Online Appendix for:
"Volatile Hiring: Uncertainty in Search and Matching Models"*

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This appendix contains supplemental material for the article "Volatile Hiring: Uncertainty in Search and Matching Models."

Any references to equations, figures, tables or sections that are not preceded by a capital letter refer to the main article.

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Appendix OA Derivations and proofs

OA .1 Derivation of Nash-bargained wage

Let V_t and U_t denote the value of an employed and an unemployed worker, respectively. That is,

$$V_t = w_t + \beta E_t \left[(1-\delta+\delta f_{t+1}) V_{t+1} + \delta(1-f_{t+1}) U_{t+1} \right],$$
$$U_t = \chi + \beta E_t \left[f_{t+1} V_{t+1} + (1-f_{t+1}) U_{t+1} \right].$$

Thus, the surplus to the household of accepting a job is given by

$$S_t = V_t - U_t = w_t - \chi + \beta E_t \left[(1-\delta)(1-f_{t+1}) S_{t+1} \right]. \tag{OA .1}$$

Similarly, the surplus to the firm of hiring a worker is simply the firm value, J_t, (which is repeated for convenience)

$$J_t = \bar{x} z_t - w_t + \beta(1-\delta) E_t [J_{t+1}]. \tag{OA .2}$$

Nash bargaining sets the wage, w_t, to maximize the Nash product such that

$$w_t = \arg\max \{ J_t^{1-\omega} S_t^{\omega} \},$$

where ω represents the bargaining power of the worker. The first order condition is given by

$$(1-\omega) S_t = J_t \omega. \tag{OA .3}$$

Using the first order condition in equation (OA .3) together with equations (OA .1) and (OA .2) gives

$$(1-\omega)(w_t-\chi)+\beta(1-\delta)\omega E_t[(1-f_{t+1})J_{t+1}]=\omega(\bar{x}z_t-w_t)+\beta(1-\delta)\omega E_t[J_{t+1}].$$

Solving this equation for w_t gives the expression for the Nash-bargained wage.

OA .2 Proof of Proposition 1

To derive Proposition 1, we substitute the linear wage rule given in equation (24) into the firm value equation in (11) and iterate forward.[OA .1] Thus,

$$\begin{aligned}J_t &= \bar{x}z_t-w_t+\beta(1-\delta)E_tJ_{t+1}\\ &= E_t\Sigma_{j=0}^{\infty}\beta^j(1-\delta)^j(1-\omega)(\bar{x}z_{t+j}-\chi).\end{aligned}$$

Next, use the law of motion for productivity (8).

$$\begin{aligned}J_t = &-\frac{(1-\omega)\chi}{1-\beta(1-\delta)}\\ &+(1-\omega)\bar{x}z_t\\ &+\beta(1-\delta)(1-\omega)[\bar{x}((1-\rho_z)+\rho_z z_t)]\\ &+\beta^2(1-\delta)^2(1-\omega)[\bar{x}((1-\rho_z)+\rho_z(1-\rho_z)+\rho_z^2 z_t)]\\ &+\beta^3(1-\delta)^3(1-\omega)[\bar{x}((1-\rho_z)+\rho_z(1-\rho_z)+\rho_z^2(1-\rho_z)+\rho_z^3 z_t)]\\ &+\cdots\end{aligned}$$

[OA .1] We rule out exploding paths, such that

$$\lim_{j\to\infty}[\beta(1-\delta)]^jE_t[J_{t+j}]=0,\quad t=0,1,\ldots$$

Now simplify the infinite sum,

$$J_t = -\frac{(1-\omega)\chi}{1-\beta(1-\delta)} + \frac{(1-\omega)\bar{x}z_t}{1-\beta(1-\delta)\rho_z} + \frac{\frac{\beta(1-\delta)(1-\omega)(1-\rho_z)\bar{x}}{1-\beta(1-\delta)}}{1-\beta(1-\delta)\rho_z},$$

and collect terms,

$$J_t = -\frac{(1-\omega)\chi}{1-\beta(1-\delta)} + \frac{(1-\omega)\bar{x}z_t}{1-\beta(1-\delta)\rho_z} + \frac{\beta(1-\delta)(1-\omega)(1-\rho_z)\bar{x}}{(1-\beta(1-\delta))(1-\beta(1-\delta)\rho_z)}.$$

This final line corresponds to equation (25). □

OA .3 Proof of Proposition 2

The firm value is in this case given by

$$J(z) = \frac{(1-\omega)(xz-\zeta)}{1-\beta(1-\delta)} - \frac{\beta\omega\kappa\theta(z)}{1-\beta(1-\delta)}.$$

Suppose that $J(z)$ is (weakly) convex in the vicinity of some $z > 0$. That is

$$tJ(z_1) + (1-t)J(z_2) \geq J(z),$$

for some $z_1 > 0$ and $z_2 > 0$ and any $t \in (0,1)$ such that $z = tz_1 + (1-t)z_2$. Then by definition

$$\frac{(1-\omega)(xz-\zeta)}{1-\beta(1-\delta)} - \frac{\beta\omega\kappa(t\theta(z_1)+(1-t)\theta(z_2))}{1-\beta(1-\delta)} \geq \frac{(1-\omega)(xz-\zeta)}{1-\beta(1-\delta)} - \frac{\beta\omega\kappa\theta(z)}{1-\beta(1-\delta)},$$

or simply

$$(t\theta(z_1) + (1-t)\theta(z_2)) \leq \theta(z).$$

That is, $\theta(z)$ must be weakly concave in the vicinity of z.

The free-entry condition implies that

$$\theta(z) = \left(\frac{\psi}{\kappa} J(z)\right)^{\frac{1}{\alpha}},$$

which implies that $\theta(z)$ is a strictly convex function in the vicinity of z. As this is a contradiction, $J(z)$ must be strictly concave for all $z > 0$, which implies that $\theta(z)$ must be strictly convex for all $z > 0$. $\square$

Appendix OB Risk aversion in the standard model

Our main analysis assumes that the representative household is risk neutral. This assumption carried the benefit of making the analysis of option-value considerations, respectively their absence, more transparent. Allowing for risk aversion introduces a number of complexities in the form of additional transmission channels and interaction effects. In this section, we give an indication of the direction in what direction they go. We highlight, in particular, that risk aversion alters the predictions of the canonical search-and-matching (SaM) model for the effects of uncertainty shocks on economic activity in two primary ways, relative to the risk-neutral benchmark. First, the interaction of investor risk aversion and search-frictions in the pricing of firm equity gives rise to non-zero, adverse pure uncertainty effects; even when the wage function is linear. Second, if households are risk averse, regular Nash bargaining over wages may dampen this uncertainty-induced recession.

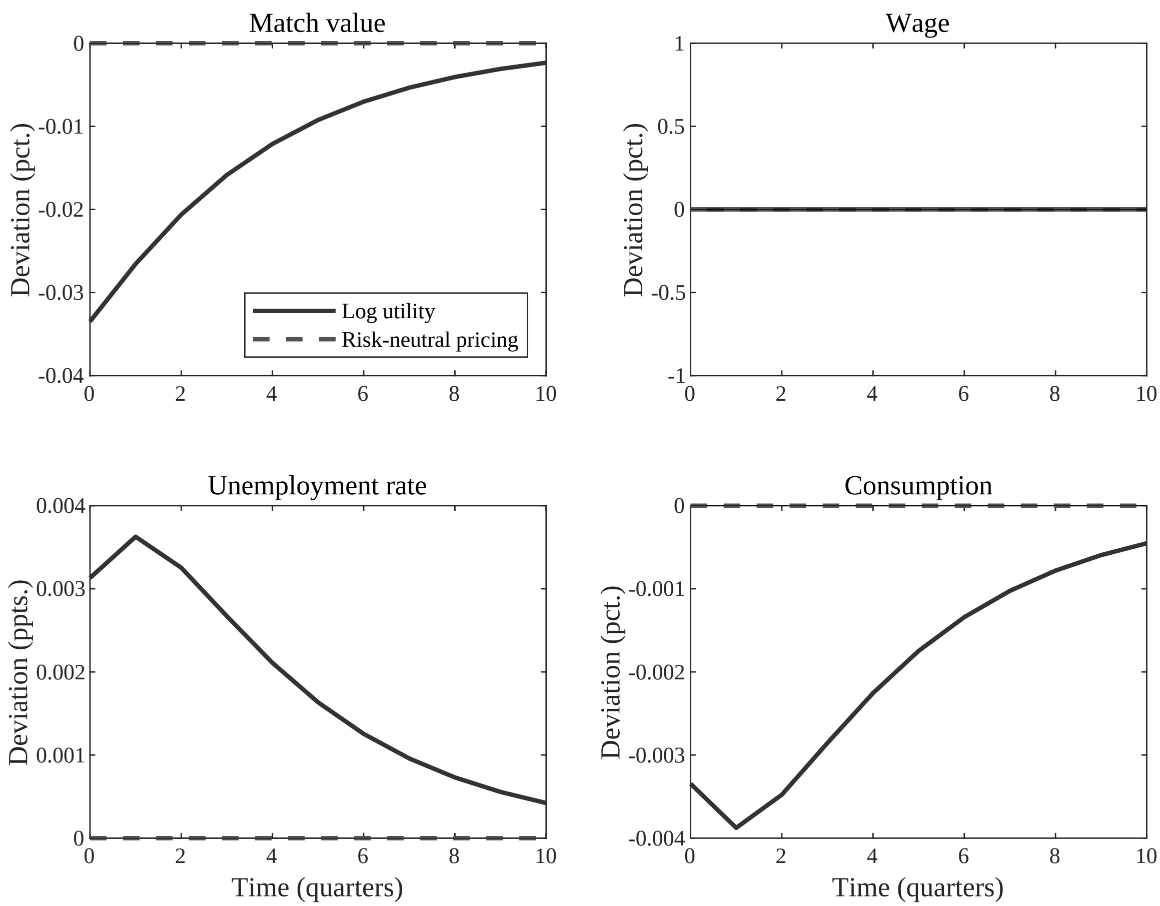
To derive these results, we proceed in two steps. First we suppose that wages are a linear function of current productivity, as in equation (24); later we will add Nash bargaining. In the main text, which assumed risk neutrality, we emphasized that under this specification, the stream of expected dividends from a match is unaffected by an increase in uncertainty (cf. Proposition 1). But risk aversion makes the representative household value any given dividend stream differently when uncertainty increases. In particular, assume that the representative household values consumption according to a log utility function. The equation pinning down the period-t firm value incorporates stochastic discounting of the continuation value in the expectation term is

$$J_t = \bar{x}z_t - w_t + (1-\delta)E_t\left[\frac{u'(c_{t+1})}{u'(c_t)}J_{t+1}\right]. \tag{OB .1}$$

Moreover, as in Leduc and Liu (2016), households can save not only in form of equity, but also risk-free government bonds that pay a gross real interest rate R_t. As such, the system of equations is augmented by a standard bond Euler equation.

