Superstar Teams:
The Micro Origins and Macro Implications of Coworker Complementarities

Lukas B. Freund

University of Cambridge

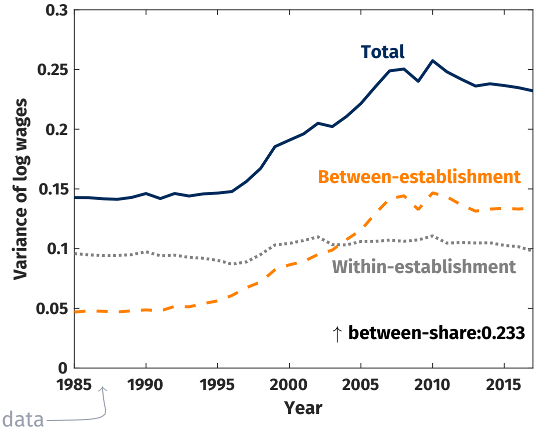
December 4, 2023

Minneapolis Fed Junior Scholar Conference

Wage inequality has risen – and firms appear to play a key role

"the variance of firm [wages] explains an increasing share of total inequality in a range of countries"

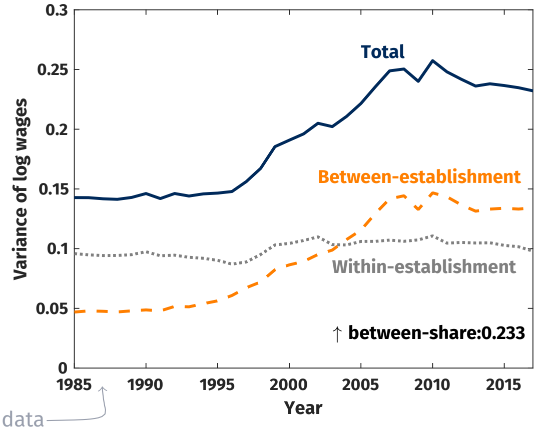
[Song-Price-Guvenen-Bloom-von Wachter, 2019]



German matched employer-employee data

Wage inequality has risen – and firms appear to play a key role

Applied question: what is/are the causal driver(s)? implications?



German matched employer-employee data

Paper offers a structural explanation for the “firming up” of inequality

- **Motivation:** wage inequality is increasingly a *between-firm* phenomenon ▸ Data

- **Firm:** an organized collection of workers performing tasks for production (team)

Paper offers a structural explanation for the "firming up" of inequality

- **Motivation:** wage inequality is increasingly a *between-firm* phenomenon ▸ Data

- **Firm:** an organized collection of workers performing tasks for production (team)

- **Hypothesis** for ↑ wage inequality between firms:
  1. the set of tasks that each worker knows how to perform very well has narrowed
  2. firms become more vulnerable to "weak links" in teams (complementarities ↑)
  3. individuals of similar talent increasingly work together (positive assortative matching ↑)
  4. this generates greater between-firm wage dispersion

Roadmap & main contributions

1. **Develop task-based theory of firm production with teams**

   $\rightarrow$ **analytical microfoundation for coworker complementarities**

Roadmap & main contributions

1. Develop task-based theory of firm production with teams

   $\rightarrow$ **analytical microfoundation for coworker complementarities**

2. **Embed into equilibrium search model & take to data**

   $\rightarrow$ coworker complementarities $\uparrow \Rightarrow$ coworker talent sorting $\uparrow$

   $\Rightarrow$ **identification result** $\rightarrow$ measure complementarities using wages (panel data)

Roadmap & main contributions

▸ Literature

1. Develop task-based theory of firm production with teams

   $\rightarrow$ analytical **microfoundation for coworker complementarities**

2. Embed into equilibrium search model & take to data

   $\rightarrow$ coworker complementarities $\uparrow \Rightarrow$ coworker talent sorting $\uparrow$

   $\Rightarrow$ **identification result** $\rightarrow$ measure complementarities using wages (panel data)

3. **Quantitatively analyze trends using structural model + DE panel data**

   $\rightarrow$ complementaries $\approx$ doubled since 1990

   $\rightarrow$ **$\approx$ 40% of $\uparrow$ between-firm wage inequality share due to $\uparrow$ complementarities**

   $\rightarrow$ implications for aggregate productivity, coworker job ladders, ...

Theory

Overview of environment

▸ Schematic illustration

- Many workers and many firms
- **Ex-ante homogeneous firms** assemble – hire workers & assign tasks – teams
- **Workers are heterogeneous in productivity**
  $\longrightarrow$ workforce composition is source of ex-post differences across firms
- Analyze in 2 steps:
  1. organization of production: allocate workers' time across tasks
  2. team formation: hire multiple workers

Step 1: production in a single team of given composition

- Firm w/ **1 team of $n \in \mathbb{Z}_{++}$ workers** produces output from **unit continuum of tasks $\mathcal{T}$**

$$\ln Y = \int_{\mathcal{T}} \ln q(\tau) d\tau \tag{1}$$

- **Workers:** each supplies 1 unit of time, has task-specific productivities $\{z_i(\tau)\}_{\tau \in \mathcal{T}}$
- **Task production:**

$$y_i(\tau) = z_i(\tau) l_i(\tau) \tag{2}$$

$$1 = \int_{\mathcal{T}} l_i(\tau)\ d\tau \tag{3}$$

- **Task aggregation:**

$$q(\tau) = \sum_{i=1}^{n} y_i(\tau) \tag{4}$$

Firm's optimization problem

▸ Extension to communication frictions

- **Firm solves mini-planner problem:** assign tasks to max. production

$$\mathcal{L}(\cdot) = Y + \lambda \underbrace{\left[\left(\int_{\mathcal{T}} \ln q(\tau) d\tau \right) - \ln Y \right]}_{\text{tasks} \rightarrow \text{output}} + \int_{\mathcal{T}} \lambda(\tau) \underbrace{\left(\sum_{i=1}^{n} y_i(\tau) - q(\tau) \right)}_{\text{task aggregation}} d\tau$$

$$+ \sum_{i=1}^{n} \lambda_i^L \underbrace{\left(\int_{\mathcal{T}} \frac{y_i(\tau)}{z_i(\tau)} d\tau - 1 \right)}_{\text{time constraint + task production}} + \text{non-negativity constraints}$$

- **FOCs** imply tasks are assigned by *comparative* advantage

$$\lambda(\tau) = \min_i \left\{ \frac{\lambda_i^L}{z_i(\tau)} \right\} \tag{5}$$

Tractability: leverage insight from trade literature

▸ PDFs ▸ Multivariate Fréchet

Assumption: Distribution of worker-task efficiencies

$\{z_i(\tau)\}_{\tau \in \mathcal{T}} \overset{\text{iid}}{\sim}$ **Fréchet:** $\Pr\{z_i(\tau) \leq z\} = \exp\left(- \left(\frac{z}{\iota x_i} \right)^{-1/\chi} \right)$

- Low-dimensional representation of worker-task productivity distribution
  - **"talent" type** $x_i \in [0, 1]$
  - **parameter** χ: **specialization** $\sim$ productivity dispersion across tasks

Illustration of worker-task productivity distribution

- $\Pr\{z_i(\tau) \leq z\} = \exp\left(-\left(\frac{z}{\iota x_i}\right)^{-1/\chi}\right)$
  - **"talent" type** $x_i \in [0, 1]$

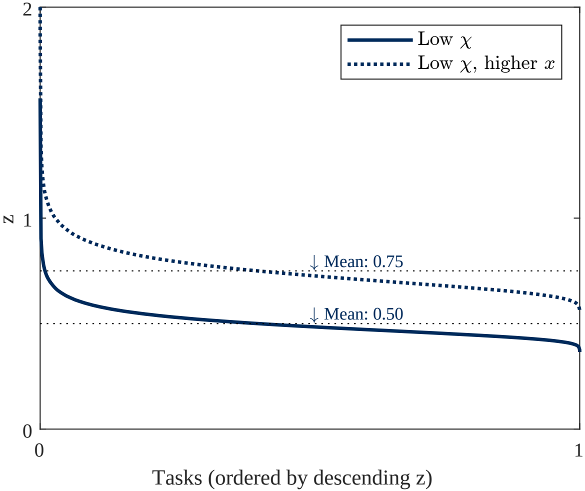
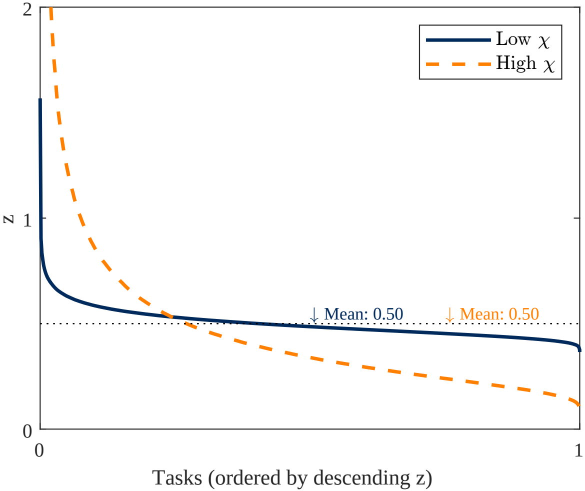


Illustration of worker-task productivity distribution

- $\Pr\{z_i(\tau) \leq z\} = \exp\left(-\left(\frac{z}{\iota x_i}\right)^{-1/\chi}\right)$
  - "talent" type $x_i \in [0, 1]$
  - **parameter χ: specialization**



Tractability: leverage insight from trade literature

- $\{z_i(\tau)\}_{\tau \in \mathcal{T}} \overset{\text{iid}}{\sim}$ Fréchet
- Low-dimensional representation of worker-task productivity distribution
  - "talent" type $x_i \in [0, 1]$
  - parameter χ: specialization $\sim$ productivity dispersion across tasks
- **Max-stability property** allows closed-form characterization of key objects
  - e.g., $\lambda(\tau) \sim$ Weibull

Optimal organization: 3 features

▸ Graphic: task assignment

Solution of firm's mini-planner problem implies:

1. **Complete division of labor**
2. Tasks assigned by **comparative advantage**
   - i's task set $\mathcal{T}_i = \left\{ \tau \in \mathcal{T} : \frac{z_i(\tau)}{\lambda_i^L} \geq \max_{k \neq i} \frac{z_k(\tau)}{\lambda_k^L} \right\}$
3. ***i*'s share of tasks $\uparrow$ in *i*'s talent, $\downarrow$ in coworkers' talent**
   - i's task share $\pi_i = \left(x_i^{\frac{1}{1+\chi}}\right) \left(\sum_{k=1}^{n} (x_k)^{\frac{1}{1+\chi}}\right)^{-1}$

Optimal organization: comparative statics for task shares

Lemma: Task shares

Suppose that $x_i > x_j$. Then

1. i performs a strictly larger share of tasks than j for $\chi < \infty$, ie $\pi_i < \pi_j$;
2. **the difference in task shares is decreasing in** χ, with $\pi_i/\pi_j \to x_i/x_j$ as $\chi \to 0$, and $\pi_i/\pi_j \to 1$ as $\chi \to \infty$.

$\Rightarrow$ Deeper specialization raises the share of tasks performed by low-x team member(s)

Micro-founded CES production function

Proposition: Aggregation result

The vector of talent types $(x_1, \ldots, x_n)$ is a sufficient statistic for team output Y, s.t.

$$f(x_1, \cdots, x_n) = \underbrace{n^{1+\chi}}_{\text{efficiency gains}} \times \underbrace{\left(\frac{1}{n} \sum_{i=1}^{n} (x_i)^{\frac{1}{1+\chi}} \right)^{1+\chi}}_{\text{complementarity}} .$$

vs. no-division-of-labor: $f(x_1, \ldots, x_n) = n \times (\frac{1}{n} \sum_i^n x_i)$

1. Standard **efficiency gains** $\nearrow \chi$

Micro-founded CES production function

▸ Taylor approx. ▸ Quality & task mismatch ▸ Example

Proposition: Aggregation result

The vector of talent types $(x_1, \ldots, x_n)$ is a sufficient statistic for team output Y, s.t.

$$f(x_1, \cdots, x_n) = \underbrace{n^{1+\chi}}_{\text{efficiency gains}} \times \underbrace{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i)^{\frac{1}{1+\chi}}\right)^{1+\chi}}_{\text{complementarity}},$$

with elasticity of complementarity $\gamma := \frac{\partial \ln(f_j/f_i)}{\partial \ln(x_i/x_j)} = \frac{\chi}{1+\chi}$.

2 **Coworker complementarity** $\nearrow \chi$: output more 'vulnerable' to lowest-x member(s)

Step 2: matching – quantitative model

▸ Equilibrium equations

- **Embed $f(\cdot)$ into frictional equilibrium matching model**
  - random-search + multi-worker firms *[Herkenhoff-Lise-Menzio-Phillips 2022]*
  - infinitely lived, risk-neutral agents; no entry/exit
  - no learning, i.e. types are invariant
  - $n_{\max} = 2$: sharp decreasing returns in production
  - employment states: unemp., employed alone, employed with $x' \in [0, 1]$
  - Nash bargaining w/ continuous renegotiation
  - baseline: no OJS – considered in extension
- Stationary equilibrium

Matching – stationary equilibrium

▸ Details

- HJ-Bellman equations $\rightarrow$ **values & matching policies** ▸ HJBs
- Flows between/**distribution** over types $\times$ employment states ▸ KFEs

Definition: Stationary equilibrium

A stationary equilibrium consists of a tuple of value functions and a distribution of agents, such that

1. the value functions satisfy the HJB equations given the distribution;
2. the distribution is stationary given the policy fn's implied by the value fn's.

Step 2: matching – quantitative model

- Embed $f(\cdot)$ into frictional equilibrium matching model
- **2 main results**
  1. mechanism: complementarities $\rightarrow$ sorting
  2. identification: measuring complementarities

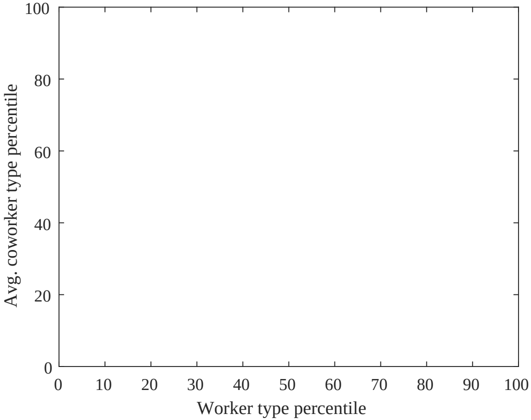
Step 2: matching – quantitative model

- Embed $f(\cdot)$ into frictional equilibrium matching model
- 2 main results
  1. **mechanism: complementarities $\rightarrow$ sorting**
  2. identification: measuring complementarities

Matching – intuition

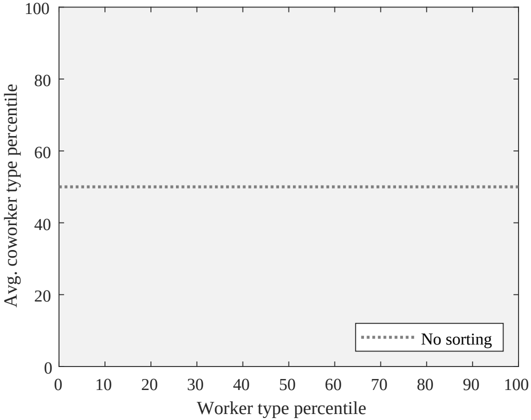
▸ Simple model ▸ Lemma ▸ Simple vs. quantitative

- In equilibrium, who tends to work with whom?



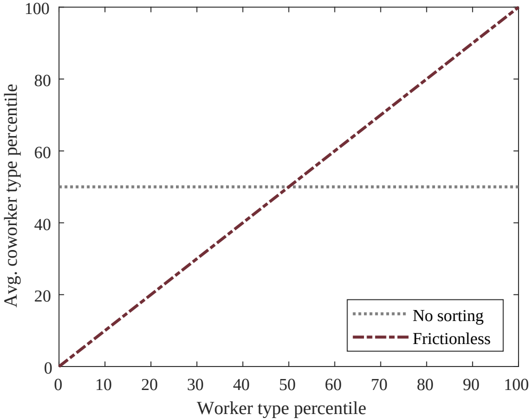
Matching – intuition

- No complementarities: everyone matches with everyone



Matching – intuition

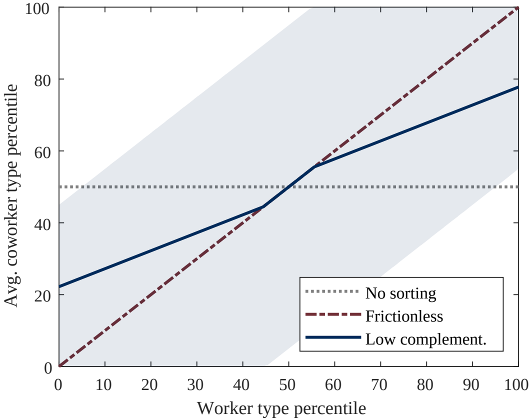
- Complementarities + frictionless: pure positive assortative matching



Matching – intuition

- **Search frictions:** trade-off **team match quality vs. cost of searching**

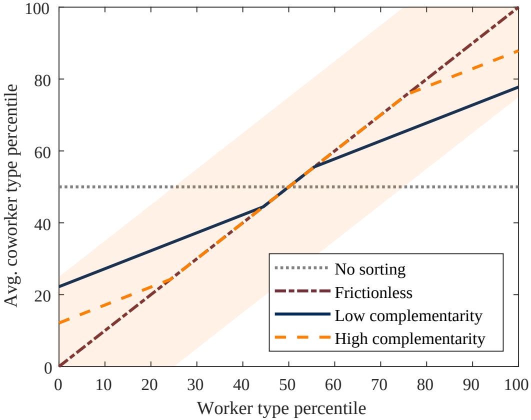
  $\Rightarrow$ range of acceptable matches



Matching – intuition

- $\chi \uparrow \Rightarrow$ *smaller* acceptable range

  1. **coworker sorting** $\uparrow$
  2. **between-firm share of var(wages)** $\uparrow$



Step 2: matching – quantitative model

- Embed $f(\cdot)$ into frictional equilibrium matching model
- 2 main results
  1. mechanism: complementarities $\rightarrow$ sorting
  2. **identification: measuring complementarities**

Identification result

- **Strategy:** recover χ directly, rather than inferring from sorting

  $\Rightarrow$ How to quantify $\frac{\partial^2 f(x,x')}{\partial x \partial x'}$?

