

Superstar Teams: The Micro Origins and Macro Implications of Coworker Complementarities

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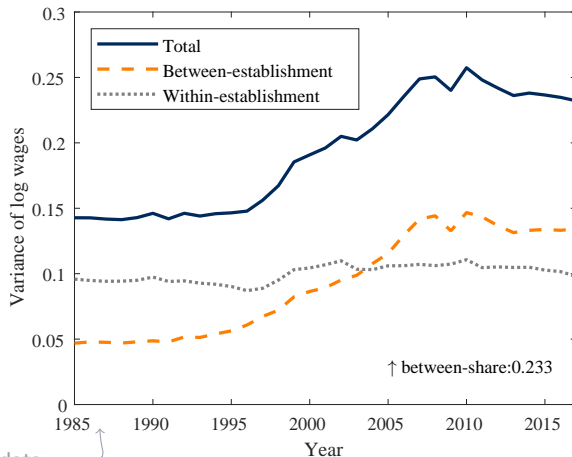
University of Cambridge

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Motivation: important role of firms in the evolution of labor market inequality

“the variance of firm [wages] explains an increasing share of total inequality in a range of countries” [Bloom et al., 2019]



German matched employer-employee data

Paper offers a structural explanation for the “firming up” of inequality

- **Motivation:** wage inequality increasingly a *between-firm* phenomenon

[e.g., Card et al., 2013; Bloom et al., 2019]

► Data

- **Hypothesis:**

- 1 the set of tasks that each worker can perform very well has narrowed
- 2 firms become more vulnerable to “talent bottlenecks” in teams
- 3 increasingly, individuals of similar talent tend to collaborate
- 4 this fosters greater between-firm wage dispersion

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3 main steps & contributions

① Develop tractable theory of team production

→ analytical **microfoundation for coworker complementarity**

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\Rightarrow **identification result** → measure coworker complementarity

③ Quantitatively analyze trends using structural model + DE panel data

→ complementarity \approx doubled since 1990

→ \approx **40% of \uparrow between-firm wage inequality share due to complementarity \uparrow**

→ implications for aggregate productivity, coworker job ladders, ...

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- **Hypothesis:**

- 1 the set of tasks that each worker can perform very well has narrowed (**specialization** ↑)
- 2 firms become more vulnerable to “talent bottlenecks” in teams (**complementarities** ↑)
- 3 increasingly, individuals of similar talent tend to collaborate (**sorting** ↑)
- 4 this fosters greater between-firm wage dispersion (**firming up inequality**)

- **Headline result:** explains $\approx 40\%$ of ↑ between-firm wage share of inequality

Relation & contributions to 3 strands of literature

- **Wage inequality: structural model of \uparrow firm-level inequality due to technological Δ**
Technology: Katz & Murphy, 1992; Krusell et al., 2000; Autor, Levy & Murnane, 2003; **Jones, 2009**; Deming, 2017; Acemoglu & Restrepo, 2018; Alon, 2018; Neffke, 2019; Jones, 2021; Atalay et al., 2021
Firms: **Card et al., 2013**; Barth et al., 2016; Alvarez et al., 2018; **Bloom et al., 2019**; Aeppli & Wilmers, 2021; Criscuolo et al. 2021; Hakanson et al., 2021; Sorkin & Wallskog, 2021; Kleinman, 2022
- **Firm organization: tractable model of team production \rightarrow frictions & quantification**
Firms: Lucas, 1978; Rosen, 1982; Becker & Murphy, 1992; **Kremer, 1993**; Kremer & Maskin, 1996; **Garicano, 2000**; **Garicano & Rossi-Hansberg, 2006**; Kohlhapp, 2022; Kuhn et al., 2022; Minni, 2023; Bassi et al., 2023
Task assignment: Costinot & Vogel, 2010; **Acemoglu & Restrepo, 2018**; Ocampo, 2021; Adenbaum, 2022
Teams: Akcigit et al., 2018; **Jarosch et al., 2021**; Herkenhoff et al., 2022; Pearce, 2022
- **Frictional labor market sorting: endogenize & measure complementarities**
Shimer & Smith, 2000; Cahuc et al., 2006; Eeckhout & Kircher, 2011/2018; Hagedorn et al., 2017; de Melo, 2018; Chade & Eeckhout, 2020; **Herkenhoff et al., 2022**; Lindenlaub & Postel-Vinay, 2023

① Theoretical model

② Model meets data

③ Analysis of historical trends

- evidence on specialization ↑, complementarities ↑, sorting ↑
- model-based quantitative analysis of “firming up inequality”
- if time: productivity, mismatch, coworker job ladders

④ Concluding discussion: framework development & future directions

Theory



Overview of the model environment

► Schematic illustration

- Many workers and many firms
- Ex-ante homogeneous firms assemble teams to produce output from tasks
 - 1 allocate workers' time across tasks
 - 2 hire workers
- Workers are heterogeneous in productivity
- **Analyze in 2 steps:**
 - 1 production given team
 - 2 team formation

Step 1: production in a single team of given composition

- Firm w/ **1 team of** $n \in \mathbb{Z}_{++}$ **workers** produces output from **unit continuum of tasks** \mathcal{T}

$$\ln Y = \int_{\mathcal{T}} \ln q(\tau) d\tau \quad (1)$$

- Workers:** each supplies 1 unit of time, has task-specific productivities $\{z_i(\tau)\}_{\tau \in \mathcal{T}}$
- Task production:**

$$y_i(\tau) = z_i(\tau) l_i(\tau) \quad (2)$$

$$1 = \int_{\mathcal{T}} l_i(\tau) d\tau \quad (3)$$

- Division of labor:**

$$q(\tau) = \sum_{i=1}^n y_i(\tau) \quad (4)$$

Firm's optimization problem

► Extension to communication frictions

- **Firm solves mini-planner problem:** assign tasks to max. production

$$\begin{aligned} \mathcal{L}(\cdot) = & Y + \lambda \left[\underbrace{\left(\int_{\mathcal{T}} \ln q(\tau) d\tau \right)}_{\text{tasks} \rightarrow \text{output}} - \ln Y \right] + \int_{\mathcal{T}} \lambda(\tau) \underbrace{\left(\sum_{i=1}^n y_i(\tau) - q(\tau) \right)}_{\text{division of labor}} d\tau \\ & + \sum_{i=1}^n \left\{ \int_{\mathcal{T}} \lambda_i(\tau) \underbrace{\left(z_i(\tau) l_i(\tau) - y_i(\tau) \right)}_{\text{task production}} d\tau + \lambda_i^l \underbrace{\left(1 - \int_{\mathcal{T}} l_i(\tau) d\tau \right)}_{\text{time constraint}} + \text{non-negativity constraints} \right\} \end{aligned}$$

- **FOCs** imply tasks are assigned by *comparative* advantage

$$\lambda(\tau) = \min_i \underbrace{\left\{ \frac{\lambda_i^l}{z_i(\tau)} \right\}}_{\lambda_i(\tau)} \quad (5)$$

Tractability: leverage insight from trade literature

► Illustration

► Multivariate Fréchet

Assumption: Distribution of worker-task efficiencies

$\{z_i(\tau)\}_{\tau \in \mathcal{T}} \stackrel{\text{iid}}{\sim}$ **Fréchet**:

$$\Pr\{z_i(\tau) \leq z\} = \exp\left(-\left(\frac{z}{\iota x_i}\right)^{-1/\chi}\right).$$

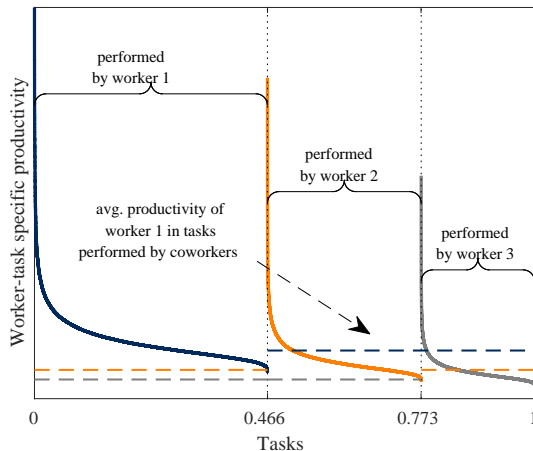
- Low-dimensional representation of worker-task productivity distribution
 - “talent” type $x_i \in [0, 1]$
 - parameter χ : **specialization** \sim productivity dispersion across tasks
- Max-stability property allows closed-form characterization of key objects
 - e.g., $\lambda(\tau)$ is distributed Weibull

Optimal organization: illustration

1 tasks assigned by comparative advantage

- i 's task set

$$\mathcal{T}_i = \left\{ \tau \in \mathcal{T} : \frac{z_i(\tau)}{\lambda_i^L} \geq \max_{k \neq i} \frac{z_k(\tau)}{\lambda_k^L} \right\}$$

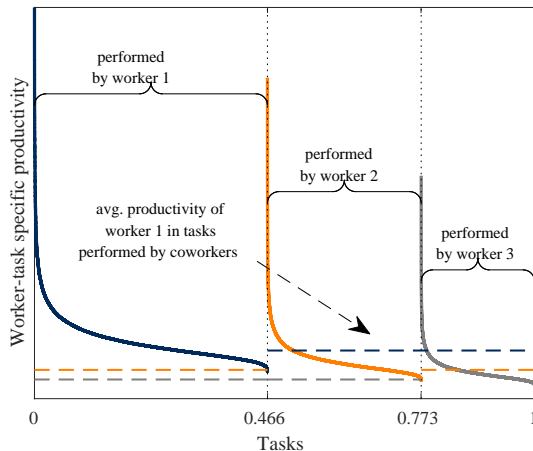


Optimal organization: illustration

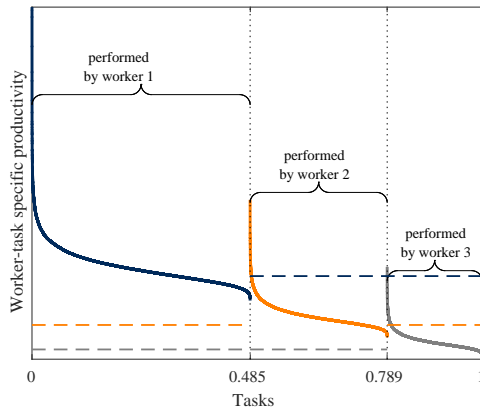
- ① tasks assigned by comparative advantage
- ② i 's share of tasks \uparrow in i 's talent, \downarrow in coworkers' talent

- i 's task share

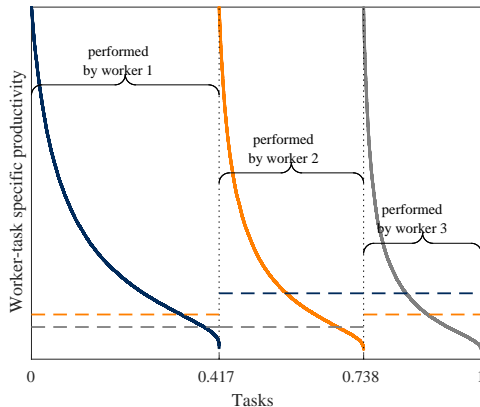
$$\pi_i = (x_i^{\frac{1}{1+\chi}}) (\sum_{k=1}^n (x_k)^{\frac{1}{1+\chi}})^{-1}$$



Optimal organization: illustration – low vs. high χ



(a) Low specialization (χ)



(b) Higher specialization (χ)

Optimal organization: task shares and χ

Lemma: Task shares

Suppose that $x_i > x_j$. Then

- 1 i performs a strictly larger share of tasks than j for $\chi < \infty$, i.e., $\pi_i < \pi_j$;
- 2 **the difference in task shares is decreasing in χ** ;
- 3 the ratio of task shares approaches $\frac{x_i}{x_j}$ as $\chi \rightarrow 0$;
- 4 task shares are equalized as $\chi \rightarrow \infty$.

Micro-founded CES production function

Proposition: Aggregation result

The vector of talent types (x_1, \dots, x_n) is a sufficient statistic for team output Y , s.t.

$$f(x_1, \dots, x_n) = \underbrace{n^{1+\chi}}_{\text{efficiency gains}} \times \underbrace{\left(\frac{1}{n} \sum_{i=1}^n (x_i)^{\frac{1}{1+\chi}} \right)^{1+\chi}}_{\text{complementarity}}.$$

vs. no-division-of-labor: $f(x_1, \dots, x_n) = n \times (\frac{1}{n} \sum_{i=1}^n x_i)$

① Standard **efficiency gains** $\nearrow \chi$

Micro-founded CES production function

► Taylor approx.

► Quality & task mismatch

► Example

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with elasticity of complementarity $\gamma = \frac{\chi}{1+\chi}$, where $\chi \sim$ specialization.

② **Coworker complementarity** ↗ χ : output more 'vulnerable' to lowest- x member

Step 2: matching – quantitative model

[▶ Equilibrium equations](#)

- **Embed $f(\cdot)$ into frictional equilibrium matching model**
 - random-search + multi-worker firms *[Herkenhoff et al., 2022]*
 - employment states: unemp., employed alone, employed with $x' \in [0, 1]$
 - Nash bargaining
- **2 main results**
 - 1 matching: complementarities \rightarrow sorting
 - 2 identification theorem: measuring complementarities

Matching – stationary equilibrium

- HJ-Bellman equations → **values & matching policies**
- Flows between/**distribution** over types \times employment states

[► HJBs](#)[► KFEs](#)

Definition: Stationary equilibrium

A stationary equilibrium consists of a tuple of value functions and a distribution of agents, such that

- 1 the value functions satisfy the HJB equations given the distribution;
- 2 the distribution is stationary given the policy functions implied by the value functions.

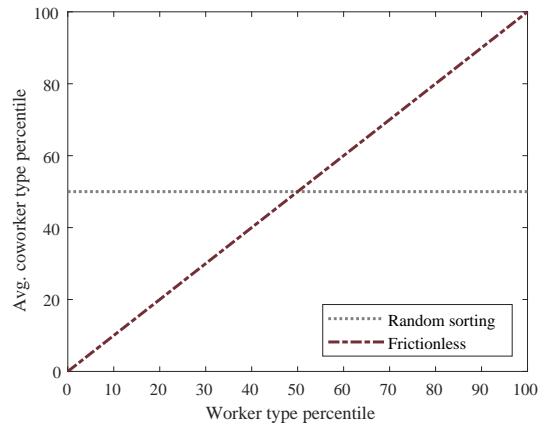
Matching – intuition

▶ Simple model

▶ Lemma

▶ Simple vs. quantitative

- In equilibrium, who tends to work with whom?



