# Superstar Teams: The Micro Origins and Macro Implications of Coworker Complementarities[*]

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August 19, 2023

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## Abstract

This paper proposes a model of the firm as a "team assembly technology," with the aim of explaining why differences between firms represent a large and growing dimension of wage inequality. In the model, firms assign tasks to workers who vary in overall talent and task-specific skills. Hiring is subject to search frictions. Worker-task specialization not only reinforces the potential gains from team production, but also endogenously generates coworker complementarity: the least capable team member's talent disproportionately influences joint output. In equilibrium, employers hire workers of similar talent, and those with superstar teams pull away in terms of pay. The key model mechanisms are validated using rich administrative micro data. A theory-informed measure of coworker complementarity doubles from the mid-1980s to the 2010s, mirroring a shift toward greater task complexity. According to a structural estimation exercise, this rise explains around 40% of the empirically observed increase in the between-firm share of wage inequality in Germany. Additionally, the model sheds light on how the interaction between specialization and labor market frictions influences total factor productivity.

**Keywords:** coworker complementarity, firms, inequality, matching, teams, technology.

# 1    Introduction

Firms play an important role in the evolution of wage inequality. An extensive empirical literature documents that, across many advanced economies, a prominent feature of wage dispersion and its rise between the 1980s and today is the large and increased share attributable to differences in average pay between employers. A key reason is that gaps across firms in the quality of their workforces have widened. Yet, theories that capture and explain these facts are scarce.[1]

In this paper, I propose a novel framework that rationalizes empirical patterns of firm-level inequality in workforce quality and wages, linking them to the nature of production. The framework centers on a conception of the firm as a "team assembly technology." This perspective emphasizes that production involves multiple workers with varied skills, whose interdependence is shaped by technological forces, and interprets labor market sorting as endogenous variation in who is hired to work alongside whom. I highlight the determinants and macroeconomic implications of a specific form of coworker interdependence: complementarity across coworkers' talents, which refers to the degree to which the marginal productivity of one employee's talent is increasing in that of colleagues. I argue that shifts in the nature of work have reinforced coworker complementarity, making it increasingly advantageous for highly capable workers to collaborate among themselves and leading to increased labor market segregation between workers of greater and lesser productivity, respectively, as they cluster in different firms.

The analysis proceeds in three main steps. First, I build and characterize an equilibrium model of firms that produce with teams of heterogeneous workers hired in a frictional labor market. This model provides a tractable, task-based microfoundation for coworker complementarities and delineates their influence on the wage distribution within and between firms. Second, I develop a method to estimate these complementarities and implement it using German matched employer-employee data. I find that complementarities have doubled in magnitude since the mid-1980s, mirroring a shift in the nature of production away from routine tasks toward more complex requirements. Third, I structurally estimate the model, with moments recovered from the micro data serving to impose discipline. Counterfactual exercises using the estimated model show that the rise in complementarities can account for 40% of the empirically observed growth in the between-firm share of wage inequality. Overall, the paper thus contributes theory, tools, and evidence that jointly facilitate a parsimonious, quantitative interpretation of changes in the nature of production and wage distribution within and between firms.

Elaborating on the first step, the model has three main ingredients. First, workers are heterogeneous in their overall talent, modeled in terms of absolute advantage, as well as the tasks in

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[1] Key references on firm-level inequality include Card *et al.* (2013) for Germany and Song *et al.* (2019) for the U.S., while Criscuolo *et al.* (2021) provide a cross-country analysis. The literature is discussed in more detail below.

which they enjoy a comparative advantage. Such "worker-task specialization" contrasts with an environment where a talented employee excels – and is thus more productive than a less capable coworker – across all tasks. Second, in the spirit of Garicano (2000), firms' role is to coordinate the division of labor. Each firm hires multiple workers and then assigns them tasks to maximize production. Third, labor services are traded in frictional markets, specifically a random-search environment à la Herkenhoff *et al.* (2022). Consequently, team assembly involves a trade-off between the productivity gain from forming a well-matched team and search costs.

These ingredients jointly give rise to an equilibrium in which some firms have teams composed of highly talented workers, while others mostly employ less capable ones. Worker-task specialization endogenously generates coworker complementarity. The output of a highly talented worker can be significantly affected by a less capable peer, because the former then spends more time on tasks they are relatively inefficient at. Absent frictions, any degree of complementarity leads to pure positive assortative matching (cf. Kremer, 1993). Due to search costs, however, workers and firms agree on a range of mutually acceptable matches and the strength of coworker complementarity influences firms' hiring behavior. If worker-task specialization and hence complementarity is pronounced, firms avoid teaming up highly capable but expensive workers with less capable peers, even if this requires expending time on search.

In this environment, technological shocks that reinforce worker-task specialization raise between-firm inequality at the macro level. And indeed, empirically different employers have not only diverged in workforce composition and remuneration; *how* firms produce using labor inputs has also changed fundamentally. Employees are performing fewer routine tasks, which everyone is similarly productive at, instead confronting more cognitive non-routine, or complex, problems. Relatedly, an increasing "burden of knowledge" (Jones, 2009) and declining within-firm communication costs necessitate and facilitate specialization. These trends have previously been tied to an increased prevalence of team production (Jones, 2021).[2] The model reveals that they also strengthen coworker complementarity, thereby amplifying gaps between firms in workforce quality and pay.

I analytically characterize the two key relationships underpinning this mechanism: the strength of coworker complementarity is increasing in the degree of worker-task specialization; and stronger complementarity fosters firm-level inequality in equilibrium. I show that the probabilistic, extreme-value formulation of technological heterogeneity commonly employed in trade and spatial economics (e.g., Eaton and Kortum, 2002) can be leveraged to derive a tractable,

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[2]Regarding changes in the task content of production, see, e.g., Acemoglu and Autor (2011) and the references in Footnote 5. Jones (2009), Bloom *et al.* (2014), and Neffke (2019) discuss shifts in learning and communication costs. As a concrete example, in the medical sector rapid advances in knowledge and technology have necessitated more narrowly focused expertise. In 2020 the American Board of Medical Specialities issued 126 speciality and sub-speciality certificates, nearly twice as many as recorded in 1980.

aggregative team production function. Specifically, the vector of team members' absolute advantage types is a sufficient statistic for team output, and the elasticity of substitution between coworkers' talents is endogenously pinned down by the specialization parameter. Stronger complementarity puts a greater weight on the talent of the least-capable team member(s) in determining output. As a result of this spillover effect, it is costlier when a highly capable employee is teamed up with a less capable coworker. Furthermore, in a stylized version of the matching model, I derive in closed-form that stronger firm-level complementarity leads to greater worker segregation and between-firm wage inequality at the macro-level.

In a second step, I empirically estimate the evolution of coworker complementarities over time, as well as validating key model mechanisms using cross-sectional variation. Directly measuring coworker complementarities in production is infeasible in general data environments. I therefore leverage the theoretical model to derive a moment, coworker wage complementarity, that is informative about production complementarity and which can be measured using a matched employer-employee panel. I implement this approach using such data for Germany, supplemented with multiple waves of survey micro data tracking the task content of production.[3] A key finding is that coworker wage complementarity has approximately doubled since 1990, alongside rising task complexity. Moreover, in the cross-section, occupations performing more complex tasks exhibit stronger complementarities, which in turn is associated with more pronounced coworker sorting – consistent with the model's predictions.

