

7-12

## FOURIER COSINE INTEGRAL REPRESENTATIONS

Represent  $f(x)$  as an integral (10).

$$7. f(x) = \begin{cases} 1 & \text{if } 0 < x < 1 \\ 0 & \text{if } x > 1 \end{cases}$$



Vi skal her finde cosine interallet

$$f(x) = \int_0^{\infty} A(w) \cos(wx) dw$$

$$A(w) = \frac{2}{\pi} \int_0^{\infty} f(v) \cdot \cos(wv) dv$$

Vi kan her sætte den øvre grænse til 1 fordi alt efter giver 0

$$A(w) = \frac{2}{\pi} \cdot \int_0^1 \cos(wv) dv$$

$$A(w) = \frac{2}{\pi} \cdot \left( \underbrace{\sin(wv)}_w \bigg|_{v=0}^{v=1} \right)$$

$$A(w) = \frac{2}{\pi} \cdot \frac{\sin(w)}{w}$$

$$f(x) = \frac{2}{\pi} \cdot \int_0^{\infty} \frac{\sin(w) \cdot \cos(wx)}{w} dw$$

Represent  $f(x)$  as an integral (11).

$$17. f(x) = \begin{cases} 1 & \text{if } 0 < x < 1 \\ 0 & \text{if } x > 1 \end{cases}$$

Her skal vi finde sine integrallet

$$f(x) = \int_0^{\infty} B(w) \cdot \sin(wx) \, dw$$

$$B(w) = \frac{2}{\pi} \int_0^{\infty} f(v) \cdot \sin(wv) \, dv$$

Her sættes den øvre grænse igen til 1 fordi  $f(x)$  er 0 efter dette

$$B(w) = \frac{2}{\pi} \cdot \int_0^1 \sin(wv) \, dv$$

$$B(w) = \frac{2}{\pi} \cdot \left( -\frac{\cos(wv)}{w} \right) \Big|_{v=0}^{v=1}$$

$$B(w) = \frac{2}{\pi} \cdot \left( \frac{\cos(0) - \cos(w)}{w} \right)$$

$$B(w) = \frac{2}{\pi} \cdot \left( \frac{1 - \cos(w)}{w} \right)$$

$$f(x) = \frac{2}{\pi} \cdot \int_0^{\infty} \left( \frac{1 - \cos(w)}{w} \right) \cdot \sin(wx) \, dw$$

## 2-11

FOURIER TRANSFORMS BY  
INTEGRATION

Find the Fourier transform of  $f(x)$  (without using Table III in Sec. 11.10). Show details.

$$3. f(x) = \begin{cases} 1 & \text{if } a < x < b \\ 0 & \text{otherwise} \end{cases}$$

Fourier transformen kan findes som:

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{\infty} e^{-i\omega x} f(x) dx$$

Her kan grænserne sættes til  $a$  og  $b$  eftersom at  $f(x)$  er 0 udenfor dette interval

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \cdot \int_a^b e^{-i\omega x} dx$$

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \cdot \left( \frac{e^{-i\omega x}}{-i\omega} \right) \Big|_{x=a}^{x=b}$$

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \cdot \left( \frac{e^{-i\omega a} - e^{-i\omega b}}{i\omega} \right)$$

$$\underline{\underline{F(\omega) = \frac{1}{i\omega \cdot \sqrt{2\pi}} \cdot (e^{-i\omega a} - e^{-i\omega b})}}$$

$$5. f(x) = \begin{cases} e^x & \text{if } -a < x < a \\ 0 & \text{otherwise} \end{cases}$$

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{\infty} e^{-i\omega x} f(x) dx$$

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-a}^a e^{-i\omega x} \cdot e^x dx$$

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-a}^a e^{-i\omega x + x} dx$$

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-a}^a e^{x \cdot (1 - i\omega)} dx$$

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \cdot \left( \frac{e^{x \cdot (1 - i\omega)}}{1 - i\omega} \right) \Big|_{x=-a}^{x=a}$$

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \cdot \left( \frac{e^{a \cdot (1 - i\omega)} - e^{-a \cdot (1 - i\omega)}}{1 - i\omega} \right)$$