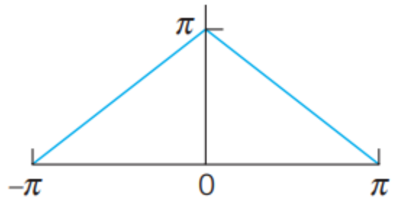
Find the Fourier series of the given function $f(x)$, which is assumed to have the period 2π. Show the details of your work. Sketch or graph the partial sums up to that including $\cos 5x$ and $\sin 5x$.

17.



$$f(x) = \begin{cases} x+\pi & -\pi \le x < 0 \\ \pi - x & 0 \le x \le \pi \end{cases}$$

Vi har her en lige funktion hvilket betyder vi kan sætte b_n=0

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$= \frac{1}{2\pi} \cdot \left(\int_{-\pi}^{0} x+\pi \, dx + \int_{0}^{\pi} \pi - x \, dx \right)$$

$$= \frac{1}{2\pi} \cdot \left(\left(\frac{1}{2}x^2 + \pi x\right)\Big|_{-\pi}^{0} + \left(\pi x - \frac{1}{2}x^2\right)\Big|_{0}^{\pi} \right)$$

$$= \frac{1}{2\pi} \cdot \left(-\left(\frac{1}{2}(-\pi)^2 + \pi\cdot(-\pi)\right) + \pi\cdot\pi - \frac{1}{2}\pi^2 \right)$$

$$= \frac{1}{2\pi} \cdot \left(-\frac{\pi^2}{2} + \pi^2 + \pi^2 - \frac{1}{2}\pi^2 \right)$$

$$= \frac{1}{2\pi} \cdot \left(2\pi^2 - \pi^2 \right)$$

$$= \frac{1}{2\pi} \cdot \left(\pi^2 \right)$$

$$a_0 = \frac{\pi}{2}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \cos(xn)\, dx$$

$$a_n = \frac{1}{\pi} \cdot \left(\int_{-\pi}^{0} (x+\pi) \cdot \cos(xn)\, dx + \int_{0}^{\pi} (\pi - x) \cdot \cos(xn)\, dx \right.$$

$$a_n = \frac{1}{\pi} \cdot \left(\int_{-\pi}^{0} x \cdot \cos(xn)\, dx + \int_{-\pi}^{0} \pi \cdot \cos(xn)\, dx + \int_{0}^{\pi} \pi \cdot \cos(xn)\, dx - \int_{0}^{\pi} x \cdot \cos(xn)\, dx \right.$$

$$\int \cos(xn) \cdot x\, dx = \frac{1}{n} \cdot \sin(xn) \cdot x - \int \frac{1}{n} \sin(xn)\, dx$$

$$= \frac{\sin(xn) \cdot x + \frac{1}{n} \cdot \cos(xn)}{n} + C$$

$$a_n = \frac{1}{\pi} \cdot \left(\left(\frac{\sin(xn) \cdot x + \frac{1}{n} \cdot \cos(xn)}{n} \right) \Bigg|_{-\pi}^{0} + \frac{\pi}{n} \cdot \sin(xn) \Big|_{-\pi}^{0} + \frac{\pi}{n} \cdot \sin(xn) \Big|_{0}^{\pi} + \left(\frac{-\sin(xn) \cdot x - \frac{1}{n} \cdot \cos(xn)}{n} \right) \Bigg|_{0}^{\pi} \right)$$

$$a_n = \frac{1}{\pi} \left(\frac{1}{n^2} \cdot \left(\left(\sin(xn) \cdot xn + \cos(xn) \right) \Big|_{-\pi}^{0} \right) + \frac{1}{n^2} \left(\left(-\sin(xn) \cdot xn - \cos(xn) \right) \Big|_{0}^{\pi} \right) \right)$$

$$a_n = \frac{1}{n^2 \pi} \left(\cos(0) - \cos(-\pi n) - \cos(\pi n) + \cos(0) \right)$$

$$a_n = \frac{1}{n^2 \pi} \cdot \left(1 - \cos(-\pi n) - \cos(\pi n) + 1 \right)$$

$$a_n = \frac{2 - 2 \cdot \cos(\pi n)}{n^2 \cdot \pi}$$

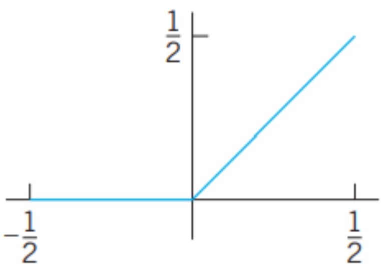
$$a_n = \frac{2 - 2 \cdot (-1)^n}{n^2 \cdot \pi}$$

$$S_5 = \frac{\pi}{2} + \frac{4}{\pi}\cos(x) + \frac{4}{9}\cos(3x) + \frac{4}{25\pi}\cos(5x)$$

11.2

Are the following functions even or odd or neither even nor odd?

13.