In step three, I combine theory and empirics to provide a quantitative assessment. The model is calibrated to fit micro- and macro-moments of the German economy around 1990 and in the 2010s. The structural estimation targets the evidence on coworker complementarities obtained from the micro data while deliberately treating between-firm inequality and coworker matching patterns as untargeted moments. The model still matches them well. In my baseline parameterization, the estimated model predicts a 16 percentage point increase in the between-establishment share of the variance of log wages, accounting for two thirds of the 23 percentage point rise in the data. Counterfactual analyses imply that if coworker complementarity had not strengthened, then the between-share of wage inequality would have increased by less than half as much. Thus, the rise in coworker complementarity estimated in the micro data can account for 40% of the empirically observed total rise in between-firm inequality.[4]

Lastly, the model suggests that rising between-firm inequality need not reflect worsening frictions such as product or wage-setting power. The microfounded production function highlights that more pronounced worker-task specialization not only yields productivity gains. By amplify-

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[3]For robustness purposes and cross-sectional analyses, I also consider comprehensive micro data for Portugal.
[4]The remainder is mostly attributed to mechanical effects of skill-biased technological change and, to a lesser extent, to declining search frictions. I also conduct robustness check that consider the influence of occupational composition effects (e.g., due to outsourcing) and on-the-job search.

ing complementarity, it also makes positively assortative coworker matching a more important precondition for efficient production. Conversely, misallocation due to search frictions is costlier. Quantitatively, the estimated model implies that coworker mismatch costs are non-trivial but have not increased much, accounting for 1.8% of per-capita output in 1990 and 2.2% in the 2010s. Crucially, though, the gap between efficient and realized labor productivity would have been 2.6 percentage points greater in 2010 absent the observed rise in coworker sorting.

LITERATURE. This paper relates to five literatures. First, I connect two strands of research on wage equality. On the one hand, studies of technological change examine how changes in the nature of work – notably, a shift from routine to cognitive and interactive non-routine tasks – affect skill prices.[5] On the other hand, numerous recent papers, notably Card *et al.* (2013) and Song *et al.* (2019), employ statistical and reduced-form approaches to document firms' significant and growing role in explaining wage inequality, a key factor being greater labor market segregation.[6] Models that could explain this elevated importance of firms are lacking, however. The challenge is that most influential theories of inequality center on the aggregate production function and wage distribution, while canonical models of labor market sorting lack a well-defined notion of within-firm worker heterogeneity (Krusell *et al.*, 2000; Shimer and Smith, 2000; Acemoglu and Restrepo, 2018). I help fill this gap by contributing a quantitative model that incorporates within- and between-firm worker heterogeneity. Indeed, once endogenous coworker complementarities are accounted for, the transformation of work studied in the first strand also helps rationalize rising between-firm inequality.

Second, I add to the literature on firm organization that studies the implications of within-firm division of labor for productivity and inequality. Recent contributions include Porzio (2017), Adhvaryu *et al.* (2020), Caliendo *et al.* (2020), Tian (2021), Adenbaum (2022), Bloesch *et al.* (2022), Kohlhepp (2022), Minni (2022) and Kuhn *et al.* (2023). My model is especially connected to and builds on the theory of knowledge-based hierarchies articulated in Garicano (2000) and Garicano and Rossi-Hansberg (2006), describing the organization of production and the distribution of wages as jointly determined by the equilibrium assignment of individuals to firms and to tasks within firms. While hierarchical models have a "vertical" focus – complementarities between managers and their endogenous number of supervisees – mine considers complementarities more generally and lends itself to quantification, at the cost of treating the span of control as exogenous. I also build on Kremer (1993) who argues that what I call coworker complementar-

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[5]See, e.g., Autor *et al.* (2003), Spitz-Oener (2006), Deming (2017), Atalay *et al.* (2020), Autor *et al.* (2022).

[6]See, among others, Kremer and Maskin (1996), Barth *et al.* (2016), Helpman *et al.* (2017), Haltiwanger and Spletzer (2020), Criscuolo *et al.* (2021), Håkanson *et al.* (2021), Sorkin and Wallskog (2021), and Wilmers and Aeppli (2021). Concurrently, firm productivity has grown increasingly dispersed (Andrews *et al.*, 2019; Decker *et al.*, 2020; De Ridder, 2022). Highlighting a spatial dimension, Kleinman (2022) documents the role of multi-region service firms, theoretically rationalized as the result of declining within-firm communication frictions.

ity should be pronounced in environments that are more "complex," though an empirical or quantitative analysis is missing.[7] Both sets of papers assume frictionless labor markets. Overall, I contribute to this literature by providing a micro-foundation for coworker complementarities, showing that their macro implications are crucially shaped by the interaction with labor market frictions, bringing the theory to micro data, and performing a quantitative assessment.

Third, the paper relates to theoretical models of task assignment, including Costinot and Vogel (2010), Acemoglu and Restrepo (2018), Martinez (2021) and Ocampo (2022). I contribute to this literature by deriving how Eaton and Kortum's (2002) probabilistic formulation of technological heterogeneity in terms of extreme-value distributions facilitates aggregation across a continuum of tasks in the context of a discrete number of producers characterized by multidimensional skill heterogeneity.[8] The resulting constant-elasticity-of-substitution production function makes it possible to connect what are typically qualitative task assignment theories with more quantitatively oriented macro models that rely on a tractable production function.

Fourth, I relate to the literature on labor market matching in random-search environments, including Shimer and Smith (2000), Eeckhout and Kircher (2011), Hagedorn *et al.* (2017), Lopes de Melo (2018). These models all share the Beckerian premise that labor market sorting is shaped by complementarities across productivity types. Instead of considering complementarities between firm and worker attributes in one-worker-one-firm matches, as is typical, I examine complementarities across coworkers' attributes in multi-worker firms. In that respect, the matching block of my model largely follows Herkenhoff *et al.* (2022), which is the first model where (frictional) sorting is between workers within teams rather than between workers and firms.[9] While Herkenhoff *et al.* (2022) primarily focus on coworker learning, they also conjecture that stronger production complementarity can explain increased labor market segregation. I offer an explicit explanation for such a change in production technology by deriving a task-based microfoundation for complementarity, document empirical support for this explanation, and quantitatively evaluate the importance of this mechanism.[10] More generally, the literature usu-

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[7]In Kremer's (1993) model, complementarities are stronger when the production process is complex in the sense of comprising many distinct tasks. I sever the mechanical link between team size and complementarities by endogenizing the number of tasks assigned to each team member, as well as isolating the implications of changing coworker complementarities from those of increasing returns to firm-level labor quality. The latter are critical to the predictions about inequality due to an increasingly convex wage function in Kremer (1993).

[8]Deming's (2017) study of social skills provides inspiration for an analogy to trade, but the setup does not yield an aggregative team production function, nor does it focus on coworker complementarities or equilibrium matching.

[9]Boerma *et al.* (2021) resolve the major challenge of solving a (frictionless) model of sorting of heterogeneous firms with teams of heterogeneous workers when technology is submodular, instead of supermodular as in this paper. Their quantitative analysis does not match the empirically observed rise in the between-firm share of wage inequality, which is a key contribution of the present paper.

[10]Chade and Eeckhout (2020) provide an alternative theory of assortative but stochastic matching with teams. A version of their model in which knowledge spillovers in downstream markets affect firms' hiring choices likewise predicts, qualitatively, that increased complementarities tilt wage inequality toward the between-firm component.

ally treats complementarity as an exogenous parameter, which constrains enquiry regarding the underlying forces that drive changes in sorting. This paper instead endogenizes (coworker) complementarities by modelling the underlying task assignment problem faced by organizations. I also develop an approach to directly measure them in micro data and provide evidence supporting the key premise of this literature, that sorting is tied to production complementarities.

