AI and Scale: A Quantitative Task-Based Theory of Automation

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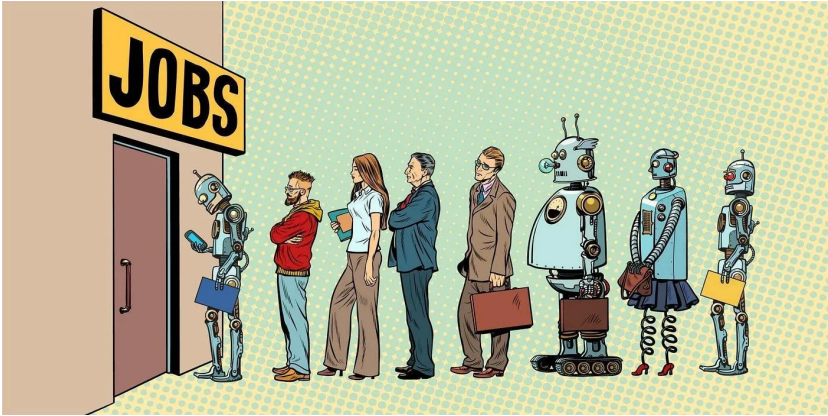
The Automation Potential of AI?



AI Automation and Scale?

Task-based Theories of Automation focus on relative marginal costs of production

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Recent progress in AI driven by scaling training (fixed) costs:

	Model Size	Training Data	Training Compute
AlexNet (2012)	60M pars.	1.2M images	$\sim 10^{18}$ FLOPs
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Economics of AI automation to account for task-level fine-tuning costs and relevance of scale?

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- CV adoption predicted to raise labor productivity by 1.5% over 10-50 years

Roadmap

Theory

Application to AI Scaling Laws

Quantitative Results

Conclusion

Firms and Production

Production Function:

- Firm needs to produce unit continuum varieties ν of task types $\tau \in \mathcal{T}$:

$$y = a \left(\sum_{\tau \in \mathcal{T}} \xi_\tau^{\frac{1}{\lambda}} \int_0^1 y_\tau(\nu)^{\frac{\lambda-1}{\lambda}} \, d\nu \right)^{\frac{\lambda}{\lambda-1}}$$

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- Machines: at price r with marginal productivity $z_{K,\tau}$ and fixed productivity $z_{K,\tau}^f$ across task-τ varieties

Cost Minimization Problem

Firm problem given scale y and factor prices (w, r):

- Choose variety output $\boldsymbol{y} \equiv \left([y_\tau(\nu)]_0^1\right)_{\tau \in \mathcal{T}}$ and input choice $\boldsymbol{x} \equiv \left([x_\tau(\nu)]_0^1\right)_{\tau \in \mathcal{T}}$ with

$$y = a \left(\sum_{\tau \in \mathcal{T}} \xi_\tau^{\frac{1}{\lambda}} \int_0^1 y_\tau(\nu)^{\frac{\lambda-1}{\lambda}} \, d\nu \right)^{\frac{\lambda}{\lambda-1}}$$

such that

$$C(w, r; y) = \min_{\boldsymbol{y}, \boldsymbol{x}} \sum_{\tau \in \mathcal{T}} \int_0^1 \left(c_\tau(\nu) \, y_\tau(\nu) + c_\tau^f(\nu) \right) d\nu$$

and that marginal and fixed costs of production $c_\tau(\nu)$ and $c_\tau^f(\nu)$ satisfy

$$c_\tau(\nu) = \begin{cases} \frac{w}{z_\tau(\nu)}, & \text{if } x_\tau(\nu) = \ell_\tau(\nu) \\ \frac{r}{z_{K,\tau}}, & \text{if } x_\tau(\nu) = k_\tau(\nu) \end{cases} \qquad c_\tau^f(\nu) = \begin{cases} 0, & \text{if } x_\tau(\nu) = \ell_\tau(\nu) \\ \frac{r}{z_{K,\tau}^f}, & \text{if } x_\tau(\nu) = k_\tau(\nu) \end{cases}$$

Automation Decisions: Variety Level

Task type τ: given firm-level marginal costs c variety-level relative labor productivity $z_\tau(\nu)/Z_\tau$

$$x_\tau(\nu) = \begin{cases} \ell_\tau(\nu), & \text{if } \widehat{z}_\tau \leq \frac{z_\tau(\nu)}{Z_\tau} \\ k_\tau(\nu), & \text{if } \frac{z_\tau(\nu)}{Z_\tau} \leq \widehat{z}_\tau \end{cases}$$

where machine comparative advantage $\widehat{z}_\tau$ defined by

$$\widehat{z}_\tau = \left[\left(\frac{z_{K,\tau}/Z_\tau}{r/w} \right)^{\lambda-1} + SA_\tau(w, r; y)^{\lambda-1} \right]^{\frac{1}{\lambda-1}}$$

and where scale advantage $SA_{j\tau}$ depends on the ratio of (variable) labor costs to fixed costs

$$SA_{j\tau}(w, r; y) \equiv \left(\frac{1}{1-\lambda} \frac{z^f_{K,\tau}/Z_\tau \times y_{L,\tau}}{r/w} \right)^{\frac{1}{1-\lambda}} \qquad y_{L,\tau} \equiv \xi_\tau \left(\frac{w/Z_\tau}{c} \right)^{-\lambda} \frac{y}{a}$$

Automation Decisions: Comparative and Scale Advantages

Machines with No Fixed Costs: automation determined by relative marginal costs

$$\lim_{z^f_{K,\tau} \to \infty} \widehat{z}_\tau = \frac{z_{K,\tau}/Z_\tau}{r/w}$$

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Automation share: task-τ's share of automated varieties for firm j

$$\varphi_{j\tau} \equiv G\left(\widehat{z}_\tau\right)$$

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Automation Decisions: Task Level

Task-τ marginal cost for the firm: given by c_τ / a where

$$c_\tau^{1-\lambda} = \overbrace{\underbrace{\left(\frac{w}{Z_\tau}\right)^{1-\lambda}}_{\text{intensive}} \underbrace{G_{\lambda-1}\left(\widehat{z}_\tau\right)}_{\text{extensive}}}^{\text{labor}} + \overbrace{\underbrace{\left(\frac{r}{z_{K,\tau}}\right)^{1-\lambda}}_{\text{intensive}} \underbrace{G\left(\widehat{z}_\tau\right)}_{\varphi_\tau \equiv \text{extensive}}}^{\text{machines}}$$

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Firm-j machine and labor shares in variable costs + fixed/variable ratio:

$$\omega_{K,\tau} \equiv \frac{r\, k_{j\tau}}{c_\tau y_\tau} = \left(\frac{r / z_{K,\tau}}{c_\tau}\right)^{1-\lambda} G\left(\widehat{z}_\tau\right)$$

$$\omega_\tau^f \equiv \frac{c_\tau^f}{c_\tau y_\tau} = \frac{r / z_{K,\tau}^f}{c_\tau y_\tau}\, G\left(\widehat{z}_\tau\right)$$

Graphical Depiction

Automation Decisions: Firm Level

Firm-level costs: characterized by fixed point problem

$$C(w, r; y) = c\frac{y}{a} + c^f \qquad c^{1-\lambda} = \sum_{\tau} \xi_\tau c_\tau^{1-\lambda} \qquad c^f \equiv \sum_{\tau} \frac{r}{z_{K,\tau}^f} G(\widehat{z}_\tau)$$

where task-level MC c_τ and AI comparative advantage $\widehat{z}_\tau$ compatible with firm-level MC c

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Share of machine in firm-level variable costs and fixed-to-variable cost ratios

$$\omega_K \equiv \frac{r\,k}{c\,(y/a)} = \mathbb{E}_\tau\left[\omega_{K,\tau}\right] \qquad \omega^f \equiv \frac{c^f}{c\,(y/a)} = \mathbb{E}_\tau^f\left[\omega_\tau^f\right]$$

where $\mathbb{E}_\tau[\cdot]$ and $\mathbb{E}_\tau^f[\cdot]$ expectation w.r.t task-τ share in variable ψ_τ and fixed $\psi_\tau \frac{\omega_\tau^f}{\omega^f}$ costs

$$\psi_\tau \equiv \frac{c_\tau y_\tau}{c\,(y/a)} \equiv \xi_\tau \left(\frac{c_\tau}{c}\right)^{1-\lambda}$$

Special Case I: No Fixed Costs

Elasticities of cost:

$$\varepsilon \equiv \frac{\partial \log C}{\partial \log y} = 1 \qquad \Rightarrow \qquad CRS$$

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Firm and Task-level Substitution Elasticities:

$$\sigma = \mathbb{E}_\tau^L \left[\sigma_\tau \right] \qquad \sigma_\tau \equiv \frac{1}{\Omega_K} \frac{\partial \log \ell_\tau}{\partial \log r}\bigg|_{y,w} = \lambda + \underbrace{\frac{1}{\omega_K} \widetilde{G}_{\lambda-1}(\widehat{z}_\tau)}_{\text{automation}}$$

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Tasks with higher comparative advantage are more substitutable:

$$\widehat{z}_{\tau'} < \widehat{z}_\tau \quad \Leftrightarrow \quad \sigma_{\tau'} < \sigma_\tau$$

Special Case I: Illustration of Case Without Fixed Costs

Toy Example: consider $\lambda = 0.5$, $\theta = 1.3$, and $|\mathcal{T}| = 5$ with $\xi_\tau = 0.01$ for $\tau \leq 4$.

$$Z_\tau = 1, \qquad \boldsymbol{z}_K = \left(1,\ e^{-10},\ e^{-20},\ e^{-30},\ 0\right), \qquad \boldsymbol{z}_K^f \equiv (\infty,\ \infty,\ \infty,\ \infty,\ 0)$$

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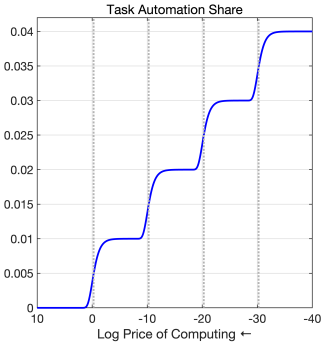

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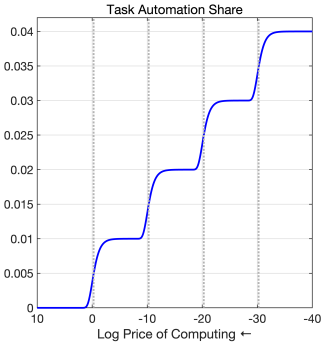