▸ Discussion

Identification result

- **Q:** How to quantify $\frac{\partial^2 f(x,x')}{\partial x \partial x'}$?

- **Theory:** wage level for worker x with coworker x'

▸ Wage equation

$$w(x|x') = \omega\big(f(x,x') - f(x')\big) + g(x) - h(x')$$

where g and h are strictly increasing functions that reflect outside options

Identification result

Beyond benchmark: scatterplot

- **Q:** How to quantify $\frac{\partial^2 f(x,x')}{\partial x \partial x'}$?
- **Theory:** wage level for worker x with coworker x'

Wage equation

$$w(x|x') = \omega \times (f(x,x') - f(x')) + g(x) - h(x')$$

Proposition: Identification result

Coworker complementarities (CC) in production are proportional to CC in wages:

$$\frac{\partial^2 f(x,x')}{\partial x \partial x'} \propto \boxed{\frac{\partial^2 w(x|x')}{\partial x \partial x'}}$$

can measure this

Model Meets Data

Bringing the model to panel micro data for Germany

▸ SIEED

- **Primary data: SIEED matched-employer employee panel for W Germany** ▸ Processing
  - 1.5% sample of establishments + entire biographies of associated workers; social security information on employer, daily wage, occupation, demographics
  - initially focus on 2010-2017, later extend to 1985-2017

- **Overview:**
  1. mapping model objects to data
  2. model calibration
  3. validation – brief today, more in paper!

Mapping theory to data – types: workers, teams & coworkers

▸ Implementation

- **Worker types** from 2-way FE wage regressions *[Abowd et al., 1999 – AKM]*
  - pre-estimation k-means clustering $\rightarrow$ deal with limited mobility bias *[Bonhomme et al., 2019]*
  - robustness: non-param. ranking algo instead of AKM *[Hagedorn et al., 2017]*

$\Rightarrow$ Worker "type" $\hat{x}_i$: decile rank of worker FE

Mapping theory to data – types: workers, teams & coworkers

▸ Implementation

- **Worker types** from 2-way FE wage regressions *[Abowd et al., 1999 – AKM]*
  - pre-estimation k-means clustering $\rightarrow$ deal with limited mobility bias *[Bonhomme et al., 2019]*
  - robustness: non-param. ranking algo instead of AKM *[Hagedorn et al., 2017]*

$\Rightarrow$ Worker "type" $\hat{x}_i$: decile rank of worker FE

- **"Representative coworker type"** $\hat{x}_{-it}$: average $\hat{x}_i$ of coworkers in same establishment-year

▸ Discussion

Evidence on coworker complementarity (2010-2017)

▸ Robustness ▸ B-o-E calc. γ ▸ Peer effects

- **Regression:**

$$\frac{w_{it}}{\bar{w}_t} = \beta_0 + \beta_1 \hat{x}_i + \beta_2 \hat{x}_{-it} + \overset{\text{coworker complementarity}}{\beta_c} \left(\hat{x}_i \times \hat{x}_{-it}\right)$$

$$+ \psi_{j(it)} + \nu_{o(i)t} + \xi_{s(i)t} + \epsilon_{it}$$

Evidence on coworker complementarity (2010-2017)

▸ Robustness ▸ B-o-E calc. γ ▸ Peer effects

- **Regression:**

$$\frac{w_{it}}{\bar{w}_t} = \beta_0 + \beta_1 \hat{x}_i + \beta_2 \hat{x}_{-it} + \underbrace{\beta_c}_{\text{coworker complementarity}} (\hat{x}_i \times \hat{x}_{-it}) + \psi_{j(it)} + \nu_{o(i)t} + \xi_{s(i)t} + \epsilon_{it}$$

	$\hat{\beta}_c$	Non-parametric FD method
Coworker complementarity	**0.0091***** (0.00035)	0.0097
Type ranking	Economy	Economy
Obs. (100,000s)	4,410	4,410

Notes. Regressions include FEs for employer; occupation-year; industry-year. Employer-clustered standard errors in parentheses. Observations weighted by the inverse employment share of the respective type and (rounded) coworker type cell. FD: finite differences.

Model parameterization: overview of methodology

▸ Details ▸ Identification validation exercises

- **Calibrate** the model to the W German economy (2010-2017), at monthly frequency
  1. externally calibrated: discount rate, team-benefit, bargaining power
  2. offline estimation: job separation hazard
  3. online estimation (indirect inference): meeting rate, unemp. flow benefit, production
     - targets: $\hat{\beta}_c$, total wage variance, avg. wage level, replacement rate, job finding rate
- **Macro moments of interest are untargeted**
  - **production complementarity informed by $\hat{\beta}_c$**
  - untargeted: coworker sorting, between-firm wage inequality

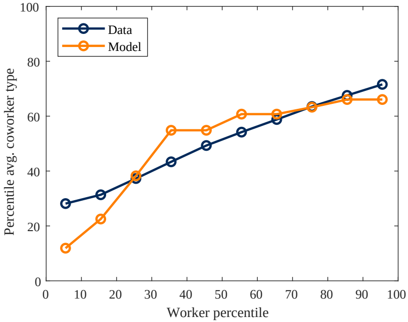
Calibration results & model validation

- Recovered elasticity of complementarity γ equal to 0.84 ▸ Parameterization

- ✓**Match coworker sorting patterns**
  - $\rho_{xx} = 0.53$ (vs. 0.62 in data)

- ✓**Match between-firm wage inequality**
  - between-share 0.56 (vs. 0.57 in data)
  - adjust for small-n bias ▸ Details

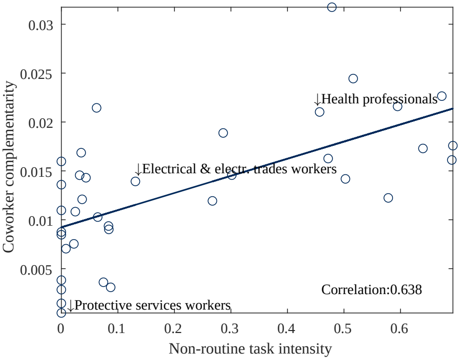
- Extensive validation exercises ▸ Details



Occupations: task complexity ⇒ complementarity ⇒ sorting

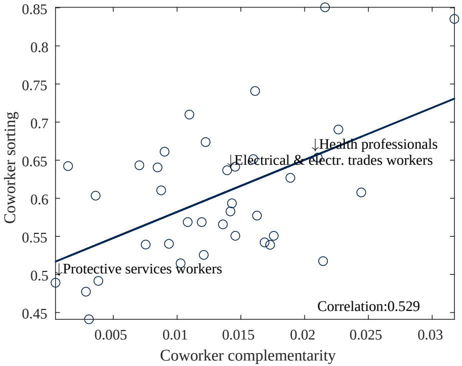
▸ More validation

- ↑ Non-routine abstract task intensity ⇒ ↑ coworker complementarity



Notes. Quadros de Pessoal microdata. Analysis at ISCO-08-2d level.

- ↑ Coworker complementarity ⇒ ↑ coworker sorting



Notes. Quadros de Pessoal microdata.

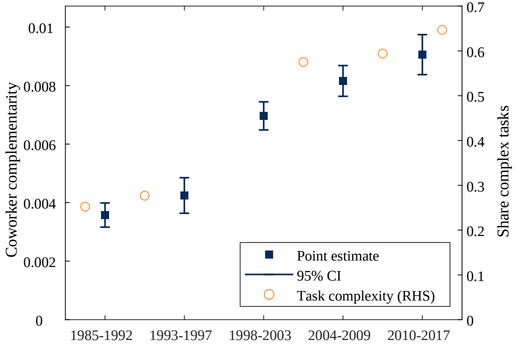
Historical Trends

Coworker complementarity has strengthened over time

▸ Schooling ▸ Peer effect trends

- **Theory:** specialization (χ) $\uparrow$ is associated with coworker complementarity $\uparrow$

✓**Coworker complementarity has more than doubled between 1985-1992 and 2010-2017**



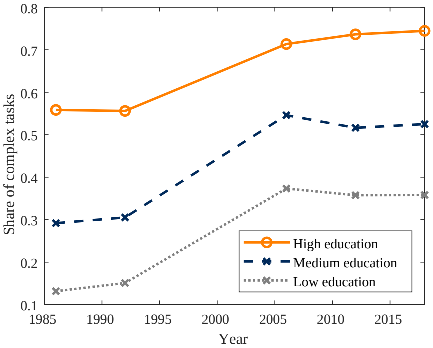
...consistent with descriptive evidence of specialization (χ) ↑...

▸ Occ. movements

- **Example:** science *[Jones, 2009/2021; Pearce, 2022]*

▸ Details

- **Task complexity ↑**: "extensive margin" of χ
  - DE longitudinal task survey ▸ BIBB
  - "complex": cognitive non-routine (e.g., organizing, researching)
- Grigsby (2023)

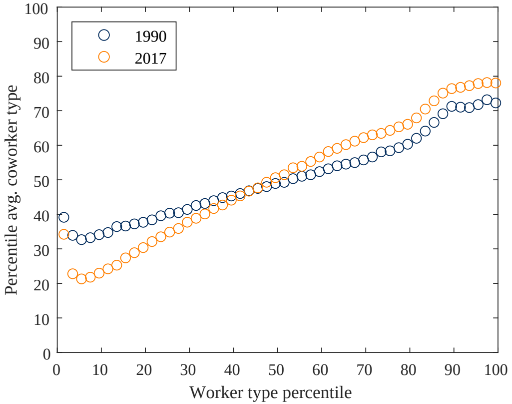


...and talent sorting has intensified

▸ Details

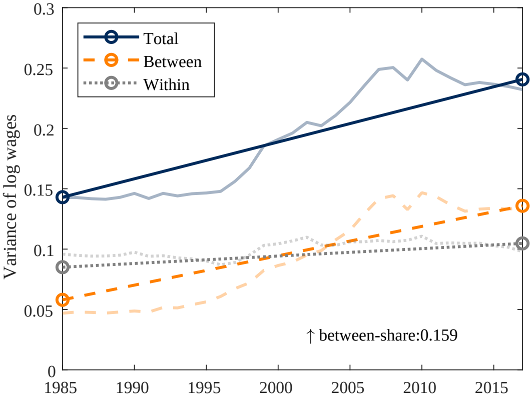
- **Theory:** complementarity ↑ is associated with talent sorting ↑

✓ **Coworker matching has become more positively assortative**



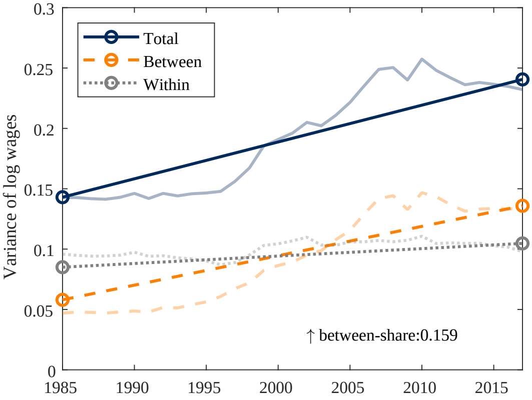
Model matches *changes* in firm-level wage distribution

- **Re-calibrate model** for '85-'92
- ✓**Model replicates empirical rise of between-share**
  - 68% of ↑ between-share in data



Model matches *changes* in firm-level wage distribution – why?

- Re-calibrate model for ’85-’92
- ✓Model replicates empirical rise of between-share
- **Reflects several parameters changing**
    - elasticity of compl.: 0.43 (vs. 0.84)
    - job arrival & separation ↑ in ’10-’17
    - ...



Complementarity $\uparrow$ explains $\approx$ 40% of observed between-share $\uparrow$

- **Q:** How much of $\uparrow$ between-firm share of wage var. is due to $\uparrow$ complementarities?

Complementarity $\uparrow$ explains $\approx$ 40% of observed between-share $\uparrow$

- **Q:** How much of $\uparrow$ between-firm share of wage var. is due to $\uparrow$ complementarities?
- **Counterfactual:** between-firm share in 2010s absent $\chi \uparrow$

Complementarity $\uparrow$ explains $\approx$ 40% of observed between-share $\uparrow$

- **Q:** How much of $\uparrow$ between-firm share of wage var. is due to $\uparrow$ complementarities?
- **Counterfactual:** between-firm share in 2010s absent $\chi \uparrow$
- **A:** $\chi \uparrow$ **accounts for 59%** of model-predicted $\Delta \leftrightarrow$ **$\approx$40% of empirical Δ**

	Δ model	Implied % Δ model due to Δ parameter
Model baseline	0.16	-
Cf.: fix period-1 complementarity	0.065	59

Complementarity $\uparrow$ explains $\approx$ 40% of observed between-share $\uparrow$

- **Q:** How much of $\uparrow$ between-firm share of wage var is due to $\uparrow$ complementarities?
- **Counterfactual:** between-firm share in 2010s absent $\chi\uparrow$
- **A:** $\chi\uparrow$ **accounts for 59% of model-predicted $\Delta \leftrightarrow \approx$40% of empirical Δ**
- Finding **robust** to checks
  - outsourcing; declining search frictions; OJS; matching on task-specific skills

Jump

Implications for aggregate productivity

▸ Effects of χ ↑: random vs eqm

- **Production complementarities imply coworker sorting matters for agg productivity**
  - $f(x_1, \cdots, x_n) = n^{1+\chi} \times \left(\frac{1}{n} \sum_{i=1}^{n} (x_i)^{\frac{1}{1+\chi}} \right)^{1+\chi}$

Implications for aggregate productivity

▸ Effects of χ $\uparrow$: random vs eqm

- **Production complementarities imply coworker sorting matters for agg productivity**
  - $f(x_1, \cdots, x_n) = n^{1+\chi} \times \left(\frac{1}{n} \sum_{i=1}^{n} (x_i)^{\frac{1}{1+\chi}} \right)^{1+\chi}$
- **Search** frictions induce misallocation $\sim$ coworker **mismatch**

Implications for aggregate productivity

▸ Effects of χ ↑: random vs eqm

- **Production complementarities imply coworker sorting matters for agg productivity**
  - $f(x_1, \cdots, x_n) = n^{1+\chi} \times \left(\frac{1}{n} \sum_{i=1}^{n} (x_i)^{\frac{1}{1+\chi}} \right)^{1+\chi}$
- **Search** frictions induce misallocation $\sim$ coworker **mismatch**
- **Quantify** mismatch costs: compare eqm outcome vs to productivity under pure PAM
  $\Rightarrow$ 2010s gap: 2.05%

Implications for aggregate productivity

▸ Effects of χ $\uparrow$: random vs eqm

- **Production complementarities imply coworker sorting matters for agg productivity**
  - $f(x_1, \cdots, x_n) = n^{1+\chi} \times \left(\frac{1}{n} \sum_{i=1}^{n} (x_i)^{\frac{1}{1+\chi}} \right)^{1+\chi}$
- **Search** frictions induce misallocation $\sim$ coworker **mismatch**
- **Quantify** mismatch costs: compare eqm outcome vs to productivity under pure PAM
  $\Rightarrow$ 2010s gap: 2.05%
- **Trends:** $\uparrow$ talent sorting limited $\uparrow$ in mismatch costs given $\chi \uparrow$
  $\Rightarrow$ no-reallocation counterfactual: productivity gap 4.65%

Implications for aggregate productivity

Effects of χ $\uparrow$: random vs eqm

- **Production complementarities imply coworker sorting matters for agg productivity**
  - $f(x_1, \cdots, x_n) = n^{1+\chi} \times \left(\frac{1}{n} \sum_{i=1}^{n} (x_i)^{\frac{1}{1+\chi}} \right)^{1+\chi}$
- **Search** frictions induce misallocation $\sim$ coworker **mismatch**
- **Quantify** mismatch costs: compare eqm outcome vs to productivity under pure PAM
  $\Rightarrow$ 2010s gap: 2.05%
- **Trends:** $\uparrow$ talent sorting limited $\uparrow$ in mismatch costs given $\chi \uparrow$
  $\Rightarrow$ no-reallocation counterfactual: productivity gap 4.65%
- **Implications for development?** *[Donovan-Lu-Schoellman, 2023]*

Extensions

Overview of extensions, robustness checks, and other implications

- **Robustness checks**
  - declining search frictions Jump
  - outsourcing & within-occupation analysis Jump
  - OJS Jump
  - ’horizontal’ complementarity Jump
- **Productivity dispersion** Jump
- **Person-level inequality** Jump
- **“Coworker job ladders”** Jump

Conclusion

Conclusion: firms form & organize teams – matters for macro

- **This paper:**
  1. **task-based microfoundation for coworker complementarity**
     $\Rightarrow$ specialization + team production $\rightarrow$ complementarity
  2. **measurement** of complementarities
     $\Rightarrow$ empirical support for model mechanisms
  3. structural & quantitative explanation for the **"firming up" of inequality**
     $\Rightarrow$ role of **increased complementarities**
     $\Rightarrow$ increased **talent sorting** helped keep TFP close to potential
- Step to broader agenda: firms as "team assemblies" in macro
  - structural interpretation of "firm effects"; importance of organizational capacity when knowledge is specialized; persistence due to complementarities; innovation; ...

