Volatile Hiring: Uncertainty in Search and Matching Models[*]

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Abstract

In search-and-matching models, the nonlinear nature of search frictions increases average unemployment rates during periods with higher volatility. These frictions are not, however, by themselves sufficient to raise unemployment following an increase in perceived uncertainty; though they may do so in conjunction with the common assumption of wages being determined by Nash bargaining. Importantly, option-value considerations play no role in the standard model with free entry. In contrast, when the mass of entrepreneurs is finite and there is heterogeneity in firm-specific productivity, a rise in perceived uncertainty robustly increases the option value of waiting and reduces job creation.

Keywords: Uncertainty, search frictions, unemployment, option value.

JEL Classification: E24, E32, J64.

1 Introduction

There is a large empirical literature which demonstrates that uncertainty is time-varying and that increased volatility negatively affects macroeconomic activity; even an increase in perceived uncertainty has been shown to lead to negative outcomes.[1] Understanding the mechanisms behind these empirical results is not trivial. In fact, several models would predict the opposite. Precautionary motives call forth a rise in savings, which in many macroeconomic models would be associated with increased investment. Also, limited liability means that firm owners' payoff function is convex, which implies that uncertainty increases firm equity value and makes investment more attractive.

Leduc and Liu (2016) provide important contributions to both the empirical and the theoretical literature. Empirically, they show that an increase in observed perceived uncertainty leads to an increase in the unemployment rate. Moreover, they demonstrate that a standard search-and-matching model (SaM) can replicate this finding, whereas – as pointed out above – many other theoretical models cannot. In fact, they show that such a model can generate the desired result even under flexible prices and with prudent agents, a combination that, by itself, typically pushes economic activity in the opposite direction. Leduc and Liu (2016) do not bring to the surface what mechanism lies behind this result. But they conjecture that matching frictions produce the famous option value of postponing investment; increased uncertainty can make it more attractive to wait and postpone investment, including vacancy-posting, under certain conditions (cf. Bernanke (1983)).[2]

The first contribution of this paper is to provide an in-depth analysis of the effects of increased volatility in SaM models, examining in particular whether there is an option-value channel. Since job creation is very much like an irreversible investment, as the associated costs are not refundable, the option-value channel is a sensible candidate to consider. However, a key finding of this paper

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[1]On which see, among many others, Jurado *et al.* (2015), Baker *et al.* (2016), and Bloom *et al.* (2018). For excellent surveys of the literature, see Bloom (2014) and Fernández-Villaverde and Guerrón-Quintana (2020).

[2]An additional contribution of Leduc and Liu (2016) is to show the importance of embedding their SaM framework into a New Keynesian framework with price rigidities. Together with a nonlinear household utility function, these give rise to an aggregate demand channel which ensures that the model can generate quantitatively substantial effects following uncertainty shocks. This paper, however, focuses exclusively on the case with flexible prices. Even though we acknowledge that the demand channel is important for the quantitative impact of uncertainty shocks, keeping our focus on the flexible-price case makes the analysis more transparent.

is that the nonlinearities of the matching function by themselves are *not* enough for an increase in perceived uncertainty to have an effect on the economy. In fact, we show that Leduc and Liu's (2016) result that an increase in perceived uncertainty leads to an increase in the unemployment rate in a standard SaM model with flexible prices depends crucially on the assumption that wages are determined by Nash bargaining, an assumption that is often adopted in the SaM literature. However, it is not an integral part of the SaM framework; many other types of wage setting assumptions are possible. Specifically, we show that changes in perceived uncertainty have no effect on job creation when wages are linear in productivity. Thus, the standard SaM features themselves do not lead to any option value of waiting. The intuition is simple. The free-entry assumption implies that the expected value of vacancy posting is, and will always remain, equal to zero. Hence, there is no point to waiting. However, the nonlinearities of the matching friction do imply that the *average* value of labor market tightness – that is, the number of vacancies relative to the number of unemployed workers searching for a job – is elevated during periods of higher *realized* volatility. Under Nash bargaining, this improves workers' bargaining position and raises the average wage. Even if higher volatility is not realized, the increase in wages is driven by what agents *expect* to happen when perceived uncertainty increases. As a result, the firm value falls, which reduces job creation.

The literature often focuses on the impact of an increase in *perceived* uncertainty, that is, the impact that is solely due to beliefs, not to an actual increase in volatility. Our second contribution is to highlight the importance of analyzing the impact of *realized* increases in volatility (measured as the impact averaged over all possible realizations). As made clear in the preceding paragraph, one reason is that model predictions for the effect of an increase in perceived uncertainty are determined by what agents expect to happen during the period of heightened volatility. Moreover, both in the standard SaM and in modifications we consider, we find that *even if* the anticipation of uncertainty itself does have a non-zero impact on the economy, these effects are small relative to those induced by realized volatility. What is more, the effects of realized and perceived volatility can differ in sign along the IRF over at least some horizons.

Our third contribution is to demonstrate how wait-and-see considerations can be introduced into

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the SaM framework. We proceed in two steps. Our starting point is to observe that virtually all SaM models assume that there are always enough potential entrepreneurs available to drive the expected profits of job creation to zero. We first highlight that an option-value channel is in principle possible by relaxing the free-entry condition and assuming that the mass of entrepreneurs is finite.[3] However, the resulting channel is only operative under restrictive assumptions. In particular, the free-entry condition must be binding in some states, such that firms make zero profits, but not in others, such that firms make positive profits. In a second step, we therefore add heterogeneity in idiosyncratic firm-productivity alongside the assumption of a finite mass of entrepreneurs. In the resulting SaM framework, there is a time-varying measure of entrepreneurs (i.e., those with a sufficiently high productivity draw) that *always* expect to make strictly positive profits when they post a vacancy. With this relatively simple modification, the model robustly predicts that perceived uncertainty leads to a postponement of job creation.

The reason is that an expected increase in future volatility increases an unmatched entrepreneur's chance of having a productivity draw for which expected profits of vacancy-posting are positive, whereas the downside risk is not affected since unmatched entrepreneurs can always choose to stay out of the market. Importantly, the introduction of heterogeneity often leads to substantial challenges for solution methods, but this is not the case here; indeed our proposed model can be solved by (higher-order) perturbation methods.

Our study is connected to two broad strands of the economic literature, namely, analyses considering frictional labor market models and the effects of uncertainty, respectively. While each of these literatures is vast in scope, we briefly comment on the most closely related studies. In particular, Bloom (2009) offers a seminal examination of the sort of real-options effects analyzed also in this paper. His model incorporates option-value considerations in hiring and (physical) investment space due to non-convex adjustment costs. We consider a particularly prominent variant of adjustment costs, namely search frictions in the labor market, and identify the conditions under

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[3]We thus share with Coles and Kelishomi (2018) an emphasis on the restrictive nature of the standard free-entry condition Diamond-Mortensen-Pissarides framework. While their study identifies the empirical difficulties caused by adopting this assumption with respect to level shocks, we concentrate on the implications of this assumption for the effects of uncertainty shocks and, in particular, the presence or absence of option-value effects.

which an option-value effect materializes; and when it does not.

Considering studies that share this focus – and beyond Leduc and Liu (2016), discussed in detail above –, the most closely related paper is Schaal (2017).[4] That paper develops a model with multi-worker firms that are heterogeneous in productivity and which are subject to an endogenous linear hiring cost at the firm level.[5] In similarity to our proposed model, the resulting irreversibility gives rise to an option value of waiting. In contrast to our approach, the free-entry condition binds in every state of the world. The reason that the value of vacancy-posting nonetheless varies over time in Schaal's (2017) setup is that firms operate a decreasing returns to scale technology, and the free-entry condition obtains at the level of the (multi-worker) firm rather than the vacancy. Moreover, Schaal (2017) assumes directed search. Given our starting point, we instead try to stay as close as possible to the canonical Diamond-Mortensen-Pissarides assumptions of constant returns to scale in production and random search, while restoring an option-value channel.

The next section describes the SaM model of Leduc and Liu (2016) with linear utility and flexible prices. Section 3.1 reports the effects on the economy both to an increase in *perceived uncertainty* as well as to an increase in volatility that does indeed materialize. Sections 3.2-3.4 dissect the model and analyze the results in detail to reach a full understanding of the role of matching frictions and wage setting for the impact of uncertainty shocks on economic activity. Section 4 discusses our modifed SaM model in which there is an option value of postponing job creation. The last section concludes.

2   Theoretical Framework

In this section, we describe the basic search-and-matching (SaM) model. The two main differences with Leduc and Liu (2016) are that we restrict ourselves to the flexible price version of the model and assume that the representative household is risk neutral. Both assumptions are common in

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[4]Fasani and Rossi's (2018) comment on Leduc and Liu (2016) likewise discusses the results through the lens of option-value effects.

[5]For a related approach, see Riegler (2019).

the matching literature. For us they have the benefit of making the analysis more transparent. Specifically, as shown in Bernanke (1983) and our example in section 3.2, the option value of waiting does not rely on risk aversion.[6] Nor are sticky prices necessary.

2.1 Households

The representative household consists of a unit-mass of workers and of a potentially infinite number of entrepreneurs.[7] In a given period t, a worker can either be employed, n_t, or unemployed, u_t. Non-employed members of the household may find a job even within the period they get displaced. Thus, the measure of the household's members that are searching for a job in the beginning of a period is given by $u_t^s = u_{t-1} + \delta n_{t-1}$, where δ denotes an exogenous separation rate. The measure of employed individuals working in period t is therefore given by $n_t = f_t u_t^s + (1 - \delta) n_{t-1}$, where f_t denotes an endogenously determined job finding rate. The real wage is denoted by w_t. In addition to labor income, the household receives net-profits, d_t, from the corporate sector.

The household's utility depends on the amount consumed, c_t, and the mass of household members working, the latter being weighted by the disutility of working, χ, such that

$$U(c_t, n_t) = c_t - \chi n_t. \tag{1}$$

The budget constraint of the household is given by[8]

$$c_t = w_t n_t + d_t. \tag{2}$$

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[6]Risk aversion does introduce other channels through which uncertainty affects economic outcomes. For example, Freund and Rendahl (2020) explain how risk aversion can lead to a larger impact of uncertainty shocks through changes in the required risk premium. We discuss the role of risk aversion in online appendix OB .

[7]That is, there is never a shortage of entrepreneurs. As explained in section 4, whether there is or is not a binding constraint on the number of potential entrants turns out to be crucial for the presence of an option value of waiting mechanism in SaM models.

[8]To maximize transparency, we assume that the aggregate resources devoted to vacancy-posting, to be discussed shortly, are rebated back to the household.