Fifth, the paper relates to a small but growing literature that highlights how team production shapes macroeconomic outcomes. In addition to studies of innovation (Akcigit *et al.*, 2018; Ahmadpoor and Jones, 2019; Pearce, 2022), recent papers on coworker learning underscore the dynamic importance of coworker sorting for human capital growth in both aggregate and distributional perspectives, as stronger sorting deprives low human capital workers of opportunities to learn from better team members (Jarosch *et al.*, 2021; Herkenhoff *et al.*, 2022; Hong, 2022). While my model omits such spillovers, these studies underline the importance of understanding the determinants of coworker sorting. I contribute to this literature by providing a quantitative framework that explicates how worker-task specialization shapes coworker sorting, firm-level wage inequality, and allocative efficiency.

OUTLINE. Section 2 presents and characterizes the theoretical model. I derive the optimal organization of production of a team of given size and composition, then embed this setup into a general-equilibrium search environment. Section 3 discusses empirical measurement and documents patterns of complementarities and sorting. Section 4 combines theory and evidence. I structurally estimate the model and then perform counterfactuals. Section 5 concludes.

## 2 Theoretical model: team production and matching

This section develops the theoretical model and answers two questions. First, what are the origins and determinants of coworker complementarities? Second, how do such complementarities influence matching (who works with whom), productivity, and firm-level inequality?

### 2.1 Task-based microfoundation for coworker production complementarity

I start by considering a single production unit, which is a firm who employs a team of ex-ante heterogeneous workers. I study the optimal assignment of tasks to these workers and derive how production depends on their absolute advantage types. The team's composition is initially taken to be exogenous. The next sub-section then endogenizes hiring decisions and models wage determination.

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### 2.1.1 Setup

A team consists of a discrete number $n$ of workers who perform tasks required for the production of a single good. The production technology is owned by the firm. It allocates workers' time across tasks in order to maximize production. Each worker has a type $x_i \in [0, 1]$, $i = 1, ..., n$, which denotes its rank in the worker talent (or "quality") distribution, and is endowed with one unit of time that they supply inelastically. For convenience, denote the set of team members by $\mathcal{S} = \{1, ..., n\}$.

FINAL GOODS PRODUCTION. The final good (or service) is produced from a unit mass of tasks $\tau \in \mathcal{T} = [0, 1]$ according to a constant elasticity of substitution (CES) aggregator,

$$Y = \left( \int_{\mathcal{T}} q(\tau)^{\frac{\eta-1}{\eta}} d\tau \right)^{\frac{\eta}{\eta-1}}, \tag{1}$$

where $q(\tau)$ denotes the amount of task $\tau$ used in production and $\eta > 0$ is the elasticity of substitution across tasks.

TASK PRODUCTION. Tasks are fulfilled by workers. Denote the amount of task $\tau$ produced by worker $i$ as $y_i(\tau)$ – the relation between $y_i(\tau)$ and $q(\tau)$ is discussed shortly. In classical Ricardian fashion, production is linear in efficiency units of labor:

$$y_i(\tau) = a_1 z_i(\tau) l_i(\tau), \tag{2}$$

where $z_i(\tau)$ is $i$'s efficiency in producing task $\tau \in \mathcal{T}$, the parameter $a_1$ is strictly positive and controls the sensitivity of task output to worker efficiency, and $l_i(\tau)$ is the time dedicated by $i$ to task $\tau$. With this notation, the time constraint for worker $i$ is

$$1 = \int_{\mathcal{T}} l_i(\tau) d\tau. \tag{3}$$

TASK-SPECIFIC EFFICIENCIES. Workers are heterogeneous along two dimensions: vertically, as captured by their type $x_i$, and horizontally, meaning that conditional on $x_i$, a given worker may be better in some tasks than in others. I treat the task-specific efficiencies for worker $i$, $\{z_i(\tau)\}_{\tau \in \mathcal{T}}$, as the realizations of a Fréchet-distributed random variable, drawn independently for each worker $i$. This assumption parallels trade models in the tradition of Eaton and Kortum (2002) and parsimoniously captures both vertical and horizontal differentiation among workers. Thus, for all $z \geq 0$, the distribution of efficiencies for worker $i$ is

$$G_i(z) := Pr(z_i(\tau) \leq z) = \exp\left( -\left( \frac{z}{\iota x_i} \right)^{-1/\chi} \right). \tag{4}$$

**(a)** Worker-specific scale parameter   **(b)** Technological (inverse) shape parameter
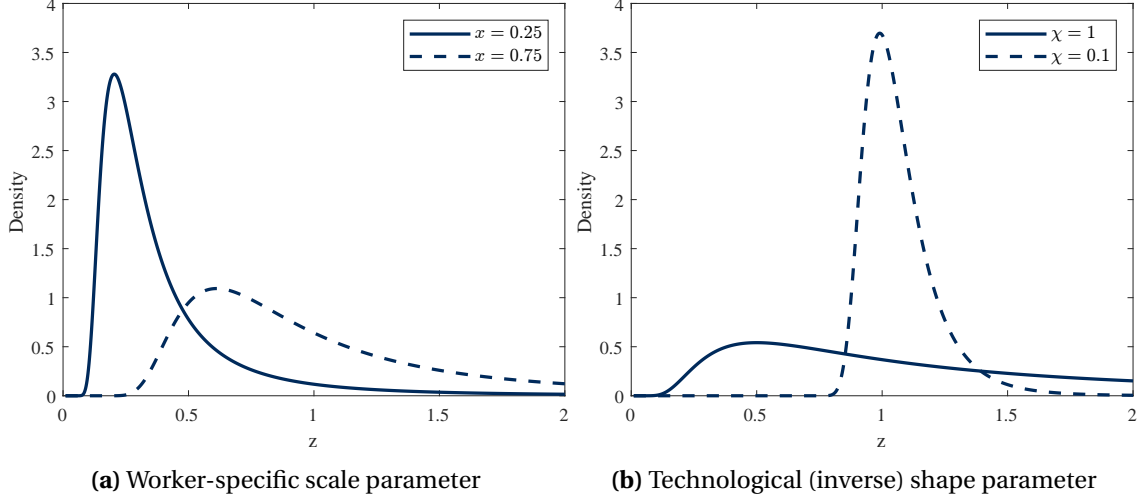
**Figure 1:** Illustration of the distribution of worker-task specific efficiencies

*Notes.* This figure illustrates how the probability density function associated with the Fréchet distribution varies with the scale parameter $x_i$ and (inverse) shape parameter $\chi$. The left panel assumes $\chi = \frac{1}{2}$, the right panel assumes $x = 1$. The illustration abstracts from the scaling factor $\iota$, which also depends on $\eta$, by setting it equal to one.

A worker's type, $x_i$, determines the scale of the worker-specific distribution. The (inverse) shape parameter, $\chi \in (0, \infty)$, determines the degree of dispersion. It is identical for all workers. Lastly, $\iota := \Gamma(1 + \chi - \eta\chi)^{\frac{1}{1-\eta}}$ is a scaling term, with $\Gamma$ denoting the Gamma function.[11]

A few remarks are in order to unpack this description, which is at the heart of the model. Equation (4) implies that workers may differ along both *absolute* and *comparative* advantage lines. Each worker can produce any task, but they are potentially heterogeneous in both their *average* task-specific production efficiency and, conditional on that average, in the distribution of efficiencies *across* different tasks. Variation in workers' talent types connotes absolute advantage, as the expectation of $z_i(\tau)$ is greater when $x_i$ is higher. In addition, there is dispersion in worker-task specific efficiencies; for a given value of $x_i$, $i$ is better in some tasks than in others. The degree of dispersion is increasing in $\chi$. Figure 1 illustrates how the distribution (density) of task-specific productivities varies with $x$ and $\chi$, respectively.