Freund and Rendahl (2020) explore this model with linear wages in depth, considering also the role of nominal rigidities and shedding light on the simultaneous operation of supply and demand channels through which uncertainty shocks then affect economic activity. Here we briefly summarize their results for the transmission of uncertainty shocks in a setting with flexible prices.

Figure OB .1: Pure uncertainty IRFs in SaM model with risk aversion and linear wage rule



Notes: The figure shows the "pure uncertainty" IRFs to a unit-increase in $\varepsilon_{\sigma,t}$ for the SaM model with a linear wage rule and allowing for risk aversion. These IRFs display how the economy responds when agents think volatility will increase, but the higher volatility actually never materializes.

The solid line in figure OB .1 indicates the pure uncertainty IRF for the firm value, wages, unemployment, and consumption. It can be seen that in this setting, when agents anticipate a persistent rise in perceived future volatility, this causes the firm value to fall. A lower firm value means that the incentives to post vacancies faced by entrepreneurs are weakened, so that it becomes more difficult for the unemployed to find a job. Unemployment consequently rises, while output and consumption contract. Thus, while search frictions *by themselves* are insufficient to raise the unemployment in response to an increase in perceived uncertainty – as visualized by the the dashed line, which describes the pure uncertainty effect in the risk-neutral case –, their *interaction with risk aversion* means that a rise in uncertainty lowers economic activity even when prices are flexible and the wage function is linear.

Driving this overall result are three distinct mechanisms. Two of them are expansionary under flexible prices, but they are dominated by a contractionary risk premium channel.[OB.1] On the one hand, two mechanisms trigger a rise in households' desire to save that leads them to value all assets, including the risky equity of intermediate goods firms, more highly when faced with a more uncertain future. The first mechanism is linked to the usual prudence motive associated with the marginal utility of consumption being convex; by Jensen's inequality, the perception of greater future volatility pushes up $E_t[u'(c_{t+1})]$. The starting point for the second expansionary mechanism is that, as described in section 3.3, search frictions mean that average unemployment is higher in periods of heightened volatility. Households *anticipating* the future to be more volatile, therefore, also *expect* average unemployment to be higher. If households aim to smooth consumption over time, this expectation reinforces their desire to save in form of both bonds and equity rather than consume in the present. At the same time, however, increased uncertainty about the future generates a stronger negative comovement between the marginal utility of consumption and the equity value; low payoffs are expected for precisely those periods where consumption will be low and, hence, when dividend income would be more valuable (and vice versa). This negative comovement is captured by a rise in the required risk premium, causing a fall in the firm value. In summary, therefore, a rise in uncertainty lowers economic activity when households are risk averse and when the wage is unresponsive to expected movements in either labor market tightness or marginal utility. The reason is that households anticipating greater future volatility require a larger risk premium to compensate them for holding the equity of firms with long-term employment relationships. More costly equity acts to suppress hiring activity.

[OB.1] In principle, the net effect of these mechanisms is *ex ante* ambiguous. Freund and Rendahl (2020) underscore that the risk premium can be large and volatile in the SaM setting, the reason being that hiring a worker is akin to investing in risky assets with long-duration payoffs.

Next, suppose that wages are not a linear function of productivity but, instead, are determined by Nash bargaining. As such, the theoretical environment we consider here corresponds to the flexible-price version of Leduc and Liu (2016, see their section 4.2.1).[OB .2] We follow Leduc and Liu (2016) in supposing that the worker's reservation value consists of a combination of unemployment benefits, ϕ, and disutility of supplying labor, χ (or, equivalently, a linear utility parameter for leisure).[OB .3] Extending equation (12), the wage is pinned down as

$$w_t^N = (1-\omega)\left(\phi + \frac{\chi}{u'(c_t)}\right) + \omega\left(\bar{x}z_t + \beta(1-\delta)\kappa E_t\left[\frac{u'(c_{t+1})}{u'(c_t)}\frac{v_{t+1}}{u^s_{t+1}}\right]\right), \tag{OB .2}$$

Relative to the risk-neutral Nash wage, household risk aversion shows up in two ways in equation (OB .2). First, the utility of leisure accrues to the worker's surplus when bargaining, but in consumption units (which is then suitably shared between the firm and the worker according to their respective bargaining power). Thus, the parameter appears in the wage equation divided by the marginal utility of consumption. Second, next-period labor market tightness is discounted with the marginal rate of substitution.

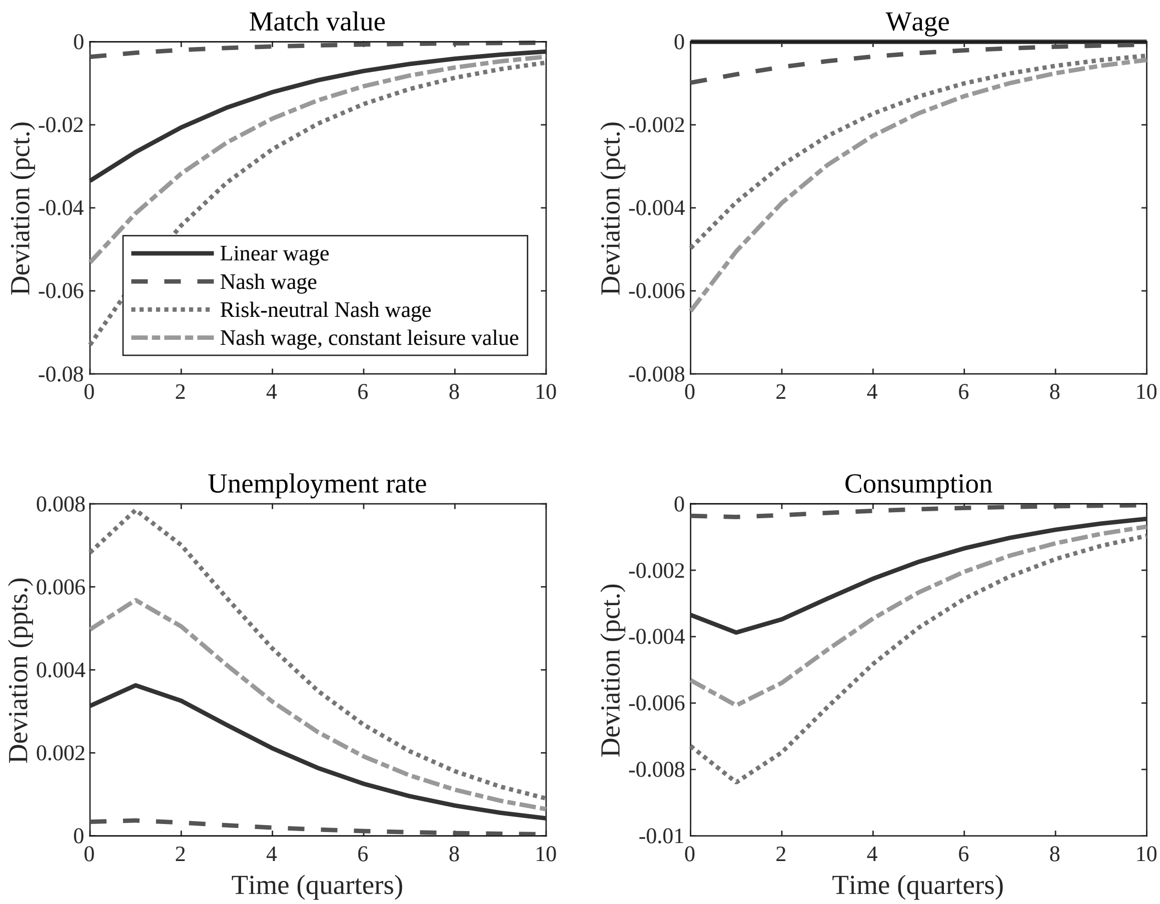
Figure OB .2 reports the effects of an increase in anticipated volatility under four wage-setting specifications: (a.) the benchmark case with linear wages (solid line); (b.) "regular" Nash bargaining (following equation (OB .2), dashed line); (c.) the wage materializing under Nash bargaining with risk neutrality (following equation (12); dashed-dotted line); and (d.) the wage materializing under Nash bargaining when the money-metric value of the utility of leisure is held at its steady-state value (dotted line). For (c.), we only assume risk neutrality in deriving the wage equation.

The figure makes clear that Nash bargaining can either dampen ("regular") or magnify (risk-neutral Nash wages) the IRFs. The difference between (a.) and (c.) is essentially the "Nash-wage

[OB .2] Different from Leduc and Liu (2016), we abstract from extrinsic wage rigidity to preserve greater transparency. On the same grounds, we also continue to assume that resources expended on vacancy posting are rebated to the household. Quantitatively, the numbers we report for the case of Nash bargaining are therefore most comparable, but do not exactly coincide with, the results reported in their online appendix, specifically the dotted line in their Figure A6.

[OB .3] As far as the calibration of the model is concerned, we also stay as close as possible to Leduc and Liu (2016). As such, under Nash bargaining we take the flow benefits of unemployment, ϕ, to be equal to 0.25, while the disutility of working, χ, is set equal to 0.5348. Given a steady-state value of consumption $\bar{c} = \bar{n} = 0.9360$, the monetary value of $(\phi + \chi/u'(c_t))$ in steady-state monetary terms is therefore 0.751, just as in the risk-neutral setting. The steady-state elasticity of labor market tightness with respect to productivity thus remains unchanged.

Figure OB .2: Pure uncertainty IRFs in SaM model with risk aversion



Notes: The figure shows the "pure uncertainty" IRFs to a unit-increase in $\varepsilon_{\sigma,t}$ for the SaM model, allowing for risk aversion and considering alternative wage specifications.

channel" explored in the main text, that is, the interaction of higher expected future labor market tightness and Nash bargaining adds to the recessionary effect of a rise in the risk premium.

