

# Superstar Teams

---

Lukas Freund

University of Cambridge

*May 24, 2024*

*Bristol Macroeconomics Workshop*

## Motivation: opening the “black box” of firms in macro

- **Most production processes involve groups of heterogeneous workers**
  - too complex for any individual to perform all tasks well
    - **firms coordinate** groups of **workers with specialized skills**
  - individuals also vary in their *overall* productivity (“**talent**”)

# Motivation: opening the “black box” of firms in macro

- **Most production processes involve groups of heterogeneous workers**
  - too complex for any individual to perform all tasks well
    - **firms coordinate** groups of **workers with specialized skills**
  - individuals also vary in their *overall* productivity (“**talent**”)
- **Q:** Who should work with whom? Market allocation? Implications?
  - classic themes in literature [*Becker-Murphy, 1992; Kremer, 1993; Garicano, 2000*]
  - but theoretically & micro oriented

## Motivation: opening the “black box” of firms in macro

- **Most production processes involve groups of heterogeneous workers**
  - too complex for any individual to perform all tasks well  
→ **firms coordinate** groups of **workers with specialized skills**
  - individuals also vary in their *overall* productivity (“**talent**”)
- **Q:** Who should work with whom? Market allocation? Implications?
  - classic themes in literature [*Becker-Murphy, 1992; Kremer, 1993; Garicano, 2000*]
  - but theoretically & micro oriented
- **This paper:** tractable model for empirical & quantitative analysis that highlights *macro* implications: **agg. productivity**, trends in labor market inequality
  - (1) theory
  - (2) measurement
  - (3) application(s)

# Modeling firms as team assemblies & core mechanism

- **What's a firm:** an organized collection of workers with task-specific skills (team)
- **Key idea:** specialization  $\Rightarrow$  coworker talent complementarities  $\Rightarrow$  sorting
- **Implications:**
  - $\uparrow$  specialization  $\Rightarrow$  talent sorting  $\uparrow$
  - input/labor market frictions lower output b/c teams are formed inefficiently

# The paper in a nutshell


- **Develop tractable theory of the firm centered on team-production & formation**
  - 1 **task-based** production
  - 2 **skill heterogeneity**: task-specificity & variable talent
  - 3 **team production**
  - 4 assembling a team takes time due to **search** frictions

# The paper in a nutshell

- **Develop tractable theory of the firm centered on team-production & formation**

- ① **task-based** production
  - ② **skill heterogeneity**: task-specificity & variable talent
  - ③ **team production**
  - ④ assembling a team takes time due to **search** frictions
- } **microfound tractable firm-level production**  
**fn. with *endogenous* complementarities**

# The paper in a nutshell

- **Develop tractable theory of the firm centered on team-production & formation**
  - 1 **task-based** production
  - 2 **skill heterogeneity**: task-specificity & variable talent
  - 3 **team production**
  - 4 assembling a team takes time due to **search** frictions

**microfound tractable firm-level production  
fn. with *endogenous* complementarities**
- **Confront theory with data**: structurally identify production fn. with micro panel data on wages+matches → estimate & test model



# The paper in a nutshell

- **Develop tractable theory of the firm centered on team-production & formation**
  - ① **task-based** production
  - ② **skill heterogeneity**: task-specificity & variable talent
  - ③ **team production**
  - ④ assembling a team takes time due to **search** frictions

} microfound tractable firm-level production  
fn. with *endogenous complementarities*
- **Confront theory with data**: structurally identify production fn. with micro panel data on wages+matches → estimate & test model
- **Main quantitative application**: deepening specialization explains  $\approx 25\%$  of  $\uparrow$  between-firm wage inequality share in DE since 1985 [cf. Card et al., 2013; Bloom et al., 2019]

# The paper in a nutshell

- **Develop tractable theory of the firm centered on team-production & formation**
  - ① **task-based** production
  - ② **skill heterogeneity**: task-specificity & variable talent
  - ③ **team production**
  - ④ assembling a team takes time due to **search** frictions

microfound tractable firm-level production  
fn. with *endogenous complementarities*
- **Confront theory with data**: structurally identify production fn. with micro panel data on wages+matches → estimate & test model
- **Main quantitative application**: deepening specialization explains  $\approx 25\%$  of  $\uparrow$  between-firm wage inequality share in DE since 1985 [*cf. Card et al., 2013; Bloom et al., 2019*]
  - theory has implications for agg. productivity, productivity dispersion, job ladders, ...

## Relation & contributions to literature

- **Firm organization:** **parsimonious team-model for quantitative applications**

**Firms:** Lucas, 1978; Rosen, 1982; Becker & Murphy, 1992; Hopenhayn & Rogerson, 1993; **Kremer, 1993**; Kremer & Maskin, 1996; **Garicano, 2000**; Klette & Kortum, 2004; **Garicano & Rossi-Hansberg, 2006**; Kohlhepp, 2022; Kuhn et al., 2022; Minni, 2022; Bassi et al., 2023

**Task assignment:** Costinot & Vogel, 2010; **Acemoglu & Restrepo, 2018**; Ocampo, 2021; Adenbaum, 2022

**Teams:** Akcigit et al., 2018; Chade & Eeckhout, 2020; **Jarosch et al., 2021**; Herkenhoff et al., 2022

- **Frictional labor market sorting:** **endogenize & measure complementarities**

Shimer & Smith, 2000; Cahuc et al., 2006; Eeckhout & Kircher, 2011/2018; Hagedorn et al., 2017; de Melo, 2018; **Herkenhoff et al., 2022**; Lindenlaub & Postel-Vinay, 2023

- **Wage inequality:** **structural model of  $\uparrow$  firm-level inequality due to technological  $\Delta$**

**Technology:** Katz & Murphy, 1992; Krusell et al., 2000; Autor, Levy & Murnane, 2003; **Jones, 2009**; Deming, 2017; Acemoglu & Restrepo, 2018; Alon, 2018; Neffke, 2019; Jones, 2021; Atalay et al., 2021

**Firms:** **Card et al., 2013**; Barth et al., 2016; Alvarez et al., 2018; **Bloom et al., 2019**; Aeppli & Wilmers, 2021; Criscuolo et al. 2021; Hakanson et al., 2021; Sorkin & Wallskog, 2021; Kleinman, 2022

# Theory

---

# Overview of environment

- **Ex-ante homogeneous firms** assemble teams of **heterogeneous workers** who vary in task specialization (*what* you're skilled at) & talent (*how* skilled you are)
  - task-based production
  - hire workers in search-frictional labor market
- **Challenge:** how to keep this tractable?
- **Analyze economy-wide organization of production in 2 steps:**
  - 1 team production: how much does a team produce if tasks are optimally allocated?
  - 2 team formation: who is matched to work with whom?

# Step 1: production in a single team of given composition

- Firm: **1 team of**  $n \in \mathbb{Z}_{++}$  **workers** produces output from **unit continuum of tasks**  $\mathcal{T}$

$$\ln Y = \int_{\mathcal{T}} \ln q(\tau) d\tau \quad (1)$$

- **Task aggregation:**

$$q(\tau) = \sum_{i=1}^n y_i(\tau) \quad (2)$$

- **Workers**  $\rightarrow$  **tasks:**  $i$  supplies 1 time unit, task-specific productivities  $\mathbf{z}_i = \{\mathbf{z}_i(\tau)\}_{\tau \in \mathcal{T}}$

$$y_i(\tau) = \mathbf{z}_i(\tau) l_i(\tau) \quad (3)$$

$$1 = \int_{\mathcal{T}} l_i(\tau) d\tau \quad (4)$$

# Firm's optimization problem

- **Firm solves mini-planner problem:**  $\max_{\mathbf{q}, \{\mathbf{l}_i\}} Y$  s.t. (1)-(4)
- **Preview:** derive & characterize *reduced-form* team production function  $f$

$$f(\mathbf{z}_1, \dots, \mathbf{z}_n) = \max Y$$

s.t. (1)-(4)

# Firm's optimization problem

- **Firm solves mini-planner problem:**  $\max Y$  s.t. (1)-(4)

$$\begin{aligned}
 \mathcal{L}(\cdot) = & Y + \lambda \left[ \underbrace{\left( \int_{\mathcal{T}} \ln q(\tau) d\tau \right)}_{\text{tasks} \rightarrow \text{output}} - \ln Y \right] + \int_{\mathcal{T}} \lambda(\tau) \underbrace{\left( \sum_{i=1}^n y_i(\tau) - q(\tau) \right)}_{\text{task aggregation}} d\tau \\
 & + \sum_{i=1}^n \lambda_i^L \underbrace{\left( \int_{\mathcal{T}} \frac{y_i(\tau)}{z_i(\tau)} d\tau - 1 \right)}_{\text{time constraint + task production}} + \text{non-negativity constraints}
 \end{aligned}$$



# Firm's optimization problem

- Firm solves mini-planner problem:**  $\max Y$  s.t. (1)-(4)

$$\begin{aligned} \mathcal{L}(\cdot) = & Y + \lambda \left[ \underbrace{\left( \int_{\mathcal{T}} \ln q(\tau) d\tau \right)}_{\text{tasks} \rightarrow \text{output}} - \ln Y \right] + \int_{\mathcal{T}} \lambda(\tau) \underbrace{\left( \sum_{i=1}^n y_i(\tau) - q(\tau) \right)}_{\text{task aggregation}} d\tau \\ & + \sum_{i=1}^n \lambda_i^L \underbrace{\left( \int_{\mathcal{T}} \frac{y_i(\tau)}{z_i(\tau)} d\tau - 1 \right)}_{\text{time constraint + task production}} + \text{non-negativity constraints} \end{aligned}$$

- FOCs** imply

$$\lambda(\tau) = \min_i \left\{ \frac{\lambda_i^L}{z_i(\tau)} \right\}$$

shadow cost of  $\tau$   $\leftarrow$   $\lambda(\tau)$

$\lambda_i^L$   $\rightarrow$  opportunity cost of  $i$ 's time

$z_i(\tau)$   $\rightarrow$  productivity of  $i$  for  $\tau$

# Tractability: leverage insight from trade literature

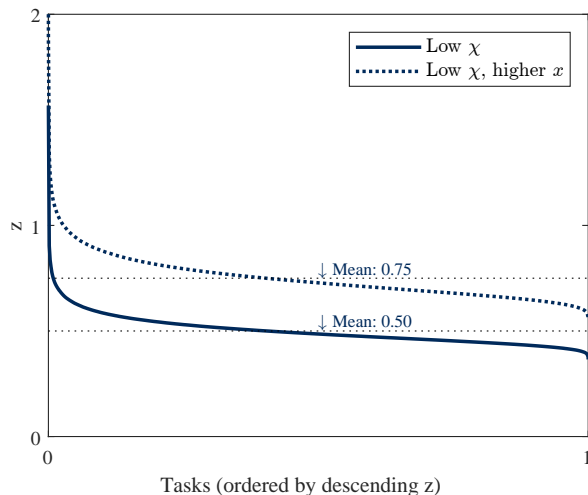
## Assumption: Multivariate Fréchet distribution of worker-task skills

$$\Pr [Z_1(\tau) \leq z_1, \dots, Z_n(\tau) \leq z_n] = \exp \left[ - \left( \sum_{i=1}^n \left( \left( \frac{z_i}{\iota X_i} \right)^{-\frac{1}{\chi}} \right)^{\frac{1}{\xi}} \right)^{\xi} \right]$$

- **Payoff #1:** low-dim. representation of worker-task skill dist.
  - $x_i$ : scale term  $\sim$  “**talent**” type
  - $\chi$ : *within*-worker skill dispersion  $\sim$  **specialization**
  - $\xi \in (0, 1]$ : correlation in  $z$ 's *across* workers  $\sim$  are team members specialized in the same or different tasks

# Illustration: talent types

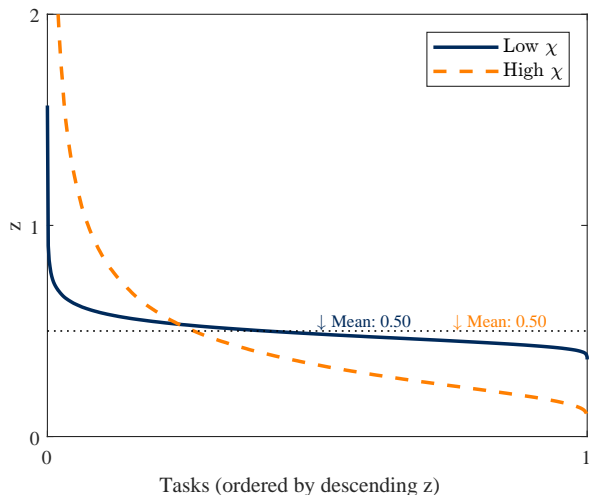
- **“Talent” type  $x_i$ :** how skilled is  $i$  conditional on whatever task they’re specialized in (absolute advantage)



# Illustration: specialization parameter

► Illustration: correlation

- Parameter  $\chi$ : *within-worker skill dispersion across tasks*  $\sim$  **specialization**



# Tractability: leverage insight from trade literature

## Assumption: Multivariate Fréchet distribution of worker-task skills

$$\Pr [Z_1(\tau) \leq z_1, \dots, Z_n(\tau) \leq z_n] = \exp \left[ - \left( \sum_{i=1}^n \left( \left( \frac{z_i}{\iota X_i} \right)^{-\frac{1}{\chi}} \right)^{\frac{1}{\xi}} \right)^{\xi} \right]$$