Matching – intuition

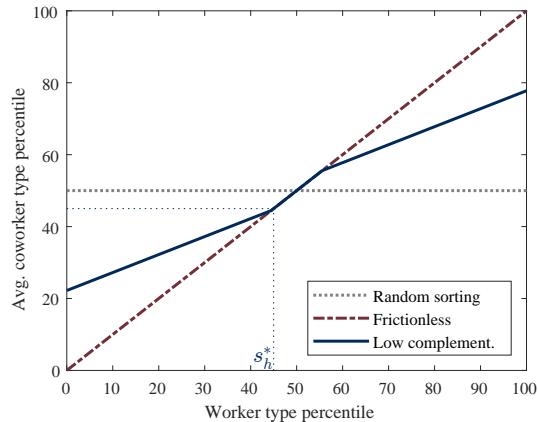
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- **Search frictions:** trade-off **team match quality vs. cost of searching**

⇒ range of acceptable matches



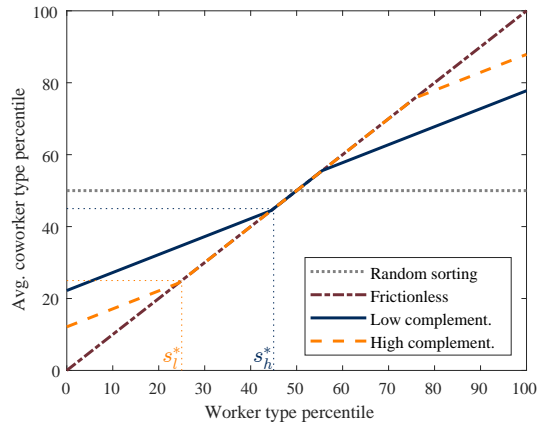
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- **Search frictions:** trade-off **team match quality vs. cost of searching**
 ⇒ range of acceptable matches
- $\chi \uparrow \Rightarrow$ *smaller* acceptable range
 - 1 **coworker sorting** \uparrow
 - 2 **between-firm share of var(wages)** \uparrow



Identification result

- **Strategy:** recover $\gamma (\rightarrow \chi)$ directly, rather than inferring from sorting
 \Rightarrow How to quantify $\frac{\partial^2 f(x, x')}{\partial x \partial x'}$?

[► Discussion](#)

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- **Theory:** wage level for worker x with coworker x'

► Wage equation

$w(x|x') = \omega \times (f(x, x') - f(x')) + \text{messy fn. of outside options that depend only on own type}$

Proposition: Identification result

Coworker complementarities (CC) in production are proportional to CC in wages:

$$\frac{\partial^2 f(x, x')}{\partial x \partial x'} \propto \frac{\partial^2 w(x|x')}{\partial x \partial x'}$$

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Model Meets Data

Bringing the model to panel micro data for Germany

► SIEED

- **Primary data: SIEED matched-employer employee panel for W Germany**
 - 1.5% sample of establishments + entire biographies of associated workers; social security information on employer, daily wage, occupation, demographics
 - initially focus on 2010-2017, later extend to 1985-2017

► Processing

Bringing the model to panel micro data for Germany

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- **Overview of steps:**
 - 1 mapping model objects to data
 - 2 model calibration
 - 3 validation – brief today, more in paper!

Mapping theory to data – types: workers, teams & coworkers

► Implementation

- **Worker types** from 2-way FE wage regressions [*Abowd et al., 1999 – AKM*]
 - pre-estimation k-means cluster → deal with limited mobility bias [*Bonhomme et al., 2019*]
 - robustness: non-param. ranking algo instead of AKM [*Hagedorn et al., 2017*]

⇒ Worker “type” \hat{x}_i : decile rank of worker FE

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⇒ Worker “type” \hat{x}_i : decile rank of worker FE

- **“Representative coworker type”** \hat{x}_{-it} : average \hat{x}_i of peers in same establishment-year

[► Discussion](#)

Empirical evidence on coworker complementarity (2010-2017)

► Robustness

► B-o-E calc. γ

- Regression:

coworker complementarity

$$\frac{w_{it}}{\bar{w}_t} = \beta_0 + \beta_1 \hat{x}_i + \beta_2 \hat{x}_{-it} + \beta_c (\hat{x}_i \times \hat{x}_{-it})$$

$$+ \psi_{j(it)} + \nu_{o(i)t} + \xi_{s(i)t} + \epsilon_{it}$$

Empirical evidence on coworker complementarity (2010-2017)

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- Regression:

$$\frac{w_{it}}{\bar{w}_t} = \beta_0 + \beta_1 \hat{x}_i + \beta_2 \hat{x}_{-it} + \overset{\text{coworker complementarity}}{\beta_c} (\hat{x}_i \times \hat{x}_{-it}) + \psi_{j(it)} + \nu_{o(i)t} + \xi_{s(i)t} + \epsilon_{it}$$

	$\hat{\beta}_c$	Non-parametric FD method
Coworker complementarity	0.0091*** (0.00035)	0.0097
Type ranking	Economy	Economy
Obs. (100,000s)	4,410	4,410

Notes. Regressions include FEs for employer; occupation-year; industry-year. Employer-clustered standard errors in parentheses. Observations weighted by the inverse employment share of the respective type and (rounded) coworker type cell.

Model parameterization: overview of methodology

[► Details](#)[► Identification validation exercises](#)

- **Calibrate** the model to the W German economy (2010-2017), at monthly frequency
 - 1 externally calibrated: discount rate, team-benefit, bargaining power
 - 2 offline estimation: job separation hazard
 - 3 online estimation (indirect inference): meeting rate, unemp. flow benefit, production
 - ⇒ **production complementarity informed by $\hat{\beta}_c$**
 - other targets: wage variance, avg. wage level, replacement rate, job finding rate
- **Key: macro moments of interest are untargeted!**
 - eg coworker sorting, between-firm wage inequality

Calibration results & model validation

- Recovered elasticity of complementarity γ equal to 0.84

► Parameterization

- ✓ **Match coworker sorting patterns**

- $\rho_{xx} = 0.53$ (vs. 0.62 in data)

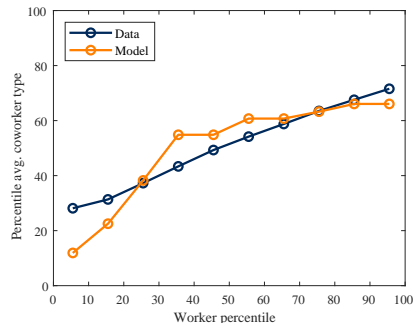
- ✓ **Match between-firm wage inequality**

- between-share 0.56 (vs. 0.57 in data)
 - adjust for small- n bias

► Details

- Extensive validation exercises

► Details



Historical Trends

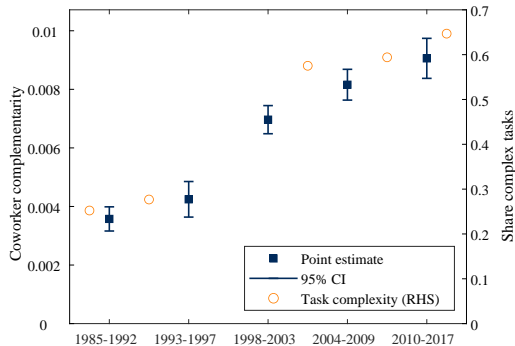


Coworker complementarity has strengthened since the mid-1980s

▶ Schooling

- **Theory:** specialization (χ) \uparrow is associated with coworker complementarity \uparrow

✓ **Coworker complementarity has more than doubled between 1985-1992 and 2010-2017**



...consistent with descriptive evidence of specialization (χ) \uparrow ...

► Occ. movements

- **Example:** science [*Jones, 2009/2021; Pearce, 2022*]

► Details

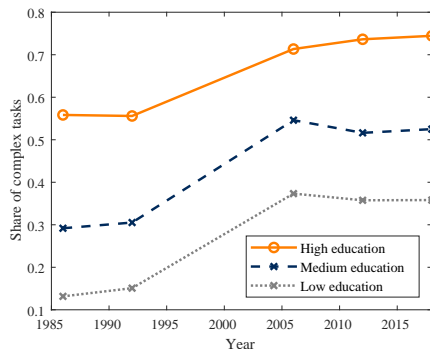
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[► Occ. movements](#)

- **Example:** science [*Jones, 2009/2021; Pearce, 2022*]

[► Details](#)

- **Task complexity \uparrow :**
“extensive margin” of χ
 - DE longitudinal task survey
 - “complex”: cognitive non-routine (e.g., organizing, researching)

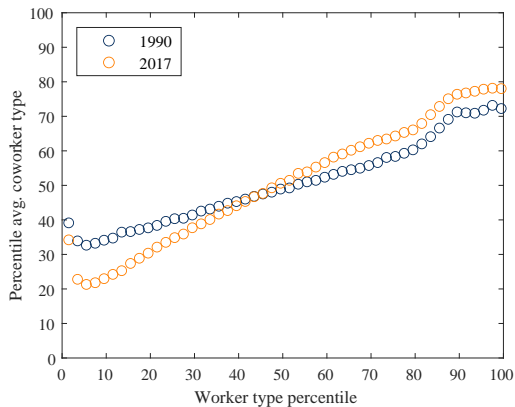
[► BIBB](#)


...and talent sorting has intensified

[Details](#)

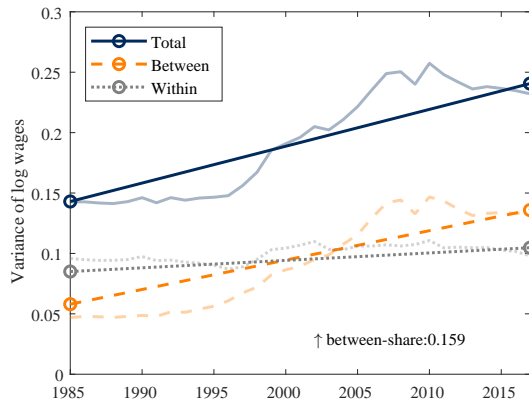
- **Theory:** complementarity \uparrow is associated with talent sorting \uparrow

✓ Coworker matching has become more positively assortative



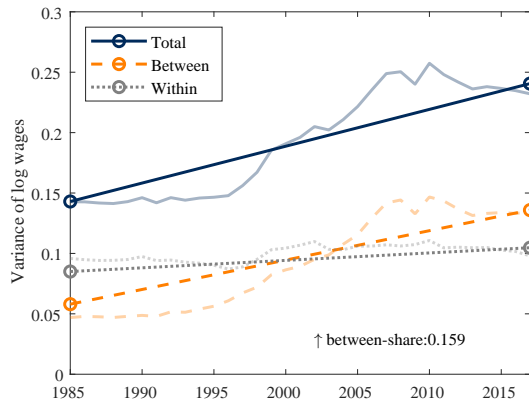
Model matches *changes* in firm-level wage distribution

- **Re-estimate parameters** for '85-'92
- ✓ Replicate empirical rise of between-share
 - 68% of \uparrow between-share in data



Model matches *changes* in firm-level wage distribution

- **Re-estimate parameters** for '85-'92
 - elasticity of compl.: 0.43 (vs. 0.84)
 - job arrival & separation \uparrow in '10-'17
- ✓ Replicate empirical rise of between-share
 - 68% of \uparrow between-share in data



Complementarity \uparrow explains $\approx 40\%$ of observed between-share \uparrow

- **Q:** How much of \uparrow between-firm share of wage variance is due to \uparrow complementarities (CC)?
- **Counterfactual:** between-firm share in 2010s absent \uparrow CC
- **A:** \uparrow CC accounts for 59% of model-predicted $\Delta \leftrightarrow \approx 40\%$ of empirical Δ

	Δ model	Implied % Δ model due to Δ parameter
Model baseline	0.16	-
Cf.: fix period-1 complementarity	0.065	59

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- Finding robust to checks, e.g., outsourcing/within-occupation analysis

[▶ Jump](#)

Implications for aggregate productivity

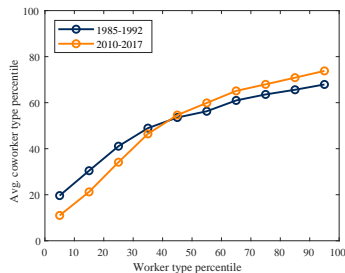
[▶ Main](#)

- **Theory:** 2-fold effect of \uparrow complementarity:

- ① cost of mismatch \uparrow for *given* allocation
- ② cost of mismatch $\uparrow \Rightarrow$ labor reallocation

- **Reallocation limited rise in mismatch costs**

- productivity gap to pure PAM: 2.05%
- no-reallocation c.f.: productivity gap 4.65%


[▶ Details](#)

- Benefits from division of labor are limited by labor market functioning

Extensions



Overview of extensions, robustness checks, and other implications

- **Robustness checks**

- declining search frictions
- outsourcing & within-occupation analysis
- OJS
- firm size

▶ Jump

▶ Jump

▶ Jump

▶ Jump

- **Productivity dispersion**

▶ Jump

- **Overall inequality**

▶ Jump

- **“Coworker job ladders”**

▶ Jump

Conclusion

Conclusions: firms form & coordinate teams

- **Takeaways:**

- 1 **specialization** fosters coworker complementarity, not only efficiency gains
⇒ specialization as a source of vulnerability
- 2 increased **complementarities** help rationalize the “firming up” wage inequality
⇒ role of teams in ‘superstar economy’
- 3 increased **talent sorting** helped keep TFP close to potential
⇒ policy implications of increased firm-level inequality not obvious

- **Contributions to framework development:** firms & technological change
 - structural interpretation of “firm effects”; impact of AI; organizational capacity

Thank You & Feedback Very Welcome!

Extra Slides

Fact 1: ↑ between-firm share of wage inequality

▶ Intro

- Large empirical literature: “firming up inequality” [e.g., Card et al., 2013; Song et al., 2019]
 - “superstar firms” [e.g., Autor et al., 2020]
- **Fact 1: ↑ wage inequality primarily due to between-component**
- Robust pattern

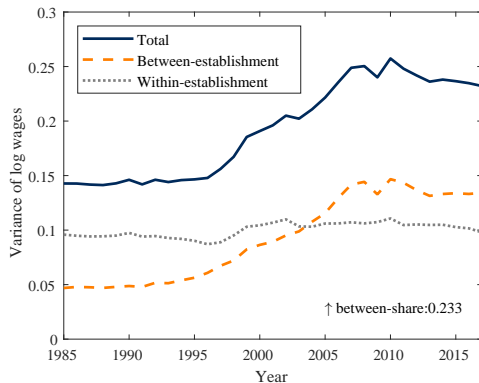
▶ Cross-country

▶ Panel est.