The possibility of a dampening effect, on the other hand, is due to the fact that whenever the household's marginal utility of consumption is elevated relative to the steady state, this lowers the wage rate. For when a recession materializes, consumption falls and the marginal utility of consumption increases. Thus, the monetary value of leisure declines, which, ceteris paribus, means that the wage is permitted to fall more than under risk neutrality. With more wage flexibility, there are smaller movements in asset prices, and the contraction is less pronounced (which, seemingly contradictory, leads to a smaller decline in wages). What is more, when uncertainty is *expected* to be high, the same channels discussed above under the heading of 'precautionary savings' raise the expected marginal utility of consumption. By the preceding logic, this mechanism lowers the expected wage, and, hence, raises expected future dividends. It therefore exerts upward pressure on the firm value, J_t, which, after all, is simply the present discounted value of dividends. Because J_t is elevated relative to the risk-neutral benchmark, vacancy creation and employment are boosted. As a result, consumption is actually higher and the marginal utility consequently lower, which raises the realized wage, other things equal.[OB .4] The dotted line in Figure OB .2 (case d.) shows this by adding a Nash bargaining solution which holds the utility component of leisure fixed in money terms over the cycle. Compared to the baseline with Nash wages, the effect of a rise in uncertainty is worse than under regular Nash bargaining or, indeed, the linear wage specification.

A final nuance arises from the fact that under regular Nash bargaining with risk aversion, the stochastic discount factor is involved in determining the worker's surplus. In terms of equation (OB .2), uncertainty shocks also propagate through the product term inside the conditional expectation involving the ratio of marginal utilities and next-period labor market tightness. In Figure OB .2, comparing the dashed-dotted line – corresponding to them model with wages determined by Nash bargaining under risk neutrality – and the dotted line – where only the money-metric value of leisure is held constant – reveals that the presence of this term exerts a positive effect on the response of the firm value to a rise in perceived future volatility. The opposite is true for the wage response. To explain why this is the case, note that similar to the precautionary savings effects discussed above, a first consequence of agents anticipating persistently higher future volatility is to put upward pressure on the expected future marginal utility, which in this instance exerts *upward* pressure on the expected future wage. Intuitively, any benefit workers derive from a tight labor market in the future counts for more. This effect lowers expected future dividends and hence the current match value, setting off the by now familiar chain of events that ends up lowering realized labor market tightness and higher unemployment. On the other hand, though, business cycle fluctuations are associated with a negative covariance between the stochastic discount factor and labor market tightness. As

[OB .4] When real wages are assumed to be extrinsically rigid, as in Leduc and Liu (2016), these effects are attenuated.

greater volatility strengthens this negative co-movement, anticipation thereof acts to *lower* the wage in both present and future, which incentivizes job creation. In principle, the net effect of these two channels is ambiguous, but the figure reveals that under the benchmark calibration it, too, serves to dampen the adverse impact of the uncertainty shock on unemployment relative to the case of risk-neutral Nash bargaining.

In summary, when households are risk averse, a rise in uncertainty may push the economy into a recession even when wages, and therefore also dividends, are linear in productivity. Nash bargaining over wages, on the other hand, may exert a dampening effect under risk aversion, provided the workers' total flow benefits from unemployment are sensitive to variations in marginal utility.

Appendix OC Two-period model with heterogeneity

This section presents and examines a two-period variant of the search-and-matching model developed in section 4.1, which features an option-value effect of waiting due to uncertainty about productivity at both the firm-specific and the aggregate levels. This two-period model affords an intuitive, graphical exposition of the key mechanisms as well as analytical results. We first suppose that the probability of an entrepreneur successfully hiring a worker upon posting a vacancy is fixed at some level $\bar{h}$. This case reveals the conditions under which an option value of waiting exists, and the underlying intuition, in a particularly transparent way. Thereafter, we consider the case in which congestion on the matching market makes the hiring rate endogenous, as in the full model.

OC .1 Constant hiring rate

Environment. There are two periods and no discounting. Entrepreneurs can produce either in period 1 or in period 2, but not both.[OC .1] That is, the separation rate is equal to 1. Production is given by $z_t + a_{i,t}$, where a_i is distributed over the interval $[-\bar{a}, +\bar{a}]$ according to the distribution F. Aggregate productivity in the first period is $z_1 = 1$, while z_2 is equal to either $1+\Delta$ ('expansion') or $1-\Delta$ ('recession'), with equal probabilities. The parameter Δ is, thus, a measure of aggregate uncertainty. Workers have no bargaining power and their outside option is zero, such that the profits of a matched entrepreneur are equal to the full value of output net of hiring costs. With the price of such output normalized to unity, and without loss of generality, let the cost of starting a firm, κ, be equal to the fixed hiring probability $\bar{h}$, so that an entrepreneur with draw $a_{i,1} = 0$ in period 1 makes zero profits.

No aggregate uncertainty. Suppose first that there is no aggregate uncertainty, that is, $z_1 = z_2 = 1$. Then the condition pinning down the cutoff productivity of the marginal entrepreneur who

[OC .1] This setup, where there are no opportunities after a production spell, resembles our assumption in the full model that entrepreneurs 'die' – in the sense of having zero value – upon separation.

is indifferent between either entering in the first period or waiting is

$$-\kappa + \bar{h}(1+\widehat{a}_{1,\Delta=0}) = \int_{-\bar{a}}^{\bar{a}} \max\{-\kappa + \bar{h}(1+a), 0\}\, dF(a), \tag{OC .1}$$

where $\widehat{a}_{1,\Delta=0}$ denotes the cutoff productivity level in the absence of aggregate uncertainty. Cancelling terms and rewriting the integral, this equation can be written as[OC .2]

$$\begin{aligned}\widehat{a}_{1,\Delta=0} &= \int_{0}^{\bar{a}} af(a)da,\\ &= \underbrace{E[a|a \geq 0]}_{\equiv a_2^*}\underbrace{\text{prob}(a \geq 0)}_{p_2} > 0,\end{aligned} \tag{OC .2}$$

where a_2^* is the expected value of a_2 conditional on being above the period-2 cutoff level, and p_2 denotes the probability of such a draw.

The fact that $\widehat{a}_{1,\Delta=0} > 0$ means that there is a standard option value of waiting due to idiosyncratic uncertainty in this model. Other things equal, the presence of idiosyncratic uncertainty encourages unmatched entrepreneurs to wait, because doing so preserves the optionality of obtaining a better draw in the future and entering; they can always not enter given a poor draw (as indicated by the max-operator), which eliminates downside risk. This mechanism affects the steady state of the main model. [OC .3] Next we will prove that this option value of waiting is amplified if the anticipated variance of z_2, Δ, is positive.

Aggregate uncertainty. The intuitive logic behind the option-value effect due to aggregate uncertainty is that in a time of high overall productivity, the level of idiosyncratic productivity needed to cover vacancy posting costs is lower, so that the probability of having such a draw is higher; at the same time, the expected value of producing conditional upon entry is higher. The opposite is true in times of low productivity, but the decrease in expected profits is lower because

[OC .2] To derive the following expressions, we use two facts: first, an entrepreneur with a draw $a_i < 0$ prefers to not enter in period 2 and, instead, makes zero profits (reflecting the max operator); second, given the fixed hiring rate we can use that $\kappa/\bar{h} = z_1 = z_2 = 1$ to simplify the expression.

[OC .3] One objective of the recalibration procedure described in section 4.2 is, then, to ensure that this option-value effect does not lower vacancy posting (and, hence, raise unemployment) in steady state – "steady state" in the sense of there being no aggregate uncertainty – below the calibration target.

entrepreneurs with low draws avoid entering in the first place and, instead, make zero profits.

To capture this intuition, we can use the same type of expression for the cutoff when there is no aggregate risk. Note first that the cutoff in period 2 satisfies $-\kappa + \bar{h}(z_2 + \widehat{a}_2) = 0$, so that $\widehat{a}_2 = -\Delta$ in good times, and $\widehat{a}_2 = +\Delta$ in bad times. Then denoting by $\widehat{a}_{1,\Delta>0}$ the cutoff productivity level in the presence of aggregate uncertainty, the cutoff equation becomes[OC .4]

$$\widehat{a}_{1,\Delta>0} = \frac{1}{2}\int_{-\Delta}^{\bar{a}} (a+\Delta) f(a) da + \frac{1}{2}\int_{+\Delta}^{\bar{a}} (a-\Delta) f(a) da.$$

The different lower limits of integration indicate that when $z_2 = 1+\Delta$, entrepreneurs with draws above $-\Delta$ will make positive profits, whereas when $z_2 = 1-\Delta$, only those with draws above $+\Delta$ will do so. Subtracting equation (OC .2) from equation (OC .3), we obtain

$$\widehat{a}_{1,\Delta>0} - \widehat{a}_{1,\Delta=0} = \frac{1}{2}\left(\int_{-\Delta}^{\bar{a}} (a+\Delta) f(a) da - \int_{0}^{\bar{a}} a f(a) da\right) + \frac{1}{2}\left(\int_{+\Delta}^{\bar{a}} (a-\Delta) f(a) da - \int_{0}^{\bar{a}} a f(a) da\right).$$

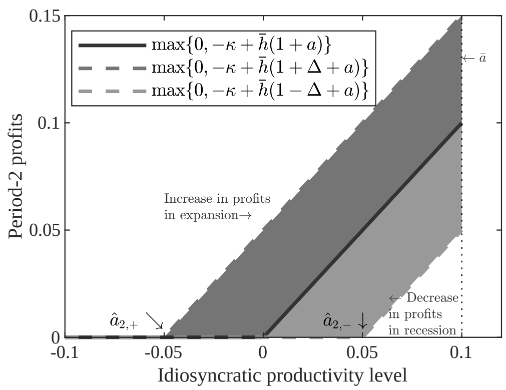
The first term in brackets is positive and the second is negative. However, the first term is larger in absolute value. Figure OC .1a illustrates this point. The dark grey area corresponds to the gain in profits when $z_2 = 1+\Delta$ rather than $z_2 = 1$; this is the first term. The lighter grey area describes the loss in profits when $z_2 = 1-\Delta$ instead; the second term. Clearly, the former area is greater than the latter. The reason is that the entrepreneur has a greater chance of making a draw that yields strictly positive profits in good times. The top-right panel OC .1b illustrates how, when deciding whether to enter or wait in the first period, the greater expected profits in the presence of aggregate risk pushes up the option value of waiting. Consequently, the cutoff level above which entrepreneurs are willing to enter in the first period is higher when $\Delta > 0$ than when $\Delta = 0$.

Analytical characterization given uniform distribution. To conclude this section, we analytically characterize the cutoff levels $\widehat{a}_{1,\Delta=0}$ and $\widehat{a}_{1,\Delta>0}$ under a particular functional form assumption

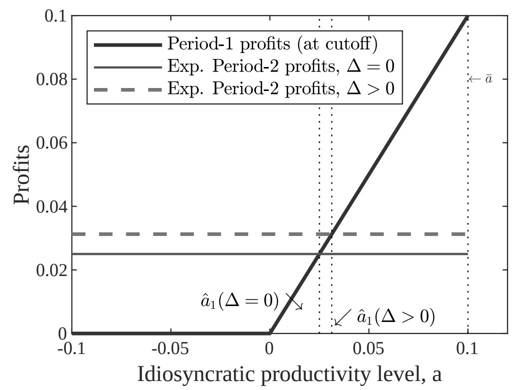
[OC .4] Deriving this expressions involves the same steps noted in Footnote OC .2, adjusted for the fact that the period-2 cutoff now is a function of the stochastic aggregate state, z_2.