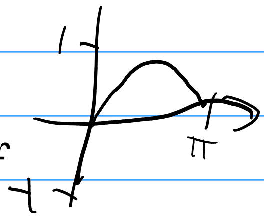
even $f(x) = f(-x)$ nej

odd $f(x) = -f(-x)$ nej

neither

Find **(a)** the Fourier cosine series, **(b)** the Fourier sine series. Sketch $f(x)$ and its two periodic extensions. Show the details.

29. $f(x) = \sin x \ (0 < x < \pi)$

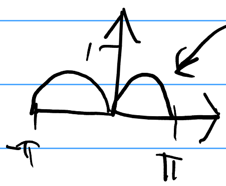


Det er nok en fejl her i stedet så anbefaler man går videre hvor den bliver løst på en nemmere måde

a

laver den først lige

$$f(x) = \begin{cases} \sin(x) & 0 \le x \le \pi \\ -\sin(x) & -\pi \le x < 0 \end{cases}$$



$b_n = 0$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$a_0 = \frac{1}{2\pi} \cdot \left(\int_{-\pi}^{0} -\sin(x)\, dx + \int_{0}^{\pi} \sin(x) dx \right)$$

$$a_0 = \frac{1}{2\pi} \left(\cos(x) \Big|_{-\pi}^{0} - \cos(x) \Big|_{0}^{\pi} \right)$$

$$a_0 = \frac{1}{2\pi} \left(1 + 1 + 1 + 1 \right)$$

$$a_0 = \frac{2}{\pi}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \cos(xn) dx$$

$$a_n = \frac{1}{\pi} \cdot \left(\int_{-\pi}^{0} -\sin(x) \cdot \cos(xn)\, dx + \int_{0}^{\pi} \sin(x) \cdot \cos(xn)\, dx \right.$$

$$a_n = \frac{1}{\pi} \left(- \int_{-\pi}^{0} \sin(x) \cdot \cos(xn) dx + \int_{0}^{\pi} \sin(x) \cdot \cos(xn)\, dx \right)$$

$$\int \sin(x) \cdot \cos(xn) dx = \int \frac{1}{2} \cdot \left(\sin(x + xn) + \sin(x - xn) \right)$$

$$a_0 = \frac{1}{L}\int_0^L f(x)\,dx$$

$$a_0 = \frac{1}{\pi}\int_0^{\pi} \sin(x)\,dx$$

$$a_0 = \frac{1}{\pi}\cdot\left(-\cos(\pi) + \cos(0)\right)$$

$$a_0 = \frac{1}{\pi}\cdot(1+1)$$

$$a_0 = \frac{2}{\pi}$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cdot\cos\left(\frac{nx\pi}{L}\right)dx$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} \sin(x)\cdot\cos(nx)\,dx$$

$$\sin(x)\cdot\cos(nx) = \frac{\sin(x+nx) + \sin(x-nx)}{2}$$

$$a_n = \frac{1}{\pi}\int_0^{\pi} \sin(x+nx) + \sin(x-nx)\,dx$$

$$a_n = \frac{1}{\pi} \cdot \left(-\frac{\cos(x+nx)}{1+n} - \frac{\cos(x-nx)}{1-n} \right) \Bigg|_0^{\pi}$$

$$a_n = \frac{1}{\pi} \cdot \left(-\frac{\cos(\pi+\pi n)}{1+n} - \frac{\cos(\pi - n\pi)}{1-n} + \frac{\cos(0)}{1+n} + \frac{\cos(0)}{1-n} \right)$$

$$a_n = \frac{1}{\pi} \cdot \left(\frac{(-1)^n}{1+n} + \frac{(-1)^n}{1-n} + \frac{1}{1+n} + \frac{1}{1-n} \right)$$

$$a_n = \frac{1}{\pi} \cdot \left(\frac{(-1)^n \cdot (1-n) + (-1)^n \cdot (1+n) + (1-n) + (1+n)}{1-n^2} \right)$$

$$a_n = \frac{1}{\pi} \cdot \left(\frac{(-1)^n \cdot \left((1-n) + (1+n) \right) + 2}{1-n^2} \right)$$

$$a_n = \frac{1}{\pi} \cdot \left(\frac{(-1)^n \cdot (2) + 2}{1-n^2} \right)$$

$$a_n = \frac{2}{\pi} \cdot \left(\frac{(-1)^n + 1}{1-n^2} \right)$$

$$a_n = \begin{cases} \frac{4}{\pi} \cdot \frac{1}{1-n^2} & n \text{ er lige} \\ 0 & n \text{ er ulige} \end{cases}$$

$$S_6(x) = \frac{2}{\pi} + \frac{4}{\pi} \cdot \left(-\frac{1}{3} \cos(2x) - \frac{1}{15} \cdot \cos(4x) - \frac{1}{35} \cdot \cos(6x) \right)$$