2.2 Search frictions

The total number of period-t matches, m_t, depends on the number of workers searching for a job, u_t^s, and the number of vacancies, v_t, as specified by a standard, Cobb-Douglas matching function

$$m_t = \psi(v_t)^{1-\alpha}(u_t^s)^{\alpha}, \tag{3}$$

which exhibits constant returns to scale. The implied hiring rate, h_t, and implied job finding rate, f_t, are then given by

$$h_t = \frac{m_t}{v_t} = \psi\theta_t^{-\alpha}, \tag{4}$$

$$f_t = \frac{m_t}{u_t^s} = \psi\theta_t^{1-\alpha}, \tag{5}$$

where θ_t indicates labor market tightness

$$\theta_t = \frac{v_t}{u_t^s}. \tag{6}$$

The law of motion for employment is given by

$$n_t = (1-\delta)n_{t-1} + m_t. \tag{7}$$

These n_t workers constitute n_t one-worker firms producing the intermediate good.

2.3 Firms and job creation

There are intermediate goods producing firms, final goods producing firms, and retail firms. Moreover, there is a potentially infinite mass of entrepreneurs with the ability to post vacancies and, thus, create one-worker firms.

Intermediate goods producers. Each of the active firms produces z_t units of output at a (relative) price equal to $\bar{x}$ in terms of the final consumption good. The only input is labor and the value of z_t is determined by the following process

$$z_t = (1 - \rho_z)\bar{z} + \rho_z z_{t-1} + \sigma_{t-1}\varepsilon_{z,t}, \tag{8}$$

$$\ln(\sigma_t) = (1 - \rho_\sigma)\ln(\sigma) + \rho_\sigma \ln(\sigma_{t-1}) + \sigma_\sigma \varepsilon_{\sigma,t}, \tag{9}$$

where $\varepsilon_{z,t}$ and $\varepsilon_{\sigma,t}$ are *iid* standard Normal processes. The steady-state value of productivity, $\bar{z}$, is normalized to unity. Uncertainty shocks are associated with changes in $\varepsilon_{\sigma,t}$. This specification of the stochastic processes is common in the literature, but deviates from Leduc and Liu (2016) in two respects. First, the process for z_t is in levels rather than in logarithms to prevent the expected value of productivity to be different from the deterministic steady-state value through a Jensen's inequality effect.[9] Second, we use the timing assumption common in the uncertainty literature (e.g., Bloom (2009); Basu and Bundick (2017) according to which volatility shocks have a delayed impact on the distribution of productivity shocks. We do so to underscore that real options effects are absent even under a timing assumption that is, in principle, favorable to wait-and-see effects (cf. Schaal (2017, footnote 12)). As in Leduc and Liu (2016), we specify the process for σ_t in logs to ensure that the standard deviation remains positive.[10]

Job/firm creation. There is an infinite number of homogeneous entrepreneurs with the ability to post vacancies and create intermediate goods producing firms. Free entry in the matching market is assumed, which implies that

$$\kappa = h_t J_t, \tag{10}$$

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[9]Quantitatively this effect is very small, but it would make the analysis of the mechanisms discussed in this paper less transparent. Although z_t could in principle turn negative with the specification in levels, we found that this does not happen even in a simulation of 10 million observations.

[10]We also tried a level process for σ_t as in Fernández-Villaverde *et al.* (2011), for instance. This does not affect the results presented in this paper.

where κ is the vacancy-posting cost, h_t the hiring rate (defined above), and J_t the beginning-of-period value of a match from the point of view of the entrepreneur ("the firm value"). The latter is given by

$$J_t = \bar{x}z_t - w_t^N + \beta E_t \left[J_{t+1}(1-\delta) \right], \tag{11}$$

where E_t denotes the mathematical expectation operator conditional on time-t information, and w_t^N is the wage, which is determined by Nash bargaining.

Nash-bargained wages   Under Nash bargaining, the wage is given by[11]

$$w_t^N = (1-\omega)\chi + \omega \left(\bar{x}z_t + \beta(1-\delta)\kappa E_t \left[\theta_{t+1} \right] \right), \tag{12}$$

where ω is the bargaining weight of the worker.[12]

Final goods producing firms and retailers.   These two sectors are not interesting for us, since we focus on the flexible price version of Leduc and Liu (2016); keeping them in the model ensures our calibration is comparable. Effectively, firms in these sectors simply produce the final goods and sell them to households earning a markup $\eta/(\eta-1)$, where η is the elasticity of substitution. This implies that the relative price of intermediate goods is $\bar{x} = (\eta-1)/\eta$.

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[11]See online appendix OA .1 for a derivation.

[12]Leduc and Liu (2016) allow for real wage rigidity to address the Shimer puzzle (cf. Shimer (2005)) and generate empirical reasonable volatilities of vacancies and unemployment. As our interest here is qualitative in nature, extrinsic rigidity makes the analysis less transparent. This is especially important regarding wage setting, since this will play a key role in understanding the results as will become clear when we discuss the results.

2.4 Key equations summarized

The relevant set of equations is the following system of four equations and four endogenous variables, J_t, w_t^N, n_t, and θ_t

$$J_t = \bar{x}z_t - w_t^N + \beta E_t\left[J_{t+1}(1-\delta)\right], \tag{13}$$

$$w_t^N = (1-\omega)\chi + \omega\left(\bar{x}z_t + \beta(1-\delta)\kappa E_t\left[\theta_{t+1}\right]\right), \tag{14}$$

$$n_t = (1 - n_{t-1} + \delta n_{t-1})f(\theta_t) + (1-\delta)n_{t-1}, \tag{15}$$

$$\kappa = h(\theta_t)J_t, \tag{16}$$

with the two additional unknown, u_t^s and v_t, given by $u_t^s = 1 - n_{t-1} + \delta n_{t-1}$, and $v_t = \theta_t u_t^s$.

2.5 Calibration and solution method

The calibration follows Leduc and Liu (2016) as closely as possible.[13] The calibrated parameter values and the associated targets/outcomes are reported in Table 1. We also use the same solution method, that is, third-order pruned perturbation.

3 Volatility in the standard search-and-matching model

The main objective of this section is to present and analyze the effect of volatility shocks in the standard search-and-matching (SaM) model. To this end, we proceed in four steps. First we illustrate IRFs of the baseline model, and outline a distinction between the total effects and those that arise purely from anticipation. As we will see, increased uncertainty generally leads to a decline in the firm value and a rise in unemployment. However, the results are more complex with respect to variables such as labor market tightness and wages. Next, we provide a simple two-period version of the model to illustrate when an option-value channel may emerge, and show that these conditions

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[13]There are some slight discrepancies that are unavoidable. For instance, with risk neutrality, the calibrated value of the disutility of labor parameter, χ, is slightly different than with log utility. Also, with utility linear in consumption there is no difference between disutility of labor and unemployment benefits and our χ parameter captures both.

Table 1: Calibrated parameters

			Wage	
Parameter	Interpretation	Source/target	Nash	Linear
β	Discount factor	Ann. interest rate of 4%	0.99	0.99
ψ	Efficiency of matching	Unemployment rate of 6.4%	0.645	0.645
$\bar{x}$	Markup	Markup of 11%	0.9	0.9
δ	Separation rate	JOLTS database	0.1	0.1
ω	Workers barg. power	Steady-state wage relation	0.5	0.915
α	Elasticity of matching	Petrongolo and Pissarides (2001)	0.5	0.5
κ	Vacancy posting cost	2 % of steady-state output	0.14	0.14
χ	Disutility of working	$\eta_{\theta,z}$ equal to 7.051	0.751	0.645
ρ_z	Persist. of agg. product.	Leduc and Liu (2016)	0.95	0.95
ρ_σ	Persist. of uncertainty	Leduc and Liu (2016)	0.76	0.76
σ_z	Std. agg. product. shock	Leduc and Liu (2016)	0.01	0.01
σ_σ	Std. uncertainty shock	Leduc and Liu (2016)	0.392	0.392

Notes. This table lists the parameter values of the baseline SaM model with both types of wage setting. The targeted value of $\eta_{\theta,z}$, the steady-state elasticity of labor market tightness with respect to aggregate productivity, is implied by Leduc and Liu's (2016) calibration for the model with Nash bargaining and linear utility. One period in the model corresponds to one quarter. Parameter values are rounded to three decimal places.

are not met in the SaM framework. Thus, the decline in economic activity revealed by the IRFs is not due to an option-value effect. Third, we show that several of the features of the baseline IRFs disappear once we dispense with Nash bargaining. In particular, a wage that is linear in productivity prohibits *any* anticipation effects. This finding provides an important insight to understand the IRFs for the model with Nash-bargained wages. Lastly, we explain just why Nash bargaining – when present – can have non-trivial implications for the transmission of uncertainty shocks.

3.1 Impulse response functions

Figure 1 plots the IRFs of an increase in the standard deviation of the productivity innovation – i.e., an uncertainty shock.[14] We plot two different types of IRFs, both of which are calculated around the stochastic steady state.[15] The first, the *total volatility* IRF, of variable x_t is the standard IRF that

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[14]More precisely, but less readably, figure 1 plots the IRFs of a unit-increase in $\varepsilon_{\sigma,t}$, that is, to the *innovation* of the time-varying standard deviation of the productivity innovation.

[15]The starting point does potentially matter in a nonlinear model. Here we follow the literature and suppose that the shock occurs after a long period during which no shocks have materialized at all (cf. Born and Pfeifer (2014)).