Figure OC .1: Two-period model

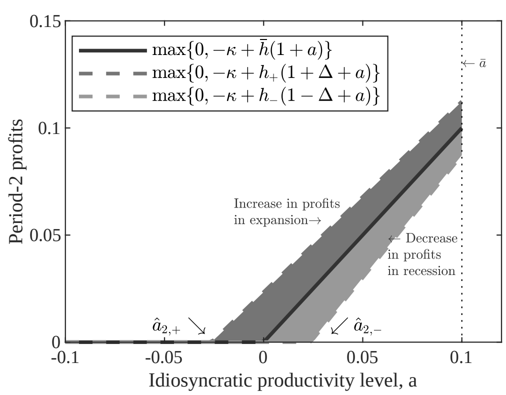
(a) Value of waiting: fixed hiring prob.



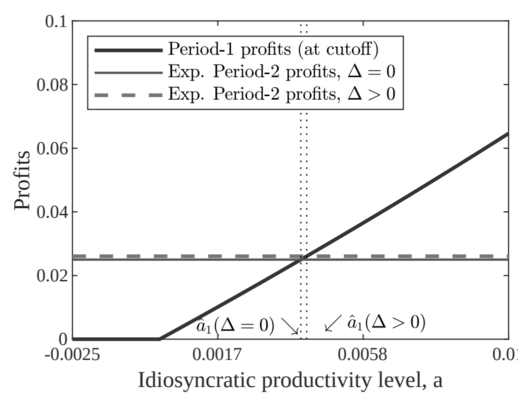
(b) Cutoff determination: fixed hiring prob.



(c) Value of waiting: variable hiring prob.



(d) Cutoff determination: variable hiring prob.



Notes: The figure shows how idiosyncratic and aggregate uncertainty both give rise to an option value of waiting in the two-period model. In the upper row, the hiring rate is fixed at $\bar{h}$ (corresponding to $\alpha = 0$), whereas the lower row allows that rate to vary according to a standard matching function ($\alpha = 0.5$). Throughout, idiosyncratic productivity is distributed uniformly over $[-0.1, +0.1]$. To ease visual comparison, the panels with fixed hiring rate assume that $\Delta = 0.05$, whereas in the case of a variable hiring rate, $\Delta = 0.15$. Finally, notice that in Panel OC .1d, period-1 profits are computed as the left-hand side of equation (OC .6), so that the slope incorporates endogenous changes in the hiring rate. To facilitate a straightforward comparison of alternative vacancy posting cost interpretations in section OD.1, the steady-state hiring rate is set equal to one. As a consequence, $h_- > 1$, which is, strictly speaking, inconsistent with h describing a probability, but intuitively just means that one entrepreneur could employ more than one worker.

about the distribution F, namely, that a_i is uniformly distributed over $[-\bar{a}, +\bar{a}]$. Then

$$p_t = \text{prob}(a \geq \widehat{a}_t) = \frac{\bar{a} - \widehat{a}_t}{2\bar{a}}, \tag{OC .3}$$

$$a_t^* = E[a | a \geq \widehat{a}_t] = \frac{\bar{a} + \widehat{a}_t}{2}. \tag{OC .4}$$

To make the connection between the two-period model and the full model very explicit, write the cutoff condition as follows:

$$-\kappa + \bar{h}\underbrace{(1 + \widehat{a}_1)}_{J_1(\widehat{a}_1)} = 0 + E_1\left[p_2(-\kappa + \bar{h}\underbrace{(z_2 + a_2^*)}_{J_2(a_2^*)}) + (1 - p_2)0\right].$$

The expectations operator E_1 conditions on the information available in period $t = 1$. The entire right-hand side describes the value of waiting, J^U. It proves instructive to re-write it as follows:

$$\begin{aligned} J^U &= -\kappa E_1[p_2] + \bar{h}E_1[p_2 J_2(a_2^*)] \\ &= -\kappa E_1[p_2] + \bar{h}\Big(E_1[p_2]E_1[J_2(a_2^*)] + \text{Cov}_1[p_2, J_2(a_2^*)]\Big). \end{aligned}$$

To characterize the expectation terms on the right-hand side, use the fact that $\widehat{a}_2 = -\Delta$ in good times, and $\widehat{a}_2 = +\Delta$ in bad times. Thus,

$$\begin{aligned} E_1[p_2] &= \frac{1}{2}\left(\frac{\bar{a} - \Delta}{2\bar{a}}\right) + \frac{1}{2}\left(\frac{\bar{a} + \Delta}{2\bar{a}}\right) = \frac{1}{2}, \\ E_1[J_2(a_2^*)] &= \frac{1}{2}\left(1 + \Delta + \frac{\bar{a} - \Delta}{2}\right) + \frac{1}{2}\left(1 - \Delta + \frac{\bar{a} + \Delta}{2}\right) = 1 + \frac{\bar{a}}{2}, \\ \text{Cov}_1[p_2, J_2(a_2^*)] &= E_1\{(p_2 - E_1[p_2])(J(a_2^*) - E_1[J(a_2^*)])\} = \frac{\Delta^2}{4\bar{a}} \geq 0. \end{aligned}$$

Substituting these expressions into equation (OC .4) and cancelling terms, we find that the cutoff-level of productivity in period $t = 1$ above which entrepreneurs enter is

$$\widehat{a}_1 = \frac{\bar{a}}{4} + \frac{\Delta^2}{4\bar{a}}. \tag{OC .5}$$

The cutoff level is positive even in the absence of aggregate uncertainty, $\widehat{a}_{1,\Delta=0} = \frac{\bar{a}}{4}$, but the option value of waiting is greater when Δ is positive due to the covariance term, with $\frac{\partial \widehat{a}_1}{\partial \Delta} = \frac{\Delta}{2\bar{a}}$. Intuitively, what this covariance term captures is the following. When the aggregate state of the economy is high, even entrepreneurs with relatively low idiosyncratic productivity can recover the costs of posting a vacancy through production, so that the probability of having a draw that is sufficiently good to incentivize entry is greater. At the same time, the expected value of producing conditional upon entry is higher. The opposite is true in times of low productivity – when only few entrepreneurs draw sufficiently high idiosyncratic productivity values – but the decrease in expected profits is lower, because entrepreneurs with low draws avoid entering in the first place and, instead, make zero profits. An increase in aggregate volatility makes the covariance term correspondingly larger. To illustrate, Figures OC .1a and OC .1b are drawn for $\bar{a} = 0.1$ and $\Delta = 0.05$, such that $\widehat{a}_{1,\Delta=0} = 0.025$ and $\widehat{a}_{1,\Delta=0} = 0.03125$.

It is interesting to note that the effect on the cutoff from a change in the period-2 variance of aggregate productivity is larger for smaller values of $\bar{a}$ and becomes ill-defined as $\bar{a}$ approaches zero. The intuitive explanation is that when idiosyncratic dispersion is small, firms are clustered around the cutoff.[OC .5] In particular, when $\bar{a}$ is close to zero, tiny changes in Δ would imply hitting the bounds of the distribution. This is not taken into account by the derivative. For this reason, it is instructive to consider the case without idiosyncratic risk.

Aggregate uncertainty but no idiosyncratic uncertainty. Suppose that $a_{i,1} = a_{i,2} = 0$ for all firms, and that there is a unit mass of firms. Then in the absence of aggregate uncertainty, all entrepreneurs are indifferent between creating a firm and not doing so. Now suppose that a fraction $\frac{1}{2}$ of all firms does create a firm in the first period. Even a tiny introduction of aggregate uncertainty regarding z_2 will induce a discontinuous jump from $\frac{1}{2}$ to 1. This is effectively what the derivative $\frac{\partial \widehat{a}_1}{\partial \Delta} = \frac{\Delta}{2\bar{a}}$ tells us.

[OC .5] A second objective of our recalibration procedure is to counteract the fact that for greater degrees of dispersion changes in the cutoff level induce smaller changes in the probability of entry and, hence, the number of jobs created. To this end, we adjust the sensitivity of entrepreneurial profits to changes in aggregate productivity upward when idiosyncratic dispersion is greater. (This is achieved by making the entrepreneurial profit share in output smaller, an aspect we abstract from in the two-period model.)

OC .2 Variable hiring rate

Instead of the hiring rate being invariant to economic conditions, suppose now that there is congestion in the labor market.

Environment. Matches are determined according to a Cobb-Douglas function, as in the full model. As the separation rate is one and 'dead' entrepreneurs are replaced by new ones, the number of vacancies posted is simply $v_t = p_t\Upsilon$, so that the hiring rate is $h_t = \psi(\Upsilon p_t)^{-\alpha}$, where Υ is the mass of entrepreneurs, ψ is the matching efficiency, and $(1-\alpha)$ the elasticity of matches with respect to vacancies. As the goal here is analytical clarity rather than targeting a particular unemployment rate, we set Υ equal to one. To ensure that in the absence of uncertainty we have that $h = \bar{h}$ and $p = 1/2 \Leftrightarrow \widehat{a} = 0$, set $\psi = \bar{h}\left(\frac{1}{2}\right)^{\alpha}$ and $\kappa/\bar{h} = 1$.[OC .6]

Graphical analysis. Three key implications of the amended description of the model environment can be understood immediately from the graphical exposition in Figure OC .1. In particular, the bottom row of panels parallels the top row in terms of their construction, but now the hiring rate is endogenous.

Firstly, as before, there exist option-value effects due to idiosyncratic uncertainty ($\widehat{a}_{1,\Delta=0} > 0$) and due to aggregate uncertainty ($\widehat{a}_{1,\Delta>0} > \widehat{a}_{1,\Delta=0}$). In particular, the figure underscores that greater anticipated uncertainty over period-2 aggregate productivity renders expected period-2 profits higher, strengthening any motive for entrepreneurs to wait due to idiosyncratic productivity dispersion. As such, endogenous matching does not alter the central, qualitative message of section OC .1. There are, however, two forces that dampen the magnitude of such option-value effects.