Kigger nu på hvordan den ser ud hvis den er ulige

$$b_n = \frac{2}{L} \int_0^L f(x) \cdot \sin\left(\frac{n x \pi}{L}\right) dx$$

$$b_n = \frac{2}{\pi} \int_0^{\pi} \sin(x) \cdot \sin(nx) \; dx$$

$$\sin(x) \cdot \sin(nx) = \frac{\cos(x - nx) - \cos(x + nx)}{2}$$

$$b_n = \frac{1}{\pi} \int_0^{\pi} \cos(x - nx) - \cos(x + nx) \; dx$$

$$b_n = \frac{1}{\pi} \cdot \left(\frac{\sin(x - nx)}{1-n} - \frac{\sin(x + nx)}{1+n} \right) \Bigg|_0^{\pi}$$

$$b_n = \frac{1}{\pi} \cdot \left(\frac{\sin(\pi - n\pi)}{1-n} - \frac{\sin(\pi + n\pi)}{1+n} - \frac{\sin(0)}{1-n} + \frac{\sin(0)}{1+n} \right)$$

$$b_n = \frac{1}{\pi}\left(\frac{\sin(\pi - n\pi)}{1-n} - \frac{\sin(\pi + n\pi)}{1+n}\right)$$

$$b_n = \frac{1}{\pi} \cdot (0)$$

$$b_n = 0$$

Tester nu når n=1 fordi at n ikke måtte være 1 i vores originale

$$b_1 = \frac{2}{\pi} \cdot \int_0^{\pi} \sin(x) \cdot \sin(x)\, dx$$

$$b_1 = \frac{2}{\pi} \cdot \left(\frac{1}{2}x - \frac{1}{4} \cdot \sin(2x)\right) \Big|_0^{\pi}$$

$$b_1 = \frac{2}{\pi} \cdot \left(\frac{1}{2}\pi - \frac{1}{4} \cdot \sin(2\pi) - 0 + \frac{1}{4} \cdot \sin(0)\right)$$

$$b_1 = \frac{2}{\pi} \cdot \left(\frac{1}{2}\pi - 0 + 0\right)$$

$$b_1 = 1$$

$$\underline{\underline{S(x) = \sin(x)}}$$

Find the steady-state oscillations of $y'' + cy' + y = r(t)$ with $c > 0$ and $r(t)$ as given. Note that the spring constant is $k = 1$. Show the details. In Probs. 14–16 sketch $r(t)$.

13. $r(t) = \sum_{n=1}^{N} (a_n \cos nt + b_n \sin nt)$

$$y'' + cy' + y = r(t)$$

$$y'' + cy' + y = \sum_{n=1}^{N} \left(a_n \cos(nt) + b_n \sin(nt)\right)$$

$$y'' + cy' + y = a_n \cos(nt) + b_n \sin(nt)$$

Gætter her på en løsning

$$y_n = A_n \cdot \cos(nt) + B_n \cdot \sin(nt)$$

$$y'_n = -cn A_n \cdot \sin(nt) + cn B_n \cdot \cos(nt)$$

$$y''_n = -n^2 \cdot A_n \cdot \cos(nt) - n^2 B_n \cdot \sin(nt)$$

$$\cos(nt) \cdot \left(-n^2 A_n + cn B_n + A_n\right) + \sin(nt) \cdot \left(-n^2 B_n - ncA_n + B_n\right) = a_n \cdot \cos(nt) + b_n \cdot \sin(nt)$$

$$a_n = -n^2 A_n + ncB_n + A_n$$

$$b_n = -n^2 B_n - ncA_n + B_n$$

$$a_n = A_n \cdot (1-n^2) + B_n \cdot n \cdot c$$

$$b_n = A_n \cdot (-cn) + B_n \cdot (1-n^2)$$

$$\begin{pmatrix} 1-n^2 & nc \\ -nc & 1-n^2 \end{pmatrix} \begin{pmatrix} A_n \\ B_n \end{pmatrix} = \begin{pmatrix} a_n \\ b_n \end{pmatrix}$$

$$A_n = \frac{\begin{vmatrix} a_n & nc \\ b_n & 1-n^2 \end{vmatrix}}{\begin{vmatrix} 1-n^2 & nc \\ -nc & 1-n^2 \end{vmatrix}} = \frac{a_n \cdot (1-n^2) - b_n \cdot nc}{(1-n^2)\cdot(1-n^2) + n^2c^2}$$

$$= \frac{a_n \cdot (1-n^2) - b_n \cdot nc}{1 + n^4 - 2n^2 + n^2c^2}$$

$$= \frac{a_n \cdot (1-n^2) - b_n \cdot nc}{1 + n^4 + n^2 \cdot (c^2-2)}$$

$$B_n = \frac{\begin{vmatrix} 1-n^2 & a_n \\ -n & b_n \end{vmatrix}}{\begin{vmatrix} 1-n^2 & n \\ -n & 1-n^2 \end{vmatrix}}$$

$$= \frac{b_n \cdot (1 - n^2) + a_n n}{1 + n^4 + n^2 \cdot (c^2 - 2)}$$

$$y_h = A_n \cdot \cos(nt) + B_n \cdot \sin(nt)$$

$$y_h = \frac{a_n \cdot (1 - n^2) - b_n \cdot n}{1 + n^4 + n^2 \cdot (c^2 - 2)} \cdot \cos(nt) + \frac{b_n \cdot (1 - n^2) + a_n n}{1 + n^4 + n^2 \cdot (c^2 - 2)} \cdot \sin(nt)$$