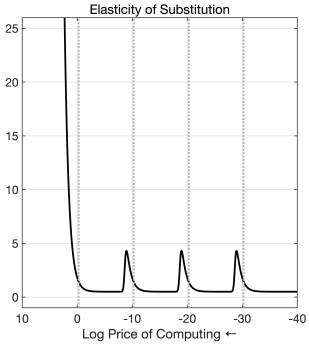

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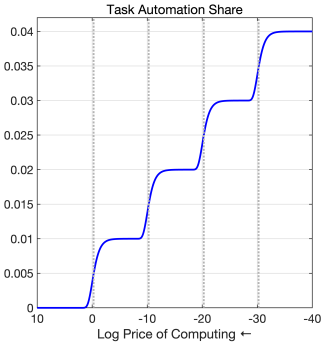

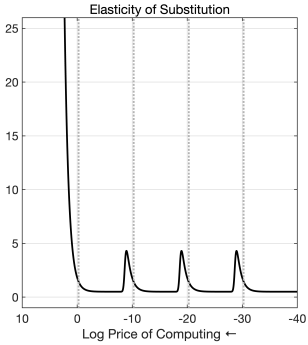

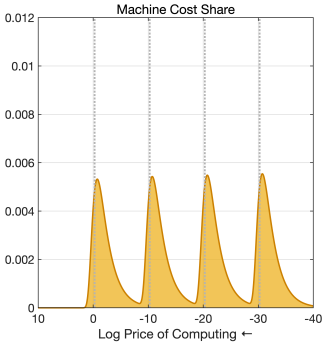

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$$\Omega_K^m \equiv \frac{\partial \log c}{\partial \log r} = \frac{\Upsilon}{\lambda} \qquad \zeta_\tau \equiv 1$$

Demand Elasticities (Firm and Task-level):

$$\eta_K = \frac{\Upsilon}{\lambda\,\omega^f}$$

$$\sigma = \mathbb{E}_\tau^L\left[\sigma_\tau\right] \qquad \sigma_\tau \equiv \frac{1}{\Omega_K}\frac{\partial \log \ell_\tau}{\partial \log r}\bigg|_{y,w} = \underbrace{\Upsilon\,\frac{1}{\Omega_K} + (1-\Upsilon)\,\frac{1}{1-\lambda}\,\frac{1}{\Omega_K}\,\widetilde{G}_{\lambda-1}(\widehat{z}_\tau)}_{\text{automation}}$$

Special Case II: No Marginal Costs

Elasticities of cost:

$$\varepsilon \equiv \frac{\partial \log C}{\partial \log y} = \frac{1}{1+\omega^f} < 1 \quad \Rightarrow \quad IRS$$

$$\Omega_K \equiv \frac{\partial \log C}{\partial \log r} = \frac{\omega^f}{1+\omega^f}$$

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Tasks with higher comparative advantage are more substitutable:

$$\widehat{z}_{\tau'} < \widehat{z}_\tau \qquad \Leftrightarrow \qquad \sigma_{\tau'} < \sigma_\tau$$

Special Case II: Illustration of Case Without Marginal Costs

Toy Example: consider $\lambda = 0.5$, $\theta = 1.3$, and $|\mathcal{T}| = 5$ with $\xi_\tau = 0.01$ for $\tau \leq 4$.

$$Z_\tau = 1, \qquad \boldsymbol{z}_K = (\infty,\ \infty,\ \infty,\ \infty,\ 0)\,, \qquad \boldsymbol{z}_K^f \equiv \left(1,\ e^{-10},\ e^{-20},\ e^{-30},\ 0\right)$$

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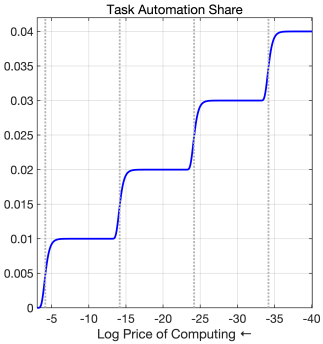

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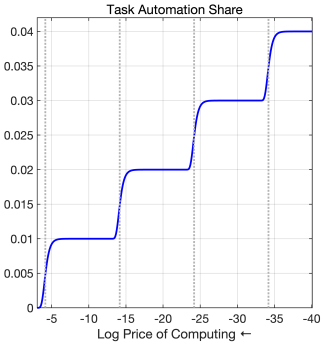

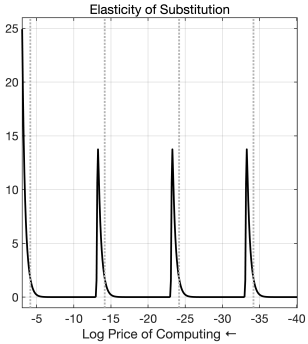

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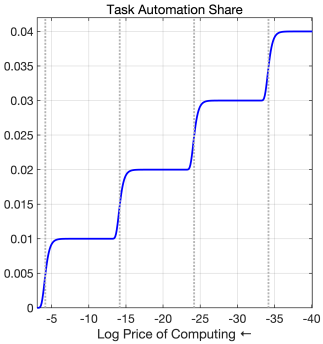

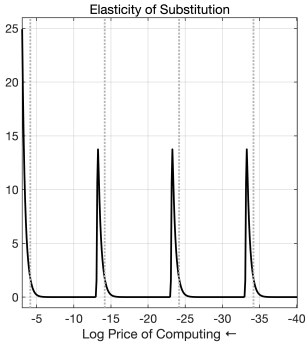

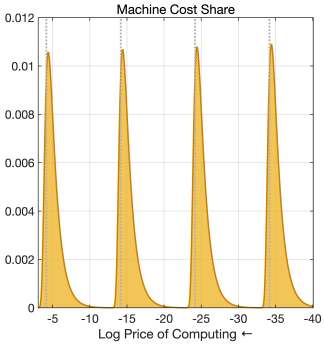

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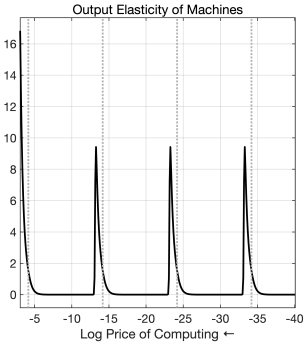

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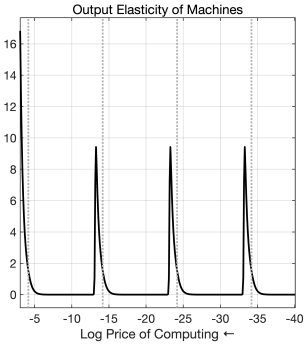

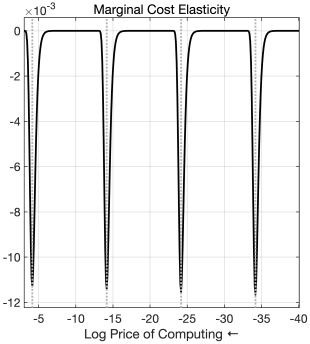

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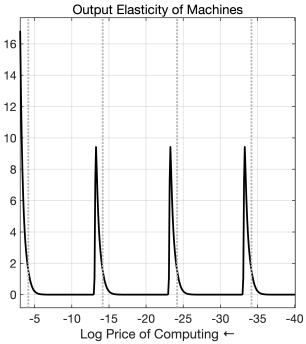

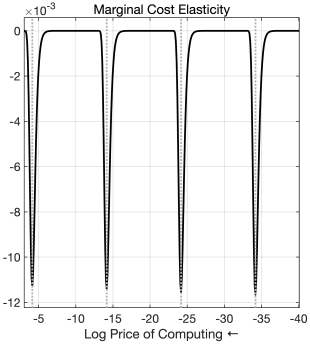

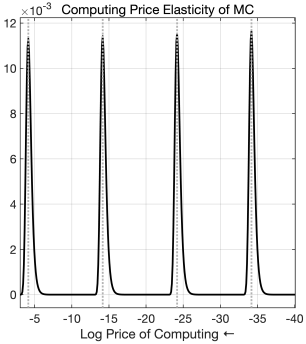

General Equilibrium Environment

Production: I industries each w. many monopol. comp. single-product firms $j \in \mathcal{I}_i$ with a_j

$$Y^{\frac{\rho-1}{\rho}} = \sum_{1 \leq i \leq I} Y_i^{\frac{\rho-1}{\rho}} \qquad Y_i^{\frac{\eta-1}{\eta}} = \sum_{\mathcal{J}_i} y_j^{\frac{\eta-1}{\eta}}$$

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- Elasticities of aggregate (total/marginal) cost and demand:
  Lashkari et al. (2024)

Aggregation

$$\overline{\varepsilon} \equiv \frac{\partial \log \overline{C}}{\partial \log r} \qquad \overline{\varepsilon}^m \equiv \frac{\partial \log \overline{c}}{\partial \log r} \qquad \overline{\eta}_K \equiv \frac{\partial \log K}{\partial \log Y}$$

$$\overline{\Omega}_K \equiv \frac{\partial \log \overline{C}}{\partial \log r} \qquad \overline{\Omega}_K^m \equiv \frac{\partial \log \overline{c}}{\partial \log r} \qquad \overline{\sigma} \equiv \frac{1}{\overline{\Omega}_K} \frac{\partial \log L}{\partial \log r}$$

GE Response to Shock In Machine Prices

Consider decline $\Delta \log r$ in price of machines

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$$\overline{L} = L\left(w, r; Y\right) \qquad \Rightarrow \qquad 0 \approx \overline{\Omega}_K \, \overline{\sigma} \, \left(\Delta \log r - \Delta \log w\right) + \overline{\eta}_L \, \Delta \log Y$$

$$1 = \mu \, \overline{c}\left(w, r; Y\right) \qquad \Rightarrow \qquad 0 \approx \overline{\Omega}_K^m \, \Delta \log r + \overline{\Omega}_L^m \, \Delta \log w + \varepsilon^m \, \Delta \log Y$$

$\overline{\Omega}_L^m$ and $\overline{\eta}_L$

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Response of wage, output, and labor share:

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$$\Delta \log w \approx -\frac{\overline{\eta}_L \overline{\Omega}_K^m - \varepsilon^m\, \overline{\Omega}_K \overline{\sigma}}{\overline{\eta}_L \overline{\Omega}_L^m + \varepsilon^m\, \overline{\Omega}_K \overline{\sigma}}\, \Delta \log r$$

$$\Delta \log Y \approx -\frac{\left(\overline{\Omega}_K^m + \overline{\Omega}_L^m\right)\overline{\Omega}_K}{\overline{\eta}_L \overline{\Omega}_L^m + \varepsilon^m\, \overline{\Omega}_K \overline{\sigma}}\, \overline{\sigma}\, \Delta \log r$$

$$\Delta \log \overline{\Omega}_L \approx \frac{\left(\overline{\Omega}_K^m + \overline{\Omega}_L^m\right)\overline{\Omega}_K}{\overline{\eta}_L \overline{\Omega}_L^m + \varepsilon^m\, \overline{\Omega}_K \overline{\sigma}} \left(\frac{\overline{\varepsilon}}{\overline{\eta}_L}\, \overline{\sigma} - 1\right) \Delta \log r$$