The key parameter with respect to which I conduct comparative statics exercises is $\chi$. Labelled "worker-task specialization," it formally controls the joint distribution of relative productivities for any given task $\tau$ across coworkers and, therefore, the strength of comparative advantage.[12] In terms of interpretation, $\chi$ captures the nature of production, comprising in

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[11]A couple of technical remarks: First, throughout the paper it is assumed that $1 + \chi(1 - \eta) > 0$, for reasons discussed when deriving the optimal organization. Beyond requiring that tasks not be too substitutable, under the maintained assumptions the precise value of $\eta$ will not influence worker-task assignment or team productivity. Second, the scaling term $\iota$ ensures that varying $\chi$ or $\eta$ does not mechanically change production levels.

[12]To be precise, $\chi$ controls two conceptually distinct distributions: the distribution of relative productivity between any two tasks for a given individual; and the joint distribution of relative productivities for any given tasks across individuals and therefore the strength of comparative advantage. What matters here is the latter, i.e., that for any

reduced-form the nature of both tasks and worker skills. Foreshadowing a more extensive discussion in Section 3.1, a rise in $\chi$ could, thus, originate from shifts in task requirements (e.g., a smaller share of tasks in which all workers are similarly productive in) or in team members' skill acquisition (e.g., they may concentrate their training on a narrower set of tasks).

ROLE OF FIRMS. Completing the model description is an account of how team production differs from multiple individuals producing the final good separately. Inside a team, collaboration in the sense of a division of labor is possible. Formally, the amount of any task that is available for final good production, $q(\tau)$, is the sum over production of that task by *all* team members,

$$q(\tau) = \sum_{i=1}^{n} y_i(\tau). \tag{5}$$

Undergirding this account is the premise that an important role of firms is to coordinate the collaboration of workers with specialized knowledge (cf. Becker and Murphy, 1992; Garicano, 2000).[13] A firm not merely incorporates the intellectual property rights, or a "recipe", for a particular product, nor is a firm merely the sum of machines and tools with which workers are equipped. Much of the knowledge required for production is intangible and embedded in individuals with limited time to learn and work, as opposed to that knowledge being codified and tradeable on markets. Hence, its mobilization and efficient use requires individuals to specialize and collaborate. An important role of firms is to coordinate this process and assign tasks to workers with a comparative advantage in performing them.

ORGANIZATIONAL OPTIMIZATION PROBLEM. This role of the firm is reflected in its optimization problem. Solving a 'mini-planner' problem, it chooses total task usage $\{q(\tau)\}_{\tau \in \mathcal{T}}$, individual task production $\{\{y_i(\tau)\}_{\tau \in \mathcal{T}}\}_{i=1}^{n}$ and individual time allocation $\{\{l_i(\tau)\}_{\tau \in \mathcal{T}}\}_{i=1}^{n}$ to maximize total production $Y$, subject to equations (1)-(5). The associated Lagrangean is

$$\mathcal{L}(\cdot) = Y + \lambda \left[ \left( \int_{\mathcal{T}} q(\tau)^{\frac{\eta-1}{\eta}} d\tau \right)^{\frac{\eta}{\eta-1}} - Y \right] + \int_{\mathcal{T}} \tilde{\lambda}(\tau) \left( \sum_{i=1}^{n} y_i(\tau) - q(\tau) \right) d\tau$$

$$+ \sum_{i=1}^{n} \left\{ \lambda_i^L \left( 1 - \int_{\mathcal{T}} l_i(\tau) d\tau \right) + \int_{\mathcal{T}} \lambda_i(\tau) \left( a_1 z_i(\tau) l_i(\tau) - y_i(\tau) \right) d\tau + \int_{\mathcal{T}} \bar{\lambda}_i(\tau) y_i(\tau) d\tau \right\},$$

where $\lambda$, $\lambda_i^L$, $\lambda_i(\tau)$, $\tilde{\lambda}(\tau)$ and $\bar{\lambda}_i(\tau)$ are Lagrange multipliers. The first four denote the shadow

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task, $\chi$ determines the slope of the relative productivity schedule for any pair of workers. The tight connection can be severed by using a multivariate Fréchet distribution (Lind and Ramondo, 2023) but the simpler version suffices for present purposes.

[13]Rivkin and Siggelkow (2003, 292), quoted in Dessein and Santos (2006) write: "The [qualitative management] literature is unified in what it perceives as the central challenge of organizational design: to divide the tasks of a firm into manageable, specialized jobs, yet coordinate the tasks so that the firm reaps the benefits of harmonious action."

values of, respectively, total production, $i$'s time, a unit of task $\tau$ produced by $i$, and a unit of task $\tau$ used in final good production. Lastly, $\bar{\lambda}_i(\tau)$ relates to a non-negativity constraint in task production.

### 2.1.2 Solving the organizational problem

Solving for the optimal production plan involves two main steps. First, we derive the demand for tasks for a given set of shadow prices, treating those as known. Then, secondly, we determine these shadow prices given the distribution of task-specific productivities across workers. We start by taking the first-order-condition (FOC) with respect to $q(\tau)$, substituting for $Y$ from equation (1) and defining $Q(\tau) := \tilde{\lambda}(\tau)q(\tau)$. This yields the planner analogue to the standard iso-elastic demand implied by the CES aggregator.

$$\frac{Q(\tau)}{\lambda Y} = \left(\frac{\tilde{\lambda}(\tau)}{\lambda}\right)^{1-\eta}. \tag{6}$$

Now integrate on both sides and use that the shadow value of all tasks used is related to total production (as derived in Appendix A.1.1),

$$Q := \int_{\mathcal{T}} Q(\tau)d\tau = \lambda Y. \tag{7}$$

This gives an expression for the shadow cost index:

$$\lambda = \left(\int_{\mathcal{T}} \tilde{\lambda}(\tau)^{1-\eta}d\tau\right)^{\frac{1}{1-\eta}}. \tag{8}$$

Who should produce which tasks, and what does that imply for the shadow costs faced by the firm, i.e., for $\{\tilde{\lambda}(\tau)\}_{\tau \in \mathcal{T}}$? To answer this question, observe first that the FOC with respect to $y_i(\tau)$ implies that $\tilde{\lambda}(\tau) = \lambda_i(\tau)$ if $y_i(\tau) > 0$. Since some worker will provide a given task $\tau$, and with task production (equation (2)) featuring constant returns to scale, cost-minimization requires that the shadow value of a task be equal to the minimum shadow cost of producing it, and that worker $i$ produce task $\tau$ if they attain this minimum:

$$\tilde{\lambda}(\tau) = \min_{i \in \mathcal{S}} \left\{\lambda_i(\tau)\right\}. \tag{9}$$

Next, we characterize the term inside $\{\cdot\}$. To relate the shadow price of producing the task to the cost of the (labor) input used, take the FOC wr.t. $l_i(\tau)$ and use it to substitute for $l_i(\tau)$ in the

task-level production function to find that

$$\lambda_i(\tau) = \frac{\lambda_i^L}{a_1 z_i(\tau)}. \tag{10}$$

Thus, the shadow price of task $\tau$ produced by worker $i$ is the ratio of the shadow value of $i$'s time and $i$'s efficiency in producing that task. Using this result in equation (9) yields

$$\tilde{\lambda}(\tau) = \min_i \left\{ \frac{\lambda_i^L}{a_1 z_i(\tau)} \right\}. \tag{11}$$

The next step is to derive what share of tasks is performed by each worker, and at what (shadow) cost the final good can be produced. The key results are summarized in the following lemma.