Extra Slides

Relation & contributions to 3 strands of literature

Roadmap

- **Wage inequality: structural model of ↑ firm-level inequality due to technological △**

  Technology: *Katz & Murphy, 1992; Krusell et al., 20000; Autor, Levy & Murnane, 2003;* ***Jones, 2009****; Deming, 2017; Acemoglu & Restrepo, 2018; Alon, 2018; Neffke, 2019; Jones, 2021; Atalay et al., 2021*

  Firms: ***Card et al., 2013****; Barth et al., 2016; Alvarez et al., 2018;* ***Bloom et al., 2019****; Aeppli & Wilmers, 2021; Criscuolo et al. 2021; Hakanson et al., 2021; Sorkin & Wallskog, 2021; Kleinman, 2022*

- **Firm organization: tractable model of team production → frictions & quantification**

  Firms: *Lucas, 1978; Rosen, 1982; Becker & Murphy, 1992;* ***Kremer, 1993****; Kremer & Maskin, 1996;* ***Garicano, 2000****;* ***Garicano & Rossi-Hansberg, 2006****; Kohlhepp, 2022; Kuhn et al., 2022; Minni, 2023; Bassi et al., 2023*

  Task assignment: *Costinot & Vogel, 2010;* ***Acemoglu & Restrepo, 2018****; Ocampo, 2021; Adenbaum, 2022*

  Teams: *Akcigit et al., 2018;* ***Jarosch et al., 2021****; Herkenhoff et al., 2022; Pearce, 2022*

- **Frictional labor market sorting: endogenize & measure complementarities**

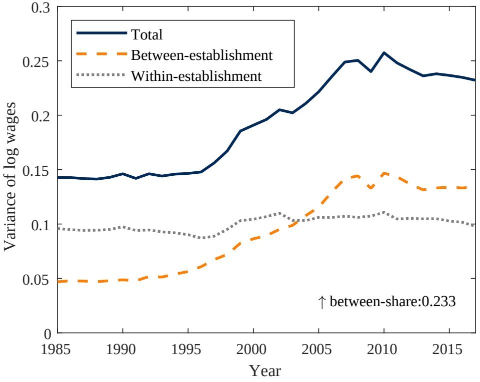
  Shimer & Smith, 2000; Cahuc et al., 2006; Eeckhout & Kircher, 2011/2018; Hagedorn et al., 2017; de Melo, 2018; Chade & Eeckhout, 2020; ***Herkenhoff et al., 2022****; Lindenlaub & Postel-Vinay, 2023*

Fact 1: ↑ between-firm share of wage inequality

▸ Intro

- Large empirical literature: "firming up inequality" *[e.g., Card et al., 2013; Song et al., 2019]*
  - "superstar firms" *[e.g., Autor et al., 2020]*
- **Fact 1: ↑ wage inequality primarily due to between-component**
- Robust pattern

▸ Cross-country ▸ Panel est. ▸ Wage resid. alternatives
▸ Within-occ ▸ Within-ind



Notes. Model-free statistical decomposition, where the "between" component corresponds to the person-weighted variance of est.-level avg. log wage.

Fact #2: talented workers increasingly collaborate

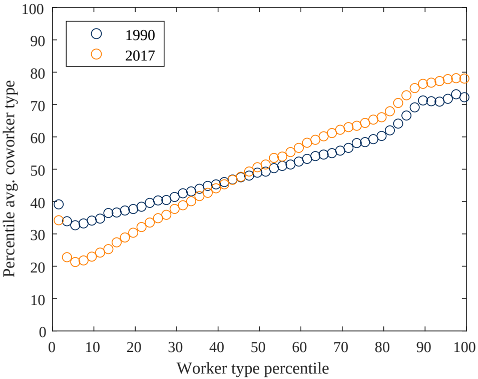
▸ Intro ▸ Var. decomp. ▸ Fact #3

- To what extent do “talented workers” tend to have “talented coworkers”?

- **Fact 2: + assortative coworker sorting ↑**
  - $\rho_{xx} = \mathrm{corr}(\hat{x}_i, \hat{x}_{-it})$: 0.43 (’85-’92) ↗ 0.62 (’10-’17)

- Robust pattern

▸ Table ▸ Within-occ. nonlinear
▸ Hakanson et al. (2021)



Fact #3: increased education premium due to workplace effects

Main

- **Fact 3: increase in return to schooling is primarily due to workplace effects**

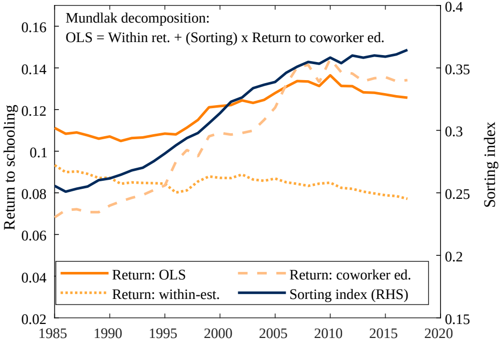
- Mundlak decomposition of year-specific OLS return to schooling:

$$\beta_t^{\text{ols}} = \beta_t^{\text{within}} + \rho_t \times \beta_t^{\text{estab.}}$$

$$\ln w_{it} = \beta_0 + \beta_t^{\text{within}} S_i + \beta_t^{\text{estab.}} \bar{S}_{j(i,t),t} + e_{it}$$

where $\bar{S}_{j(i,t),t}$ is avg. years of schooling in establishment j of worker i in year t

1. β_t^{within}: within-establishment return
2. $\beta_t^{\text{estab.}}$: return to avg. establishment schooling



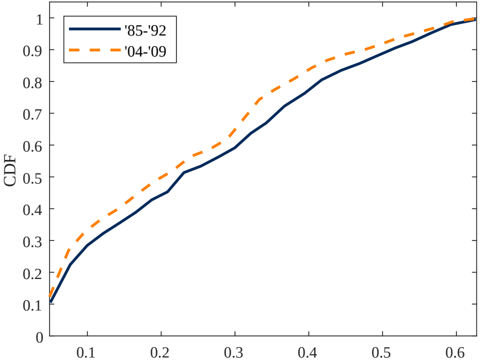
Notes. Plot of coefficients from year-by-year regressions of log wages.

Workers increasingly tend to perform similar tasks across different jobs

Back

- ✓Workers move to jobs with similar tasks, rather than randomly Comparison
- **Q:** are workers becoming *more* likely to perform similar tasks across jobs, over time?

- **Yes:** distribution of moves in ('04-'09) is stochastically dominated by that in ('85-'92)
  - uncond. average: 0.253 $\rightarrow$ 0.227: 10% decline
- Robust in regression design
  - quantile regressions: ✓at different quantiles



Brief summary of methodology: occupations and task space

▸ Details

1. **Measure task similarity between occupations**
   - $\bar{\mathbf{l}}_o = (\bar{l}_{o1}, \ldots, \bar{l}_{o|\hat{\mathcal{T}}|})$: vector of task content of occupation o, with $\bar{l}_{o\tau}$ denoting the fraction of workers in occupation o performing task $\tau \in \hat{\mathcal{T}}$, where $|\hat{\mathcal{T}}| \in \mathbb{Z}_{++}$ (i.e., discretized)
   - *distance in task space* between any two occupations o and o',

$$\varphi(\bar{\mathbf{l}}_o, \bar{\mathbf{l}}_{o'}) = \frac{1}{\pi}\cos^{-1}\left(\frac{\bar{\mathbf{l}}_o'\bar{\mathbf{l}}_{o'}}{||\bar{\mathbf{l}}_o|| \cdot ||\bar{\mathbf{l}}_{o'}||}\right) \in [0, 1]$$

   - **implementation:** BIBB longitudinal microdata ($|\hat{\mathcal{T}}| = 15$)
2. **Occupational movers:** an individual i who in period t is employed at j in occupation o counts as an *occupational mover* if in $t+1$, i is employed at $j' \neq j$ and $o' \neq o$
   - implementation: matched employer-employee micro data (entire biography!)
3. Merge $\{\varphi(\bar{\mathbf{l}}_o, \bar{\mathbf{l}}_{o'})\}_{oo'}$ into mover sample $\rightarrow$ **project moves onto task distance space**
   - harmonized occupational classification KldB1988-2d

Methodology (1): occupations and task space

Main

- **Occupations as points in task space – definitions:**
  - $\bar{\mathbf{l}}_o = (\bar{l}_{o1}, \ldots, \bar{l}_{o|\hat{\mathcal{T}}|})$: vector of task content of occupation o, with $\bar{l}_{o\tau}$ denoting the fraction of workers in occupation o performing task $\tau \in \hat{\mathcal{T}}$, where $|\hat{\mathcal{T}}| \in \mathbb{Z}_{++}$ (i.e., discretized)
  - *distance in task space* between any two occupations o and o',

$$\varphi(\bar{\mathbf{l}}_o, \bar{\mathbf{l}}_{o'}) = \frac{1}{\pi}\cos^{-1}\left(\frac{\bar{\mathbf{l}}_o'\bar{\mathbf{l}}_{o'}}{||\bar{\mathbf{l}}_o|| \cdot ||\bar{\mathbf{l}}_{o'}||}\right) \in [0, 1]$$

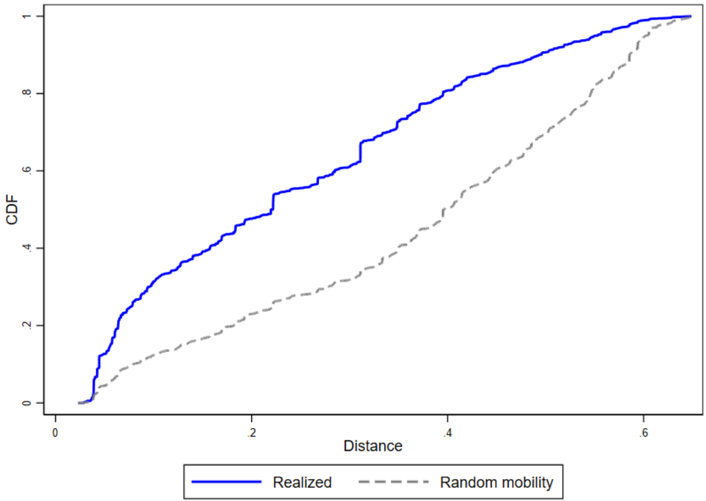
- **Implementation:** BIBB longitudinal microdata
  - 15 harmonized tasks across survey waves
  - measure $\bar{l}_{o\tau}$: share of individuals belonging to o performing τ
  - construct matrix of bilateral distances for each wave, then take *average* of $\varphi(\bar{\mathbf{l}}o, \bar{\mathbf{l}}o')$ across waves for each (o, o')
  - *note:* I thus hold the distance in task space between any two occupations fixed over time

Methodology (2): occupational movers

- Consider individual i who in period t is employed at j in occupation o. I consider i an *occupational mover* if in $t+1$, i is employed at $j' \neq j$ and $o' \neq o$.
  - only counted as occ. mover if employed in different job: cf. Kambourov and Manovskii (2008)
  - considering switch in adjacent periods: more likely that it is a *voluntary* move
- Function $\varphi(\bar{\mathbf{l}}_o, \bar{\mathbf{l}}_{o'})$ maps each $o \to o'$ move onto $[0,1]$
- Implementation: merge $\{\varphi(\bar{\mathbf{l}}_o, \bar{\mathbf{l}}_{o'})\}_{oo'}$ into SIEED
  - harmonized occupational classification KldB1988-2d
  - restrict myself to 1985-2009, because subsequent stark change in occupational classification limits comparability (missing notifications, etc.)

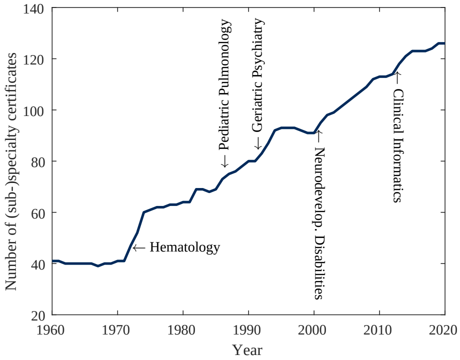
Comparison of realized movements in task space vs. random mobility

Back



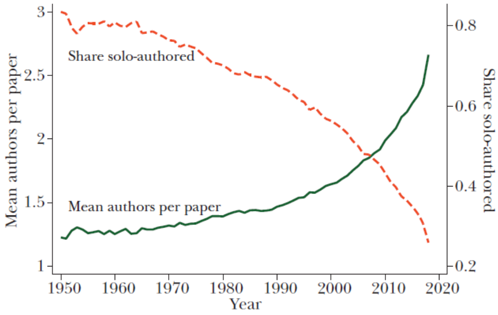
Examples: rising specialization

- **Deepening medical specialization**



Notes. Data from American Board of Medical Specialities. For each year, it shows the number of unique speciality or sub-speciality certificates that have been approved and issued at least once by that year and which are are still being issued.

- **Rise of research teams** *[Jones, 2021]*



Data sources: short description of main datasets

▸ Main ▸ Imputation procedure

- **Germany**: SIEED linked employer-employee dataset
  - *establishment* and individual data generated in administrative processes
  - built up from a 1.5% sample of all establishments, but includes comprehensive employment biographies of individuals employed at these establishments
  - worker info: (real) **daily wage**, occupation, education, ...
  - top coding (affects >50% of university-educated men in regular full-time employment) $\rightarrow$ adopt standard imputation methods (Dustmann et al., 2009; CHK, 2013)
  - Much larger sampling frame than more familiar LIAB

- **Portugal**: Quadros de Pessoal & Relatório Único, 1986-2017
  - $\approx$ universe of private sector firms and workers employed by them
  - annual panel
  - worker info: detailed earnings measures (base wage, regular benefits, irregular benefits (performance-pay, bonuses, etc.), overtime pay); no top-coding; also hours worked within the month (regular and overtime) $\Rightarrow$ (real) total hourly wage

Sample restrictions

Main

- Data cleaning $\Rightarrow$ broadly harmonized samples
- Main restrictions
  - age 20-60
  - full-time employed
  - drop agriculture, public sector, utilities industries
  - firms (and their employees) with at least 10 employees
- DE: West Germany
- Transform SIEED spell-level data into annual panel
- PRT: at least. official minimum wage

Wage Imputation procedure

- Follow imputation approach in CHK2013, building on Gartner et al. (2005) and Dustmann et al. (2009)
  1. fit a series of Tobit models to log daily wages
  2. then impute an uncensored value for each censored observation using the estimated parameters of these models and a random draw from the associated (left-censored) distribution

- Fit 16 Tobit models (4 age groups, 4 education groups) *after* having restricted the sample (to include West German men only, in particular) and I follow CHK in the specification of controls by including not only age, firm size, firm size squared and a dummy for firms with more than ten employees, but also the mean log wage of co-workers and fraction of co-workers with censored wages. Finally, following Dauth & Eppselheimer (2020) I limit imputed wages at 10 $\times$ 99th percentile.

Mapping model to data: coworker types

Main

- Defining $S_{-it} = \{k : j(kt) = j(it), k \neq i\}$ as the set of i's coworkers in year t, compute the average type of i's coworkers in year t as $\hat{x}_{-it} = \frac{1}{|S_{-it}|} \sum_{k \in S_{-it}} \hat{x}_k$.
- **Coworker group:**
  - alternative: same establishment-occupation-year cell
  - but CC arise precisely when workers are *differentiated* in their task-specific productivities
- **Averaging step:**
  - equally-weighted averaging ignores non-linearity in coworker aggregation
  - paper: show using non-linear averaging method that baseline results in bias, but it's minor in magnitude
- **Firm size variation:** averaging ensures that a single move will induce a smaller change in the *average* coworker quality in a large team than in a small one

Mapping model to data: identification strategy for χ

Main

- **Literature:** complementarities – primarily between workers and firms – usually inferred indirectly from sorting patterns
  - exception: Hagedorn-Law-Manovskii (2017)
- **This paper:** directly measure coworker complementarity in the data, recover χ structurally given $\gamma = \frac{\chi}{\chi+1}$
- Paper does *not* use microfoundation itself to measure χ, respectively γ
- Experiment: fit a (truncated) Fréchet distribution to Grigsby's (2023) non-parametric estimates of the multi-dimensional skill distribution estimated from CPS data for the U.S.
  Recover $\gamma = 0.84$ for 2006 but *very* noisy estimates
- **Ongoing work:** use the extended microfoundation to identify χ

Direct estimation of χ: proof of concept

- Grigsby (2023): only paper that provides a *cardinal* measure of skill task-specificity
- Evidence on time trends are at least qualitatively consistent with my "specialization hypothesis": cross-type average of within-type variance across specific skills grew by nearly 50% b/w 1980s and 2000s & skill transferability has declined amongst high-skill occupations
- His operationalization of worker types and tasks does *not* directly map onto my model (no identifying assumption; coarse occupational skills; US vs DE data)
- **Proof of concept:** but *suppose* we just take those data, extract moments capturing average within-worker cross-task efficiency dispersion, fit a (truncated) Fréchet, recover $\gamma = \frac{\chi}{1+\chi}$
  $\Rightarrow$ ✓ γ **similar to structural estimation result based on evidence from wage CC**

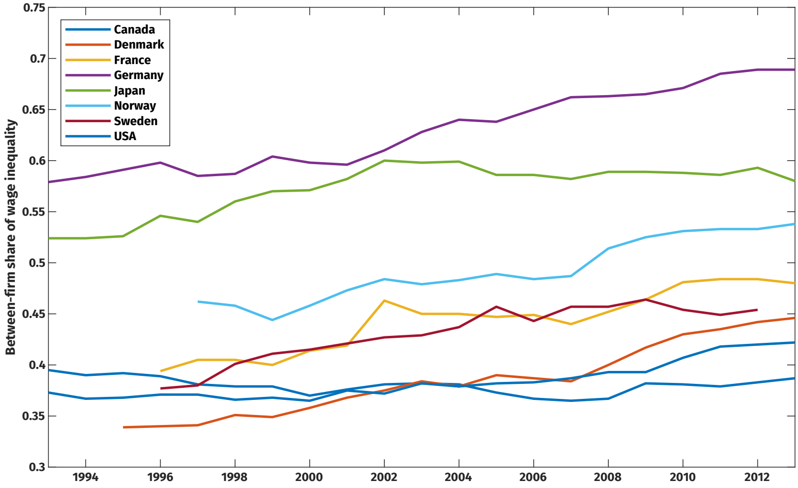
Semi-structural back-of-envelope calculation for γ

▸ Main

- Structurally recover $\gamma\frac{\chi}{\chi+1}$ by estimating $\frac{\partial^2 w(x|x')}{\partial x \partial x'}$ in the data, which was shown to be proportional to $\frac{\partial^2 f(x,x')}{\partial x \partial x'}$
- But how is $\frac{\partial^2 f(x,x')}{\partial x \partial x'}$ related to γ?
- Definitionally, $\gamma = (f f_{ij})/(f_i f_j)$ for any $i \neq j$, where subscripts denote partial derivatives
- Can we avoid full structural model? $\Rightarrow$ If have measures not only of f_{ij} but also output f and marginal products f_i
- Suppose, for any x and x', we use *wages* to back out marginal products – competitive wage determination rather bargaining! – and recover output from sum of wages divided by labor share
- Find $\gamma \approx 0.79$ – very close to structural estimate!

