# Coordination is Key: Task Assignment, Communication, and Productivity*

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*Preliminary and incomplete.*

**Abstract**

A central role for firms is to facilitate efficient production by coordinating the collaboration between specialized workers. But coordination also costs time, especially in poorly organized firms. This brief note sketches out how the team production model of Freund (2022) can be extended to allow for communication requirements that make the division of labor costly. Two simple results are established. First, the productivity advantage from high organizational quality – when division of labor is achieved without workers spending much time away from production – is greater when worker-task specialization is marked. Second, fully realizing this advantage requires forming a team composed of members of similar quality. The reason is that in a context of specialization, a deeper division of labor also fosters vulnerabilities; endogenously emerging coworker quality complementarities imply that the least capable team member carries disproportionate weight in determining output. These conjectures invite more rigorous theoretical, empirical, and quantitative evaluation.

**Keywords:** coworker complementarity, division of labor, firms, teams, technology.

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# 1 Introduction

No single individual can competently execute all the tasks necessary for producing complex goods or services. Instead, individuals specialize in specific tasks and firms gather them into teams, coordinating their collaboration. By enabling workers to pursue their comparative advantage, firms facilitate production that is more efficient than would obtain absent the division of labor. Yet, while this ideal picture arguably captures an important aspect of how modern economies produce vast and varied output, it omits obstacles to the division of labor that would seem non-trivial according to both theoretical literature — including seminal papers by Becker and Murphy (1992) and Garicano (2000) – and inspection – to anyone who has ever worked in an organization. Indeed, the division of labor requires communicating information, which is time-consuming and, hence, costly. What is more, some firms appear to be rather better at workforce coordination than others.

To shed further light on these themes, it is helpful to draw a distinction between specialization in terms of *skills* – what tasks you are good at – and specialization in terms of *time allocation* or the *division of labor* – what tasks you concentrate on doing. Productivity in a team is enhanced when these two notions of specialization are aligned: each member concentrates on taking care of those tasks they are particularly good at. *Communication* among coworkers is a way to achieve such alignment, whether by clarifying in advance who will be responsible for what tasks or by asking others for help when encountering a problem you are not familiar with. I take *organizational quality* to be a notion of how costly such communication is in terms of time spent away from production.[1]

In this short note, I sketch out how the team production model of Freund (2022) can be extended to incorporate communication time costs. I then use the extended model to make two simple points about the relationship between the skill specialization, organizational quality and productivity. The first point is that the productivity advantage from high organizational quality is greater when skill specialization is marked, as the division of labor is more valuable in such a context. The second point is that to fully realize this advantage, a firm has to additionally form a team composed of members of similar quality. The reason is that in a context of skill specialization, the deeper division of labor facilitated by higher organizational quality also fosters vulnerabilities; endogenously emerging coworker quality complementarities imply that the least capable team member carries disproportionate weight in determining output.

This note is consciously limited in scope and length. I merely sketch and underline two qualitative relationships that immediately follow from a simple extension of the baseline model,

[1] Of course, this notion captures only one particular facet of what "organizational quality" might connote, with other aspects referring to culture or learning opportunities, for instance.

which abstracts from communication costs. I believe that these conjectures are interesting enough to merit further investigation, both theoretically and empirically, but do not want to further magnify the already lengthy appendix to the main paper.[2]

In the baseline model of Freund (2022), the firm is conceptualized as a "team assembly technology": it hires well-matched teams and assigns tasks line with workers' comparative advantage. In that paper, it is argued that modelling the firm in this way and embedding it in the context of a frictional general-equilibrium labor market model, helps rationalize key patterns in labor markets and the evolution of wage inequality. In line with the idealized picture painted in the opening paragraph, the internal organizational problem is simplified, abstracting from any coordination frictions. Therefore, the division of labor is always complete, for any degree of skill specialization, with no two workers performing the same task. It is this assumption which I relax in this note, while abstracting from the general-equilibrium labor market interactions that are key to the argument of the main paper.

I reconsider the organizational problem of an individual firm when deepening the division of labor costs time due to communication requirements. I study the problem of one firm – or team, I use the two terms interchangeably here – in isolation, effectively treating the hiring process as having been completed. Each worker is characterized by a vector of task-specific skills. In order to produce, all tasks have to be completed, but a worker can ask a colleagues who are is knowledgeable about a given problem for help. Helping out in this way costs scarce time, however, not only for problem solving itself but also to communicate the solution, similar to Garicano (2000). The firm organizes production by deciding who spends how much time on what task and helping whom, taking into account these communication costs and individual workers' time constraints. Maximizing production involves a balance between a deeper division of labor that aligns time allocation with skill specialization, on the one hand, and economizing on communication time, on the other hand. The specific formulation of this problem pushes the analogy between the task assignment model and trade models à la Eaton and Kortum (2002), which was leveraged already in Freund (2022), further, in that the communication requirement take the form of iceberg costs. I interpret the extent to which communication takes away from production itself, and thus the magnitude of iceberg costs, or "organizational quality" for short. Higher organizational quality thus understood leads to a deeper division of labor under the optimal organization, other things equal. While this modelling clearly is an abstraction, it plausibly captures, for instance, how well jobs were designed in the first place.