**Lemma 1.** *Suppose that workers' task-specific efficiencies are independently Fréchet-distributed. Then:*

(i) *The shadow cost index is*

$$\lambda = a_1^{-1} \left( \sum_{i \in \mathcal{S}} \left( \frac{x_i}{\lambda_i^L} \right)^{1/\chi} \right)^{-\chi}. \tag{12}$$

(ii) *The fraction of tasks for which worker $i$ is the least-cost provider is*

$$\pi_i := Pr\{\lambda_i(\tau) \leq \min_{k \in \mathcal{S} \setminus i} \lambda_k(\tau)\} = \frac{(x_i/\lambda_i^L)^{1/\chi}}{\sum_{k \in \mathcal{S}} (x_k/\lambda_k^L)^{1/\chi}}. \tag{13}$$

(iii) *The shadow value of all tasks used in final goods production that were produced by worker $i$, defined as $Q_i := \int_{\mathcal{T}} \tilde{\lambda}(\tau) y_i(\tau) d\tau$, is a fraction $\pi_i$ of the total shadow value of tasks used:*

$$Q_i = \pi_i Q. \tag{14}$$

*Proof.* Appendix section A.1.2. □

That this type of problem – with an arbitrarily large, discrete number of potential producers of a continuum of tasks – is tractable under the assumptions made is a key insight of Eaton and Kortum (2002). A key step involves the max-stability property of the Fréchet distribution, under which the maximum of Fréchet-distributed random variables is also Fréchet. As a result, the distribution of shadow costs conditional on any task being performed by the worker with a

comparative advantage remains tractable. Using Lemma 1 and normalizing the shadow price of final goods output to unity ($\lambda = 1$), we next characterize the optimal organization of production.

### 2.1.3  Characterizing the optimal organization of production

Intuition for the properties of the optimal organization of production can be gained in three steps. First, and restating a canonical idea, the optimal organization features complete division of labor, with each worker assigned those tasks in which she has a comparative advantage (e.g., Duranton and Puga, 2004). Equation (11) indicates that any task $\tau$ is produced by whichever worker is most efficient in producing that task, as captured by $a_1 z_i(\tau)$, relative to the task-invariant, endogenous shadow cost of their time, $\lambda_i^L$.[14] Second, better workers perform more tasks, other things equal. The share of tasks produced by worker $i$ can be shown to equal $\pi_i = \left(x_i^{\frac{1}{1+\chi}}\right)\left(\sum_{k \in \mathcal{S}}(x_k)^{\frac{1}{1+\chi}}\right)^{-1}$, hence $\frac{\partial \pi_i}{\partial x_i} > 0$. Note that when team members are differentiated only in their task-specific productivities (so that $x_i = x \ \forall i \in \mathcal{S}$), then $\pi_i = 1/n$.

Third, and most importantly, this organizational structure renders the value of any worker's time a function not only of her own endowments; to the extent that $\chi > 0$, this value depends also on the environment, specifically the number and abilities of her coworkers. The shadow value of all tasks produced by worker $i$ is

$$Q_i = n^{1+\chi} a_1 x_i^{\frac{1}{1+\chi}} \tilde{x}_i^{\chi}, \tag{15}$$

where $\tilde{x}_i = \frac{1}{n} \sum_{k \in \mathcal{S}} x_k^{\frac{1}{1+\chi}}$. Other things equal, the division of labor enables a worker to focus on those tasks in which she is particularly adept, raising the total value of tasks she can produce. For a given team size, better coworkers facilitate this concentration on high-efficiency tasks. These coworker interdependencies are more marked when $\chi$ is high. Suppose, to the contrary, that a worker is equally productive in all tasks ($\chi \to 0$). Then their overall productivity is independent of the task assignment. It is, instead, when individuals are specialized in particular tasks, and comparative advantage is pronounced, that their productivity will depend on colleagues' capability. Crucially, as equation (15) underlines, the absolute gain from having better coworkers is greater for high-$x$ workers. Simply put, more output is at stake.

The following result concisely summarizes how team members' qualities jointly pin down team output.[15] Exploiting that the Fréchet assumption renders the firm's 'demand' for a worker's

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[14]Strictly speaking, we need to specify a tie-breaking rule for tasks where multiple workers have the same shadow cost. But those tasks have mass zero anyway.

[15]This aggregation result relates to the seminal paper of Houthakker (1955) and, more recently, Acemoglu and Restrepo (2018), Martinez (2021) and Ocampo (2022). In the IO literature, an important reference is Anderson *et al.* (1989). In independent work, Dvorkin and Monge-Naranjo (2019) likewise exploit properties of the Fréchet distribution to derive a microfoundation for an aggregate CES production function; their result shows that the

time an iso-elastic function of the shadow price of that time, one can show that under the optimal organizational plan, the number and quality types of team members are sufficient statistics for team output.[16]

**Proposition 1** (Aggregation result)**.** *Team output $Y$ can be written as a function of members' quality types, $f : [0,1]^n \to \mathbb{R}_+$,*

$$f(x_1, \cdots, x_n) = \underbrace{n^{1+\chi}}_{\text{efficiency gains}} \underbrace{\left( \frac{1}{n} \sum_{i=1}^{n} (a_1 x_i)^{\frac{1}{1+\chi}} \right)^{1+\chi}}_{\text{complementarity}}. \tag{16}$$

*Proof.* See Appendix Section A.1.3. □

The first term summarizes the benefits from team production relative to each worker producing all tasks themselves, which are increasing in $\chi$. Suppose that there is no division of labor, perhaps because workers are not coordinated by a firm. Then each worker produces all tasks (in proportions minimizing their individual shadow costs of producing the final good), and it is straightforward to show that total output is simply $\sum_{i=1}^{n} a_1 x_i$. The "Smithian" efficiency gains from team production relative to this scenario, which is captured by the first term of equation (16), echo a long-standing theoretical literature (e.g., Becker and Murphy, 1992; Garicano, 2000), as well as cohering with recent empirical papers on firm organization (cf. Section 1).

The second term takes the form of a CES function. Intuitively, the more *horizontally* differentiated team members are at the task level, the more limited is the scope for substitution in terms of workers' *absolute advantage*. Simply put, even the most talented worker may lose much time when performing tasks they are not specialized in.[17] The following corollary summarizes the coworker interdependencies arising from the efficient organization of production.

**Corollary 1** (Coworker complementarity)**.** *The elasticity of complementarity is $\gamma = \frac{\chi}{1+\chi}$.*

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substitutability of capital and labor then relates to the shape parameter of the Fréchet distribution of productivities.

[16]Notice that the elasticity of substitution across tasks, $\eta$, does not show up in equation (16). Technically, as noted by Eaton and Kortum (2002, Footnote 18), this holds as long as we maintain that $1 + \chi - \eta\chi > 0$, in which case $\eta$ only appears in a constant term that cancels with the scaling term $\iota$. The irrelevance of $\eta$ in that sense is a tight implication of the Fréchet, and it serves to sharply bring into relief that coworker complementarities do not hinge on the assumption that tasks combine in a Leontief fashion (e.g., Kremer, 1993). In a more general setting, the strength of coworker complementarities would also be influenced by the value of $\eta$.

[17]Proposition 1 offers a microfoundation for the reduced-form model of the relationship between individual and team outcomes assumed by Ahmadpoor and Jones (2019) in their study of team production and matching in science and invention. They postulate that team output corresponds to $\beta_n \left[ \frac{1}{n} \sum_{i=1}^{n} x_i^\zeta \right]^{1/\zeta}$, where $\beta_n$ "captures impact benefits associated with teamwork" while the second term measures by how much these benefits diminish as the gap between team members' abilities grows.

Coworker complementarity signifies that team productivity is lowered if members vary in their quality type, for a given (arithmetic) *average* value of $x_i$. In statistical terms, the constant-elasticity term in equation (16) corresponds to the power mean of the underlying $x_i$ values. When $\gamma = 0$, it reduces to the arithmetic mean, consistent with an efficiency-unit treatment of worker types. Otherwise, by the power mean inequality, the stronger the degree of complementarity, the greater is the weight on quality of the least-capable team member(s) in determining total output.