Firming up inequality: cross-country evidence

Main

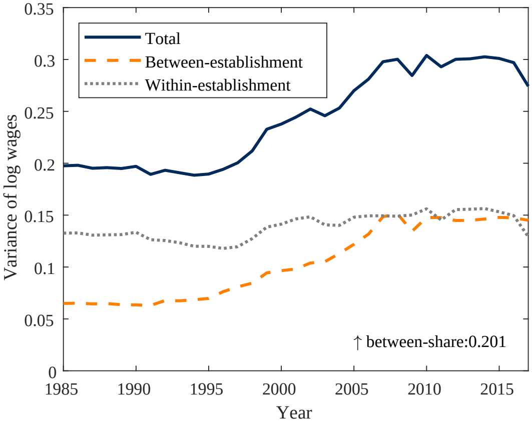


Notes. Data from Tomaskovic-Devey et al. (2020). Measures of earnings differ across countries and, for Germany, between T-D et al. and my study based on the SIEED.

Between-/within-employer wage var decomp. - panel establishments

Main

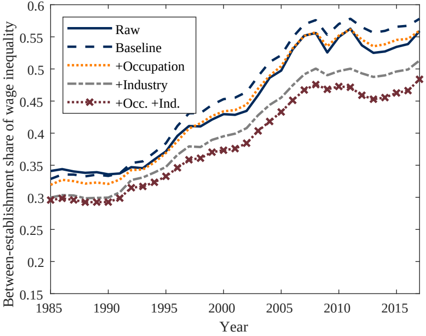
- Instead of considering *all* employers, restrict attention to “panel establishments”



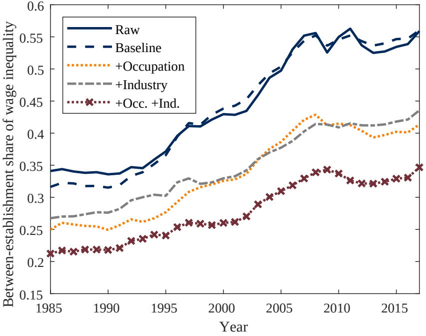
Between-/within-employer wage var. decomp. - alternative w-residuals

Main

- "With worker FEs": regress $\ln \tilde{w}_{it} = \alpha_i + X'_{it}\hat{\beta} + \epsilon_{it}$, construct $\ln w_{it} = \ln(\tilde{w}_{it} - X'_{it}\hat{\beta})$.
- "Without worker FEs": regress $\ln \tilde{w}_{it} = \alpha_0 + X'_{it}\hat{\beta} + \epsilon_{it}$, and consider residuals ϵ_{it}



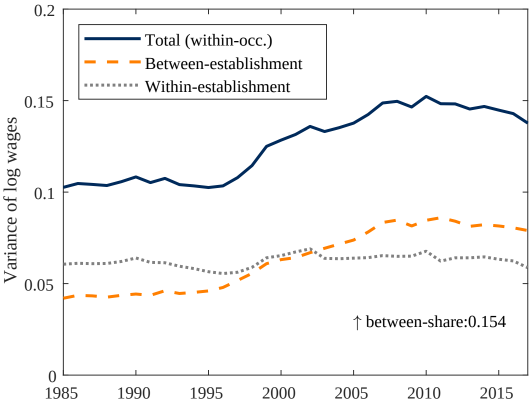
(a) With worker FEs



(b) Without worker FEs

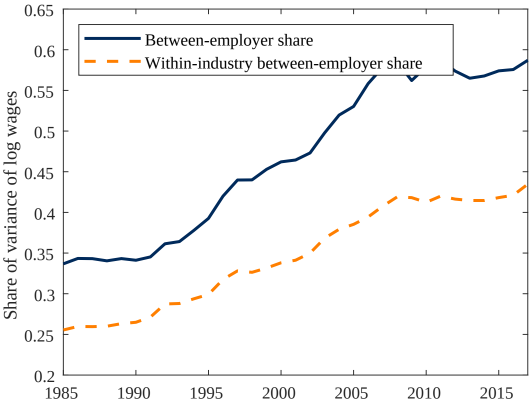
Between-/within-employer wage var. decomp. - within-Occupation

Main



Between-/within-employer wage var. decomp. - within-Industry

Main



Notes. Based on 'baseline' residualized wages.

Evolution of coworker sorting: correlation coefficient

Main

- Estimate $\hat{x}_i$ and $\hat{x}_{-it}$ separately for 5 periods
- In addition to the baseline, also consider ranking within-occupation

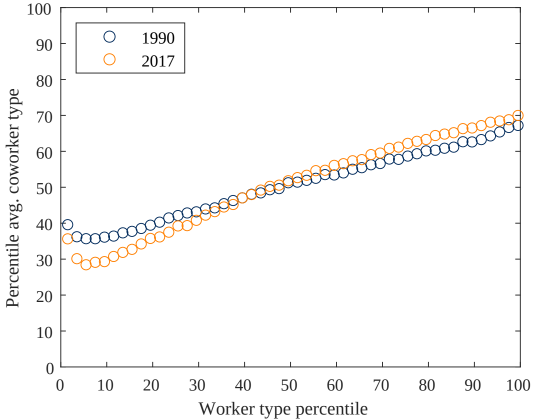
	Sorting	
Period	Spec. 1	Spec. 2
1985-1992	0.427	0.423
1993-1997	0.458	0.443
1998-2003	0.495	0.452
2004-2009	0.547	0.470
2010-2017	0.617	0.519

Notes. The column labelled "Sorting" indicates the correlation between a worker's estimated type and that of their average coworker, separately for five sample periods. Under "Spec. 1" workers are ranked economy wide (baseline), while under "Spec. 2" they are ranked within occupations.

Evolution of coworker sorting: within-occupation ranking binscatter

Main

- Reproduce non-linear sorting plot, but now $\hat{x}_i$ is based on *within-occupation* ranking



AKM-based wage variance decomposition

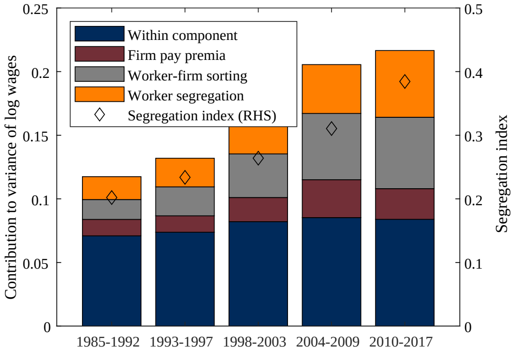
Main

$$\mathrm{Var}(w_{it}) = \underbrace{\mathrm{Var}(\alpha_i - \bar{\alpha}_{j(i,t)}) + \mathrm{Var}(\epsilon_{i,j})}_{\text{within-component}}$$

$$+ \underbrace{\mathrm{Var}(\psi_{j(it)}) + 2\mathrm{Cov}(\bar{\alpha}_{j(it)}, \psi_{j(it)}) + \mathrm{Var}(\bar{\alpha}_{j(it)})}_{\text{between-component}}$$

- $\mathrm{Var}(\psi_j)$: **firm-specific pay premia**
- $\mathrm{Cov}(\bar{\alpha}_j, \psi_j)$: **(worker-firm) sorting**
- $\mathrm{Var}(\bar{\alpha}_j)$: **(worker-worker) segregation**

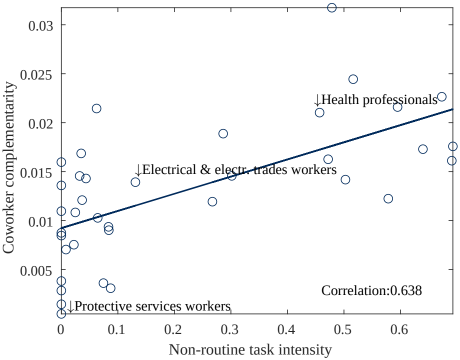
- Segregation index *[Kremer-Maskin, 1996]*: $\mathrm{Var}(\bar{\alpha}_{j(it)})/\mathrm{Var}(\alpha_i)$



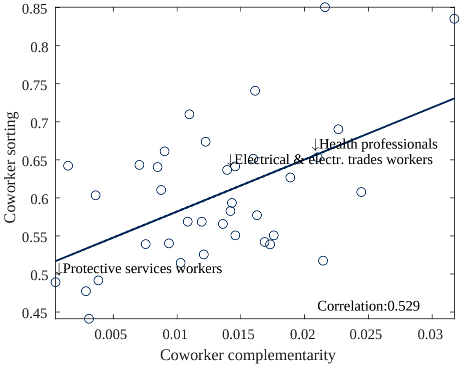
Occupations: task complexity ⇒ complementarity ⇒ sorting

Main

- ↑ Non-routine abstract task intensity ⇒ ↑ coworker wage complementarity



- ↑ Coworker wage complementarity ⇒ ↑ coworker sorting

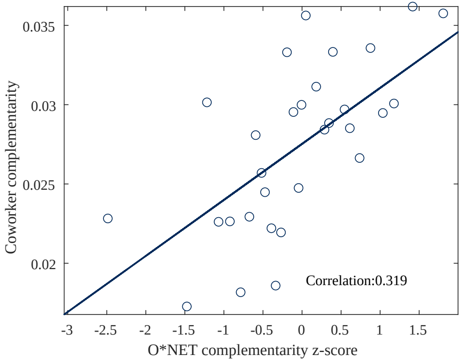


Notes. Quadros de Pessoal microdata. Horizontal axis indicates occupation's reliance on non-routine, abstract (NRA) tasks *[Mihaylov and Tidens, 2019]*.

Industries: coworker importance ⇒ complementarity ⇒ sorting

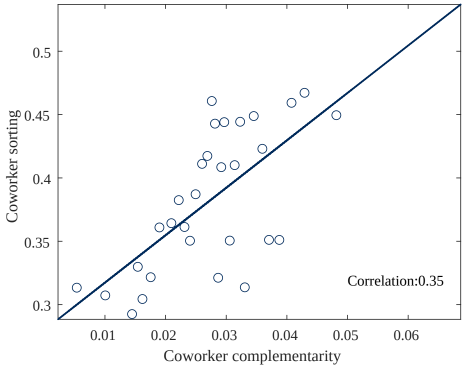
▸ Main

- ↑ **Teamwork** *[Bombardini et al., 2012]* ⇒ ↑ **coworker wage complementarity**



Notes. Horizontal axis measures the industry-level weighted mean score of an occupation-level index constructed from O*NET measuring the importance of: teamwork, impact on coworker output, communication, and contact.

- ↑ **Coworker wage complementarity** ⇒ ↑ **coworker sorting**

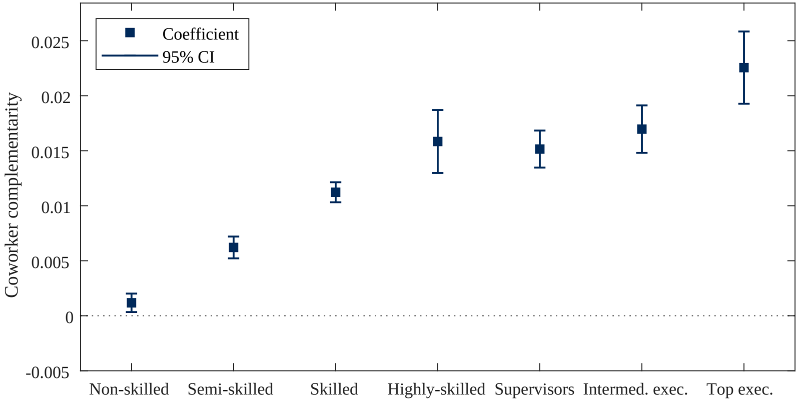


Notes. NACE-4-digit industries.

Hierarchies: complexity ⇒ complementarities

▸ Main ▸ Occupations

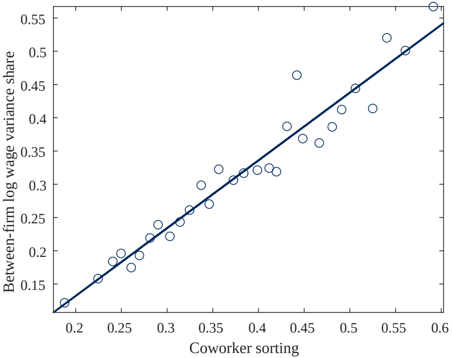
⇒ Coworker wage complementarities are (weakly) ↑ in the layer of a firm's hierarchy



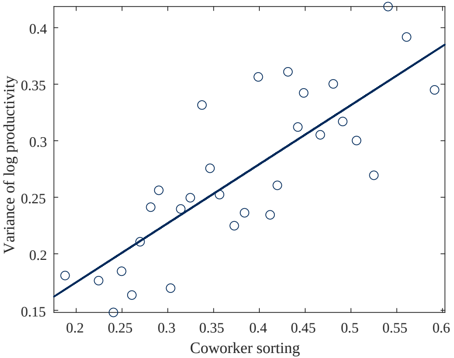
Industries: coworker sorting ⇒ between-firm inequality

▸ Main

⇒ Measures of between-firm inequality in productivity and pay are increasing in the degree of coworker sorting at the industry-level.



(a) Between-firm share of wage dispersion



(b) Productivity dispersion

Cluster-based methodology: motivation

Main

- **Standard AKM approach** estimates large number of firm-specific parameters, identified solely off worker mobility $\Rightarrow$ incidental parameters problem $\approx$ limited mobility bias $\Rightarrow$ *var*$(\psi) \uparrow$ & *cov*$(\psi, \alpha) \downarrow$
- **Bonhomme, Lamadon, and Malresa (2019**, Ecma): 2-step grouped FE estimation
  1. Recover firm classes using k-means clustering, based on similarity of earnings dist.
  2. Estimate parameters of correlated random effects model by maximum likelihood, conditional on the estimated firm classes
- **Potential advantages**
  1. mitigate limited mobility bias
     – sufficient number of workers who move between any given cluster to identify the cluster fixed effects
  2. allows relaxing sample restrictions (n-connected set restriction when estimating group-specific firm/cluster FEs)
  3. *if* also take step 2, can estimate match complementarities between firms and workers

Cluster-based methodology: implementation

- Obtain clusters by solving **weighted k-means problem**

$$\min_{k(1),\ldots,k(J),H_1,\ldots,H_K} \sum_{j=1}^{J} n_j \int (\hat{F}_j(w) - H_{K_j}(w))^2 d\mu(w),$$

  - $k(1), \ldots, k(J)$: partition of firms into K known classes; $\hat{F}_j$: empirical cdf of log-wages in firm j; n_j: average number of workers of firm j over sample period; $H_1, \ldots., H_K$: generic cdf's

- Implementation here:
  - baseline value of $K = 10$, as in BLM, but experiment with $K = 20$ and $K = 100$
  - use firms' cdf's over entire sample period on a grid of 20 percentiles
- "Half-BLM": take step (1), impute class to each worker-year observation, then estimate 2-way FE wage regression using cluster effects instead of firm effects:

$$w_{it} = \alpha_i + \sum_{k=1}^{K} \psi_k \mathbb{1}(J(i,t) = k) + \beta X'_{it} + r_{it}$$

Robustness checks: measuring coworker complementarity

Main

- Types from non-parametric ranking algorithm instead of AKM-based Jump
- Non-parametric, FD approximation: see paper
- Schooling as a non-wage measure of types Jump
- Small teams - team size variation Jump
- Lagged types: see paper

Complementarity estimates using years of schooling

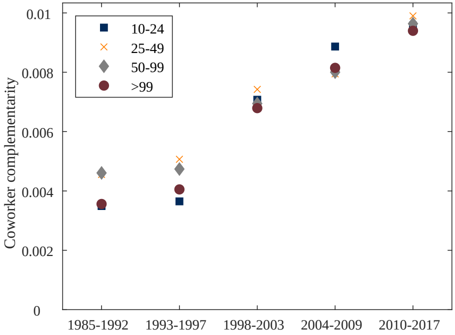
▸ Main

	'85-'92	'93-'97	'98-'03	'04-'09	'10-'17
Interaction	0.0063***	0.0060***	0.0099***	0.0112***	0.0129***
	(0.0008)	(0.0007)	(0.0008)	(0.0007)	(0.0009)
Obs. (100,000s)	3,613	2,508	2,694	3,836	4,376
R^2	0.5033	0.5451	0.5746	0.6330	0.6425

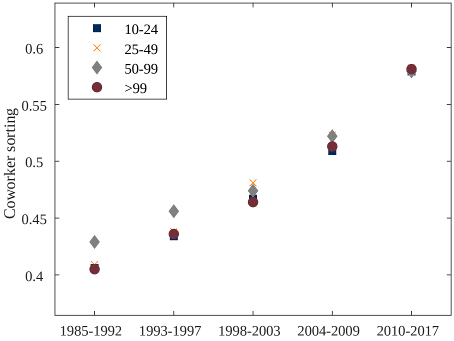
Notes. Dependent variable is the wage level over the year-specific average wage. Independent variables are a constant, years of schooling, coworker years of schooling, and the interaction between those two terms. All regressions include industry-year, occupation-year and employer fixed effects. Employer-clustered standard errors in parentheses. Observations are unweighted. The sample is unchanged from the main text, except that 96,517 observations with missing years of schooling are dropped. Observation count rounded to 100,000s.