▶ Wage resid. alternatives

▶ Within-occ

▶ Within-ind



Notes. Model-free statistical decomposition, where the “between” component corresponds to the person-weighted variance of est.-level avg. log wage.

Fact #2: talented workers increasingly collaborate

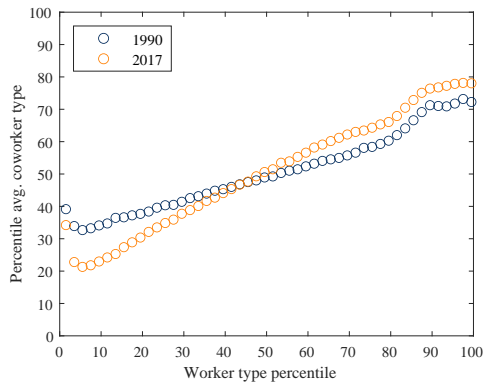
[▶ Intro](#)[▶ Var. decomp.](#)[▶ Fact #3](#)

- To what extent do “talented workers” tend to have “talented coworkers”?

- **Fact 2: + assortative coworker sorting** ↑

- $\rho_{xx} = \text{corr}(\hat{x}_i, \hat{x}_{-it})$: 0.43 ('85-'92) ↗ 0.62 ('10-'17)

- Robust pattern

[▶ Table](#)[▶ Within-occ. nonlinear](#)[▶ Hakanson et al. \(2021\)](#)

Fact #3: increased education premium due to workplace effects

[Main](#)

- **Fact 3: increase in return to schooling is primarily due to workplace effects**

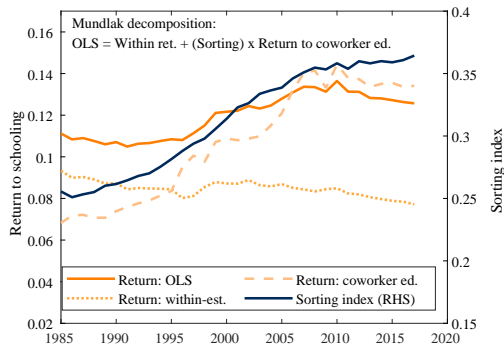
- Mundlak decomposition of year-specific OLS return to schooling:

$$\beta_t^{\text{ols}} = \beta_t^{\text{within}} + \rho_t \times \beta_t^{\text{estab.}}$$

$$\ln w_{it} = \beta_0 + \beta_t^{\text{within}} S_i + \beta_t^{\text{estab.}} \bar{S}_{j(i,t),t} + e_{it}$$

where $\bar{S}_{j(i,t),t}$ is avg. years of schooling in establishment j of worker i in year t

- 1 β_t^{within} : within-establishment return:
- 2 $\beta_t^{\text{estab.}}$: return to avg. establishment schooling:

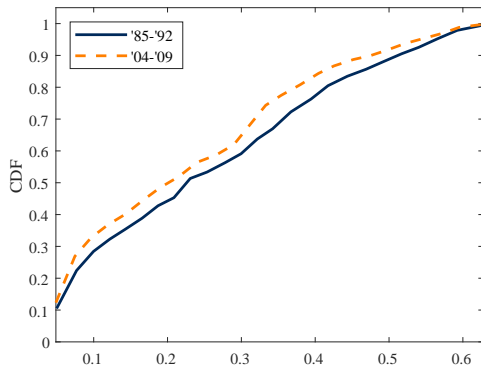


Notes. Plot of coefficients from year-by-year regressions of log wages.

Workers increasingly tend to perform similar tasks across different jobs

[▶ Back](#)[▶ Comparison](#)

- ✓ Workers move to jobs with similar tasks, rather than randomly
- **Q:** are workers becoming *more* likely to perform similar tasks across jobs, over time?
- **Yes:** distribution of moves in ('04-'09) is stochastically dominated by that in ('85-'92)
 - uncond. average: 0.253 \rightarrow 0.227: 10% decline
- Robust in regression design
 - quantile regressions: 3 at different quantiles



Brief summary of methodology: occupations and task space

[▶ Details](#)

1 Measure task similarity between occupations

- $\bar{\mathbf{l}}_o = (\bar{l}_{o1}, \dots, \bar{l}_{o|\hat{\mathcal{T}}|})$: vector of task content of occupation o , with $\bar{l}_{o\tau}$ denoting the fraction of workers in occupation o performing task $\tau \in \hat{\mathcal{T}}$, where $|\hat{\mathcal{T}}| \in \mathbb{Z}_{++}$ (i.e., discretized)
- *distance in task space* between any two occupations o and o' ,

$$\varphi(\bar{\mathbf{l}}_o, \bar{\mathbf{l}}_{o'}) = \frac{1}{\pi} \cos^{-1} \left(\frac{\bar{\mathbf{l}}_o \bar{\mathbf{l}}_{o'}}{\|\bar{\mathbf{l}}_o\| \cdot \|\bar{\mathbf{l}}_{o'}\|} \right) \in [0, 1]$$

- **implementation:** BIBB longitudinal microdata ($|\hat{\mathcal{T}}| = 15$)

2 Occupational movers: an individual i who in period t is employed at j in occupation o counts as an *occupational mover* if in $t + 1$, i is employed at $j' \neq j$ and $o' \neq o$

- implementation: matched employer-employee micro data (entire biography!)

3 Merge $\{\varphi(\bar{\mathbf{l}}_o, \bar{\mathbf{l}}_{o'})\}_{oo'}$ into mover sample \rightarrow **project moves onto task distance space**

- harmonized occupational classification KldB1988-2d

Methodology (1): occupations and task space

- **Occupations as points in task space – definitions:**

- $\bar{\mathbf{l}}_o = (\bar{l}_{o1}, \dots, \bar{l}_{o|\hat{\mathcal{T}}|})$: vector of task content of occupation o , with $\bar{l}_{o\tau}$ denoting the fraction of workers in occupation o performing task $\tau \in \hat{\mathcal{T}}$, where $|\hat{\mathcal{T}}| \in \mathbb{Z}_{++}$ (i.e., discretized)
- *distance in task space* between any two occupations o and o' ,

$$\varphi(\bar{\mathbf{l}}_o, \bar{\mathbf{l}}_{o'}) = \frac{1}{\pi} \cos^{-1} \left(\frac{\bar{\mathbf{l}}_o \bar{\mathbf{l}}_{o'}}{\|\bar{\mathbf{l}}_o\| \cdot \|\bar{\mathbf{l}}_{o'}\|} \right) \in [0, 1]$$

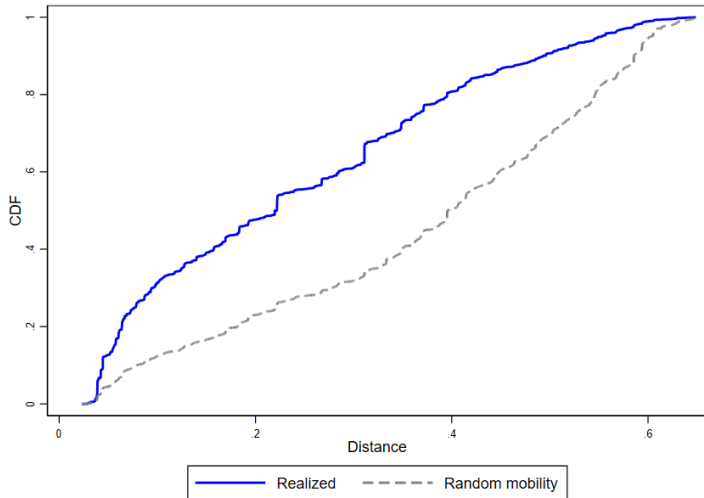
- **Implementation:** BIBB longitudinal microdata

- 15 harmonized tasks across survey waves
- measure $\bar{l}_{o\tau}$: share of individuals belonging to o performing τ
- construct matrix of bilateral distances for each wave, then take *average* of $\varphi(\bar{\mathbf{l}}_o, \bar{\mathbf{l}}_{o'})$ across waves for each (o, o')
- *note:* I thus hold the distance in task space between any two occupations fixed over time

Methodology (2): occupational movers

- Consider individual i who in period t is employed at j in occupation o . I consider i an *occupational mover* if in $t + 1$, i is employed at $j' \neq j$ and $o' \neq o$.
 - only counted as occ. mover if employed in different job: cf. Kambourov and Manovskii (2008)
 - considering switch in adjacent periods: more likely that it is a *voluntary* move
- Function $\varphi(\bar{\mathbf{l}}_o, \bar{\mathbf{l}}_{o'})$ maps each $o \rightarrow o'$ move onto $[0, 1]$
- Implementation: merge $\{\varphi(\bar{\mathbf{l}}_o, \bar{\mathbf{l}}_{o'})\}_{oo'}$ into SIEED
 - harmonized occupational classification KldB1988-2d
 - restrict myself to 1985-2009, because subsequent stark change in occupational classification limits comparability (missing notifications, etc.)

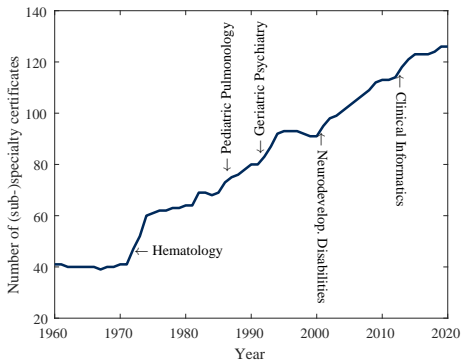
Comparison of realized movements in task space vs. random mobility

[▶ Back](#)

Examples: rising specialization

[▶ Main](#)

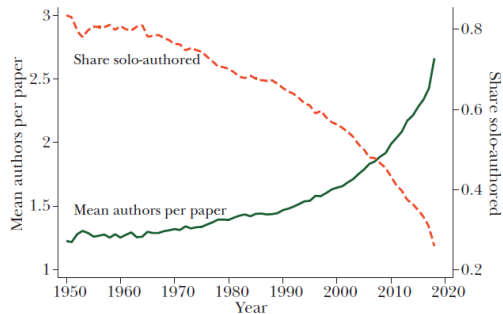
• Deepening medical specialization



Notes. Data from American Board of Medical Specialities. For each year, it shows the number of unique specialty or sub-specialty certificates that have been approved and issued at least once by that year and which are still being issued.

• Rise of research teams [Jones, 2021]

Panel A. All economics papers, 1950–2018



Data sources: short description of main datasets

[▶ Main](#)[▶ Imputation procedure](#)

- **Germany:** SIEED linked employer-employee dataset
 - *establishment* and individual data generated in administrative processes
 - built up from a 1.5% sample of all establishments, but includes comprehensive employment biographies of individuals employed at these establishments
 - worker info includes: (real) **daily wage**, occupation and education; establishment info not yet utilized
 - top coding (affects >50% of university-educated men in regular full-time employment)
→ adopt standard imputation methods (Dustmann et al., 2009; CHK, 2013)
 - Much larger sampling frame than more familiar LIAB
- **Portugal:** Quadros de Pessoal & Relatório Único, 1986-2017
 - \approx universe of private sector firms and workers employed by them
 - annual panel
 - worker information includes: detailed earnings measures (base wage, regular benefits, irregular benefits (performance-pay, bonuses, etc.), overtime pay); no top-coding; also includes information on hours worked within the month (regular and overtime) → (real)

Sample restrictions

- Data cleaning \Rightarrow broadly harmonized samples
- Main restrictions
 - age 20-60
 - full-time employed
 - drop agriculture, public sector, utilities industries
 - firms (and their employees) with at least 10 employees
- DE: West Germany
- Transform SIEED spell-level data into annual panel
- PRT: at least. official minimum wage

Wage Imputation procedure

- Follow imputation approach in CHK2013, building on Gartner et al. (2005) and Dustmann et al. (2009)
 - ① fit a series of Tobit models to log daily wages
 - ② then impute an uncensored value for each censored observation using the estimated parameters of these models and a random draw from the associated (left-censored) distribution
- Currently I fit 16 Tobit models (4 age groups, 4 education groups) *after* having restricted the sample (to include West German men only, in particular) and I follow CHK in the specification of controls by including not only age, firm size, firm size squared and a dummy for firms with more than ten employees, but also the mean log wage of co-workers and fraction of co-workers with censored wages. Finally, following Dauth & Eppselheimer (2020) I limit imputed wages at 10×99 th percentile.

Mapping model to data: coworker types

- Defining $S_{-it} = \{k : j(kt) = j(it), k \neq i\}$ as the set of i 's coworkers in year t , compute the average type of i 's coworkers in year t as $\hat{x}_{-it} = \frac{1}{|S_{-it}|} \sum_{k \in S_{-it}} \hat{x}_k$.
- **Coworker group:**
 - alternative: same establishment-occupation-year cell
 - but CC arise precisely when workers are *differentiated* in their task-specific productivities
- **Averaging step:**
 - equally-weighted averaging ignores non-linearity in coworker aggregation
 - paper: show using non-linear averaging method that baseline results in bias, but it's minor in magnitude
- **Firm size variation:** averaging ensures that a single move will induce a smaller change in the *average* coworker quality in a large team than in a small one

Mapping model to data: identification strategy for χ

[▶ Main](#)

- **Literature:** complementarities – primarily between workers and firms – usually inferred indirectly from sorting patterns
 - exception: Hagedorn-Law-Manovskii (2017)
- **This paper:** directly measure coworker complementarity in the data, recover χ structurally given $\gamma = \frac{\chi}{\chi+1}$
- Paper does *not* use microfoundation itself to measure χ , respectively γ
- Experiment: fit a (truncated) Fréchet distribution to Grigsby's (2023) non-parametric estimates of the multi-dimensional skill distribution estimated from CPS data for the U.S.
Recover $\gamma = 0.84$ for 2006 but very noisy estimates
- **Ongoing work:** use the extended microfoundation to identify χ

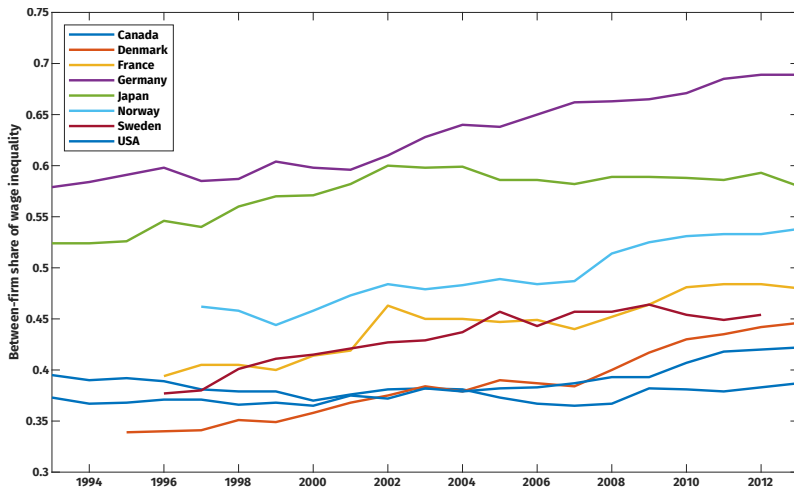
Direct estimation of χ : proof of concept

- Grigsby (2023): only paper that provides a *cardinal* measure of skill task-specificity
- Evidence on time trends are at least qualitatively consistent with my “specialization hypothesis”: cross-type average of within-type variance across specific skills grew by nearly 50% b/w 1980s and 2000s & skill transferability has declined amongst high-skill occupations
- His operationalization of worker types and tasks does *not* directly map onto my model (no identifying assumption; coarse occupational skills; US vs DE data)
- **Proof of concept:** but *suppose* we just take those data, extract moments capturing average within-worker cross-task efficiency dispersion, fit a (truncated) Fréchet, recover $\gamma = \frac{\chi}{1+\chi}$
 $\Rightarrow \checkmark \gamma$ **similar to structural estimation result based on evidence from wage CC**

Semi-structural back-of-envelope calculation for γ

- Structurally recover $\gamma \frac{\chi}{\chi+1}$ by estimating $\frac{\partial^2 w(x|x')}{\partial x \partial x'}$ in the data, which was shown to be proportional to $\frac{\partial^2 f(x, x')}{\partial x \partial x'}$
- But how is $\frac{\partial^2 f(x, x')}{\partial x \partial x'}$ related to γ ?
- Definitionally, $\gamma = (f f_{ij}) / (f_i f_j)$ for any $i \neq j$, where subscripts denote partial derivatives
- Can we avoid full structural model? \Rightarrow If have measures not only of f_{ij} but also output f and marginal products f_i
- Suppose, for any x and x' , we use wages to back out marginal products – competitive wage determination rather bargaining! – and recover output from sum of wages divided by labor share
- Find $\gamma \approx 0.79$ – very close to structural estimate!