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$$\Delta \log Y \approx -\frac{(\overline{\Omega}_K^m + \overline{\Omega}_L^m)\overline{\Omega}_K}{\overline{\eta}_L \overline{\Omega}_L^m + \varepsilon^m \overline{\Omega}_K \overline{\sigma}} \, \overline{\sigma} \, \Delta \log r = -\overline{\sigma} \, \frac{\overline{\Omega}_K}{\overline{\Omega}_L} \, \Delta \log r$$

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Wage Output Labor Share

Roadmap

Theory

Application to AI Scaling Laws

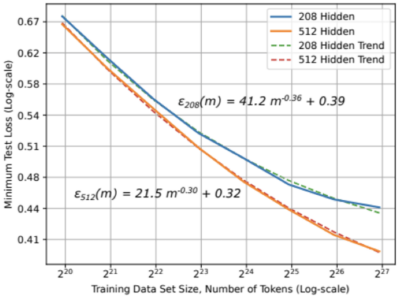
Quantitative Results

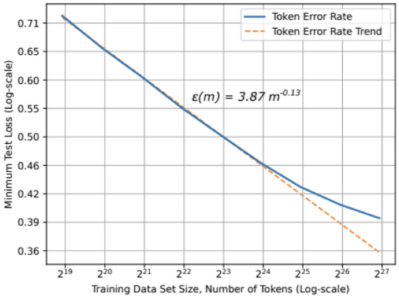
Conclusion

AI Scaling Laws

- Empirical relationship bet. AI performance and computational inputs (model size & data)

  Hestness et al. (2017); Rosenfeld (2020); Kaplan et al. (2020); Zhai et al. (2022)





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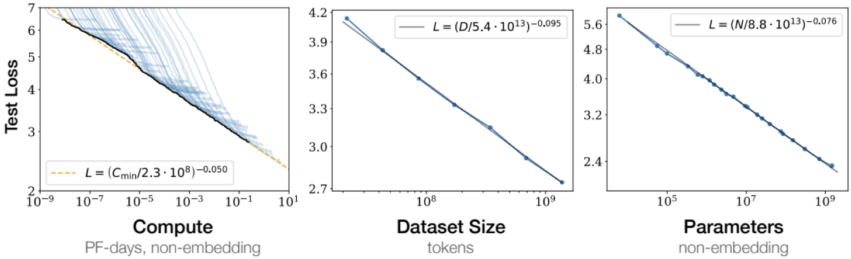

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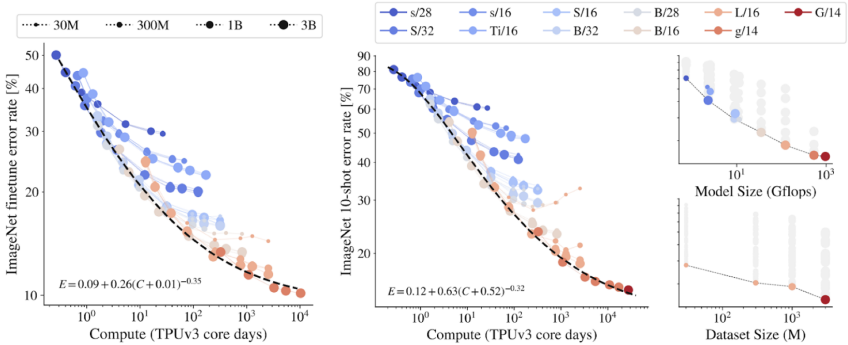

Scaling Laws Determine Comparative Advantage of AI

Fine-tuning: process of tailoring a pre-trained AI system to specific applications

- Requires inputs model $m_{i\tau}$ and data $d_{i\tau}$ size to lower cross-entropy loss $h_{i\tau}$:

  Li, et al. (2024)

$$h_{i\tau} = \mathcal{H}\left(m_{i\tau}, d_{i\tau}; \chi_{i\tau}\right) = \left[\psi_m\left(\chi_{i\tau}\right)\ m_{i\tau}\right]^{-\alpha_m(\chi_{i\tau})} + \left[\psi_d\left(\chi_{i\tau}\right)\ d_{i\tau}\right]^{-\alpha_d(\chi_{i\tau})} + \underline{h}$$

- Task complexity $\chi_{i\tau}$: number of outcomes

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Training Cost Minimization: firms choose $d_{i\tau}$, $m_{i\tau}$ to minimize training costs:

$$\frac{1}{z^f_{K\tau}} = \min_{m_{i\tau}, d_{i\tau}} 6 \cdot B \cdot m_{i\tau} d_{i\tau} \quad s.t. \quad h\left(\epsilon_{i\tau}; \chi_{i\tau}\right) = \mathcal{H}\left(m_{i\tau}, d_{i\tau}; \chi_{i\tau}\right)$$

Inference (AI marginal) costs scale with model size $m_{i\tau}$ implying $z_{K,\tau} = 6\, d_{i\tau}\, z^f_{K,\tau}$

Details

Estimating the Fine-Tuning Scaling Law

Computer Vision Fine-Tuning Experiment: estimating scaling-law parameters $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\underline{h}$

- Fine-tune SwinTransformer model varying task complexity (class number), model size, data size Details

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Parametrization:

$$\log \alpha_x(\chi_{i\tau}) \equiv \beta^{\alpha}_{x,0} + \beta^{\alpha}_{x,1} \log \chi_{i\tau} \qquad \log \psi_x(\chi_{i\tau}) \equiv \beta^{\psi}_{x,0} + \beta^{\psi}_{x,1} \log \chi_{i\tau} \qquad x \in \{m, d\}$$

Parameter	$\beta^{\psi}_{m,0}$	$\beta^{\psi}_{m,1}$	$\beta^{\alpha}_{m,0}$	$\beta^{\alpha}_{m,1}$	$\beta^{\psi}_{d,0}$	$\beta^{\psi}_{d,1}$	$\beta^{\alpha}_{d,0}$	$\beta^{\alpha}_{d,1}$	$\log \underline{h}$	δ
Value	4.29	-3.88	-1.92	-0.09	-2.50	-1.10	-0.42	-0.08	-27.06	5.11
90% CI	[3.25, 5.40]	[-4.15, -3.64]	[-1.97, -1.87]	[-0.11, -0.07]	[-3.03, -2.06]	[-1.17, -1.01]	[-0.53, -0.29]	[-0.10, -0.05]	[-38.17, -24.91]	[4.68, 5.50]

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Cross-entropy vs. error rates: estimated non-parametrically within the experiment Details

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Data

Task Complexity Survey:

Sum. Stat. | Task estimates | Validation | Sample

- Used to estimate task-level measures of error tolerance ϵ and complexity χ
- Implemented in two waves, between July and November 2023 covering 271 occupations, 445 tasks

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Sum. Stat. | Task estimates | Validation | Sample

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- Implemented in two waves, between July and November 2023 covering 271 occupations, 445 tasks

LLM-Sourced Data:

- Used to estimate comparable measures of task frequency ξ and labor productivity Z_τ

Details

Data

Task Complexity Survey:

Sum. Stat. | Task estimates | Validation | Sample

- Used to estimate task-level measures of error tolerance ϵ and complexity χ
- Implemented in two waves, between July and November 2023 covering 271 occupations, 445 tasks

LLM-Sourced Data:

- Used to estimate comparable measures of task frequency ξ and labor productivity Z_τ Details

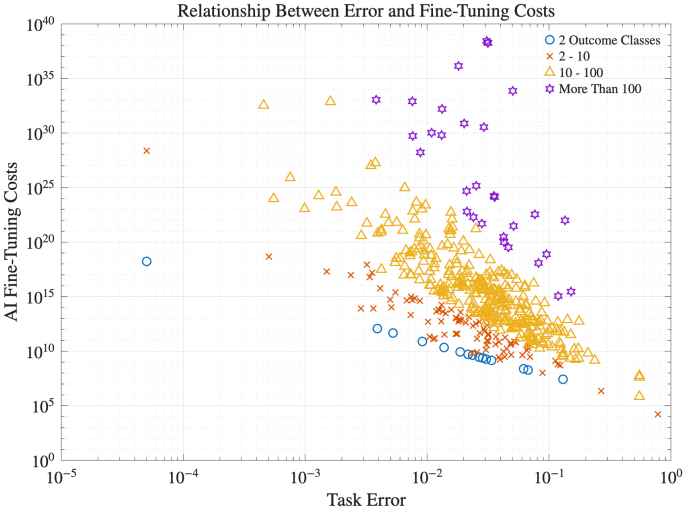
BDS (2023):

- Used to calibrate distribution of firm productivity a_j based on firm size distribution Details | Parameters

Calibrated Parameters

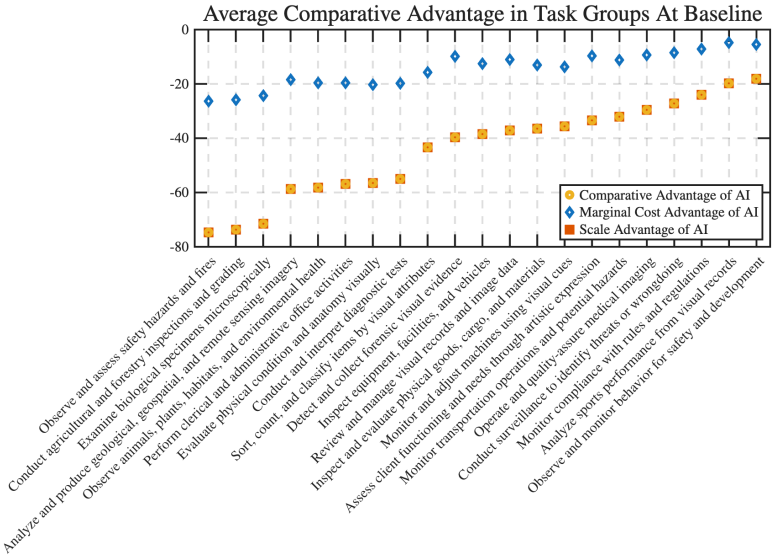
Parameter		Value	Source	Moment	Literature	Model
From Literature						
Task EoS	λ	0.5	Humlum (2019)			
Firm EoS	η	4.0				
Industry EoS	ρ	0.8				
Labor productivity dispersion	θ	1.3	Burstein, et al. (2019)			
Decline in computing prices	$\Delta \log r$	-0.31/-1.35	Del Sozzo, et al. (2025); Ho, et al. (2024)			
Calibration						
Firm productivity	ϑ	1.9	BDS			
Capital price	$\log r$	-13.2	Bonney, et al. (2024)	Fall 2023 ComputerVision adoption rate (unweighted)	0.5%	0.7%
Fixed cost shifter	$\log B$	27.5	Bonney, et al. (2024)	Fall 2023 ComputerVision adoption rate (weighted)	2.4%	2.2%

