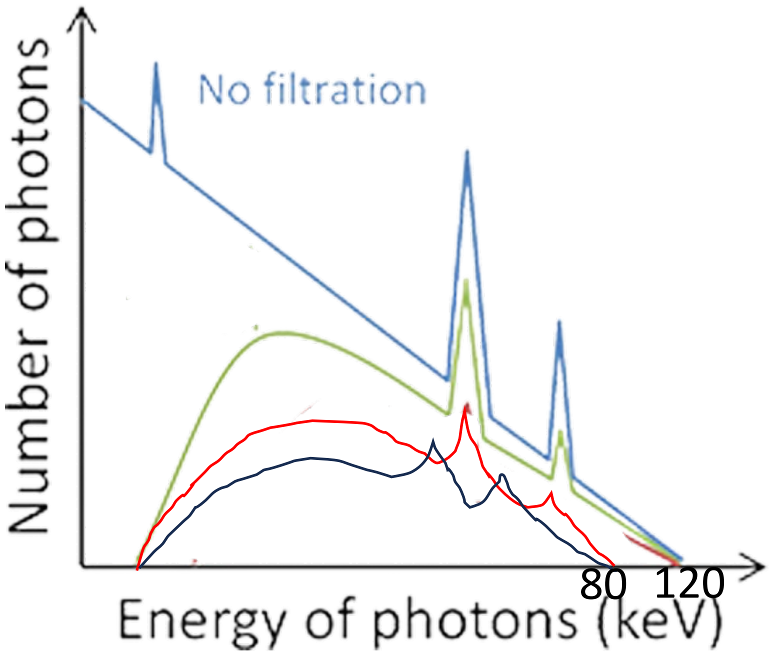
2) The contrast can be improved after image acquisition through image processing techniques (histogram equalization, histogram shift etc) or before the image acquisition by introducing a contrast agent inside the patient. The contrast is deteriorated by the amount of noise (more noise, less contrast).

3) In a CT scanner, contrast is typically measured using the Hounsfield scale, which assigns specific numerical values to different tissues based on their X-ray attenuation properties. Special phantoms including tissue equivalent elements are used, and the contrast is computed by using the equation reported in 1)

Question 3 - X-ray production (8 points)

Sketch and comment the Energy spectrum of X-ray production in the following cases:

1) Source power = 120KV, no filtration, 5 mA current, Tungsten (**2 points**)
2) Source power = 120KV, with filtration, 5 mA current, Tungsten (**2 points**)
3) Source power = 80KV, with filtration, 5 mA current, Tungsten (**2 points**)
4) Source power = 80KV, with filtration, 5 mA current, Molybdenum (**2 points**)



1) Blue curve, it represents the entire energy spectrum of photons produced in the X-ray tube. The max energy is equal to the peak Voltage between anode and cathode (120 KV)
2) Green curve, it represents 1) filtered by a low-pass energy filter that cuts very low energy X photons (useless for imaging)
3) Red curve, in this case the max energy is 80 KV and the area under the curve is lower than that one in 2) (less beam quantity)

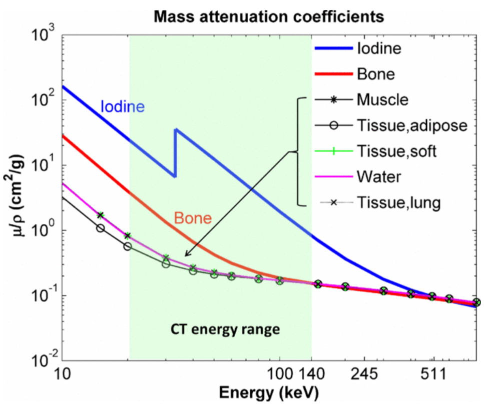
4) Black curve, it represents the energy spectrum of photons produced with an anode with lower Z (Molybdenum) than 1), 2) and 3) (Tungsten). The max energy is like in 3) the beam quantity is lower than 3) and the characteristics peaks are at lower energy compared to the Tungsten anode.