Coworker complementarity & sorting by team size

▸ Robustness

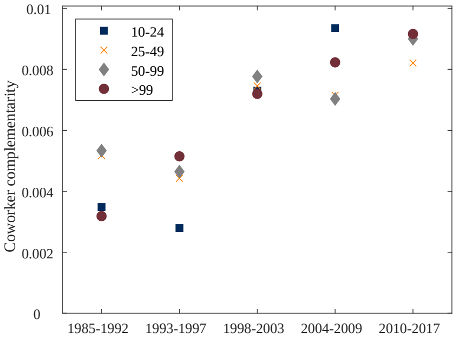


(a) Complementarity

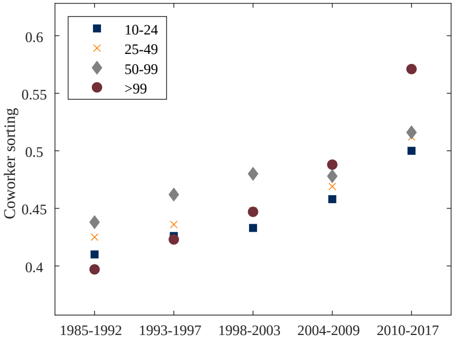


(b) Sorting

Coworker complementarity & sorting by team size – panel estab. only



(a) Complementarity

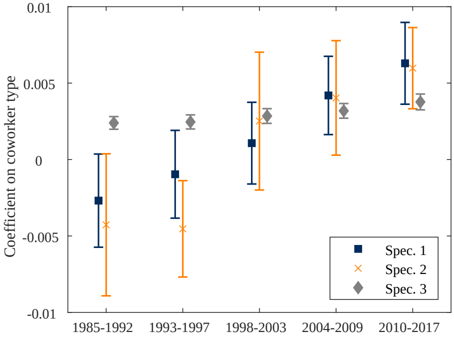


(b) Sorting

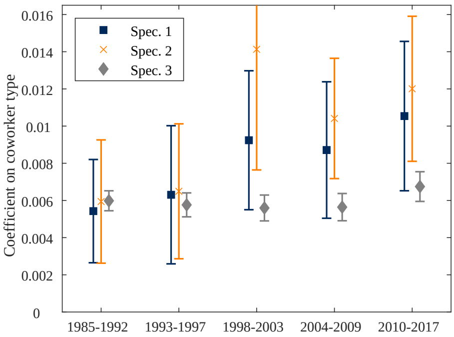
Coworker effects: log wage regression

Back: cross-section Back: time series

$$\ln w_{it} = \beta_0 + \beta_1 \hat{x}_i + \beta_2 \hat{x}_{-it} + \psi_{j(it)} + \nu_{o(i)t} + \xi_{s(i)t} + \epsilon_{it}$$



(a) AKM types



(b) NP types

Notes. Specifications vary by ranking method – within-economy (spec. 1) vs. within-occupation (spec. 2/spec.3) and coworker group definition – establishment-year (spec. 1/spec.2) vs. establishment-occupation-year (spec.3).

Sorting & complementarity based on non-parametric ranking algorithm

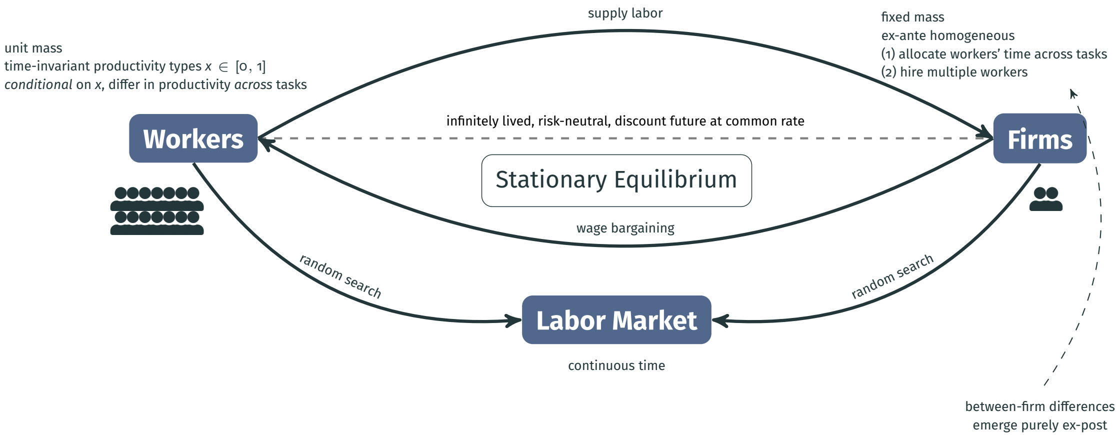
▸ Main

- Instead of ranking workers based on AKM worker FEs, use non-param. ranking algo *[Hagedorn et al., 2017]*

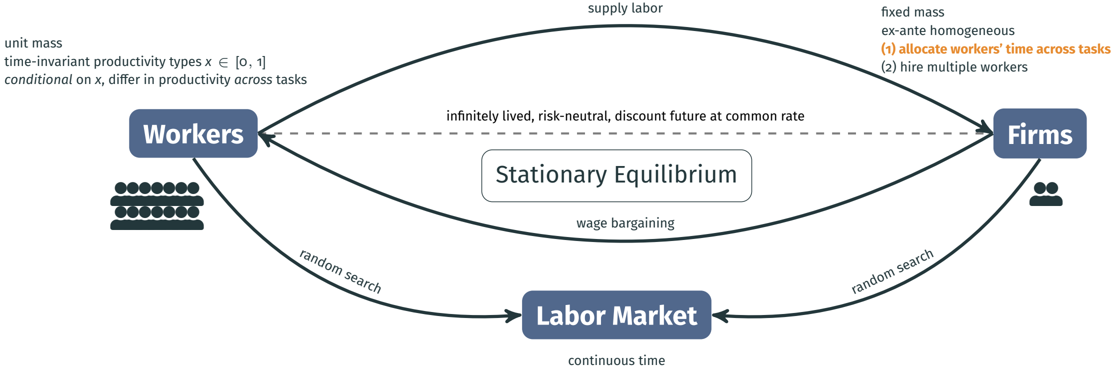
	Sorting		Complementarities	
Period	Spec. 1	Spec. 2	Spec. 1	Spec. 2
1985-1992	0.47	0.38	0.001	0.000
1993-1997	0.56	0.46	0.002	0.001
1998-2003	0.60	0.48	0.004	0.002
2004-2009	0.65	0.50	0.005	0.002
2010-2017	0.68	0.51	0.005	0.004

Notes. This table indicates, under the column "Sorting" the correlation between a worker's estimated type and that of their average coworker, separately for five sample periods. The column "Complementarities" indicates the point estimate of the regression coefficient β_C. Under "Specification 1" workers are ranked economy wide, while under "Specification 2" they are ranked within two-digit occupations. Worker rankings are based on the non-parametric method.

Overview of model environment: firm organization meets labor search

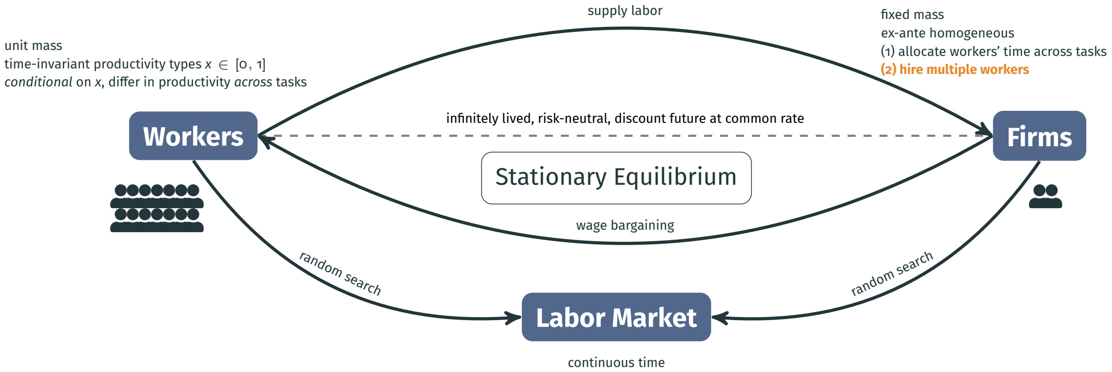


Overview of model environment: firm organization meets labor search



1. **task assignment: derive production function for *one* firm, workforce exogenous**
2. competition for talent: equilibrium matching *given* production function

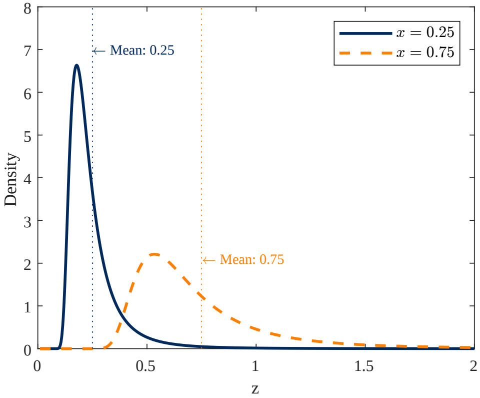
Overview of model environment: firm organization meets labor search



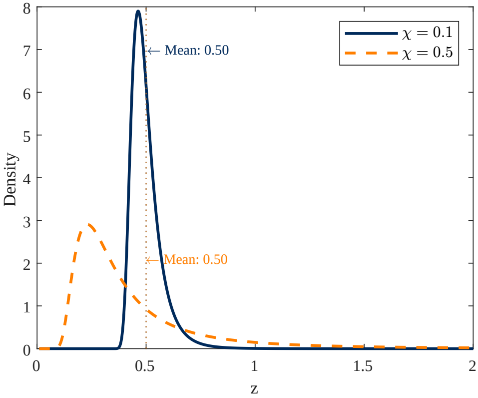
1. task assignment: derive production function for *one* firm, workforce exogenous
2. **competition for talent: equilibrium matching *given* production function**

Illustration: Fréchet distribution

▸ Main



(a) Scale parameter



(b) (Inverse) shape parameter

Taylor approximation to CES

▸ Back to team production ▸ Back to stylized model

- Analytically tractable version of the hiring block:

$$f(x_1, x_2) = x_1 + x_2 - \xi(x_1 - x_2)^2$$

- Justification: CES and link to team production model
    - in the $\kappa = 1$ special case, ξ maps onto $\frac{\chi}{\chi+1}$ (up to scale)

Remark: Second-order Taylor approximation to CES

The second-order Taylor approximation to $f(x_1, x_2) = (\frac{1}{2}x_1^\gamma + \frac{1}{2}x_2^\gamma)^{1/\gamma}$ around $(\bar{x}, \bar{x})$ with $\bar{x} = \frac{x_1+x_2}{2}$ is

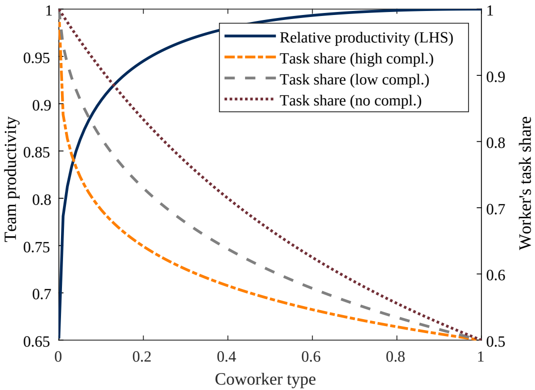
$$\bar{x} - \frac{1}{2}\underbrace{(1-\gamma)}_{\approx\xi}\frac{\sigma_x^2}{\bar{x}},$$

where $\sigma_x^2 = (\frac{x_1-x_2}{2})^2$

Quality mismatch & task mismatch

Main

- When high type is paired with a low type, she ends up “wasting time” on tasks that she is relatively less efficient in and wouldn’t have to do if teamed up optimally



Extension: team production with communication costs

▸ Main

- Assumption till now: division of labor incurs no output losses due to coordination frictions
- But implementing the division of labor may $\downarrow$ time available for task production because of *communication* requirements
  [Becker & Murphy, 1992; Deming, 2017]
- **Extension** allowing for such **coordination costs** shows:
  1. qualitative link between technology & coworker complementarities exists *unless* division of labor is prohibitively costly
  2. $\chi \uparrow \Rightarrow$ importance of organizational quality for productivity $\uparrow$
- Ongoing research: rich microdata from Fortune-100 company to describe communication behavior

Extension: specialization vs. comparative advantage

Main

- Suppose the joint distribution of task-specific productivities across coworkers satisfies

$$\Pr\left[z_i(\tau) \leq z_1, \ldots, z_n(\tau) \leq z_n\right] = \exp\left[-\left(\sum_{i=1}^{n}\left(\left(\frac{z}{\iota x_i}\right)^{-\frac{1}{\tilde{\chi}}}\right)^{\frac{1}{\xi}}\right)^{\xi}\right], \tag{6}$$

  where $\tilde{\chi}$ is the common shape parameter of the the marginal Fréchet distributions, while $\xi \in (0, 1]$ controls correlation in draws across workers

- Can derive

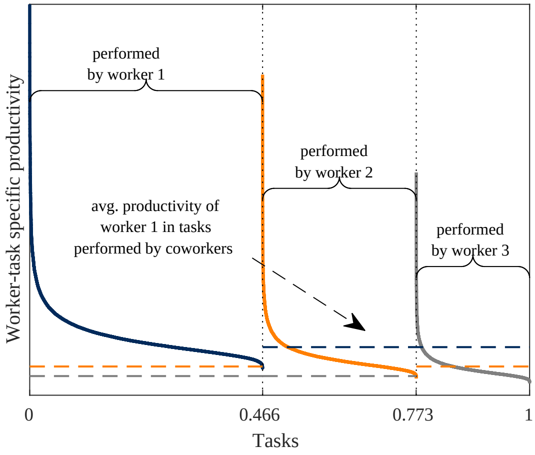
$$Y = f(\mathbf{x}; \tilde{\chi}, \xi) = n^{1+\tilde{\chi}\xi}\left(\frac{1}{n}\sum_{i=1}^{n}(a_1 x_i)^{\frac{1}{\chi\xi+1}}\right)^{\tilde{\chi}\xi+1} \tag{7}$$

- All results go through with $\chi = \tilde{\chi}\xi$

Optimal organization: illustration

Main

1. tasks assigned by comparative advantage
   - i's task set $\mathcal{T}_i = \left\{ \tau \in \mathcal{T} : \frac{z_i(\tau)}{\lambda_i^L} \geq \max_{k \neq i} \frac{z_k(\tau)}{\lambda_k^L} \right\}$

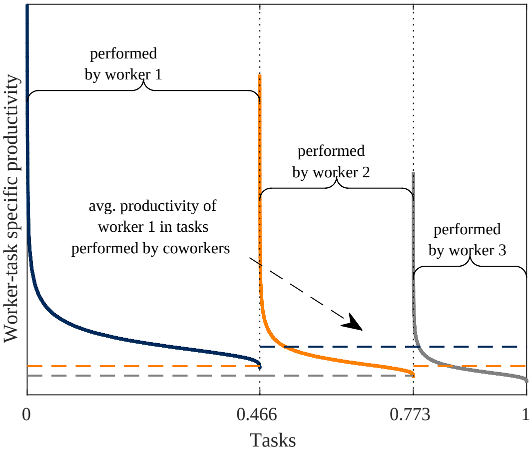


Optimal organization: illustration

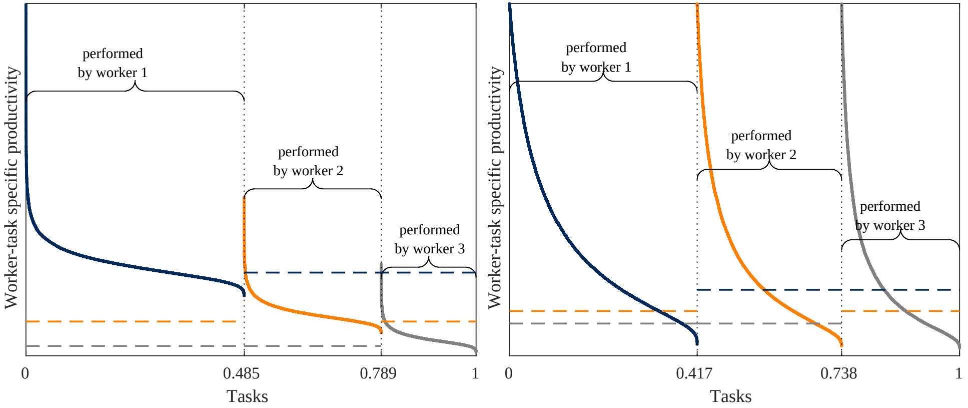
1. tasks assigned by comparative advantage

2. *i*'s share of tasks $\uparrow$ in *i*'s talent, $\downarrow$ in coworkers' talent