From Task Error/Complexity to AI Costs



Automation Rate | Task Complexity | Task Error

Comparative and Scale Advantages of AI: 2023

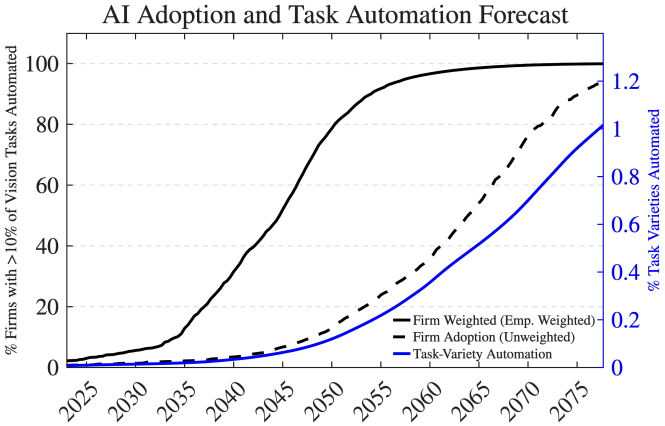


Results for 2050

Predicting Future of Computer Vision Automation

- Trends in computing price r: annual decline of 26.4% in $ cost of FLOP

  Del Sozzo, et al. (2025); Ho, et al. (2024)



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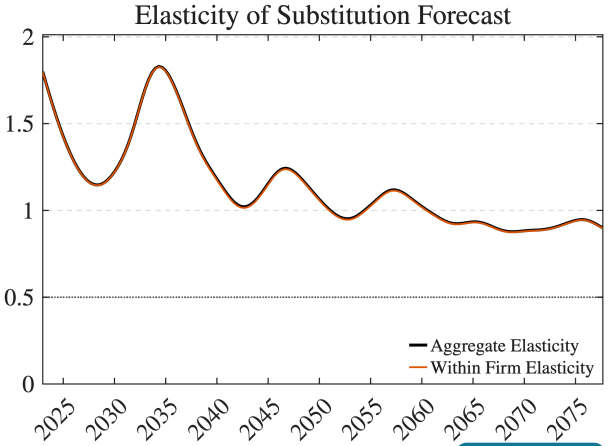

Alternative Calibration | Decline in Algorithmic + Computing Price

Predicting Future of Computer Vision Automation

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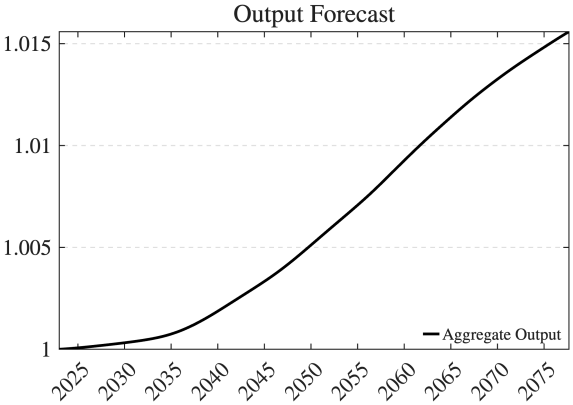

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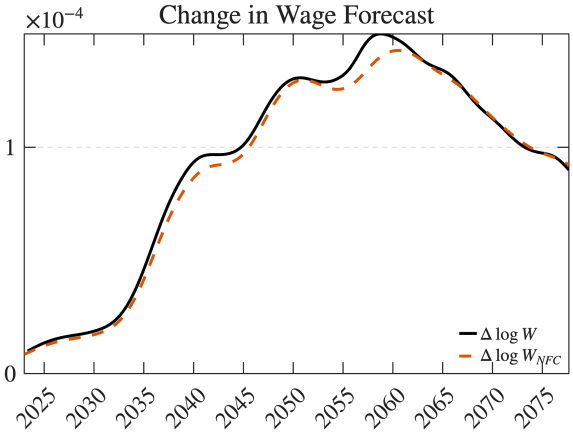

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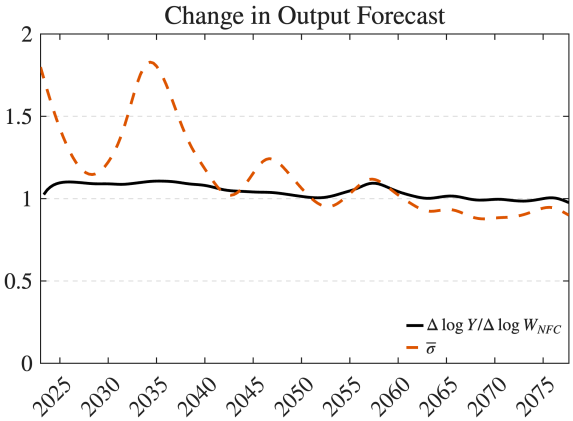

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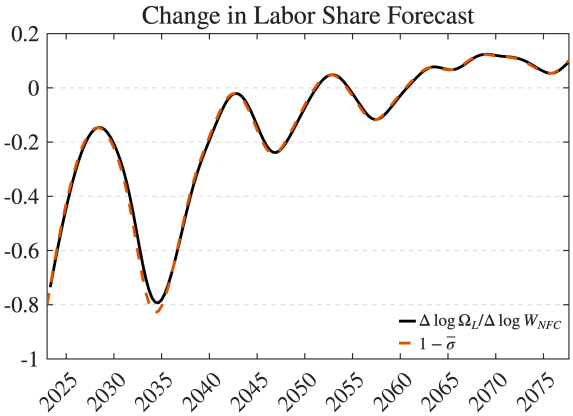
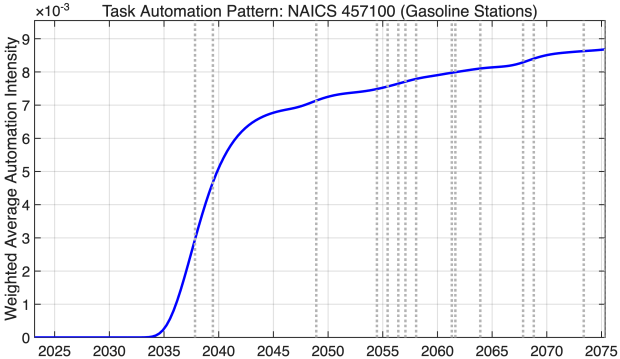


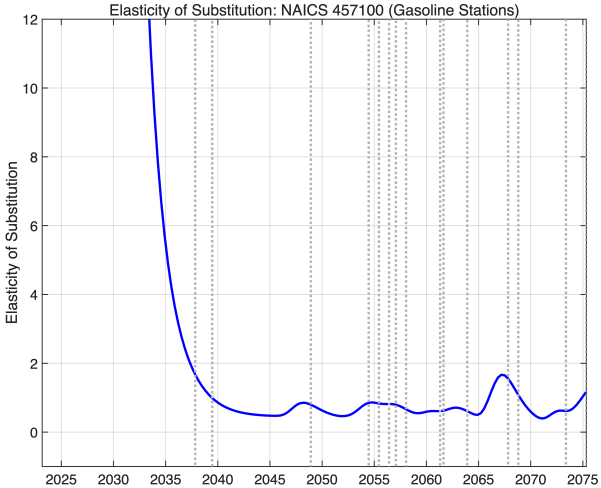
Revisiting Firm-Level Outcomes



Automated Tasks:

1: Examine merchandise to ensure that it is correctly priced and displayed and that it functions as advertised.
2: Watch for and recognize security risks and thefts and know how to prevent or handle these situations.
3: Inspect supplies, equipment, and work areas to ensure efficient service and conformance to standards.
4: Check conditions of trailers after contents have been unloaded to ensure that there has been no damage.
5: Inspect job sites to determine the extent of maintenance or repairs needed.
6: Sort materials, such as metals, glass, wood, paper or plastics, into appropriate containers for recycling.
7: Observe color of products being baked, and adjust oven temperatures, humidity, or conveyor speeds accordingly.
8: Review work throughout the work process and at completion to ensure that it has been performed properly.
9: Inspect and adjust rental items to meet needs of customer.
10: Monitor and authorize entrance and departure of employees, visitors, and other persons to guard against theft and maintain security of premises.
11: Inspect and clean food preparation areas, such as equipment, work surfaces, and serving areas, to ensure safe and sanitary food-handling practices.
12: Open, sort, and route incoming mail, answer correspondence, and prepare outgoing mail.
13: Examine shipment contents and compare with records, such as manifests, invoices, or orders, to verify accuracy.
14: Inspect loads to ensure that cargo is secure.
15: Inspect equipment for wear and for conformance to specifications.

Revisiting Firm-Level Outcomes



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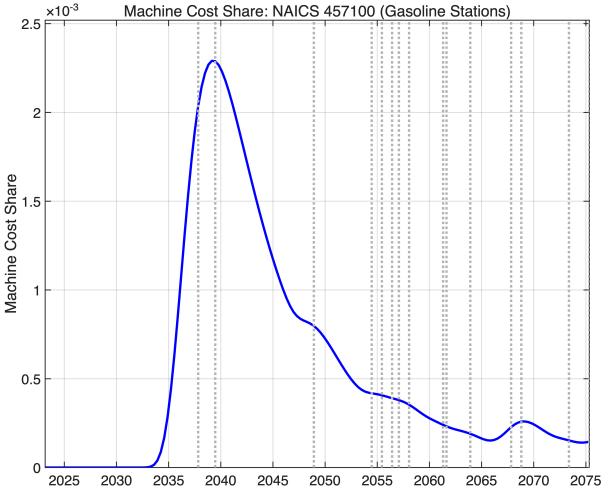
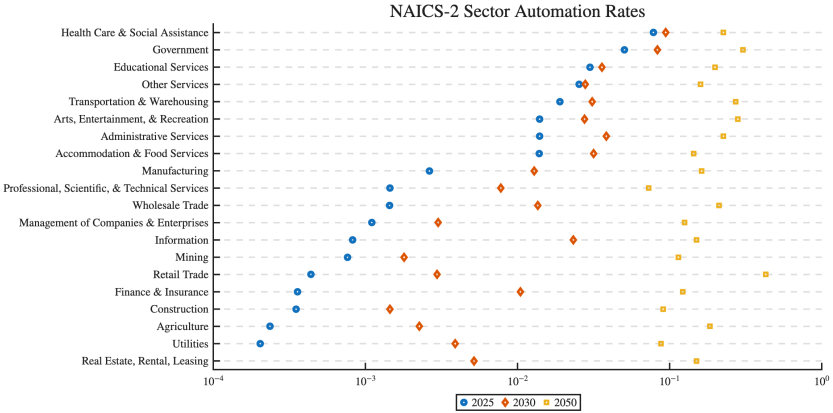