Question 4 – Image formation (4 points)

Two X-ray images are shown below. One corresponds to an X-ray beam with 140 kVp and the other to X-ray beam with 70 kVp. Explain which is which, and the reasons for the differences in image contrast and signal intensity (**4 points**).

A **B**

I labeled with A the image featuring less contrast and B the image with more contrast. A has been obtained with 140 kVp while B with 70 kVp. In fact, in X-ray based imaging the contrast depends on the difference between mass attenuation coefficient of two tissue. This difference decreases at higher energies.



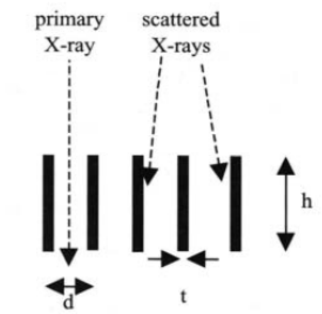
Question 5 – Instrumentation (9 points)

In an X-ray imaging device,

a) where is the anti-scatter grid positioned and what is its role (**2 points**).
b) Has the anti-scatter grid the same function as the filter? (Motivate your answer) (**2 points**)
c) Let's consider 2 anti-scattering grids, A and B, with the following geometry
   h_a= 5 mm h_b= 5 mm
   d_a= 2 mm d_b= 3 mm

t_a= 0.1 mm t_b= 0.2 mm

c.1) What can you say about the final image and dose to the patient? (**3 points**)
c.2) Which grid would you choose for your system and why? (**2 points**)



a) The anti-scatter grid is located between the patient and the detector, and it is used to reduce the amount of scatter noise reaching the detector.
b) No, they have different function. While the anti-scatter grid reduces the presence of noise in the image, the filter blocks the X-ray photons with very low energy which would not contribute to generate the image.
c) The parameter used to evaluate the anti-scatter grid is the grid ratio $\frac{h}{d}$
Higher grid ratio will reject scatter noise better than a lower grid ratio but it gives more dose to the patient because we have to increase patient exposure.

c.1) grid "a" has higher grid ratio, so more dose to patient
c.2) it is not possible to give an answer since the selection of the grid depends on different factors not illustrated in this question, such as the imaging modality (radiography, fluoroscopy, CT, etc), the anatomical site to image, dose limitation (sensitive organs, age of the patient).

Question 6 - Instrumentation (7 points)

As a clinical engineer, you must decide about new equipment in the radiology department. Two vendors propone their own systems:

1) Mobile C-Arm, with Image Intensifier, 30cmx30cm FOV, pixel matrix 512x512, 60 Hz acquisition rate
2) Fixed C-Arm, with Flat panel, 40cmx50cm FOV, pixel matrix 512x512,50 Hz acquisition rate

Write a short report where you compare the 2 systems (**7 points**)

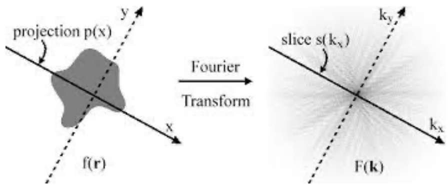
System 1) is a portable system that can be used both in different rooms and that can be stored when not necessary. This might be ideal for an Operating Room (OR), where the C-Arm is required only for some procedures while for other it would only get in the way. It has a smaller Field of View (FOV) than system 2) but it has a better spatial resolution since the physical pixel size is 0.6 mm x 0.6 mm vs. 0.78 mm x 0.98 mm. System 1) has also better temporal resolution since it is able to acquire 60 frames per second (fps) vs. 50 fps of system 2). The system 2) can be used for imaging larger anatomical areas, where the presence of smaller structures is not relevant. Since it is a fixed installation, it is ideal for a room that is used always for the same kind of clinical procedures.