   - *i*'s task share $\pi_i = \left(x_i^{\frac{1}{1+\chi}}\right)\left(\sum_{k=1}^{n}(x_k)^{\frac{1}{1+\chi}}\right)^{-1}$



Optimal organization: illustration – low vs. high χ



(a) Low specialization (χ)

(b) Higher specialization (χ)

Numerical example: setup

Main

- **H, H'**: avg productivity 100 vs. **L, L'**: avg. productivity 50
- Working alone: realized productivity equal to avg
- **Low value of** χ

	Ideation	Theory	Matlab	Stata	Write	Present	With L'	With H'	Δ
L	53	52	51	49	48	47			
H	106	104	102	98	96	94			

- **High value of** χ

	Ideation	Theory	Matlab	Stata	Write	Present	With L'	With H'	Δ
L	65	60	55	45	40	35			
H	130	120	110	90	80	70			

Numerical example: teaming up with L'

- **Low value of χ**

	Ideation	Theory	Matlab	Stata	Write	Present	With L'	With H'	Δ
L	**53**	**52**	**51**	49	48	47	52		
H	**106**	**104**	**102**	**98**	**96**	94	101.2		
L'	47	48	49	51	52	53			

- **High value of χ**

	Ideation	Theory	Matlab	Stata	Write	Present	With L'	With H'	Δ
L	**65**	**60**	**55**	45	40	35	60		
H	**130**	**120**	**110**	**90**	80	70	112.5		
L'	35	40	45	50	55	60			

Numerical example: teaming up with H'

- **Low value of χ**

	Ideation	Theory	Matlab	Stata	Write	Present	With L’	With H’	Δ
L	**53**	52	51	49	48	47	52	53	1
H	**106**	**104**	**102**	98	96	94	101.2	104	2.8
L’	47	48	49	51	52	53			
H’	94	96	98	102	104	106			

- **High value of χ**

	Ideation	Theory	Matlab	Stata	Write	Present	With L’	With H’	Δ
L	**65**	**60**	55	45	40	35	60	62.5	2.5
H	**130**	**120**	**110**	90	80	70	112.5	120	7.5
L’	35	40	45	50	55	60			
H’	70	80	90	110	120	130			

Numerical example: comparison and take-aways

Low value of χ	With L’	With H’	Δ
L	52	53	**1**
H	101.2	104	**2.8**
Δ			1.8

High value of χ	With L’	With H’	Δ
L	60	62.5	**2.5**
H	112.5	120	**7.5**
Δ			5

Takeaways

1. Higher χ: greater benefit from team production (“love for variety”)
2. Higher χ: coworker quality matters more for own realized productivity
3. Higher χ: stronger complementarity – benefit from better coworker for H grows more than that for L

Task allocation

Main

Corollary: Task shares

Suppose that $x_i > x_j$. Then (i) i performs a strictly larger share of tasks than j for $\chi < \infty$; (ii) the difference in task shares is decreasing in χ; (iii) the ratio of i's relative to j's task share approaches $\frac{x_i}{x_j}$ as $\chi \to 0$; and (iv) task shares are equalized as $\chi \to \infty$.

Proof:

All four elements of the statement immediately follow by noting that Lemma 1 (ii) implies that $\frac{\pi_i}{\pi_j} = \left(\frac{x_i}{x_j}\right)^{\frac{1}{1+\chi}}$.

Extension: multivariate Fréchet

Main Back to extensions

- Baseline: worker-task specific productivities are *independent* draws from Fréchet,

$$Pr(z_i(\tau) \leq z) = \exp\left(- \left(\frac{z}{\iota x_i} \right)^{-1/\chi} \right)$$

  - χ plays dual role: within-worker dispersion and across-coworker dispersion

- Instead, suppose that *joint* distribution across workers

$$P\left[z_i(\tau) \leq z_1, \ldots, z_n(\tau) \leq z_n\right] = \exp\left[- \left(\sum_{i=1}^{n} \left(\left(\frac{z}{\iota x_i} \right)^{-\frac{1}{\chi}} \right)^{\frac{1}{\xi}} \right)^{\xi} \right]$$

  - χ: now clearly defined as referring to one workers' productivity profile over tasks
  - $\xi \in (0, 1]$: **captures correlation in draws *across* workers**

Extended aggregation result

Lemma-1'

Proposition: Aggregation result – extended

The vector of talent types $(x_1, \ldots, x_n)$ is a sufficient statistic for team output Y, s.t.

$$Y = f(\mathbf{x}; \chi, \xi) = n^{1+\chi\xi} \left(\frac{1}{n} \sum_i (x_i)^{\frac{1}{\chi\xi+1}} \right)^{\chi\xi+1}$$

Interpretation of extended aggregation result

- **Intuition:** (for n sufficiently large), **output is greater if**
  1. **coworkers are *similar* to each other in terms of talent** ("vertical");

     as before, but also if
  2. **coworkers are more *different* from one another in terms of *what* tasks they're good at** ("horizontal")
- Dependence of Y on team composition – in vertical & horizontal dimension – is increasing in χ
  - nb: for χ large enough, output by 2 low-x workers can > that of 2 high-x workers *if* former team characterized by higher ξ
- Coworker talent complementarities now depend on $\chi \times \xi$, hence also on ξ
  - results in baseline go through with $\hat{\chi} = \chi\xi$

Dynamic setting: motivation

- **Q:** how is ξ determined?
- **Intuition:** in a *dynamic* setting, ξ is naturally endogenous to firms' hiring decisions: to maximize production, a firm will *aim* to hire workers whose $\mathbf{z}$ vector has a low correlation with existing employee(s)
- Microfoundation thus motivates departure from 'standard' setup w/ matching based on absolute advantage types only
  - implicit assumption of standard model: no hiring based on task-specific skills
  - this becomes less tenable if χ is strong

Dynamic setting: timing of horizontal match quality revelation

- An unmatched worker x randomly draws (i) a searching firm with current employee x'; and together they draw (ii) a 'horizontal match-quality shock' ξ from an exogenous distribution H
  - task-specific skills as fully observable at time of meeting ('inspection good'), i.e. no gradual learning about horizontal match takes place over time
- Given the aggregation result, (x, x', ξ) are sufficient to determine Y, and the matching decision is made

Sketch of dynamic model (1): overview

- Define

$$
\begin{aligned}
\Omega_1(x) &:= V_{f.1}(x) + V_{e.1}(x) \\
S(x) &:= \Omega_1(x) - V_{f.0} - V_u(x) \\
\Omega_2(x, x', \xi) &:= V_{f.2}(x, x', \xi) + V_{e.2}(x|x', \xi) + V_{e.2}(x'|x, \xi) \\
S(x|x', \xi) &:= \Omega_2(x, x', \xi) - \Omega_1(x') - V_u(x)
\end{aligned}
$$

- Hiring policy for second worker: $h(x|x', \xi) = 1\{S(x|x', \xi) > 0\}$
  - $h(x|x') = \Pr\{S(x|x', \xi) > 0\}$
  - $h(x) = 1\{S(x) > 0\}$
- Evolution of distribution: as before, given probabilistic definition of $h(x|x')$

Sketch of dynamic model (2): values

- Value of unmatched firm:

$$\rho V_{f.0} = (1-\omega)\lambda_{v.u} \int \frac{d_u(x)}{u} S(x)^+ dx$$

- Value function of unmatched worker

$$\rho V_u(x) = b(x) + \lambda_u \omega \left[\int \frac{d_{f.0}}{v} S(x)^+ + \int \frac{d_{m.1}(\tilde{x}')}{v} S(x|\tilde{x}', \tilde{\xi})^+ dH(\tilde{\xi}) d\tilde{x}' \right] \tag{8}$$

- Joint value of a firm with employee x

$$\rho \Omega_1(x) = f_1(x) - \delta S(x) + \lambda_{v.u}(1-\omega) \int \frac{d_u(\tilde{x}')}{u} S(\tilde{x}'|x, \tilde{\xi})^+ dH(\tilde{\xi}) d\tilde{x}' \tag{9}$$

- Joint value of firm with (x, x', ξ)

$$\rho \Omega_2(x, x', \xi) = f_2(x, x', \xi) - \delta S(x|x', \xi) - \delta S(x'|x, \xi) \tag{10}$$

Sketch of dynamic model (3): solution approach

OJS: a challenge

- Define *threshold values* of ξ for each (x, x') such that $S(x|x', \bar{\xi}(x|x')) = 0$. As this equation is quadratic in ξ, there are 0, 1 or 2 roots in the interval $[0, 1]$:
  - $0 = S(x|x', 1) + \frac{f(x,x',\bar{\xi}(x|x')) - f(x,x',1)}{\rho+2\delta}$
  - $\bar{\xi}$: $\xi \in (0, 1] : S(x|x', \bar{\xi}(x|x')) > 0$ if $\xi > \bar{\xi}(x|x')$
  - $\underline{\xi}$: $\xi \in (0, 1] : S(x|x', \underline{\xi}(x|x')) > 0$ if $\xi < \underline{\xi}(x|x')$
- Implied *conditional expected values*:
  - $\bar{\xi}^*(k) = \frac{\int_k^1 \xi dH(\xi)}{1-H(k)}$; and $\underline{\xi}^*(k) = \frac{\int_0^k \xi dH(\xi)}{H(k)}$
- Then rewrite eg equation (8)

$$\rho V_u(x) = b(x) + \lambda_u \omega \left[\int \frac{d_{f.0}}{v} S(x)^+ + \int \frac{d_{m.1}(\tilde{x}')}{v} S^*(x|\tilde{x}') \right] d\tilde{x}'$$

where $S^*(x|\tilde{x}') = \Big[H\left(\underline{\xi}\left(x|\tilde{x}'\right)\right) \times S\left(x|\tilde{x}', \underline{\xi}^*\left(\underline{\xi}\left(x|\tilde{x}'\right)\right)\right) + \left(1 - H\left(\bar{\xi}\left(x|\tilde{x}'\right)\right) \times S\left(x|\tilde{x}', \bar{\xi}^*\left(\bar{\xi}\left(x|\tilde{x}'\right)\right)\right)\right.$

Implication: new strategy to identify χ using the micro-foundation

Corollary: Identification of χ

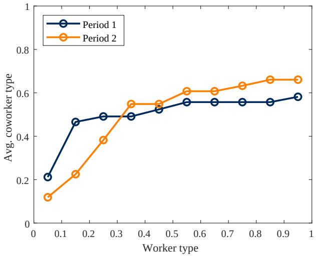
Variation in wages *conditional* on x and x' identifies χ:

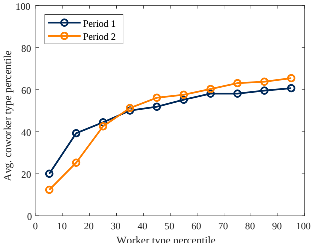
$$\frac{\overbrace{w(x|x';\xi=1)}^{w_{\max}(x|x')}}{\underbrace{w(x|x';\xi=\bar{\xi}(x|x'))}_{w_{\min}(x|x')}} = \frac{\omega\big(f(x,x';\xi=1)-f(x')\big)+g(x)-h(x')}{\omega\big(f(x,x',\xi=\bar{\xi}(x|x))-f(x')\big)+g(x)-h(x')}$$

- **Intuition:** variation in Y – and hence w – conditional on workforce composition reflects differences in horizontal match quality, ξ; the extent to which variation in ξ translates into variation in Y depends on the importance of specialization, χ.
- ✓method validated inside theoretical model

Implication: improves the quantitative fit

- *Tentative* calibration of extended model
- Extended model (bottom) produces 'smoother' average coworker profiles
  - can also see this when looking at the full conditional coworker distribution
  - also 'smoother' response to changes in χ
- Reason: matching probabilities conditional on meeting are no longer binary





Implication: new empirical avenues

- Occupation-pair specific complementarities should be greater if (i) these two occupations are distant in skill/task space, but (ii) they are commonly employed together
  - eg complementarity should be high for the pair hospital-manager/nurse
- Costs of job displacement should be greater if (i) highly specialized, and (ii) it is difficult to find coworkers with 'horizontally matching' skills
  - team production and Marshallian agglomeration effects

Extended Lemma 1

Lemma: Lemma 1 – extended

Implied task share and shadow-cost index equal

$$\pi_i = \frac{\left(x_i/\lambda_i^L\right)^{\frac{1}{\chi\xi}}}{\sum_{k=1}^{n}\left(x_i/\lambda_i^L\right)^{\frac{1}{\chi\xi}}} \quad x; \lambda = \left(\sum_{i=1}^{n}\left(\frac{x_i}{\lambda_i^L}\right)^{\frac{1}{\chi\xi}}\right)^{-\chi\xi}$$

OJS: *not* currently feasible with match-specific shocks

- **Challenge:** need to track distribution of ξ's among the (x, x') pairs, but those will be mixtures of distributions that itself depend on team combination and origin
  - consider a vacant firm attempting to poach x, if x is currently with x' and their horizontal fit is ξ; then moving decision depends on $S(x) - S(x|x', \xi)$
  - but distribution of ξ conditional on (x, x') will differ depending on how (x, x') was formed, e.g. one coming out of unemployment vs via EE moves itself
  - attempt to poach x by a firm with x'' when (x, x') is the current team: look at $S(x|x'', \xi') - S(x|x', \xi)$ $\rightarrow$ ξ' can be drawn from the unconditional distribution H but ξ comes from a distribution $H(z|x, x')$, which doesn't have a tractable form
- Idea for a solution approach – but will be different paper
  - conjecture: $H(z|x, x')$ is be a mixture of left-truncated versions of H, where the truncation points differ by (x, x'), but crucially also on the origin of that team
  - conceptually, the mixture weights should be obtainable from the ergodic distribution itself $\rightarrow$ possibility for a fixed point...

Environment: demographics & preferences & production technology

▸ Main

- **Time**: continuous
- **Agents:** workers & firms
  - unit mass of workers, types uniformly distributed $x \in \mathcal{X} = [0, 1]$
  - m_f mass of firms – ex-ante homogeneous
  - agents indexed by *ranks* of prod. dist., hence uniform type dist. *[Hagedorn et al., 2017]*
  - agents infinitely-lived, risk-neutral, common discount rate ρ, max. PV of payoffs
- **Production technology**: firms are vacant or have 1 or 2 workers
  - normalize team size to max. $n = 2 \leftarrow$ key is "existing workforce" & "potential hire"
  - convention: from x's perspective, let x' denote *coworker*
  - team production: $f(x, x') \leftarrow$ see microfoundation
  - 1-worker: $f(x)$, short for $f(x, \emptyset)$

Environment: random search & wage bargaining

- **Timing** within *dt*-intervals
  1. exogenous separation: Poisson rate δ
  2. random search & matching
  3. production & surplus sharing
- **Meeting process**: unemployed meet *some* firm at Poisson rate λ_u
  - probability for a firm to be contacted by an(y) unemployed: $\lambda_{v.u} = \lambda_u \times u$
  - baseline: no on-the-job search
- **Matching** decisions based on joint surplus b/w firm & worker(s): privately efficient
- **Surplus sharing**: firm bargains with potential new hire, taking into account coworker complementarities; worker bargaining power ω
  - continuous renegotiation, as if each worker is marginal (i.e., outside option: unemp.)

Stationary equilibrium

▸ Equilibrium equations

- Formally, after defining (i.) HJBs for unemployed & vacant & surplus values, and (ii.) Kolmogorov Forward Equations (KFEs) describing the evolution of the distribution of agents across states:

> **Definition:**
>
> A stationary search equilibrium is a tuple of value functions together with a stationary distribution of agents across states such that (i.) the value functions satisfy the HJB Equations given the distribution; and (ii.) the distribution satisfies the KFEs given the policy functions implied by the value functions.