- Payoff #1: low-dim. representation of worker-task skill dist.
- **Payoff #2:** max-stability property allows closed-form characterization of key objects

# Micro-founded production function

[▶ Lemma](#)

## Proposition: Reduced-form production function

Talent types  $(x_1, \dots, x_n)$ ,  $\chi$  and  $\xi$  are sufficient statistic for team output  $Y$ :

$$Y = f(x_1, \dots, x_n; \chi, \xi)$$

- Proof sketch

[▶ Details](#)

- solving firm's mini-planner problem implies task-demand and assignment of tasks by comparative advantage with complete division of labor
- Fréchet distribution allows closed-form characterization of distribution of shadow costs  $\lambda(\tau)$  conditional on this assignment
- integrate over tasks and workers, normalizing  $\lambda = 1$

# Micro-founded production function: characterization

## Proposition: Reduced-form production function

$$f(x_1, \dots, x_n; \chi, \xi) = n^{1+\chi\xi} \times \left( \frac{1}{n} \sum_{i=1}^n (x_i)^{\frac{1}{1+\chi\xi}} \right)^{1+\chi\xi}$$



vs. no-division-of-labor:  $f(x_1, \dots, x_n) = n \times (\frac{1}{n} \sum_i^n x_i)$

- 1 **Output depends on team composition** –  $f$  is not additively separable

# The whole is greater than the sum of its parts...

## Proposition: Reduced-form production function

$$f(x_1, \dots, x_n; \chi, \xi) = \underbrace{n^{1+\chi\xi}}_{\text{efficiency gains}} \times \left( \frac{1}{n} \sum_{i=1}^n (x_i)^{\frac{1}{1+\chi\xi}} \right)^{1+\chi\xi}$$

### 2 Efficiency gains from teamwork – large when workers have skills that are *specialized* ( $\chi$ ) in *different* ( $\xi$ ) tasks

- no-division-of-labor:  $f(\cdot) = n \times (\frac{1}{n} \sum_i^n x_i)$
- intuition: gains from tasks being assigned by comparative advantage

► task assignment



## ...but output is lowered by talent dispersion

### Proposition: Reduced-form production function

$$f(x_1, \dots, x_n; \chi, \xi) = \underbrace{n^{1+\chi\xi}}_{\text{efficiency gains}} \times \underbrace{\left( \frac{1}{n} \sum_{i=1}^n (x_i)^{\frac{1}{1+\chi\xi}} \right)^{1+\chi\xi}}_{\text{talent complementarity}},$$

### 3 Coworker talent complementarities – strong when workers have skills that are *specialized* ( $\chi$ ) in *different* ( $\xi$ ) tasks

- reduced-form elasticity of complementarity  $\gamma := \frac{\partial \ln(f_j/f_i)}{\partial \ln(x_i/x_j)} = \frac{\chi\xi}{1+\chi\xi}$
- greater dispersion in team members' talent reduces output, other things equal
- $\partial f(\cdot) / \partial x_i \partial x_{-i} > 0$

## ...but output is lowered by talent dispersion

### Proposition: Reduced-form production function

$$f(x_1, \dots, x_n; \chi, \xi) = \underbrace{n^{1+\chi\xi}}_{\text{efficiency gains}} \times \underbrace{\left( \frac{1}{n} \sum_{i=1}^n (x_i)^{\frac{1}{1+\chi\xi}} \right)^{1+\chi\xi}}_{\text{talent complementarity}},$$

### 3 Coworker talent complementarities are strong when workers have skills that are *specialized* ( $\chi$ ) in *different* ( $\xi$ ) tasks

- intuition: greater specialization means higher- $\chi$  team members can't easily perform more tasks as their skill rapidly diminishes

► task assignment

# Key takeaways

① ↑ **Skill specialization** *endogenously* generates ↑ **coworker talent complementarities**

# Firm organization meets frictional matching

- So team composition matters for productivity, especially when  $\chi$  is high
- **How is team composition determined?**
- Exploit tractability of team production fn.  $f$  to **integrate team production with dynamic eqm. matching**
  - team composition determined by tradeoff between complementarities and search costs
  - preview: dynamic setting will also be key for identification of  $f$  with panel data

# Frictional matching into teams: environment

- **Random-search + multi-worker firms** *[Herkenhoff-Lise-Menzio-Phillips, 2024]*
  - teams instead of one-worker firms *[e.g., Shimer-Smith, 2000; Hagedorn-Law-Manovskii, 2017]*
- **Production technology:**  $f(\cdot)$  with  $n \in \{0, 1, 2\}$
- **Employment states:** unemp., employed alone, employed with  $x' \in \mathcal{X}$
- **Nash wage bargaining** with continuous renegotiation
- $\xi$ : **match-specific shock** observed by firms & workers before match decision
  - microfoundation: coworkers' task specialization matters for  $f$ , not only  $x$
  - $\xi$  is endogenously determined

# Matching – stationary equilibrium

[▶ Details](#)

- HJ-Bellman equations → **values & matching policies** [▶ HJBs](#)
- Flows between/**distribution** over types  $\times$  employment states [▶ KFEs](#)

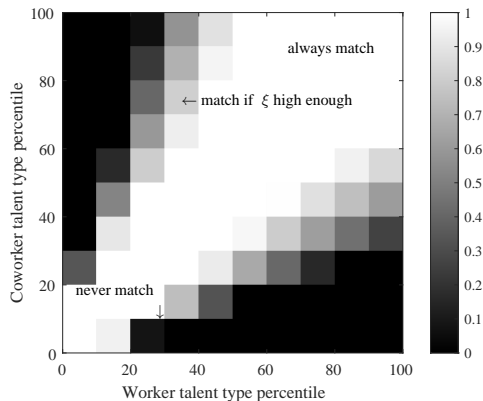
## Definition: Stationary equilibrium

A stationary eqm. consists of a value functions & a distribution of agents, s.t.

- 1 the value functions satisfy the HJB equations given the distribution;
- 2 the distribution is stationary given the policy fn's implied by the value fn's.

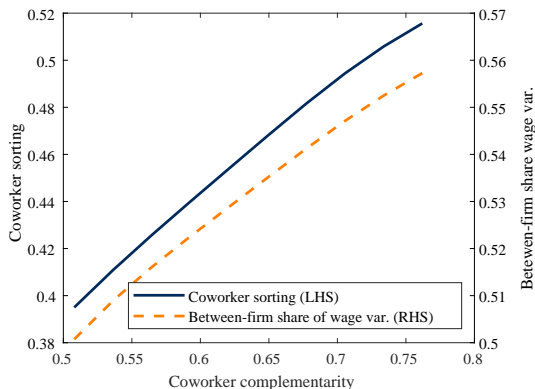
## Mechanism: conditional matching probabilities for given $\chi$

- What is the probability that a firm with a type- $x$  worker would hire a type- $x'$  worker conditional on meeting?



## Mechanism: more positive assortative matching as $\chi \uparrow$

- What is the correlation between talent  $x$  & coworker talent  $x'$ ,  $\rho_{xx}$ ?





# Key takeaways

- 1 ↑ Skill specialization endogenously generates ↑ coworker talent complementarities
- 2 ↑ **Talent complementarities** lead to ↑ **positive assortative matching**

## Model Meets Data

---

# Taking the model to (micro) data

- **Goal:** test core mechanisms & analyze quantitative properties of the model
- **Steps:**
  - ➊ matched employer-employee panel data
  - ➋ map model objects to data
  - ➌ estimate & quantitatively analyze model
  - ➍ validate core model mechanisms (*brief today*)

# Panel micro data for Germany

- **Primary data: SIEED matched-employer employee panel for W Germany**
  - 1.5% sample of establishments + entire biographies of associated workers; social security information on employer, daily wage, occupation, demographics
  - initially focus on 2010-2017, later extend to 1985-2017
  - selection: f-t employees aged 20-60,  $\geq 10$  observations per establishment-year
- **What do we need to know to quantitatively analyze the model?**
  - some parameters – e.g. labor market separation and meeting rates – can be  $\sim$  directly read off micro data or are informed by literature (e.g., discount rate)
  - $x$  and  $x'$  can be recovered using applied micro methods
  - key challenge:  $\chi$

## Mapping theory to data: worker & coworker types

- **Theory:** wage monotonically  $\uparrow$  in  $x$ , so can measure using panel dimension
  - Implementation: fixed effect (FE) wage regressions
    - AKM model [Abowd et al., 1999]: job switching identifies time-invariant “worker types,” controlling for possibility that some employers pay more to all workers
      - pre-est. k-means clustering to address limited mobility bias [Bonhomme et al., 2019]
    - AKM misspecified according to structural model, so also use a non-param. ranking algo [Hagedorn et al., 2017] that’s consistent  $\rightarrow$  similar worker types
- $\Rightarrow$  **Worker  $i$ ’s talent type  $\hat{x}_i$ : decile rank of  $i$ ’s FE within 2d-occupation**

## Mapping theory to data: worker & coworker types

- **Theory:** wage monotonically  $\uparrow$  in  $x$ , so can measure using panel dimension
- Implementation: fixed effect (FE) wage regressions
  - AKM model [Abowd et al., 1999]: job switching identifies time-invariant “worker types,” controlling for possibility that some employers pay more to all workers
    - pre-est. k-means clustering to address limited mobility bias [Bonhomme et al., 2019]
  - AKM misspecified according to structural model, so also use a non-param. ranking algo [Hagedorn et al., 2017] that’s consistent  $\rightarrow$  similar worker types

$\Rightarrow$  **Worker  $i$ ’s talent type  $\hat{x}_i$ : decile rank of  $i$ ’s FE within 2d-occupation**

- **“Representative coworker type”  $\hat{x}_{-it}$ :** avg.  $\hat{x}$  of workers in same estab.-yr.
- Then can compute, e.g.,  $\hat{\rho}_{xx} = \text{corr}(\hat{x}_i, \hat{x}_{-it})$

► Discussion

## Measurement: a useful identification result

- Within-worker skill dispersion  $\chi$  isn't easily observable
- **Theory:** Proposition 1 ties  $\chi$  to  $\frac{\partial^2 f(\cdot)}{\partial x \partial x'}$ , which given  $x$  &  $x'$  we can identify from  $w(x|x')$

### Proposition: Measuring complementarities

Coworker complementarities (CC) in production are proportional to CC in wages:

$$\frac{\partial^2 f(x, x', \xi)}{\partial x \partial x'} \propto \frac{\partial^2 w(x|x', \xi)}{\partial x \partial x'}.$$

$$\Rightarrow \frac{\partial^2 \bar{f}(x, x')}{\partial x \partial x'} \approx \boxed{\frac{\partial^2 \bar{w}(x|x')}{\partial x \partial x'}} \quad \leftarrow \text{can measure this}$$

► Proof sketch

## Key takeaways

- ①  $\uparrow$  Skill specialization endogenously generates  $\uparrow$  coworker talent complementarities
- ②  $\uparrow$  Talent complementarities lead to  $\uparrow$  + assortative coworker matching
- ③ **Complementarities can be identified from panel micro data on wages and matches**



# Mapping theory to data: coworker complementarity

- Want to approximate  $\frac{\partial^2 \bar{w}(x|x')}{\partial x \partial x'}$
- **Estimating polynomial equation:**

$$\frac{w_{it}}{\bar{w}_t} = \beta_0 + \beta_1 \hat{x}_i + \beta_{11} \hat{x}_i^2 + \beta_2 \hat{x}_{-it} + \beta_{22} \hat{x}_{-it}^2 + \beta_c (\hat{x}_i \times \hat{x}_{-it}) \\ + \psi_{j(it)} + \nu_{o(i)t} + \xi_{s(i)t} + \epsilon_{it}$$

- alternative: non-parametric approximation

# Evidence on coworker complementarity (2010-2017)

► Robustness

► B-o-E calc.  $\gamma$ 

► Peer effects

- Estimating equation:

coworker complementarity

$$\frac{w_{it}}{\bar{w}_t} = [\dots] + \overset{\text{coworker complementarity}}{\hat{\beta}_c} (\hat{x}_i \times \hat{x}_{-it}) + [\dots]$$

	$\hat{\beta}_c$	Non-parametric FD method
Coworker complementarity	<b>0.0058***</b>	0.0075
Obs. (1000s)	4,410	4,410

Notes. Regressions include FEs for employer; occupation-year; industry-year. Employer-clustered standard errors in parentheses. Observations weighted by the inverse employment share of the respective type and (rounded) coworker type cell. FD: finite differences.