[2] I have also obtained access to rich data on high-frequency communication patterns, hierarchical structure and job tasks inside a large Fortune-500 company. Contrary to initial hopes, it was not possible to enrich the data in a way that would allow measuring or proxying for productivity, which would have facilitated an empirical evaluation of the theoretical conjectures made here. Should this data access situation improve over the medium-term, this note may be extended into a full paper.

Given this environment, I derive that output under the optimal organizational plan has a closed-form analytical solution not only in the frictionless case — as used in the main paper — but also in a special case of the model with communication costs, namely when all workers have identical absolute advantage types. Together with numerical solutions, I use this simple result to establish the following two points.

The first result describes the relationship between organizational quality, labor productivity and the extent to which workers' task-specific skills are differentiated. It states that the productivity advantage from high organizational capacity is strictly increasing in the degree of skill specialization. The reason is simple. When worker-task skill specialization is limited, there is little to gain from workers helping each other out in the first place, and so it makes little difference whether communication is costly or not; each worker just performs most tasks themselves, maximizing the time available for production. On the other hand, when skills are very task-specific, then being able to implement a deep division of labor affords significant efficiency benefits, as workers do not waste time solving problems that take them much longer to solve compared to a coworker with suitable specialized knowledge. Productivity can be much higher, other things equal, when there is little task overlap across workers and each performs mostly those tasks in which they have a comparative advantage. Through the lens of the model, high organizational quality means that these benefits can be reaped without much time being lost on communication. Under low organizational quality, the division of labor is shallow, on the other hand, even if the *potential* gains from deepening it are high – too much time would be lost on talking.

The second result underlines the complexity of organizing production efficiently when worker-task specialization is high: a deeper division of labor also renders production vulnerable to low performance of an individual team members. To see this point, suppose that there are differences in team members' absolute advantage types. This would naturally be the case if hiring was modelled as being subject to search frictions. Specialization implies coworker quality complementarities, in the sense of team quality being supermodular (Freund, 2022): the quality of the least capable team member carries disproportionate weight. Consequently, for production to be organized optimally across the entire economy, team members need to be matched together in a positively assortative manner.

In summary, the two results together indicate a dual challenge if production involving a specialized workforce — as is typically required in high-value added processes, from semi-conductor manufacturing to scientific innovation — is to be organized efficiently. Both within-firm barriers to labor specialization, labelled "organizational quality" in the model, and labor market frictions, which prevent assortative matching in the labor market, can lower total output below the production possibility frontier. Neither obstacle is of similar importance when worker-task specialization is limited.

This brief note primarily engages with the literature on firm organization. On the theoretical side, the interplay between coordination gains and communication costs has been considered in seminal papers such as Becker and Murphy (1992), Radner (1993), Bolton and Dewatripont (1994), and Garicano (2000). Here I highlight how the importance of organizational quality for output is shaped by the nature of work, and specifically the extent of worker-task specialization. Technically, this is accomplished by re-casting an Eaton and Kortum (2002) type trade model as a frictional task assignment planning problem; in this context new insights are derived. The closest relationship is to Deming (2017), who uses a similar setup to highlight the complementarity between cognitive skill and social skills at work. Here, I contribute a tight analytical characterization by considering an organizational planning problem and taking the limit of within-firm worker quality dispersion. This approach reveals an interaction between organizational quality and worker-task specialization in production.

Recent progress in the literature has instead been primarily empirical in nature, and some of the findings echo the theoretical results highlighted here. Minni (2022) makes use of personnel records at a large multinational firm alongside exogenous variation in manager rotation across teams to provide evidence of the critical role of managers in matching workers to jobs within the firm. Coraggio *et al.* (2022) highlight firm-level variation in measures of worker-job match quality, which are positively correlated with measures of firm performance. Managerial quality specifically appears to be a key role in the extent to which firms are able to achieve an efficient allocation of their human capital inputs. Kohlhepp (2022) avails himself of detailed data from hair salons to find that firms with lower organizational costs have a competitive advantage in the product market. Kuhn *et al.* (2023) use German survey data to document large firm heterogeneity in the degree of coordination. Firms with more coordinated work processes are more productive; crucially, they also suffer more negative consequences after unexpected worker shortfalls. Lastly, Bassi *et al.* (2023) collect novel survey data on manufacturing firms in Uganda and conclude that barriers to within-firm labor specialization are an important driver of low productivity.

I next sketch the model, provide an analysis centered around the two main findings, and end on a brief concluding discussion.

## 2 Model

This section first describes the model environment and then derives the optimal organizational plan.

## 2.1 Model environment and organizational planning problem

A firm consists of a discrete number $n$ of workers who perform tasks required for the production of a single final good. The production technology is owned by the firm, which aims to maximize production and is characterized by an organizational quality type $m \in [0, 1]$. Each worker is characterized by a vector of task-specific skills, $\mathbf{z}_i$, defined over a unit interval of tasks $\tau \in \mathcal{T} = [0, 1]$, and is endowed with one unit of time that they supply inelastically. For convenience, denote the set of team members by $\mathcal{S} = \{1, ..., n\}$.