Industry AI Adoption Rates



Roadmap

Theory

Application to AI Scaling Laws

Quantitative Results

Conclusion

Conclusion

Theory: multi-industry quantitative task-based model of automation

- Potential task-level increasing returns in automation decisions
- Automation determined by patterns of AI marginal cost and scale advantage

Application to Computer Vision AI Automation:

- AI Scaling Law + Task Complexity Survey $\Rightarrow$ AI cost structure
- LLM Data $\Rightarrow$ worker comparative advantage

Results:

- S-curve of adoption of CV over time with declining computing prices
- Scale advantage explains lion's share of cross-task patterns of comparative advantage

Roadmap

Appendix

Appendix

Properties of Production Function

Elasticities of cost function:

$$\varepsilon \equiv \frac{\partial \log C}{\partial \log y} = \frac{c\,(y/a)}{C}$$

$$\Omega_K \equiv \frac{\partial \log C}{\partial \log r} = \frac{r\,k}{C}$$

Properties of Production Function

Elasticities of cost function:

$$\varepsilon \equiv \frac{\partial \log C}{\partial \log y} = \frac{c\,(y/a)}{C} = \frac{1}{1+\omega^f} \leq 1 \qquad \Rightarrow \qquad (C/I)RS$$

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$$\eta_K \equiv \frac{\partial \log k}{\partial \log y}$$

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Elasticities of marginal cost: $\widetilde{G}\,(x) \equiv \frac{x\,G(x)}{G(x)}$

$$\varepsilon^m \equiv \frac{\partial \log c}{\partial \log y} = -\frac{1}{\lambda}\Upsilon \qquad \Upsilon \equiv \frac{\frac{\lambda}{1-\lambda}\,\omega^f\,\mathbb{E}^f_\tau\left[\zeta_\tau\,\widetilde{G}(\widehat{z}_\tau)\right]}{1+\frac{\lambda}{1-\lambda}\,\omega^f\,\mathbb{E}^f_\tau\left[\zeta_\tau\,\widetilde{G}(\widehat{z}_\tau)\right]} \qquad \zeta_\tau \equiv \left(\frac{SA_\tau}{\widehat{z}_\tau}\right)^{\lambda-1}$$

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Elasticities of demand: $\widetilde{G}_{\lambda-1}(x) \equiv -\frac{x\,G_{\lambda-1}(x)}{G_{\lambda-1}(x)}, \quad \mathbb{E}^L_\tau[\cdot] \equiv \sum_\tau \psi_\tau \frac{\omega_{L,\tau}}{\omega_L}.$

$$\eta_K \equiv \frac{\partial \log k}{\partial \log y} = \frac{1}{\varepsilon}\,\frac{\Omega^m_K}{\Omega_K}$$

$$\sigma \equiv \frac{1}{\Omega_K}\,\frac{\partial \log \ell}{\partial \log r} = \lambda\,\frac{\Omega^m_K}{\Omega_K} + \frac{1}{\Omega_K}\mathbb{E}^L_\tau[\widetilde{G}_{\lambda-1}(\widehat{z}_\tau)] + \frac{1-\Omega^m_K}{\Omega_K}\,\frac{\lambda}{1-\lambda}\,\mathbb{E}^L_\tau[\zeta_\tau\,\widetilde{G}_{\lambda-1}(\widehat{z}_\tau)]$$

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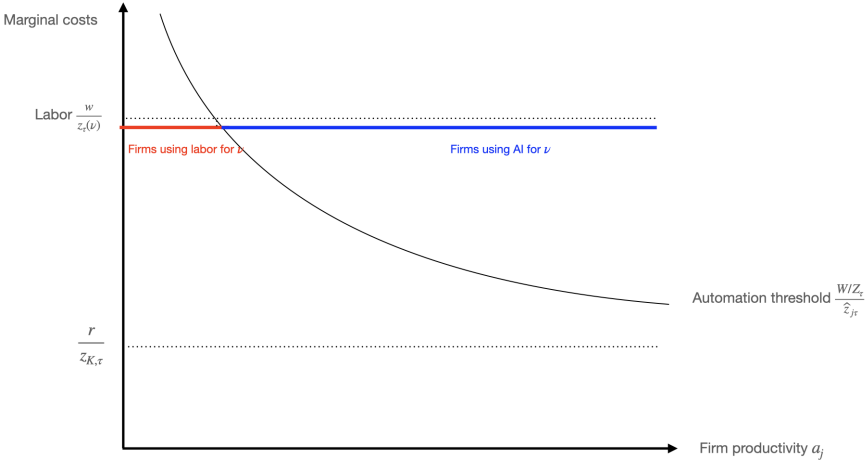
Automation



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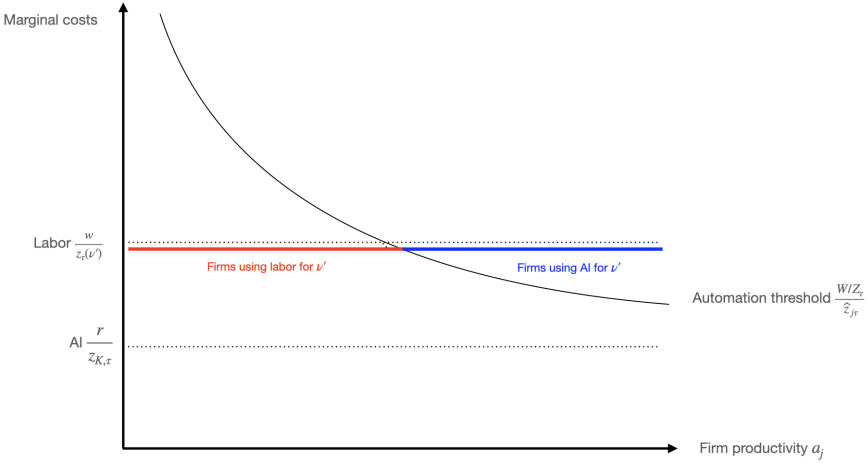
Automation



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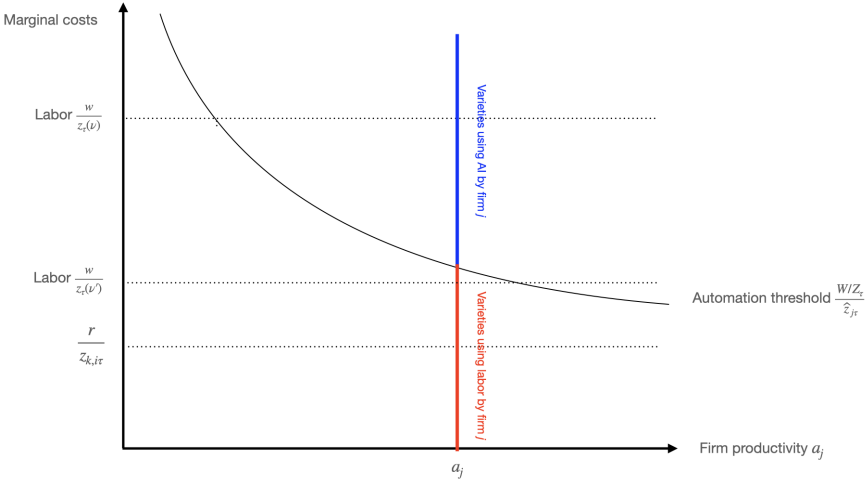
Automation



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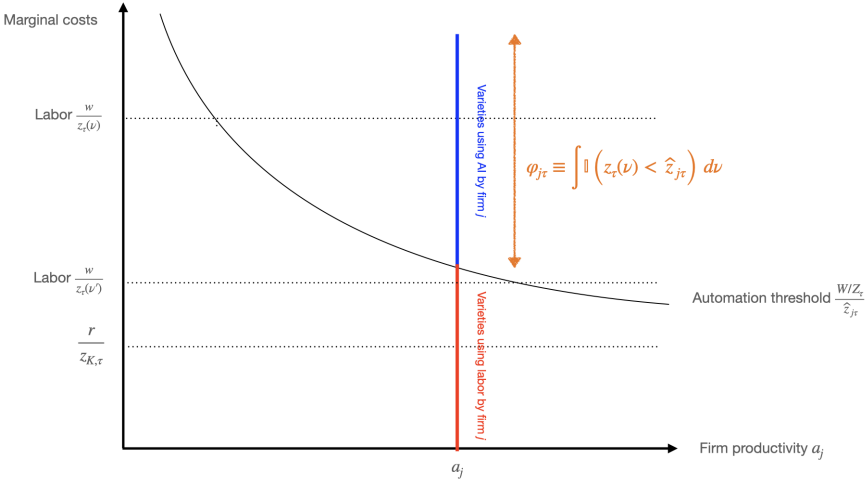
Automation



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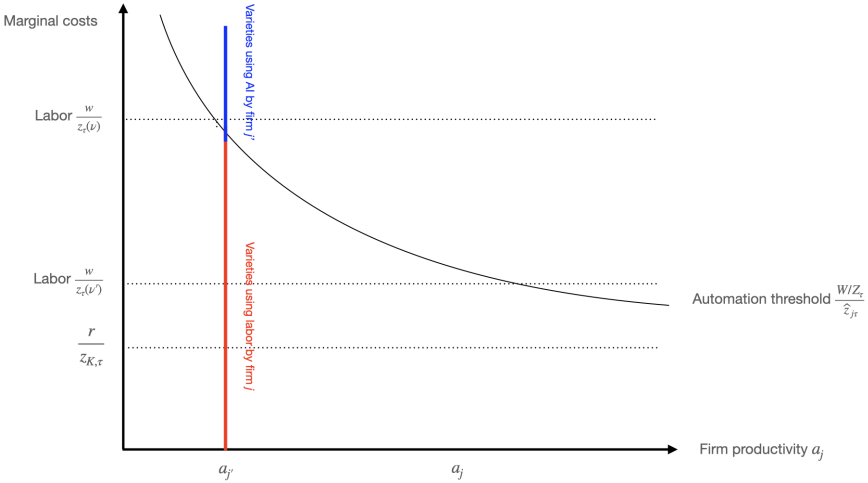