# Robustness checks: measuring coworker complementarity

► Main

- Types from non-parametric ranking algorithm instead of AKM-based
- Schooling as a non-wage measure of types
- Lagged types
- Small teams
- Movers
- Non-parametric, finite-differences approximation
- Excluding managers
- Log specification

► Jump

► Jump

► Jump

► Jump

► Jump

► Jump

► Jump

► Jump

# Quantifying the model: approach

► Parameter values

► Identification validation

► Within-industry

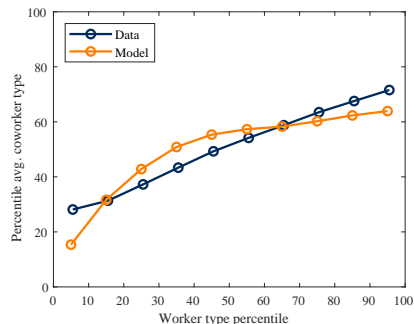
- **Calibrate** the model to the W German economy (2010-2017)
  - 1 externally calibrated: discount rate,  $\bar{n}$ , bargaining power
  - 2 offline estimation: job separation hazard
  - 3 **online estimation** (indirect inference): meeting rate, unemp. flow benefit, production
    - targets:  $\hat{\beta}_c$ , total wage variance, avg. wage level, replacement rate, job finding rate
- **Production function disciplined by  $\hat{\beta}_c$** 
  - set  $\chi$  to align regression estimates for this reduced-form moment using model-generated data with empirical estimate
- **Macro moments of interest are untargeted:** sorting, between-firm wage inequality

► Monte-Carlo

# Calibration results & model validation

► Between-share adjustment method

- ✓ **Match coworker sorting patterns**
  - $\rho_{xx} = 0.43$  (vs. 0.62 in data)
- ✓ **Match between-firm wage inequality**
  - between-share 0.48 (vs. 0.57 in data)
- Extensive **validation** of core mechanisms



# Validation of core model mechanisms

[► Details](#)

- Non-wage implications of complementarities: **direction of EE moves** [► Details](#)
  - $\Delta$  coworker talent positively correlated with own talent
- Cross-sectional analysis: **variation across occupations/industries** [► Details](#)
  - task-based proxy for  $\chi \uparrow \Rightarrow$  coworker talent complementarity  $\uparrow$
  - coworker talent complementarity  $\uparrow \Rightarrow$  coworker talent sorting  $\uparrow$
- Work-in-progress analyses
  - $\xi \uparrow \Rightarrow$  coworker talent complementarity  $\uparrow$
  - $\chi \uparrow \Rightarrow$  wage dispersion conditional on  $(x, x') \uparrow$
  - wage effects induced by separation from a worker  $\leftrightarrow$  Jaeger-Heining (2022)

## **Firming Up Inequality**

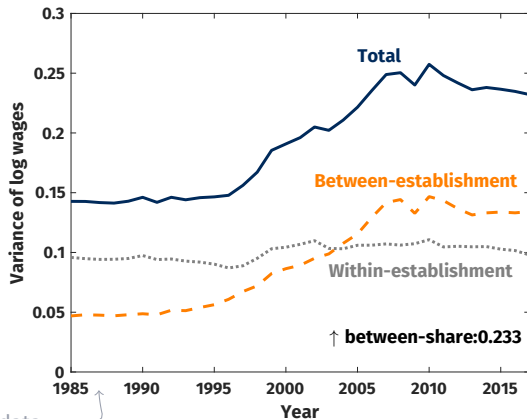
---

# Wage inequality has risen – and firms appear to play a key role

[Details](#)

*“the variance of firm [wages] explains an increasing share of total inequality in a range of countries”*

*[Song-Price-Guvenen-Bloom-von Wachter, 2019]*



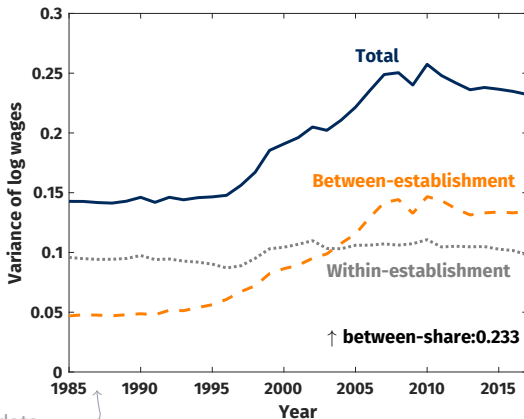
German matched employer-employee data



# Wage inequality has risen – and firms appear to play a key role

[Details](#)

**Applied question:** what is/are the causal driver(s)? implications?



German matched employer-employee data

# Hypothesis

- **Hypothesis:**

- ① the set of tasks any one worker can perform very well has narrowed: specialization ↑
- ② coworker talent complementarities ↑
- ③ individuals of similar talent increasingly work together
- ④ this generates greater between-firm wage dispersion

# Hypothesis

- **Hypothesis:**

- ① the set of tasks any one worker can perform very well has narrowed: specialization  $\uparrow$
- ② coworker talent complementarities  $\uparrow$
- ③ individuals of similar talent increasingly work together
- ④ this generates greater between-firm wage dispersion

- **Approach:**

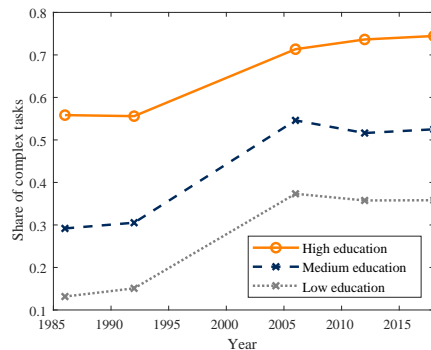
- evaluate descriptive evidence
- use structural model for counterfactual exercises quantifying this mechanism

# Qualitative evidence suggests specialization ( $\chi$ ) $\uparrow$

► Occ. movements

► Science

- **Task complexity  $\uparrow$ :**  
“extensive margin” of  $\chi$ 
  - DE longitudinal task survey [► BIBB](#)
  - “complex”: cognitive non-routine (e.g., organizing, researching)
- Literature points in similar direction
  - Grigsby ('23): transferability  $\downarrow$  among occ w high/social/manual skills
  - Jones ('09): burden of knowledge
  - Jovanovic-Rousseau ('08): proliferation of occupation codes



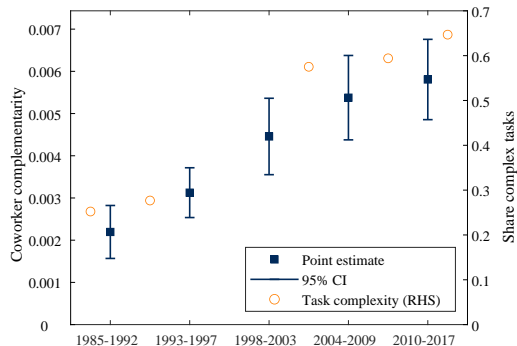
# Coworker talent complementarity has strengthened

► Schooling

► Peer effect trends

- **Theory:** specialization ( $\chi$ )  $\uparrow$  is associated with coworker complementarity  $\uparrow$

✓ **Coworker complementarity has more than doubled between 1985-1992 and 2010-2017**

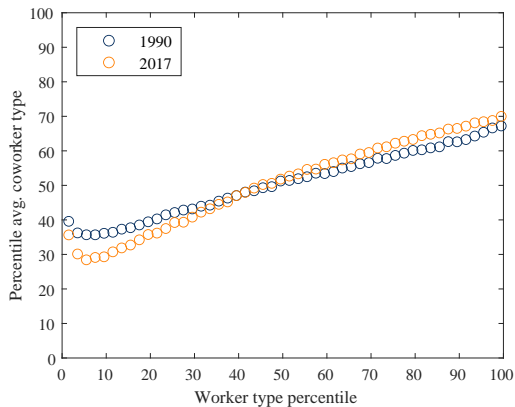


# Talent sorting has intensified

[Details](#)

- **Theory:** complementarity  $\uparrow$  is associated with talent sorting  $\uparrow$

✓ Coworker matching has become more positively assortative

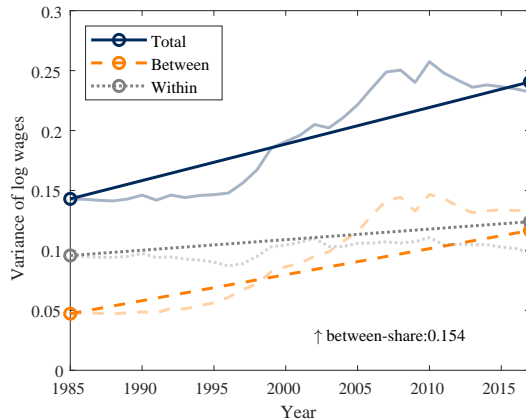


## Key takeaways

- 1 ↑ Skill specialization endogenously generates ↑ coworker talent complementarity
- 2 ↑ Coworker complementarity leads to ↑ positive assortative coworker sorting
- 3 Complementarities can be disciplined using panel micro data on wages and matches
- 4 **Coworker complementarity has doubled** since 1985 & **talent sorting has intensified**

# Model matches *changes* in firm-level wage distribution

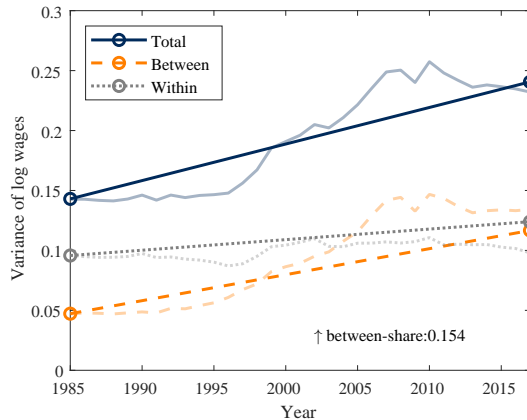
- How quantitatively important are these developments in accounting for the firming up of inequality?
- **Re-calibrate model: '85-'92**
- ✓ **Model replicates untargeted rise of between-share in data**
  - 2/3 of  $\uparrow$  between-share in data, ('85-'92)  $\rightarrow$  ('10-'17)





# Model matches changes in firm-level wage distribution – why?

- ✓ Model replicates untargeted rise of between-share in data
- **Reflects several parameters changing**
  - $\chi$ : XX in '85-'92 (vs. XX in '10-'17)
  - job arrival & separation  $\uparrow$
  - ...



## Complementarity $\uparrow$ explains $\approx 25\text{-}40\%$ of observed between-share $\uparrow$

- **Q:** How much of  $\uparrow$  between-firm share of wage var. is due to  $\uparrow$  complementarities?

## Complementarity $\uparrow$ explains $\approx 25\text{-}40\%$ of observed between-share $\uparrow$

- **Q:** How much of  $\uparrow$  between-firm share of wage var. is due to  $\uparrow$  complementarities?
- **Counterfactual:** between-firm share in 2010s absent  $\chi \uparrow$  since '85-'92

## Complementarity $\uparrow$ explains $\approx 25\text{-}40\%$ of observed between-share $\uparrow$

- **Q:** How much of  $\uparrow$  between-firm share of wage var. is due to  $\uparrow$  complementarities?
- **Counterfactual:** between-firm share in 2010s absent  $\chi \uparrow$  since '85-'92
- **A:**  $\chi \uparrow$  **accounts for 58%** of model-predicted  $\Delta \leftrightarrow \approx 38\%$  of empirical  $\Delta$
- **Robustness** exercises: 25-40%

	$\Delta$ model	Implied % $\Delta$ model due to $\Delta$ parameter
Model	0.154	-
Cf.: fix period-1 complementarity	0.065	58

# Overview of model robustness checks

- Declining search frictions
- Within-industry calibration
- Economy-wide vs. within-occupation analysis
- OJS
- Increased talent dispersion

► Jump

► Jump

► Jump

► Jump

► Jump

## Key takeaways

- 1 ↑ Skill specialization endogenously generates ↑ coworker talent complementarity
- 2 ↑ Coworker complementarity leads to ↑ positive assortative coworker sorting
- 3 Complementarities can be disciplined using panel micro data on wages and matches
- 4 Coworker complementarity has doubled since 1985 & talent sorting has intensified
- 5 **This explains a substantial share of ↑ between-firm wage inequality share**

## Other applications

---

# Overview of extensions & other implications

- **Agg. productivity:** complementarities + search frictions  $\Rightarrow$  mismatch costs

► Jump

- **Productivity dispersion:** talent sorting  $\Rightarrow$  firm-level productivity gaps

► Jump

- **“Coworker job ladders”:** workers, especially talented ones, tend to switch toward jobs with better coworkers

► Jump

- **Person-level inequality:** segregation by itself *need not* cause greater person-level inequality, but it may in interaction with, e.g., product market frictions or fairness norms

► Jump



# Conclusion

---

## Conclusion: firms form & organize teams – matters for macro

- **Main idea:** if individuals have specialized skills, firms assemble teams of ‘complementary’ coworkers, generating systematic sorting patterns
- **This paper:**
  - 1 **task-based microfoundation for firm-level production fn. with skill complementarities**  
⇒ specialization + team production → complementarities
  - 2 **measurement** combining reduced-form micro evidence with model structure  
⇒ between-firm differences w/o assuming ex-ante heterogeneity
  - 3 structural explanation for the **“firming up” of inequality**  
⇒ role of **increased complementarities**

Thank You!

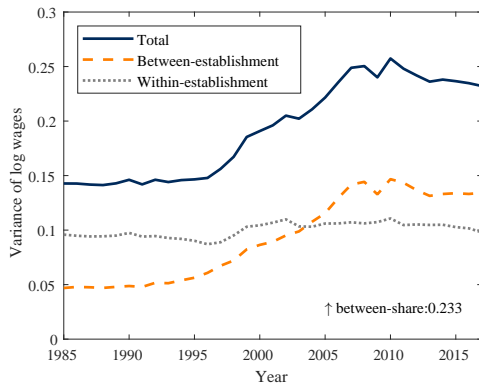
## Extra Slides

---

# Fact #1: ↑ between-firm share of wage inequality

[▶ Intro](#)

- Large empirical literature: “firming up inequality” [e.g., Card et al., 2013; Song et al., 2019]
  - “superstar firms” [e.g., Autor et al., 2020]
- **Fact 1: ↑ wage inequality primarily due to between-component**
- Robust pattern

[▶ Cross-country](#)
[▶ Panel est.](#)
[▶ Wage resid. alternatives](#)
[▶ Within-occ](#)
[▶ Within-ind](#)


Notes. Model-free statistical decomposition, where the “between” component corresponds to the person-weighted variance of est.-level avg. log wage.