A key take-away from this task-based microfoundation of team production is, therefore, that greater worker-task specialization not only enhances the scope for efficiency gains from collaboration. Output also becomes more vulnerable to relatively lower-quality team members. Specifically, larger values of $\chi$ render *coworker mismatch* – dispersion in team members' quality – more costly in terms of output. In short, the composition of a team becomes more important for productivity. The next section therefore examines how firms choose to assembly teams.

## 2.2 Complementarity and endogenous team formation

Having previously considered one team of given composition, suppose now that there are many workers and many potential employers. What teams of workers would different multi-worker firms hire – who does, in fact, work with whom – and how do matching patterns and wage distribution respond to changes in $\chi$? To answer this set of questions, in this section I embed the aggregative team production model into a general-equilibrium environment where firms and employees match on a search-frictional labor market.

To motivate the inclusion of search frictions, it is instructive to take a step back and ask what matching patterns would emerge in a frictionless labor market. The answer is that for $\chi > 0$, the equilibrium – and efficient – allocation features pure positive assortative matching (PAM). In the absence of frictions, zero within-firm worker quality dispersion is therefore an *equilibrium outcome*. Indeed, for any initial value $\chi_0 > 0$, an increase to $\chi_1 > \chi_0$ does not alter matching patterns. Since production is supermodular for $\chi > 0$, this result immediately follows from Becker (1973), extended to the many-to-one case by Kremer (1993). Specifically, when a worker's marginal productivity is increasing in coworkers' quality, a feasible Pareto improvement exists unless all team mates of a worker of type $x$ are likewise of type $x$. Although both low-$x$ and high-$x$ workers are more productive in the presence of a better coworker, complementarities mean that the benefit is differentially larger for the high type. In the absence of frictions, even the tiniest such differential is sufficient to yield pure PAM. The equilibrium joint distribution of team members is degenerate and matching characterized by a single-valued function $\mu(x) = x$ that deterministically relates worker and coworker types.

This strong prediction points to two distinct reasons for incorporating search.[18] First, the prediction of pure positive sorting is counterfactual. The correlation between coworkers' types would need to equal one, which is not true empirically (see for instance Table 1). Second, variation in coworker types conditional on a worker's own type is essential to empirically measure worker types and coworker complementarities. Search frictions align the structural model more closely with the empirical analysis by generating stochastic matching. In their presence, some degree of "mismatch" between team members' types is tolerated, since waiting is costly. How much mismatch there will be in equilibrium crucially depends on the strength of complementarities and, thus, on $\chi$.

### 2.2.1  Environment

I embed the team production function into a version of Herkenhoff *et al.*'s (2022) matching model. To focus on the key mechanism of changing production complementarities, I abstract from learning and, in the baseline, from on-the-job search (considered in Section 4.4). These simplifications afford a tight characterization of the equilibrium properties in a stylized version of the model, and permit a transparent derivation of a novel measurement result that indicates how the strength of complementarities can be disciplined using micro data.[19]

DEMOGRAPHICS, PREFERENCES AND PRODUCTION. Time is continuous. All agents are infinitely-lived, risk-neutral, and maximize the present value of payoffs, discounted at a common rate $\rho \in (0,1)$. There is a unit mass of workers, denoted $d_w = 1$, who are either employed ($e$) or unemployed ($u$). Workers are ex-ante heterogeneous, as just described. For ease of exposition, I often refer to a "worker $x$" instead of a "worker of type $x$." Noting that the production function (16) is increasing in each argument, I follow Hagedorn *et al.* (2017) in treating $x$ as a worker's rank in the underlying productivity distribution, so that the distribution of worker types is uniform. The production function thus maps from an ordinal to a cardinal space.[20] There is a unit mass of firms, $d_f = 1$, that are either idle, with mass $d_{f,0}$, or actively producing in a match. I write down the model assuming that all firms are ex-ante homogeneous, the idea being that the firm matters, namely through team assembly, but its value is tightly linked to the team it employs.[21]

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[18] In principle, a whole host of other frictions could be considered, including information and screening costs or spatial restrictions. Search frictions are arguably the most conventional route to obtain stochastic sorting and they are particularly useful for quantitative analyses, as we can bring to bear micro data on labor market transitions to discipline search costs.

[19] My notation differs from Herkenhoff *et al.*'s (2022) and seeks to maximize comparability with the standard model of matching between workers and *one-worker* firms, especially the notation in Hagedorn *et al.* (2017).

[20] As noted by Hagedorn et al. (2017, 2.1.1.), if the non-rank distribution of workers is $\Phi$ and the production function which takes the non-rank type of worker as an input is $\hat{f}(\hat{x})$, then $f(x) = \hat{f}(\Phi^{-1}(x))$. The logic straightforwardly extends to multiple workers.

[21] A Working Paper version incorporated ex-ante heterogeneity in firm productivity, underscoring that results proved here do not hinge on assuming otherwise, but the gain in generality was outweighed by a loss in clarity.

Production requires a firm and either one or two employees. That is, the maximum team size is $n = 2$. This assumption of sharply decreasing returns is evidently unrealistic and made for computational reasons. The state space explodes with larger team sizes, as complementarities mean that we need to keep track of every team members possible type. A team size of two is sufficient, however, to study how coworker complementarities shape between-firm inequality. Section 3.2 discusses the mapping between theory and data in light of this assumption.[22]

Denote the production function of a single-worker firm $f_1(x) : [0, 1] \to \mathbb{R}_+$, and that of a two-worker firm as $f_2(x, x') : [0, 1]^2 \to \mathbb{R}_+$. Note that I indicate the team size state as a subscript to aid orientation when there is scope for confusion (but suppress it otherwise), and I refer to the coworker type by $x'$. A key value-added from the aggregation result in Proposition 1 is that we do not need to keep track of worker-task specific efficiencies or worker-task assignment, which happens 'in the background', instead we only need to keep track of workers' quality types.

POPULATION COMPOSITION. Let $d_{m.1}(x)$ denote the measure of producing matches consisting of a firm and one worker of type $x$, while $d_{m.2}(x, x')$ is the corresponding measure of matches with an additional coworker $x'$. The following adding-up property holds for workers of type $x$:

$$d_w(x) = d_u(x) + d_{m.1}(x) + \int d_{m.2}(x, \tilde{x}')d\tilde{x}', \tag{17}$$

so that the total measure of type $x$, $d_w(x)$, is the sum of type-$x$ workers who are unemployed, $d_u(x)$, in one-worker matches or in two-worker matches. The aggregate unemployment rate is obtained by integrating over the first of these three terms, $u = \int d_u(x)dx$. Similarly for firms,

$$d_f = d_{f.0} + \int d_{m.1}(x)dx + \frac{1}{2} \int \int d_{m.2}(x, x')dxdx'. \tag{18}$$

Dividing by 2 in the last term avoids double-counting, $d_{m.2}(x, x')$ being symmetric.

TIMING. Within any increment of time, each matched worker may be exogenously separated from their employer at an exogenous Poisson rate $\delta$. There are no endogenous separations. Next, unmatched workers and all firms engage in random search and matching decisions. Upon meeting, the types $x$ are perfectly observed, but the task-specific productivities are revealed only after the hiring decision is made. Then production and surplus sharing happen.

SEARCH AND BARGAINING. Every unemployed worker engages in search. So do all firms with empty spots, the total number of which is $v = d_{f.0} + \int d_{m.1}(x)dx$. It is not permissible to replace a worker with a better matched candidate, so two-worker firms do not search. Apart from being hard to reconcile with labor market policies in Germany – the economy on which

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[22]Appendix section C.2.3 shows that allowing for a maximum team size equal to 3 leads to qualitatively identical and quantitatively very similar predictions as the baseline model.

