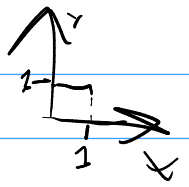
11.7

7–12 FOURIER COSINE INTEGRAL REPRESENTATIONS

Represent $f(x)$ as an integral (10).

7. $f(x) = \begin{cases} 1 & \text{if } 0 < x < 1 \\ 0 & \text{if } x > 1 \end{cases}$



$$f(x) = \int_0^\infty A(w)\cos(wx)\,dw$$

$$A(w) = \frac{2}{\pi}\int_0^\infty f(v)\cdot\cos(wv)\,dv$$

$$A(w) = \frac{2}{\pi}\cdot\int_0^1 \cos(wv)\,dv$$

$$A(w) = \frac{2}{\pi}\cdot\left(\frac{\sin(wv)}{w}\Big|_{v=1}^{v\to 0}\right)$$

$$A(w) = \frac{2}{\pi}\cdot\frac{\sin(w)}{w}$$

$$f(x) = \frac{2}{\pi}\cdot\int_0^\infty \frac{\sin(w)\cdot\cos(wx)}{w}\,dw$$

16–20 FOURIER SINE INTEGRAL REPRESENTATIONS

Represent $f(x)$ as an integral (11).

17. $f(x) = \begin{cases} 1 & \text{if } 0 < x < 1 \\ 0 & \text{if } x > 1 \end{cases}$

$$f(x) = \int_0^\infty B(w) \cdot \sin(wx)\, dw$$

$$B(w) = \frac{2}{\pi} \int_0^\infty f(v) \cdot \sin(wv)\, dv$$

$$B(w) = \frac{2}{\pi} \cdot \int_0^1 \sin(wv)\, dv$$

$$B(w) = \frac{2}{\pi} \cdot \left(-\frac{\cos(wv)}{w} \Big|_{v=0}^{v=1} \right)$$

$$B(w) = \frac{2}{\pi} \cdot \left(\frac{\cos(0) - \cos(w)}{w} \right)$$

$$B(w) = \frac{2}{\pi} \cdot \left(\frac{1 - \cos(w)}{w} \right)$$

$$f(x) = \frac{2}{\pi} \cdot \int_0^\infty \frac{(1 - \cos(w))}{w} \cdot \sin(wx)\, dw$$

2–11 FOURIER TRANSFORMS BY INTEGRATION

Find the Fourier transform of $f(x)$ (without using Table III in Sec. 11.10). Show details.

3. $f(x) = \begin{cases} 1 & \text{if } a < x < b \\ 0 & \text{otherwise} \end{cases}$

$$F(w) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{\infty} e^{-iwx} f(x)\, dx$$

$$F(x) = \frac{1}{\sqrt{2\pi}} \cdot \int_{a}^{b} e^{-iwx}\, dx$$

$$F(x) = \frac{1}{\sqrt{2\pi}} \cdot \left(\frac{e^{-iwx}}{-iw} \Big|_{x=a}^{x=b} \right)$$

$$F(x) = \frac{1}{\sqrt{2\pi}} \cdot \left(\frac{e^{-iwa} - e^{-iwb}}{iw} \right)$$

$$F(x) = \frac{1}{iw \cdot \sqrt{2\pi}} \cdot \left(e^{-iwa} - e^{-iwb} \right)$$

5. $f(x) = \begin{cases} e^x & \text{if } -a < x < a \\ 0 & \text{otherwise} \end{cases}$

$$F(x) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{\infty} e^{-iwx} f(x)\, dx$$

$$F(x) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-a}^{a} e^{-iwx} \cdot e^{x}\, dx$$

$$F(x) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-a}^{a} e^{-iwx+x}\, dx$$

$$F(x) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-a}^{a} e^{x\cdot(1-iw)}\, dx$$

$$F(x) = \frac{1}{\sqrt{2\pi}} \cdot \left(\frac{e^{x\cdot(1-iw)}}{1-iw} \Bigg|_{x=-a}^{x=a} \right)$$

$$F(x) = \frac{1}{\sqrt{2\pi}} \cdot \left(\frac{e^{a\cdot(1-iw)} - e^{-a\cdot(1-iw)}}{1-iw} \right)$$