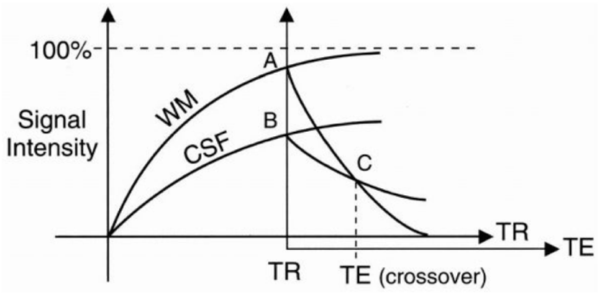
**Exercice 1**

Consider the figure below, where $T_1$ and $T_2$ curves are plotted simultaneously for convenience after a RF transmission with flip angle $\alpha$=90°. Assume the following values:

- For white matter (WM), $T_1$ = 250 ms, $T_2$ = 50 ms
- For cerebrospinal fluid (CSF): $T_1$ = 1000 ms, $T_2$ = 100 ms
- $T_R$=1.5 s.

Also assume a net magnetization $M_0$=150 mA/m for both WM and CSF




a. Calculate the ratio between the signal intensities of WM and CSF (point A/point B respectively) (**3 points**)
b. Calculate the crossover point C where WM and CSF have identical $T_E$ weighting (**5 points**)
c. Calculate the ratio of the signal CSF/WM for $T_E$ = 25ms and $T_E$ = 100 ms and make some considerations about contrast (**7 points**)
d. What does a longer $T_R$ cause on image contrast? (**2 points**)
e. What does a longer $T_E$ cause on image contrast? (**2 points**)

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a. In a rotating frame of reference, we know that $M_z(t) = M_0\left(1 - e^{-\frac{t}{T_1}}\right)$

$$\frac{M_{ZWM}}{M_{ZCSF}} = \frac{\cancel{M_0}\left(1 - e^{-\frac{T_R}{T_{1WM}}}\right)}{\cancel{M_0}\left(1 - e^{-\frac{T_R}{T_{1CSF}}}\right)} = 1.284$$

b. In a rotating frame of reference, we know that FID=$M_0\left(1 - e^{-\frac{T_R}{T_1}}\right)e^{-\frac{t}{T_2}}$

Here we want $FID_{WM}$=$FID_{CSF}$

$$\cancel{M_0}\left(1 - e^{-\frac{T_R}{T_{1WM}}}\right)e^{-\frac{T_E}{T_{2WM}}} = \cancel{M_0}\left(1 - e^{-\frac{T_R}{T_{1CSF}}}\right)e^{-\frac{T_E}{T_{2CSF}}}$$

By substituting what we found in point a

$$1.284 \cdot e^{-\frac{T_E}{T_{2WM}}} = e^{-\frac{T_E}{T_{2WM}}}$$

$$ln(1.284) - \frac{T_E}{50} = -\frac{T_E}{100}$$

$$ln(1.284) = T_E \frac{2-1}{100}$$

$$T_E = 100 \cdot ln(1.284) = 25\ ms$$

c. We have to compute the ratio $\frac{FID_{CSF}}{FID_{WM}}$

When $T_E$=25ms

$$\frac{\cancel{M_0}\left(1 - e^{-\frac{T_R}{T_{1CSF}}}\right) e^{-\frac{T_E}{T_{2CSF}}}}{\cancel{M_0}\left(1 - e^{-\frac{T_R}{T_{1WM}}}\right) e^{-\frac{T_E}{T_{2WM}}}} = \frac{\left(1 - e^{-\frac{1500}{1000}}\right) e^{-\frac{25}{100}}}{\left(1 - e^{-\frac{1500}{250}}\right) e^{-\frac{25}{50}}} = 1$$

When $T_E$=100ms

$$\frac{\left(1 - e^{-\frac{1500}{1000}}\right) e^{-\frac{100}{100}}}{\left(1 - e^{-\frac{1500}{250}}\right) e^{-\frac{100}{50}}} = 2.11$$

With $T_E$=25ms we cannot distinguish CSF signal from WM signal
By increasing $T_E$=100ms CSF signal becomes hyperintense with respect to WM signal

d. Considering FID=$M_0\left(1 - e^{-\frac{T_R}{T_1}}\right) e^{-\frac{t}{T_2}}$ a longer $T_R$ reduces $T_1$ weighting in favor either of proton density or $T_2$ weighting.