I will estimate the model – such replacement hiring would distract from the central trade-off between match quality and search costs. Unemployed workers contact a firm at an exogenous Poisson rate $\lambda_u$. Hence, for any searching firm the probability of meeting an unemployed worker is $\lambda_{v.u} = \frac{\lambda_u u}{v}$. Wages are continuously renegotiated and workers' bargaining power is $\omega \in [0,1]$. Negotiation takes the generalized Nash form, concerns the entire surplus, and the firm treats each employee as marginal, so that their outside option is unemployment. This protocol ensures that all matching decisions are privately efficient and can be characterized by parties' joint surplus, which in turn only depends on individual types (cf. Bilal *et al.*, 2022). Moreover, the wage is unaffected by the order with which workers join a team.

### 2.2.2 Definition and sharing of joint surplus

Next, I define the agents' joint value and match surplus functions and describe the protocol by which this surplus is shared. Denote the joint value of a match between a firm and a type-$x$ worker as

$$\Omega_1(x) = V_{f.1}(x) + V_{e.1}(x), \tag{19}$$

where $V_{e.1}(x)$ is $x$'s value of being employed alone at a firm, whose value in turn is $V_{f.1}(x)$. Then the surplus generated by such a match is

$$S(x) = \Omega_1(x) - V_{f.0} - V_u(x), \tag{20}$$

where $V_u(x)$ is the value of unemployment for $x$ and $V_{f.0}$ is the value of an idle firm.

Similarly, the joint value of a firm with team $(x, x')$ is

$$\Omega_2(x, x') = V_{f.2}(x, x') + V_{e.2}(x|x') + V_{e.2}(x'|x), \tag{21}$$

with $V_{e.2}(x|x')$ denoting the value of $x$ being employed in a team if the coworker is of type $x'$. Hence, the surplus generated when a firm that already has employee $x'$ hires a type $x$ worker is:

$$S(x|x') = \Omega_2(x, x') - \Omega_1(x') - V_u(x). \tag{22}$$

This surplus will rise in the output generated by the team, yet fall in both sides' outside values. $S(x|x')$ is not, in general, symmetric in the two arguments even if $\Omega_2(x, x')$ is, because it matters for both outside options whether type $x$ is the potential employee or type $x'$ is (unless $x = x'$).

Turning to surplus sharing, the wage of a worker of type $x$ employed alone satisfies

$$(1 - \omega)\big(V_{e.1}(x) - V_u(x)\big) = \omega\big(V_{f.1}(x) - V_{f.0}\big), \tag{23}$$

In a two-worker firm, the wage $w(x|x')$ of a type-$x$ worker with a coworker of type $x'$ satisfies

$$(1 - \omega)\big(V_{e.2}(x|x') - V_u(x)\big) = \omega\big(V_{e.2}(x'|x) + V_{f.2}(x, x') - V_{e.1}(x') - V_{f.1}(x')\big). \tag{24}$$

### 2.2.3 Equilibrium conditions

I focus on a stationary equilibrium. This equilibrium is characterized by a set of Hamilton-Jacobi-Bellman equations (HJBs) that describe agents' optimal matching strategies together with Kolmogorov-Forward equations (KFEs) that pin down the stationary distributions.

OPTIMALITY. Starting with unmatched agents, the asset value of an idle firm satisfies

$$\rho V_{f.0} = (1 - \omega)\lambda_{v.u} \int \frac{d_u(x)}{u} S(x)^+ dx, \tag{25}$$

where for any $r$, I let $r^+ = \max\{r, 0\}$ indicate the optimal choice, to ease notation. The discounted value thus corresponds to the weighted conditional expectation of the firm's share of the match surplus generated with an unemployed worker times the unconditional probability of meeting any unmatched worker. The value of such an unemployed worker $x$ is

$$\rho V_u(x) = b(x) + \lambda_u \omega \left[ \int \frac{d_{f.0}}{v} S(x)^+ + \int \frac{d_{m.1}(\tilde{x}')}{v} S(x|\tilde{x}')^+ d\tilde{x}' \right]. \tag{26}$$

Notice that we need to take into account the worker's flow value from being unmatched, $b(x)$, as well as differentiating between the worker meeting an unmatched firm or a one-worker firm.

What about the matched agents? To understand the model mechanics, it is instructive to consider the joint value of firm and worker(s) – instead of jumping straight to surplus values – starting with that of a firm employing a pair of workers $x$ and $x'$:

$$\rho\Omega_2(x, x') = f_2(x, x') + \delta\big[\big(-\Omega_2(x, x') + \Omega_1(x) + V_u(x')\big) + \big(-\Omega_2(x, x') + \Omega_1(x') + V_u(x)\big)\big]. \tag{27}$$

The discounted value contains the flow value of production, and at rate $\delta$ either $x$ or $x'$ leaves the firm. Notice that in case of separation, the change in values is equal to the surplus $S(x'|x)$ if $x'$ leaves and equal to $S(x|x')$ if $x$ is separated.

Lastly, and importantly, the joint value of a firm with a type-$x$ employee satisfies

$$\rho\Omega_1(x) = f_1(x) + \delta\big[-\Omega_1(x) + V_u(x) + V_{f.0}\big] \tag{28}$$
$$+ \lambda_{v.u} \int \frac{d_u(\tilde{x}')}{u}\big(-\Omega_1(x) + V_{e.2}(x|\tilde{x}') + V_{f.2}(x,\tilde{x}')\big)^+ d\tilde{x}'.$$

The discounted value thus includes the flow value of production. At rate $\delta$ the match is exogenously destroyed. But with probability $\lambda_{v.u}$, the firm is contacted by an unemployed worker. The conditions under which such a meeting leads to a match are key. What type of worker is a firm with employee $x$ willing to hire, and conversely, which job would the candidate $x'$ be willing to accept? A match is formed if the sum of values accruing to the firm and $x$ from teaming up with $x'$ exceed their joint value if $x'$ is not hired. The latter value comprises, amongst others, the option value of waiting to meet a worker other than $x'$ who is a better match. Since $\big(-\Omega_1(x) + V_{e.2}(x|x') + V_{f.2}(x,x')\big)$ is proportional to $S(x'|x)$ – as Appendix A.2.2 shows – an equivalent statement is that a worker $x'$ will be hired if the joint surplus from doing so is positive.

POPULATION DYNAMICS. In stationary equilibrium, the inflows and outflows into different states balance each other. These flows interact with the value functions as hiring decisions are pinned down by the surplus values:

$$h(x) = \mathbf{1}\{S(x) > 0\}, \tag{29}$$
$$h(x|x') = \mathbf{1}\{S(x|x') > 0\}. \tag{30}$$

These policy functions describe, respectively, whether an unmatched firm is willing to hire a worker $x$; and whether a firm that already employs worker $x'$ is willing to hire a worker $x$.[23] The bargaining protocol ensures that decisions are mutually agreed upon, i.e., they are privately efficient. As the KFEs are straightforward but lengthy, they are collected in Appendix A.2.1.

### 2.2.4 Equilibrium definition and objects

In stationary equilibrium, agents' values must be consistent with the distributions to which they give rise, and vice versa. Appendix A.2.2 derives the recursions for the surplus values, see specifically equations (A.14) and (A.17), so that the optimality conditions are fully described in terms of the values of unmatched agents and surpluses. A formal definition follows.

**Definition 1.** *A stationary search equilibrium is a tuple of value functions, $\big(V_u(x), V_{f.0}, S(x), S(x|x')\big)$, together with a distribution of agents across states, $\big(d_{m.1}(x), d_{m.2}(x,x')\big)$, such that (i.) the value*

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[23]I impose that matching only takes place when the relevant surplus is strictly greater than zero; since the indifference case occurs for a measure zero of agents, this assumption has no bearing on the result.

*functions satisfy the HJBs* (25), (26), (A.14) *and* (A.17) *given the distributions; and (ii.) the sta-*
*tionary distributions satisfy the KFEs* (A.11)-(A.12) *given the policy functions implied by the value*
*functions according to equations* (29)-(30).

