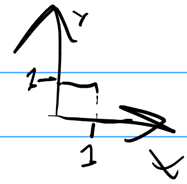
11,7)

7–12 **FOURIER COSINE INTEGRAL REPRESENTATIONS**

Represent $f(x)$ as an integral (10).

7. $f(x) = \begin{cases} 1 & \text{if} \quad 0 < x < 1 \\ 0 & \text{if} \quad x > 1 \end{cases}$



Vi skal her finde cosine interallet

$$f(x) = \int_0^{\infty} A(w)\cos(wx)\,dw$$

$$A(w) = \frac{2}{\pi}\int_0^{\infty} f(v)\cdot\cos(wv)\,dv$$

Vi kan her sætte den øvre grænse til 1 fordi alt efter giver 0

$$A(w) = \frac{2}{\pi}\cdot\int_0^{1}\cos(wv)\,dv$$

$$A(w) = \frac{2}{\pi}\cdot\left(\frac{\sin(wv)}{w}\Bigg|_{v=0}^{v=1}\right)$$

$$A(w) = \frac{2}{\pi}\cdot\frac{\sin(w)}{w}$$

$$f(x) = \frac{2}{\pi}\cdot\int_0^{\infty}\frac{\sin(w)\cdot\cos(wx)}{w}\,dw$$

16–20 **FOURIER SINE INTEGRAL REPRESENTATIONS**

Represent $f(x)$ as an integral (11).

17. $f(x) = \begin{cases} 1 & \text{if} \quad 0 < x < 1 \\ 0 & \text{if} \quad x > 1 \end{cases}$

Her skal vi finde sine integrallet

$$f(x) = \int_0^{\infty} B(w) \cdot \sin(wx)\, dw$$

$$B(w) = \frac{2}{\pi} \int_0^{\infty} f(v) \cdot \sin(wv)\, dv$$

Her sættes den øvre grænse igen til 1 fordi f(x) er 0 efter dette

$$B(w) = \frac{2}{\pi} \cdot \int_0^{1} \sin(wv)\, dv$$

$$B(w) = \frac{2}{\pi} \cdot \left(\left[-\frac{\cos(wv)}{w} \right]_{v=0}^{v=1} \right)$$

$$B(w) = \frac{2}{\pi} \cdot \left(\frac{\cos(0) - \cos(w)}{w} \right)$$

$$B(w) = \frac{2}{\pi} \cdot \left(\frac{1 - \cos(w)}{w} \right)$$

$$f(x) = \frac{2}{\pi} \cdot \int_0^{\infty} \left(\frac{1 - \cos(w)}{w} \right) \cdot \sin(wx)\, dw$$

2–11 FOURIER TRANSFORMS BY INTEGRATION

Find the Fourier transform of $f(x)$ (without using Table III in Sec. 11.10). Show details.

3. $f(x) = \begin{cases} 1 & \text{if} \quad a < x < b \\ 0 & \text{otherwise} \end{cases}$

Fourier transformen kan findes som:

$$F(x) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{\infty} e^{-iwx} f(x)\, dx$$

Her kan grænserne sættes til a og b eftersom at f(x) er 0 udenfor dette interval

$$F(x) = \frac{1}{\sqrt{2\pi}} \cdot \int_{a}^{b} e^{-iwx}\, dx$$

$$F(x) = \frac{1}{\sqrt{2\pi}} \cdot \left(\frac{e^{-iwx}}{-iw} \Bigg|_{x=a}^{x=b} \right)$$

$$F(x) = \frac{1}{\sqrt{2\pi}} \cdot \left(\frac{e^{-iwa} - e^{-iwb}}{iw} \right)$$

$$\underline{\underline{F(x) = \frac{1}{iw \cdot \sqrt{2\pi}} \cdot \left(e^{-iwa} - e^{-iwb} \right)}}$$

5. $f(x) = \begin{cases} e^x & \text{if} \quad -a < x < a \\ 0 & \text{otherwise} \end{cases}$

$$F(x) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{\infty} e^{-iwx} f(x)\, dx$$

$$F(x) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-a}^{a} e^{-iwx} \cdot e^{x}\, dx$$

$$F(x) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-a}^{a} e^{-iwx+x}\, dx$$

$$F(x) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-a}^{a} e^{x\cdot(1-iw)}\, dx$$

$$F(x) = \frac{1}{\sqrt{2\pi}} \cdot \left(\frac{e^{x\cdot(1-iw)}}{1-iw} \Bigg|_{x=-a}^{x=a} \right)$$

$$F(x) = \frac{1}{\sqrt{2\pi}} \cdot \left(\frac{e^{a\cdot(1-iw)} - e^{-a\cdot(1-iw)}}{1-iw} \right)$$