The total amount of the final good produces is the linearly additive sum of what each team member produces in their respective role,

$$Y = \sum_{i \in \mathcal{S}} Y_i, \tag{1}$$

where $Y_i$ is the total output produced by worker $i$ from combining tasks according to a CES aggregator,

$$Y_i = \left( \int_{\mathcal{T}} q_i(\tau)^{\frac{\eta-1}{\eta}} d\tau \right)^{\frac{\eta}{\eta-1}}, \tag{2}$$

where $q_i(\tau)$ is the amount of task $\tau$ used by worker $i$ in production, and $\eta > 0$ is the elasticity of substitution across tasks.[3]

In a classical Ricardian fashion, task-level production is linear in efficiency units of labor,

$$y_i(\tau) = z_i(\tau) l_i(\tau), \tag{3}$$

where $l_i(\tau)$ is the time dedicated by $i$ to that task.

Each worker can use their time either in the *production* of tasks or *communicating*. Thus, the time constraint for worker $i \in \mathcal{S}$ reads

$$1 = \int_{\mathcal{T}} l_i(\tau) d\tau + c_i, \tag{4}$$

where $c_i$ is the total amount of time $i$ spends *communicating*, which is the sum spent communi-

[3] We may think it is more plausible for the $Y_i$'s to be imperfectly substitutable. While this is true, a key point of the model is that complementarities between workers' qualities can emerge even without imposing that the outputs of different job roles' are complements.

cating with each one of her coworkers

$$c_i = \sum_{n \in S \setminus i} c_{in}. \tag{5}$$

Communication does not directly contribute to production, indeed reduces the time available for problem solving. We will see that it can nonetheless serve a productive purpose, by ensuring that whatever time is spent in production makes the best possible use of individuals' task-specific skills.

In the spirit of trade model à la Eaton and Kortum (2002), the worker-task specific efficiencies for worker $i$, $\{z_i(\tau)\}_{\tau \in \mathcal{T}}$, are treated probabilistically as the realizations of a Fréchet-distributed random variable, drawn independently for each worker $i$. Thus, for all $z \geq 0$, the distribution of efficiencies for worker $i$ is

$$G_i(z) := Pr(z_i(\tau) \leq z) = \exp\left( - \left( \frac{z}{\iota x_i} \right)^{-1/\chi} \right). \tag{6}$$

This assumption parsimoniously captures both vertical and horizontal differentiation among workers. A worker's *absolute* advantage type, $x_i$, determines the scale of the worker-specific distribution. The (inverse) shape parameter, $\chi \in (0, \infty)$, determines the degree of dispersion, and thus the importance of *comparative* advantage. It is identical for all workers. Lastly, $\iota := \Gamma(1 + \chi - \eta\chi)^{\frac{1}{1-\eta}}$ is a scaling term, with $\Gamma$ denoting the Gamma function. Throughout the paper, the standard assumption is made that $1 + \chi(1 - \eta) > 0$.

The firm's role in this model is to facilitate the efficient division of labor. Concretely, we suppose that workers obtain tasks from each other – in a modern knowledge economy, we might think of information and answers to problems encountered in production. The total quantity of task $\tau$ used by some worker $n$ is

$$q_n(\tau) = \sum_{i \in \mathcal{S}} q_{in}(\tau), \tag{7}$$

where $q_{in}(\tau)$ is the quantity of task $\tau$ produced by worker $i$ and used for production by worker $n$,[4] alongside an adding-up constraint

$$\sum_{n \in S} q_{in}(\tau) \leq y_i(\tau). \tag{8}$$

However, such "task trade" (Deming, 2017) is potentially costly, because it requires communi-

[4]To aid orientation and where applicable we use the subscript $n$ for a 'user' of tasks and $i$ for the 'producer'.

cation that, per equation (4), reduces the amount of time available for task production. For the solution to some task $\tau$ solved by worker $i$ to be usable by coworker $n$, the former has to explain it to the latter. Specifically, suppose that

$$c_{in} = \int_{\mathcal{T}} \frac{q_{in}(\tau)}{z_i(\tau)} \times \left(\frac{1}{m} - 1\right) d\tau \tag{9}$$

if $i \neq n$ and naturally, $c_{ii} = 0$. This effectively means that the time spent by $i$ communicating with $n$ is proportional to the time spent by $i$ solving problems for $n$.[5] As in Garicano (2000), it is assumed that the 'helping' cost is incurred by the helper alone; this is arguably counterfactual but important for tractability. The *extent* to which the division of labor absorbs communication time depends on how well the team is organized, as determined by $m$: At one extreme, when $m = 0$ the division of labor would leave workers with no time to actually produce – so it will be optimal not to implement said division of labor – while at the other extreme $m = 1$ corresponds to a world in which coordination through communication is cost-free.[6] Going forward, for ease of notation define $m_{in} = m$ if $i \neq n$ and $m_{in} = 1$ otherwise.[7]

