

Superstar Teams

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Society for Economic Dynamics Conference, Barcelona

June 27-29 2024

Motivation: firms form & organize teams of heterogeneous workers

- **Most production processes are too complex for 1 person to perform *all* tasks well**
 - → individuals have **heterogeneous, task-specific skills**
 - → firms facilitate the division of labor among >1 workers (**“team production”**)

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- **How is production with het. workers organized & what are the macro implications?**
 - classic question [*e.g., Kremer, 1993; Garicano, 2000*] but literature is theoretical & qualitative

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- **This paper:**
 - 1 **theory** that is tractable
 - 2 **measurement** with micro data
 - 3 **quantify macro implications** for agg. productivity & labor market inequality

Intuition: skill specificity → complementarities → sorting

- **Environment:** the firm as an organized collection of heterogeneous workers
 - 1 **task-based production**
 - 2 **multi-dim. skill heterogeneity**
 - talent ~ absolute advantage
 - skill specificity ~ *dispersion* in individual task-specific skills
 - 3 **teams**
 - 4 **search**

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- **Mechanism:** when skills are task-specific and tasks optimally assigned...
 - ⇒ ...team production is advantageous
 - ⇒ ...production features **coworker talent complementarities** → incentives for **talent sorting** → firm-level inequality in productivity & wages

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 - ⇒ ...production features **coworker talent complementarities** → incentives for **talent sorting** → firm-level inequality in productivity & wages
- **Application:** skill specificity ↑ can explain the **“firming up of inequality”**

[cf. Card et al., 2013; Bloom et al., 2019; ...]

Today: theory & one application

► Literature

- ❶ **Develop tractable theory of the firm centered on team production & formation**
 - **microfound task-based production fn. with endogenous coworker complementarities**
 - tractable enough to endogenize team formation via search
- ❷ **Confront theory with data**
 - identification with micro panel data on wages+matches → estimate & validate model
- ❸ **Quantitative application: structural explanation for “firming up inequality”**
 - ↑ skill specificity explains $\approx 25\%$ of ↑ between-firm wage inequality share in DE since '85
 - application 2: search frictions lower agg. productivity due to costly coworker mismatch

Environment: high-level overview

- Continuums of workers & firms, infinitely-lived & risk-neutral
- **Ex-ante identical firms**
 - hire $n \in \{0, 1, 2\}$ workers through sequential random search [cf. HLMP, 2024]
 - assign n workers to continuum of tasks $\mathcal{T} = [0, 1]$ that get combined into final good
[cf. Acemoglu-Restrepo, 2018]
- **Heterogeneous workers:** worker i has task-specific skills $\{z_i(\tau)\}_{\tau \in \mathcal{T}}$

⇒ **Analysis:**

- 1 **microfound tractable firm-level production function** ← **focus today**
- 2 integrate into search environment & analyze who is matched with whom

Environment: firm-level organization of production

- **Final good:** combine **unit continuum of tasks** \mathcal{T} into output

$$\ln Y = \int_{\mathcal{T}} \ln q(\tau) d\tau \quad (1)$$

- **Task-level aggregation** for task τ across n workers

$$q(\tau) = \sum_{i=1}^n y_i(\tau) \quad (2)$$

- **Worker-level task production:** i produces τ with skill $z_i(\tau)$, given 1 time unit

$$y_i(\tau) = z_i(\tau) l_i(\tau) \quad (3)$$

$$1 = \int_{\mathcal{T}} l_i(\tau) d\tau \quad (4)$$

Firm's optimization problem

- **Firm solves mini-planner problem:** $\max_{\mathbf{q}, \{\mathbf{y}_i\}, \{\mathbf{l}_i\}} Y$ s.t. (1)-(4)
- **Preview:** derive & characterize *reduced-form* team production function f

$$f(\mathbf{z}_1, \dots, \mathbf{z}_n) = \max Y$$
$$\text{s.t. (1)-(4)}$$

Firm's optimization problem

- **Firm solves mini-planner problem:** $\max Y$ s.t. (1)-(4)

$$\begin{aligned}
 \mathcal{L}(\cdot) = & Y + \lambda \left[\underbrace{\left(\int_{\mathcal{T}} \ln q(\tau) d\tau \right)}_{\text{tasks} \rightarrow \text{output}} - \ln Y \right] + \int_{\mathcal{T}} \lambda(\tau) \underbrace{\left(\sum_{i=1}^n y_i(\tau) - q(\tau) \right)}_{\text{task aggregation}} d\tau \\
 & + \sum_{i=1}^n \lambda_i^L \underbrace{\left(\int_{\mathcal{T}} \frac{y_i(\tau)}{\mathbf{z}_i(\tau)} d\tau - 1 \right)}_{\text{time constraint + task production}} + \text{non-negativity constraints}
 \end{aligned}$$

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- FOCs imply

$$\lambda(\tau) = \min_i \left\{ \frac{\lambda_i^L}{z_i(\tau)} \right\}$$

shadow cost of τ \leftarrow $\lambda(\tau)$

λ_i^L \rightarrow opportunity cost of i 's time

$z_i(\tau)$ \rightarrow i 's skill for τ

Firm's optimization problem

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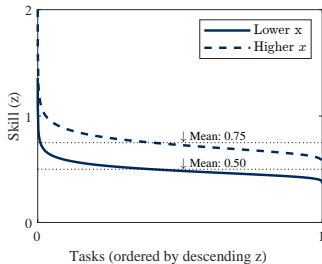
- **FOCs** imply **task assignment by comparative advantage**, complete division of labor

$$\lambda(\tau) = \min_i \left\{ \frac{\lambda_i^L}{z_i(\tau)} \right\}$$

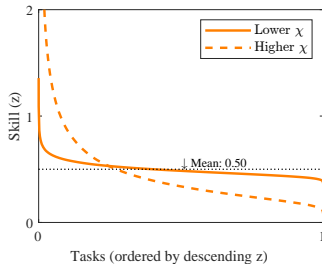
Parametrized distribution of task-specific skills: “Fréchet-ing things up”

$$\Pr [z_1(\tau) \leq z_1, z_2(\tau) \leq z_2] = \exp \left[- \left(\sum_{i=1}^{n=2} \left(\left(\frac{z_i}{\iota X_i} \right)^{-\frac{1}{\chi}} \right)^{\frac{1}{\xi}} \right)^{\xi} \right]$$

(a) x_i : talent (scale)



(b) χ : skill specificity (1/shape)



Micro-founded production function

► Lemma

Proposition: Reduced-form production function

Talent types \mathbf{x} and coworker distance ξ are sufficient statistics for team output Y given parameter χ :

$$Y = f(x_1, \dots, x_n, \xi; \chi)$$

- Fréchet max-stability property allows closed-form characterization of key objects, e.g. distribution of $\lambda(\tau) \rightarrow$ integrate over *continuum* of tasks
- **Benchmark** without division of labor: $Y = n \times \left(\frac{1}{n} \sum_{i=1}^n x_i\right)$

Gains from team production are increasing in skill specificity

Proposition: Reduced-form production function

$$f(\mathbf{x}, \xi; \chi) = \underbrace{n^{1+\chi\xi}}_{\text{efficiency gains}} \times \left(\frac{1}{n} \sum_{i=1}^n (x_i)^{\frac{1}{1+\chi\xi}} \right)^{1+\chi\xi}$$

- 1 Value of **team production** increasing in skill specificity (χ)

► Intuition: task assignment

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► Intuition: task assignment

- **realized team advantage greater when coworkers are good at different tasks** (ξ)