Figure 1: IRFs for uncertainty shock in standard SaM model under Nash Bargaining
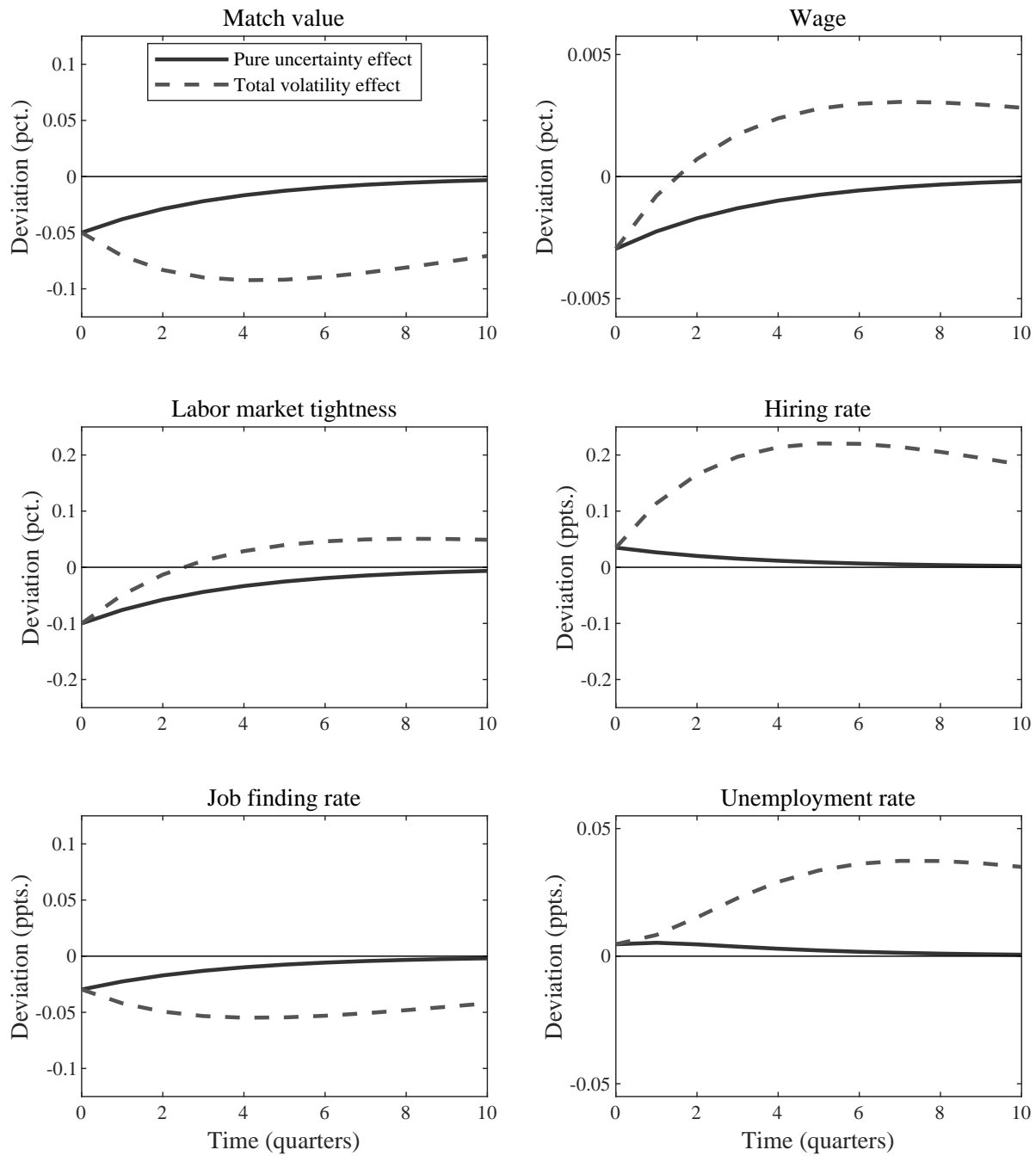
Notes: The "total volatility" IRFs plot the change in the period-0 expected values of the indicated variables in response to a unit-increase in $\varepsilon_{\sigma,t}$. The "pure uncertainty" IRFs display how the economy responds when agents think volatility will increase, but the higher volatility actually never materializes.

plots $E_\tau[x_{\tau+j}]$, where τ is the period the shock occurs and $j = 0, 1, \cdots$. These IRFs describe what happens on average (or, equivalently, in *expectation*) during periods of enhanced volatility. Whereas the impact of a period-τ shock on variables in subsequent periods does *not* depend on realizations of future shocks in linear models, this is not the case in nonlinear models. This means that one has to integrate over all possible future realizations to calculate the *expected* impact of a period-τ shock.[16]

The second IRF, the *pure uncertainty* IRF, plots the response to the economy when agents perceive an increase in future volatility, but this increase never materializes. For an IRF that uses the stochastic steady state as the starting point, this means that agents think σ_t is higher than normal during the period following the shock and act accordingly, but period after period, z_t, still takes on its steady-state value. Thus, the pure uncertainty IRF measures the effects of an increase in *purely anticipated* uncertainty. The resulting effects arise solely due to agents' responses to changed expectations about the future. These changed expectations are described by the total volatility IRF. Thus, the latter type of IRFs are essential to understand the first kind (and vice versa).[17]

The key observations about figure 1 are the following. First, the value of a firm falls and the unemployment rate increases. This is true for both types of IRFs. Second, there are important qualitative differences between the two types of IRFs. Specifically, whereas the pure uncertainty IRFs follow the usual monotone pattern, the total volatility IRFs display an inverted u-shape. Moreover, for the wage rate and tightness variable, the two types of IRFs even have different signs at some horizons. Note that both types of IRFs take on negative values initially for these two variables.[18] However, for the wage rate and the labor market tightness, the response of the total volatility IRFs turns positive soon after the shock occurs whereas this is not the case for the pure uncertainty IRF. As explained in the next section, this observation is important to understand why the firm value drops in the matching model with Nash bargaining when volatility increases or is

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[16]These total volatility IRFs are calculated using the technique of Andreasen *et al.* (2018).

[17]This relationship is also important to understand first-order moment "news" shocks. For example, how the economy responds to news that productivity will be higher in the future requires understanding how the economy responds to higher productivity.

[18]By construction, the two IRFs take on the same value in the period when the shock occurs, since our timing assumption implies that uncertainty shocks have no effect in that period. Thus, the first-period responses for both types of IRFs are purely based on expectations on what will happen and those expectations are the same.

anticipated to increase.

3.2 The evasive option value

To assist understanding the IRFs presented in the last section, we make use of a very simple example to illustrate why and when an increase in uncertainty increases the option value of postponing investment. As we will see, there cannot be an option-value channel operating within this framework, and the simple example developed here is useful to make this point as clearly as possible.

Option value of postponing investment.   The option value to wait is most transparent under risk neutrality, as risk aversion will add additional aspects to the analysis, such as precautionary savings and changes in risk premia. Thus, we consider a risk neutral agent. This agent can choose between the following two investment paths. The first possibility consists of investing immediately and earning a known return R_1 in the first period and a stochastic return R_2 in the second period. The latter return will only become known in period 2. Alternatively, the agent can postpone making a decision. In this case, she would instead bring the money to the bank in the first period and earn a return equal to $R^* < R_1$. In the second period, the agent will invest in the project only if $R_2 > R^* \geq 0$. The expected values of the two strategies – *commit* and *wait* – are given by

$$J_{\text{commit}} = R_1 + \beta E[R_2], \tag{17}$$

$$J_{\text{wait}} = R^* + \beta E[\max\{R_2, R^*\}]. \tag{18}$$

How does increased volatility, i.e., an increase in the standard deviation of R_2, affect the entrepreneur's choice when we keep the *expected value* of R_2 the same? It obviously does not affect the value of J_{commit}. However, it increases the value of J_{wait}. The reason is that by waiting the entrepreneur is ensured of a minimum return, namely R^*, but she benefits from the higher upward potential of the investment project.

We want to highlight two features that are important. First, the decision is *irreversible*. That

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is, if the entrepreneur starts the project in period 1, then she cannot unwind the project in period 2 and get a refund. Second, the projects are *mutually exclusive*. That is, the entrepreneur has to adopt either the commit or the wait strategy. While we will elaborate more on these aspect below, Bernanke (1983) provides a more general treatment.

Option value of waiting in search-and-matching models. For comparison purposes, consider a two-period version of the standard SaM model.[19] An entrepreneur who invests by creating a vacancy in period 1 faces the cash flow

$$-\kappa + h_1 \underbrace{(R_1 + \beta E[R_2])}_{J_1}, \tag{19}$$

where R_t is now equal to profits net of wages. For an entrepreneur who waits we have

$$0 + \beta E[\max\{-\kappa + h_2 \underbrace{R_2}_{J_2}, 0\}], \tag{20}$$

where we have assumed – for simplicity – that the rate of return on the alternative period 1 investment is equal to 0. Investments are irreversible in the SaM model, since κ is paid upfront. Does this mean that individual entrepreneurs in SaM models have a benefit of waiting when the expected volatility of period 2 profits increases keeping its expected value constant? The answer is no. First, the free-entry condition implies that expected profits are equal to zero in every time period and in every state of the world; that is, $-\kappa + h_t J_t = 0$, $t = 1, 2$. Since profits from vacancy-posting are expected to always be equal to zero, the upward potential that increased the value of waiting in the example discussed above does not exist here. That is, with free entry the last two equation can be

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[19]Note that what matters for economic activity in the standard SaM model with exogenous separation are the expected profits from posting a vacancy, not those associated with an existing match. That is, we consider the decision of an unmatched entrepreneur.

written as

$$-\kappa + h_1 J_1 = 0, \tag{21}$$

$$\beta E[\max\{-\kappa + h_2 J_2, 0\}] = \beta E[\max\{0,0\}] = 0. \tag{22}$$

Thus, although job creation is irreversible, it is not sufficient to generate an option-value channel.[20]

Key for the result that the expected value of vacancy posting is zero is that investing now and waiting are *not* mutually exclusive. That is, posting a vacancy this period does not prevent vacancies being posted next period. It would not make a difference if these choices were mutually exclusive for the entrepreneur herself, that is, if one assumed that each entrepreneur can be involved in one project only. The reason is that there are always *other* entrepreneurs who can pursue the alternative choices, exhausting all positive profits. Thus, what is mutually exclusive applies to the economy *as a whole*, and not to individual agents.

In section **??**, we will show that it is possible for the SaM model to have an option-value mechanism if one assume that each matched entrepreneur cannot be involved in more than one project *and* the mass of entrepreneurs is finite. This is sufficient to create an environment in which projects are mutually exclusive and expected profits are potentially positive.

Figure 1 demonstrates that there is one aspect of the properties of the SaM model developed in section 2 that is quite different from the analysis based on the simple two-period setup. Specifically, figure 1 documents that the value of a match, J_t, declines in response to an anticipated uncertainty shock, whereas the value of investing early in the two-period model, J_1, remains unaffected.

One might conjecture that the reason behind this decline in J_t is an increase in the option value

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[20]Irreversibility refers to the posting costs. Here, we have assumed that the entrepreneur who invests in period 1 will have some positive cash flow equal, $R_2 \geq 0$, in period 2 for sure. That is, there is no incentive to end the relationship early endogenously. But R_2 could be negative, for example, with sticky wages. Allowing for endogenous discontinuation means that the net present value of the surplus flows accruing to the entrepreneur investing in period 1 is given by

$$-\kappa + h_1 R_1 + \beta E[\max\{R_2, 0\}] = -\kappa + h_1 J_1 = 0. \tag{23}$$

The convexity introduced by endogenous job destruction implies that an increase in anticipated uncertainty would *raise* the value of J_1, which in turn would lead to an increase in vacancies in period 1. That is, the outcome is the opposite of that predicted by an option value of waiting mechanism.

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of waiting (Leduc and Liu, 2016, p. 21). But note that J_t in the matching model corresponds to J_{commit} in the simple model; that is, to the value of investing *now*. In contrast, the idea of the option value to wait is that the value of the strategy that involves waiting and potentially investing later increases. In the terminology of our stylized setup, an increase in uncertainty leads to an increase in J_{wait}, not to a decrease in J_{commit}.