For one, the sensitivity of the value of waiting to aggregate uncertainty is lessened. As Panel OC .1c shows, both the *increase* in profits in an expansion and the *decrease* in profits in a recession are muted relative to the case of a fixed hiring rate. Intuitively, in an expansion, congestion on the entrepreneurs' side of the matching market means that the hiring rate is lower than in steady state or when the hiring rate is fixed (as reflected in the flatter slope of the dashed profit line). Hence, the increase in profits is muted, as more vacancies posted remain unfilled. The opposite is true in a

[OC .6] In a model with positive wages, we would set $\kappa/\bar{h} = Q$, where Q denotes steady-state profits.

recession, when the probability of a successful match conditional on posting a vacancy is greater. Because the lower hiring rate in an expansion applies to a wider range of idiosyncratic productivity values, the net effect is to limit the rise in expected profits due to aggregate uncertainty. Therefore, endogenous matching dampens the rise in the option value of waiting when uncertainty over future aggregate productivity is elevated.

More importantly still, for any given change in the value of waiting, the increase in the period-1 cutoff and, thus, the impact of period-2 uncertainty on period-1 economic activity, is much smaller than when the hiring rate is fixed. For any given increase in expected period-2 profits due to either idiosyncratic or aggregate uncertainty, the equilibrium change in the period-1 cutoff and, hence, vacancy posting, is much smaller (Panel OC .1d). The reason is that any increase in the cutoff lowers the share of entrepreneurs entering and, hence, the amount of congestion. The endogenous rise in the hiring rate makes entering more attractive, leading to a partial offsetting of the increase in the cutoff that pushed up the hiring rate in the first place. The converse holds for a fall in the cutoff. This general equilibrium dampening force is suppressed when the hiring rate is constant.

While the graphical analysis communicates the central implications of allowing for matching frictions, for the interested reader we next consider again in detail the different configurations of idiosyncratic and aggregate uncertainty. We start with the case where idiosyncratic dispersion is zero but aggregate uncertainty is positive, as in the benchmark search-and-matching model.

Aggregate uncertainty but no idiosyncratic uncertainty. Suppose first that $a_{i,1} = a_{i,2} = 0$ for all firms. As in the case of a fixed hiring rate, in the absence of aggregate uncertainty all entrepreneurs are indifferent between creating a firm and not doing so. Suppose that initially a fraction $\frac{1}{2}$ does. In a marked difference from the previous case, now the introduction of a tiny amount of aggregate uncertainty does *not* lead to a discontinuous jump from $\frac{1}{2}$ to 1. To the contrary, when Δ is positive (but small), the entry probability p_2 and, hence, the hiring probability h_2 adjusts such that expected profits are equal to zero both when $z_2 = 1 + \Delta$ and when $z_2 = 1 - \Delta$.[OC .7] As such,

[OC .7] As emphasized in section **??**, this argument presumes that the constraint on the available number of entrepreneurs is sufficiently slack – respectively, the shocks sufficiently small –so that movements in the hiring rate can render expected profits equal to zero.

there is no option value of waiting and p_1 is still equal to $1/2$. Thus, the endogenous adjustment of the hiring probability plays a key role in ensuring that the free-entry condition holds in every state of the world.

Idiosyncratic uncertainty but no aggregate uncertainty. Next, consider the option value of waiting due to firm-level dispersion but absent aggregate uncertainty. With a variable hiring rate, the cutoff equation becomes

$$-\kappa + \underbrace{\psi p_1^{-\alpha}}_{h_1}(1+\widehat{a}_1) = p_2 \left(-\kappa + \underbrace{\psi p_2^{-\alpha}}_{h_2}(1+\widehat{a}_2) \right) > 0,$$

and it implies that the value of waiting in period 1 still is positive. For notice that our calibration ensures that when $\Delta = 0$, the period-2 cutoff is always equal zero, with half of all entrepreneurs entering, so that $h_2 = \bar{h}$. But the conditional expected value is positive provided $\bar{a} > 0$, that is, $a_2^* > \widehat{a}_2 = 0$.[OC .8] Thus, an increase in idiosyncratic dispersion, that is in $\bar{a}$, unambiguously raises the right-hand side by raising a_2^*. For the left-hand side to increase also, it must be that $\widehat{a}_1$ increases, as p_1 is strictly decreasing in $\bar{a}$ when $\widehat{a}_1 > 0$.

In contrast to the case with fixed matching probabilities, however, $\widehat{a}_1$ increases more gradually as the degree of idiosyncratic dispersion increases. The reason is that a variable hiring rate introduces a natural dampening effect: when the cutoff is higher and more entrepreneurs choose to wait, the hiring probability increases, which makes entering more attractive again. Specifically, whereas in the case of a fixed hiring rate, $\frac{\partial J_1(\widehat{a}_1)}{\partial \widehat{a}_1} = \bar{h}$, we now have

$$\frac{\partial J_1(\widehat{a}_1)}{\partial \widehat{a}_1} = h_1 + (1+\widehat{a}_1)\frac{\partial h_1}{\partial \widehat{a}_1}, \qquad \text{where}$$
$$\frac{\partial h_1}{\partial \widehat{a}_1} = \frac{\alpha}{2\bar{a}} \left(\frac{\bar{a} - \widehat{a}_1}{2\bar{a}} \right)^{-(\alpha+1)}.$$

As such, a small increase in $\widehat{a}_1$ following a rise in idiosyncratic uncertainty is sufficient for the

[OC .8] The period-2 cutoff level, $\widehat{a}_2$, satisfies $-\kappa + \psi p_2^{-\alpha}(1+\widehat{a}_2) = 0$ and given the uniform distribution we have that $p_2 = \frac{\bar{a}-\widehat{a}_2}{2\bar{a}}$.

left-hand side to be equal to the – now greater – right-hand side.[OC .9]

Idiosyncratic and aggregate uncertainty. Finally, when both $\bar{a} > 0$ and $\Delta > 0$, the cutoff equation with endogenous matching is

$$-\kappa + h_1(1 + \widehat{a}_1) = E_1\left[p_2(-\kappa + h_2(z_2 + a_2^*))\right]. \tag{OC .6}$$

As in the case without aggregate uncertainty that we just considered, for any given change in $\widehat{a}_1$, the fact that h_1 is an increasing function of $\widehat{a}_1$ introduces a first dampening force relative to the model with fixed hiring rate. What about the magnitude of the value of waiting?

Consider the right-hand side of equation (OC .6), that is, J^U. In period 2, we have the following two expressions that implicitly define the cutoff levels (recall that p_2 is a function of the cutoff itself).

$$-\kappa + h_{2,+}(1 + \Delta + \widehat{a}_{2,+}) = 0 \Leftrightarrow \widehat{a}_{2,+} = \frac{\kappa}{\psi}\left(\frac{\bar{a} - \widehat{a}_{2,+}}{2\bar{a}}\right)^{\alpha} - 1 - \Delta,$$
$$-\kappa + h_{2,-}(1 - \Delta + \widehat{a}_{2,+}) = 0 \Leftrightarrow \widehat{a}_{2,-} = \frac{\kappa}{\psi}\left(\frac{\bar{a} - \widehat{a}_{2,-}}{2\bar{a}}\right)^{\alpha} - 1 + \Delta.$$

Thus, relative to the case with fixed hiring rate, the cutoff in an expansion, $\widehat{a}_{2,+}$, is higher, while the cutoff in a recession, $\widehat{a}_{2,-}$, is lower. The reason is that the effective hiring costs to be covered by revenue over the duration of a match are greater (smaller) in an expansion (recession), when the hiring probability is lower (higher).

[OC .9] This relationship is non-linear, insofar as a change in the cutoff of a given size leads to a more modest change in the entry probability and, hence, hiring rate for a more dispersed distribution.

We can rewrite the value of waiting as a a function of these cutoffs.

$$
\begin{aligned}
J^U &= \frac{1}{2}E\left[p_{2,+}\left(-\kappa+\psi(p_{2,+})^{-\alpha}(1+\Delta+a)\right)|a \geq \widehat{a}_{2,+}\right] \\
&+\frac{1}{2}E\left[p_{2,-}\left(-\kappa+\psi(p_{2,-})^{-\alpha}(1-\Delta+a)\right)|a \geq \widehat{a}_{2,-}\right], \\
&= \frac{1}{2}\left[-\kappa\left(\frac{\bar{a}-\widehat{a}_{2,+}}{2\bar{a}}\right)+\left(\psi\left(\frac{\bar{a}-\widehat{a}_{2,+}}{2\bar{a}}\right)^{1-\alpha}\left(1+\Delta+\frac{\bar{a}+\widehat{a}_{2,+}}{2}\right)\right)\right] \\
&+\frac{1}{2}\left[-\kappa\left(\frac{\bar{a}-\widehat{a}_{2,-}}{2\bar{a}}\right)+\left(\psi\left(\frac{\bar{a}-\widehat{a}_{2,-}}{2\bar{a}}\right)^{1-\alpha}\left(1-\Delta+\frac{\bar{a}+\widehat{a}_{2,-}}{2}\right)\right)\right].
\end{aligned}
$$

While no explicit solution is possible, the expressions nevertheless reveal very intuitive implications of making the hiring rate endogenous.[OC .10] Recall that in the case of a fixed hiring rate (nested here for $\alpha = 0$), the option value of waiting was increasing in the anticipated variance of period-2 productivity, Δ, because the entry probability and the conditional expected value of a match co-varied positively, that is, $\mathrm{Cov}_1[p_2, J_2(a_2^*)] > 0$. This same effect is still operative here, but its quantitative magnitude is dampened, because even though in an expansion the probability of entry is higher, congestion externalities mean that the hiring probability is lower, which limits the conditional expected value of entering. The converse holds true in a recession. That is, $\mathrm{Cov}_1[h_2, J_2(a_2^*)] < 0$. The expressions also underscore that since the hiring rate only ever moves due to changes in the entry probability, this mechanism can only ever dampen but not completely eliminate the option value of waiting due to aggregate uncertainty, provided that $\bar{a} > 0$.

In summary, the two-period model illustrates the existence of option-value considerations in a model with a finite mass of entrepreneurs and firm-specific productivity. Allowing for congestion in the matching market is crucial because it ensures that introducing even the tiniest amount of aggregate uncertainty does not lead to a discontinuous rise in the value of waiting. At the other end of the extreme, in the absence of heterogeneity in firm-specific productivity (and presuming a sufficiently large number of potential entrepreneurs), such endogenous variations in the hiring probability entirely eliminate any option-value effects. The model with a variable hiring rate, as in

[OC .10] An additional mechanism is that the rise in the hiring rate in a recession is greater than its decline in a boom – the matching function being concave – so that aggregate uncertainty raises the expected hiring rate.

the standard search-and-matching environment, and a finite mass of entrepreneurs that vary in terms of their productivity draws represents a potentially attractive middle way.

Appendix OD Robustness exercises

This appendix section reports the results of several robustness checks regarding the novel search-and-matching (SaM) model with a finite mass of entrepreneurs and firm-specific productivity.