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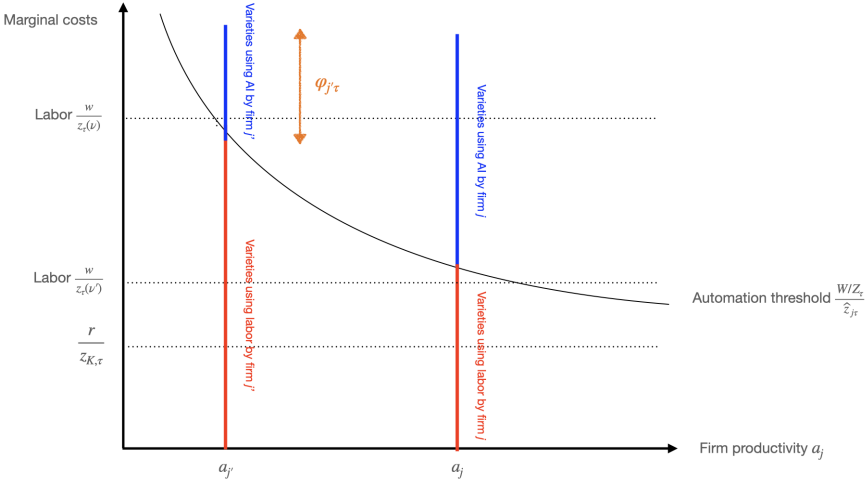

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Aggregation

Firm-level price and output:

$$p_j = \frac{\eta}{\eta-1}\,\frac{c_j}{a_j} \qquad\qquad y_j = J_i^{\frac{\rho-\eta}{1-\eta}}\, I^{\frac{\rho}{1-\rho}} \left(\frac{c_j}{a_j}\right)^{-\eta} \overline{c}_i^{\eta-\rho}\,\overline{c}^{\rho}\,Y$$

Industry and aggregate average costs: we have $1 = \frac{\eta}{\eta-1}\overline{c}$

$$\overline{c}_i^{1-\eta} = \frac{1}{J_i}\sum_{j\in\mathcal{J}_i}\left(\frac{c_j}{a_j}\right)^{1-\eta} \qquad\qquad \overline{c}^{1-\rho} = \sum_i J_i^{\frac{1-\rho}{1-\eta}}\,\overline{c}_i^{1-\rho}$$

Aggregation

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Firm and industry shares in variable costs:

$$\Psi_{j,i} \equiv \frac{c_j y_j/a_j}{J_i\overline{c}_i Y_i} = \frac{1}{J_i}\left(\frac{c_j/a_j}{\overline{c}_i}\right)^{1-\eta} \qquad\qquad \Psi_i \equiv J_i^{\frac{1}{1-\eta}}\frac{\overline{c}_i Y_i}{\overline{c}\,Y} = J_i^{\frac{1-\rho}{1-\eta}}\left(\frac{\overline{c}_i}{\overline{c}}\right)^{1-\rho}$$

Aggregation

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Shares of skill-group and machines in aggregate variable costs:

$$\overline{\omega}_L \equiv \frac{w\,L}{\overline{c}\,Y} = \sum_{i,j}\Psi_i\Psi_{j,i}\,\omega_{L,j} \qquad \overline{\omega}_K \equiv \frac{r\overline{K}-\overline{C}^f}{\overline{c}\,Y} = \sum_{i,j}\Psi_i\Psi_{j,i}\,\omega_{K,j}$$

Firm-Level Scale Elasticities

Firm-level elasticities of cost and marginal cost:

$$\varepsilon_j \equiv \frac{\partial \log C_j}{\partial \log y_j} = \frac{c_j y_j}{C_j a_j} = \frac{1}{1+\omega_{f,j}} \qquad \varepsilon_j^m \equiv \frac{\partial \log c_j}{\partial \log y_j} = \frac{\omega_j^f \, \mathbb{E}_{\tau|j}^f \left[\theta \log \varphi_{j\tau} \, \zeta_{j\tau}\right]}{1-\lambda-\lambda \, \omega_j^f \, \mathbb{E}_{\tau|j}^f \left[\theta \log \varphi_{j\tau} \, \zeta_{j\tau}\right]}$$

with $\mathbb{E}_{\tau|j}^f [\cdot]$ task average w.r.t to dist. of fixed costs $\psi_{\tau,j} \, \omega_{j\tau}^f / \omega_j^f$ and $\zeta_{j\tau}$ importace of fixed costs:

$$\zeta_{j\tau} \equiv \frac{(1-\lambda) \, z_{K,\tau} / z_{K,\tau}^f y_{K,j\tau}}{1+(1-\lambda) \, z_{K,\tau} / z_{K,\tau}^f y_{K,j\tau}}$$

Firm-level scale elasticities of demand:

$$\varepsilon_{K,j} \equiv \frac{\partial \log k_j}{\partial \log y_j} = \frac{\omega_{K,j} - \lambda \varepsilon_j^m \, \omega_{L,j}}{\omega_{K,j} + \omega_j^f} - \theta \left(1 + \lambda \varepsilon_j^m\right) \frac{\omega_j^f}{\omega_{K,j} + \omega_j^f} \mathbb{E}_{\tau|j}^f \left[\log \varphi_{j\tau}\right]$$

$$\varepsilon_{L,j} \equiv \frac{\partial \log \ell_{s,j}}{\partial \log y_j} = \left(1 + \lambda \varepsilon_j^m\right) \left(1 + \theta \, \frac{\omega_j^f}{\omega_{L,j}} \, \mathbb{E}_{\tau|j}^f \left[\log \varphi_{j\tau}\right]\right)$$

back

Aggregate-Level Scale Elasticities

Industry- and aggregate-level scale elasticities of cost:

$$\overline{\varepsilon}_i^m \equiv \frac{1}{\eta}\left(\mathbb{E}_{j|i}\left[\left(1+\eta\,\varepsilon_j^m\right)^{-1}\right]-1\right) \qquad \overline{\varepsilon}^m \equiv \frac{1}{\rho}\left(\mathbb{E}_i\left[\left(1+\rho\,\overline{\varepsilon}_i^m\right)^{-1}\right]-1\right)$$

Pass-through distribution:

$$\Psi_{j,i}^p \equiv \Psi_{j,i}\frac{1+\eta\,\overline{\varepsilon}_i^m}{1+\eta\,\varepsilon_j^m} \qquad \Psi_i^p \equiv \Psi_i\frac{1+\rho\,\overline{\varepsilon}^m}{1+\rho\,\overline{\varepsilon}_i^m}$$

Industry- and aggregate-level scale elasticities of demand:

$$\overline{\eta}_{K,i} \equiv \frac{1}{\overline{\omega}_{K,i}}\mathbb{E}_{j|i}^p\left[\omega_{K,j}\varepsilon_{K,j}\right] \qquad \overline{\eta}_{L,i} \equiv \frac{1}{\overline{\omega}_{L,i}}\mathbb{E}_{j|i}^p\left[\omega_{L,j}\varepsilon_{L,j}\right]$$

$$\overline{\eta}_K \equiv \frac{1}{\overline{\omega}_K}\mathbb{E}_i^p\left[\overline{\omega}_{K,i}\overline{\varepsilon}_{K,i}\right] \qquad \overline{\eta}_L \equiv \frac{1}{\overline{\omega}_s}\mathbb{E}_{j|i}^p\left[\omega_{s,j}\varepsilon_{s,j}\right]$$

where $\mathbb{E}_{j|i}^p[\cdot]$ and $\mathbb{E}_i^p[\cdot]$ denote the pass-through-weighted averages within and across industries

back

Aggregate-Level Substitution Elasticities

Aggregate-level substitution elasticities of demand:

$$\overline{\sigma} = \mathbb{E}_j^c \left[\frac{\Omega_{L,j}}{\Omega_L} \frac{\Omega_{K,j}}{\Omega_K} \sigma_j \right] - \mathbb{E}_i \left[\frac{\eta \overline{\varepsilon}}{1+\eta \overline{\varepsilon}_i^m} \mathbb{C}_{j|i}^p \left(\frac{\omega_{L,j} \varepsilon_{L,j}}{\Omega_L}, \frac{\omega_{K,j} \varepsilon_{K,j}}{\Omega_K} \right) \right] - \frac{\rho \overline{\varepsilon}}{1+\rho \overline{\varepsilon}^m} \mathbb{C}_i^p \left(\frac{\overline{\omega}_{L,i} \overline{\varepsilon}_{L,i}}{\Omega_s}, \frac{\overline{\omega}_{K,i} \overline{\varepsilon}_{K,i}}{\Omega_K} \right)$$

where $\mathbb{E}_j^c [\cdot]$ denotes the total-cost-weighted average across all firms under the distribution $\frac{\overline{\varepsilon}}{\varepsilon_j} \Psi_i \Psi_{ji}$, $\mathbb{C}_{j|i}^p (\cdot, \cdot)$ and $\mathbb{C}_i^p (\cdot, \cdot)$denote the within-industry and across-industry covariances under the pass-through distributions $\Psi_{j,i}^p$ and Ψ_i^p, respectively, and where $\overline{\varepsilon} \equiv 1/(1 + \overline{\omega}^f)$ denotes the aggregate scale elasticity of costs.

back

Deriving $\overline{\Omega}_L^m$ and $\overline{\eta}_L$

Industry- and aggregate-level scale elasticities of cost:

$$\overline{\Omega}_L^m = \frac{\overline{\eta}_K - \overline{\Omega}_K^m}{1-\overline{\Omega}_K}\left(\frac{\overline{\varepsilon}}{\overline{\eta}_K} - \overline{\Omega}_K\right)$$

$$\overline{\eta}_L = \frac{1}{1-\overline{\Omega}_K}\left(\overline{\varepsilon} - \overline{\Omega}_K\overline{\eta}_K\right)$$

back

Cost Structure of AI

AI Cost Minimization: firms choose $d_{i\tau}$, $m_{i\tau}$ to minimize training costs $\equiv 6 \cdot m_{i\tau} d_{i\tau}$

- FLOPs during training may be approximated by $m_{i\tau} d_{i\tau}$

  Rosenfeld (2021)

- Scalar 6 arises from number of passes in standard fine-tuning process

  Sevilla, et al. (2022)

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Implied AI fixed (training) and marginal (inference) productivity:

$$z^f_{K,\tau} = \frac{1}{6}\left(\frac{1}{\psi_m}\frac{\alpha_d}{\alpha_m+\beta_d}\right)^{\frac{1}{\alpha_m}}\left(\frac{1}{\psi_d}\frac{\alpha_m}{\alpha_m+\alpha_d}\right)^{\frac{1}{\alpha_d}}(h_{i\tau}-\underline{h})^{\frac{1}{\alpha_m}+\frac{1}{\alpha_d}}$$

$$z_{K,\tau} = \left(\frac{1}{\psi_m}\frac{\alpha_d}{\alpha_m+\alpha_d}\right)^{\frac{1}{\alpha_m}}(h_{i\tau}-\underline{h})^{\frac{1}{\alpha_m}}$$

back

Estimating the Entropy Function

Computer Vision Fine-Tuning Experiment: estimating $h\left(\epsilon_{i\tau}; \chi_{i\tau}\right)$