3.3   If it is not an option value, what is it?

Our discussion above made clear that the environment for the entrepreneur in the SaM model does not satisfy the conditions that generate an option value of postponing job creation. But it is still the case that volatility shocks lower the match value and increase the unemployment rate. The question is why does this happens, and whether the reason still can be given some option-value interpretation.

Note that in the two-period model, we assumed that $E[R_2]$, i.e. expected profits, remain the same when we increased the expected volatility. The same is true for expected values of future productivity in the full dynamic models. Thus, it must be the case that the behavior of wages is essential for understanding the results in figure 1.

The Nash bargaining assumption adopted in Leduc and Liu (2016) is just one of many possibilities and it is not an essential characteristic of the matching mechanism. Nash bargaining introduces feedback between wages and market tightness. That is, wages are higher when more vacancies are expected to be posted, and wages in turn affect match value and, thus, vacancy posting. This feedback makes the model somewhat harder to understand.

Now, an important objective of this paper is to understand the role of matching frictions for the impact of volatility shocks, both when they do and when they do not realize. As will become clear below, we do need a model with more than two periods. But it will help if we strip the model to its bare essentials. Those essentials are, firstly, that neither workers nor entrepreneurs find a match with probability one. And, secondly, that both sides face congestion effects, that is, the probability of finding a match decreases if more of your type are searching; that is, the matching function is concave in both arguments. Nash bargaining is not one of those essentials. To better understand

the role of uncertainty in SaM, we first consider the case in which not only the expected value of productivity but also the expected value of profits remains unchanged when volatility increases. This can be easily accomplished if one assumes that wages are a linear function of current productivity, z_t, only.[21] Specifically,

$$w_t = \omega \bar{x} z_t + (1 - \omega) \chi. \tag{24}$$

Matching frictions and anticipated volatility changes. Under this wage rule one can derive a useful, analytical expression for J_t.

Proposition 1. *Suppose that wages are set by the linear wage rule given in equation (24), then*

$$J_t = \frac{(1 - \omega)\bar{x}}{1 - \beta(1 - \delta)\rho_z} z_t - \frac{(1 - \omega)\chi}{1 - \beta(1 - \delta)} + \frac{\beta(1 - \delta)(1 - \omega)(1 - \rho_z)\bar{x}}{(1 - \beta(1 - \delta))(1 - \beta(1 - \delta)\rho_z)}. \tag{25}$$

Proof. See online appendix OA . □

Thus, J_t is a linear function of z_t. The formula directly makes clear that an increase in *anticipated* uncertainty has no effect on J_t. If the anticipated increase in volatility does not materialize then J_t will not change in subsequent periods either even when agents continue to anticipate higher uncertainty in the future. Consequently, none of the other variables will be affected either as is documented in figure 2 which plots the two types of IRFs for an increase in uncertainty under the linear wage rule.[22]

The fact that the IRFs associated with an anticipated increase in volatility are zero in every

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[21]This linear specification can be motivated by an alternating-offers game (Freund and Rendahl, 2020). A key aspect of this game is that separation is not a credible threat. Consequently, agreement is reached within the period and market tightness does not affect the outcome. As long as agreement has not been reached, the worker is not working. The parameter χ captures the utility of not working during the negotiations. See Hall and Milgrom (2008) for details. Also, this wage coincides exactly with that of Jung and Kuester (2011), in which the Nash product, $(w_t - \chi)^\omega (\bar{x} z_t - w_t)^{1-\omega}$, is maximized.

[22]The parameters of the version with linear wages are chosen to make it comparable to that with Nash bargaining. Specifically, we choose the outside option χ such that the elasticity of labor market tightness with respect to productivity is unchanged relative to Nash bargaining. To this end, we exploit the close relationship between that elasticity and the fundamental surplus, $\bar{x}\bar{z} - \chi$, as defined by Ljungqvist and Sargent (2017). Given the remaining parameters, the bargaining weight ω is then pinned down by the steady-state version of equation (24). Parameter values are given in Table 1.

Figure 2: IRFs for uncertainty shock in standard SaM model with linear wage rule
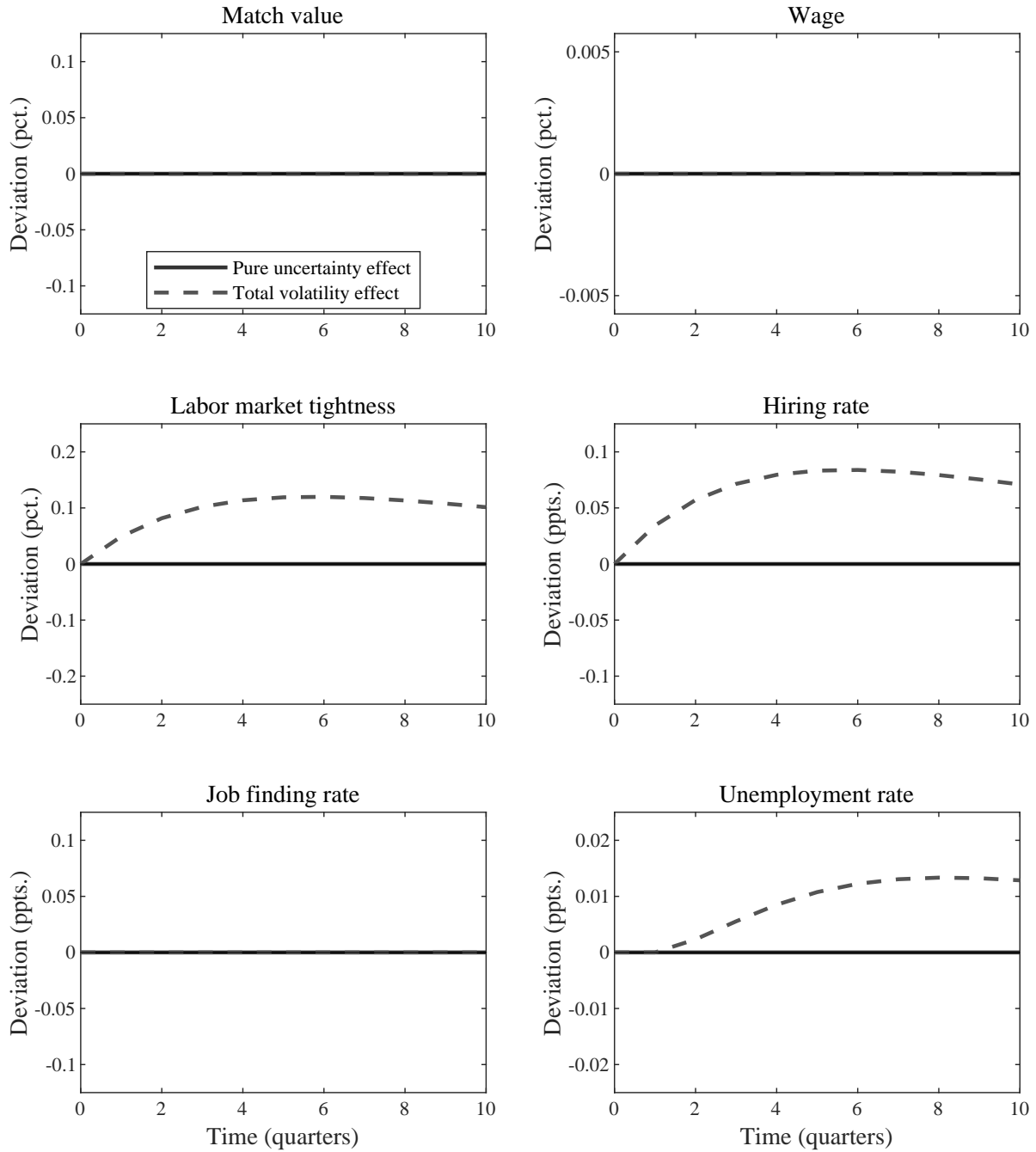
Notes: The "total volatility" IRFs plot the change in the period-0 expected values of the indicated variables in response to a unit-increase in $\varepsilon_{\sigma,t}$. The "pure uncertainty" IRFs display how the economy responds when agents think volatility increases, but the higher volatility actually never materializes.

period allows us to draw a strong conclusion. That is, the nonlinearity of the matching function by itself does *not* generate an employment effect in response to an increase in anticipated uncertainty. Consequently, there is also no option-value channel associated with the pure anticipation effect of an increase in uncertainty.

But increased volatility will make J_t more volatile, which in turn renders matching probabilities more volatile too. We therefore explore next whether the nonlinearities of the matching function could be such that increases in volatility affect the expected values of employment during the period of elevated uncertainty.

Matching frictions and realized volatility changes. How can we expect an increase in the standard deviation of productivity shocks to affect values of key variables in the model during the period of higher volatility? Given the linearity of J_t, the total volatility IRF of J_t will also be zero. However, as demonstrated by figure 2, there are increases in the expected values of market tightness, θ_t, the hiring rate, h_t, and the unemployment rate, u_t. But it has no effect on expected values of the job finding rate, f_t. To understand these results recall the expressions for θ_t, h_t, and f_t

$$ h_t = \frac{\kappa}{J_t}, \quad \theta_t = \left(\frac{\psi}{\kappa} J_t \right)^{\frac{1}{1-\alpha}}, \text{and} \quad f_t = \psi \left(\frac{\psi}{\kappa} J_t \right)^{\frac{1-\alpha}{\alpha}}. $$

Recall that α is the curvature parameter in the matching function. The hiring rate, h_t, is a convex function of J_t, for any value of $\alpha \in (0,1)$. For our linear wage function this means that it is also convex in z_t. Consequently, a rise in volatility then leads to an increase in expected values. To see why reductions in J_t matter more for the hiring rate, h_t, than increases in J_t, just consider a drop in J_t to (almost) zero and an increase of the same size. The first change will push h_t towards infinity whereas the second event simply halves the hiring rate.

Tightness, θ_t, is also a convex function of z_t for any value of $\alpha \in (0,1)$. When J_t is small, for instance, an increase in J_t leads to small increases in vacancies. The reason is that small values of J_t are associated with low values of v_t. This implies a high marginal "productivity" of the matching function so that small changes in the level of v_t are sufficient to restore the equilibrium conditions.

By contrast, f_t can either be a convex or a concave function of z_t depending on the value of α. Our results are based on $\alpha = 1/2$ in which case the job finding rate is linear in J_t and, thus, in z_t. This explains why the total volatility IRF for f_t is zero at all forecast horizons. The reason for the ambiguity and the dependence on the value of α is that the hiring rate is inversely related to J_t but the job finding rate is inversely related to the hiring rate. Whether f_t is a convex or concave function of J_t depends on which inverse relationship is stronger.