Skill specificity implies that productivity is lowered by talent dispersion

Proposition: Reduced-form production function

$$f(\mathbf{x}, \xi; \chi) = \underbrace{n^{1+\chi\xi}}_{\text{efficiency gains}} \times \underbrace{\left(\frac{1}{n} \sum_{i=1}^n (x_i)^{\frac{1}{1+\chi\xi}} \right)^{1+\chi\xi}}_{\text{talent complementarity}},$$

1 Value of team production increasing in skill specificity (χ)

► Intuition

- *realized* team advantage greater when coworkers are good at different tasks (ξ)

2 **Coworker talent complementarities** increasing in skill specificity (χ)

► Intuition

- $\frac{\partial(\partial f(\cdot)/\partial x_i \partial x_{-i})}{\partial \chi} > 0$

Surplus max. determines which teams are formed

- Embed $f(\cdot)$ into search-frictional matching model *[similar to Herkenhoff-Lise-Menzio-Phillips (2024), but with multi-dim. skills and w/o OJS]*
- Joint value of firm with worker x , $\Omega_1(x)$, satisfies:

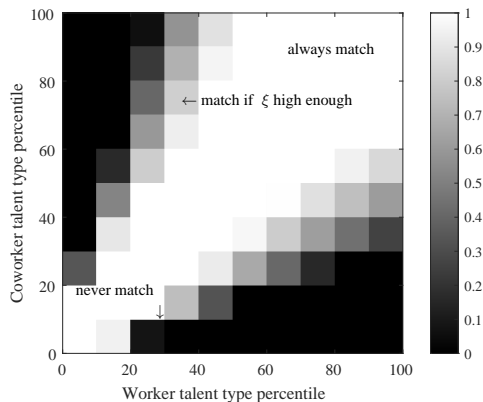
$$\begin{aligned} \rho\Omega_1(x) = & f(x) + \delta[-\Omega_1(x) + V_u(x) + V_{f.o}] \\ & + \lambda_{v.u} \int \int \frac{d_u(\tilde{x}')}{u} \max \left\{ \underbrace{-\Omega_1(x) + V_{e.2}(x|\tilde{x}', \tilde{\xi}) + V_{f.2}(x, \tilde{x}', \tilde{\xi})}_{(1-\omega)S(\tilde{x}'|x, \tilde{\xi})}, 0 \right\} dH(\tilde{\xi}) d\tilde{x}' \end{aligned}$$

- $V_u(x)$: value for unemp. worker; $V_{f.o}$: value for vacant firm; $d_u(x)$: density of unemployed workers of type x ; $u = \int d_u(x)dx$; ω : worker bargaining wgt; δ : sep. rate; $\lambda_{v.u}$: rate of vacancy meeting unmatched worker
- Surplus $S(x|x', \xi)$ reflects production complementarities

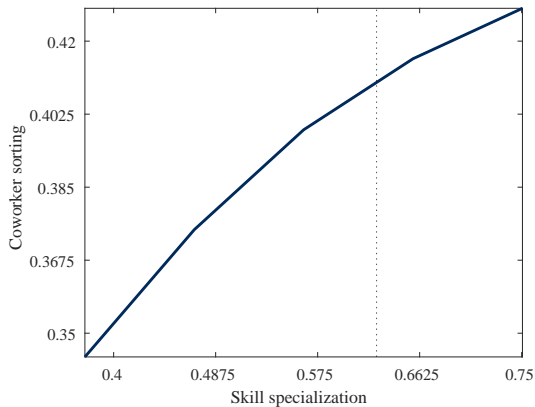
$$S(x|x', \xi)(\rho + 2\delta) = f(x, x', \xi) - \rho(V_u(x) + V_u(x') + V_{f.o}) + \delta S(x) - (\rho + \delta)S(x')$$

Equilibrium properties: conditional matching probabilities for given χ

- Team composition determined by tradeoff between **match quality vs. search costs**
 \Rightarrow matching probabilities $h(x'|x) = P \{S(x'|x, \xi) > 0\}$



Comparative 'statics': more positive assortative matching as $\chi \uparrow$



Roadmap & key takeaways

Theory

- ① **Skill specificity** *endogenously* generates **coworker complementarities**
- ② **Talent complementarities** lead to **positive assortative matching**

Next: confront theory with data

Taking the model to the data: overview

- **Data:** SIEED matched-employer employee panel for W Germany
 - production unit: establishment
- **Approach:**
 - worker i 's talent type $\hat{x}_i \approx$ rank in wage FE dist. ▶ Details
 - “representative coworker type” \hat{x}_{-it} : avg. \hat{x} of workers in same estab.-yr. ▶ Details
 - some param's from literature (e.g. discount rate ρ , bargaining weight ω) or estimated offline (e.g. job separation hazard δ)
 - indirect inference: meeting rate, unemp. flow benefit, production
 - targets: total wage variance, avg. wage level, replacement rate, job finding rate
- **Novel identification strategy:** χ can be recovered from $\frac{\partial^2 \bar{w}(x|x')}{\partial x \partial x'}$ ▶ Details
- ✓ Match untargeted moments like talent sorting ▶ Details
- ✓ Extensive validation of core model mechanism ▶ Details

Roadmap & key takeaways

Theory

- ① Skill specificity endogenously generates coworker complementarities
- ② Talent complementarities lead to positive assortative matching

Model Meets Data

- ③ The model, estimated with DE micro data, endogenously generates large ex-post firm differences

Next: application(s)

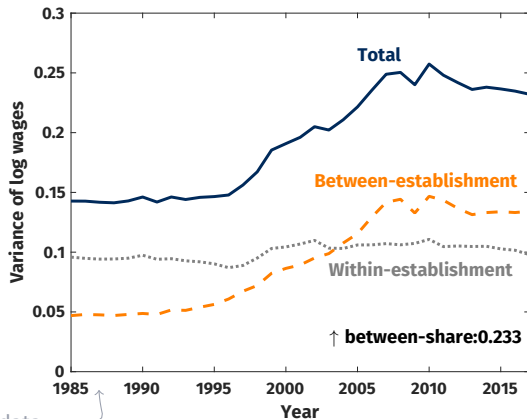
- **Today:** structural explanation for the “firming up of inequality”

Wage inequality has risen – and firms appear to play a key role

[Details](#)

“the variance of firm [wages] explains an increasing share of total inequality in a range of countries”

[Song-Price-Guvenen-Bloom-von Wachter, 2019]

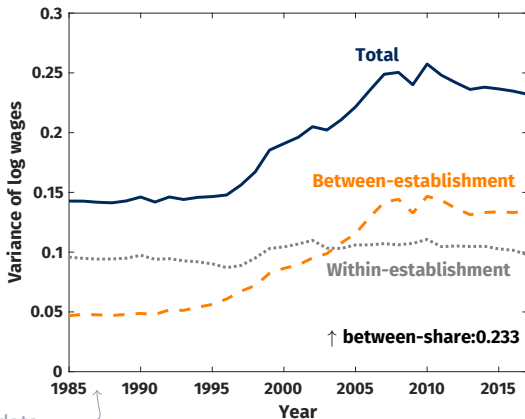


German matched employer-employee data →

Applied question

[Details](#)

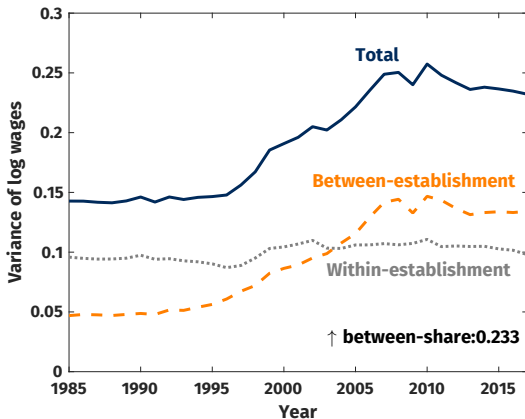
Applied question: what is/are the causal driver(s)? implications?