Question 7 – Image reconstruction (6 points)

State the Central slice theorem and explain how it's used for CT image reconstruction (**6 points**)

The Central slice theorem states that

"The 1-dimensional Fourier transform "P_θ" of a projection function f(x,y), "p_θ" (Radon Transform) is equal to a 2-dimensional Fourier transform of the function f(x,y) evaluated at the slice passing through the origin in the frequency domain and parallel to P_θ"



In CT image reconstruction, the Central Slice Theorem is used in filtered back projection (FBP), which is one of the most common methods for reconstructing CT images from projection data.

Here, it's a possible implementation of FBP

- A projection p_θ of the object is computed
- The Fourier Transform of p_θ is calculated thus obtaining P_θ
- A ramp filter is applied to P_θ
- The inverse Fourier transform is applied to P_θ thus obtaining a filtered projection p^*_θ
- A filtered sinogram is obtained
- The filtered sinogram is back projected

The steps are repeated for each θ

Ultrasounds (US)

Question 8 – Basic principles (6 points)

a. Write the relationship between the reflection coefficient R and the acoustic impedance Z in case of perpendicular reflection (**1 point**)
b. Compute R at the interface Z_{muscle}-Z_{fat} knowing that the density (ρ) and US speeds are: (**3 points**)
   - ρ_{muscle}=1.07 g·cm^{-3}
   - ρ_{fat}=0.92 g·cm^{-3}
   - c_{muscle}=1600 m·s^{-1}
   - c_{fat}=1450 m·s^{-1}
c. How does US intensity decrease in depth? Explain it by writing the equation and explaining all variables/constants involved (**2 points**)

a. Given Z_1 and Z_2 the impendence of tissue 1 and tissue 2 respectively, R is

$$R = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$$

b. We know that $Z = \rho \cdot c$ and 1 kg/m^{2}s = 1 Rayl

$$Z_{muscle} = 1.07\frac{10^{-3}Kg}{10^{-6}m^3} \cdot 1600\frac{m}{s} = 1.71 \cdot 10^6 Rayls$$
$$Z_{fat} = 0.92\frac{10^{-3}Kg}{10^{-6}m^3} \cdot 1450\frac{m}{s} = 1.33 \cdot 10^6 Rayls$$
$$R = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2 = 0.0154$$

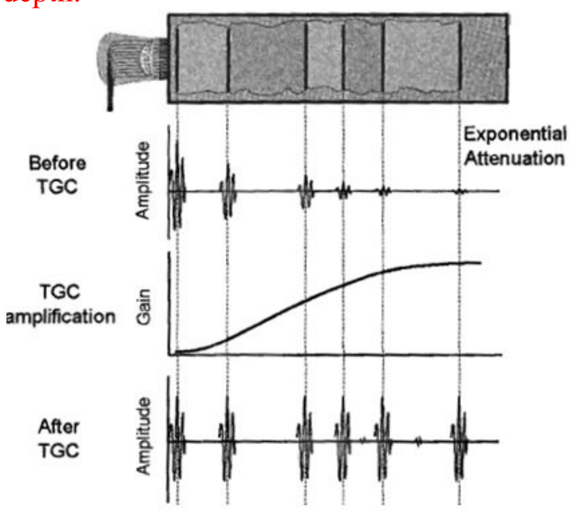
c. The attenuation is given by $I_2 = I_1 e^{-\mu f x}$
With **μ** Attenuation coefficient (tissue dependent)
f frequency of the US wave
x depth of the tissue
The equations shows that the intensity decreases in depth (x) as a function of f and **μ**

Question 9 – Instrumentation (6 points)