Firming up inequality: cross-country evidence

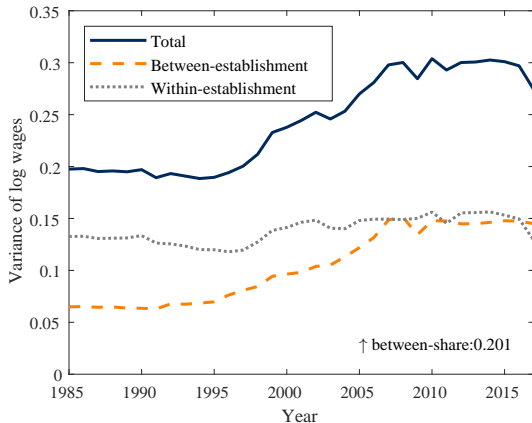
[▶ Main](#)

Notes. Data from Tomaskovic-Devey et al. (2020). Measures of earnings differ across countries and, for Germany, between T-D et al. and my study based on the SIEED.

Between-/within-employer wage var decomp. - panel establishments

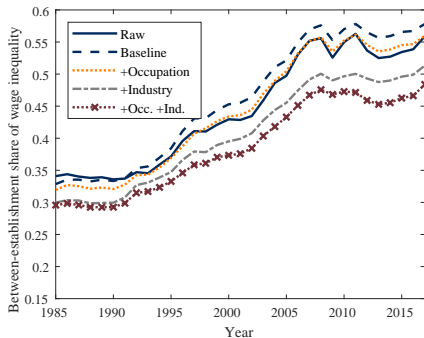
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- Instead of considering *all* employers, restrict attention to “panel establishments”

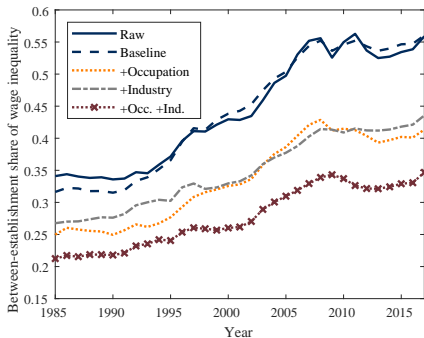


Between-/within-employer wage var. decomp. - alternative w-residuals

- “With worker FEs”: regress $\ln \tilde{w}_{it} = \alpha_i + X'_{it}\hat{\beta} + \epsilon_{it}$, construct $\ln w_{it} = \ln(\tilde{w}_{it} - X'_{it}\hat{\beta})$.
- “Without worker FEs”: regress $\ln \tilde{w}_{it} = \alpha_0 + X'_{it}\hat{\beta} + \epsilon_{it}$, and consider residuals ϵ_{it}

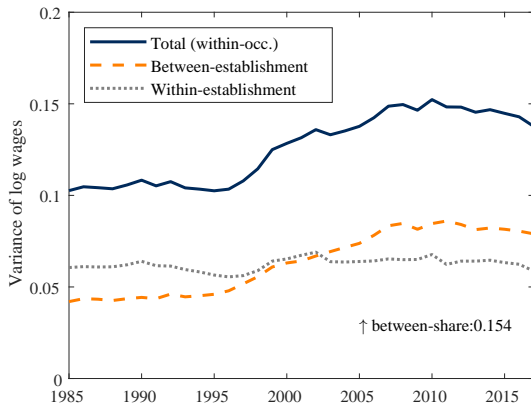


(a) With worker FEs

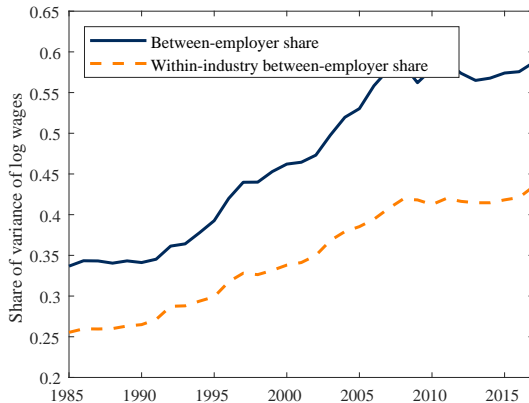


(b) Without worker FEs

Between-/within-employer wage var. decomp. - within-Occupation

[▶ Main](#)

Between-/within-employer wage var. decomp. - within-Industry

[▶ Main](#)

Notes. Based on 'baseline' residualized wages.

Evolution of coworker sorting: correlation coefficient

- Estimate \hat{x}_i and \hat{x}_{-it} separately for 5 periods
- In addition to the baseline, also consider ranking within-occupation

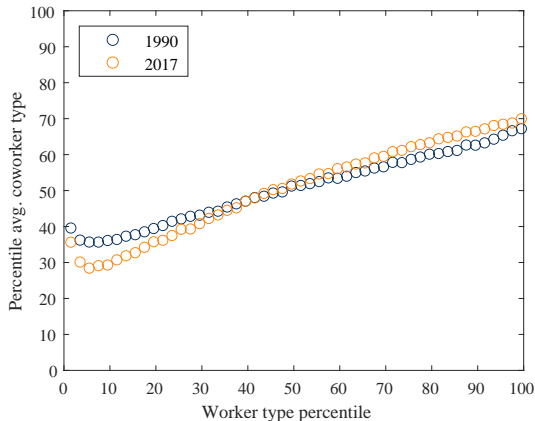
Period	Sorting	
	Spec. 1	Spec. 2
1985-1992	0.427	0.423
1993-1997	0.458	0.443
1998-2003	0.495	0.452
2004-2009	0.547	0.470
2010-2017	0.617	0.519

Notes. The column labelled “Sorting” indicates the correlation between a worker’s estimated type and that of their average coworker, separately for five sample periods. Under “Spec. 1” workers are ranked economy wide (baseline), while under “Spec. 2” they are ranked within occupations.

Evolution of coworker sorting: within-occupation ranking binscatter

[▶ Main](#)

- Reproduce non-linear sorting plot, but now \hat{x}_i is based on *within-occupation* ranking



AKM-based wage variance decomposition

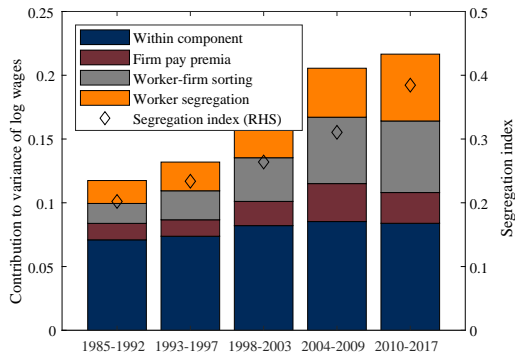
- AKM-based var. decomp. [Song et al., 2019]

$$\text{Var}(w_{it}) = \underbrace{\text{Var}(\alpha_i - \bar{\alpha}_{j(i,t)}) + \text{Var}(\epsilon_{i,j})}_{\text{within-component}} + \underbrace{\text{Var}(\psi_{j(it)}) + 2\text{Cov}(\bar{\alpha}_{j(it)}, \psi_{j(it)}) + \text{Var}(\bar{\alpha}_{j(it)})}_{\text{between-component}}$$

- $\text{Var}(\psi_j)$: **firm-specific pay premia**
- $\text{Cov}(\bar{\alpha}_j, \psi_j)$: **(worker-firm) sorting**
- $\text{Var}(\bar{\alpha}_j)$: **(worker-worker) segregation**

- Segregation index [Kremer-Maskin, 1996]:

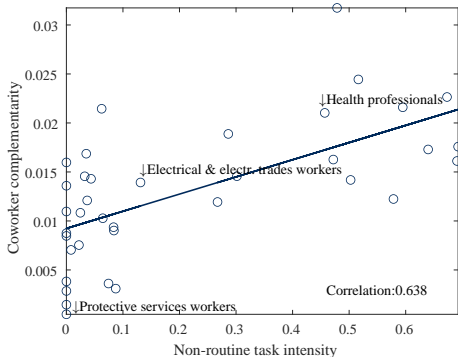
$$\text{Var}(\bar{\alpha}_{j(it)}) / \text{Var}(\alpha_i)$$



Occupations: task complexity \Rightarrow complementarity \Rightarrow sorting

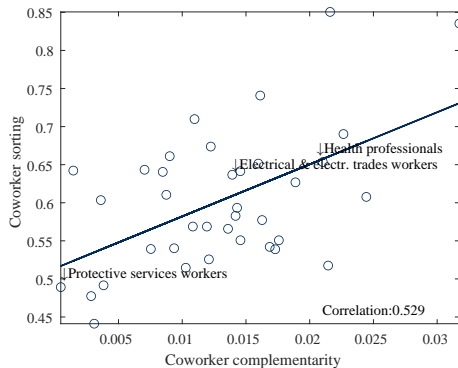
[Main](#)

- \uparrow **Non-routine abstract task intensity**
 $\Rightarrow \uparrow$ **coworker wage complementarity**



Notes. Quadros de Pessoal microdata. Horizontal axis indicates occupation's reliance on non-routine, abstract (NRA) tasks [Mihaylov and Tidens, 2019].

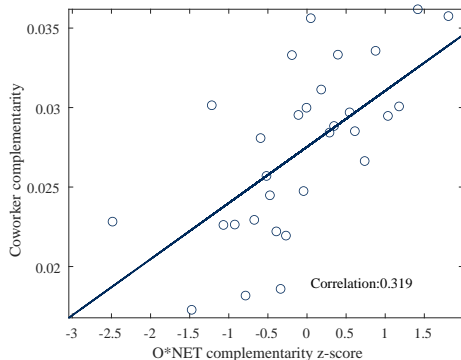
- \uparrow **Coworker wage complementarity**
 $\Rightarrow \uparrow$ **coworker sorting**



Industries: coworker importance \Rightarrow complementarity \Rightarrow sorting

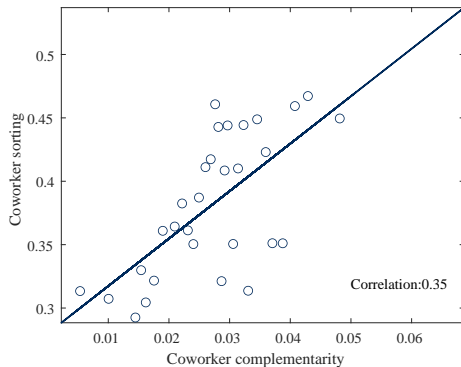
[▶ Main](#)

- \uparrow **Teamwork** [*Bombardini et al., 2012*]
 $\Rightarrow \uparrow$ **coworker wage complementarity**



Notes. Horizontal axis measures the industry-level weighted mean score of an occupation-level index constructed from O*NET measuring the importance of: teamwork, impact on coworker output, communication, and contact.

- \uparrow **Coworker wage complementarity**
 $\Rightarrow \uparrow$ **coworker sorting**

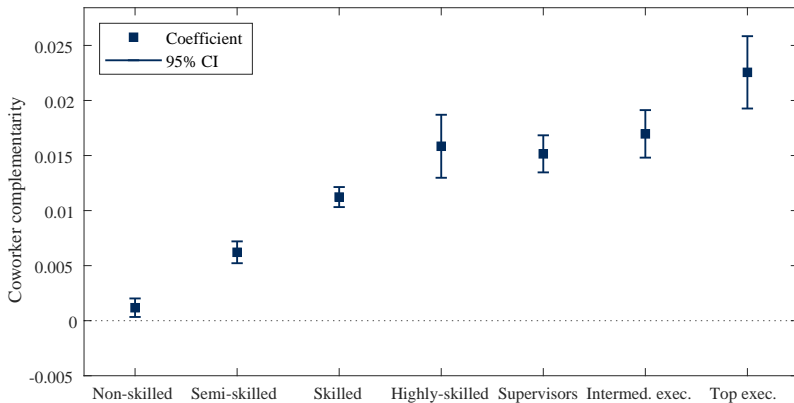


Notes. NACE-4-digit industries.