German matched employer-employee data

Preview of argument

- 1 The set of tasks any one worker can perform well has narrowed: skill specificity \uparrow
- 2 Coworker complementarities \uparrow
- 3 Individuals of similar talent increasingly work together
- 4 This generates greater between-firm wage dispersion

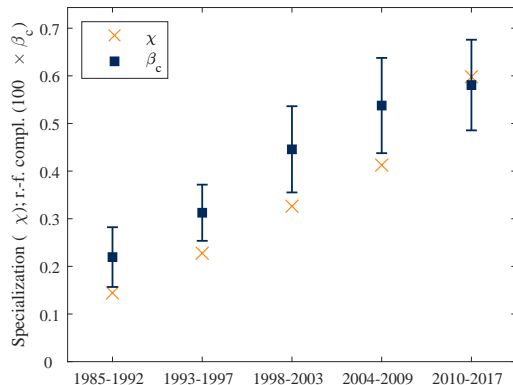


Estimate model for several periods: skill specialization \uparrow

► Schooling

► Peer effect trends

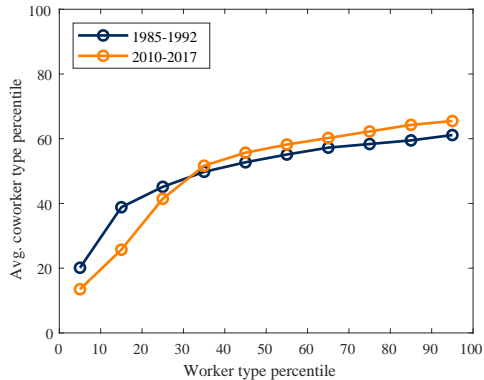
- Estimate: **skill specificity** $\chi \uparrow$
- Consistent with independence evidence
 - Grigsby (2024) estimates
 - evidence on Δ task composition: decline in routine (“low- χ ”) tasks
 - rise of team production in science due to the “burden of knowledge” [Jones, 2009] & growing importance of social skills [Deming, 2017]



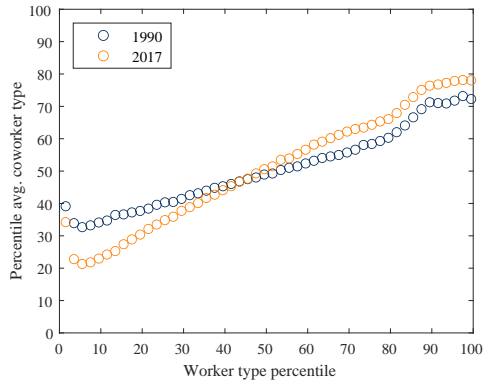
Talent sorting has intensified: theory & data

[Details](#)

(a) Theory

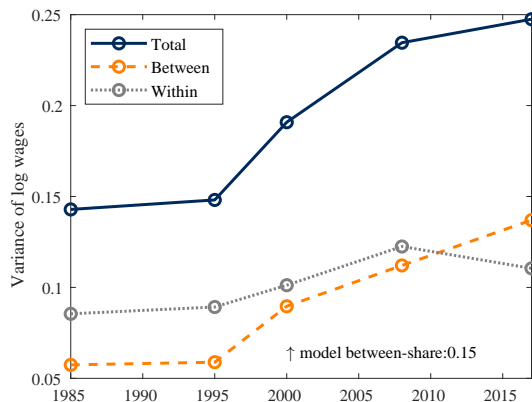


(b) Data



Model matches *changes* in firm-level wage distribution

- Model replicates **untargeted rise of between-share in data**
 - $\sim 2/3$ of \uparrow between-share in data, ('85-'92) \rightarrow ('10-'17)



Skill specificity $\chi \uparrow$ explains $\approx 25\text{-}40\%$ of observed between-share \uparrow

- **Q:** How much of \uparrow between-firm share of wage var. is due to $\chi \uparrow$?
- **Counterfactual:** between-firm share in 2010s absent $\chi \uparrow$ since '85-'92
- **A:** $\chi \uparrow$ **accounts for 58%** of model-predicted $\Delta \leftrightarrow \approx 38\%$ of empirical Δ
- **Robustness** exercises: 25-40%

	Δ model	Implied % Δ model due to Δ parameter
Model	0.15	
Cf.: χ '85-'92	0.065	58

Roadmap & key takeaways

Theory

- ① Skill specificity *endogenously* generates coworker complementarities
- ② Talent complementarities lead to positive assortative matching

Model Meets Data

- ③ Estimated model endogenously generates realistic ex-post firm heterogeneity

Applications

- ④ **Increased skill specificity – leading to stronger complementarities and, hence, sorting – explains a substantial share of \uparrow between-firm wage inequality share**
- ⑤ Enhanced sorting crucial to realize productivity gains from \uparrow skill specialization

Conclusion

Conclusion: firms form & organize teams – matters for macro

- **Main idea:** if workers have specialized skills, firms assemble teams of complementary coworkers, with macro implications for productivity & inequality
- **Today:**
 - 1 **theory:** task-based firm-level production fn. with endog. skill complementarities
⇒ skill specificity + teams → production complementarities
 - 2 **measurement** combining reduced-form micro evidence with model structure
⇒ endogenously generated between-firm differences in productivity & pay
 - 3 **quantitative** application to explain **macro implications**
⇒ **rising skill specificity** contributed to the “**firming up**” of inequality

Thank You!

Extra Slides

Relation & contributions to literature

- **Firm organization: task-based microfoundation for complementarities**
Firms & teams: Lucas, 1978; Becker & Murphy, 1992; **Kremer, 1993**; Kremer & Maskin, 1996; **Garicano, 2000**; **Garicano & Rossi-Hansberg, 2006**; Porzio, 2017; **Jarosch et al., 2021**; Kuhn et al., 2023
Task assignment: Costinot & Vogel, 2010; **Acemoglu & Restrepo, 2018**; Ocampo, 2021
- **Multi-dim. skill heterogeneity: parsimonious parametrization for teams model**
 Gathman-Schoenberg, 2010; Lindenlaub, 2017; Guvenen et al., 2020; Baley et al., 2022; Grigsby, 2024
- **Frictional labor market sorting: endogenize & measure complementarities**
 Shimer & Smith, 2000; Cahuc et al., 2006; Eeckhout & Kircher, 2011/2018; Hagedorn et al., 2017; de Melo, 2018; Lindenlaub & Postel-Vinay, 2023; **Herkenhoff et al., 2024**
- **Wage inequality: structural model of \uparrow firm-level inequality due to \uparrow specialization**
Technology: Katz & Murphy, 1992; Krusell et al., 2000; Autor et al., 2003; Acemoglu & Restrepo, 2018
Firms: **Card et al., 2013**; Barth et al., 2016; Alvarez et al., 2018; **Bloom et al., 2019**; Sorkin & Wallskog, 2023

Endogenous team composition: frictional matching

[► Details](#)

- Integrate $f(\cdot)$ with search-frictional dynamic matching into teams
- **Main features** of search block: *[similar to Herkenhoff-Lise-Menzio-Phillips, 2024]*
 - random search with multi-worker firms
 - employment states: unemp., employed alone, employed with one coworker
 - Nash wage bargaining with continuous renegotiation
- But introduce **multi-dim. skills** in tractable fashion:
 - microfoundation: if $\chi > 0$, both talent composition \mathbf{x} and differentiation ξ matter for Y
 - ξ is a **match-specific shock** observed by firms + workers before match decision
- **Stationary equilibrium**

[► Details](#)

Measurement: a useful identification result

[▶ Main](#)
[▶ Identification validation](#)
[▶ Monte Carlo](#)