e. Longer $T_E$ increases $T_2$

**Exercice 2**

a. **Write the relationship between magnetic momentum $\vec{\mu}$ of a sub-atomic particle and the external, static magnetic field $\overrightarrow{B_0}$(1 points)**
b. **Explain how can the equilibrium of $\vec{\mu}$ be altered? (2 points)**
c. **In order to achieve what you explained in (b), which components are introduced in the instrumentation, and which/why is the preferred configuration? (4 points)**
d. **Sketch a diagram of the MRI scanner in closed bore configuration and label each relevant component (3 points)**

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a. $\frac{d\vec{\mu}}{dt} = \vec{\mu} \times \gamma \vec{B}$

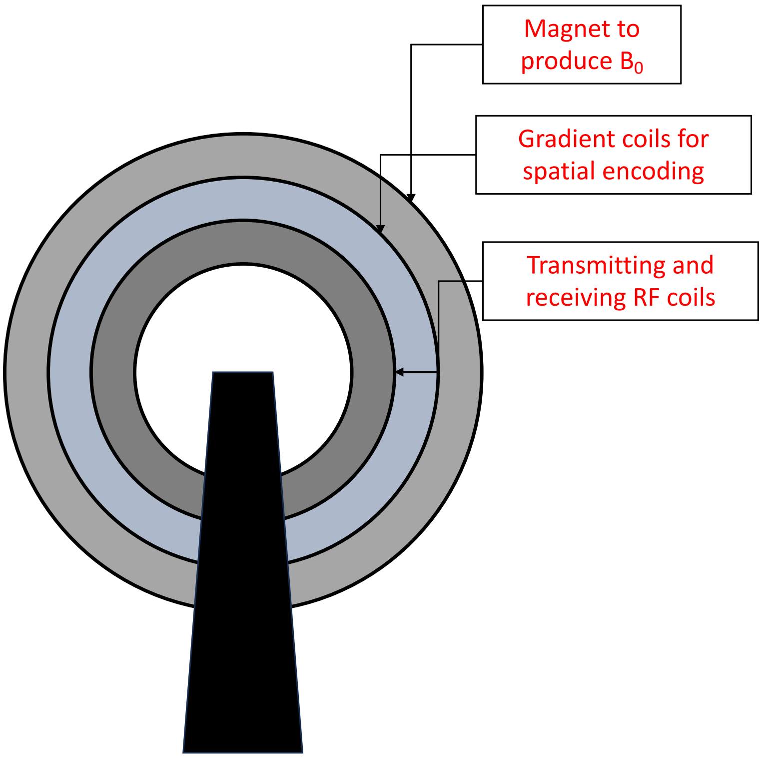
b. In order to alter the equilibrium of $\vec{\mu}$, we need to provide a quantity ΔE of energy to the system. The ΔE required from spins to transit from low energy to high energy is equal to absorbing a photon with energy equal to $\hbar\boldsymbol{\gamma B}_0$. This is obtained by generating an electromagnetic field with frequency $|\omega_0| = \gamma|B_0|$, known as resonance frequency, which is in the band of radiofrequencies (RF).

$$E = \mu B_0 cos\theta = -\gamma\, J B_0 cos\theta \Longrightarrow E = -\gamma\, \hbar m B_0$$

If $I = \frac{1}{2}$ we have 2 possible energy levels

$$\begin{cases} E = \frac{1}{2}\gamma\, \hbar B_0 \\ E = -\frac{1}{2}\gamma\, \hbar B_0 \end{cases} \Longrightarrow \Delta E = E_{up} - E_{down} = \gamma\, \hbar B_0$$

c. In order to generate the EM field with frequency $|\omega_0|$ we use an RF-coil to produce a time-varying excitation field B1. The B1 field is produced by driving alternate electrical currents through specialized RF-transmit coils. The configuration of RF coils can be such that it produces linearly polarized (LP) EM waves or circularly polarized (CP) EM waves. The CP configuration is obtained by setting cols in quadrature transmission. CP is the preferred choice to reduce heat production and increase the efficiency.


Magnet to produce $B_0$
Gradient coils for spatial encoding
Transmitting and receiving RF coils

d.