## Fact #2: talented workers increasingly collaborate

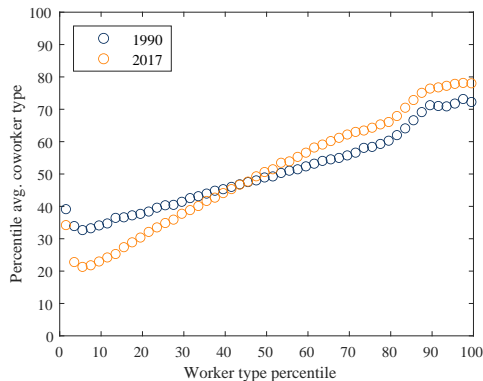
[▶ Intro](#)[▶ Main](#)[▶ Var. decomp.](#)[▶ Fact #3](#)

- To what extent do talented workers tend to have talented coworkers?

- **Fact 2: + assortative coworker sorting** ↑

- $\rho_{xx} = \text{corr}(\hat{x}_i, \hat{x}_{-it})$ : 0.43 ('85-'92) ↗ 0.62 ('10-'17)

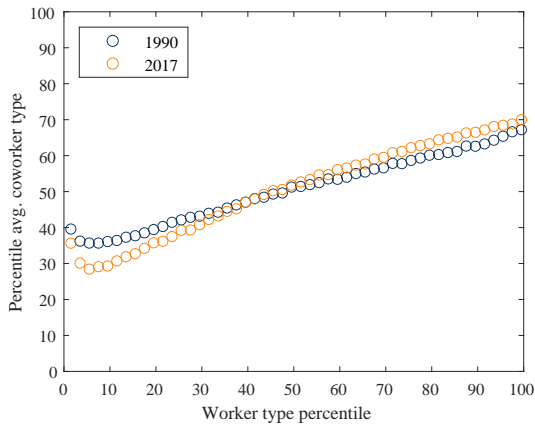
- Robust pattern

[▶ Table](#)[▶ Within-occ. nonlinear](#)[▶ Hakanson et al. \(2021\)](#)

# Evolution of coworker sorting: within-occupation ranking

[▶ Main](#)

- The most talented within each occupation – the best engineer, PA, economist, manager, ... – tend to work together, and increasingly so



# Fact #3: increased education premium due to workplace effects

[Main](#)

- **Fact 3: increase in return to schooling is primarily due to workplace effects**

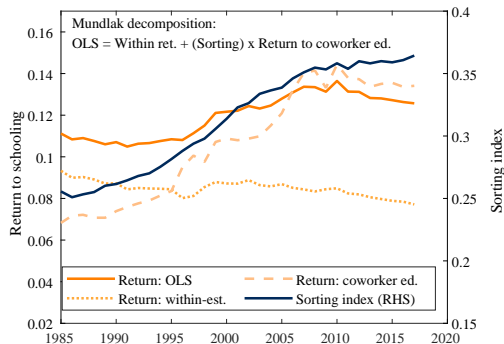
- Mundlak decomposition of year-specific OLS return to schooling:

$$\beta_t^{\text{ols}} = \beta_t^{\text{within}} + \rho_t \times \beta_t^{\text{estab.}}$$

$$\ln w_{it} = \beta_0 + \beta_t^{\text{within}} S_i + \beta_t^{\text{estab.}} \bar{S}_{j(i,t),t} + e_{it}$$

where  $\bar{S}_{j(i,t),t}$  is avg. years of schooling in establishment  $j$  of worker  $i$  in year  $t$

- 1  $\beta_t^{\text{within}}$ : within-establishment return
- 2  $\beta_t^{\text{estab.}}$ : return to avg. establishment schooling

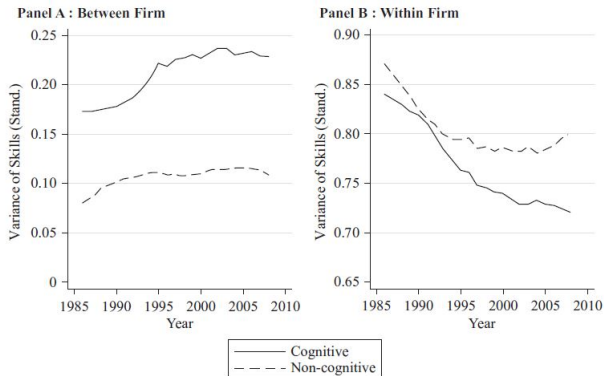


Notes. Plot of coefficients from year-by-year regressions of log wages.

## Evidence from the literature: Hakanson et al. (2021)

[▶ Main](#)

- *Direct* measures of cognitive and non-cognitive skills across Swedish firms during 1986–2008, using test data from military enlistment

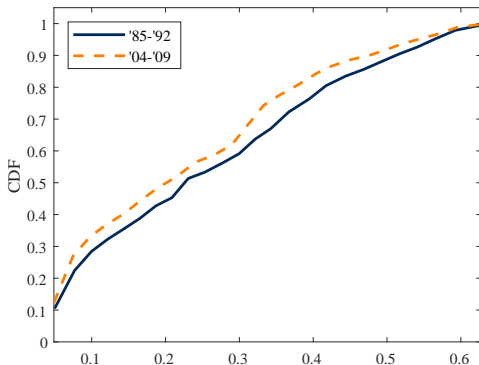




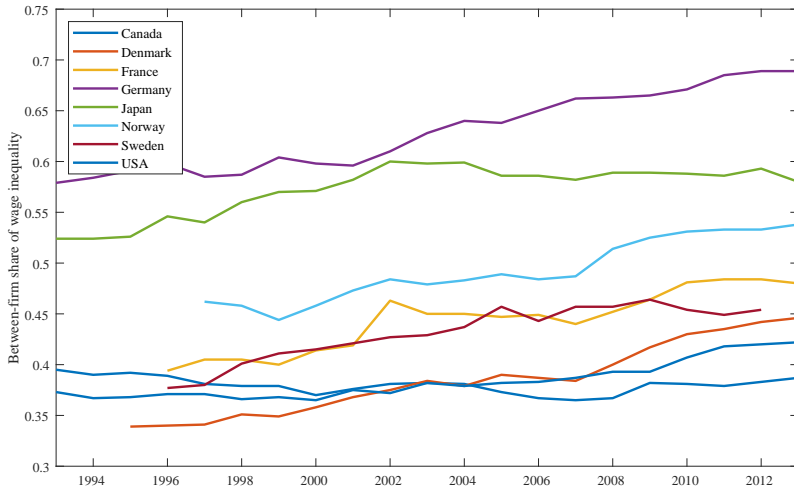
# Workers increasingly tend to perform similar tasks across different jobs

[▶ Back](#)[▶ Comparison](#)

- ✓ Workers move to jobs with similar tasks, rather than randomly
  - **Q:** are workers becoming *more* likely to perform similar tasks across jobs, over time?
- 
- **Yes:** distribution of moves in ('04-'09) is stochastically dominated by that in ('85-'92)
    - uncond. average: 0.253  $\rightarrow$  0.227: 10% decline
  - Robust in regression design
    - quantile regressions: ✓ at different quantiles



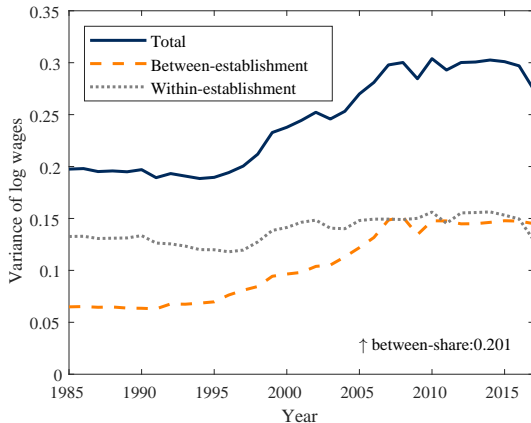
# Firming up inequality: cross-country evidence

[▶ Main](#)

Notes. Data from Tomaskovic-Devey et al. (2020). Measures of earnings differ across countries and, for Germany, between T-D et al. and my study based on the SIEED.

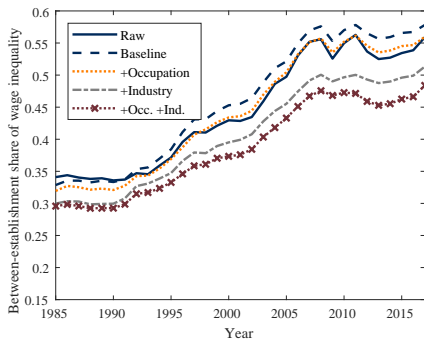
# Between-/within-employer wage var decomp. - panel establishments

- Instead of considering *all* employers, restrict attention to “panel establishments”

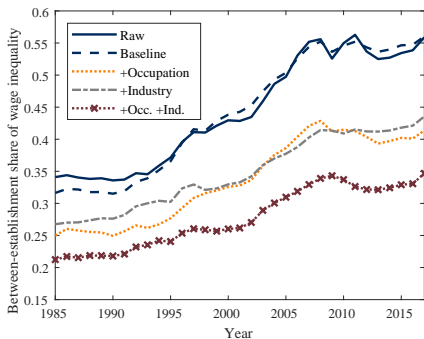


# Between-/within-employer wage var. decomp. - alternative w-residuals

- “With worker FEs”: regress  $\ln \tilde{w}_{it} = \alpha_i + X'_{it}\hat{\beta} + \epsilon_{it}$ , construct  $\ln w_{it} = \ln(\tilde{w}_{it} - X'_{it}\hat{\beta})$ .
- “Without worker FEs”: regress  $\ln \tilde{w}_{it} = \alpha_0 + X'_{it}\hat{\beta} + \epsilon_{it}$ , and consider residuals  $\epsilon_{it}$

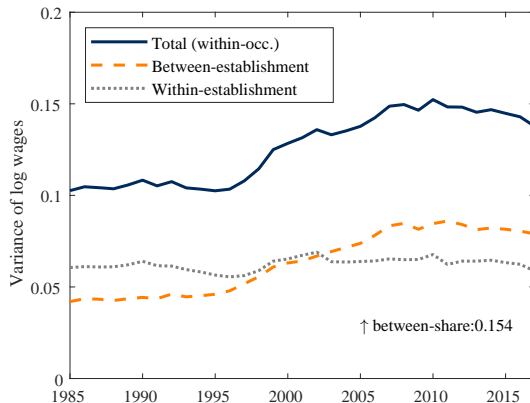


(a) With worker FEs

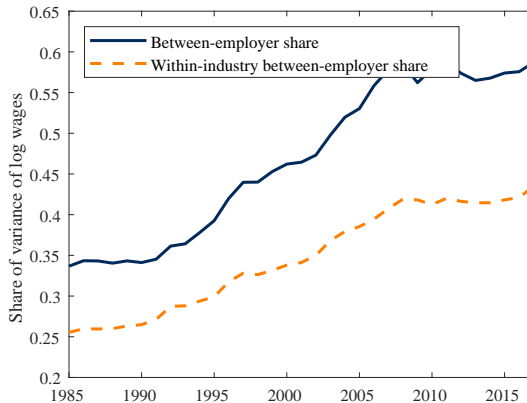


(b) Without worker FEs

# Between-/within-employer wage var. decomp. - within-Occupation

[▶ Main](#)

# Between-/within-employer wage var. decomp. - within-Industry

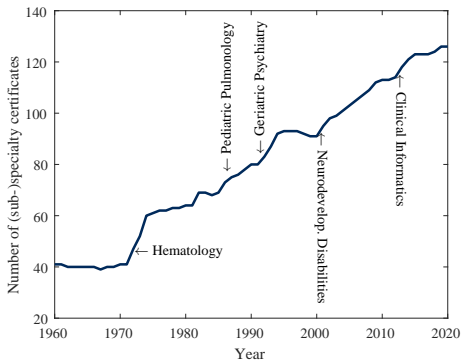
[▶ Main](#)[▶ Other moments](#)

Notes. Based on 'baseline' residualized wages.

# Examples: rising specialization

[▶ Main](#)

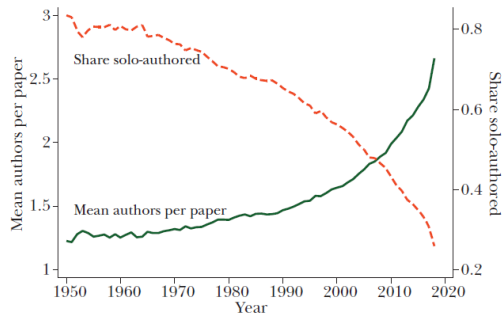
## • Deepening medical specialization



Notes. Data from American Board of Medical Specialties. For each year, it shows the number of unique specialty or sub-specialty certificates that have been approved and issued at least once by that year and which are still being issued.