- **Challenge:** skill specialization χ not directly observable
 - literature doesn't offer cardinal measures of specificity [*exception: Grigsby, 2024*]
 - could infer χ from sorting, but v indirect & liable to misattribution
- **Theory guides measurement:** Proposition 1 ties χ to $\frac{\partial^2 f(\cdot)}{\partial x \partial x'}$, which given prior measures of x and x' & accounting for selection on ξ , we can recover from $w(x|x')$

$$\frac{\partial^2 f(x, x', \xi)}{\partial x \partial x'} \propto \frac{\partial^2 w(x|x', \xi)}{\partial x \partial x'}.$$

$$\Rightarrow \frac{\partial^2 \bar{f}(x, x')}{\partial x \partial x'} \sim \frac{\partial^2 \bar{w}(x|x')}{\partial x \partial x'}$$

can measure this

[▶ Proof sketch](#)

Reduced-form regression to identify χ (2010-2017)

► Robustness

► Main

- Approximate $\frac{\partial^2 \bar{w}(x|x')}{\partial x \partial x'}$ using **regression with interaction term**

$$\frac{w_{it}}{\bar{w}_t} = \beta_0 + \beta_1 \hat{x}_i + \beta_{11} \hat{x}_i^2 + \beta_2 \hat{x}_{-it} + \beta_{22} \hat{x}_{-it}^2 + \beta_c (\hat{x}_i \times \hat{x}_{-it}) + \psi_{j(it)} + \nu_{o(i)t} + \xi_{s(i)t} + \epsilon_{it}$$

	$\hat{\beta}_c$	Non-parametric FD method
Coworker complementarity	0.0058***	0.0075
Obs. (1000s)	4,410	4,410

Notes. Regressions include FEs for employer; occupation-year; industry-year. Employer-clustered standard errors in parentheses. Observations weighted by the inverse employment share of the respective type and (rounded) coworker type cell. FD: finite differences.

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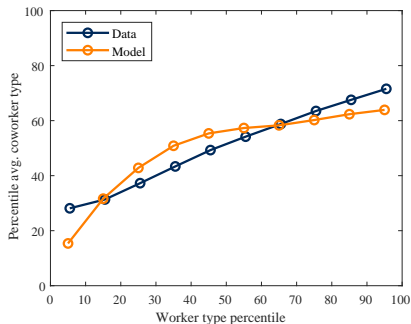
Notes. Regressions include FEs for employer; occupation-year; industry-year. Employer-clustered standard errors in parentheses. Observations weighted by the inverse employment share of the respective type and (rounded) coworker type cell. FD: finite differences.

- Estimation of structural model:** replicate semi-structural regression with model-generated data, infer χ from matching empirical $\hat{\beta}_c$

Quantitative properties of estimated model: untargeted moments

► B-S adj. method

- ✓ Match talent sorting patterns
 - $\rho_{xx} = 0.43$ (vs. 0.62 in data)
- ✓ Match between-firm wage inequality
 - between-share 0.48 (vs. 0.57 in data)



⇒ **Model endogenously generates ex-post firm differences**

Validation of core model mechanisms

[▶ Main](#)

- **Team-production functions in science** *[cf. Ahmadpoor-Jones, 2019]* [▶ Details](#)
 - ✓ talent complementarities stronger *precisely* when teamwork more valuable
- **Cross-sectional variation across occupations/industries** [▶ Details](#)
 - ✓ task-based proxy for $\chi \uparrow \rightarrow$ estimated talent complementarity \uparrow
 - ✓ estimated talent complementarity $\uparrow \rightarrow$ coworker talent sorting \uparrow
- **Direction of EE moves:** non-wage implications of complementarities [▶ Details](#)
 - ✓ Δ coworker talent positively correlated with own talent
- **Heterogeneous effects of coworker deaths** *[cf. Jaeger-Heining, 2022]* [▶ Details](#)
 - ✓ wage gains from coworker death *if* coworker specialized in different tasks ($\xi \uparrow$)

What's the value-added of the micro-founded production function?

- **Concern:** the microfoundation isn't used for measurement — i.e. measure $z_i(\tau)$'s directly and then 'aggregate up' to recover complementarities – so what's the point?
- **Value-added #1:** very tractable formalization of team production with multi-dimensional skills
 - it's not obvious *ex ante* that team production with multi-dim. skills can be represented in this way, nor how this can be incorporated into a search framework
- **Value-added #2:** relative to a reduced-form CES function with talent x (1-dimensional) [*e.g. Herkenhoff et al., 2024*]
 - ① offers explanation for why talent complementarities may vary & change over time – in
 - ② the two models are not observationally equivalent
 - benefit from team production is also increasing with χ , hence this term co-moves with talent complementarities (and it affects sorting differently)
 - selection effects due to ξ : when we observe low and high x workers together, they are likely to be a good match in terms of their task-specific skills [*cf. Borovickova-Shimer, 2024*]

Lemma

Lemma: Lemma

Implied task share and shadow-cost index equal

$$\pi_i = \frac{(x_i/\lambda_i^L)^{\frac{1}{\chi\xi}}}{\sum_{k=1}^n (x_k/\lambda_k^L)^{\frac{1}{\chi\xi}}} \quad x; \lambda = \left(\sum_{i=1}^n \left(\frac{x_i}{\lambda_i^L} \right)^{\frac{1}{\chi\xi}} \right)^{-\chi\xi}$$

Intuition: features of optimal organization

- **What is the intuition for these properties?**
- Solution of firm's mini-planner problem implies:
 - ① Complete division of labor, with tasks assigned by comparative advantage
 - i 's task set $\mathcal{T}_i = \left\{ \tau \in \mathcal{T} : \frac{z_i(\tau)}{\lambda_i^L} \geq \max_{k \neq i} \frac{z_k(\tau)}{\lambda_k^L} \right\}$
 - classic source of efficiency gains
 - ② i 's share of tasks \uparrow in i 's talent, \downarrow in coworkers' talent
 - i 's task share $\pi_i = (x_i^{\frac{1}{1+\chi\xi}}) (\sum_{k=1}^n (x_k)^{\frac{1}{1+\chi\xi}})^{-1}$

Intuition: comparative statics for task shares

- Suppose that $x_i > x_j$. Then
 - 1 i performs a strictly larger share of tasks than j for $\chi < \infty$



Intuition: comparative statics for task shares

- Suppose that $x_i > x_j$. Then
 - ① i performs a strictly larger share of tasks than j for $\chi < \infty$
 - ② the difference in task shares is decreasing in χ



\Rightarrow **Greater skill specialization implies a larger share of tasks is performed by relatively less talented team members** – more talented coworkers can't easily compensate

Surplus sharing protocol

- The wage of a worker of type x employed alone satisfies

$$(1 - \omega)(V_{e.1}(x) - V_u(x)) = \omega(V_{f.1}(x) - V_{f.o}), \quad (5)$$

- The wage $w(x|x', \xi)$ of a type- x worker with a coworker of type x' given shock ξ satisfies

$$(1 - \omega)(V_{e.2}(x|x', \xi) - V_u(x)) = \omega(V_{e.2}(x'|x, \xi) + V_{f.2}(x, x', \xi) - V_{e.1}(x') - V_{f.1}(x')). \quad (6)$$

HJB: unmatched

[▶ Main](#)

- Unmatched firm:

$$\rho V_{f.o} = (1 - \omega) \lambda_{v.u} \int \frac{d_u(x)}{u} S(x)^+ dx, \quad (7)$$

- Unmatched worker:

$$\rho V_u(x) = b(x) + \lambda_u \omega \left[\frac{d_{f.o}}{v} S(x)^+ + \int \int \frac{d_{m.1}(\tilde{x}')}{v} S(x|\tilde{x}', \tilde{\xi})^+ dH(\tilde{\xi}) d\tilde{x}' \right] \quad (8)$$