- Experiment varies task complexity (class number), achieved error, and cross-entropy loss

Details

Estimating the Entropy Function

Computer Vision Fine-Tuning Experiment: estimating $h(\epsilon_{i\tau}; \chi_{i\tau})$

- Experiment varies task complexity (class number), achieved error, and cross-entropy loss

Details

Parametrization:

$$\log h(\epsilon_{i\tau}; \chi_{i\tau}) \equiv \beta_0^h + \beta_1^h \log \epsilon_{i\tau} + \beta_2^h \log \chi_{i\tau} + \beta_3^h \log \epsilon_{i\tau} \times \log \chi_{i\tau}$$
$$+ \beta_3^h \log \epsilon_{i\tau} \times \log \chi_{i\tau} + \beta_4^h \frac{1}{\chi_{i\tau}} + \beta_5^h \epsilon_{i\tau} \times \log \epsilon_{i\tau}$$

Estimated Parameters of the Mapping from Error to Cross-Entropy Loss

Parameter	β_1^h	β_2^h	β_3^h	β_0^h	β_4^h	β_5^h
Estimate	0.28	0.21	0.13	0.51	-1.75	0.36

back

Roadmap

Appendix

Appendix

Task estimates from survey data

Task accuracy: worst error rate at which worker can complete task & still be considered qualified

Task complexity: number of possible choices/categories in outcome

Residualize outcomes $y_{o\tau,\ell}$ for occupation o, task τ, respondent i:

$$y_{o\tau,\ell} = \alpha + \beta \cdot Duration_{o\tau,i} + \gamma_{o\tau} + \Gamma' \boldsymbol{X_{o\tau,i}} + \epsilon_{o\tau,i}$$

- $\boldsymbol{X_{o\tau,i}}$ includes confidence level, language, age, gender, experience, & income fixed effects
- $\gamma_{o\tau}$ are occupation-task estimates excluding effect of respondent characteristics

Aggregate $\gamma_{o\tau}$ using occupation share in industry by wage bill using CPS

Comparison with other measures | Correlation with O*NET importance | Back

Relationship Between Variables

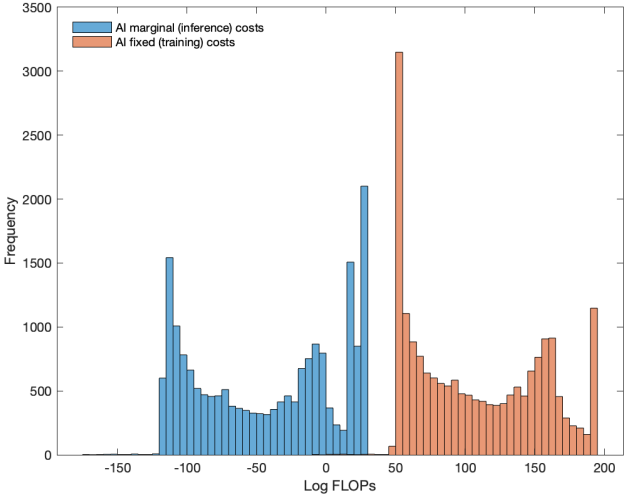
	Log CV Task Characteristics				
	(1)	(2)	(3)	(4)	(5)
	Max. Error	Typ. Error	Complexity	Frequency	Time Use
Log O*NET Importance	-1.584**	-1.059	1.091	0.725	-0.202
	(0.494)	(0.628)	(0.727)	(0.778)	(0.236)
N	505	504	418	505	505
Element FE	✓	✓	✓	✓	✓
2-digit SOC FE	✓	✓	✓	✓	✓

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

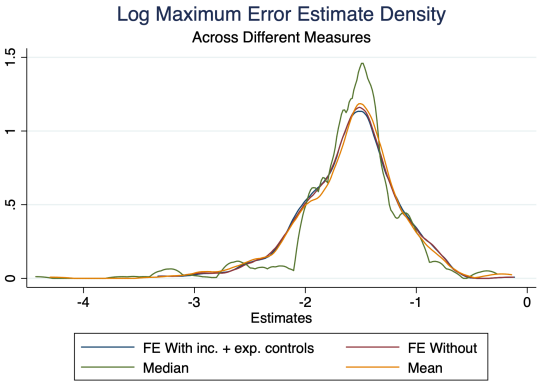
Back

Distribution of Estimated AI Costs



Back

Task accuracy kernel densities under different measures



Back

Correlation of survey data with O*NET Importance

	Log CV Task Characteristics				
	(1)	(2)	(3)	(4)	(5)
	Max. Error	Typ. Error	Complexity	Frequency	Time Use
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2-digit SOC FE	✓	✓	✓	✓	✓

Standard errors in parentheses

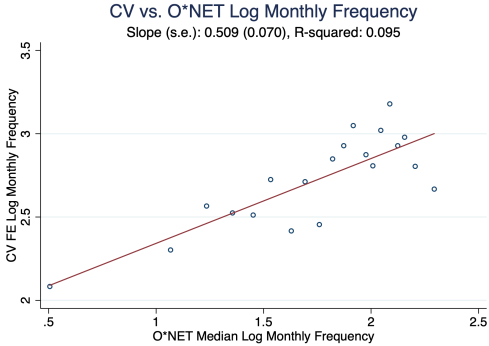
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Back

Validation of Survey Data with O*NET

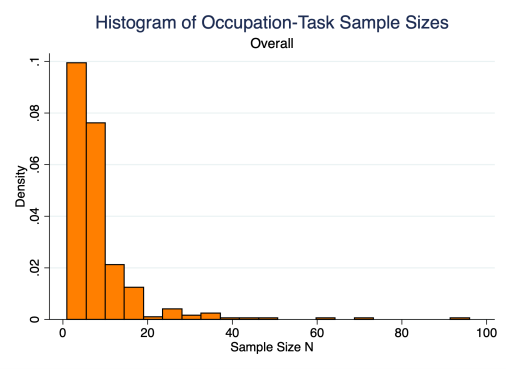
Task frequency: both O*NET and our survey measure monthly occurence of tasks

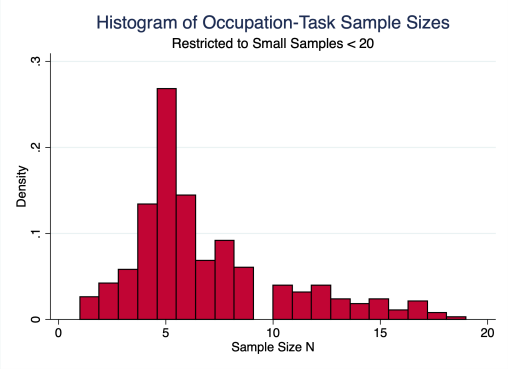
- Survey question: "How often do workers make judgments for the visual part of this task?"
- O*NET question: "How often do workers perform this task?"



Back

Survey Sample Sizes





Back

Firm Productivity Calibration

Suppose productivity a_{ij} of firm j in industry i follow a Pareto distribution:

$$G(a) = 1 - \left(\frac{\underline{a}_i}{a}\right)^{\psi}$$

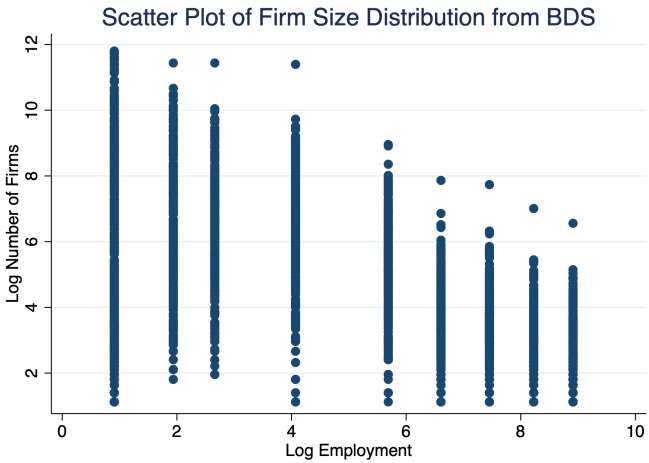
Since $\ell_{ij} \propto a_{ij}^{\eta-1}$,

Scatter plot in BDS

$$\log G_{ij}^{-1} = -\frac{\psi}{\eta - 1} \log \ell_{ij} + \psi \log \underline{a}_i$$

Back

Scatter Plot of Firm Size using BDS



Back

LLM Generated Labor Productivity

AI Model: GPT-5

System and User Prompt: Generates task frequency and time use for all O*NET tasks

Prompt | Data Validation

- Task Frequency:

  Calculate task frequencies at the O*NET task and industry level

$$\xi_{i\tau} = \sum \frac{w_{o,i} l_{o,i}}{\sum_{o',i} w_{o',i} l_{o',i}} \cdot \xi_{o,\tau} \qquad \xi_{o,\tau} = \frac{Freq_{\tau,o}}{\sum_{\tau'} Freq_{\tau',o}}$$

- Task-Level Labor Productivity:

  Calculate productivity for set of vision tasks ($\mathcal{T}_V$) at the O*NET task level

  Calculate productivity for non-vision tasks at the GWA (Generalized Work Activity) level

$$Z_\tau = \frac{Freq_\tau}{TimeUse_\tau \times w_o} \qquad \tau \in \mathcal{T}_V$$

$$Z_{GWA} = \frac{1}{|GWA|} \sum_{\tau \in GWA} \frac{Freq_\tau}{TimeUse_\tau \times w_o} \qquad GWA \not\subset \mathcal{T}_V$$

Back

LLM Prompt

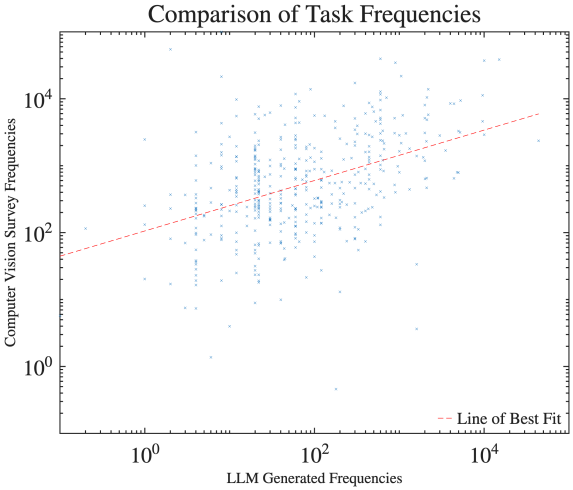
System Prompt: You are a(n) {occupation} participating in a research study about job tasks. You will be asked to answer questions about job tasks associated with your occupation. Think through your answer carefully. You must return a strictly positive (non-zero) numeric answer. Provide no explanation for your answer. Keep in mind that the following tasks are associated with your occupation: {List of Tasks}

Question 1: How often does a worker in your occupation perform this task? Answer with a single positive (non-zero) number representing the number of times per month.