## • Rise of research teams [Jones, 2021]

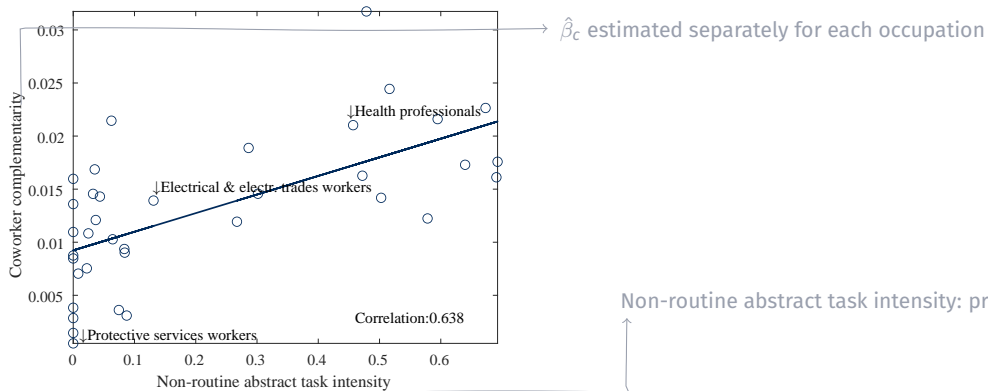
Panel A. All economics papers, 1950–2018



# X-sectional validation (occ's): tasks $\Rightarrow$ complementarity

[► More validation](#)

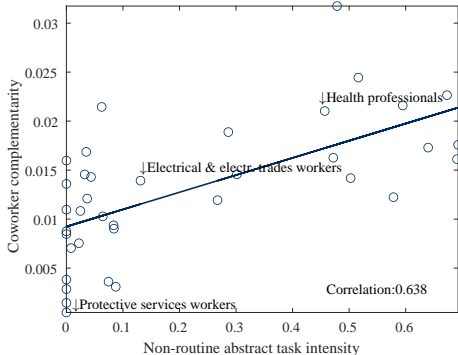
- **↑ Non-routine abstract task intensity**  
 $\Rightarrow$  **↑ coworker talent complementarity**



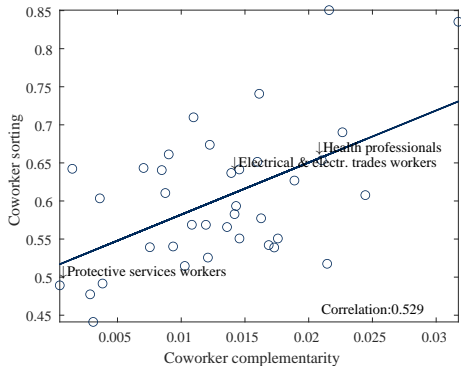


# X-sectional validation (occ's): tasks $\Rightarrow$ complementarity $\Rightarrow$ sorting

- $\uparrow$  Non-routine abstract task intensity  
 $\Rightarrow \uparrow$  coworker talent complementarity



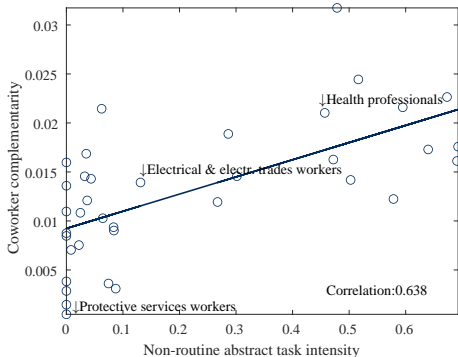
- $\uparrow$  **Coworker talent complementarity**  
 $\Rightarrow \uparrow$  **coworker sorting**



# Occupations: task complexity $\Rightarrow$ complementarity $\Rightarrow$ sorting

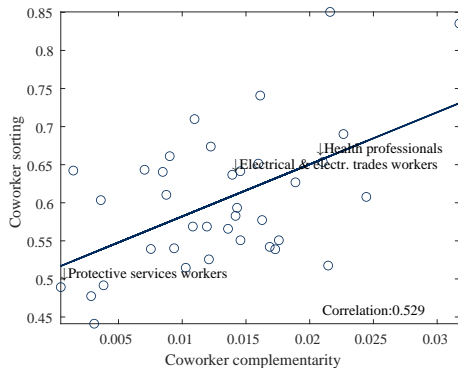
[Main](#)

- $\uparrow$  **Non-routine abstract task intensity**  
 $\Rightarrow \uparrow$  **coworker wage complementarity**



Notes. Quadros de Pessoal microdata. Horizontal axis indicates occupation's reliance on non-routine, abstract (NRA) tasks [Mihaylov and Tidens, 2019].

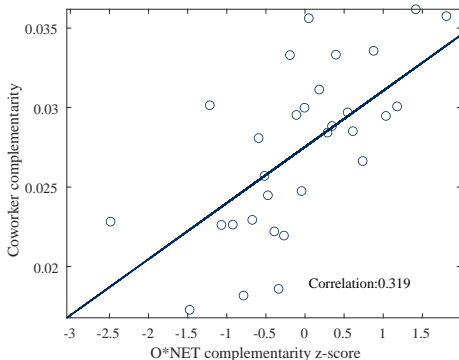
- $\uparrow$  **Coworker wage complementarity**  
 $\Rightarrow \uparrow$  **coworker sorting**



# Industries: coworker importance $\Rightarrow$ complementarity $\Rightarrow$ sorting

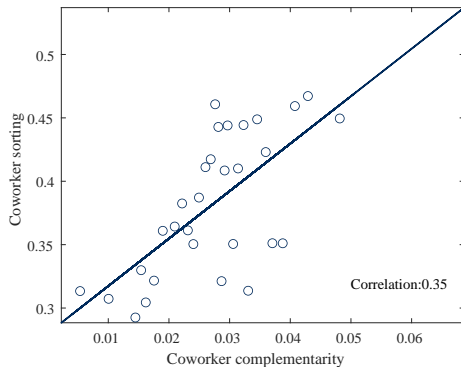
[▶ Main](#)

- $\uparrow$  **Teamwork** [Bombardini et al., 2012]  
 $\Rightarrow \uparrow$  **coworker wage complementarity**



Notes. Horizontal axis measures the industry-level weighted mean score of an occupation-level index constructed from O\*NET measuring the importance of: teamwork, impact on coworker output, communication, and contact.

- $\uparrow$  **Coworker wage complementarity**  
 $\Rightarrow \uparrow$  **coworker sorting**

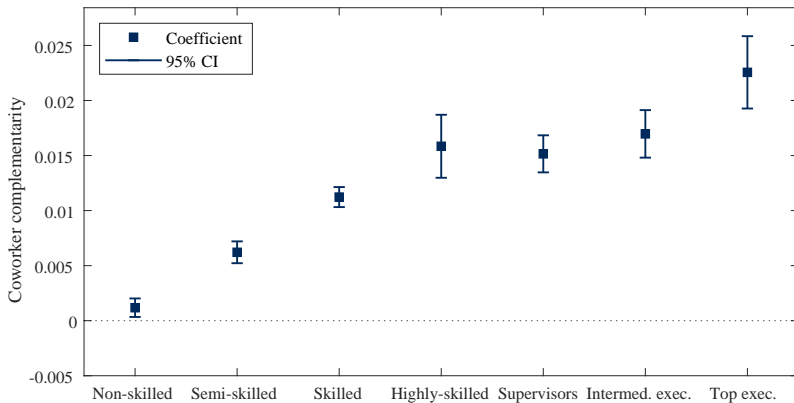


Notes. NACE-4-digit industries.

# Hierarchies: complexity $\Rightarrow$ complementarities

[▶ Main](#)[▶ Occupations](#)

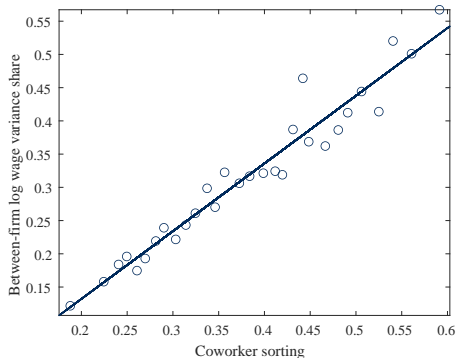
$\Rightarrow$  Coworker wage complementarities are (weakly)  $\uparrow$  in the layer of a firm's hierarchy



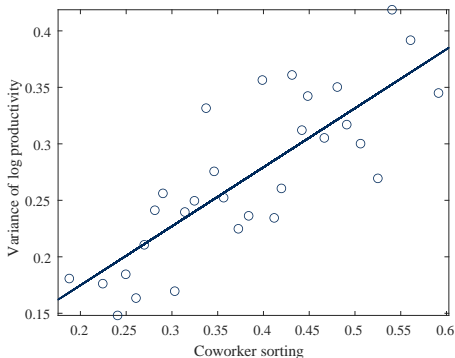
# Industries: coworker sorting $\Rightarrow$ between-firm inequality

[▶ Main](#)

$\Rightarrow$  Measures of between-firm inequality in productivity and pay are increasing in the degree of coworker sorting at the industry-level.



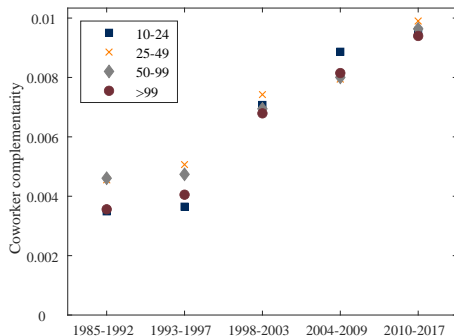
**(a)** Between-firm share of wage dispersion



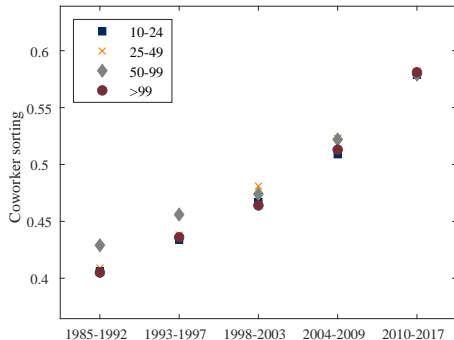
**(b)** Productivity dispersion

# Coworker complementarity & sorting by team size

► Robustness

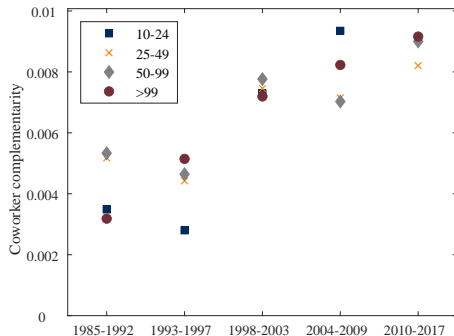


(a) Complementarity

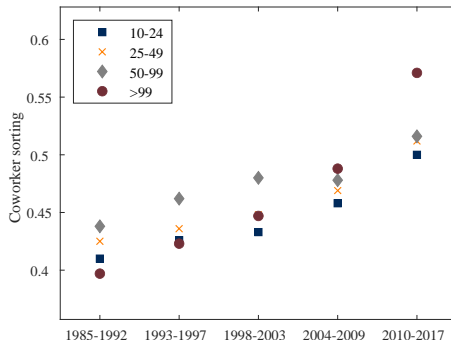


(b) Sorting

# Coworker complementarity & sorting by team size – panel estimab. only



(a) Complementarity

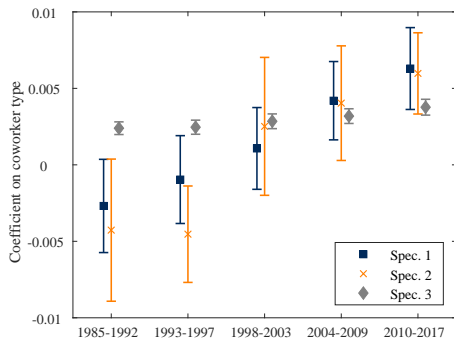


(b) Sorting

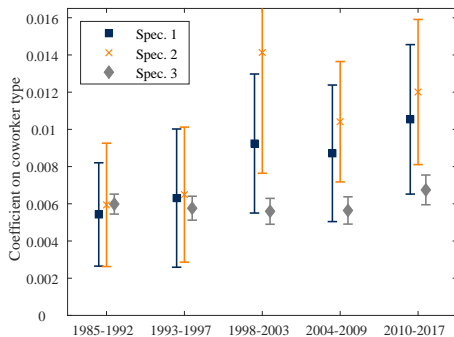
# Coworker effects: log wage regression

[▶ Back: cross-section](#)
[▶ Back: time series](#)

$$\ln w_{it} = \beta_0 + \beta_1 \hat{x}_i + \beta_2 \hat{x}_{-it} + \psi_{j(it)} + \nu_{o(i)t} + \xi_{s(i)t} + \epsilon_{it}$$



(a) AKM types



(b) NP types

Notes. Specifications vary by ranking method – within-economy (spec. 1) vs. within-occupation (spec. 2/spec.3) and coworker group definition – establishment-year (spec. 1/spec.2) vs. establishment-occupation-year (spec.3).



# Sorting & complementarity based on non-parametric ranking algorithm

- Instead of ranking workers based on AKM worker FEs, use non-param. ranking algo  
[Hagedorn et al., 2017]

Period	Sorting		Complementarities	
	Spec. 1	Spec. 2	Spec. 1	Spec. 2
1985-1992	0.47	0.38	0.001	0.000
1993-1997	0.56	0.46	0.002	0.001
1998-2003	0.60	0.48	0.004	0.002
2004-2009	0.65	0.50	0.005	0.002
2010-2017	0.68	0.51	0.005	0.004

Notes. This table indicates, under the column "Sorting" the correlation between a worker's estimated type and that of their average coworker, separately for five sample periods. The column "Complementarities" indicates the point estimate of the regression coefficient  $\beta_C$ . Under "Specification 1" workers are ranked economy wide, while under "Specification 2" they are ranked within two-digit occupations. Worker rankings are based on the non-parametric method.