Joint values

- Joint value of firm with x and x', ξ

$$\rho\Omega_2(x, x', \xi) = f_2(x, x', \xi) - \delta S(x|x', \xi) - \delta S(x'|x, \xi) \quad (9)$$

- Joint value of firm with x

$$\begin{aligned} \rho\Omega_1(x) = & f_1(x) + \delta [-\Omega_1(x) + V_u(x) + V_{f.o}] \\ & + \lambda_{v.u} \int \int \frac{d_u(\tilde{x}')}{u} \underbrace{(-\Omega_1(x) + V_{e.2}(x|\tilde{x}', \tilde{\xi}) + V_{f.2}(x, \tilde{x}', \tilde{\xi}))}_{(1-\omega)S(\tilde{x}'|x, \tilde{\xi})}^+ dH(\tilde{\xi}) d\tilde{x}'. \end{aligned} \quad (10)$$

HJB: surpluses

- Surplus of coalition of firm with worker x

$$(\rho + \delta)S(x) = f_1(x) - \rho(V_u(x) + V_{f.o}) + \lambda_{v.u}(1 - \omega) \int \frac{d_u(\tilde{x}')}{u} S(\tilde{x}'|x, \tilde{\xi})^+ dH(\tilde{\xi})\tilde{x}'. \quad (11)$$

- Surplus from adding x to x' with xi

$$S(x|x', \xi)(\rho + 2\delta) = f_2(x, x', \xi) - \rho(V_u(x) + V_u(x') + V_{f.o}) + \delta S(x) - (\rho + \delta)S(x'). \quad (12)$$

KFE: unemployed

$$\delta \left(d_{m.1}(x) + \int d_{m.2}(x, \tilde{x}') d\tilde{x}' \right) = d_u(x) \lambda_u \left(\int \frac{d_{f.o}}{v} h(x, \tilde{y}) + \int \frac{d_{m.2}(\tilde{x}')}{v} h(x|\tilde{x}') d\tilde{x}' \right). \quad (13)$$

KFE: one-worker matches

$$d_{m.1}(x) \left(\delta + \lambda_{v.u} \int \frac{d_u(\tilde{x}')}{u} h(\tilde{x}'|x) d\tilde{x}' \right) = d_u(x) \lambda_u \frac{d_{f.o}}{v} h(x) + \delta \int d_{m.2}(x, \tilde{x}') d\tilde{x}'. \quad (14)$$

KFE: two-worker matches

$$2\delta d_{m.2}(x, x') = d_u(x)\lambda_u \frac{d_{m.1}(x')}{v} h(x|x') + d_u(x')\lambda_u \frac{d_{m.1}(x)}{v} h(x'|x). \quad (15)$$

Matching – stationary equilibrium

[▶ Main](#)

- HJ-Bellman equations → **values & matching policies**
- Flows between/**distribution** over types \times employment states

[▶ HJBs](#)[▶ KFEs](#)

Definition: Stationary equilibrium

A stationary eqm. consists of a production function, value functions & a distribution of agents, s.t.

- 1 the production function is consistent with the optimal assignment of tasks;
- 2 the value functions satisfy the HJB equations given the distribution;
- 3 the distribution is stationary given the policy fn's implied by the value fn's.

Mapping theory to data: worker & coworker types

[▶ Main](#)

- **Theory:** wage monotonically \uparrow in x , so can measure using panel dimension
- **Implementation:** standard methods
 - pragmatic approach: AKM fixed effect (FE) wage regressions [Abowd et al., 1999] with pre-est. k-means clustering to address limited mobility bias [Bonhomme et al., 2019]
 - theory-consistent: non-param. ranking algo [Hagedorn et al., 2017] \rightarrow similar ranking

\Rightarrow **Worker i 's talent type \hat{x}_i : decile rank of i 's FE within 2d-occupation**

- **“Representative coworker type” \hat{x}_{-it} :** avg. \hat{x} of workers in same estab.-yr.

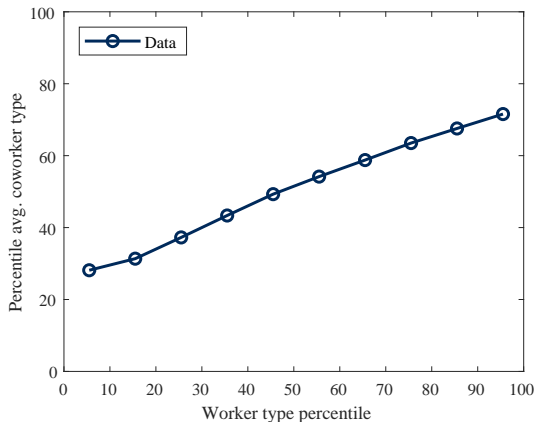
[▶ Discussion](#)

Mapping model to data: coworker types

- Defining $S_{-it} = \{k : j(kt) = j(it), k \neq i\}$ as the set of i 's coworkers in year t , compute the average type of i 's coworkers in year t as $\hat{x}_{-it} = \frac{1}{|S_{-it}|} \sum_{k \in S_{-it}} \hat{x}_k$.
- **Coworker group:**
 - alternative: same establishment-occupation-year cell
 - but CC arise precisely when workers are *differentiated* in their task-specific productivities
- **Averaging step:**
 - equally-weighted averaging ignores non-linearity in coworker aggregation
 - paper: show using non-linear averaging method that baseline results in bias, but it's minor in magnitude
- **Firm size variation:** averaging ensures that a single move will induce a smaller change in the *average* coworker quality in a large team than in a small one

Mapping theory to data: talent sorting in the data

- Measures of \hat{x}_i and \hat{x}_{-it} sufficient to measure empirical talent sorting



Measurement: a useful identification result

[▶ Main](#)
[▶ Non-separable case: scatterplot](#)

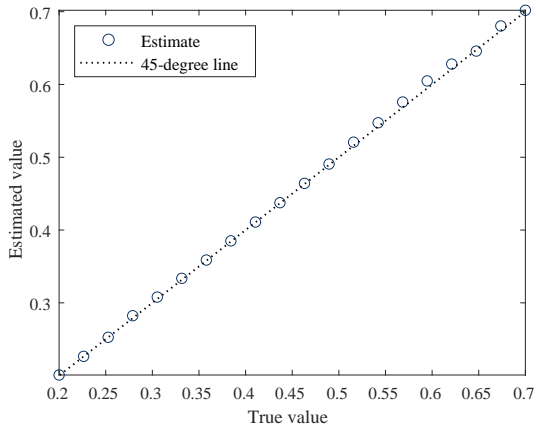
- **Q:** How to quantify $\frac{\partial^2 f(x, x')}{\partial x \partial x'}$?
- **Proposition:** production complementarities are proportional to wage compl.
- **Proof sketch:** wage level for worker x with coworker x'

$$w(x|x', \xi) = \omega f(x, x', \xi) + g(x) - h(x')$$

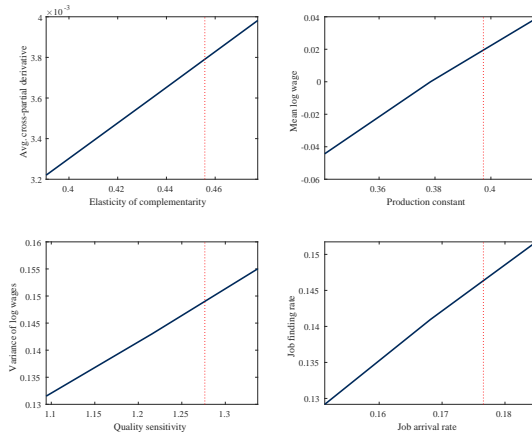
where $g : [0, 1] \rightarrow \mathbb{R}$ and $h : [0, 1] \rightarrow \mathbb{R}$ are strictly increasing

\Rightarrow *outside options are separable: affect level of wage but not the cross-partial*

Monte Carlo study

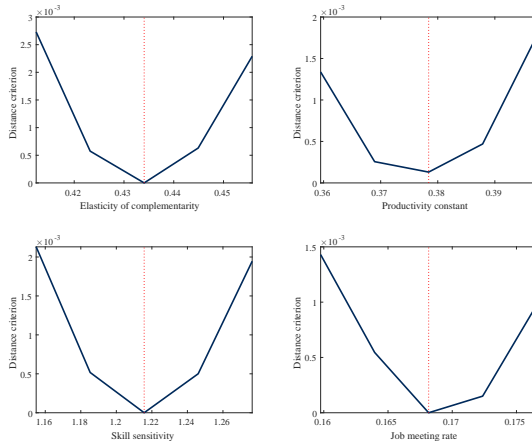
[▶ Main](#)

Identification validation exercise 1

[▶ Main](#)

Notes. This figure plots the targeted moment against the relevant parameter, holding constant all other parameters.