The firm's problem is to maximize team output subject to the time and technological constraints. The associated Lagrangian is

$$\begin{aligned}
\mathcal{L}(\cdot) = \sum_{i \in \mathcal{S}} \Bigg\{ & Y_i + \lambda_i^L \left[ 1 - \int_{\mathcal{T}} \left( l_i(\tau) + \sum_{n \in \mathcal{S}} \frac{q_{in}(\tau)}{z_i(\tau)} \left( \frac{1}{m_{in}} - 1 \right) \right) d\tau \right] \\
& + \int_{\mathcal{T}} \lambda_i(\tau) \Big( z_i(\tau) l_i(\tau) - y_i(\tau) \Big) d\tau + \int_{\mathcal{T}} \hat{\lambda}_i(\tau) \Big( y_i(\tau) - \sum_{n \in \mathcal{S}} q_{in}(\tau) \Big) d\tau \\
& + \int_{\mathcal{T}} \tilde{\lambda}_i(\tau) \Big( \sum_{j \in \mathcal{S}} q_{ji}(\tau) - q_i(\tau) \Big) d\tau + \lambda_i \left[ \left( \int_0^1 q_i(\tau)^{\frac{\eta-1}{\eta}} d\tau \right)^{\frac{\eta}{\eta-1}} - Y_i \right] \\
& + \check{\lambda}_i \left( \int_{\mathcal{T}} \lambda_i(\tau) y_i(\tau) d\tau - \int_{\mathcal{T}} \tilde{\lambda}_i(\tau) q_i(\tau) d\tau \right) + \int_{\mathcal{T}} \bar{\lambda}_i(\tau) y_i(\tau) d\tau + \sum_{j \in \mathcal{S}} \int_{\mathcal{T}} \bar{\lambda}_{ji}(\tau) q_{ji}(\tau) d\tau \Bigg\}.
\end{aligned}$$

Here, the various $\lambda$ terms denote Lagrange multipliers, with the last two ($\bar{\lambda}_i$ and $\bar{\lambda}_{ji}$) corresponding o non-negativity constraints on the production and use of tasks. In addition, to simplify the problem, in the second line we substituted already for communication time.

Importantly, and as indicated in the second-to-last line, we also impose a form of incentive

[5]Implicitly, but plausibly, we assume that greater efficiency in production also means greater efficiency in explaining solutions.

[6]This is the case considered in Freund (2022).

[7]A key difference to Garicano's (2000) setup is that here workers *know* each others' task-specific skills and so it is clear whom to ask for help upon encountering a problem. By contrast, in Garicano (2000), matching problems to coworkers is costly, and so workers ask for help until they find someone who can solve the problem.

compatibility constraint by requiring that

$$\int_{\mathcal{T}} \lambda_i(\tau) y_i(\tau) d\tau = \int_{\mathcal{T}} \tilde{\lambda}_i(\tau) q_i(\tau) d\tau. \tag{10}$$

This means that for each worker, the shadow value of all tasks used must be equal to the shadow value of all tasks produced by that worker. This assumption is consistent with an allocation of tasks such that workers whose welfare is increasing in their individual final goods output $Y_i$ (say, due to a suitable wage compensation arrangement) would voluntarily undertake the task trades.[8]

## 2.2 Solving for the optimal organization

To solve for efficient production, we start by deriving the demand for tasks for a given set of shadow prices. The first-order condition (FOC) with respect to worker $i \in \mathcal{S}$'s usage of task $\tau \in \mathcal{T}$, $q_i(\tau)$, reads

$$\tilde{\lambda}_i(\tau) = \lambda_i \left( \int_{\mathcal{T}} q_i(\tau)^{\frac{\eta-1}{\eta}} d\tau \right)^{\frac{\eta}{\eta-1}-1} q_i(\tau)^{-1/\eta} \tag{11}$$

Using the expression for job-level output (2) and letting $Q_i(\tau) = \tilde{\lambda}_i(\tau) q_i(\tau)$ denote the shadow value of $i$'s use of task $\tau$, we obtain

$$\frac{Q_i(\tau)}{\lambda_i Y_i} = \left( \frac{\tilde{\lambda}_i(\tau)}{\lambda_i} \right)^{1-\eta}. \tag{12}$$

Integrating on both sides and using

$$Q_i \equiv \int_{\mathcal{T}} Q_i(\tau) d\tau = \lambda_i Y_i, \tag{13}$$

we get the following expression for the shadow price index for worker $i$:

$$\lambda_i = \left( \int_{\mathcal{T}} \tilde{\lambda}_i(\tau)^{1-\eta} d\tau \right)^{\frac{1}{1-\eta}}. \tag{14}$$

Finally, notice that equation (13) implies that real final goods output of worker $i$ is the sum

[8] Clearly, this treatment of the incentive problem could be refined, but it suffices to make the points of this short note.

total of the shadow values of tasks used by $i$ divided by the worker-specific shadow cost index:

$$Y_i = \frac{Q_i}{\lambda_i}. \tag{15}$$

Next, consider task production and shadow costs. Treating the task-specific productivities as known, the FOC with respect to $q_{in}(\tau)$ – the quantity of task $\tau$ produced by $i$ and used by $n$ – is

$$\tilde{\lambda}_n(\tau) + \bar{\lambda}_{in}(\tau) = \hat{\lambda}_i(\tau) + \frac{\lambda_i^L}{z_i(\tau)}\left(\frac{1}{m_{in}} - 1\right) \tag{16}$$