# Coworker complementarity: excluding managers

[▶ Robustness overview](#)

- **Concern** regarding complementarity estimates: driven by managers?
  - only managers benefit from team quality, e.g. via larger span of control
  - the only coworkers that matter are managers

Period	Baseline	Exclude as recipients	Exclude entirely
1985-1992	0.0036***	0.0036***	0.0038***
1993-1997	0.0042***	0.0041***	0.0043***
1998-2003	0.0070***	0.0074***	0.0076***
2004-2009	0.0082***	0.0084***	0.0092***
2010-2017	0.0091***	0.0097***	0.0093***

Notes. Managed are defined based on KldB-1988-3d, as in Jarosch et al. (2023).

## Coworker complementarity: movers

[▶ Robustness overview](#)

- Consider sub-samples of job movers, job movers with contiguous employment spells ( $t \rightarrow t + 1$ ), and job movers with non-contiguous E spells ( $t \rightarrow t + s$ ,  $s > 1$ )
- Caveat: annual panel given data size, no direct observation of U/N spells in SIEED

Period	Baseline	All movers	Contig. E spells	Non-contig. E spells
1985-1992	0.0043***	0.0043***	0.0045***	0.0039***
1993-1997	0.0049***	0.0052***	0.0052***	0.0051***
1998-2003	0.0078***	0.0085***	0.0083***	0.0082***
2004-2009	0.0090***	0.0107***	0.0104***	0.0102***
2010-2017	0.0088***	0.0103***	0.0101***	0.0090***
Obs. in '10-'17 (1000s)	4,410	538	355	375

Notes. Unweighted observations. Regressions include FEs for employer; occupation-year; industry-year. Employer-clustered standard errors in parentheses.

## Coworker complementarity: finite-differences approximation

[▶ Robustness overview](#)

- Regression approach imposes strong functional form assumptions on approximated empirical wage function  $\hat{w}(x|x')$ 
  - ofc, mirrored inside structural model when calibrating
- Alternative: construct non-parametric  $\hat{w}(x|x')$ , then use finite-difference methods to compute the cross-partial derivative (but w/o FE controls)

Period	Regression	Non-parametric FD method
1985-1992	0.0036	0.0073
1993-1997	0.0042	0.0074
1998-2003	0.0070	0.0081
2004-2009	0.0082	0.0120
2010-2017	0.0091	0.0098

## Coworker complementarity: lagged types

[▶ Robustness overview](#)

- Concern with both regression approach and non-parametric FD approach: mechanical relationship between wages (“LHS”) and (within-period time-invariant) worker types, which are estimated from wages themselves (“RHS”)
- Robustness check #1: years of schooling as type measure [▶ Jump](#)
- Robustness check #2: assign to each individual  $i$  in periods  $p \in \{2, 3, 4, 5\}$  the FE estimated for  $i$  in period  $p - 1$ ; re-compute worker deciles and average coworker types,  $\hat{x}_i^{p-1}$  and  $\hat{x}_{-it}^{p-1} = (|S_{-it}|)^{-1} \sum_{k \in S} \hat{x}_k^{p-1}$ ; re-estimate wage regression
- Results (see paper): magnitude of estimated  $\hat{\beta}_c$  around 50% smaller when using lagged types, but evolution over time similar to baseline

# Complementarity estimates using years of schooling

[▶ Robustness overview](#)

	'85-'92	'93-'97	'98-'03	'04-'09	'10-'17
Interaction	0.0063*** (0.0008)	0.0060*** (0.0007)	0.0099*** (0.0008)	0.0112*** (0.0007)	0.0129*** (0.0009)
Obs. (1000s)	3,613	2,508	2,694	3,836	4,376
$R^2$	0.5033	0.5451	0.5746	0.6330	0.6425

*Notes.* Dependent variable is the wage level over the year-specific average wage. Independent variables are a constant, years of schooling, coworker years of schooling, and the interaction between those two terms. All regressions include industry-year, occupation-year and employer fixed effects. Employer-clustered standard errors in parentheses. Observations are unweighted. The sample is unchanged from the main text, except that 96,517 observations with missing years of schooling are dropped. Observation count rounded to 1000s.

# Within-industry empirical analysis

[► Overview: robustness](#)
[► Within-industry calibration](#)

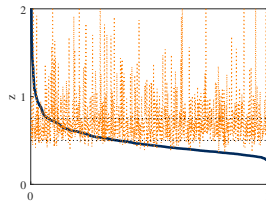
Sample Period	Baseline				Within-industry avg.			
	$\sigma_w^2$	$\sigma_w^2/\sigma_w^2$	$\rho_{xx}$	$\hat{\beta}_c$	$\sigma_w^2$	$\sigma_w^2/\sigma_w^2$	$\rho_{xx}$	$\hat{\beta}_c$
1	0.143	0.337	0.427	0.0036	0.125	0.249	0.333	0.00283
2	0.148	0.391	0.458	0.0042	0.125	0.288	0.351	0.00342
3	0.191	0.456	0.495	0.0070	0.150	0.324	0.369	0.00585
4	0.234	0.547	0.547	0.0082	0.168	0.388	0.405	0.00738
5	0.241	0.568	0.617	0.0091	0.171	0.412	0.464	0.00823

Notes. Within-industry avg. is person-year weighted average across OECD STAN-A38 (2-digit) industries.

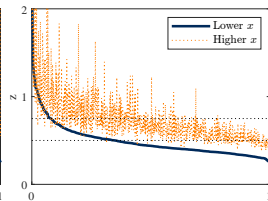
# Illustration: specialization parameter

[▶ Back](#)

- $\xi$ : controls correlation of coworkers' task-specific productivities



**(c)** Independent draws  
( $\xi = 1$ )



**(d)** Correlated draws  
( $\xi \rightarrow 0$ )



# Proof sketch

[▶ Main](#)

- ① Derive  $G(p) := \Pr\{\tilde{\lambda}(\tau) \leq p\}$  given  $G_i(p) := \Pr\{\lambda_i(\tau) \leq p\}$ , using FOC and max-stability property
- ② Use  $G(p)$  + standard CES shadow price index to solve (int. by sub.) for

$$\lambda = \left( \int_{\mathcal{T}} \tilde{\lambda}(\tau)^{1-\eta} d\tau \right)^{\frac{1}{1-\eta}}. \quad (5)$$

- ③ Use  $G(p)$  and  $G_i(p)$  to derive probability that  $i$  produces some task  $\tau$ , which by LLN (continuum assumption!) is equal to share of tasks produced,  $\pi_i$
- ④ Relate  $\lambda_i^L$  to value of all tasks produced by the worker,  $\lambda_i^L = \pi_i \lambda Y$
- ⑤ Normalize  $\lambda = 1$ , then algebra yields  $Y = f(x_1, \dots, x_n; \chi)$

# Lemma

## Lemma: Lemma

Implied task share and shadow-cost index equal

$$\pi_i = \frac{(x_i/\lambda_i^L)^{\frac{1}{\chi\xi}}}{\sum_{k=1}^n (x_k/\lambda_k^L)^{\frac{1}{\chi\xi}}} \quad x; \lambda = \left( \sum_{i=1}^n \left( \frac{x_i}{\lambda_i^L} \right)^{\frac{1}{\chi\xi}} \right)^{-\chi\xi}$$

# Intuition: features of optimal organization

[▶ Main](#)
[▶ Extension to communication frictions](#)

- **What is the intuition for these properties?**
- Solution of firm's mini-planner problem implies:
  - ① **Complete division of labor**, with tasks assigned by **comparative advantage**
    - $i$ 's task set  $\mathcal{T}_i = \left\{ \tau \in \mathcal{T} : \frac{z_i(\tau)}{\lambda_i^L} \geq \max_{k \neq i} \frac{z_k(\tau)}{\lambda_k^L} \right\}$
    - **classic source of efficiency gains**
  - ②  $i$ 's share of tasks **↑ in  $i$ 's talent**, **↓ in coworkers' talent**
    - $i$ 's task share  $\pi_i = (x_i^{\frac{1}{1+\chi\xi}}) \left( \sum_{k=1}^n (x_k)^{\frac{1}{1+\chi\xi}} \right)^{-1}$

# Intuition: comparative statics for task shares

- Suppose that  $x_i > x_j$ . Then
  - 1  $i$  performs a strictly larger share of tasks than  $j$  for  $\chi < \infty$

Low  $\chi$



## Intuition: comparative statics for task shares

- Suppose that  $x_i > x_j$ . Then
  - ①  $i$  performs a strictly larger share of tasks than  $j$  for  $\chi < \infty$
  - ② the difference in task shares is decreasing in  $\chi$



⇒ **Greater skill specialization implies a larger share of tasks is performed by relatively less talented team members** – more talented coworkers can't easily compensate

## Surplus max. determines which worker types a firm w/ worker $x$ hires ► Main

- Joint value of firm with worker  $x$ ,  $\Omega(x)$ , satisfies:

$$\rho\Omega(x) = f(x) + \delta[-\Omega(x) + V_u(x) + V_{f.o}] + (1 - \omega)\lambda_{v.u} \int \frac{d_u(\tilde{x}')}{u} \max\{S(\tilde{x}'|x), 0\} d\tilde{x}'$$

- $V_u(x)$ : value for unemp. worker;  $V_{f.o}$ : value for vacant firm;  $S(x)$ : surplus from zero-worker firm hiring  $x$
- $d_u(x)$ : density of unemployed workers of type  $x$ ;  $u = \int d_u(x)dx$
- $\omega$ : worker bargaining wgt;  $\delta$ : sep. rate;  $\lambda_{v.u}$ : rate of vacancy meeting unmatched worker

# Surplus max. determines which worker types a firm w/ worker $x$ hires ► Main

- Joint value of firm with worker  $x$ ,  $\Omega(x)$ , satisfies:

$$\rho\Omega(x) = f(x) + \delta[-\Omega(x) + V_u(x) + V_{f.o}] + (1 - \omega)\lambda_{v.u} \int \frac{d_u(\tilde{x}')}{u} \max\{S(\tilde{x}'|x), 0\} d\tilde{x}'$$

- $V_u(x)$ : value for unemp. worker;  $V_{f.o}$ : value for vacant firm;  $S(x)$ : surplus from zero-worker firm hiring  $x$
  - $d_u(x)$ : density of unemployed workers of type  $x$ ;  $u = \int d_u(x) dx$
  - $\omega$ : worker bargaining wgt;  $\delta$ : sep. rate;  $\lambda_{v.u}$ : rate of vacancy meeting unmatched worker
- Surplus  $S(x|x')$  reflects complementarities – and hiring decisions reflect surplus

$$S(x|x')(\rho + 2\delta) = f(x, x') - \rho(V_u(x) + V_u(x') + V_{f.o}) + \delta S(x) - (\rho + \delta)S(x')$$

$$h(x|x') = \mathbf{1}\{S(x|x') > 0\}$$

# HJB: unmatched

[▶ Main](#)

- Unmatched firm:

$$\rho V_{f.o} = (1 - \omega) \lambda_{v.u} \int \frac{d_u(x)}{u} S(x)^+ dx \quad (6)$$

- Unmatched worker:

$$\rho V_u(x) = b(x) + \lambda_u \omega \left[ \int \frac{d_{f.o}}{v} S(x)^+ + \int \frac{d_{m.1}(\tilde{x}')}{v} S(x|\tilde{x}')^+ d\tilde{x}' \right] \quad (7)$$



# HJB: surpluses

- Surplus of coalition of firm with worker  $x$

$$(\rho + \delta)S(x) = f(x) - \rho(V_u(x) + V_{f.o}) + \lambda_{v.u}(1 - \omega) \int \frac{d_u(\tilde{x}')}{u} S(\tilde{x}'|x)^+ d\tilde{x}' \quad (8)$$

- Surplus from adding  $x$  to  $x'$

$$S(x|x')(\rho + 2\delta) = f(x, x') - \rho(V_u(x) + V_u(x') + V_{f.o}) + \delta S(x) - (\rho + \delta)S(x') \quad (9)$$

## KFE: unemployed

$$\delta \left( d_{m.1}(x) + \int d_{m.2}(x, \tilde{x}') d\tilde{x}' \right) = d_u(x) \lambda_u \left( \int \frac{d_{f.o}}{v} h(x, \tilde{y}) + \int \frac{d_{m.2}(\tilde{x}')}{v} h(x|\tilde{x}') d\tilde{x}' \right). \quad (10)$$

## KFE: one-worker matches

$$d_{m.1}(x) \left( \delta + \lambda_{v.u} \int \frac{d_u(\tilde{x}')}{u} h(\tilde{x}'|x) d\tilde{x}' \right) = d_u(x) \lambda_u \frac{d_{f.o}}{v} h(x) + \delta \int d_{m.2}(x, \tilde{x}') d\tilde{x}'. \quad (11)$$

## KFE: two-worker matches

$$2\delta d_{m.2}(x, x') = d_u(x)\lambda_u \frac{d_{m.1}(x')}{v} h(x|x') + d_u(x')\lambda_u \frac{d_{m.1}(x)}{v} h(x'|x). \quad (12)$$

# Measurement: a useful identification result

[▶ Main](#)
[▶ Non-separable case: scatterplot](#)

- **Q:** How to quantify  $\frac{\partial^2 f(x, x')}{\partial x \partial x'}$ ?
- **Proposition:** production complementarities are proportional to wage compl.
- **Proof sketch:** wage level for worker  $x$  with coworker  $x'$

[▶ Wage eq.](#)

$$w(x|x', \xi) = \omega f(x, x', \xi) + g(x) - h(x')$$

where  $g : [0, 1] \rightarrow \mathbb{R}$  and  $h : [0, 1] \rightarrow \mathbb{R}$  are strictly increasing