We now turn our attention to the effect of uncertainty on the employment rate, n_t. We repeat its law of motion for convenience.

$$n_t = (1-\delta)n_{t-1} + (1-(1-\delta)n_{t-1})f_t.$$

Although f_t always becomes more volatile, its expected value remains the same when $\alpha = 1/2$. But the total volatility IRFs indicate that this higher volatility is associated with a higher unemployment rates and, thus, lower employment rates. Why does an increase in the volatility of f_t reduce the expected future values of n_t? The reason is that the higher values of the job finding rate are expected to occur during expansions when fewer workers are searching for a job. Consequently, the impact on the employment rate will be smaller. By contrast, the lower values of the job finding rate will have a bigger impact because they are expected to occur during recessions when lots of workers are searching for a job.[23] Note that the effect is non-monotone. In the period of the shock, the mass of searching workers, $1-(1-\delta)n_{t-1}$, is fixed and, hence, a higher volatility of f_t has no effect on expected employment. In the next few periods, this mass is still close to its steady-state value. But as time goes on, the asymmetric effect becomes more important when z_t shocks push unemployment either up or down. This explains the inverted u-shaped pattern for the unemployment IRF.

Why increased uncertainty might reduce unemployment due to matching frictions. When $\alpha = 1/2$, then the increased volatily has no effect on the average value of f_t. However, when $\alpha < 1/2$, then f_t is a convex function of z_t, which implies that the expected values of the job finding

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[23]See Hairault *et al.* (2010) and Jung and Kuester (2011).

rate increases. So the question arises whether at low values of α a rise in uncertainty leads to an expected decrease in unemployment because the increase in the average job finding rate dominates the downward effect on unmployment discussed above. Figure 3 plots the results when $\alpha = 0.2$. Since f_t is now a convex function of z_t, the period of higher volatility correspond to higher average job finding rates. Initially – as the unemployment rate is still close to its steady-state value – this does indeed push the unemployment rate down. This result illustrates that matching frictions by themselves can even lead to decreases in the unemployment rate, although the value of α has to lower than values typically assumed in the literature (cf. Petrongolo and Pissarides (2001)). Of course, an anticipated increase in volatility will still have no effect when $\alpha < 1/2$. This is another example that illustrates how the two types of higher volatility experiments generate quite different outcomes.

3.4   The non-trivial implications of Nash bargaining

The analysis above indicates that the nonlinearities of the SaM model can generate a rich set of results to volatility shocks, even when the expected match value J_t is not affected. It also makes clear, however, that matching frictions by themselves do not give a reason why the economy should respond to *anticipated* increases in uncertainty, that is, when agents believe a period of higher volatility lies ahead, but it never materializes.

These "pure uncertainty" IRFs play an important role in the literature, because they would provide the theoretical counterpart of changes in empirical measures of "perceived" uncertainty like the ones used in Leduc and Liu (2016). As shown in section 3.1, with Nash bargaining such an anticipated increase in uncertainty does lead to a reduction in match value and a recession. What is it about Nash Bargaining that changes the results discussed above? The answer actually follows quite directly from the results for the linear wage rule and the expression of the Nash-bargained wage rate, which we repeat here for convenience,

$$w_t^N = \omega \bar{x} z_t + (1 - \omega)\chi + \omega \beta (1 - \delta) E_t [\kappa \theta_{t+1}]$$

Figure 3: IRFs for uncertainty shock in standard SaM model with linear wage rule & low α
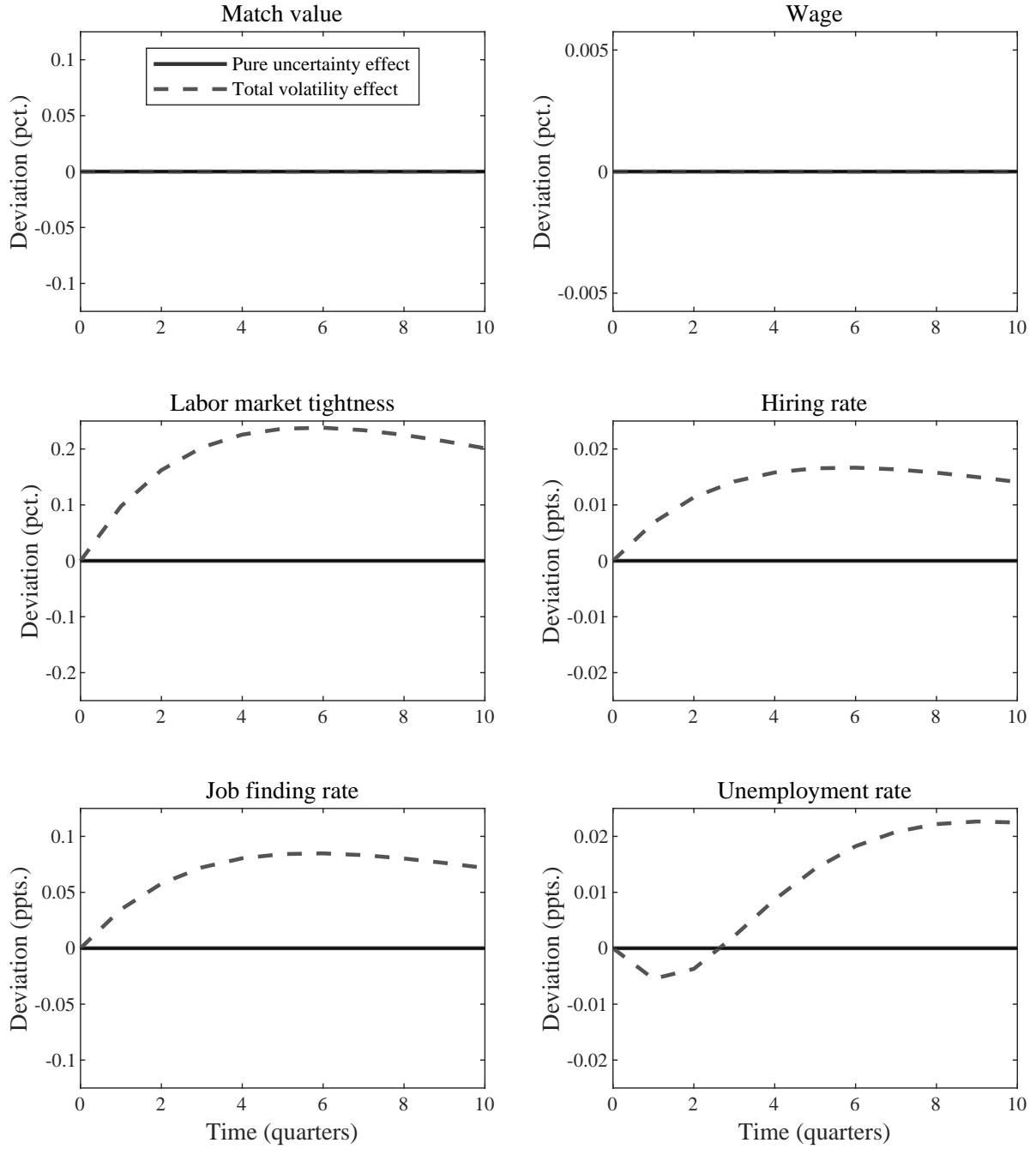
This expression makes clear that the the wage does not only increase with the period-t benefits of not working, χ, and with current-period firm revenues, $\bar{x}z_t$. A higher expected value of future tightness likewise implies a higher wage rate this period.[24]

Now, as discussed above, search frictions, and specifically the convexity of tightness, mean that the higher volatility in J_t increases the expected values of future tightness. With Nash bargaining, this expectation translates into higher current wage rates.[25] Higher current wages lead to a reduction in match value. The following proposition proves more formally that the match value J is concave in productivity under Nash bargaining.

Proposition 2. *Suppose that productivity is constant, $z_t = z_{t+1} = \cdots = z$, and wages are set by Nash bargaining, then $J(z)$ is a strictly concave function, and $\theta(z)$ is a strictly convex function.*

Proof. See online appendix OA . □

Intuitively, the free-entry condition together with the nonlinearity of the matching function ensures that θ is a convex function of J. Moreover, as the Nash bargained wage depends positively and linearly on tightness, the wage function is also convex in J. The concavity of J then follows from the convexity of the wage function.[26]

The concavity of $J(z_t)$ implies that its expected value should decrease if z_t becomes more volatile. But the story does not end here. The reduction in J_t leads to an immediate reduction in vacancy posting, which in turn puts an immediate *downward* effect on tightness and a reduction in the job finding rate. If one considers a period with an anticipated increase in volatility that

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[24]To gain intuition for why this is the case, notice that using the free-entry condition, we can write

$$\kappa\theta_{t+1} = \kappa\frac{v_{t+1}}{u_{t+1}^s} = \kappa\frac{f_{t+1}}{h_{t+1}} = f_{t+1}J_{t+1}.$$

That is, what matters for wage setting in terms of forward-looking behavior is the expected value of the product of next period's job finding rate and next period's firm value.

[25]In online appendix OB we show that if households are risk averse, then additional interaction effects come into play. In particular, whenever the marginal utility of consumption is elevated, this lowers the Nash-bargained wage rate.

[26]The above reasoning relies on the properties of J, and not productivity z. Thus to complete the argument it ought to be noticed that tightness will always be a convex function of z *unless J is sufficiently concave*. However, if this is the case, labor market tightness as well as wages are concave functions of z. Since w enters the match value negatively, this would imply that J must be convex, which is a contradiction. Thus J is concave in z. See section 3.3 for a discussion of a similar relationship between θ and z even when J is a linear function of z.

never materializes, then the expected increase in tightness due to higher volatility of J_t will never materialize either. Consequently, there is just the the downward effect on firm value, tightness, and the wage rate, consistent with the IRFs given in Figure 1. There will be an instantaneous jump down in these variables and a gradual return towards the (stochastic) steady state. What about the total volatility effect? For tightness we have the effect that works through the wage rate, that is, tightness drops because of the expected increase in the wage rate. This channel is strongest just after the shock and then leads to a monotonically declining effect. But we also have the effect from the nonlinearity of the matching function which implies that tightness is a convex function. This latter feature gives rise to an upward effect on average tightness during periods of higher volatility. The result is a non-monotone effect that is small at first.[27] Initially, the negative effect must dominate, but expected tightness becomes positive after two periods when it is overturned by the effect working through the nonlinearity of the matching function. The wage rate IRF leads the change in the expected value for tightness which follows directly from equation (12). The firm value is simply the mirror image of the wage rate since expected productivity actually does not change.

Note that it must be the case that the total volatility IRF for tightness turns positive at some point. If it would never turn positive, then the wage response would not turn positive either, which means that firm value would not have dropped; but then tightness should not have fallen in the first place.