Identification validation exercise 2



Notes. This figure plots the distance function $\mathcal{G}(\psi_i, \psi_i^*)$ when varying a given parameter ψ_i around the estimated value ψ_i^* . The remaining parameters are allowed to adjust to minimize \mathcal{G} .

Robustness: reduced-form coworker complementarity

[▶ Main](#)

- Types from non-parametric ranking algorithm instead of AKM-based
- Schooling as a non-wage measure of types
- Lagged types
- Small teams
- Movers
- Non-parametric, finite-differences approximation
- Excluding managers
- Log specification

[▶ Jump](#)[▶ Jump](#)[▶ Jump](#)[▶ Jump](#)[▶ Jump](#)[▶ Jump](#)[▶ Jump](#)[▶ Jump](#)

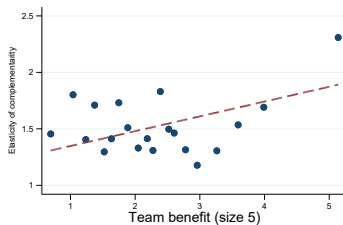
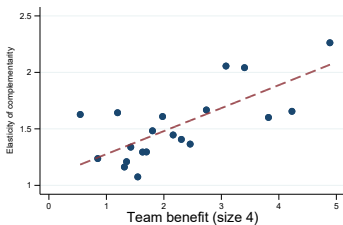
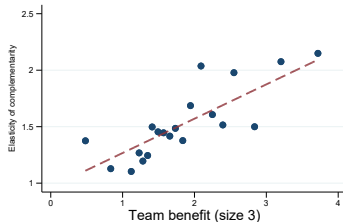
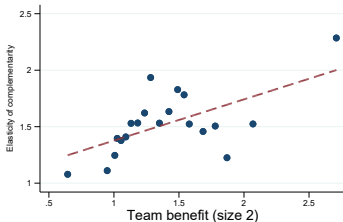
Estimation results (2010-2017)

Parameter	Description	Target	Value	m	\hat{m}
χ	Specialization	$\hat{\beta}_c$	0.67	0.0058	0.0058
a_0	Production, constant	Avg. wage (norm.)	0.29	1	1
a_1	Production, scale	Var. log wage	1.71	0.241	0.241
b_1	Replacement rate, scale	Replacement rate	0.60	0.63	0.63
λ_u	Meeting hazard	Job finding rate	0.22	0.162	0.162
δ	Separation hazard	Job loss rate	0.008	0.008	0.008
ω	Worker bargaining weight	External	0.50		
\bar{n}	Effective team size	External	25		

Validation: Production functions estimated by Ahmadpoor-Jones (2019)

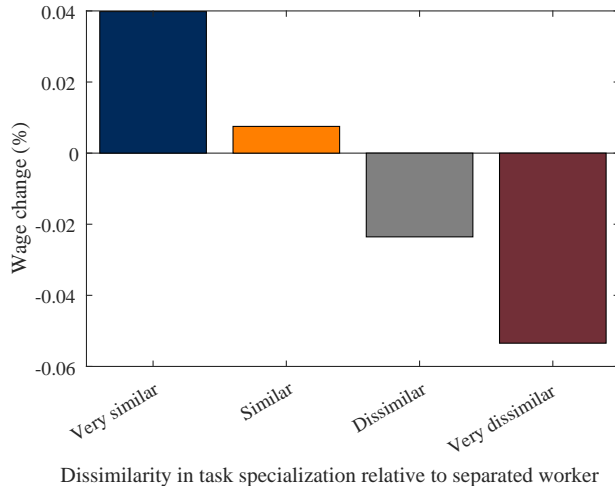
[▶ Main](#)

Complementarity vs. team benefit (Patents)



Notes. Source data from Ahmadpoor and Jones (2019, PNAS). Own calculations. Binscatter plot for subsample with complementarity ≤ 5 .

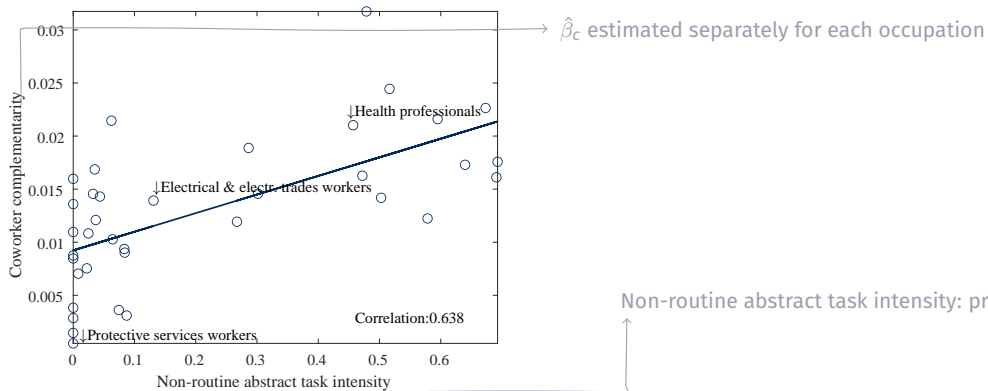
Validation: Structural interpretation of Jaeger-Heining (2022)

[▶ Main](#)

X-sectional validation (occ's): tasks \Rightarrow complementarity

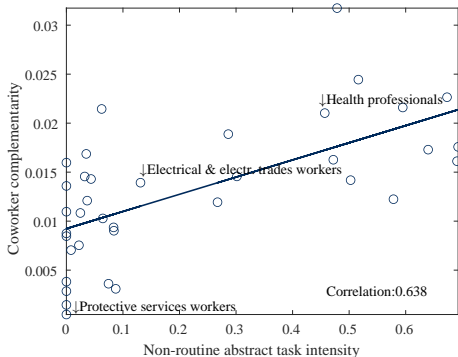
[Main](#)

- **↑ Non-routine abstract task intensity**
 \Rightarrow **↑ coworker talent complementarity**

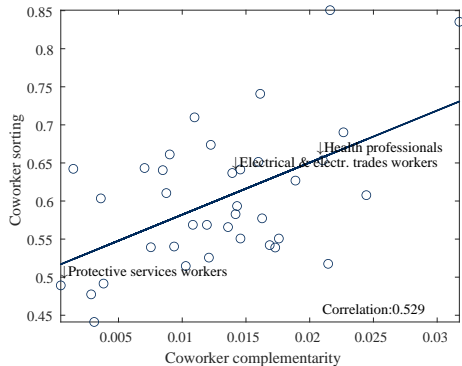


X-sectional validation (occ's): tasks \Rightarrow complementarity \Rightarrow sorting