$\Rightarrow$  *outside options are separable: affect level of wage but not the cross-partial*

# Mapping model to data: coworker types

- Defining  $S_{-it} = \{k : j(kt) = j(it), k \neq i\}$  as the set of  $i$ 's coworkers in year  $t$ , compute the average type of  $i$ 's coworkers in year  $t$  as  $\hat{x}_{-it} = \frac{1}{|S_{-it}|} \sum_{k \in S_{-it}} \hat{x}_k$ .
- **Coworker group:**
  - alternative: same establishment-occupation-year cell
  - but CC arise precisely when workers are *differentiated* in their task-specific productivities
- **Averaging step:**
  - equally-weighted averaging ignores non-linearity in coworker aggregation
  - paper: show using non-linear averaging method that baseline results in bias, but it's minor in magnitude
- **Firm size variation:** averaging ensures that a single move will induce a smaller change in the *average* coworker quality in a large team than in a small one

# Mapping model to data: identification strategy for $\chi$

[▶ Main](#)

- **Literature:** complementarities – primarily between workers and firms – usually inferred indirectly from sorting patterns
  - exception: Hagedorn-Law-Manovskii (2017)
- **This paper:** directly measure coworker complementarity in the data, recover  $\chi$  structurally given  $\gamma = \frac{\chi}{\chi+1}$
- Paper does *not* use microfoundation itself to measure  $\chi$ , respectively  $\gamma$
- Experiment: fit a (truncated) Fréchet distribution to Grigsby's (2023) non-parametric estimates of the multi-dimensional skill dist. estimated from CPS data
  - recover  $\gamma = 0.84$  for 2006 but *very* noisy estimates
- **Ongoing work:** use the extended microfoundation to identify  $\chi$

[▶ Jump](#)[▶ Jump](#)

## Direct estimation of $\chi$ : proof of concept

- Grigsby (2023): only paper that provides a *cardinal* measure of skill task-specificity
- Evidence on time trends are qualitatively consistent with “specialization hypothesis”: cross-type average of within-type variance across specific skills grew by nearly 50% b/w 1980s and 2000s & skill transferability has declined amongst high-skill occupations
- His operationalization of worker types and tasks does *not* directly map onto my model (no identifying assumption; coarse occupational skills; US vs DE data)
- **Proof of concept:** but *suppose* we just take those data, extract moments capturing average within-worker cross-task efficiency dispersion, fit a (truncated) Fréchet, recover  $\gamma = \frac{\chi}{1+\chi}$   
 $\Rightarrow \checkmark \gamma$  **similar to structural estimation result based on evidence from wage CC**



# Semi-structural back-of-envelope calculation for $\gamma$

[▶ Main](#)

- Structurally recover  $\gamma \frac{x}{x+1}$  by estimating  $\frac{\partial^2 w(x|x')}{\partial x \partial x'}$  in the data, which was shown to be proportional to  $\frac{\partial^2 f(x, x')}{\partial x \partial x'}$
- But how is  $\frac{\partial^2 f(x, x')}{\partial x \partial x'}$  related to  $\gamma$ ?
- Definitionally,  $\gamma = (f f_{ij}) / (f_i f_j)$  for any  $i \neq j$
- Can we avoid full structural model?  $\Rightarrow$  If have measures not only of  $f_{ij}$  but also output  $f$  and marginal products  $f_i$
- Suppose, for any  $x$  and  $x'$ , we use wages to back out marginal products – competitive wage determination rather bargaining! – and recover output from sum of wages divided by labor share
- Find  $\gamma \approx 0.79$  – very close to structural estimate!

# Overview of validation exercises: direction EE transitions & cross-section

[▶ Main](#)

- 2 additional types of validation exercise:

- ✓ **EE transitions** reallocate workers to more + assortative matches
- do model-implied relationships also hold in **cross-section**?

[▶ Details](#)

①  $\chi \uparrow \Rightarrow$  coworker complementarity  $\uparrow$

② coworker complementarity  $\uparrow \Rightarrow$  + assortative matching  $\uparrow$

can test predictions *because*  
we have measures of comple-  
mentarity!

- Implementation of cross-sectional exercises: rich Portuguese micro data

- universe of private-sector actors, employer-employee data & income statements

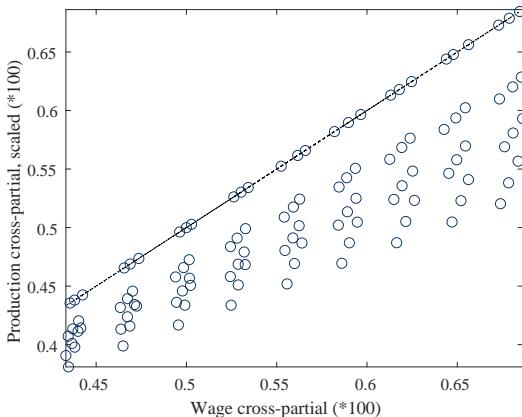
- Cross-sectional exercises:

- ✓ **Hierarchies**
- ✓ **Industries**
- ✓ **Occupations**

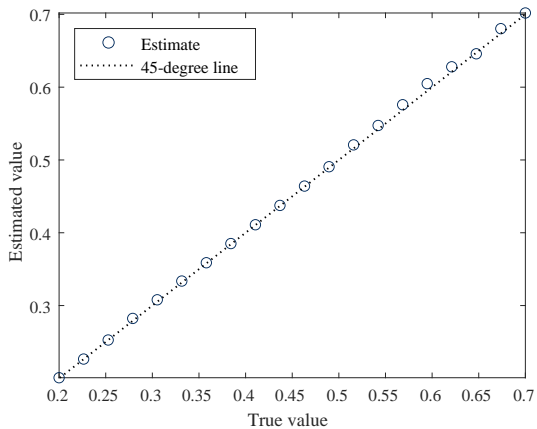
[▶ Details](#)[▶ Details](#)[▶ Details](#)

# Wage and production cross-partials beyond the benchmark

- Solve model for many combinations of  $\chi$ ,  $\lambda_e$  and  $b$
- Compare FD approx of  $f_{xx'}(x, x')$ , scaled by  $\omega$ , and  $w_{xx'}(x|x')$
- Main parameter driving wedge:  $\lambda_e$



# Monte Carlo study

[▶ Main](#)

# Implications for aggregate productivity

[► Effects of  \$\chi\$  ↑: random vs eqm](#)

- **Production complementarities imply coworker sorting matters for agg productivity**

$$\circ f(x_1, \dots, x_n) = n^{1+\chi\xi} \times \left( \frac{1}{n} \sum_{i=1}^n (x_i)^{\frac{1}{1+\chi\xi}} \right)^{1+\chi\xi}$$

# Implications for aggregate productivity

[► Effects of  \$\chi\$  ↑: random vs eqm](#)

- **Production complementarities** imply coworker sorting matters for agg productivity

- $f(x_1, \dots, x_n) = n^{1+\chi\xi} \times \left( \frac{1}{n} \sum_{i=1}^n (x_i)^{\frac{1}{1+\chi\xi}} \right)^{1+\chi\xi}$

- **Search** frictions induce misallocation  $\sim$  coworker **mismatch**

# Implications for aggregate productivity

► Effects of  $\chi$  ↑: random vs eqm

- **Production complementarities imply coworker sorting matters for agg productivity**

$$\circ f(x_1, \dots, x_n) = n^{1+\chi\xi} \times \left( \frac{1}{n} \sum_{i=1}^n (x_i)^{\frac{1}{1+\chi\xi}} \right)^{1+\chi\xi}$$

- **Search** frictions induce misallocation  $\sim$  coworker **mismatch**
- **Quantify** mismatch costs: compare eqm outcome vs to productivity under pure PAM  
 $\Rightarrow$  2010s gap: 2.05%, similar for earlier periods

# Implications for aggregate productivity

► Effects of  $\chi$  ↑: random vs eqm

- **Production complementarities imply coworker sorting matters for agg productivity**

$$\circ f(x_1, \dots, x_n) = n^{1+\chi\xi} \times \left( \frac{1}{n} \sum_{i=1}^n (x_i)^{\frac{1}{1+\chi\xi}} \right)^{1+\chi\xi}$$

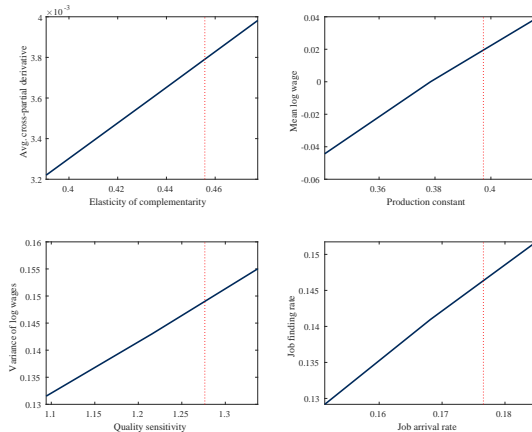
- **Search** frictions induce misallocation  $\sim$  coworker **mismatch**
- **Quantify** mismatch costs: compare eqm outcome vs to productivity under pure PAM  
 $\Rightarrow$  2010s gap: 2.05%, similar for earlier periods
- **Trends:**  $\uparrow$  talent sorting limited  $\uparrow$  in mismatch costs given  $\chi \uparrow$   
 $\Rightarrow$  no-reallocation counterfactual: productivity gap 4.65%



# Parameterization, including estimation results (2010s)

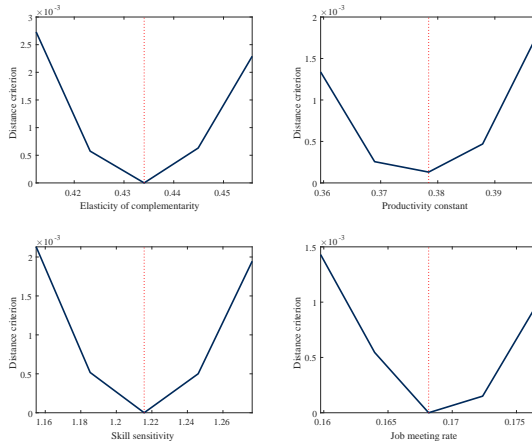
Parameter	Description	Targeted moment	Value	$m$	$\hat{m}$
$\chi$	Specialization	$\hat{\beta}_c$	0.67	0.0058	0.0058
$a_0$	Production, constant	Avg. wage (norm.)	0.29	1	1
$a_1$	Production, scale	Var. log wage	1.71	0.241	0.241
$b_1$	Replacement rate, scale	Replacement rate	0.60	0.63	0.63
$\delta$	Separation hazard	Job loss rate	0.008	0.008	0.008
$\lambda_u$	Meeting hazard	Job finding rate	0.22	0.162	0.162
$\rho$	Discount rate	External	0.008		
$\omega$	Worker bargaining weight	External	0.50		
$\bar{n}$	Effective team size	External	25		

# Identification validation exercise 1

[▶ Main](#)

*Notes.* This figure plots the targeted moment against the relevant parameter, holding constant all other parameters.

# Identification validation exercise 2



*Notes.* This figure plots the distance function  $\mathcal{G}(\psi_i, \psi_i^*)$  when varying a given parameter  $\psi_i$  around the estimated value  $\psi_i^*$ . The remaining parameters are allowed to adjust to minimize  $\mathcal{G}$ .

# What about shifts in the talent distribution?

[▶ Robustness overview](#)

- **Q:** could  $\uparrow$  economy-wide  $x$ -dispersion,  $\sigma_x$ , also explain  $\uparrow$  firm-level inequality?
  - Kremer-Maskin (1996): as the dispersion of skills (talent)  $\uparrow$  in the economy, the relative dispersion of talent within firms  $\downarrow$
- **Obs. #1:** Hakanson et al. (2021): direct skill measures (from military enlistment tests) point to Flynn effect but not to increased dispersion (in cognitive test scores)
- **Obs. #2:** model exercise
  - method: instead of rank interpretation, i.e.  $x \sim U$ , we separately parameterize  $\sigma_x$  by assuming a  $\mathcal{N}$  distribution
  - finding:  $\sigma_x \uparrow \Rightarrow \rho_{xx} \uparrow, \sigma_w \uparrow$ , and  $\sigma_{\bar{w}}/\sigma_w \uparrow$  – *but* no measured increase in  $\hat{\beta}_c$
- **Conclusion:** empirically unclear whether  $\sigma_x \uparrow$ , and if so, this would not explain the observed increase in coworker complementarity, i.e. latter is a distinct channel

# Training policies in a team production context (w-i-p)

[► Overview](#)

- “Training policies”  $\sim$  non-parametric perturbations of the talent distribution
  - left-tail intervention: give everyone in 1st decile productivity of those in 2nd decile
  - right-tail intervention: give everyone in 9th decile productivity of those in 10th decile
- Team production: effect of “training policies” partially via coworker spillovers!
- Relative effectiveness of left-tail vs right-tail intervention:
  - 1 the stronger are coworker complementarities, the relatively greater are the realized productivity gains from a left-tail intervention, b/c low- $x$  tend to be weak links
  - 2 but raising the productivity of coworkers of workers with high productive potential generates greater gains – and with sorting, those coworkers are themselves high- $x$
- Equilibrium: relative effectiveness of left-/right-tail training depends on both forces
  - tentative quantitative finding: right-tail intervention boosts average productivity by more *but* left-tail training also lowers inequality