4   A search-and-matching model with option value

Section 3.2 showed that even though job creation is an irreversible investment, the standard search-and-matching (SaM) model does not have the other ingredient needed to generate an option-value channel – the mutual exclusivity of investment projects – since the choice to create a job this period does not restrict job creation in the future. In this section we propose an amended SaM model according to which elevated uncertainty does raise the value of waiting. Before specifying that

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[27]If the shock occurs in period τ, then $z_{\tau+1}$ will be more volatile because $\varepsilon_{z,\tau+1}$ is more volatile. But $z_{\tau+2}$ will be more volatile because $\varepsilon_{z,\tau+2}$ *and* $z_{\tau+1}$ will be more volatile.

model, we briefly discuss a simple experiment which demonstrates that an option-value channel is possible, in principle, simply by assuming that the mass of available entrepreneurs is *finite*. This example clarifies the crucial role of the free-entry condition in eliminating the option-value channel. It also serves as a stepping stone to understanding our proposed model.

Consider the model of section 2 with a linear wage rule. Productivity is constant and the economy starts out in steady state. We assume that the mass of entrepreneurs, while finite, is large enough for the steady state to be unaffected. In period t, the economy encounters the following increase in anticipated volatility. Aggregate productivity in some state (or states) of period $t+1$ is (expected to be) sufficiently great for the profits associated with posting a vacancy in that state to be strictly positive. In particular, there are simply not enough unmatched entrepreneurs available in the entire economy for these profits to be exhausted due to entry. That is, the free-entry condition no longer holds in that state, as

$$h_{t+1}J_{t+1} - \kappa > 0,$$

whereas it holds with equality in all other states.[28]

In period t, an idle entrepreneur is now faced with the choice of either posting a vacancy immediately, or waiting in the hope of entering when profits are strictly positive. As long as profits in period t fall short of the expected profits in period $t+1$, waiting is obviously a dominant strategy. Vacancies in period t therefore decline, and the hiring rate increases. This remains true until profits in period t are exactly equal to the expected profits of entry in period $t+1$. That is, until the arbitrage condition

$$h_t J_t - \kappa = E_t[h_{t+1}J_{t+1} - \kappa] > 0,$$

is satisfied. Thus, the (expectation of) positive profits available in period $t+1$, caused by a shortage

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[28]Note, in particular, that the prospect of a large *fall* in aggregate productivity leaves expected profits unaffected, since such a shock lead to an increase in the slack of the constraint on the available number of entrepreneurs.

of available entrepreneurs, gives rise to positive profits in period t; profits that are generated by a rise in the hiring rate, h_t, which is accomplished through a decline in vacancies. In short, a perceived increase in future uncertainty can give rise to an option value of waiting and a decline in economic activity in the present, if that increased volatility means that the constraint on the number of available is expected to be binding in some future state of the world.

While this simple extension of the baseline model is sufficient to give rise to an option-value channel, is suffers from several disadvantages. Firstly, for an option-value mechanism to operate in this environment, one had to postulate the existence of states of the world in which there is literally nobody left to create jobs, regardless of how great the associated profits are. That seems implausible. More broadly, the distribution of shocks must be such that the free-entry condition is binding in some states but not in others. That is, the presence of option-value effects is sensitive to assumptions about the size of shocks. Further, once a state characterized by sufficiently high aggregate productivity is expected to materialize, the value of waiting becomes positive for *all* entrepreneurs.[29] Finally, the requirement that the constraint on the number of entrepreneurs be occasionally binding complicates the numerical analysis.

4.1 A model with firm heterogeneity

So what can be done? Clearly, we have to maintain the assumption of a finite mass of entrepreneurs, lest free entry drive expected profits to zero in all states of the world, eliminating the possibility of an option-value channel. At the same time, it is desirable to have an internal solution. This can be accomplished by having heterogeneity in productivity among idle entrepreneurs. This simple modification gives rise to a framework in which there are *always* idle entrepreneurs, but only *some* that find it profitable to enter the matching market. The measure that finds it profitable to do so is endogenous and time-varying. At the same time, the option value of waiting remains present since higher uncertainty gives entrepreneurs upward potential, whereas they are shielded from downward

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[29]That is, until the arbitrage condition is satisfied, at which *all* entrepreneurs are indifferent between entering across periods/states.

risk, precisely because they can always choose not to post vacancies.

4.1.1 Setup

Introducing heterogeneity into the main framework necessitates some additional assumptions. In particular, there is a finite, and constant, mass of potential entrepreneurs, Υ. In every period, each unmatched entrepreneur receives an *iid* productivity draw, a, from the cumulative distribution function, $F(a)$, with mean zero. The distribution is uniform on the interval $A = [-\bar{a}, \bar{a}]$, with $\bar{a} = \sqrt{3}\sigma_a$, where σ_a is the standard deviation of a.[30]

If an entrepreneur is successful in creating a new firm, the idiosyncratic productivity draw, a, is realized and lasts permanently throughout the match. As in the baseline model, the firm is then only dissolved by exogenous separation, which occurs at a rate δ. In case of separation the entrepreneur "dies" and gets replaced.[31] If, on the other hand, the entrepreneur is unsuccessful in creating a firm – by choice or by chance – the same entrepreneur receives a new productivity draw is made in the subsequent period. Thus, "death" can only occur after a match has taken place. As a consequence, only entrepreneurs with a high enough value for a will find it worthwhile to pay the cost of posting a vacancy. Others may instead find it more beneficial to wait for the opportunity of receiving a better draw in the future. That is, there is scope for an option value of waiting, without having to rely on there being states of the world in which there are no entrepreneurs left who conceivably could post further vacancies.

With idiosyncratic productivity shocks the firm value is given by[32]

$$J_t(a) = (1 - \omega)(\bar{x}(z_t + a) - \chi) + \beta(1 - \delta)E_t[J_{t+1}(a)]. \tag{26}$$

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[30]Results with a Normal distribution are similar and are reported in online appendix OD.2.

[31]It should be noted that as entrepreneurs in a match may *die* – and will therefore *not* receive a prospective new draw of a – there is no "accidental option value" emerging in case of separation. This assumption is made to allow for a transparent analysis, in which the option value has no accidental benefits, but can only be realized by choice, and carries no noteworthy quantitative implications.

[32]The notation is somewhat simplified in that it does not specify that a is the draw that the entrepreneur received in the period the match was created. Over time, firm level productivity, $z_t + a$, only varies with aggregate productivity, z_t. We also do not add a subscript to indicate that the level of a is firm specific.

In the baseline model, the free-entry condition ensured that the value of an idle entrepreneur is always zero. In the current case, by contrast, the measure of entrepreneurs, Υ, is finite, and the value of being an idle entrepreneur prior to the revelation of the idiosyncratic draw, J_t^U, is non-negative and given by

$$J_t^U = \int_A \max\left\{\beta E_t[J_{t+1}^U], h_t J_t(a) + (1-h_t)\beta E_t[J_{t+1}^U] - \kappa\right\} dF(a) \tag{27}$$

$$= \int_A \max\left\{0, h_t(J_t(a) - \beta E_t[J_{t+1}^U]) - \kappa\right\} dF(a) + \beta E_t[J_{t+1}^U]. \tag{28}$$

Define $\hat{a}_t$ as the productivity cut-off that renders an entrepreneur indifferent between entering or not; that is,

$$h_t(J_t(\hat{a}_t) - \beta E_t[J_{t+1}^U]) - \kappa = 0. \tag{29}$$

Moreover, denote a_t^* as the expected value of a conditional on a being above the cutoff level, and p_t as the probability of such a draw. That is,

$$p_t = 1 - F(\hat{a}_t), \tag{30}$$

$$a_t^* = \frac{1}{p_t} \int_{\hat{a}_t}^{\overline{a}} a \, dF(a) = E_t[a|a \geq \hat{a}_t]. \tag{31}$$

Then the value of an idle entrepreneur can be written as

$$J_t^U = p_t\left(h_t(J_t(a_t^*) - \beta E_t[J_{t+1}^U]) - \kappa\right) + \beta E_t[J_{t+1}^U]. \tag{32}$$

Lastly, the number of vacancies are given by

$$v_t = p_t(\Upsilon - (1-\delta)n_{t-1}). \tag{33}$$

Thus, in contrast to the previous framework, the firm value is now provided by equation (26), and

the free-entry condition is replaced by equation (29); the equations for h_t, f_t, n_t, as well as the exogenous processes remain the same.

Before providing a qualitative analysis of the option-value channel it is necessary to touch upon some aspects of the calibration (see section 4.2 for additional details). In particular, our ambition is to keep the heterogenous-firm version as close as possible to the baseline, and for both frameworks to coincide – at least with respect to the key variables – at the steady state. In the baseline framework the cut-off level $\hat{a}$ is, by construction, zero. Thus, we calibrate the model such that the steady-state value of $\hat{a}$ remains at zero for any value of σ_a. Given the symmetry of the distribution this implies that $p = 0.5$. Moreover, following equation (33), and imposing the steady-state values of vacancies, v, and employment, n, from the baseline model, one finds that Υ must be set as

$$\Upsilon = 2v + (1 - \delta)n. \tag{34}$$

Another salient implication of this choice of p is that at the steady state, the measure of idle entrepreneurs posting a vacancies is equally large as that of idle entrepreneurs that are not. Thus, the constraint on the number of entrepreneurs is unlikely to be binding even for fairly large shocks, and we will proceed under the assumption that it indeed never is.

4.1.2   An option value of waiting

The emergence of an option-value channel in this framework is intuitive and visible even in the absence of aggregate risk. We first explain how the channel emerges only due to idiosyncratic risk, and then discuss how a similar effect arises from aggregate volatility. In online appendix OC , we furthermore develop a two-period version of this model with heterogeneous productivity levels which is helpful in providing some graphical intuition as well as some analytical results.

Idiosyncratic risk.   Suppose that there is no aggregate risk and that the cross sectional dispersion in productivity is zero; that is, $\sigma_a = 0$. Provided that there is a sufficient amount of available entrepreneurs to exhaust all (excess) profits of entry, the first term in equation (28) must equal zero.

That is,

$$\max\left\{0, h(J(0) - \beta J^U) - \kappa\right\} = 0, \tag{35}$$

where we dropped time subscripts given the absence of aggregate uncertainty. Consequently, $J^U = 0$, and the above equation simply replicates the free-entry condition in the standard SaM model. Thus, with $\sigma_a = 0$, the heterogeneous-firm model nests the baseline.