- \uparrow Non-routine abstract task intensity
 $\Rightarrow \uparrow$ coworker talent complementarity



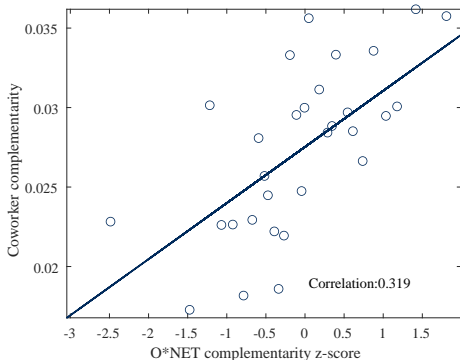
- \uparrow **Coworker talent complementarity**
 $\Rightarrow \uparrow$ **coworker sorting**



Industries: coworker importance \Rightarrow complementarity \Rightarrow sorting

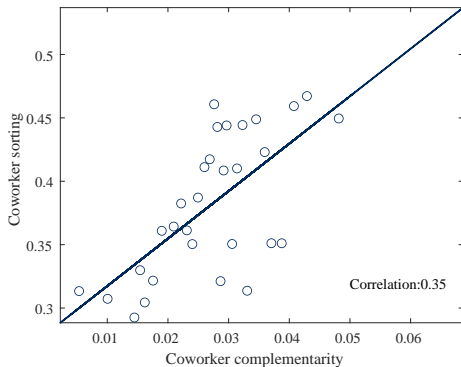
[▶ Main](#)

- \uparrow **Teamwork** [Bombardini et al., 2012]
 $\Rightarrow \uparrow$ **coworker wage complementarity**



Notes. Horizontal axis measures the industry-level weighted mean score of an occupation-level index constructed from O*NET measuring the importance of: teamwork, impact on coworker output, communication, and contact.

- \uparrow **Coworker wage complementarity**
 $\Rightarrow \uparrow$ **coworker sorting**



Notes. NACE-4-digit industries.

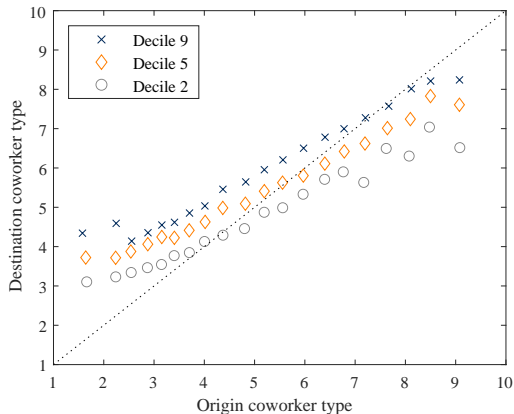
EE transitions in theory and data

- **Theoretical prediction:** EE transitions move workers in surplus-maximizing direction
 $\Rightarrow \Delta \hat{x}_{-it} = \hat{x}_{-i,t} - \hat{x}_{-i,t-1}$ should be *positively* correlated with \hat{x}_i
 - $h_{2.1}(x, x'' | x') = 1$ – worker x in a two-worker firm with coworker x'' would move to an employer that currently has one employee of type x' – if $S(x|x') - S(x|x'') > 0$
- **Empirical analysis:** use SIEED *spell* data to create worker-originMonth-destinationMonth-originJob-destinationJob panel, with information on characteristics of origin and destination job
 - subsample period 2008-2013 (huge panel at monthly frequency)
 - count as “EE” if employer change between two adjacent months
- **Regression analysis:** regress $\Delta \hat{x}_{-it}$, scaled by std. σ_{Δ} of coworker quality changes, on *own* type and *origin* coworker type

$$\frac{\Delta \hat{x}_{-it}}{\sigma_{\Delta}} = \beta_0 + \beta_1 \hat{x}_i + \beta_2 \hat{x}_{-i,t-1} + \epsilon_{it}$$

Empirical coworker sorting changes due to EE moves

- **EE transitions push toward greater coworker sorting:** for given origin, higher x-workers move to places with better coworkers than lower-x workers do
- Limitation: empirically, EE transitions “move up” low types more than theory predicts
- “**Coworker job ladder**” with both absolute and type-specific dimension?
- **Next:** change in the job ladder [e.g., Haltiwanger-Spetzler, 2021]



Evidence that EE *increasingly* reallocate toward PAM: in data & model

	Data		Model	
<i>Change in coworker type</i>	'85-'92	'10-'17	Period-1	Period-2
Own type	0.0883 ^{***} (0.000799)	0.118 ^{***} (0.000918)	0.214	0.270
Controls	Year FEs, Origin	Year FEs, Origin	Origin	Origin
N	196,098	282,718	∞	∞
adj. R ²	0.284	0.204		

Table 1: Change in coworker type due to EE moves positively related to own type – increasingly so

Notes. For the data columns, individual-level clustered standard errors are given in parentheses. Model counterparts are computed simulation-free in population. Dependent variable is scaled throughout by the standard deviation of the change in coworker type.

Fact #1: ↑ between-firm share of wage inequality

▶ Intro

▶ Intro

- Large empirical literature: “firming up inequality” [e.g., Card et al., 2013; Song et al., 2019]
 - “superstar firms” [e.g., Autor et al., 2020]
- **Fact 1: ↑ wage inequality primarily due to between-component**
- Robust pattern

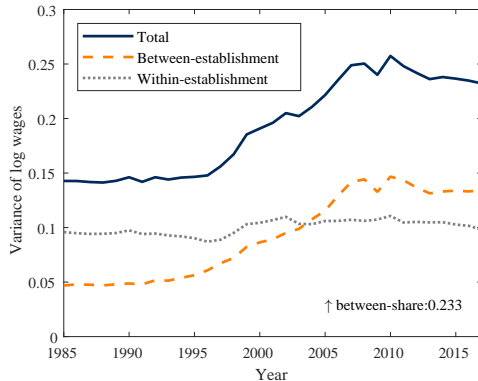
▶ Cross-country

▶ Panel est.

▶ Wage resid. alternatives

▶ Within-occ

▶ Within-ind



Notes. Model-free statistical decomposition, where the “between” component corresponds to the person-weighted variance of est.-level avg. log wage.

Fact #2: talented workers increasingly collaborate

► Intro

► Main

► Var. decomp.

► Fact #3

- To what extent do talented workers tend to have talented coworkers?

- Fact 2: + assortative coworker sorting** ↑

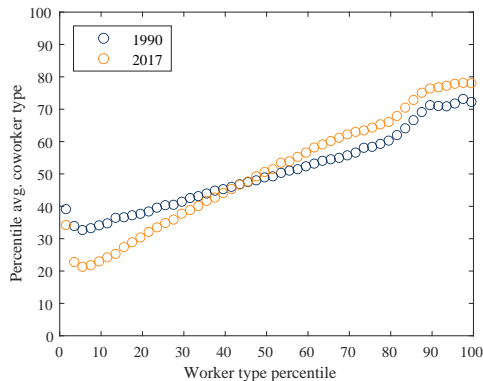
- $\rho_{xx} = \text{corr}(\hat{x}_i, \hat{x}_{-it})$: 0.43 ('85-'92) ↗ 0.62 ('10-'17)

- Robust pattern

► Table

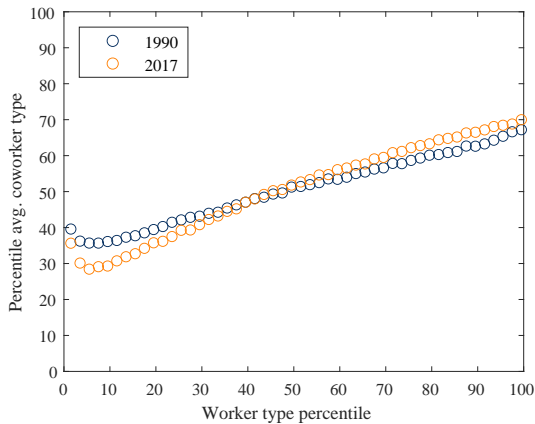
► Within-occ. nonlinear

► Hakanson et al. (2021)