- Needs to be computed **numerically**
  - agents' expectations & decisions must conform w/ population dynamics to which they give rise; as distribution evolves, so do agents' expectations

Environment: firm & worker states

- Distribution across states for a **worker** type x:

$$d_w(x) = d_u(x) + d_{m.1}(x) + \int d_{m.2}(x, \tilde{x}')d\tilde{x}'$$

  - $d_u(x)$: 'density' of unemployed of type x
  - $d_m(x)$, shorthand for $d_m(x, \emptyset)$: 'density' of matches w/ x as only worker
  - $d_m(x, x')$: 'density' of "team matches" b/w x and x'

- Distribution across states for a **firm** type y:

$$d_f = d_{f.0} + \int d_{m.1}(x)dx + \frac{1}{2}\int\int d_{m.2}(x, x')dxdx'.$$

  - $\frac{1}{2}$: account for 1 firm having 2 workers

- **Aggregates** can be backed out, e.g. $u = \int d_u(x)dx$

Environment: surplus sharing

- **One-worker firm:**

$$(1-\omega)\big(V_{e.1}(x) - V_u(x)\big) = \omega\big(V_{f.1}(x) - V_{f.0}\big) \tag{11}$$

Two-worker firm

$$(1-\omega)\big(V_{e.2}(x|x') - V_u(x)\big) = \omega\big(V_{e.2}(x'|x) + V_{f.2}(x,x') - V_{e.1}(x') - V_{f.1}(x')\big) \tag{12}$$

Surplus max. determines which worker types a firm w/ worker x hires Main

- Joint value of firm with worker x, $\Omega(x)$, satisfies:

$$\rho\Omega(x) = f(x) + \delta\left[-\Omega(x) + V_u(x) + V_{f.0}\right] + (1-\omega)\lambda_{v.u}\int \frac{d_u(\tilde{x}')}{u}\max\{S(\tilde{x}'|x), 0\}d\tilde{x}'$$

  - $V_u(x)$: value for unemp. worker; $V_{f.0}$: value for vacant firm; $S(x)$: surplus from zero-worker firm hiring x
  - $d_u(x)$: density of unemployed workers of type x; $u = \int d_u(x)dx$
  - ω: worker bargaining wgt; δ: sep. rate; $\lambda_{v.u}$: rate of vacancy meeting unmatched worker

Surplus max. determines which worker types a firm w/ worker x hires ▸ Main

- Joint value of firm with worker x, $\Omega(x)$, satisfies:

$$\rho\Omega(x) = f(x) + \delta\Big[- \Omega(x) + V_u(x) + V_{f.0}\Big] + (1-\omega)\lambda_{v.u} \int \frac{d_u(\tilde{x}')}{u} \max\{S(\tilde{x}'|x), 0\} d\tilde{x}'$$

  - $V_u(x)$: value for unemp. worker; $V_{f.0}$: value for vacant firm; $S(x)$: surplus from zero-worker firm hiring x
  - $d_u(x)$: density of unemployed workers of type x; $u = \int d_u(x)dx$
  - ω: worker bargaining wgt; δ: sep. rate; $\lambda_{v.u}$: rate of vacancy meeting unmatched worker

- Surplus $S(x|x')$ reflects complementarities – and hiring decisions reflect surplus

$$S(x|x')(\rho + 2\delta) = f(x, x') - \rho\big(V_u(x) + V_u(x') + V_{f.0}\big) + \delta S(x) - (\rho+\delta)S(x')$$

$$h(x|x') = \mathbf{1}\{S(x|x') > 0\}$$

HJB: unmatched

Main

- Unmatched firm:

$$\rho V_{f.0} = (1-\omega)\lambda_{v.u} \int \frac{d_u(x)}{u} S(x)^+ dx \tag{13}$$

- Unmatched worker:

$$\rho V_u(x) = b(x) + \lambda_u \omega \left[\int \frac{d_{f.0}}{v} S(x)^+ + \int \frac{d_{m.1}(\tilde{x}')}{v} S(x|\tilde{x}')^+ d\tilde{x}' \right] \tag{14}$$

HJB: surpluses

- Surplus of coalition of firm with worker x

$$(\rho+\delta)S(x) = f(x) - \rho\big(V_u(x) + V_{f.0}\big) + \lambda_{v.u}(1-\omega)\int \frac{d_u(\tilde{x}')}{u} S(\tilde{x}'|x)^+ d\tilde{x}' \tag{15}$$

- Surplus from adding x to x'

$$S(x|x')\big(\rho+2\delta\big) = f(x,x') - \rho\big(V_u(x) + V_u(x') + V_{f.0}\big) + \delta S(x) - (\rho+\delta)S(x') \tag{16}$$

KFE: unemployed

$$\delta\left(d_{m.1}(x)+\int d_{m.2}(x,\tilde{x}')d\tilde{x}'\right)=d_u(x)\lambda_u\left(\int\frac{d_{f.0}}{v}h(x,\tilde{y})+\int\frac{d_{m.2}(\tilde{x}')}{v}h(x|\tilde{x}')d\tilde{x}'\right). \tag{17}$$

KFE: one-worker matches

$$d_{m.1}(x)\left(\delta + \lambda_{v.u}\int \frac{d_u(\tilde{x}')}{u} h(\tilde{x}'|x)d\tilde{x}'\right) = d_u(x)\lambda_u \frac{d_{f.0}}{v} h(x) + \delta \int d_{m.2}(x,\tilde{x}')d\tilde{x}'. \tag{18}$$

KFE: two-worker matches

$$2\delta d_{m.2}(x,x') = d_u(x)\lambda_u \frac{d_{m.1}(x')}{v} h(x|x') + d_u(x')\lambda_u \frac{d_{m.1}(x)}{v} h(x'|x). \tag{19}$$

Lemma: monotonicity of unemployment value and wage in x

Main

Lemma: Monotonicity

Assume that $\frac{f_1(x)}{\partial x} > 0$, $\frac{\partial f_2(x,x')}{\partial x} > 0$, and $\omega > 0$. Then: (i) The value of unemployment $V_u(x)$ is monotonically increasing in x, and (ii) so is the wage function $w(x|x')$.

Proof:

See paper appendix. Key: surplus representation.

Wage equation

▸ Main

- NB: here allow for ex-ante firm heterogeneity $\Rightarrow$ measurement result extends

$$
\begin{aligned}
w(x|y,x') = {} & \rho V_u(x) + \omega\big[f(x,y,x') - \rho\left(V_u(x) + V_u(x') + V_{f.o}(y)\right) \\
& + \delta S(x|y) - (\rho+\delta)S(x'|y)\big] - \delta\omega S(x|y) \\
= {} & \omega\big(f(x,y,x') - f(x',y)\big) + (1-\omega)\rho V_u(x) \\
& - \omega(1-\omega)\lambda_{v.u}\int \frac{d_u(\tilde{x}'')}{u} S(\tilde{x}''|y,x')^{+} d\tilde{x}''.
\end{aligned}
$$

Characterization using a stylized model: setup

Main

Intuition for how coworker complementarities shape matching can be gained from a stylized model $\rightarrow$ closed-form solutions

- **Simplified setup:**
  - no ex-ante firm heterogeneity; mass of firms $m_f = \frac{1}{2}$
  - production: $f(x, x') = x + x' - \xi(x - x')^2$, where ξ controls complementarity

    Approx. of microfounded team prod. fn.
  - no production with 1 employee & abstract from team production benefits
  - firm has no bargaining power, workers each receive outside option plus half the surplus
- **Explicit search costs**: no discounting, guaranteed match ($M_u = M_f = 1$); but type-invariant worker search costs c
  - supermodularity in f suffices for PAM [Atakan, 2006]

Characterization using a stylized model: timing

▸ Frictionless stage-2 outcome

0. Each firm is randomly paired with one worker $x' \in \mathcal{X}$
   - remaining: mass $\frac{1}{2}$ of uniformly distributed workers; mass $\frac{1}{2}$ of firms with one employee

1. Each (firm + x') unit is randomly paired with a worker $x \in \mathcal{X} \to$ decision:
   a. **match**: form a team
      + produce & share production value
      – no further actions and zero payoff in stage 2

      or
   b. **search**: don't form a team
      – workers pay search cost c
      + all have opportunity to re-match in stage 2, s.t.

2. frictionless matching b/w unmatched firms & workers; production
   - pure PAM: x works with x ($\leftarrow$ deterministic coupling $\mu(x) = x$)
   - payoffs *given* pure PAM: $w^*(x) = x$ and $v^* = 0$

Characterization using a stylized model: timing

▸ Frictionless stage-2 outcome

0. Each firm is randomly paired with one worker $x' \in \mathcal{X}$
   - remaining: mass $\frac{1}{2}$ of uniformly distributed workers; mass $\frac{1}{2}$ of firms with one employee
1. Each (firm + x') unit is randomly paired with a worker $x \in \mathcal{X} \rightarrow$ decision:

   a. **match**: form a team
   + produce & share production value
   – no further actions and zero payoff in stage 2

   or

   b. **search**: don't form a team
   – workers pay search cost c
   + all have opportunity to re-match in stage 2, s.t.

2. frictionless matching b/w unmatched firms & workers; production
   - pure PAM: x works with x ($\leftarrow$ deterministic coupling $\mu(x) = x$)
   - payoffs *given* pure PAM: $w^*(x) = x \quad$ and $\quad v^* = 0$

Characterization using a stylized model: timing

▸ Frictionless stage-2 outcome

0. Each firm is randomly paired with one worker $x' \in \mathcal{X}$
   - remaining: mass $\frac{1}{2}$ of uniformly distributed workers; mass $\frac{1}{2}$ of firms with one employee
1. Each (firm + x') unit is randomly paired with a worker $x \in \mathcal{X} \rightarrow$ decision:
   a. **match**: form a team
      + produce & share production value
      – no further actions and zero payoff in stage 2

      or
   b. **search**: don't form a team
      – workers pay search cost c
      + all have opportunity to re-match in stage 2, s.t.
2. frictionless matching b/w unmatched firms & workers; production
   - pure PAM: x works with x ($\leftarrow$ deterministic coupling $\mu(x) = x$)
   - payoffs *given* pure PAM: $w^*(x) = x$ and $v^* = 0$

Characterization using a stylized model: stage-1 matching decision

- A firm with employee x' that meets a worker of type $x \in \mathcal{X}$ decides to hire her, i.e. $h(x, x') = 1$, if

$$\underbrace{\overbrace{f(x,x')}^{\text{match}} - \overbrace{\left[w^*(x) + w^*(x') + v^* - 2c^w\right]}^{\text{search}}}_{\equiv S(x,x')} > 0$$

- **Threshold distance** s^* s.t. $h(x, x') = 1 \Leftrightarrow |x' - x| < s^*$
- Threshold distance satifies: $s^* = \sqrt{2c/\xi}$
  - **greater complementarities (ξ) render the matching set *narrower***
  - greater search costs (c) render the matching set *wider*

Characterization using a stylized model: corollary

Main

Corollary: Stylized model

For a given threshold s, which is *decreasing* in χ:

1. the coworker correlation is: $\rho_{xx} = (2s+1)(s^2-1)^2$;
2. the average coworker type is

$$\hat{\mu}(x) = \begin{cases} \frac{x+s^*}{2} & \text{for} \quad x \in [0, s^*) \\ x & \text{for} \quad x \in [s^*, 1-s^*] \\ \frac{1+x-s^*}{2} & \text{for} \quad x \in (1-s^*, 1]. \end{cases}$$

3. the between-firm share of the variance of wages is decreasing in s

Frictionless matching: assignment and payoffs

▸ Main

- Working backwards, pin down frictionless payoffs that determine the outside option
- The equilibrium of the frictionless model can be derived in many ways
- Equilibrium assignment and payoffs:
  - PAM: $\mu(x) = x$ *given supermodular* $f(x_1, x_2)$
  - wage schedule obtained from integrating over FOC

$$w^*(x) = \int_0^x f_x(\tilde{x}, \mu(\tilde{x}))d\tilde{x}$$

    where integration constant is zero due to $f(0, 0) = 0$
  - firm payoffs in this formulation are 0
- Given $f(x_1, x_2) = x_1 + x_2 - \gamma(x_1 - x_2)^2$, with $\gamma > 0$, we have

$$\mu(x) = x$$

$$w^*(x) = x \quad \text{and } v^* = 0$$

Characterization results: conditional distribution

▸ Main

Lemma: Conditional type distribution

Given a threshold distance s, the conditional distribution of coworkers for $x \in \mathcal{X}$ is

$$\Phi(x'|x) = \begin{cases} 0 & \text{for } x' < \sup\{0, x-s\} \\ \frac{x-\sup\{0,x-s\}}{\inf\{x+s,1\}-\sup\{0,x+s\}} & \text{for } x' \in [\sup\{0, x-s,\}, \inf\{x+s, 1\}] \\ 1 & \text{for } x' > \inf\{x+s, 1\} \end{cases}$$

Characterization results: between-share of wage variance

Main

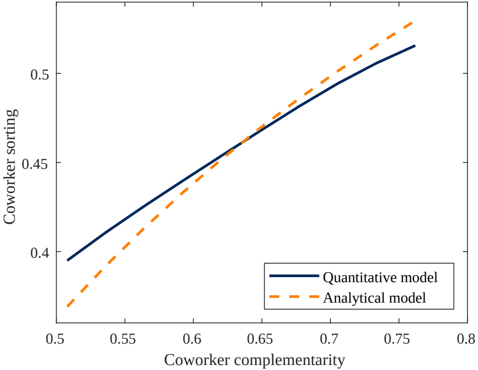
Corollary:

Given a threshold distance s and a value of γ, the between-firm share of the variance of wages is equal to

$$\frac{-\frac{13\,\gamma^2\,s^5}{2400}+\frac{\gamma^2\,s^4}{80}+\frac{5\,s^3}{36}-\frac{s^2}{6}+\frac{1}{12}}{\frac{\gamma^2\,s^4}{45}-\frac{4897\,\gamma^2\,s^5}{10800}-\frac{\gamma^2\,s^6}{324}+\frac{19\,\gamma^2\,s^5\,\ln(2)}{30}+\frac{1}{12}}.$$

Matching patterns: comparison of stylized model vs. quantitative

Back



(a) Comparison to quantitative model

Overview of validation exercises: direction EE transitions & cross-section

Main

- 2 additional types of validation exercise:
  - ✓**EE transitions** reallocate workers to more + assortative matches Details
  - do model-implied relationships also hold in **cross-section**?
    1. $\chi \uparrow \Rightarrow$ coworker complementarity $\uparrow$
    2. coworker complementarity $\uparrow \Rightarrow$ + assortative matching $\uparrow$

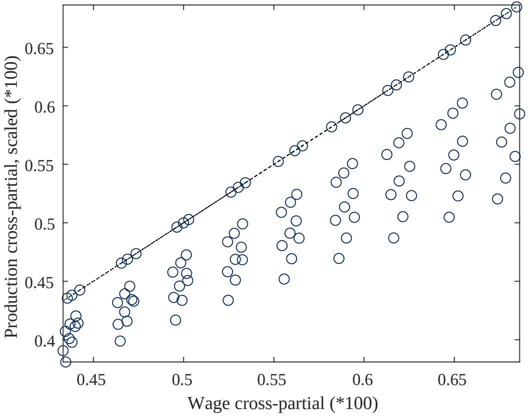
can test predictions *because* we have measures of complementarity!

- Implementation of cross-sectional exercises: rich Portuguese micro data
  - universe of private-sector actors, employer-employee data & income statements
- Cross-sectional exercises:
  - ✓ **Hierarchies** Details
  - ✓ **Industries** Details
  - ✓ **Occupations** Details

Wage and production cross-partials beyond the benchmark

▸ Main

- Solve model for many combinations of χ, λ_e and b
- Compare FD approx of $f_{xx'}(x, x')$, scaled by ω, and $w_{xx'}(x|x')$
- Main parameter driving wedge: λ_e



Parameterization, including estimation results

▸ Main

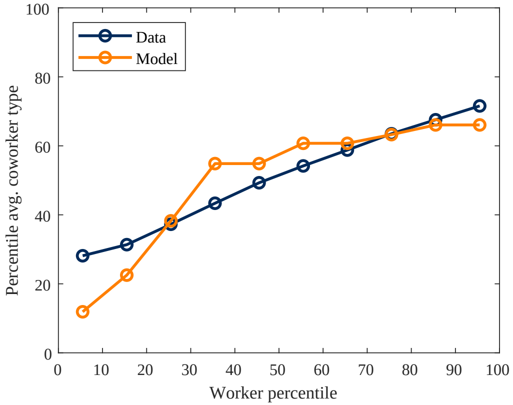
Parameter	Description	Targeted moment	Value	m	$\hat{m}$
γ	Elasticity of complementarity	$\hat{\beta}_c$	**0.837**	0.0091	0.0091
a_0	Production, constant	Avg. wage (norm.)	0.239	1	1
a_1	Production, scale	Var. log wage	1.557	0.241	0.241
b_1	Replacement rate, scale	Replacement rate	0.664	0.63	0.63
δ	Separation hazard	Job loss rate	0.008	0.008	0.008
λ_u	Meeting hazard	Job finding rate	0.230	0.162	0.162
ρ	Discount rate	External	0.008		
ω	Worker bargaining weight	External	0.50		
a_2	Production, team advantage	External	1.10		

Notes. This table lists for each of the estimated parameters, the targeted moment, the estimated value, and moment values in data ($\hat{m}$) and model (m). In the quantitative model, the production function includes a con-

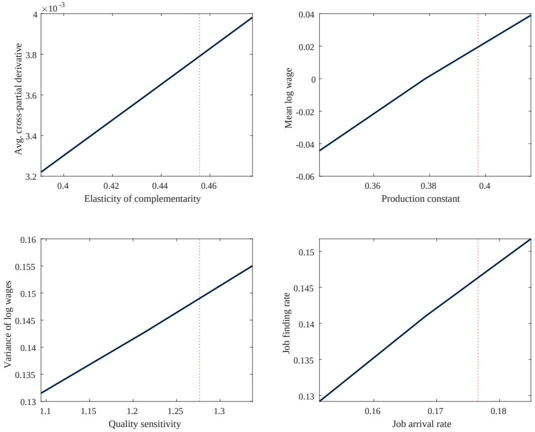
Validation: model reproduces empirical coworker sorting patterns

Main

- Key untargeted moment (1): **coworker sorting patterns**
- Coworker correlation matches data well, $\rho_{xx} = 0.53$ (vs. 0.62 in the data)
- Model slightly underestimates the quality of coworkers at both bottom and top
  - OJS will help

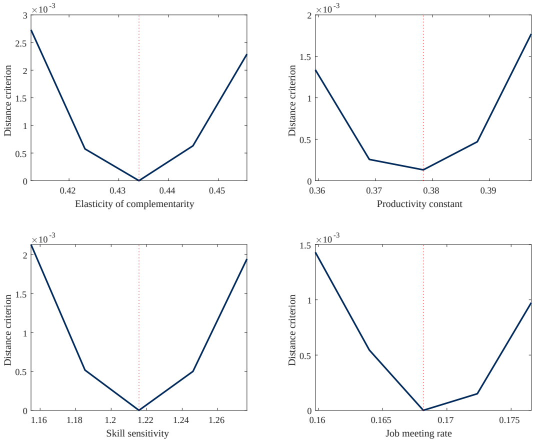


Identification validation exercise 1



Notes. This figure plots the targeted moment against the relevant parameter, holding constant all other parameters.

Identification validation exercise 2



Notes. This figure plots the distance function $\mathcal{G}(\psi_i, \psi^*_{-i})$ when varying a given parameter ψ_i around the estimated value ψ^*_i. The remaining parameters are allowed to adjust to minimize $\mathcal{G}$.