OD.1 Stochastic vs. non-stochastic hiring specification

The model in section 4.1 assumes that in any period t, an unmatched entrepreneur can post a single vacancy, which costs κ units of final goods. The vacancy turns into a match with probability h_t. Otherwise the vacancy remains unfilled and the entrepreneur faces a new decision about whether to remain on the sidelines or post a vacancy at the beginning of the following period. We will refer to this specification as "stochastic hiring." As it is probably the most common way of formulating the SaM model, used in particular also by Leduc and Liu (2016, esp. their eqn. 22), we adopt it as our baseline. An alternative specification supposes that the entrepreneur posts $1/h_t$ vacancies, each costing κ units of the final good, and then creates one job with certainty. We will refer to this case as "non-stochastic hiring."

In the canonical SaM model with risk-neutral entrepreneurs, the two specifications yield isomorphic equilibrium conditions; only the suggested interpretations potentially differ. Either way, in equilibrium, labor market tightness — and, hence, h_t — adjusts such that the expected profit from a new job is equal to the expected cost of hiring a worker. The value of waiting is zero by construction. Under stochastic hiring, it is most natural to write $h_t J_t = \kappa$, meaning that in equilibrium the value of a match discounted by the probability of a vacancy being filled is equal to the fixed cost of posting that vacancy. Under non-stochastic hiring, writing $J_t = \kappa(1/h_t)$ is more natural, in which case equilibrium requires the value of a match to be equal to the expected cost of posting a vacancy until it is filled.

In our modified framework, entrepreneurs are also risk neutral. Nevertheless, the quantitative model outcomes are somewhat different for these two different hiring specifications. In brief, the reason is that under stochastic hiring, expected profits from waiting now and posting a vacancy later are affected by the hiring probability covarying negatively with the match value; the same is not true under non-stochastic hiring. We next discuss these properties in some more detail.

The intuition is most easily described with reference to the two-period model. Recall that in our baseline, the indifference condition is (cf. equation (OC .6))

$$-\kappa + h_1(1+\widehat{a}_1) = E_1\left[p_2(-\kappa + h_2(z_2 + a_2^*))\right].$$

In contrast, when entrepreneur can post multiple vacancies, the condition is

$$-\kappa/h_1 + (1+\widehat{a}_1) = E_1\left[p_2(-\kappa/h_2 + (z_2 + a_2^*))\right].$$

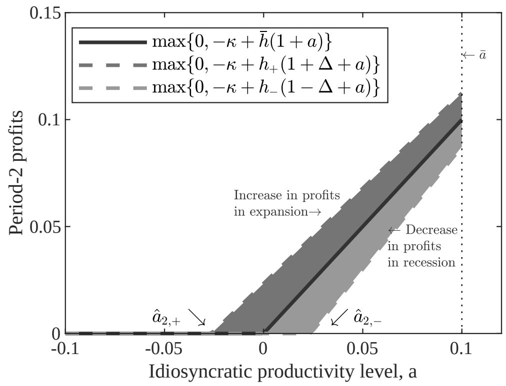
Now, the level of idiosyncratic productivity above which an entrepreneur would rather enter than wait (i.e., not produce at all in the two-period setting), $\widehat{a}_2$, and, hence, the conditional expected value of a draw, a_2^*, are the same across the alternative hiring cost specifications. Instead, the key difference between these two equations is that in the former case, the *expectation* on the right-hand side, conditional on period-1 information, includes a product term involving h_2 and z_2, so that their negative co-movement lowers the value of waiting. By contrast, since under non-stochastic hiring $1/h_2$ multiplies a term that is invariant to uncertainty regarding z_2 (viz., the fixed costs κ), this same mechanism is not operative.[OD.1]

The upper row of panels in Figure OD.1 uses the graphical apparatus set out in section OC to describe the value of waiting due to aggregate uncertainty under the two alternative specifications. As is clearly visible, the expected value of waiting – the difference between the increase in profits in an expansion and the decrease in profits in a recession when integrating over all possible productivity draws above the respective cutoffs – is greater under the alternative, non-stochastic hiring specification. The reason is that the benefit from a higher idiosyncratic productivity draw in an expansion is not scaled down by greater congestion that lowers the probability of that draw

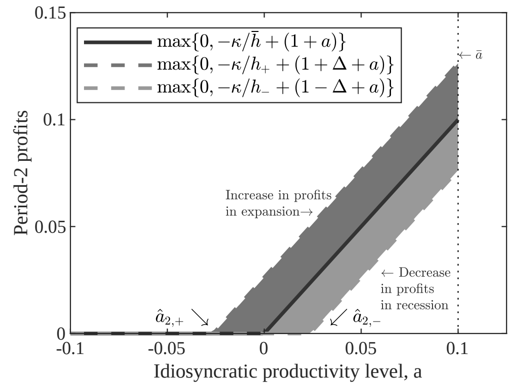
[OD.1] For completeness, notice that the division of κ by h_2 under non-stochastic hiring affects the expected value of hiring costs in the presence of aggregate uncertainty. We consider a quantitative assessment of the two specifications below.

Figure OD.1: Stochastic vs. non-stochastic hiring specification

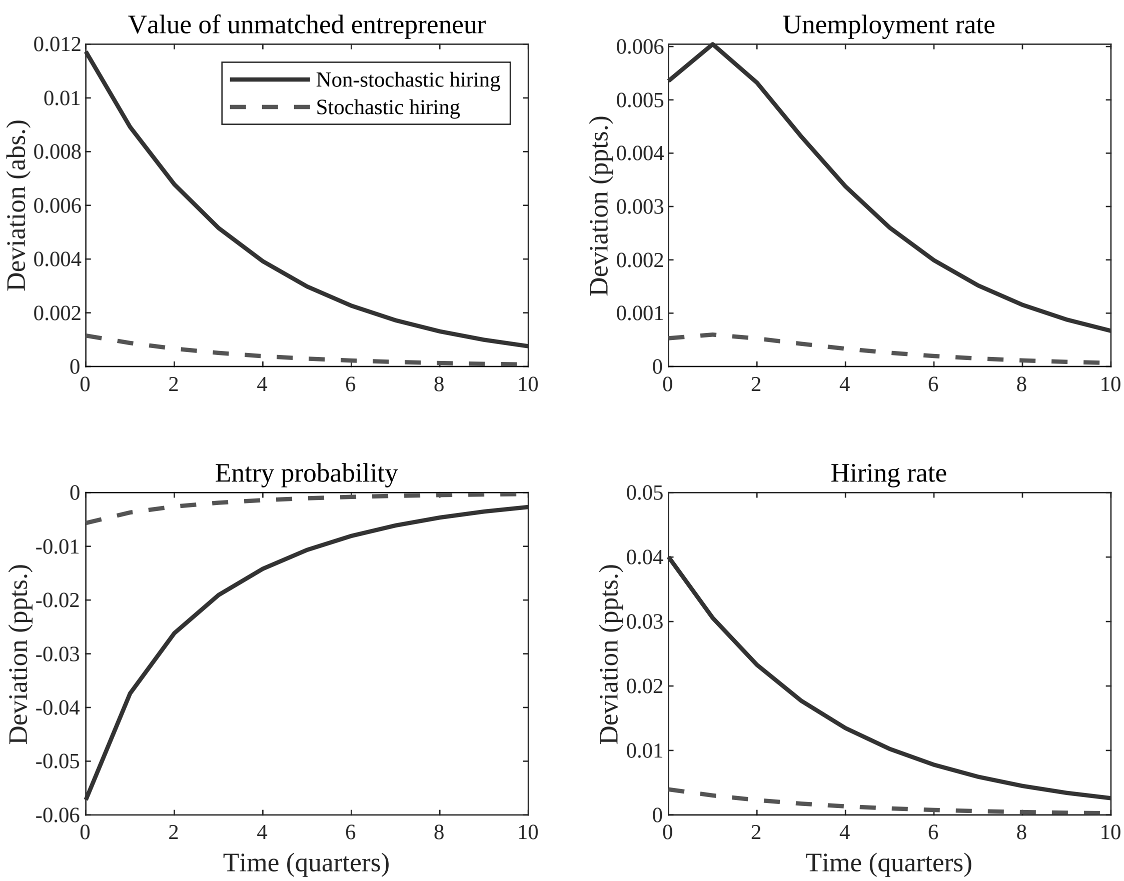
(a) 2-period model: stochastic hiring



(b) 2-period model: non-stochastic hiring



(c) Pure uncertainty IRFs in the full model, $\sigma_a = 0.001$



Notes: Panels **??** and OD.1b show how the value of waiting is determined in a stylized, two-period setting. The left panel corresponds to Figure OC .1c and relies on the "stochastic hiring" assumption. The right panel instead captures the "non-stochastic" specification. To make the visual comparison straightforward, the steady-state hiring rate, h, is set equal to one; the no-aggregate-uncertainty line is then the same for both specifications. For the full, infinite-horizon model, Panel OD.1c plots the usual pure uncertainty IRFs to a unit-increase in $\varepsilon_{\sigma,t}$.

actually yielding profits in a filled match. The opposite holds in a recession, but then these relatively higher probability of hiring boosts expected profits only over a smaller range of productivity draws. (This is the flipside of the argument presented in section OC .2, of course.) Thus, the comparison of alternative hiring specifications informs us that firm size potentially matters for the transmission of uncertainty shocks even under constant returns to scale in production, insofar as the option-value effect will be stronger when an entrepreneur can post several vacancies instead of just one.

How do these different specifications shape the predictions of the full, infinite-horizon model for the effects on economic activity of an anticipated increase in volatility?[OD.2] The solid line in Figure OD.1c graphs the pure uncertainty IRFs for the value of an unmatched entrepreneur, unemployment, the entry probability and the hiring rate, when $\sigma_a = 0.001$ for the non-stochastic hiring specification. The dashed line represents the stochastic hiring benchmark. In either model, the recalibration procedure described in section 4.2 is applied, which facilitates comparison. Consistent with the reasoning developed in the context of the two-period model, the uncertainty effects are several magnitudes larger given the non-stochastic hiring specification than in the baseline. The analysis thus indicates that the results in the main text, which are based on the stochastic hiring assumption, represent a conservative, lower bound for the effects of uncertainty shocks.

[OD.2] To be explicit, under either specification the equation pinning down the cutoff level $\widehat{a}$ for which an entrepreneur is indifferent between going to the matching market is

$$-\kappa/h_t = J_t(\widehat{a}_t) - \beta E_t[J^U_{t+1}].$$

But under the non-stochastic hiring specification the beginning-of-period t of an unmatched entrepreneur is given by

$$J^U_t = \beta E_t[J^U_{t+1}] + p_t\left(-\frac{\kappa}{h_t} + (J_t(a^*_t) - \beta E_t[J^U_{t+1}])\right),$$

with steady-state value $J^U = \frac{p(-\kappa/h+J(a^*))}{1-\beta(1-p)}$. Previously, we had

$$J^U_t = \beta E_t[J^U_{t+1}] + p_t\left(-\kappa + h_t(J_t(a^*_t) - \beta E_t[J^U_{t+1}])\right),$$

and $J^U = \frac{p(-\kappa+h*J(a^*))}{1-\beta(1-ph)}$.