Hierarchies: complexity \Rightarrow complementarities

[Main](#)[Occupations](#)

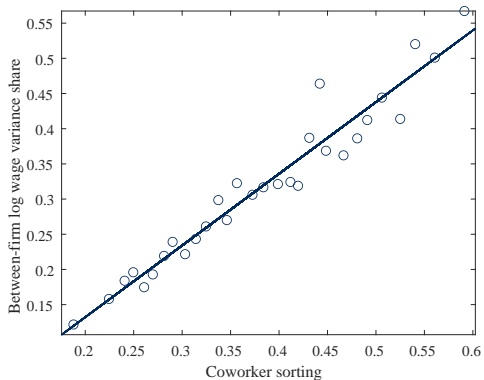
\Rightarrow Coworker wage complementarities are (weakly) \uparrow in the layer of a firm's hierarchy



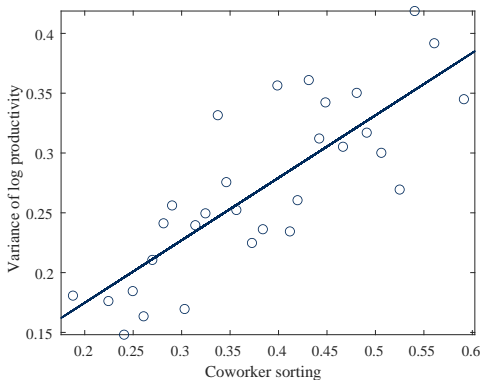
Industries: coworker sorting \Rightarrow between-firm inequality

[▶ Main](#)

\Rightarrow Measures of between-firm inequality in productivity and pay are increasing in the degree of coworker sorting at the industry-level.



(a) Between-firm share of wage dispersion



(b) Productivity dispersion

Cluster-based methodology: motivation

- **Standard AKM approach** estimates large number of firm-specific parameters, identified solely off worker mobility \Rightarrow incidental parameters problem \approx limited mobility bias $\Rightarrow \text{var}(\psi) \uparrow$ & $\text{cov}(\psi, \alpha) \downarrow$
- **Bonhomme, Lamadon, and Malresa (2019, Ecma)**: two-step grouped fixed-effects estimation
 - 1 Recover firm classes using k-means clustering, based on the similarity of their earnings distribution
 - 2 Estimate parameters of correlated random effects model by maximum likelihood, conditional on the estimated firm classes
- **Potential advantages**
 - 1 mitigate limited mobility bias
 - sufficient number of workers who move between any given cluster to identify the cluster fixed effects
 - 2 allows relaxing sample restrictions (n -connected set restriction when estimating

Cluster-based methodology: implementation

- Obtain clusters by solving **weighted k-means problem**

$$\min_{k(1), \dots, k(J), H_1, \dots, H_K} \sum_{j=1}^J n_j \int (\hat{F}_j(w) - H_{K_j}(w))^2 d\mu(w),$$

- $k(1), \dots, k(J)$: partition of firms into K known classes;
 - \hat{F}_j : empirical cdf of log-wages in firm j
 - n_j : average number of workers of firm j over sample period
 - H_1, \dots, H_K : generic cdf's
- Implementation here:
 - baseline value of $K = 10$, as in BLM, but experiment with $K = 20$ and $K = 100$
 - use firms' cdf's over entire sample period on a grid of 20 percentiles
- “Half-BLM”: take step (1), impute class to each worker-year observation, then estimate 2-way FE wage regression using cluster effects instead of firm effects:

Robustness checks: complementarity regression

[▶ Main](#)

- Types from non-parametric ranking algorithm instead of AKM-based
- Non-parametric, FD approximation
- Schooling as a non-wage measure of types
- Lagged types

[▶ Jump](#)[▶ Jump](#)[▶ Jump](#)[▶ Jump](#)

Complementarity estimates using years of schooling

	Dependent variable: wage				
	'85-'92	'93-'97	'98-'03	'04-'09	'10-'17
Interaction	0.0063 0.0008	0.0060 0.0007	0.0099 0.0008	0.0112 0.0007	0.0129 0.0009
Fixed effects	Yes	Yes	Yes	Yes	Yes
Obs. (100,000s)	3,613	2,508	2,694	3,836	4,376
R^2	0.5033	0.5451	0.5746	0.6330	0.6425

Notes. Dependent variable is the wage level over the year-specific average wage. Independent variables are a constant, years of schooling, coworker years of schooling, and the interaction between those two terms. All regressions include industry-year, occupation-year and employer fixed effects. Employer-clustered standard errors are given in parentheses. Observations are unweighted. The sample is unchanged from the main text, except that 96,517 observations with missing years of schooling are dropped. Observation count rounded to 100,000s.

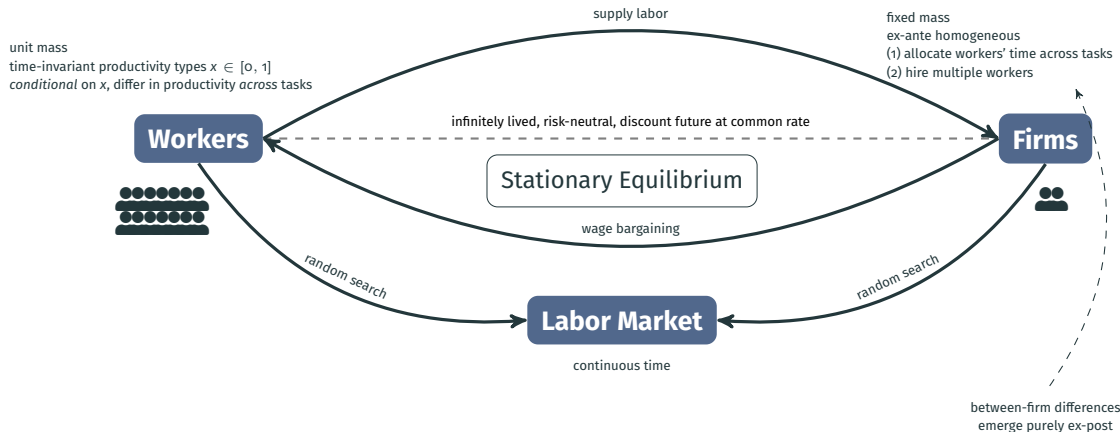
Sorting & complementarity based on non-parametric ranking algorithm

- Instead of ranking workers based on AKM worker FEs, use non-parametric ranking algorithm *[Hagedorn et al., 2017]*

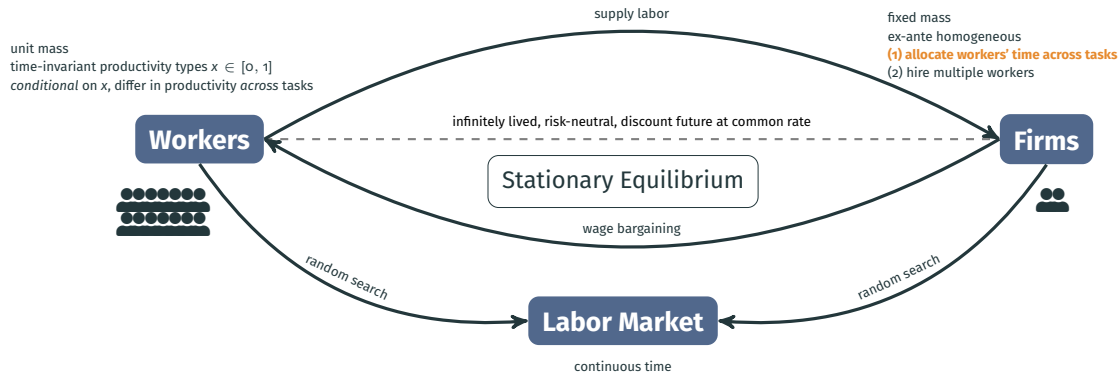
Period	Sorting		Complementarities	
	Spec. 1	Spec. 2	Spec. 1	Spec. 2
1985-1992	0.47	0.38	0.001	0.000
1993-1997	0.56	0.46	0.002	0.001
1998-2003	0.60	0.48	0.004	0.002
2004-2009	0.65	0.50	0.005	0.002
2010-2017	0.68	0.51	0.005	0.004

Notes. This table indicates, under the column “Sorting” the correlation between a worker’s estimated type and that of their average coworker, separately for five sample periods. The column “Complementarities” indicates the point estimate of the regression coefficient β_c . Under “Specification 1” workers are ranked economy wide,

Overview of model environment: firm organization meets labor search



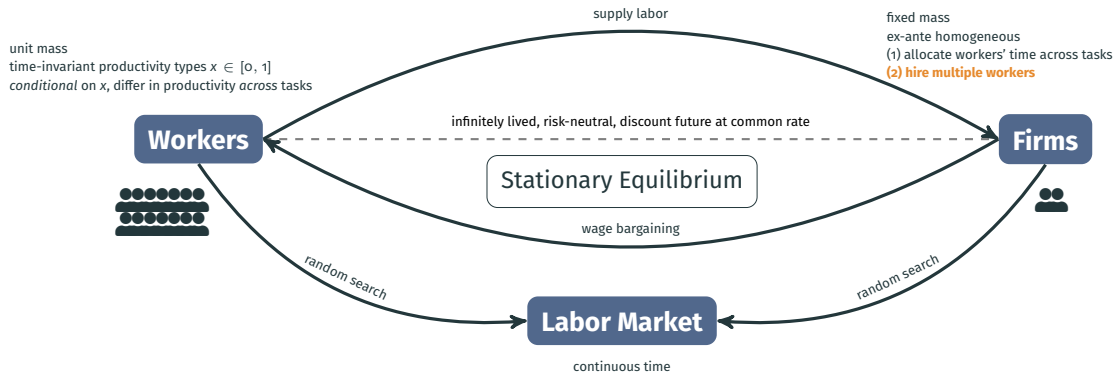
Overview of model environment: firm organization meets labor search



• Analyze in 2 steps

- 1 task assignment: derive production function for *one* firm, treating workforce as exogenous

Overview of model environment: firm organization meets labor search



• Analyze in 2 steps

- 1 task assignment: derive production function for *one* firm, treating workforce as exogenous

Illustration: Fréchet distribution

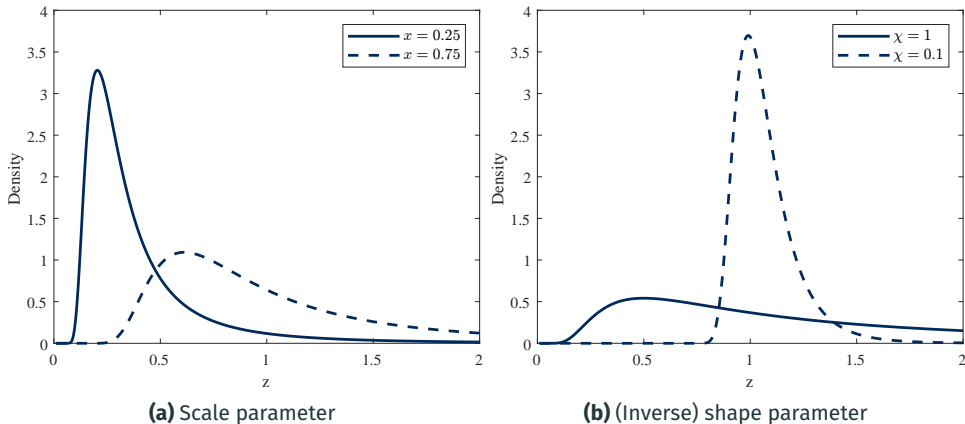


Figure 1: Illustration of properties of Fréchet distribution

Taylor approximation to CES

[▶ Back to team production](#)
[▶ Back to stylized model](#)

- Analytically tractable version of the hiring block:

$$f(x_1, x_2) = x_1 + x_2 - \xi(x_1 - x_2)^2$$

where parameter ξ controls the degree of complementarity

- Justification: CES and link to team production model
 - in the $\kappa = 1$ special case, ξ maps onto $\frac{\chi}{\chi+1}$ (up to scale)

Remark (Second-order Taylor approximation to CES)

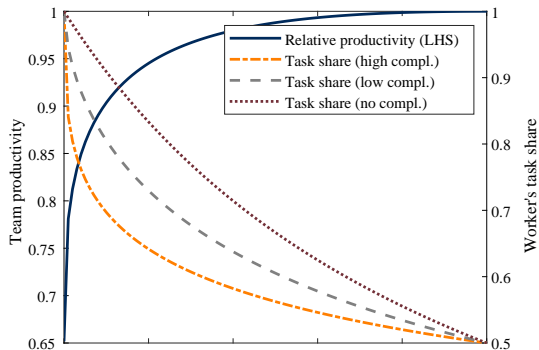
The second-order Taylor approximation to $f(x_1, x_2) = (\frac{1}{2}x_1^\gamma + \frac{1}{2}x_2^\gamma)^{1/\gamma}$ around (\bar{x}, \bar{x}) with $\bar{x} = \frac{x_1+x_2}{2}$ is

$$\bar{x} - \underbrace{\frac{1}{2}(1-\gamma)}_{\approx \xi} \frac{\sigma_x^2}{\bar{x}},$$

where $\sigma_x^2 = (\frac{x_1-x_2}{2})^2$.

Quality mismatch & task mismatch

- When the high-quality worker is paired with a low-quality worker, she ends up “wasting time” on tasks that she is relatively less efficient in and wouldn’t have to do if teamed up optimally.