Now if $\bar{\lambda}_{in}(\tau) > 0$, then $\tilde{\lambda}_n(\tau) < \hat{\lambda}_i(\tau) + \frac{\lambda_i^L}{z_i(\tau)}\left(\frac{1}{in} - 1\right)$ and $q_{in}(\tau) = 0$. If $q_{in}(\tau) > 0$, then $\bar{\lambda}_{in}(\tau) = 0$ and $\tilde{\lambda}_n(\tau) = \hat{\lambda}_i(\tau) + \frac{\lambda_i^L}{z_i(\tau)}\left(\frac{1}{m_{in}} - 1\right)$. Intuitively, the shadow price of an additional unit of $\tau$ used by $n$ must be equal to the shadow price of $i$ providing it, accounting for the communication time involved in explanation.

Since $n$ will obtain this good from *some* team member,

$$\tilde{\lambda}_n(\tau) = \min_i \left\{\hat{\lambda}_i(\tau) + \frac{\lambda_i^L}{z_i(\tau)}\left(\frac{1}{m_{in}} - 1\right)\right\}. \tag{17}$$

The FOC with respect to production of $\tau$ by $i$, $y_i(\tau)$, is

$$\hat{\lambda}_i(\tau) + \bar{\lambda}_i(\tau) = \lambda_i(\tau), \tag{18}$$

so that $\hat{\lambda}_i(\tau) = \lambda_i(\tau)$ if $y_i(\tau) > 0$. Since $y_i(\tau) > 0$ if $q_{in}(\tau) > 0$, we can rewrite equation (17) as

$$\tilde{\lambda}_n(\tau) = \min_i \left\{\lambda_i(\tau) + \frac{\lambda_i^L}{z_i(\tau)}\left(\frac{1}{m_{in}} - 1\right)\right\}. \tag{19}$$

Worker $i$ produces task $\tau$ if she achieves this minimum for some $n \in \mathcal{S}$.[9]

Next, we characterize $\lambda_i(\tau)$. Suppose that $i$ does produce $\tau$, such that $y_i(\tau) > 0$. Then the FOC for $l_i(\tau)$ is

$$\begin{aligned} -\lambda_i^L + \lambda_i(\tau) z_i(\tau) &= 0 \\ \Leftrightarrow \lambda_i(\tau)\frac{y_i(\tau)}{l_i(\tau)} &= \lambda_i^L, \end{aligned} \tag{20}$$

[9]Notice that if $= 0$, so that any division of labor is fruitless, then for any $n \in \mathcal{S}$ this least-cost provider $i$ will simply be herself.

Substituting for $\lambda_i(\tau)$ in equation (19) we find that

$$\tilde{\lambda}_n(\tau) = \min_i \Bigg\{ \underbrace{\frac{\lambda_i^L}{z_i(\tau)}}_{\text{shadow cost of production}} + \underbrace{\frac{\lambda_i^L}{z_i(\tau)}\left(\frac{1}{m_{in}} - 1\right)}_{\text{shadow cost of communication}} \Bigg\} \tag{21}$$

$$= \min_i \left\{ \frac{\lambda_i^L}{z_i(\tau) m_{in}} \right\}, \tag{22}$$

where the term inside $\{\cdot\}$ defines $\tilde{\lambda}_{in}(\tau)$ as the shadow price of $i$ providing task $\tau$ to $n$.

Next, we exploit the assumption on the distribution of task-level production efficiencies to pin down the worker-specific shadow price index, $\lambda_i$. That is, treating the individual $z_i(\tau)$ terms as unknown (but the distributional parameters as known), the probability that the shadow price of $i$ providing task $\tau$ to $n$ is less than $p$ is

$$G_{in}(p) \equiv \Pr\left\{ \frac{\lambda_i^L}{z_i(\tau)_{in}} \le p \right\} = 1 - \exp\left( - x_i^{1/\chi} \left( \frac{\lambda_i^L}{\iota p_{in}} \right)^{-1/\chi} \right). \tag{23}$$

Using this result, the probability distribution $G_n(\tau)$ of shadow costs of $n$ obtaining task $\tau$ can be shown to be

$$G_n(p) \equiv \Pr\{\tilde{\lambda}_n(\tau) \le p\} = 1 - \exp((-\iota p)^{1/\chi} \Phi_n), \tag{24}$$

where

$$\Phi_n = \sum_{i \in \mathcal{S}} x_i^{1/\chi} \left( \frac{\lambda_i^L}{m_{in}} \right)^{-1/\chi} \tag{25}$$

indexes the shadow cost of tasks that $n$ can use if tasks are assigned optimally.