OD.2 Normal distribution

Our baseline specification of the model with heterogeneity in entrepreneurial productivity assumes that firm-level productivity has a uniform distribution in the interval $[-\sqrt{3}\sigma_a, \sqrt{3}\sigma_a]$. Methodologically, our baseline assumption has the advantage that in the recalibration step (cf. section 4.2) we can analytically solve for the elasticity of labor market tightness with respect to productivity. In addition, the uniform distribution ensured consistency between the full model and the analytical, two-period model presented in section OC . This appendix section shows that the results are highly comparable when, instead, firm-level productivity draws comes from a normal distribution, that is, when $a \sim \mathcal{N}(0, \sigma_a^2)$.

Focusing on the implications of firm-level dispersion in the absence of aggregate uncertainty first, the solid lines in the upper panels of Figure OD.2 report the steady-state value of an unmatched entrepreneur, J^U, as well as the unemployment rate as a function of σ_a. To ease comparison, the dashed line repeats the results under a uniform distribution. Unsurprisingly, the qualitative patterns are the same. The only model equations that are directly affected by functional choices for $F(a)$ are the expressions for $p = \text{prob}[a \geq \widehat{a}]$ and $a^* = E[a \geq \widehat{a}]$. In either case, greater values of σ_a increase the value of remaining unmatched, as doing so preserves the option of getting another, better draw in the future. Without recalibrating the model parameters, this effect lowers lower vacancy posting and, hence, increases steady-state unemployment.[OD.3]

Next, Figure OD.2b reports the IRFs for an increase in anticipated future aggregate volatility under the two alternative assumptions. Again the results are highly comparable; the effect sizes are just slightly more pronounced under the assumption of a uniform distribution. These quantitative differences further illustrate the point made in section 4.1 that aggregate uncertainty shocks have a smaller impact for larger values of σ_a when the hiring rate is endogenous.[OD.4] With a 'tighter' distribution, here in the sense of the normal distribution having greater kurtosis than the uniform,

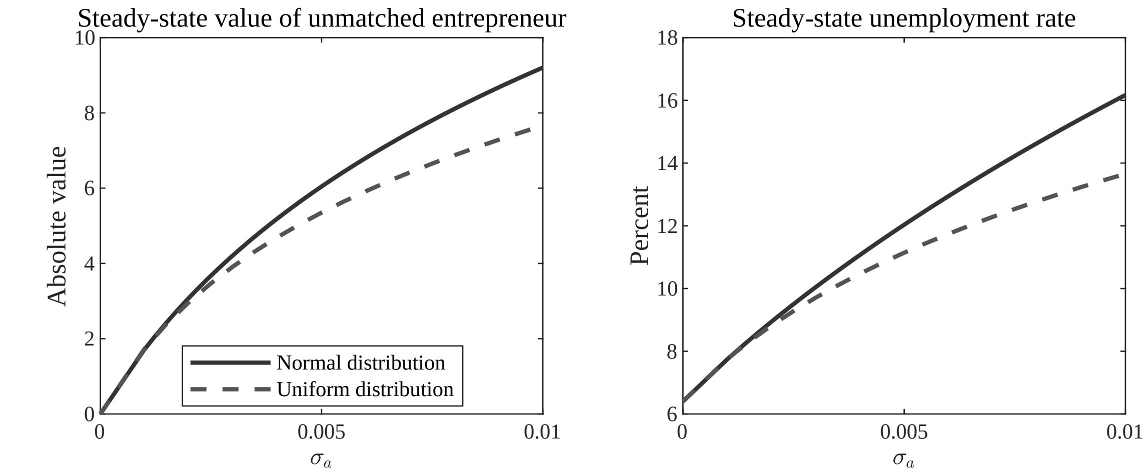
[OD.3] Quantitatively, the strength of this mechanism increases slightly more with σ_a when assuming a normal distribution. For even though idiosyncratic uncertainty raises the conditional expected value of a draw by more under a uniform distribution, the mass of marginal entrepreneurs (i.e., near the indifference point) is greater under a normal distribution.

[OD.4] This result rests, of course, on the assumption that in steady state the cutoff is at or near the center of the normal distribution.

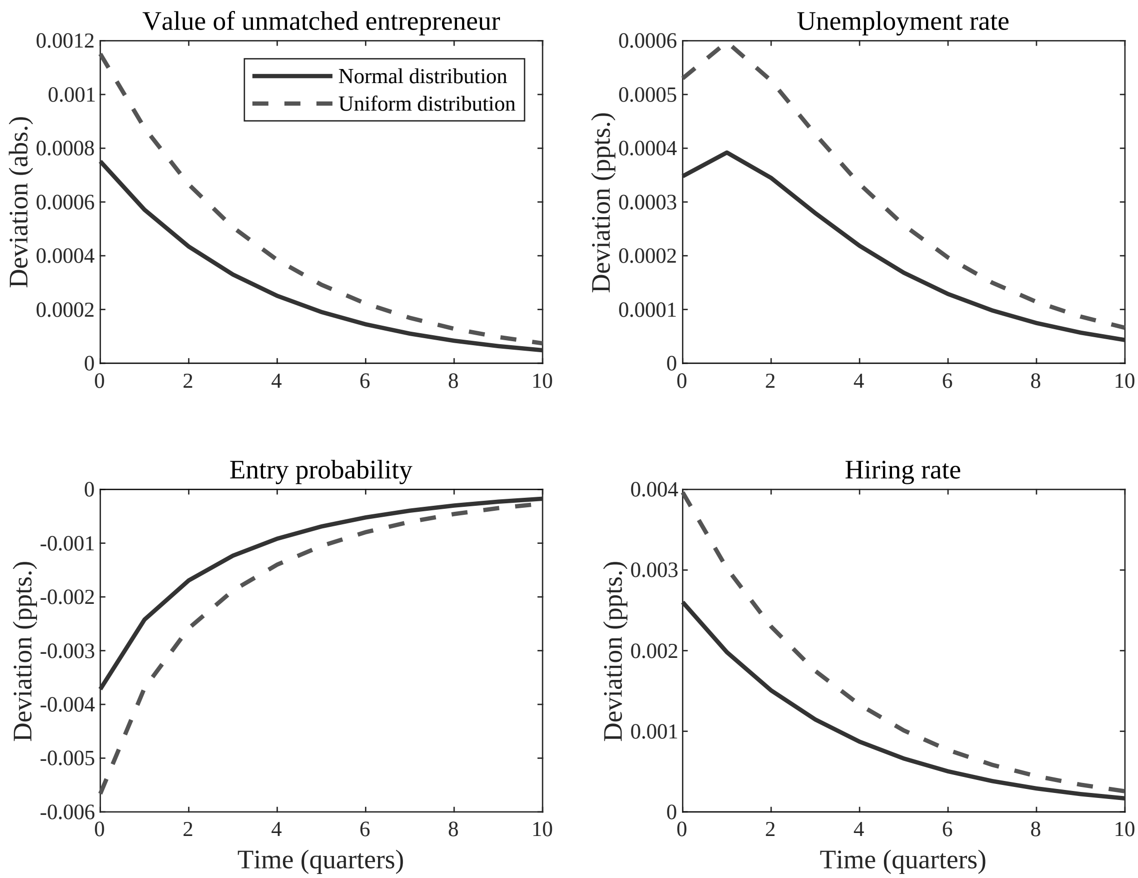
entrepreneurs are more similar. Accordingly, we are closer to the benchmark case where profits are completely eroded due to variations in the hiring rate due to (expected) movements in aggregate productivity.

Figure OD.2: Uncertainty effects with normal distribution for firm-level productivity

(a) Steady-state effects of greater idiosyncratic dispersion



(b) Pure uncertainty IRFs ($\sigma_a = 0.001$)