Extension: team production with communication costs

- Assumption till now: division of labor incurs no output losses due to coordination frictions
- But implementing the division of labor may \downarrow time available for task production because of *communication* requirements

[Becker & Murphy, 1992; Deming, 2017]

- **Extension** allowing for such **coordination costs** shows:
 - ① qualitative link between technology & coworker complementarities exists *unless* division of labor is prohibitively costly
 - ② $\chi \uparrow \Rightarrow$ importance of organizational quality for productivity \uparrow
- Ongoing research: rich microdata from Fortune-100 company to describe communication behavior

Extension: specialization vs. comparative advantage

[▶ Main](#)

- Suppose the joint distribution of task-specific productivities across coworkers satisfies

$$\Pr [z_i(\tau) \leq z_1, \dots, z_n(\tau) \leq z_n] = \exp \left[- \left(\sum_{i=1}^n \left(\left(\frac{z}{\iota x_i} \right)^{-\frac{1}{\tilde{\chi}}} \right)^{\frac{1}{\xi}} \right)^{\xi} \right], \quad (6)$$

where $\tilde{\chi}$ is the common shape parameter of the the marginal Fréchet distributions, while $\xi \in (0, 1]$ controls correlation in draws across workers

- Can derive

$$Y = f(\mathbf{x}; \tilde{\chi}, \xi) = n^{1+\tilde{\chi}\xi} \left(\frac{1}{n} \sum_{i=1}^n (a_i x_i)^{\frac{1}{\tilde{\chi}\xi+1}} \right)^{\tilde{\chi}\xi+1} \quad (7)$$

- All results go through with $\chi = \tilde{\chi}\xi$

Numerical example: setup

- **H, H'**: avg productivity 100 vs. **L, L'**: avg. productivity 50
- Working alone: realized productivity equal to avg
- **Low value of χ**

	Ideation	Theory	Matlab	Stata	Write	Present	With L'	With H'	Δ
L	53	52	51	49	48	47			
H	106	104	102	98	96	94			

- **High value of χ**

	Ideation	Theory	Matlab	Stata	Write	Present	With L'	With H'	Δ
L	65	60	55	45	40	35			
H	130	120	110	90	80	70			

Numerical example: teaming up with L'

- Low value of χ

	Ideation	Theory	Matlab	Stata	Write	Present	With L'	With H'	Δ
L	53	52	51	49	48	47	52		
H	106	104	102	98	96	94	101.2		
L'	47	48	49	51	52	53			

- High value of χ

	Ideation	Theory	Matlab	Stata	Write	Present	With L'	With H'	Δ
L	65	60	55	45	40	35	60		
H	130	120	110	90	80	70	112.5		
L'	35	40	45	50	55	60			

Numerical example: teaming up with H'

- Low value of χ

	Ideation	Theory	Matlab	Stata	Write	Present	With L'	With H'	Δ
L	53	52	51	49	48	47	52	53	1
H	106	104	102	98	96	94	101.2	104	2.8
L'	47	48	49	51	52	53			
H'	94	96	98	102	104	106			

- High value of χ

	Ideation	Theory	Matlab	Stata	Write	Present	With L'	With H'	Δ
L	65	60	55	45	40	35	60	62.5	2.5
H	130	120	110	90	80	70	112.5	120	7.5
L'	35	40	45	50	55	60			
H'	70	80	90	110	120	130			

Numerical example: comparison and take-aways

Low value of χ	With L'	With H'	Δ
L	52	53	1
H	101.2	104	2.8
Δ			1.8

High value of χ	With L'	With H'	Δ
L	60	62.5	2.5
H	112.5	120	7.5
Δ			5

Takeaways

- ① Higher χ : greater benefit from team production (“love for variety”)
- ② Higher χ : coworker quality matters more for own realized productivity
- ③ Higher χ : stronger complementarity – benefit from better coworker for H grows more than that for L

Task allocation

Corollary: Task shares

Suppose that $x_i > x_j$. Then (i) i performs a strictly larger share of tasks than j for $\chi < \infty$; (ii) the difference in task shares is decreasing in χ ; (iii) the ratio of i 's relative to j 's task share approaches $\frac{x_i}{x_j}$ as $\chi \rightarrow 0$; and (iv) task shares are equalized as $\chi \rightarrow \infty$.

Proof:

All four elements of the statement immediately follow by noting that Lemma 1 (ii) implies that $\frac{\pi_i}{\pi_j} = \left(\frac{x_i}{x_j}\right)^{\frac{1}{1+\chi}}$.

Extension: multivariate Fréchet

- Baseline: worker-task specific productivities are *independent* draws from Fréchet,

$$G_i(z) := \Pr(z_i(\tau) \leq z) = \exp \left(- \left(\frac{z}{\iota X_i} \right)^{-1/\chi} \right)$$

- χ plays dual role: within-worker dispersion and (within-firm) across-worker dispersion

- Instead, suppose that *joint* distribution across workers

$$P[z_i(\tau) \leq z_1, \dots, z_n(\tau) \leq z_n] = \exp \left[- \left(\sum_{i=1}^n \left(\left(\frac{z}{\iota X_i} \right)^{-\frac{1}{\chi}} \right)^{\frac{1}{\xi}} \right)^{\xi} \right]$$

- χ : now clearly defined as referring to one workers' productivity profile over tasks
- $\xi \in (0, 1]$: captures correlation in draws *across* workers

Revised aggregation result

- Revised Lemma 1: implied task share and shadow-cost index

$$\pi_i = \frac{(x_i/\lambda_i^L)^{\frac{1}{\chi\xi}}}{\sum_{k=1}^n (x_k/\lambda_k^L)^{\frac{1}{\chi\xi}}}$$

$$\lambda = \left(\sum_{i=1}^n \left(\frac{a_1 x_i}{\lambda_i^L} \right)^{\frac{1}{\chi\xi}} \right)^{-\chi\xi}$$

Proposition: Aggregation result

Team output Y can be written as a function

$$Y = f(\mathbf{x}; \chi, \xi) = n^{1+\chi\xi} \left(\frac{1}{n} \sum_i (a_1 x_i)^{\frac{1}{\chi\xi+1}} \right)^{\chi\xi+1}$$

Dynamic setting

- **Intuition:** in a *dynamic* setting, ξ is naturally endogenous to firms' hiring decisions: to maximize production, a firm will *aim* to hire workers whose skill vector has a low correlation with existing employee(s)
- **Timing:**
 - an unmatched worker x randomly draws (i) a searching firm with current employee x' , and together they draw (ii) a 'horizontal match-quality shock' ξ from an exogenous distribution H
 - we treat the task-specific skills as fully observable at time of meeting ('inspection good'), i.e. no gradual learning about horizontal match takes place over time
 - given the aggregation result, (x, x', ξ) are sufficient to pin down Y – in the 'background', the firm optimally organizes production – and the matching decision is made

Sketch of dynamic model (1)

- Define

$$\Omega_1(x) = V_{f,1}(x) + V_{e,1}(x)$$

$$S(x) = \Omega_1(x) - V_{f,0} - V_u(x)$$

$$\Omega_2(x, x', \xi) = V_{f,2}(x, x', \xi) + V_{e,2}(x|x', \xi) + V_{e,2}(x'|x, \xi)$$

$$S(x|x', \xi) = \Omega_2(x, x', \xi) - \Omega_1(x') - V_u(x)$$

- Hiring policy for second worker: $h(x|x', \xi) = 1\{S(x|x', \xi) > 0\}$
 - $h(x|x') = \Pr\{S(x|x', \xi) > 0\}$
 - $h(x) = 1\{S(x) > 0\}$
- Evolution of distribution: as before, given probabilistic definition of $h(x|x')$

Sketch of dynamic model (2): values

- Value of unmatched firm:

$$\rho V_{f.o} = (1 - \omega) \lambda_{v.u} \int \frac{d_u(x)}{u} S(x)^+ dx$$

- Value function of unmatched worker

$$\rho V_u(x) = b(x) + \lambda_u \omega \left[\int \frac{d_{f.o}}{v} S(x)^+ + \int \frac{d_{m.1}(\tilde{x}')}{v} S(x|\tilde{x}', \tilde{\xi})^+ dH(\tilde{\xi}) d\tilde{x}' \right] \quad (8)$$

- Joint value of a firm with employee x

$$\rho \Omega_1(x) = f_1(x) - \delta S(x) + \lambda_{v.u} (1 - \omega) \int \frac{d_u(\tilde{x}')}{u} S(\tilde{x}'|x, \tilde{\xi})^+ dH(\tilde{\xi}) d\tilde{x}' \quad (9)$$

- Joint value of firm with (x, x', ξ)

Sketch of dynamic model (3): solution approach

► OJS: a challenge

- Define *threshold values* of ξ for each (x, x') such that $S(x|x', \bar{\xi}(x|x')) = 0$. As this equation is quadratic in ξ , there are 0, 1 or 2 in the interval $[0, 1]$:

- $0 = S(x|x', 1) + \frac{f(x, x', \bar{\xi}(x|x')) - f(x, x', 1)}{\rho + 2\delta}$
- $\bar{\xi}$: $\xi \in (0, 1] : S(x|x', \bar{\xi}(x|x')) > 0$ if $\xi > \bar{\xi}(x|x')$
- $\underline{\xi}$: $\xi \in (0, 1] : S(x|x', \underline{\xi}(x|x')) > 0$ if $\xi < \underline{\xi}(x|x')$

- Implied *conditional expected values*:

- $\bar{\xi}^*(k) = \frac{\int_k^1 \xi dH(\xi)}{1 - H(k)}$
- $\underline{\xi}^*(k) = \frac{\int_0^k \xi dH(\xi)}{H(k)}$

- Then rewrite eg equation (8)

$$\rho V_u(x) = b(x) + \lambda_u \omega \left[\int \frac{d_{f,0}}{v} S(x)^+ + \int \frac{d_{m,1}(\tilde{x}')}{v} S^*(x|\tilde{x}') \right] d\tilde{x}' \quad (11)$$

where $S^*(x|\tilde{x}') = \left[H(\underline{\xi}(x|\tilde{x}')) \times S(x|\tilde{x}', \underline{\xi}^*(\underline{\xi}(x|\tilde{x}'))) + (1 - H(\bar{\xi}(x|\tilde{x}'))) \times S(x|\tilde{x}', \bar{\xi}^*(\bar{\xi}(x|\tilde{x}')) \right]$

Environment: demographics & preferences & production technology

- **Time:** continuous
- **Agents:** workers & firms
 - unit mass of workers, types uniformly distributed $x \in \mathcal{X} = [0, 1]$
 - m_f mass of firms – ex-ante homogeneous
 - agents indexed by *ranks* of their respective productivity distribution, hence uniform type distribution [Hagedorn et al., 2017]
 - all agents are infinitely-lived, risk-neutral, share a common discount rate ρ , max. the present value of payoffs
- **Production technology:** firms are vacant or have 1 or 2 workers
 - normalize team size to max. $n = 2 \leftarrow$ key is “existing workforce” & “potential hire”
 - convention: from x 's perspective, let x' denote *coworker*
 - team production: $f(x, x') \leftarrow$ see microfoundation

Environment: random search & wage bargaining

- **Timing** within dt -intervals
 - ① exogenous separation: Poisson rate δ
 - ② random search & matching
 - ③ production & surplus sharing
- **Meeting process:** unemployed meet *some* firm at Poisson rate λ_u
 - probability for a firm to be contacted by an(y) unemployed: $\lambda_{v,u} = \lambda_u \times u$
 - baseline: no on-the-job search
- **Matching** decisions based on joint surplus b/w firm & worker(s): privately efficient
[cf. Bilal-Engbom-Mongey-Violante, 2021]
- **Surplus sharing:** firm bargains with potential new hire, taking into account coworker complementarities; worker bargaining power ω

Stationary equilibrium

- Formally, after defining (i.) HJBs for unemployed & vacant & surplus values, and (ii.) Kolmogorov Forward Equations (KFEs) describing the evolution of the distribution of agents across states:

Definition:

A stationary search equilibrium is a tuple of value functions together with a stationary distribution of agents across states such that (i.) the value functions satisfy the HJB Equations given the distribution; and (ii.) the distribution satisfies the KFEs given the policy functions implied by the value functions.

- Needs to be computed **numerically**
 - agents' expectations & decisions must conform w/ population dynamics to which they give rise; as distribution evolves, so do agents' expectations

Environment: firm & worker states

- Distribution across states for a **worker** type x :

$$d_w(x) = d_u(x) + d_{m.1}(x) + \int d_{m.2}(x, \tilde{x}') d\tilde{x}'$$

- $d_u(x)$: ‘density’ of unemployed of type x
- $d_m(x)$, shorthand for $d_m(x, \emptyset)$: ‘density’ of matches w/ x as only worker
- $d_m(x, x')$: ‘density’ of “team matches” b/w x and x'

- Distribution across states for a **firm** type y :

$$d_f = d_{f.o} + \int d_{m.1}(x) dx + \frac{1}{2} \int \int d_{m.2}(x, x') dx dx'.$$

- $\frac{1}{2}$: account for 1 firm having 2 workers

- **Aggregates** can be backed out, e.g. $u = \int d_u(x) dx$

Environment: surplus sharing

- **Desiderata:**

- ① hiring decisions are based on the joint value to all parties affected and, thus, constrained Pareto efficiency in allocations
- ② order of hiring does not matter
- ③ wages continuously renegotiated

- **One-worker firm:**

$$(1 - \omega)(V_{e.1}(x) - V_u(x)) = \omega(V_{f.1}(x) - V_{f.0}) \quad (13)$$

Two-worker firm

$$(1 - \omega)(V_{e.2}(x|x') - V_u(x)) = \omega(V_{e.2}(x'|x) + V_{f.2}(x, x') - V_{e.1}(x') - V_{f.1}(x')) \quad (14)$$

HJB: unmatched

[▶ Main](#)

- Unmatched firm:

$$\rho V_{f.o} = (1 - \omega) \lambda_{v.u} \int \frac{d_u(x)}{u} S(x)^+ dx \quad (15)$$

- Unmatched worker:

$$\rho V_u(x) = b(x) + \lambda_u \omega \left[\int \frac{d_{f.o}}{v} S(x)^+ + \int \frac{d_{m.1}(\tilde{x}')}{v} S(x|\tilde{x}')^+ d\tilde{x}' \right] \quad (16)$$

Key hiring decision

- **Notation:**

- Value functions for unemployed worker, $V_u(x)$; worker x employed with coworker x' , $V_e(x|x')$
- Value functions for vacant firm, $V_{f.o}$; firm producing with x and x' : $V_f(x, x')$
- $d_u(x)$: density of unemployed workers of type x

- **Key question:** which type(s) of workers is a firm that already has one worker willing to hire – trading off match quality vs. cost of searching

- HJB for the **joint value** of a firm with worker x , $\Omega(x)$

$$\begin{aligned} \rho\Omega(x) = & f(x) + \delta[-\Omega(x) + V_u(x) + V_{f.o}] \\ & + \lambda_{v.u} \int \frac{d_u(\tilde{x}')}{u} \underbrace{\max\{-\Omega(x) + V_e(x|\tilde{x}') + V_f(x, \tilde{x}'), 0\}}_{=(1-\omega)S(\tilde{x}'|x)^+} d\tilde{x}' \end{aligned}$$

HJB: surpluses

- Surplus of coalition of firm with worker x

$$(\rho + \delta)S(x) = f_1(x) - \rho(V_u(x) + V_{f.o}) + \lambda_{v.u}(1 - \omega) \int \frac{d_u(\tilde{x}')}{u} S(\tilde{x}'|x)^+ d\tilde{x}' \quad (17)$$

- Surplus from adding x to x'

$$S(x|x')(\rho + 2\delta) = f_2(x, x') - \rho(V_u(x) + V_u(x') + V_{f.o}) + \delta S(x) - (\rho + \delta)S(x') \quad (18)$$

KFE: unemployed

$$\delta \left(d_{m.1}(x) + \int d_{m.2}(x, \tilde{x}') d\tilde{x}' \right) = d_u(x) \lambda_u \left(\int \frac{d_{f.o}}{v} h(x, \tilde{y}) + \int \frac{d_{m.2}(\tilde{x}')}{v} h(x|\tilde{x}') d\tilde{x}' \right). \quad (19)$$

KFE: one-worker matches

$$d_{m.1}(x) \left(\delta + \lambda_{v.u} \int \frac{d_u(\tilde{x}')}{u} h(\tilde{x}'|x) d\tilde{x}' \right) = d_u(x) \lambda_u \frac{d_{f.o}}{v} h(x) + \delta \int d_{m.2}(x, \tilde{x}') d\tilde{x}'. \quad (20)$$

KFE: two-worker matches

$$2\delta d_{m.2}(x, x') = d_u(x)\lambda_u \frac{d_{m.1}(x')}{v} h(x|x') + d_u(x')\lambda_u \frac{d_{m.1}(x)}{v} h(x'|x). \quad (21)$$

Lemma: monotonicity of unemployment value and wage in x

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Lemma: Monotonicity

Assume that $\frac{f_1(x)}{\partial x} > 0$, $\frac{\partial f_2(x, x')}{\partial x} > 0$, and $\omega > 0$. Then: (i) The value of unemployment $V_u(x)$ is monotonically increasing in x , and (ii) so is the wage function $w(x|x')$.