Hence, the integral in equation (14) can be simplified as

$$\lambda_i = \left( \int_0^\infty p^{1-\eta} dG_i(p) \right)^{\frac{1}{1-\eta}}$$
$$= (\Phi_i)^{-\chi} \tag{26}$$

Exploiting the properties of the Fréchet distribution, it can be shown that the fraction of tasks for which $i$ is the minimum cost provider for $n$ is

$$\pi_{in} \equiv \Pr\{\lambda_{in}(\tau) \leq \min_{k \in \mathcal{S} \setminus i} \lambda_{kn}(\tau)\} = \frac{x_i^{1/\chi}(\lambda_i^L/m_{in})^{-1/\chi}}{\Phi_n}. \tag{27}$$

Define the shadow value of all deliveries from $i$ to $n$ as

$$Q_{in} = \int_{\mathcal{T}} \tilde{\lambda}_n(\tau) q_{in}(\tau) d\tau. \tag{28}$$

Then since the distribution of $\tilde{\lambda}_n$ is the same regardless of which team member ultimately provides the tasks – another key implication of the Fréchet distribution – and since the fraction of tasks provided to $n$ by $i$ is $\pi_{in}$, we get:

$$\begin{aligned} Q_{in} &= \pi_{in}\left(\int_{\mathcal{T}} \tilde{\lambda}_n(\tau) q_n(\tau) d\tau\right) = \pi_{in} \int_{\mathcal{T}} Q_n(\tau) d\tau = \pi_{in} Q_n, \\ \Leftrightarrow Q_{in} &= \pi_{in} Q_n, \end{aligned} \tag{29}$$

so that $\pi_{in}$ is also the fraction of the shadow value of tasks used by $n$ that is provided by $i$.

Next, define the shadow value of all tasks produced by worker $i$ as

$$Y_i^p = \int_{\mathcal{T}} \lambda_i(\tau) y_i(\tau) d\tau. \tag{30}$$

We can relate the shadow value of $i$'s labor to this production value by integrating over equation (20) and using the worker's time constraint (4), which yields

$$\lambda_i^L = Y_i^p. \tag{31}$$

Now use equation (29) together with the expressions for $\pi_{in}$ and $\Phi_n$ to get

$$Q_{in} = \frac{x_i^{1/\chi}(\lambda_i^L/m_{in})^{-1/\chi}}{\sum_{k \in \mathcal{S}} x_k^{1/\chi}\left(\lambda_k^L/m_{kn}\right)^{-1/\chi}} Q_n. \tag{32}$$

Together with equations (10) and (31), and summing across all $n$'s, we obtain

$$\sum_{n \in \mathcal{S}} Q_{in} = \sum_{n \in \mathcal{S}} \frac{x_i^{1/\chi}(\lambda_i^L/m_{in})^{-1/\chi}}{\sum_{k \in \mathcal{S}} x_k^{1/\chi}\left(\lambda_k^L/_{kn}\right)^{-1/\chi}} \lambda_n^L \tag{33}$$

Together with equations (7), (31) and again (10), we obtain the following system of $n-1$ independent equations which together with one normalization pin down the shadow values of the labor provided by each worker:

$$\lambda_i^L = \sum_{n\in\mathcal{S}} \frac{x_i^{1/\chi}(\lambda_i^L/m_{in})^{-1/\chi}}{\sum_{k\in\mathcal{S}} x_k^{1/\chi}(\lambda_k^L/m_{kn})^{-1/\chi}}\lambda_n^L. \tag{34}$$

It can be shown that a solution for the vector of labor shadow values exists and is unique up to the normalization of one shadow price (Alvarez and Lucas, 2007).

# 3 Analysis and main results

I next describe two implications of the model of team production with communication costs.

Before doing so, it is instructive to take the following, preliminary step to describe $Y_i$ as a function of absolute advantage ($x$), skill specialization ($\chi$) and an (endogenous) measure of the depths of the division of labor. From equation (15) and using that $\lambda_i^L = Q_i$, as implied by equations (31) and (10), final goods output of worker $i$ can be expressed as

$$Y_i = \frac{\lambda_i^L}{\lambda_i}. \tag{35}$$

Moreover, from equation (29) and substituting $\Phi_n = \lambda_n^{1/\chi}$ gives

$$Q_{in} = \frac{x_i^{1/\chi}(\lambda_i^L/m_{in})^{-1/\chi}}{\lambda_n^{1/\chi}}Q_n,$$

which together with $\pi_{in} = Q_{in}/Q_n$ implies

$$\pi_{in} = m_{in}^{1/\chi}x_i^{1/\chi}\left(\lambda_i^L/\lambda_n\right)^{-1/\chi}$$

Evaluating this expression at $i=n$, using equation (35) and solving for output gives

$$Y_i = x_i\pi_{ii}^{-\chi}. \tag{36}$$

Thus, any worker's output is the product of her own skill and a function of the share of tasks she obtains from herself. The latter is a measure of the depth of the division of labor.[10] Intuitively, and other things equal, the more an organization permits a worker to focus on those tasks in

[10]To be clear, this result is well-known in the trade literature.

which she has a comparative advantage, the more productive she will be. Hence, a worker's productivity is pinned down not solely by her *endowments* but also on the *organization of work*, as reflected in the assignment of tasks.

When $m < 1$, there will be *task overlap* as opposed to a *perfect division of labor*. As a result, each worker performs some tasks that she would not perform by herself absent communication costs. Because communication takes time away from actual production, some tasks get performed in which worker $i$ does not have a comparative advantage (in the sense of minimizing $\frac{\lambda_i^L}{z_i \tau}$).

## 3.1 Specialization and organizational quality

The first result describes how worker-task specialization ($\chi$) mediates the relationship between labor productivity and organizational quality ($m$).