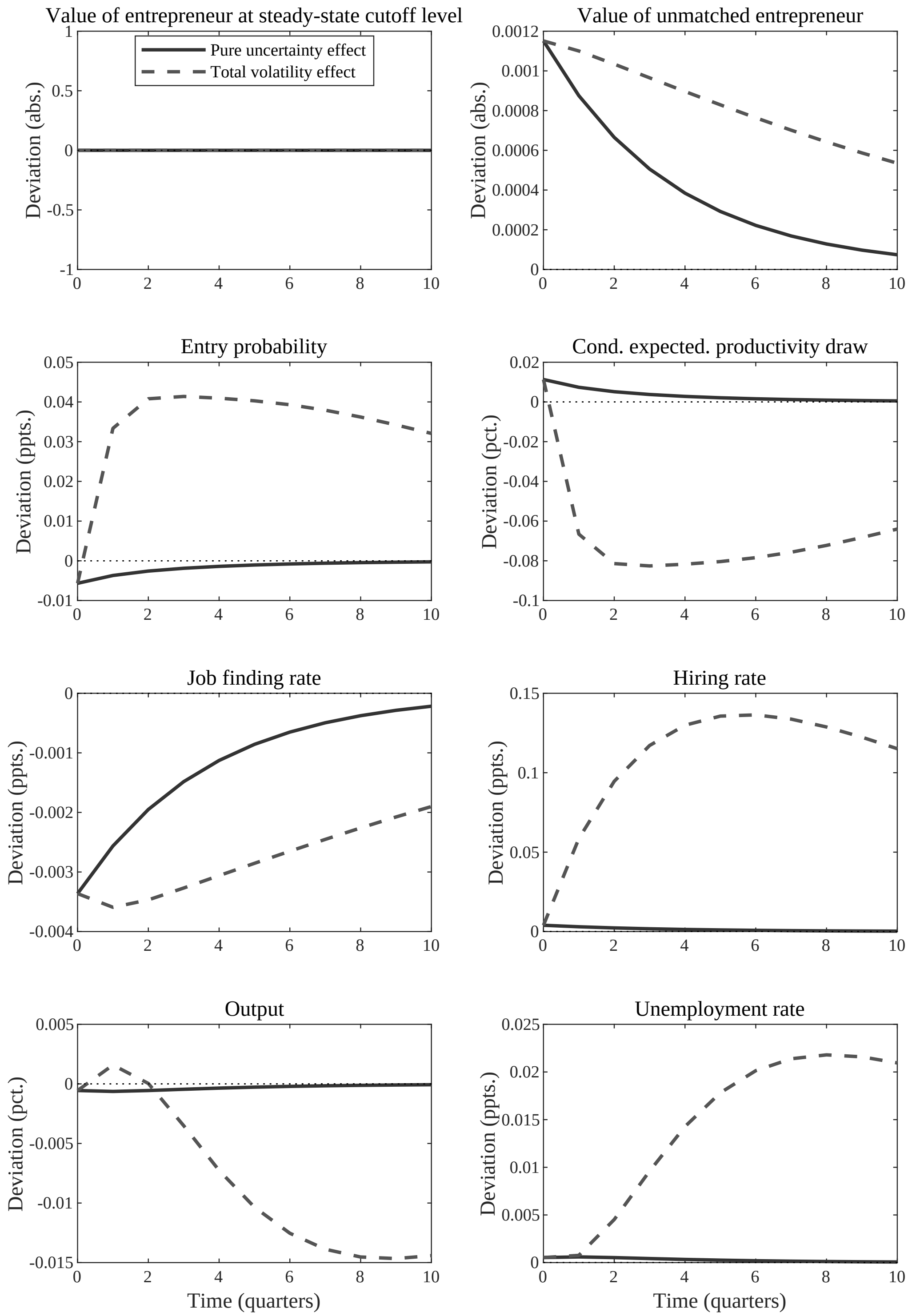


Notes: The two panels in Figure OD.2a describe the steady-state effects of increasing firm-level dispersion, as measured by σ_a, assuming a normal (solid line) or uniform (dashed line) respectively. The panels in Figure OD.2b reports pure uncertainty IRFs to a unit-increase in $\varepsilon_{\sigma,t}$ under these two assumptions, imposing $\sigma_a = 0.001$ and $P = 0.5$. In the latter case, the model parameters are recalibrated following the procedure described in section 4.2.

OD.3 Different degree of cross-sectional dispersion

Figure 5 indicated pure uncertainty and total volatility IRFs conditional on the standard deviation of idiosyncratic productivity shocks, σ_a, being equal to 0.003. Figure OD.3 reports the same IRFs when $\sigma_a = 0.001$ instead. A comparison of the two figures reveals that the magnitude of uncertainty effects is generally larger for the higher value of σ_a, in line with what is shown in figure 5 for the unemployment rate, specifically. Interestingly, the composition effect – following a volatility shock the average productivity of active firms initially rises – is now sufficiently strong for the total volatility effect on output to reach positive territory, albeit only very briefly, before the negative impact of rising unemployment kicks in.

Figure OD.3: IRFs for uncertainty shock in SaM model with cross-sectional dispersion; $\sigma_a = 0.001$



Notes: The "total volatility" IRFs plot the change in the period-0 expected values of the indicated variables in response to a unit-increase in $\varepsilon_{\sigma,t}$. The model parameters are recalibrated following the procedure described in section 4.2. In particular, $\chi = 0.682$, $\omega = 0.885$, and $\bar{z} = 0.999$.

OD.4 Different steady-state entry probability

In the baseline analysis, we calibrated the model such that in steady state half of the available entrepreneurs actually post a vacancy. As noted in the main text, this choice has a number of practical advantages. For completeness, we report the results of a robustness exercise in which the steady-state value for P is 0.2 instead of 0.5.

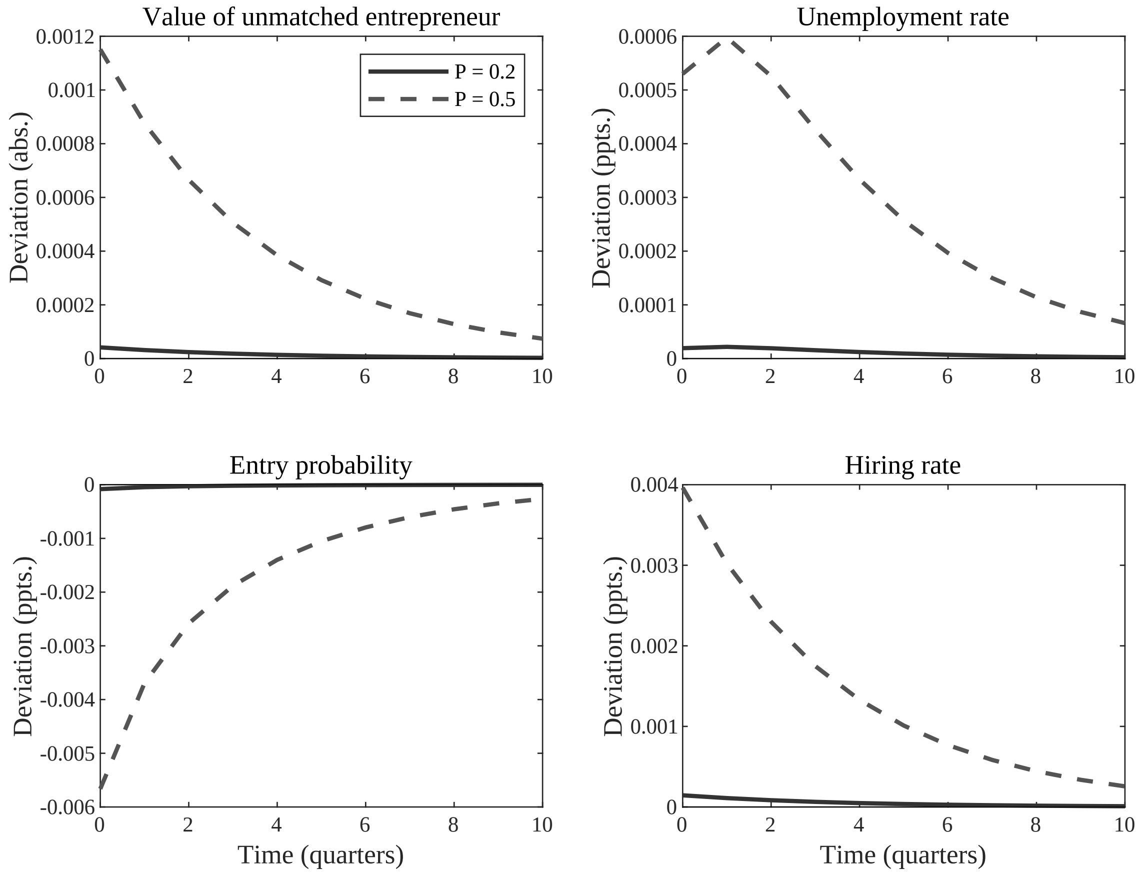
Figure OD.4 compares the pure uncertainty IRFs for the value of an unmatched entrepreneur and unemployment, as well as the entry probability and the hiring rate, when $\sigma_a = 0.001$ for both $P = 0.2$ (solid line) and $P = 0.5$ (dashed line; as in figure OD.3).[OD.5] Clearly, the value of waiting due to greater anticipated uncertainty rises by less in the latter case, and accordingly unemployment worsens by less also. The reason is that a lower steady-state entry probability is associated with a larger mass of potential entrepreneurs, Υ. As such, for any given degree of firm-level dispersion we are closer to the benchmark SaM model with an infinite mass of potential entrepreneurs. In that latter model, the option value of waiting, and thus also pure uncertainty effects given risk neutrality and linear wages, are zero. In more economic terms, with more potential entrepreneurs, adjustments in the hiring rate in both good and bad productivity states are sufficient to leave expected profits nearly invariant to changed in anticipated volatility.

Imposing a lower steady state probability of entry means that the admissable range of σ_a is wider when recalibrating the model as described in section 4.2 (as the worker bargaining power parameter ω hits the lower limit of zero for higher values of σ_a when P is lower, that is, when the mass of potential entrepreneurs is greater). In this spirit, the solid line in Figure OD.4b indicates the impact effect of an uncertainty shock as a function of σ_a when $P = 0.2$. The dashed line refers to the same statistic for a fixed value $\sigma_a = 0.001$ but with $P = 0.5$. This analysis makes clear that for a lower steady-state entry probability, the option-value effect due to anticipated greater future volatility matches that obtained with a higher steady-state entry probability for a greater degree of firm-level dispersion.

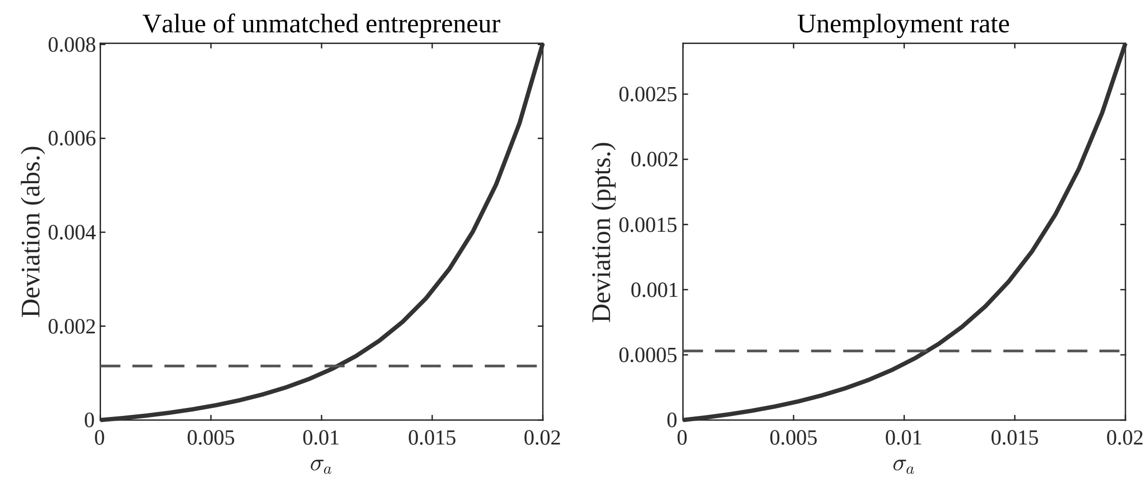
[OD.5] Given our recalibration procedure, steady-state unemployment is the same across the different specifications.

Figure OD.4: Uncertainty effects with lower steady-state entry probability

(a) Pure uncertainty IRFs ($\sigma_a = 0.001$)



(b) Impact effects of uncertainty shock for different σ_a values ($P = 0.2$)



Notes: The upper panel reports pure uncertainty IRFs to a unit-increase in $\varepsilon_{\sigma,t}$, assuming that $\sigma_a = 0.001$ and that the steady-state entry probability is either $P = 0.2$ (solid line) or $P = 0.5$ (dashed line). The bottom panel plots the impact effect in the case of $P = 0.2$ as a function of σ_a (solid line). Here, the dashed line describes the impact effect for a fixed value of $\sigma_a = 0.001$ and with $P = 0.5$. The model parameters are recalibrated following the procedure described in section 4.2.

References for online appendix

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