Suppose instead that $\sigma_a > 0$. If $\hat{a}_t$ was unaffected by this alteration (remaining at zero), so would the hiring rate, h_t. However, the presence of cross-sectional dispersion in productivity implies that a^* – i.e. the expected value of the idiosyncratic component conditional upon entry – must rise above zero. This means that the value of waiting, $\beta E_t[J^U_{t+1}]$, is positive as well. Consequently, an entrepreneur with $a = 0$ now prefers to wait in the hope of getting a better draw next period. Consequently, $\hat{a}_t$ will increase until the hiring rate has dropped sufficiently so that the expected profits of vacancy posting at the new cut-off level equals the value of waiting.[33]

Would it not be possible that changes in the hiring rate drive the expected value of waiting to zero? No. If that would be true, then the expected profits of vacancy-posting are equal to zero for an entrepreneur with $a = \hat{a}_t$. With idiosyncratic dispersion, however, this agent has some probability of receiving a draw for a in the future that exceeds $\hat{a}_t$ in which case the expected profits must be strictly positive.[34] The more cross-sectional dispersion, as indicated by σ_a, the larger the difference between $\hat{a}$ and a^*; that is, the stronger the option value of waiting due to idiosyncratic risk, the lower p_t, and the higher the unemployment rate. The leftmost graph in figure 4 illustrates this relationship between σ_a and the steady state level of the unemployment rate. As can be seen, the mechanism is powerful; an increase in the standard deviation of a from zero to 0.01 (that is, one percent of the

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[33]It is of course essential that entrepreneurs have the option to *not* post a vacancy in the future. That is, expected profits are always bounded below at zero. This leads to a convex payoff function and Jensen's inequality then implies that uncertainty raises expected values.

[34]With idiosyncratic dispersion, a^*_t could be equal to $\hat{a}_t$, but only if $\hat{a}_t$ is at the upperbound of the distribution, that is, when nobody would want to post vacancies. This does not happen for any of our parameterizations, because changes in matching probabilities always ensure an interior solution for $\hat{a}_t$.

Figure 4: Cross-sectional dispersion and steady-state properties
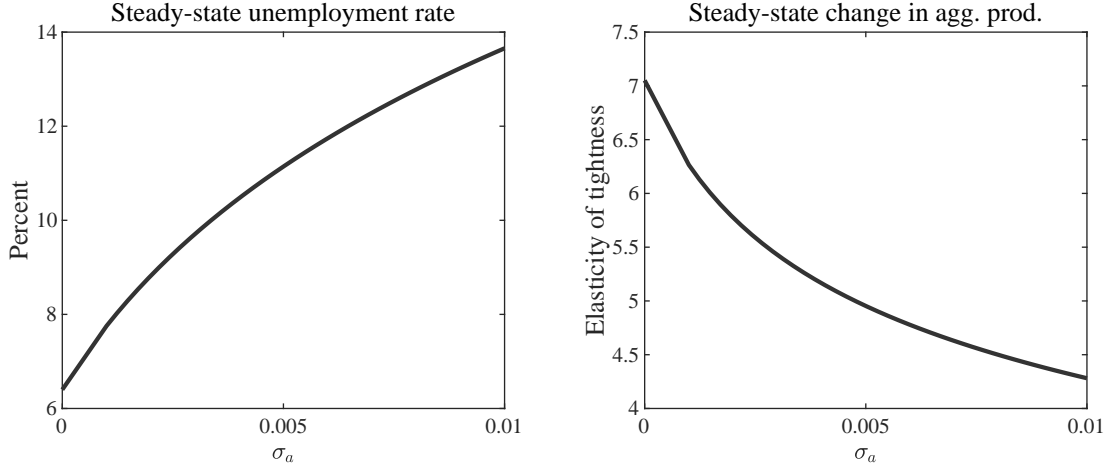
Notes: The panels display key moment properties as a function of the amount of cross-sectional dispersion. The left panel indicates the steady-state unemployment rate, while the right panel plots the steady-state elasticity of labor market tightness with respect to aggregate productivity. All other model parameters are kept fixed and are equal to the ones given in table 1 for the case with the linear wage rule and no cross-sectional dispersion.

output level without idiosyncratic dispersion) increases the steady-state unemployment rate from 6.4% to almost 14%.

Aggregate risk. The presence of aggregate risk also gives rise to an option value of waiting mechanism, which operates similarly, but not identically nor independently, to the above mechanism. To understand the nuance, notice that a higher value for z_{t+1} would increase the value of $J_{t+1}(a)$, while a lower value for z_{t+1} would result in a decline. When wages are linear in productivity and entrepreneurs die after an exogenous separation, which is the case in this framework, the increase and decrease in J_{t+1} exactly offset each other. Nevertheless there still is an option value of waiting. The reason is as follows. The increase in z_{t+1} generally leads to a reduction in the cutoff value $\hat{a}_{t+1}$ (and, hence, in a^*_{t+1}), since *total* productivity, $z_{t+1} + \hat{a}_{t+1}$, will anyway increase. Similarly, the decrease in z_{t+1} generally leads to an increase in $\hat{a}_{t+1}$ (and a^*_{t+1}). Consequently, the probability of entering and thereby benefiting from an increase in J_{t+1} is higher than that of the decrease.[35]

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[35]Phrased in another way, the higher J_{t+1} is multiplied by a higher value of p_{t+1} than the lower J_{t+1}. See online appendix OC for an intuitive, graphical exposition using a 2-period version of the model.

Therefore, an anticipated increase in future aggregate volatility increases the conditional expected value of a match, $J_{t+1}(a^*_{t+1})$, which thereby raises the value of waiting, J^U_{t+1}; the option-value channel materializes.

It ought to be noted that the presence of cross-section dispersion – alongside, of course, the finite measure of entrepreneurs – is necessary for this mechanism to operate *at all*.[36] Indeed, entrepreneurs can only "benefit" from a higher match value *if* the profits of entry can be positive. Absent cross-sectional dispersion (and with a potentially infinite measure of entrepreneurs), the hiring rate would otherwise adjust to ensure that the expected profits of entry were zero in all time periods, and in all states of the world, and the option-value channel would close down. Of course, this reasoning simply echoes the key results of section 3.2, but serves as a useful reminder. The rightmost graph in figure 4 shows the relationship between the amount of cross-sectional dispersion, σ_a, and the steady-state elasticity of labor market tightness with respect to aggregate productivity. Less dispersion implies a higher elasticity, which reflects the fact that dispersion dampens the movements in the hiring rate.

4.2   Recalibration scheme

An insight from the previous section is that the amount of cross-sectional dispersion, σ_a, alters some of the key properties of the model. In particular, a higher value of σ_a is associated with a higher steady-state unemployment rate for a given value of aggregate productivity. Yet, a key element of the calibration strategy of Leduc and Liu (2016) and adopted here is that the theoretical steady-state unemployment rate matches its empirical counterpart. In addition, the volatility of the hiring rate is declining in σ_a. In view of this, we pursue a recalibration strategy which ensures that *irrespective* of the chosen value of σ_a, the model economy matches key empirical targets and features a comparable degree of aggregate volatility to the baseline model of section 2. Specifically, (*i*) the steady-state values of all endogenous variables are unchanged; and (*ii*) the steady-state elasticity of labor market

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[36]That is, unless we rely on a shock distribution that render the free-entry condition occasionally binding, as discussed above.

tightness with respect to aggregate productivity equals the baseline.

The key parameter to obtain the latter target is χ, which controls the value of the worker's outside option during bargaining. By choosing larger values for χ when σ_a is higher, we reduce the contemporaneous surplus, $\bar{x}(z_t + a - \chi)$, which renders the model variables more volatile – offsetting the lower volatility implied by a wider cross-sectional distribution. Next, for the steady-state rate of unemployment to be the same as in the baseline model, the steady-state value of the cutoff level $\hat{a}$ must be equal to zero for the different values of σ_a considered.[37] The key parameter to accomplish this is the worker bargaining power, ω. When χ is increased, the share that accrues to the entrepreneur, i.e., $1 - \omega$, must increase to ensure the same level of steady state vacancy posting.

Lastly, the steady-state total productivity of the *average* firm is given by $\bar{z} + a^*$. Since a^* increases with σ_a, we adjust the value of $\bar{z}$ downward to compensate for this effect. A benefit of this approach is that the steady-state value of $\left(J(\hat{a}) - \beta J^U\right)$, i.e., the difference in the value of a match at the cutoff relative to the value of an unmatched entrepreneur, is the same across economies.[38] Since that term plays a key role in driving the dynamics of the model, this aspect of the recalibration procedure assists with the interpretation of the results.

Our recalibration scheme imposes a natural range for the values of σ_a. As σ_a increases, we need to increase χ and lower ω. Above a value of $\sigma_a = 0.003$, ω quickly approaches its natural lower bound of 0. As this is a fairly low value, we adopt it as a benchmark.[39] Following our recalibration procedure, when $\sigma_a = 0.003$, we set $\chi = 0.757$, $\omega = 0.636$, and $\bar{z} = 0.997$. The remaining parameters are unchanged and available in table 1, section 2.5, while the mass of entrepreneurs Υ given $p = 0.5$ is equal to 1.11.

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[37]Recall that Υ is set such that the fraction of entrepreneurs that enters the matching market is $1/2$ in the economy without cross-sectional dispersion. Online appendix OD.4 provides the results when this fraction is equal to 0.2 instead.

[38]The calibration strategy involves setting the fraction of output spent on vacancy posting costs in steady-state, $\kappa v/(\bar{z} a^* n)$, equal to 2%. The adjustment of $\bar{z}$ ensures that κ is the same across economies, which together with the fact that h is calibrated to be the same across economies means that $J(\hat{a}) - \beta J^U$ is the same across economies.

[39]The amount of idiosyncratic dispersion is small relative to the degree of cross-sectional productivity dispersion observed in the real world. See, for example, Sterk *et al.* (2020). It is not surprising that our framework with ex-ante identical entrepreneurs cannot generate the observed differences which are likely to arise from numerous factors besides those present in a simplified model as we consider here.

4.3 Numerical results

Figure 5 plots the IRFs for a volatility shock given $\sigma_a = 0.003$, that is, the upper bound permissible given the recalibration procedure.[40] The following observations immediately stand out. First, and consistent with the preceding qualitative discussion, an increase in anticipated aggregate uncertainty causes a recession even though wages are linear in productivity. The anticipation of heightened future volatility increases the value of waiting, which in turn reduces entry and vacancy-posting, lowering the job finding rate and, ultimately, pushes up the unemployment rate. Second, the total volatility effects are much larger than the pure uncertainty effects. This result strengthens our recommendation, expressed in the context of the baseline model, to consider both types of IRFs when studying the implications of time-varying volatility. Indeed, the total volatility IRFs strongly resemble those obtained in the absence of firm heterogeneity, and for similar reasons; the nonlinearities in the matching function generate a persistent rise in both the unemployment rate and the hiring rate, as discussed in section 3.