Question 2: How many hours per month does a worker actively spend performing this task? Exclude background or implicit activities (e.g., general awareness, compliance, or principles). Answer with a single strictly positive (non-zero) number representing the hours per month.

Back

LLM Data Validation: Computer Vision Survey



Back

AI Scaling Law Experiment

AI Model: SwinTransformer fine-tuned on unseen classes

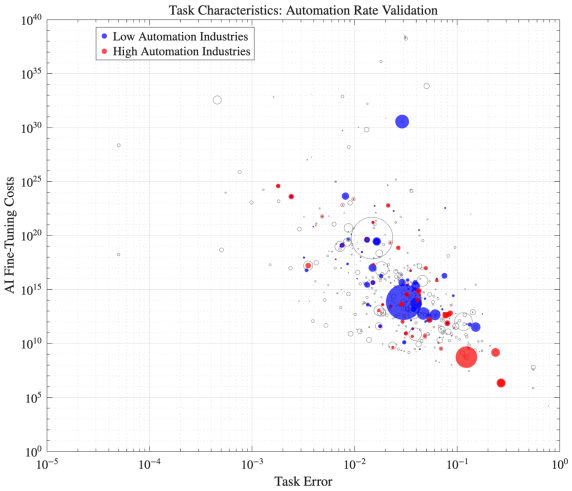
Dataset: ImageNet

Experimental setup: 80 configurations varying:

- Data size: 5 levels (13/class to 1,300/class)
- Model size: 4 levels (7.3K to 87.8M parameters)
- Task complexity: 4 levels (2 to 500 classes)
- Performance tracked across training steps

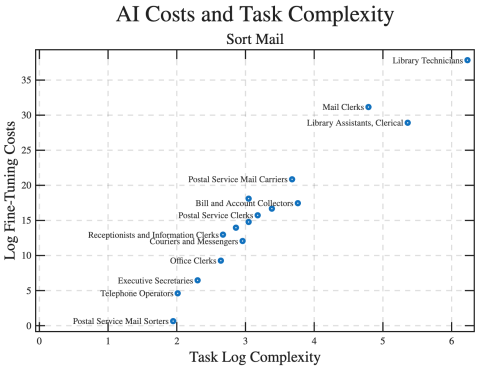
Back

Scaling Law Costs by Automation Rate



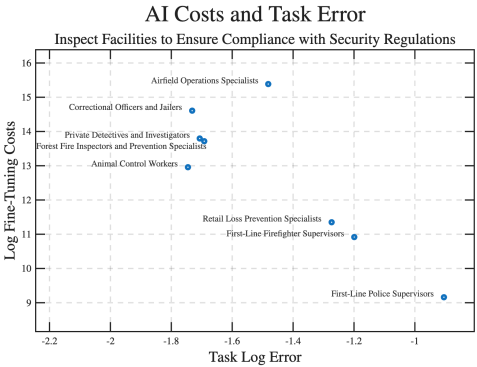
Back

Task Complexity Example



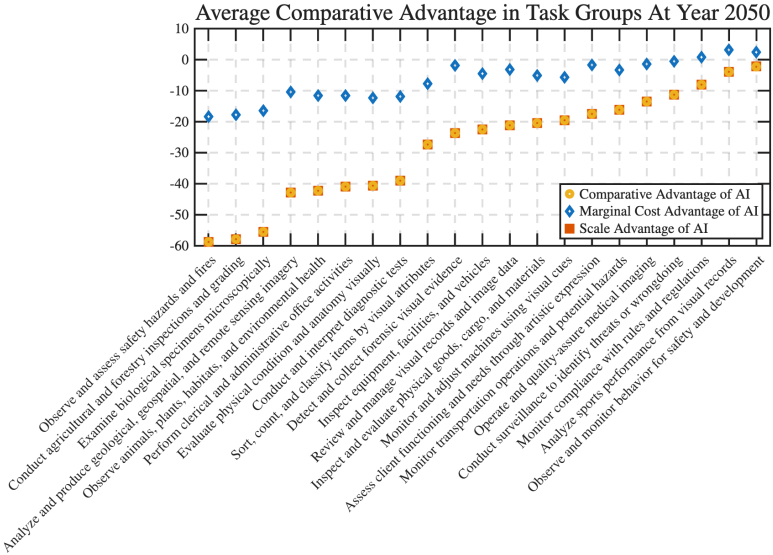
Back

Task Error Example



Back

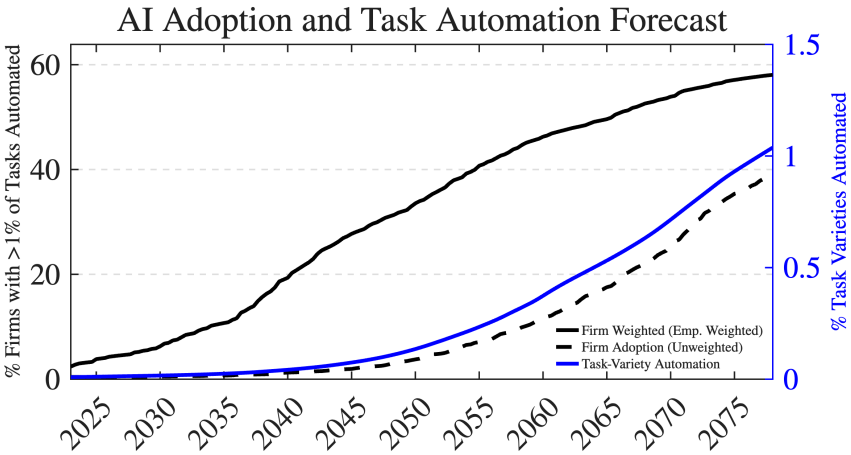
Comparative and Scale Advantage of AI: 2050



Back

Predicting Future of Computer Vision Automation

- Trends in computing price r: annual decline of 26.4% in $ cost of FLOP
  Del Sozzo, et al. (2025); Ho, et al. (2024)



Back

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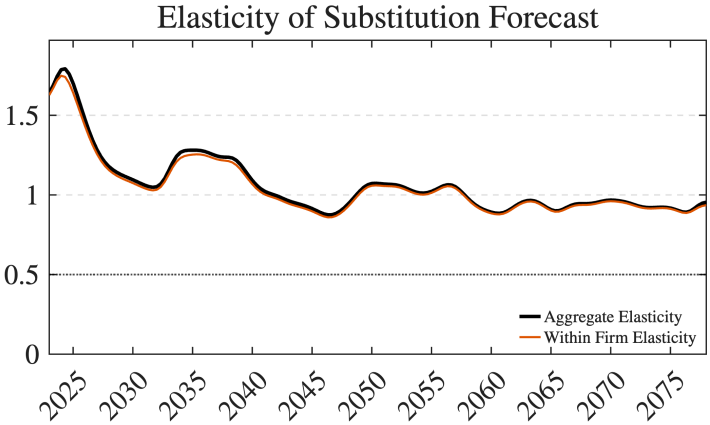

Back

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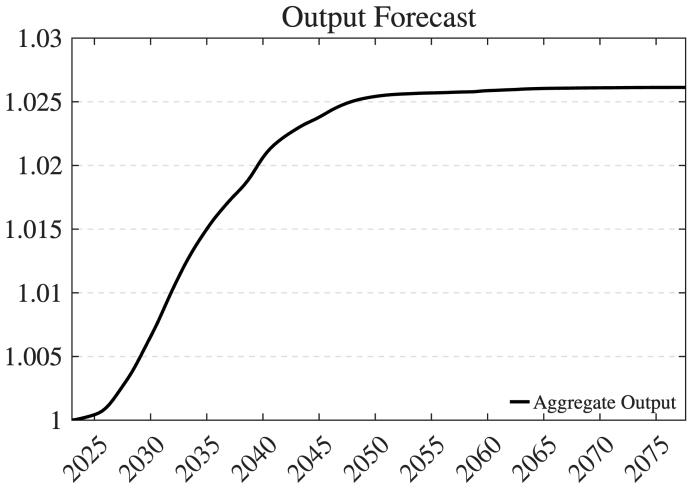

Back

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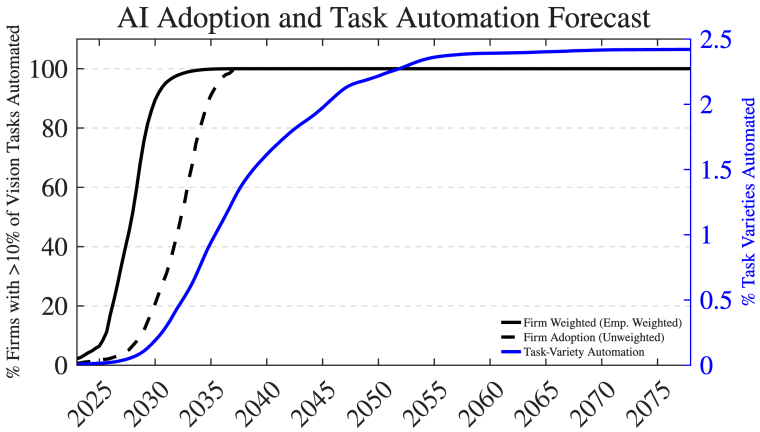

Back

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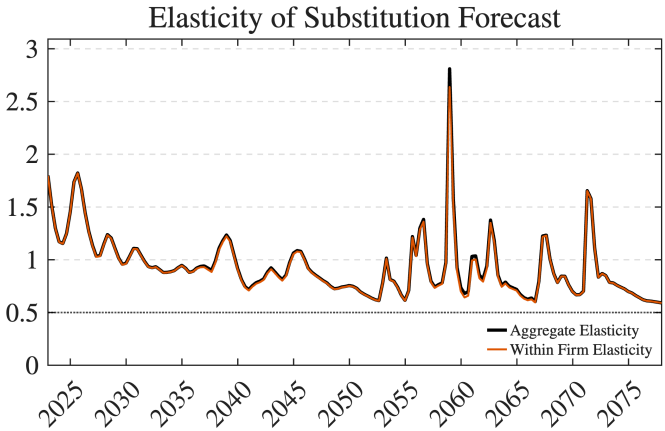

Back