Proof:

See paper appendix. Key: surplus representation.

Wage equation

- NB: here allow for ex-ante firm heterogeneity \Rightarrow measurement result extends

$$\begin{aligned}
 w(x|y, x') &= \rho V_u(x) + \omega [f(x, y, x') - \rho (V_u(x) + V_u(x') + V_{f.o}(y))] \\
 &\quad + \delta S(x|y) - (\rho + \delta) S(x'|y)] - \delta \omega S(x|y) \\
 &= \omega (f(x, y, x') - f(x', y)) + (1 - \omega) \rho V_u(x) \\
 &\quad - \omega(1 - \omega) \lambda_{v.u} \int \frac{d_u(\tilde{x}'')}{u} S(\tilde{x}''|y, x')^+ d\tilde{x}'' .
 \end{aligned}$$

Characterization using a stylized model: setup

Intuition for how coworker complementarities shape matching can be gained from a stylized model → closed-form solutions

- **Simplified setup:**

- no ex-ante firm heterogeneity; mass of firms $m_f = \frac{1}{2}$
- production: $f(x, x') = x + x' - \xi(x - x')^2$, where ξ controls complementarity
 - ▶ Approx. of microfounded team prod. fn.
- no production with 1 employee & abstract from team production benefits
- firm has no bargaining power, workers each receive outside option plus half the surplus

- **Explicit search costs:** no discounting, guaranteed match ($M_u = M_f = 1$); but type-invariant worker search costs c
 - supermodularity in f suffices for PAM [Atakan, 2006]

Characterization using a stylized model: timing

► Frictionless stage-2 outcome

- Each firm is randomly paired with one worker $x' \in \mathcal{X}$
 - remaining: mass $\frac{1}{2}$ of uniformly distributed workers; mass $\frac{1}{2}$ of firms with one employee
- 1 Each (firm + x') unit is randomly paired with a worker $x \in \mathcal{X} \rightarrow$ decision:
 - a. **match**: form a team
 - + produce & share production value
 - no further actions and zero payoff in stage 2

or
 - b. **search**: don't form a team
 - workers pay search cost c
 - + all have opportunity to re-match in stage 2, s.t.
- 2 frictionless matching b/w unmatched firms & workers; production
 - pure PAM: x works with x (\leftarrow deterministic coupling $\mu(x) = x$)
 - payoffs given pure PAM: $w^*(x) = x$ and $v^* = 0$

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 - pure PAM: x works with x (\leftarrow deterministic coupling $\mu(x) = x$)
 - payoffs given pure PAM: $w^*(x) = x$ and $v^* = 0$

Characterization using a stylized model: stage-1 matching decision

- A firm with employee x' that meets a worker of type $x \in \mathcal{X}$ decides to hire her, i.e. $h(x, x') = 1$, if

$$\underbrace{f(x, x')}_{\text{match}} - \underbrace{\left[w^*(x) + w^*(x') + v^* - 2c^w \right]}_{\text{search}} > 0$$

$\equiv S(x, x')$

- Threshold distance** s^* s.t. $h(x, x') = 1 \Leftrightarrow |x' - x| < s^*$
- Threshold distance satisfies: $s^* = \sqrt{2c/\xi}$
 - greater complementarities (ξ) render the matching set narrower**
 - greater search costs (c) render the matching set *wider*

Characterization using a stylized model: corollary

Corollary: Stylized model

For a given threshold s , which is *decreasing* in χ :

- 1 the coworker correlation is: $\rho_{xx} = (2s + 1)(s^2 - 1)^2$;
- 2 the average coworker type is

$$\hat{\mu}(x) = \begin{cases} \frac{x+s^*}{2} & \text{for } x \in [0, s^*) \\ x & \text{for } x \in [s^*, 1 - s^*] \\ \frac{1+x-s^*}{2} & \text{for } x \in (1 - s^*, 1]. \end{cases}$$

- 3 the between-firm share of the variance of wages is decreasing in s

Frictionless matching: assignment and payoffs

- Working backwards, pin down frictionless payoffs that determine the outside option
- The equilibrium of the frictionless model can be derived in many ways
- Equilibrium assignment and payoffs:
 - PAM: $\mu(x) = x$ given *supermodular* $f(x_1, x_2)$
 - wage schedule obtained from integrating over FOC

$$w^*(x) = \int_0^x f_x(\tilde{x}, \mu(\tilde{x})) d\tilde{x}$$

where integration constant is zero due to $f(0, 0) = 0$

- firm payoffs in this formulation are 0
- Given $f(x_1, x_2) = x_1 + x_2 - \gamma(x_1 - x_2)^2$, with $\gamma > 0$, we have

$$\mu(x) = x$$

$$w^*(x) = x \quad \text{and} \quad v^* = 0$$

Characterization results: conditional distribution

Lemma: Conditional type distribution

Given a threshold distance s , the conditional distribution of coworkers for $x \in \mathcal{X}$ is

$$\Phi(x'|x) = \begin{cases} 0 & \text{for } x' < \sup\{0, x - s\} \\ \frac{x - \sup\{0, x - s\}}{\inf\{x + s, 1\} - \sup\{0, x - s\}} & \text{for } x' \in [\sup\{0, x - s\}, \inf\{x + s, 1\}] \\ 1 & \text{for } x' > \inf\{x + s, 1\} \end{cases}$$

Characterization results: between-share of wage variance

Corollary:

Given a threshold distance s and a value of γ , the between-firm share of the variance of wages is equal to

$$\frac{-\frac{13\gamma^2 s^5}{2400} + \frac{\gamma^2 s^4}{80} + \frac{5s^3}{36} - \frac{s^2}{6} + \frac{1}{12}}{\frac{\gamma^2 s^4}{45} - \frac{4897\gamma^2 s^5}{10800} - \frac{\gamma^2 s^6}{324} + \frac{19\gamma^2 s^5 \ln(2)}{30} + \frac{1}{12}}.$$

Overview of validation exercises: direction EE transitions & cross-section

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- 2 additional types of validation exercise:

- ✓ **EE transitions** reallocate workers to more + assortative matches
- do model-implied relationships also hold in **cross-section**?

[▶ Details](#)

① $\chi \uparrow \Rightarrow$ coworker complementarity \uparrow

② coworker complementarity $\uparrow \Rightarrow$ + assortative matching \uparrow

can test predictions *because*
we have measures of comple-
mentarity!

- Implementation of cross-sectional exercises: rich Portuguese micro data

- universe of private-sector actors, employer-employee data & income statements

- Cross-sectional exercises:

- ✓ **Hierarchies**
- ✓ **Industries**
- ✓ **Occupations**

[▶ Details](#)[▶ Details](#)[▶ Details](#)

Parameterization, including estimation results

[▶ Main](#)

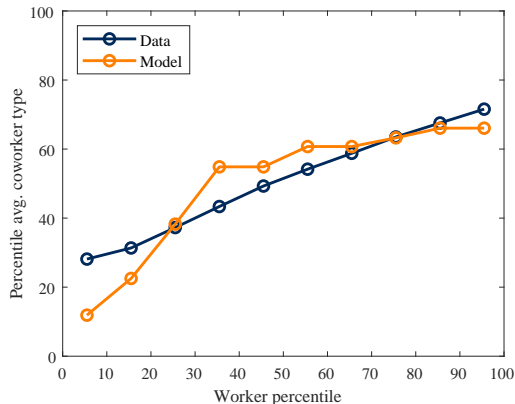
Parameter	Description	Targeted moment	Value	m	\hat{m}
γ	Elasticity of complementarity	$\hat{\beta}_c$	0.837	0.0091	0.0091
a_0	Production, constant	Avg. wage (norm.)	0.239	1	1
a_1	Production, scale	Var. log wage	1.557	0.241	0.241
b_1	Replacement rate, scale	Replacement rate	0.664	0.63	0.63
δ	Separation hazard	Job loss rate	0.008	0.008	0.008
λ_u	Meeting hazard	Job finding rate	0.230	0.162	0.162
ρ	Discount rate	External	0.008		
ω	Worker bargaining weight	External	0.50		
a_2	Production, team advantage	External	1.10		

Notes. This table lists for each of the estimated parameters, the targeted moment, the estimated value, and moment values in data (\hat{m}) and model (m). In the quantitative model, the production function includes a con-

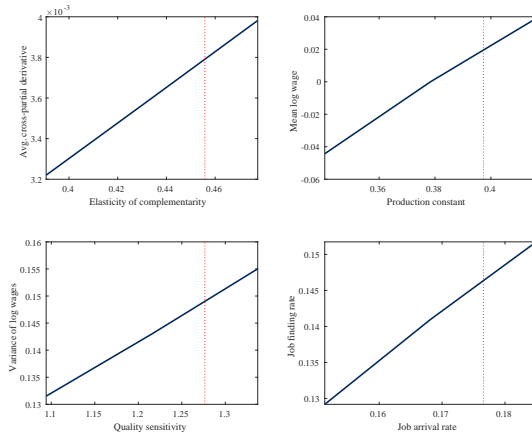
Validation: model reproduces empirical coworker sorting patterns

[▶ Main](#)

- Key untargeted moment (1):
coworker sorting patterns
- Coworker correlation matches data well, $\rho_{xx} = 0.53$ (vs. 0.62 in the data)
- Model slightly underestimates the quality of coworkers at both bottom and top
 - OJS will help

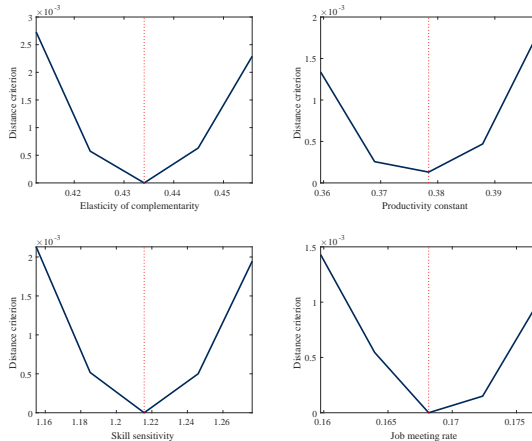


Identification validation exercise 1

[▶ Main](#)

Notes. This figure plots the targeted moment against the relevant parameter, holding constant all other parameters.

Identification validation exercise 2



Notes. This figure plots the distance function $\mathcal{G}(\psi_i, \psi_i^*)$ when varying a given parameter ψ_i around the estimated value ψ_i^* . The remaining parameters are allowed to adjust to minimize \mathcal{G} .

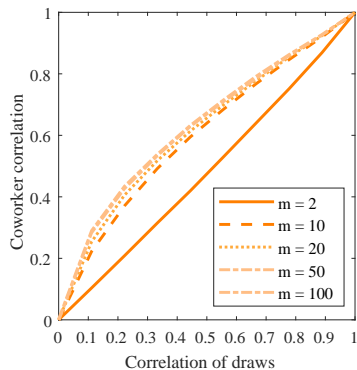
Between-share adjustment procedure (1)

- **Problem.** For any degree of coworker sorting less than unity, i.e. $\rho_{xx} < 1$, the level of the between-share in a model with team size $m = 2$ will be biased upward relative to the case of $m > 2$ and, in particular, $m \rightarrow \infty$ (LLN...)
 - implication 1: upward bias in level
 - implication 2: downward bias in estimated \uparrow between-share as sorting increases
- **Propose** statistical adjustment method
- Consider a random vector $X = (X_1, X_2, \dots, X_m)'$ whose distribution is described by a Gaussian copula over the unit hypercube $[0, 1]^m$, with an $m \times m$ dimensional correlation matrix $\Sigma(\rho^c)$, which contains ones on the diagonal and the off-diagonal elements are all equal to ρ_c
- Interpretation. Each vector of observations drawn from the distributions of X , $x_j = (x_{1j}, x_{2j}, \dots, x_{mj})'$, describes the types of workers in team of size m , indexed by j

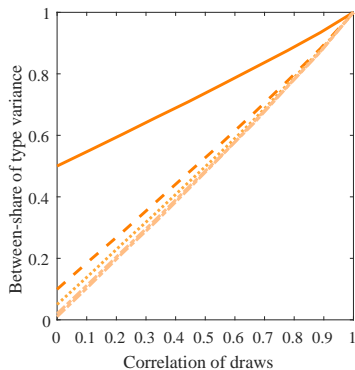
Between-share adjustment procedure (2)

- Since the marginals of the Gaussian copula are simply continuous uniforms defined over the unit interval, the variance of the union of all draws is just $\frac{1}{12}$
- The mean of the elements of X is itself a random variable, \bar{X} . That is, for some realization x_j , we can define $\bar{x}_j = \frac{1}{m} \sum_{i=1}^m x_{ij}$
- The variance of \bar{X} will be $\frac{1}{m^2} \left(\frac{m}{12} + m(m-1) \left(\frac{\rho_c}{12} \right) \right)$
- So $\sigma_{x, \text{between-share}}^2(\rho_c, m) = \frac{1}{m} \left(1 + (m-1)\rho_c \right)$
- Correction-factor = $\frac{1}{2} \left(1 + \rho_c \right) - \frac{1}{\hat{m}} \left(1 + (\hat{m}-1)\rho_c \right)$ where the empirical average size is \hat{m}

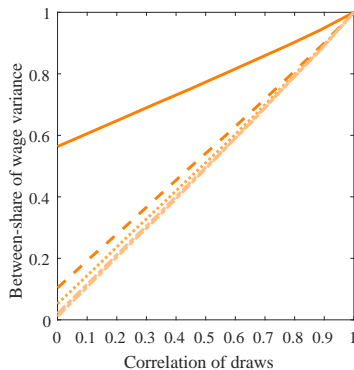
Between-share adjustment procedure (3)



(a) Coworker sorting



(b) Between-share of type var.



(c) Between-share of wage var.