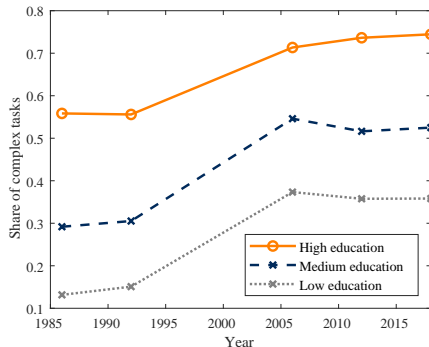
Evolution of coworker sorting: within-occupation ranking

- The most talented within each occupation – the best engineer, PA, economist, manager, ... – tend to work together, and increasingly so



Task composition changes

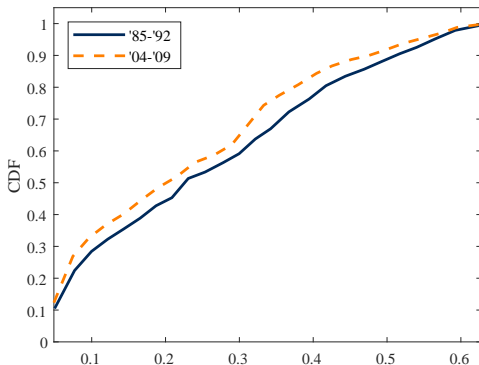
- **Task complexity** \uparrow :
“extensive margin” of χ
 - DE longitudinal task survey [▶ BIBB](#)
 - “complex”: cognitive non-routine (e.g., organizing, researching)



Workers increasingly tend to perform similar tasks across different jobs

[▶ Back](#)[▶ Comparison](#)

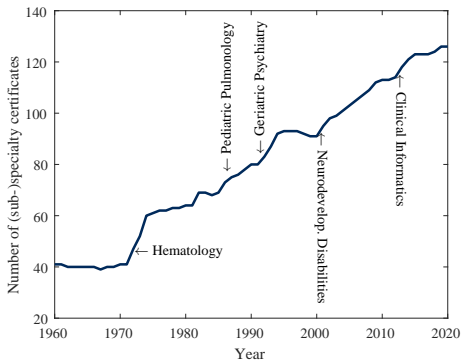
- ✓ Workers move to jobs with similar tasks, rather than randomly
- **Q:** are workers becoming *more* likely to perform similar tasks across jobs, over time?
- **Yes:** distribution of moves in ('04-'09) is stochastically dominated by that in ('85-'92)
 - uncond. average: 0.253 → 0.227: 10% decline
- Robust in regression design
 - quantile regressions: ✓ at different quantiles



Examples: rising specialization

[▶ Main](#)

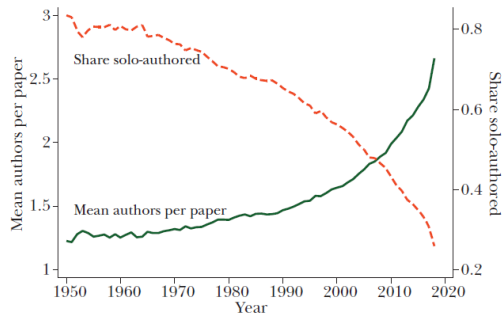
• Deepening medical specialization



Notes. Data from American Board of Medical Specialties. For each year, it shows the number of unique specialty or sub-specialty certificates that have been approved and issued at least once by that year and which are still being issued.

• Rise of research teams [Jones, 2021]

Panel A. All economics papers, 1950–2018



Overview of model robustness checks

- Declining search frictions
- Within-industry calibration
- Economy-wide vs. within-occupation analysis
- OJS
- Increased talent dispersion

► Jump

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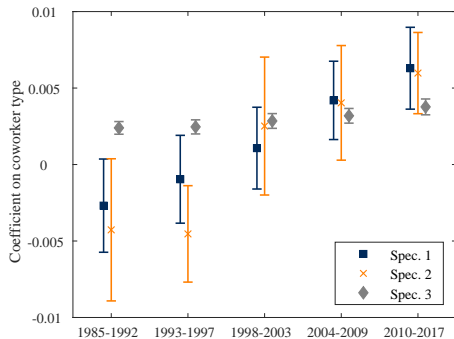
► Jump

Coworker effects: log wage regression

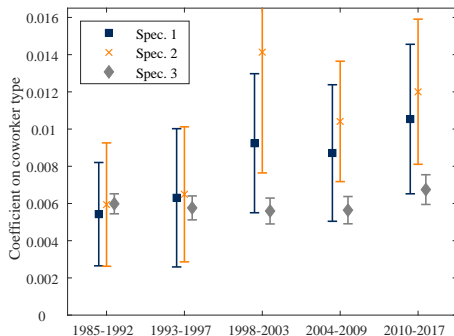
[▶ Back: cross-section](#)
[▶ Back: time series](#)

$$\ln w_{it} = \beta_0 + \beta_1 \hat{x}_i + \beta_2 \hat{x}_{-it} + \psi_{j(it)} + \nu_{o(i)t} + \xi_{s(i)t} + \epsilon_{it}$$

(a) AKM types



(b) NP types



Notes. Specifications vary by ranking method – within-economy (spec. 1) vs. within-occupation (spec. 2/spec.3) and coworker group definition – establishment-year (spec. 1/spec.2) vs. establishment-occupation-year (spec.3).

Implications for aggregate productivity

► Conclusion

- **Production complementarities imply sorting matters for agg productivity, but search frictions induce misallocation**

Implications for aggregate productivity

- **Production complementarities** imply sorting matters for agg productivity, but search frictions induce misallocation
- **Quantify** mismatch costs: compare eqm outcome to productivity under pure talent-PAM and different values of ξ – given param's for 2010s

Implications for aggregate productivity

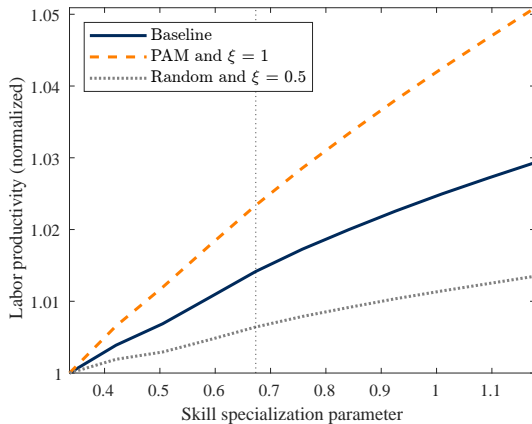
- **Production complementarities imply sorting matters for agg productivity, but search frictions induce misallocation**
- **Quantify** mismatch costs: compare eqm outcome to productivity under pure talent-PAM and different values of ξ – given param's for 2010s

	Labor productivity
Baseline (norm.)	100
PAM + $\xi = 1$	102.6
PAM	101.1
$\xi = 1$	101.4

- Eliminating mismatch would yield **productivity gains** but of **limited magnitude**

Reaping benefits of specialization requires well-functioning labor markets

“The benefits of the division of labor are limited by the functioning of the labor market”



Key takeaways

- ① Skill specialization endogenously generates coworker talent complementarities
- ② Talent complementarities lead to + assortative coworker matching
- ③ This fosters ex-post heterogeneity across firms
- ④ Enhanced talent sorting is crucial to realize the productivity gains from deepening skill specialization