Between-share adjustment procedure (1)

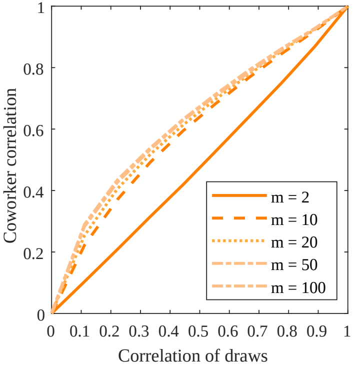
▸ Main

- **Problem.** For any degree of coworker sorting less than unity, i.e. $\rho_{xx} < 1$, the level of the between-share in a model with team size $m = 2$ will be biased upward relative to the case of $m > 2$ and, in particular, $m \to \infty$ (LLN...)
  - implication 1: upward bias in level
  - implication 2: downward bias in estimated $\uparrow$ between-share as sorting increases
- Propose **statistical adjustment method**
- Consider a random vector $X = (X_1, X_2, \ldots, X_m)'$ whose distribution is described by a Gaussian copula over the unit hypercube $[0, 1]^m$, with an $m \times m$ dimensional correlation matrix $\Sigma(\rho^c)$, which contains ones on the diagonal and the off-diagonal elements are all equal to ρ_c
- Interpretation. Each vector of observations drawn from the distributions of X, $x_j = (x_{1j}, x_{2j}, \ldots, x_{mj})'$, describes the types of workers in team of size m, indexed by j

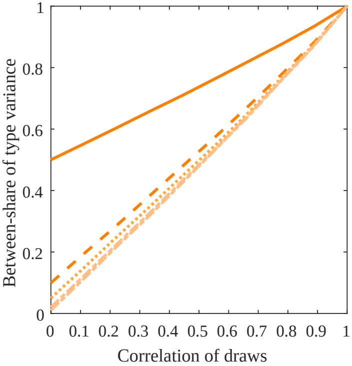
Between-share adjustment procedure (2)

- Since the marginals of the Gaussian copula are simply continuous uniforms defined over the unit interval, the variance of the union of all draws is just $\frac{1}{12}$
- The mean of the elements of X is itself a random variable, $\bar{X}$. That is, for some realization x_j, we can define $\bar{x}_j = \frac{1}{m}\sum_{i=1}^{m} x_{ij}$
- The variance of $\bar{X}$ will be $\frac{1}{m^2}\left(\frac{m}{12} + m(m-1)(\frac{\rho_c}{12})\right)$
- So $\sigma^2_{x,\text{between-share}}(\rho_c, m) = \frac{1}{m}\left(1 + (m-1)\rho_c\right)$
- Correction-factor $= \frac{1}{2}\left(1 + \rho_c\right) - \frac{1}{\hat{m}}\left(1 + (\hat{m}-1)\rho_c\right)$ where the empirical average size is $\hat{m}$

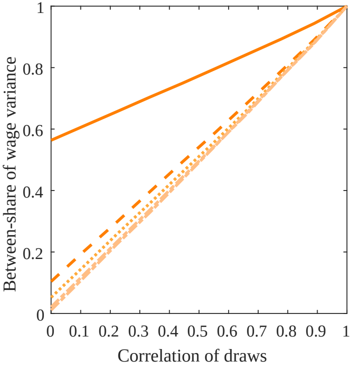
Between-share adjustment procedure (3)



(a) Coworker sorting



(b) Between-share of type var.



(c) Between-share of wage var.

The effect of declining search frictions

▸ Main

- **Model-based concern:** reduction in search frictions could also explain ↑ coworker sorting
- Yes: job arrival & separation rates estimated to ↑ from p1 to p2
- **Counterfactual analysis:** explains 6% of empirically observed ↑ in between-employer share of wage variance

	Δ model	Implied % Δ model due to Δ parameter
Model 1: baseline	0.159	-
Cf. a: fix period-1 complementarity	0.065	59
Cf. b: fix period-1 search frictions	0.150	6

Outsourcing & within-occupation ranking analysis

Main

- **Concern:** confounding shifts in labor boundary of firm, e.g. outsourcing
- **Address this concern in multiple steps:**
  1. empirically rank workers *within* occupation ("good engineer vs. mediocre engineer")
  2. empirically re-estimate coworker sorting & complementarity (lower but similar ↑)
  3. re-estimate model for both periods & re-do counterfactual exercises
- **Result:** qualitatively & quantitatively similar findings

	Δ model	Implied % Δ model due to Δ parameter
Model 2: within-occ. ranking	0.198	-
Cf. a: fix period-1 comp.	0.076	61.47

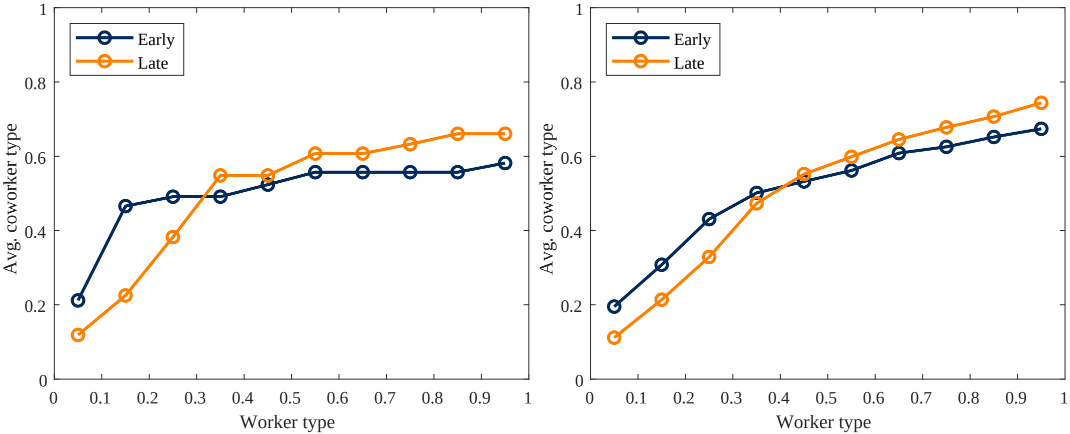
Extension: model with OJS

▸ Main

- Baseline model abstracted from OJS
  - transparent trade-off, connection to analytical results
- **Consider extension to OJS:** employed worker meet vacancies at Poisson rate λ_e
  - wages both off and on the job are continuously renegotiated under Nash bargaining, with unemployment serving as the outside option *[cf. di Addario et al., 2021]*
  - re-estimate, with empirical labor market flows disciplining λ_e
- Qualitative question: is coworker sorting outcome robust, even if workers can switch to better job after accepting job out of unemployment?
- **Analyses:**
  1. coworker sorting patterns & changes
  2. additional model validation: direction of EE flows in model & data

Model-implied coworker sorting patterns: without and with OJS

Main



(a) Baseline **(b)** OJS

EE transitions in theory and data

Main

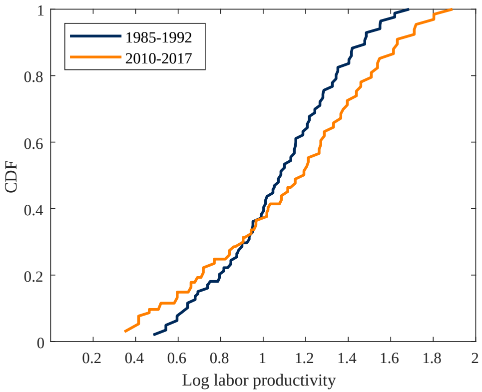
- **Theoretical prediction:** EE transitions move workers in surplus-maximizing direction $\Rightarrow \Delta\hat{x}_{-it} = \hat{x}_{-i,t} - \hat{x}_{-i,t-1}$ should be *positively* correlated with $\hat{x}_i$
  - $h_{2.1}(x, x''|x') = 1$ – worker x in a two-worker firm with coworker x'' would move to an employer that currently has one employee of type x' – if $S(x|x') - S(x|x'') > 0$
- **Empirical analysis**: use SIEED *spell* data to create worker-originMonth-destinationMonth-originJob-destinationJob panel, with information on characteristics of origin and destination job (e.g., coworker quality)
  - subsample period 2008-2013 (huge panel at monthly frequency)
  - count as "EE" if employer change between two adjacent months
- **Regression analysis:** regress $\Delta\hat{x}_{-it}$, scaled by std. σ_Δ of coworker quality changes, on *own* type and *origin* coworker type

$$\frac{\Delta\hat{x}_{it}}{\sigma_\Delta} = \beta_0 + \beta_1\hat{x}_i + \beta_2\hat{x}_{-i,t-1} + \epsilon_{it}$$

Productivity dispersion

Main

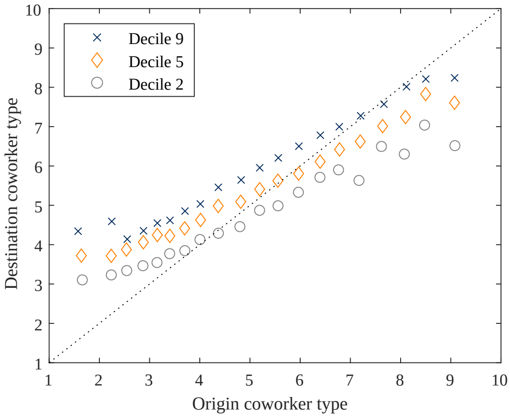
- Firm dynamics literature: increased firm-level productivity dispersion *[Autor et al., 2020; de Ridder, 2023]*



Empirical coworker sorting changes due to EE moves

Main

- **EE transitions push toward greater coworker sorting:** for any given origin, higher *x*-workers move to workplaces with better coworkers than lower-*x* workers do
- *But* in data EE transitions "move up" low types more than theory predicts
- "**Coworker job ladder**" with both absolute and type-specific dimension?
- **Next:** change in the job ladder *[e.g., Haltiwanger-Spetzler, 2021]*



Evidence that EE *increasingly* reallocate toward PAM: in data & model

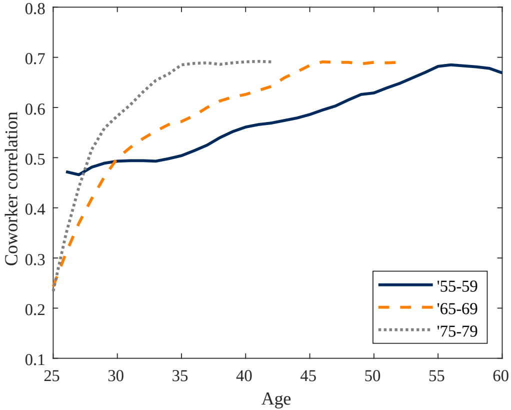
	Data		Model	
Change in coworker type	'85-'92	'10-'17	Period-1	Period-2
Own type	**0.0883*****	**0.118*****	**0.214**	**0.270**
	(0.000799)	(0.000918)		
Controls	Year FEs, Origin	Year FEs, Origin	Origin	Origin
N	196,098	282,718	∞	∞
adj. R^2	0.284	0.204		

Table 1: Change in coworker type due to EE moves positively related to own type – increasingly so

Notes. For the data columns, individual-level clustered standard errors are given in parentheses. Model counterparts are computed simulation-free in population. Dependent variable is scaled throughout by the standard deviation of the change in coworker type.

Related: life-cycle patterns of sorting consistent with cohort effects

- Coworker sorting increases over life-cycle
- Coworker sorting higher in recent cohorts
    - consistent with each cohort becoming more specialized
    - but also with improvement in search frictions over time, which primarily affects sorting of young people
- Currently don't have life-cycle effects in my model, but could be incorporated...



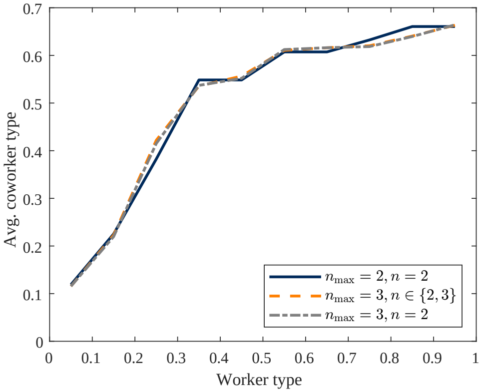
Notes. Figure displays the correlation of own type with average coworker type, by age, for three separate cohorts.

Extension to $n_{max=3}$

Main

- Baseline model imposes $n_{\max} = 2$ for reasons of (i) tractability and (ii) transparency

- Can extend to $n_{\max} = 3$ (or $n_{\max} = 4$) & find that results are robust

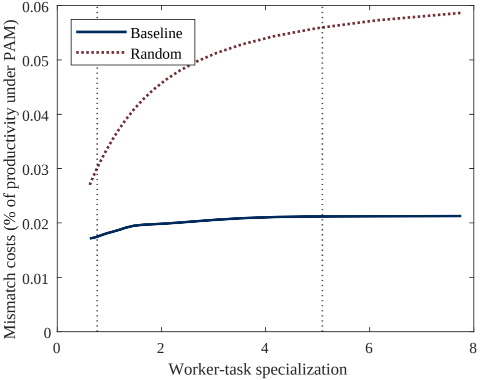


Productivity costs of complementarity & labor market functioning

▸ Main

"The benefits of the division of labor are limited by the functioning of the labor market"

- Microfoundation: $\uparrow \chi \Rightarrow \uparrow$ efficiency *benefit* from teamwork *but* also $\uparrow$ mismatch *costs*
- **Q:** how does the gap to potential vary depending on labor market structure?
- **A:** under random sorting, productivity gap due to misallocation $\uparrow$ more sharply as $\chi \uparrow$
- Outside model: severe labor mkt frictions (e.g., dev'ing countries [Donovan et al., 2023]) may inhibit specialization [cf. Atencio et al., 2023; Bassi et al., 2023]



Implications for overall inequality?

Main

- **Coworker complementarities do not necessarily ↑ variance of person-level wages**
  - (un-)surprising? Varriance decomposition perspective vs. common intuition/question
  - counterfactual: variance of log wages 0.2166 in 2010 under 1990-complementarities vs. 0.210
  - (i) **reallocation effect**, (ii) valuation effect, (iii) outside option effect

- Several mechanisms though through which ↑ sorting could ↑ person-level inequality
  1. regulation or norms that lead to within-firm wage compression *[Akerlof-Yellen, 1990]*
  2. coworker learning *[Jarosch et al., 2021; HLMP, 2023]*
  3. increasing returns to labor quality *[Kremer, 1993]*

Evolution of the German task structure

Main

- Employment Surveys (ES) carried out by the German Federal Institute for Vocational Training (BIBB)
  - detailed information on tasks performed at work
  - individual-level, with consistent occupation codes
  - repeated cross-sections ranging from 1985/86 to 2018
  - large sample sizes (20,000-30,000 per wave)
- Methodology to study evolution of task content of production follows Spitz-Oener (2006), Antonczyk et al. (2009), Rohrbach-Schmidt & Tiemann (2013)
  - task classification
  - sample harmonization (West Germany, aged 15 to 65, employed)

Task classification

- Focus on Δ in usage of abstract/complex (non-routine, non-manual) tasks vs. “rest” (manual & routine)

  [Autor and Handel, 2009; Acemoglu & Autor, 2011; Rohrbach-Schmidt & Tiemann, 2013]

- Index of complex tasks for worker i in period t *[Antonczyk et al., 2009]*

$$T_{it}^{\text{complex}} = \frac{\text{number of activities performed by } i \text{ in task category "complex" in sample year } t}{\text{total number of activities performed by } i \text{ in sample year } t}$$

Task classification	Task name	Description
Complex	investigating organizing researching programming teaching consulting promoting	gathering information, investigating, documenting organizing, making plans, working out operations, decision making researching, evaluating, developing, constructing working with computers, programming teaching, training, educating consulting, advising promoting, consulting, advising
Other	repairing, buying, accommodating, caring, cleaning, protecting, measuring, operating, manufacturing, storing, writing, calculating	

Increase in aggregate task complexity, driven by within-occupation $\uparrow$

- Aggregate task intensity & decompose period-by-period change into:
  1. between component: Δ occupational employment shares
  2. within component: Δ task content within occupations

	Total	Between	Within	Within-share
1986 level	0.252			
1986-1992	0.025	0.002	0.022	0.906
1992-2006	0.298	0.057	0.241	0.809
2006-2012	0.019	0.002	0.017	0.890
2012-2018	0.053	0.028	0.025	0.476
Total change	0.395	0.089	0.306	0.775

Notes. Decompose changes in the usage of abstract tasks between periods t and $t-1$ according to $\Delta \bar{T}_t^{\text{abstract}} = \sum_o \omega_{o,t-1}(\bar{T}_{t,o}^{\text{abstract}} - \bar{T}_{t-1,o}^{\text{abstract}}) + \sum_o(\omega_{o,t} - \omega_{o,t-1})\bar{T}_{t,o}^{\text{abstract}}$ where $\bar{T}_{t,o}^{\text{abstract}}$ measures the average usage of abstract tasks by members of occupation o in period t and $\omega_{o,t}$ is the period- t employment share of occupation.

Large variation in task complexity across occupations

- Aggregate individual responses to 2-d occupation level, using 2012 & 2018 waves
  - 2012 & 2018: $\leftarrow$ ISCO-08 codes available

Occupation	$\bar{T}_o^{\text{complex}}$
Business and administration professionals	0.859
Legal, social and cultural professionals	0.830
Business and administration associate professionals	0.827
Administrative and commercial managers	0.820
Teaching professionals	0.807
...	...
Drivers and mobile plant operators	0.214
Agricultural, forestry and fishery labourers	0.193
Market-oriented skilled forestry, fishery and hunting workers	0.168
Food preparation assistants	0.131
Cleaners and helpers	0.124

Notes. Top-5 and bottom-5 ISCO-08 2-digit occupations when ranked by $\bar{T}_o^{\text{abstract}}$ in pooled 2012 & 2018 waves.

Evidence from the literature: Hakanson et al. (2021)

▸ Main

- *Direct* measures of cognitive and non-cognitive skills across Swedish firms during 1986–2008, using test data from military enlistment