The effect of declining search frictions

- **Model-based concern:** reduction in search frictions could also explain \uparrow coworker sorting
- Yes: job arrival & separation rates estimated to \uparrow from p1 to p2
- **Counterfactual analysis:** explains 6% of empirically observed \uparrow in between-employer share of wage variance

	Δ model	Implied % Δ model due to Δ parameter
Model 1: baseline	0.159	-
Cf. a: fix period-1 complementarity	0.065	59
Cf. b: fix period-1 search frictions	0.150	6

Outsourcing & within-occupation ranking analysis

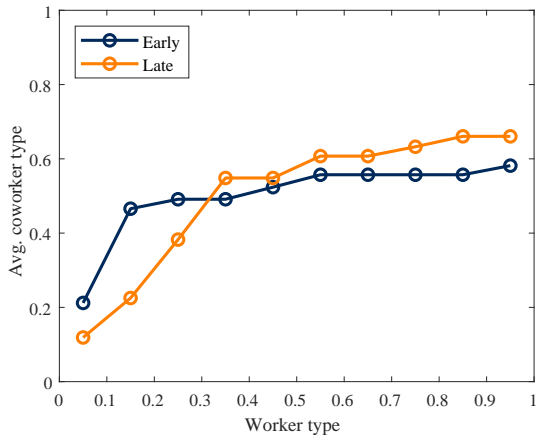
- **Concern:** confounding shifts in labor boundary of firm, e.g. outsourcing
- **Address this concern in multiple steps:**
 - ① empirically rank workers *within* occupation (“good engineer vs. mediocre engineer”)
 - ② empirically re-estimate coworker sorting & complementarity (lower but similar \uparrow)
 - ③ re-estimate model for both periods & re-do counterfactual exercises
- **Result:** qualitatively & quantitatively similar findings

	Δ model	Implied % Δ model due to Δ parameter
Model 2: within-occ. ranking	0.198	-
Cf. a: fix period-1 comp.	0.076	61.47

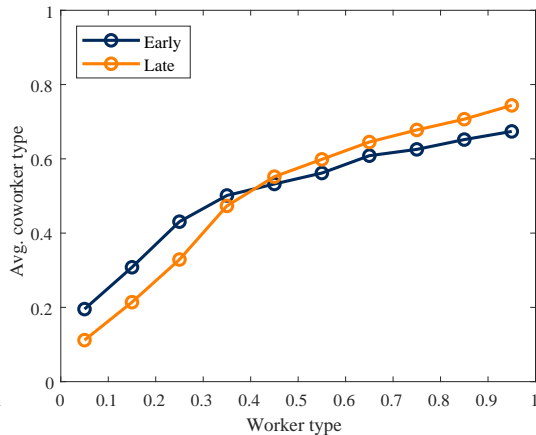
Extension: model with OJS

- Baseline model abstracted from OJS
 - transparent trade-off, connection to closed-form matching results
- **Consider extension to OJS:** employed worker meet vacancies at Poisson rate λ_e
 - wages both off and on the job are continuously renegotiated under Nash bargaining, with unemployment serving as the outside option *[cf. di Addario et al., 2021]*
 - re-estimate, with empirical labor market flows disciplining λ_e
- Qualitative question: is coworker sorting outcome robust, even if workers can switch to better job after accepting job out of unemployment?
- **Analyses:**
 - 1 coworker sorting patterns & changes
 - 2 additional model validation: direction of EE flows in model & data

Model-implied coworker sorting patterns: without and with OJS

[Main](#)

(a) Baseline



(b) OJS

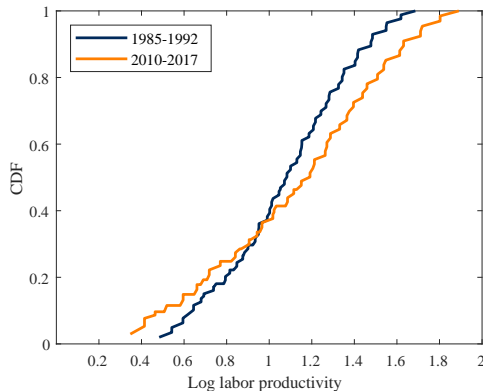
EE transitions in theory and data

- **Theoretical prediction:** EE transitions move workers in surplus-maximizing direction
 $\Rightarrow \Delta \hat{x}_{-it} = \hat{x}_{-i,t} - \hat{x}_{-i,t-1}$ should be *positively* correlated with \hat{x}_i
 - $h_{2.1}(x, x'' | x') = 1$ – worker x in a two-worker firm with coworker x'' would move to an employer that currently has one employee of type x' – if $S(x|x') - S(x|x'') > 0$
- **Empirical analysis:** use SIEED *spell* data to create worker-originMonth-destinationMonth-originJob-destinationJob panel, with information on characteristics of origin and destination job (e.g., coworker quality)
 - subsample period 2008-2013 (huge panel at monthly frequency)
 - count as “EE” if employer change between two adjacent months
- **Regression analysis:** regress $\Delta \hat{x}_{-it}$, scaled by std. σ_{Δ} of coworker quality changes, on *own* type and *origin* coworker type

$$\frac{\Delta \hat{x}_{it}}{\sigma_{\Delta}} = \beta_0 + \beta_1 \hat{x}_i + \beta_2 \hat{x}_{-i,t-1} + \epsilon_{it}$$

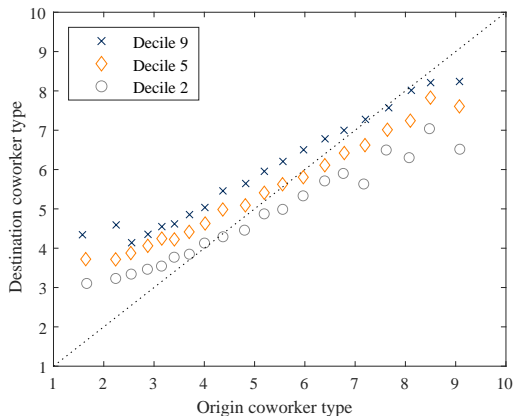
Productivity dispersion

- Firm dynamics literature: increased firm-level productivity dispersion [Autor et al., 2020; de Ridder, 2023]



Empirical coworker sorting changes due to EE moves

- **EE transitions push toward greater coworker sorting:** for any given origin, higher x-workers move to workplaces with better coworkers than lower-x workers do
- *But* in data EE transitions “move up” low types more than theory predicts
- “**Coworker job ladder**” with both absolute and type-specific dimension?
- **Next:** change in the job ladder [e.g., Haltiwanger-Spetzler, 2021]



Evidence that EE *increasingly* reallocate toward PAM: in data & model

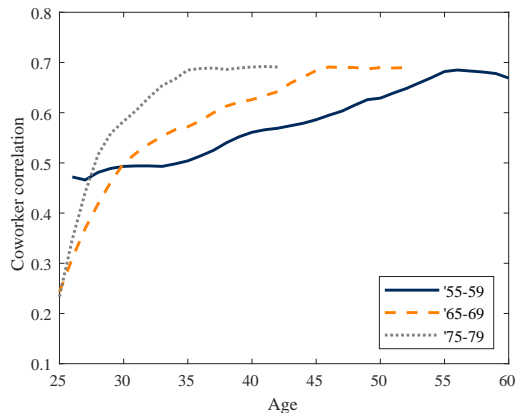
	Data		Model	
<i>Change in coworker type</i>	'85-'92	'10-'17	Period-1	Period-2
Own type	0.0883 ^{***} (0.000799)	0.118 ^{***} (0.000918)	0.214	0.270
Controls	Year FEs, Origin	Year FEs, Origin	Origin	Origin
N	196,098	282,718	∞	∞
adj. R^2	0.284	0.204		

Table 1: Change in coworker type due to EE moves positively related to own type – increasingly so

Notes. For the data columns, individual-level clustered standard errors are given in parentheses. Model counterparts are computed simulation-free in population. Dependent variable is scaled throughout by the standard deviation of the change in coworker type.

Related: life-cycle patterns of sorting consistent with cohort effects

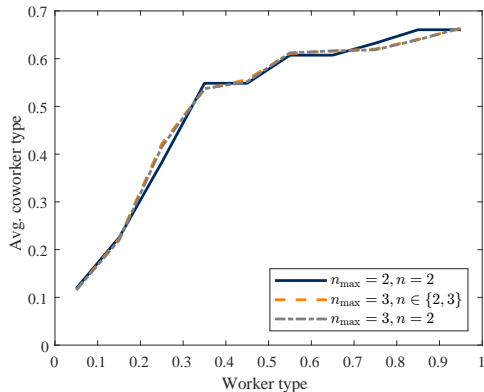
- Coworker sorting increases over life-cycle
- Coworker sorting higher in recent cohorts
 - consistent with each cohort becoming more specialized
 - but also with improvement in search frictions over time, which primarily affects sorting of young people
- Currently don't have life-cycle effects in my model, but could be incorporated...



Notes. Figure displays the correlation of own type with average coworker type, by age, for three separate cohorts.

Extension to $n_{\max}=3$

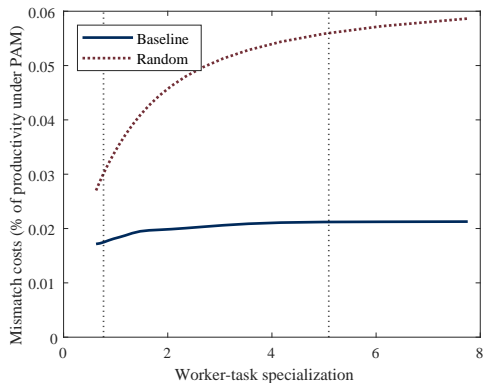
- Baseline model imposes $n_{\max} = 2$ for reasons of (i) tractability and (ii) transparency
- Can extend to $n_{\max} = 3$ (or $n_{\max} = 4$) & find that results are robust



Productivity costs of complementarity & labor market functioning

“The benefits of the division of labor are limited by the functioning of the labor market”

- Microfoundation: $\uparrow \chi \Rightarrow \uparrow$ efficiency *benefit* from teamwork *but* also \uparrow mismatch costs
- **Q:** how does the gap to potential vary depending on labor market structure?
- **A:** under random sorting, the productivity gap due to misallocation \uparrow more sharply as $\chi \uparrow$
- Outside model: severe labor mkt frictions (e.g. developing countries [Donovan et al. 2022])



Implications for overall inequality?

- **Coworker complementarities do not necessarily \uparrow variance of person-level wages**
 - (un-)surprising? AKM-variance decomposition perspective vs. common intuition/question
 - counterfactual: variance of log wages 0.2166 in 2010 under 1990-complementarities vs. 0.210
 - mechanisms: (i) reallocation effect, (ii) valuation effect, (iii) outside option effect
- Importance of within-firm wage compression: if high, then \uparrow coworker sorting also pushes up *overall* inequality (currently conjecture only!)
- Mechanisms: coworker learning [*Jarosch-Oberfield-RossiHansberg, 2021*], increasing returns to labor quality [*Kremer, 1993*], fairness [*Akerlof-Yellen, 1990*], sharing in monopoly rents, ...
 \Rightarrow future research

Evolution of the German task structure

- Employment Surveys (ES) carried out by the German Federal Institute for Vocational Training (BIBB)
 - detailed information on tasks performed at work
 - individual-level, with consistent occupation codes
 - repeated cross-sections ranging from 1985/86 to 2018
 - large sample sizes (20,000-30,000 per wave)
- Methodology to study evolution of task content of production follows Spitz-Oener (2006), Antonczyk et al. (2009), Rohrbach-Schmidt & Tiemann (2013)
 - task classification
 - sample harmonization (West Germany, aged 15 to 65, employed)

Task classification

- Focus on Δ in usage of abstract/complex (non-routine, non-manual) tasks vs. “rest” (manual & routine)

[Autor and Handel, 2009; Acemoglu & Autor, 2011; Rohrbach-Schmidt & Tiemann, 2013]

- Index of complex tasks for worker i in period t *[Antonczyk et al., 2009]*

$$T_{it}^{\text{complex}} = \frac{\text{number of activities performed by } i \text{ in task category "complex" in sample year } t}{\text{total number of activities performed by } i \text{ in sample year } t}$$

Task classification	Task name	Description
Complex	investigating organizing researching programming teaching consulting promoting	gathering information, investigating, documenting organizing, making plans, working out operations, decision making researching, evaluating, developing, constructing working with computers, programming teaching, training, educating consulting, advising promoting, consulting, advising
Other	repairing, buying, accommodat- ing, caring, cleaning, protect- ing, measuring, operating, man- ufacturing, storing, writing, cal- culating	

Increase in aggregate task complexity, driven by within-occupation \uparrow

- Aggregate task intensity & decompose period-by-period change into:
 - 1 between component: Δ occupational employment shares
 - 2 within component: Δ task content within occupations

	Total	Between	Within	Within-share
1986 level	0.252			
1986-1992	0.025	0.002	0.022	0.906
1992-2006	0.298	0.057	0.241	0.809
2006-2012	0.019	0.002	0.017	0.890
2012-2018	0.053	0.028	0.025	0.476
Total change	0.395	0.089	0.306	0.775

Notes. Decompose changes in the usage of abstract tasks between periods t and $t - 1$ according to $\Delta \bar{t}_t^{\text{abstract}} = \sum_o \omega_{o,t-1}(\bar{t}_{t,o}^{\text{abstract}} - \bar{t}_{t-1,o}^{\text{abstract}}) + \sum_o (\omega_{o,t} - \omega_{o,t-1})\bar{t}_{t,o}^{\text{abstract}}$ where $\bar{t}_{t,o}^{\text{abstract}}$ measures the average usage of abstract tasks by members of occupation o in period t and $\omega_{o,t}$ is the period- t employment share of occupation.

Large variation in task complexity across occupations

- Aggregate individual responses to 2-d occupation level, using 2012 & 2018 waves
 - 2012 & 2018: ← ISCO-o8 codes available

Occupation	$\bar{T}_o^{\text{complex}}$
Business and administration professionals	0.859
Legal, social and cultural professionals	0.830
Business and administration associate professionals	0.827
Administrative and commercial managers	0.820
Teaching professionals	0.807
...	...
Drivers and mobile plant operators	0.214
Agricultural, forestry and fishery labourers	0.193
Market-oriented skilled forestry, fishery and hunting workers	0.168
Food preparation assistants	0.131
Cleaners and helpers	0.124

Notes. Top-5 and bottom-5 ISCO-o8 2-digit occupations when ranked by $\bar{T}_o^{\text{abstract}}$ in pooled 2012 & 2018 waves.

Evidence from the literature: Hakanson et al. (2021)

[▶ Main](#)

- *Direct* measures of cognitive and non-cognitive skills across Swedish firms during 1986–2008, using test data from military enlistment