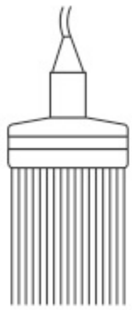
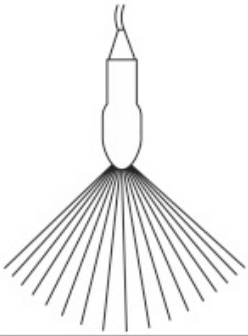
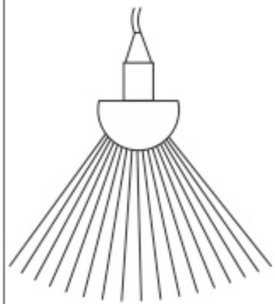
a. Which kind of "special" amplifier is used in US instrumentation and how does it work? (**1 point**)
b. How can the piezoelectric elements be arranged on the probe and how is the field of view affected? (**4 points**)
c. An US devise has two transducers with different frequencies:
   - Transducer 1, f_1= 4 Mhz
   - Transducer 2, f_2= 24 Mhz

   Which transducer is ideal to perform measurement on the derma layer and why? (**1 point**)

   a. In US a Time Gain Compensator is used to amplify the reflected US waves as a function of depth. This is necessary due to the exponential loss of intensity with depth.



   b. Ultrasound probes can be single-element, linear arrays (up to 512 elements), phased arrays. Single-element probes have a fixed focal depth determined by the design of the transducer either with an acoustic lens in front of the element or a concave piezoelectric. In such probes, the transducer is mechanically pivoted to sweep out two-dimensional sector B-scans. Linear arrays are focused by changing the number of fired elements. Phased arrays are focused electronically by changing the firing time (phase) of fired elements. In both cases the focus can be adjusted.

	Linear Array	Sector (Phased) Array	Curved Linear Array
			
Frequency range	Higher (8–15 MHz)	Lower (2–6 MHz)	Higher (2–12 MHz)
Depth of imaging	Superficial (1–4 cm)	Deeper (4–8 cm)	Intermediate (2–6 cm)
Field of view	Linear, limited (depends on footprint)	Trapezoidal, wider at depth, narrow at the surface	Trapezoidal , wide at surface and depth

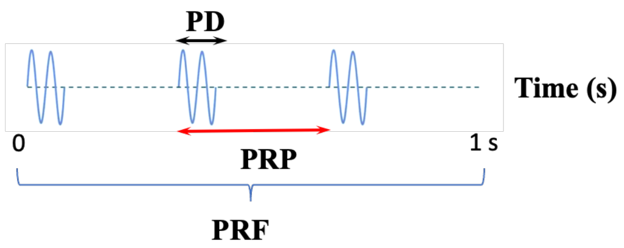
c. The ideal transducer is the second (f_2) since it has higher frequency that means a short penetration depth and higher axial resolution.

Question 10 – Image formation (5 points)

a. Explain why pulsed US waves (PW) are preferred to continuous US waves (PW) to build an image and describe the most important parameters (**4 points**)
b. Compute the near field depth of an US beam generated by (f=frequency, D=piezoelement diameter) assuming an average speed in soft tissues of 1540 m/s for
   1. f=2MHz, D= 10 mm
   2. f=2MHz, D= 5 mm
   3. f=4MHz, D= 10 mm
   4. f=4MHz, D= 5 mm

   Comment on the results (**1 point**)

a. In order to be able to listen to reflected waves in US imaging without overlaps with transmitted waves, we need to establish a receiving time during which US waves are not transmitted. This approach is called pulsed wave ultrasounds and it allow to build an image with higher efficiency. The most relevant parameters are:
   1. pulse duration, **PD**, time during which the pulse is generated;
   2. spatial pulse length, **SPL**, the distance in space traveled by ultrasound during one pulse;
   3. pulse repetition period, **PRP**, amount of time between the start of one pulse and the start of the next pulse;
   4. pulse repetition frequency **PRP**, # of pulses per unit time (1 second)
   5. duty factor: ratio between the pulse duration and the pulse repetition period



b. We know that $L = \frac{d^2}{4\lambda} = \frac{d^2 f}{4c}$

1. L=3.25 cm
2. L=0.81 cm
3. L=6.50 cm
4. L=1.62 cm

By changing the frequency or the diameter of the transducer, the near field length