To establish this relationship in the most transparent fashion, consider the special case where there is no vertical differentiation, in the sense that all team members are of the same quality: $x_i = x \ \forall i \in \mathcal{S}$. Such an allocation would naturally obtain if the hiring margin was endogenized and labor markets were frictionless, since for $\chi > 0$ coworker qualities are complementary (cf. Freund, 2022, 2.2).[11]

Under this assumption, we can establish the following closed-form result:

**Lemma 1.** *Suppose that $x_i = x \ \forall i \in \mathcal{S}$. Then team output as a function of $x$ is*

$$Y = f(x; \chi, m) = nx\left(1 + (n-1)m^{1/\chi}\right)^{\chi}. \tag{37}$$

*Proof.* Assuming that $x_i = x \ \forall i \in \mathcal{S}$, which by symmetry implies that $\lambda_i^L = \lambda_L \ \forall i \in \mathcal{S}$. Then since $\lambda_i = \Phi_i^{-\chi}$ and

$$\begin{aligned}
\Phi_i &= \sum_{i \in \mathcal{S}} x_i^{1/\chi}\left(\frac{\lambda_i^L}{m_{in}}\right)^{-1/\chi} \\
&= \left(\frac{x}{\lambda^L}\right)^{1/\chi} + \sum_{k \in S \setminus i}\left(\frac{x}{\lambda^L}\right)^{1/\chi} m^{1/\chi} \\
&= \left(\frac{x}{\lambda^L}\right)^{1/\chi}\left(1 + (n-1)m^{1/\chi}\right),
\end{aligned}$$

it follows that each worker's shadow cost index of production is the same.

[11] The result is straightforward to validate numerically also when there is within-firm worker quality heterogeneity.

Considering the tasks shares, each worker will use weakly more tasks from herself but obtain the same share from each of her coworkers:

$$\begin{aligned}\pi_{in} &= \frac{x_i^{1/\chi}(\lambda_i^L/m_{in})^{-1/\chi}}{\Phi_n} \\ &= \frac{1}{1+(n-1)m^{1/\chi}}.\end{aligned}$$

Notice that for $m = 0$, $\pi_{ii} = 1$ and for $m = 1$, $\pi_{ii} = \frac{1}{n}$.

Substituting for $\pi_{ii}$ in equation (36) and summing over workers completes the proof. □

It is then straightforward to see that the effect of an improvement in organizational quality $m$ on labor productivity is positive for $\chi > 0$ and strictly increasing in $\chi$. Specifically, it follows that:

**Corollary 1.** *Suppose that $x_i = x \ \forall i \in \mathcal{S}$. Then the elasticity of per-capita output with respect to $m$ is*

$$\epsilon_{y,m} = \frac{(n-1)m^{1/\chi}}{1+(n-1)m^{1/\chi}}. \tag{38}$$

Interpreted through the lens of the model, this relationship obtains for the following reason. For any given value of $\chi$, a higher organizational capacity $m$ enables a deeper division of labor; at the margin, it is more likely that the worker optimally asks a more knowledgeable colleague for help to resolve some problem. The gain of this reduction in task overlap, with the worker concentrating on those tasks in which she has a higher efficiency $z$, is greater when worker-task specific productivities are more dispersed, that is when $\chi$ is high. Simply put, organization is more valuable when specialization is high.

## 3.2 Organizational quality and coworker complementarities

The second result explains how a deeper division of labor, facilitated by high organizational capacity, generates not only productivity gains, for any value of $\chi$, but also reinforces vulnerabilities in the sense of coworker complementarities.

To see this, I first re-state the aggregation result of Freund (2022), which obtains under the special case of $m = 1$.

**Lemma 2** (Aggregation result)**.** *When $m = 1$, team output $Y$ can be written as a function of* *members' quality types,* $f : [0,1]^n \rightarrow \mathbb{R}_+$,

$$f(x_1, \cdots, x_n) = \underbrace{n^{1+\chi}}_{\textit{efficiency gains}} \underbrace{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i)^{\frac{1}{1+\chi}}\right)^{1+\chi}}_{\textit{complementarity}}. \tag{39}$$

As the second term takes the form of a CES function, it is immediate how the elasticity of complementarity – the inverse of the elasticity of substitution – is equal to $\gamma = \frac{\chi}{1+\chi}$. Concretely, this means that when $\chi$ is high, and for a given (arithmetic) *average* value of $x_i$, productivity is lowered if members vary in their quality type.