A few subtleties are worth pointing out. For one, in the presence of idiosyncratic dispersion, aggregate output is no longer proportional to $z_t n_t$. The composition of the sample of producing firms matters, as they vary in their individual productivity levels. Specifically, changes in the number of vacancies posted occur through changes in the cutoff level, which in turn affects the average productivity of producing firms. Following a volatility shock, the value of waiting rises on impact due to anticipation effects. The associated *increase* in the average productivity level of those firms that do enter dampens, but does not overturn, the reduction in output due to the fall in employment – an effect that is absent in the model with homogeneous entrepreneurs.[41] However, in the case of total volatility effects, this dampening effect is short-lived. The sharp rise in the unemployment rate and the associated increase in vacancy posting (through an increased entry probability) takes hold, whereupon the average productivity of entrants declines. Consequently, output not only falls

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[40]See online appendix OD.3 for the same set of IRFs given $\sigma_a = 0.001$. Additionally, figure 6 plots the impact and maximum total volatility effect on the unemployment rate, specifically, as a function of σ_a.

[41]Figure OD.3 in the online appendix illustrates that this effect can, in principle, lead to a small initial increase in output when $\sigma_a = 0.001$.

Figure 5: IRFs for uncertainty shock in SaM model with cross-sectional dispersion; $\sigma_a = 0.003$
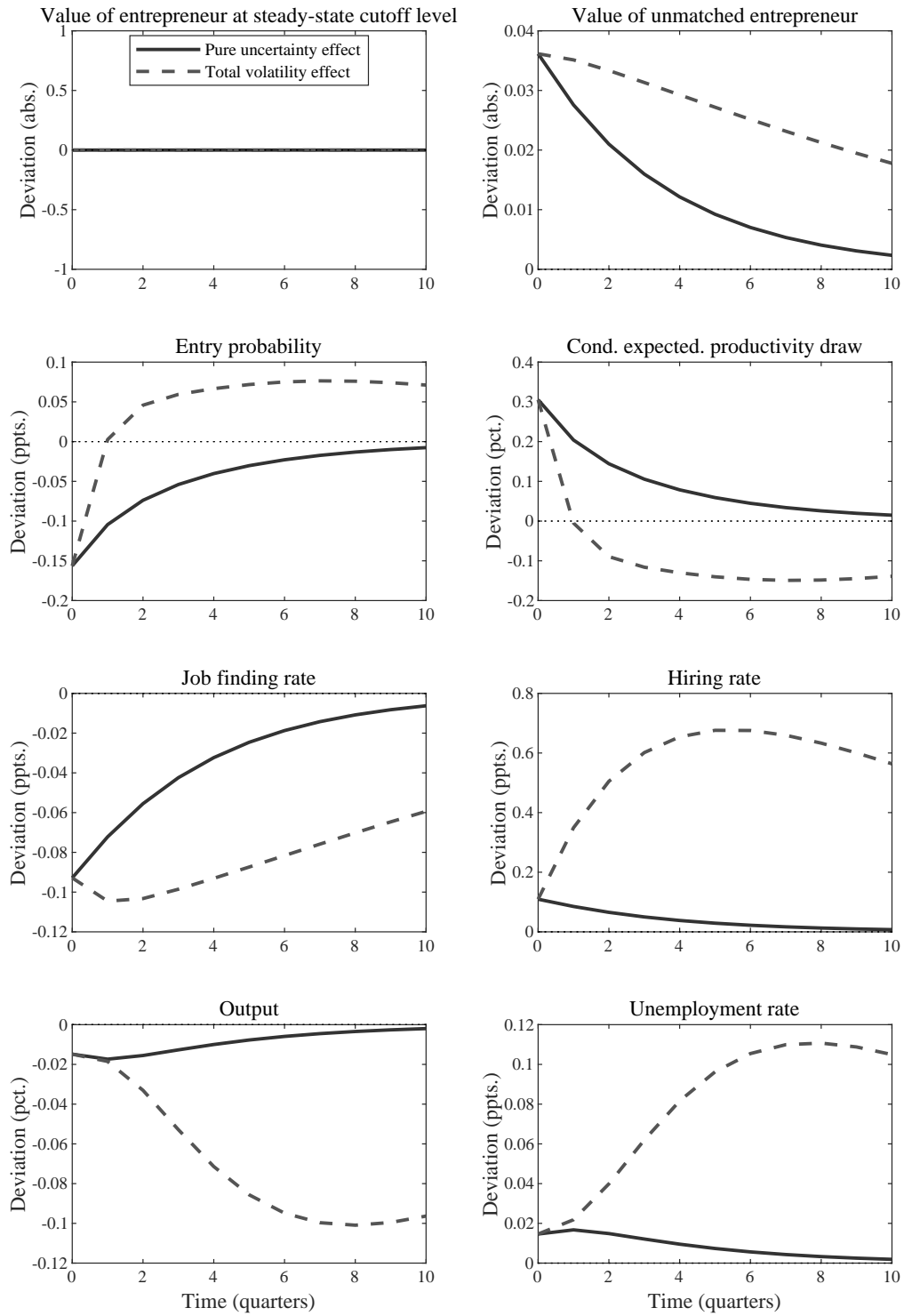
Notes: The "total volatility" IRFs plot the change in the period-0 expected values of the indicated variables in response to a unit-increase in $\varepsilon_{\sigma,t}$. The "pure uncertainty" IRFs display how the economy responds when agents think volatility will increase, but the higher volatility actually never materializes.

because of the decline in employment, but also due to composition effects.

Moreover, uncertainty shocks have non-zero effects on the job finding rate. This result stands in contrast to the baseline model (with linear wages), according to which both the pure and the total volatility effects on the job finding rate are equal to zero in expectation when the matching elasticity, α, is equal to 0.5. Here, instead, the presence of a wait-and-see mechanism – specifically the associated reduction in the entry probability – causes the job-finding rate to decline when perceived uncertainty rises. The total volatility effect on the job-finding rate is likewise negative, larger, and more persistent. To see why, recall from the discussion in section 3, that the hiring rate is a convex function of z_t. Equation (29) makes clear that the observed increase in the value of an unmatched entrepreneur introduces an additional positive effect on the hiring rate, and thus a negative effect on the average job finding rate.

Quantitative comparison.   To evaluate the quantitative impact of uncertainty shocks in the current framework, we compare our results with those described in section 3.1 for the standard SaM model with free entry and Nash bargaining (recall that this wage-setting assumption is the key reason why pure uncertainty shocks have non-zero effects in that model).

Figure 6 illustrates the total volatility effect of an uncertainty shock on the unemployment rate, both on impact (left graph) and at the maximum (right graph) along the IRF. The effect on impact is entirely due to anticipation, and the maximum total volatility effect occurs after roughly eight quarters. The horizontal lines in the two graphs indicate, for comparison purposes, the same statistics obtained in the baseline model with Nash bargaining. Recall that in the model with heterogeneity we adopted the linear wage rule and deliberately chose the mass of entrepreneurs, Υ, such that the constraint on their number is never binding. Hence, with barely any cross-sectional dispersion, there should be no quantitatively significant anticipation effects due to a volatility shock. The figure reveals that, indeed, for very small values of σ_a, we are essentially back to the model of section 2 with linear wages.

Nonetheless, even with still relatively little cross-sectional dispersion, volatility shocks can

Figure 6: Cross-sectional dispersion and aggregate uncertainty effects
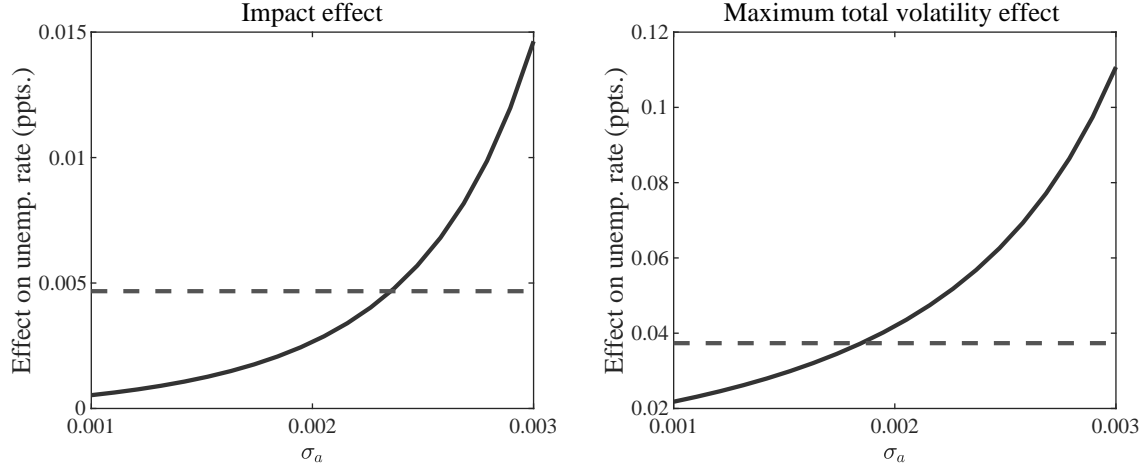
Notes: The panels display the initial impact and the maximum total volatility impact of a unit-increase in $\varepsilon_{\sigma,t}$ as a function of the amount of cross-sectional dispersion, σ_a. Other model parameters are recalibrated to make the economies with different values of σ_a comparable.

generate a substantial effect on unemployment. Thus, a value of σ_a equal to 0.003 implies that the entrepreneur with the most productive draw for a is just one percent more productive than the entrepreneur with the least productive draw. In spite of that, both the initial pure uncertainty effect as well as the maximum total volatility effect are more than double what is generated in the baseline model with Nash bargaining.

Robustness checks. In online appendix OD, we discuss the results of several robustness exercises. Most importantly, our baseline specification of the model assumes that an entrepreneur can post only one vacancy and then creates a job with probability h_t ("stochastic hiring"). An alternative would be to suppose that the entrepreneur posts $1/h_t$ vacancies and then creates one job with certainty ("non-stochastic hiring"). In the standard SaM model with risk-neutral entrepreneurs, these two options generate the exact same model properties. In our modified framework, entrepreneurs are also risk neutral and the two different specifications imply the same qualitative properties. Quantitatively, however, when there is both aggregate and idiosyncratic uncertainty, a model with non-stochastic hiring generates a substantially stronger option-value effect due to elevated volatility than implied by our baseline specification.

5 Concluding remarks

The option value of waiting to invest in the presence of uncertainty strikes many as a plausible mechanism to rationalize the empirical finding that elevated uncertainty negatively impacts economic activity. Moreover, the popularity of the search-and-matching (SaM) literature underscores the usefulness of modeling job creation as an investment. Yet, we showed that the usual assumption in that literature of there being a "potentially infinite number" of entrepreneurs to take advantage of opportunities in the matching market eliminates any grounds for wait-and-see behavior. The standard SaM model, therefore, cannot be used to rationalize the effects of uncertainty shocks in terms of an option-value channel. If, on the other hand, there is a limit on the number of potential entrepreneurs and they vary in their idiosyncratic productivity levels – two modifications that are both plausible and can be introduced into the model in a tractable manner – the model properties completely change. In particular, an increase in perceived volatility then does indeed robustly increase the option value of waiting, causing a reduction in job creation and higher unemployment.

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