# The effect of declining search frictions

- $\downarrow$  search frictions could also explain  $\uparrow$  coworker sorting
- Job arrival & separation rates estimated to  $\uparrow$  from p1 to p2
- **Counterfactual analysis:** explains 6% of model-implied  $\uparrow$  in between-employer share of wage variance

	$\Delta$ model	Implied % $\Delta$ model due to $\Delta$ parameter
Model 1: baseline	0.159	-
Cf. a: fix period-1 complementarity	0.065	59
Cf. b: fix period-1 search frictions	0.150	6

# Outsourcing & within-occupation ranking analysis

- **Concern:** confounding shifts in labor boundary of firm, e.g. outsourcing
- **Address this concern in multiple steps:**
  - ① empirically rank workers *within* occupation (“good engineer vs. mediocre engineer”)
  - ② empirically re-estimate coworker sorting & complementarity (lower but similar  $\uparrow$ )
  - ③ re-estimate model for both periods & re-do counterfactual exercises
- **Result:** qualitatively & quantitatively similar findings

	$\Delta$ model	Implied % $\Delta$ model due to $\Delta$ parameter
Model 2: within-occ. ranking	0.198	-
Cf. a: fix period-1 comp.	0.076	61.47

# Robustness: model with OJS - brief overview

[▶ Main](#)

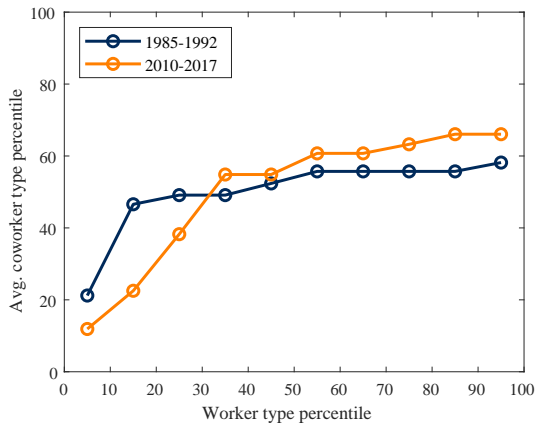
- **Baseline:** no job-to-job transitions – but is main story robust when workers can switch to better job after accepting job out of unemployment?
  - two opposing effects from increased complementarities
- **Extension:** employed workers also meet vacancies at Poisson rate  $\lambda_e$
- **Main findings:**
  - better fit to empirical sorting patterns in cross-section [▶ Comparison](#)
  - contribution of  $\uparrow$  complementarities to  $\uparrow$  firm-level wage inequality slightly smaller, more attributed to  $\uparrow$  labor market transitions
    - conservative estimates (endogenous search effort, forward-looking wage specification)
- Opens door to thinking about coworker complementarities and **job ladders** [▶ Jump](#)



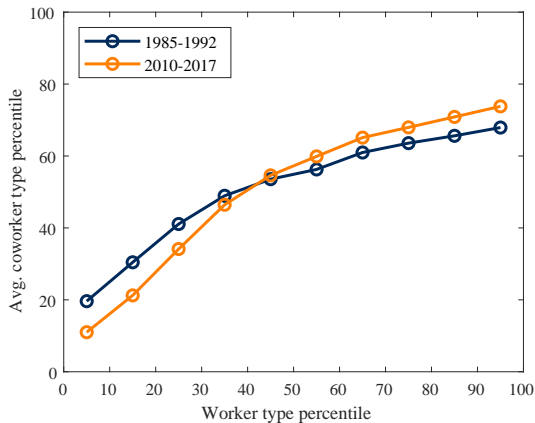
## Extension: model with OJS

- Baseline model abstracted from OJS
  - transparent trade-off, connection to analytical results
- **Consider extension to OJS:** employed worker meet vacancies at Poisson rate  $\lambda_e$ 
  - wages both off and on the job are continuously renegotiated under Nash bargaining, with unemployment serving as the outside option *[cf. di Addario et al., 2021]*
  - re-estimate, with empirical labor market flows disciplining  $\lambda_e$
- Qualitative question: is coworker sorting outcome robust, even if workers can switch to better job after accepting job out of unemployment?
- **Analyses:**
  - 1 coworker sorting patterns & changes
  - 2 additional model validation: direction of EE flows in model & data

# Model-implied coworker sorting patterns: without and with OJS

[▶ Main](#)

(a) Baseline



(b) OJS

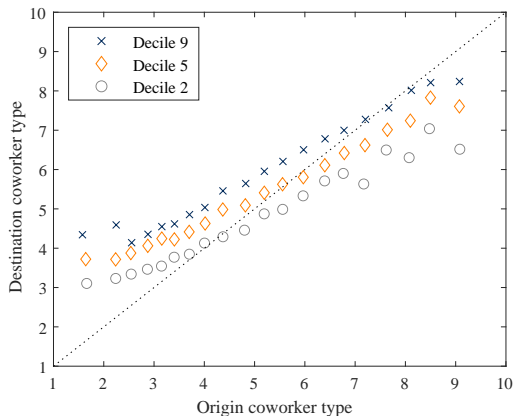
# EE transitions in theory and data

- **Theoretical prediction:** EE transitions move workers in surplus-maximizing direction  
 $\Rightarrow \Delta \hat{x}_{-it} = \hat{x}_{-i,t} - \hat{x}_{-i,t-1}$  should be *positively* correlated with  $\hat{x}_i$ 
  - $h_{2.1}(x, x'' | x') = 1$  – worker  $x$  in a two-worker firm with coworker  $x''$  would move to an employer that currently has one employee of type  $x'$  – if  $S(x|x') - S(x|x'') > 0$
- **Empirical analysis:** use SIEED *spell* data to create worker-originMonth-destinationMonth-originJob-destinationJob panel, with information on characteristics of origin and destination job
  - subsample period 2008-2013 (huge panel at monthly frequency)
  - count as “EE” if employer change between two adjacent months
- **Regression analysis:** regress  $\Delta \hat{x}_{-it}$ , scaled by std.  $\sigma_{\Delta}$  of coworker quality changes, on *own* type and *origin* coworker type

$$\frac{\Delta \hat{x}_{-it}}{\sigma_{\Delta}} = \beta_0 + \beta_1 \hat{x}_i + \beta_2 \hat{x}_{-i,t-1} + \epsilon_{it}$$

# Empirical coworker sorting changes due to EE moves

- **EE transitions push toward greater coworker sorting:** for any given origin, higher x-workers move to workplaces with better coworkers than lower-x workers do
- *But* in data EE transitions “move up” low types more than theory predicts
- **“Coworker job ladder”** with both absolute and type-specific dimension?
- **Next:** change in the job ladder [e.g., Haltiwanger-Spetzler, 2021]



## Evidence that EE *increasingly* reallocate toward PAM: in data & model

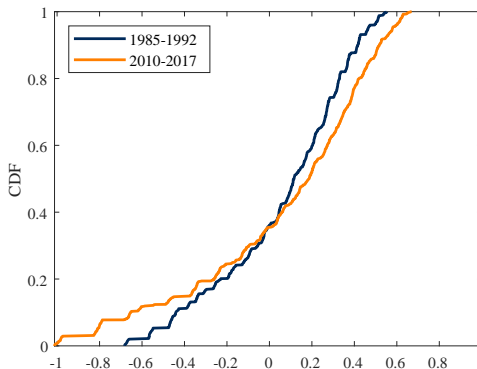
	Data		Model	
<i>Change in coworker type</i>	'85-'92	'10-'17	Period-1	Period-2
Own type	<b>0.0883</b> <sup>***</sup> (0.000799)	<b>0.118</b> <sup>***</sup> (0.000918)	<b>0.214</b>	<b>0.270</b>
Controls	Year FEs, Origin	Year FEs, Origin	Origin	Origin
<i>N</i>	196,098	282,718	∞	∞
adj. <i>R</i> <sup>2</sup>	0.284	0.204		

**Table 1:** Change in coworker type due to EE moves positively related to own type – increasingly so

*Notes.* For the data columns, individual-level clustered standard errors are given in parentheses. Model counterparts are computed simulation-free in population. Dependent variable is scaled throughout by the standard deviation of the change in coworker type.

# Productivity dispersion

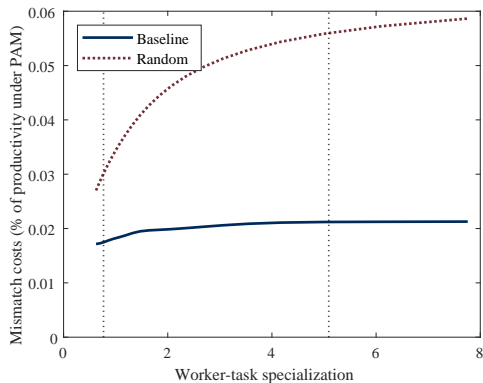
- Firm dynamics literature: increased firm-level productivity dispersion [Autor et al., 2020; de Ridder, 2024]; correlated with wage & talent dispersion [Berlingieri et al., 2017; Sorkin-Wallskog, 2020]



# Productivity costs of complementarity & labor market functioning

“The benefits of the division of labor are limited by the functioning of the labor market”

- Microfoundation:  $\uparrow \chi \Rightarrow \uparrow$  efficiency *benefit* from teamwork *but* also  $\uparrow$  mismatch costs
- **Q:** how does the gap to potential vary depending on labor market structure?
- **A:** under random sorting, productivity gap due to misallocation  $\uparrow$  more sharply as  $\chi \uparrow$
- Outside model: severe labor mkt frictions (e.g., dev'ing countries [Donovan et al., 2023]) may inhibit specialization [cf. Atencio et al., 2023; Bassi et al., 2023]



# Implications for overall inequality?

- **Coworker complementarities do not necessarily  $\uparrow$  variance of person-level wages**
  - (un-)surprising? Variance decomposition perspective vs. common intuition [Kremer, 1993]
  - (i) **reallocation effect**, (ii) valuation effect, (iii) outside option effect
- Several mechanisms though through which  $\uparrow$  sorting could  $\uparrow$  person-level inequality
  - 1 regulation or norms that lead to within-firm wage compression [Akerlof-Yellen, 1990]
  - 2 coworker learning [Jarosch et al., 2021; HLMP, 2023]
  - 3 increasing returns to labor quality [Kremer, 1993]



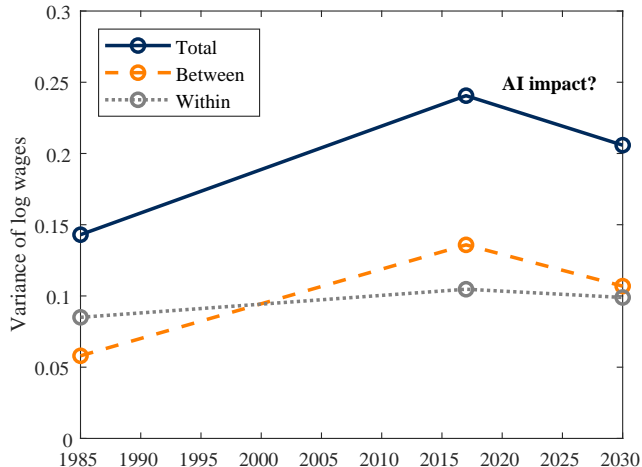
# Implications for AI impact on labor markets (1): overview

[▶ Main](#)

- Literature moving toward richer way of thinking about tech. change. Here: shifts in across-worker productivity differences *and* in specialization/interdependence
- Example: **AI** – early/conjectural evidence on impact of LLMs etc
  - *absolute* adv. less important (“leveller”) [e.g., Brynjolfsson et al., 2023]
  - *comparative* adv. less important (e.g., can interpret medical imaging w/o radiologist)
- **Illustrative model counterfactual:** relative to the 2010s
  - ① everyone's productivity  $\uparrow$  by equivalent to 20% of lowest type's productivity – so in proportional terms, lower-x benefit more
  - ② coworker complementarity  $\downarrow$  by 20%

## Implications for AI impact on labor markets (2): illustrative exercise

- Model prediction: AI could lead to reversal of historical trends –  $\sigma_W^2 \downarrow$ ,  $\sigma_{\bar{W}}^2 \downarrow$ ,  $\rho_{XX} \downarrow$  – along side a productivity boom
  - driven by  $\downarrow$  firm-level wage inequality



## Implications for AI impact on labor markets (3): conjectures

[▶ Main](#)

- Literature moving toward richer way of thinking about tech. change. Here: shifts in across-worker productivity differences *and* in specialization/interdependence
- Example: AI – early/conjectural evidence on impact of LLMs etc
  - *absolute* adv. less important? (“leveller”) [e.g., Brynjolfsson et al., 2023]
  - *comparative* adv. less important? (e.g., can interpret medical imaging w/o radiologist)
- Illustrative model counterfactual: **AI could lead to reversal of historical trends...**
- **...and perhaps also...**
  - ① **...flatter organizations**, with managerial span of control ↑
  - ② **...↓ barriers to entry** for self-employment/start-ups
  - ③ **...easier job transitions** & shorter training durations

# Within-industry calibration: overview

[▶ Main](#)[▶ Data moments](#)

- Baseline: calibration and evaluation based on economy-wide moments
- But the model does not incorporate between-industry differences in, e.g., production technology
- Alternative considered here: target  $\hat{\beta}_c$  and  $\sigma_w^2$  computed as within-industry average, evaluate against within-industry trends
  - keep other targets (e.g., job separation) as before

# Within-industry calibration: model fit & counterfactual

- Counterfactual:  $\chi \uparrow$  explains 83% of model-implied  $\uparrow$  in between-share