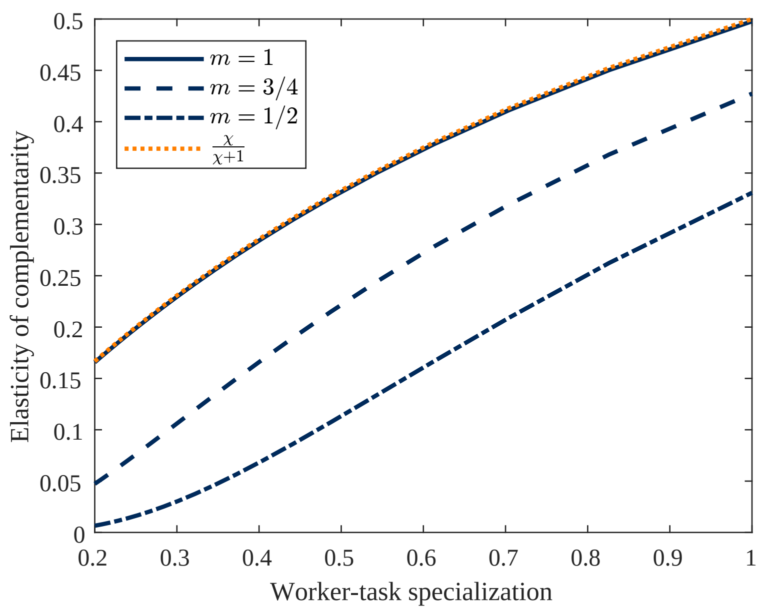


**Figure 1:** Coworker complementarities are increasing in specialization and organizational quality

*Notes.* This figure plots the elasticity of complementarity as a function of $\chi$, for three different values of $m$. For the solid, dashed, and dashed-dotted line, the elasticity $\gamma$ is computed numerically, by solving for the optimal organization for different values of $\{x_i\}_{i=1}^{n}$ on a grid, and then computing the gradients numerically. The dashed line indicates the exact value of $\gamma$ given the closed-form solution under $m = 1$. The size is set to $n = 2$.

If we relax the assumption that $m = 1$, the mapping between workers' quality types and $Y$ can no longer be cast in closed-form, but the elasticity of complementarity can, of course, still be computed numerically. Figure 1 plots $\gamma$ as a function of $\chi$ for three alternative values of $m$. Two observations stand out. First, coworker complementarity is increasing in $\chi$ even if $m < 1$. This implies that the key relationship between worker-task specialization in complementarity in Freund (2022) is robust to imperfect division of labor, as generated by $m < 1$. Second, and more pertinently in the present note, the degree of complementarity is *increasing* in $m$ for any given value of $\chi$. Simply put, when communication is costly each worker perform a greater share of tasks than if communication is cost-free, resulting in lower interdependence across coworkers.

The model thus implies that greater organizational quality is associated with productivity gains, because each worker is more enabled to focus on those tasks in which they have a comparative advantage, but this also implies a greater "vulnerability" to low-ability team members. This theoretical result echoes the spirit of the empirical results of Kuhn *et al.* (2023).[12]

# 4 Concluding discussion: some future research possibilities

In summary, in this short note I considered a version of the team production model of Freund (2022) extended to incorporate communication costs. I showed that the productivity advantage from high organizational capacity – facilitating within-firm division of labor without much time being lost on communication – is increasing in worker-task specialization; and that a deeper division of labor strengthens coworker complementarities, making output vulnerable to below-average quality team members. Simply put, it is difficult to organize production efficiently in a context of specialization, requiring both within-firm coordination capacities and access to well-matched team members.

The note merely provides a sketch of these ideas, with the hope of stimulating further research in this direction. On the theoretical side, it would be interesting to endogenize the span of control by incorporating a managerial time constraint – trading off team size $n$ and time dedicated to facilitating efficient collaboration, for instance – as well as explicitly modelling firm heterogeneity. Regarding the latter, and going beyond the model, the brief analysis here suggests, for instance, that ex-ante differences in organizational or managerial capacity should be amplified in terms of firm performance outcomes as worker-task specialization increases over time.

Empirically, the literature has made progress in identifying the important role of worker-job matching in determining productivity, leaning on rich and innovative micro data (see overview in Section 1). In light of the conjecture formulated in this note, it would be interesting to see whether the importance of this role is greater when the problems or tasks confronted are complex and workers are specialized. In addition, both for the questions considered in this paper and the related, recent literature on coworker learning (Nix, 2020; Jarosch *et al.*, 2019; Herkenhoff *et al.*, 2022), direct evidence on within-team interactions linked to outcomes would be very valuable.[13]

Lastly, the dual obstacles to efficient production which this note has shown to arise when requirements are complex and worker skills specialized may have some implications for macro topics in both advanced and developing economy contexts. Regarding the former, these obstacles

[12] Of course, an increase in $m$ unambiguously increases total output even if worker quality types are dispersed. The claim here is merely that those gains are more muted than they would be absent the coworker complementarity effect.

[13] Some progress in this direction is made in Emanuel *et al.* (2023), albeit in a narrow work context.

point to an additional reason for the existence of "superstar firms," beyond such factors as winner-takes-it-all product market structures. As is well-illustrated by the unique position of firms such as TSMC or ASML in the semi-conducting space, a different reason may be that the combination of many specialized inputs that need to be assembled may just be very hard to pull off, so few attempts succeed. In the developing economy context these considerations may be salient in a different way. Bassi *et al.* (2023) underscore barriers to within-firm division of labor using survey data from Uganda, and labor market frictions are well-understood to be more severe in poorer countries (Donovan *et al.*, 2023). Insofar as production of goods and services with greater value-added also involve more worker-task specialization, as seems plausible, the results in this note suggest that these within-firm and input market frictions represent obstacles to climbing up the value chain.

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