The equilibrium needs to be computed numerically because of a non-trivial general-equilibrium
interaction: Agents' expectations and matching decisions must conform with the distribution
to which they give rise, yet as that distribution evolves, so do agents' expectations over future
meeting probabilities and, hence, their optimal actions.

EQUILIBRIUM OBJECTS. The model implies specific coworker sorting matching patterns
as well as a decomposition of wage inequality into a between- and within-employer compo-
nent. Consistent with the focus on multi-worker firms, and for the remainder of the paper,
I consider only workers employed in a team to compute these moments. As a preliminary
step, note that the joint distribution of worker types in teams is characterized by the density
$\phi(x, x') = \frac{1}{e_2} d_{m.2}(x, x')$, where $e_2 = \int \int d_{m.2}(x, x') dx dx'$ is total employment in teams. Hence,
the unconditional density of types in teams is $\phi(x) = \int_0^1 \phi(x, \tilde{x}') d\tilde{x}'$, while the distribution
of coworker types conditional on type is $\phi(x'|x) = \frac{\phi(x,x')}{\int \phi(x,\tilde{x}') d\tilde{x}'}$. The corresponding CDFs are
denoted $\Phi(x)$ and $\Phi(x'|x)$.

To track coworker sorting, consider two different objects. The first is the coworker correlation
coefficient,

$$\rho_{xx} = \frac{\int \int (x - \bar{x})(x' - \bar{x}) d\Phi(x'|x) d\Phi(x)}{\int (x - \bar{x})^2 d\Phi(x)}, \tag{31}$$

where $\bar{x} = \int x d\Phi(x)$ is the average worker type among those employed in teams. To provide
a more disaggregated picture I also compute the mapping between worker type and average
coworker type, which is the natural analogue of a deterministic matching function in setting
with stochastic matching:

$$\hat{\mu}(x) = \int \tilde{x}' d\Phi(\tilde{x}'|x). \tag{32}$$

Next, the average log wage is given by

$$\ln \bar{w} = \int \int \ln w(x|x') d\Phi(x'|x) d\Phi(x),$$

and the total variance of log wages as well as the variance of firm-level average log wages are

$$\sigma_w^2 = \int \int \left( \ln w(x|x') - \ln \bar{w} \right)^2 d\Phi(x'|x) d\Phi(x), \tag{33}$$

$$\sigma_{\bar{w}}^2 = \int \int \left( \ln \bar{w}(x, x') - \ln \bar{w} \right)^2 \phi(x, x') dx dx' \tag{34}$$

where $\ln \bar{w}(x, x') = \frac{1}{2} \left( \ln w(x|x') + \ln w(x'|x) \right)$ is the average log wage in a firm with team $(x, x')$. Hence, the between-employer share of wage inequality is $\sigma_{\bar{w}}^2 / \sigma_w^2$.

### 2.2.5 Elucidating the mechanism in a simplified environment

The key trade-off shaping within- and between-firm worker heterogeneity relates to the matching decision to be taken when a firm that already has a type-$x$ employee meets an unmatched worker of type $x'$. This decision, encapsulated in the policy function $h(x'|x)$, balances match quality considerations and search costs.[24] If the hire is made, output $f_2(x, x')$ is produced and shared. Else, the firm with its employee produces $f_1(x)$, the unmatched worker receives the flow value of unemployment, $b(x')$, and both sides search for another production partner with whom they can generate more surplus.

The strength of coworker production complementarity determines in how far team output is influenced by the match quality between $x$ and $x'$. Suppose that worker-task specialization ($\chi$) and, hence, coworker complementarity is pronounced, and consider a firm with a good worker $x$ and a less capable potential hire $x'$. Then the surplus from such a match is low, because $x$'s potentially high productivity would be dragged down by collaborating with $x'$. It is preferable for both sides to keep searching. Consequently, in a high-$\chi$ economy, workers of similar quality are matched together but not workers that are distant in quality space. Hence, there is positive assortative matching. If, however, $\chi$ is low, then $x$ can perform a greater share of tasks without this substantially lowering the average efficiency with which she performs these tasks, mismatch is less costly, and consequently the degree of coworker sorting is low.

To further clarify this mechanism, I briefly sketch and describe key findings from a stylized version of the model, set out in Appendix A.3. While not suited for quantitative analysis, it qualitatively preserves the same mechanism operative in the full model and the equilibrium can be characterized analytically. I adapt the setup of Eeckhout and Kircher (2011) to the context of team formation. Instead of search costs arising implicitly, because agents discount the future, it is assumed that workers incur a fixed search cost if they reject a match and get paired with

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[24]The decision of an unmatched firm and an unmatched worker is quite trivial, as the surplus is generally positive across potential matching combinations for plausible parameterization.
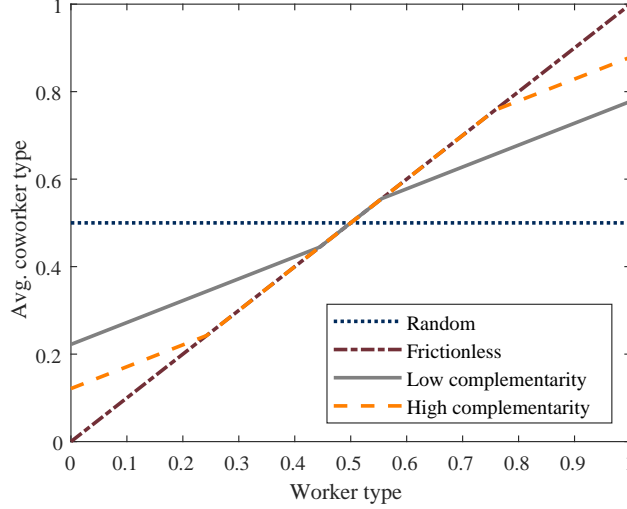
**Figure 2:** Illustration of matching patterns in the stylized matching model

*Notes.* This figure plots, for each worker type $x$, the expected coworker type, $\hat{\mu}(x)$. Each line represents an alternative equilibrium assignment, as summarized by the threshold $s^*$. Specifically, "Random" corresponds to $s^* = 1$; "frictionless" to $s^* = 0$; "Low complementarity" to $s^* = 0.25$; and "High complementarity" to $s^* = 0.125$.

their optimal coworker type in return. In this environment, the matching decision obeys a simple threshold rule; a firm with a type-$x$ employee will hire a type-$x'$ worker if and only if the absolute type distance, $|x - x'|$, is lower than a threshold value $s$. This threshold is decreasing in the strength of complementarity and increasing in the cost of search. I derive closed-form expressions for the conditional match distribution and firm-level average wages, yielding the following predictions.

**Corollary 2** (Coworker complementarity and distributional outcomes)**.** *In the simplified model, a marginal increase in coworker complementarity lowers the equilibrium threshold $s^*$, other things equal, which leads to the following outcomes.*

(i) *The correlation between coworkers' types is higher, specifically $\rho_{xx} = (2s^* + 1)((s^*)^2 - 1)^2$.*

(ii) *The average coworker type is lower for types of quality below $s^*$, greater for types above $1 - s^*$, and unchanged for intermediate types. Specifically, the expected coworker type is*

$$\hat{\mu}(x) = \begin{cases} \frac{x+s^*}{2} & \text{for} & x \in [0, s^*) \\ x & \text{for} & x \in [s^*, 1 - s^*] \\ \frac{1+x-s^*}{2} & \text{for} & x \in (1 - s^*, 1]. \end{cases}$$

(iii) *The between-firm share of the variance of wages, $\sigma_{\bar{w}}^2 / \sigma_w^2$, is higher.*

Figure 2 graphically illustrates the equilibrium matching outcomes under four alternative parameterizations. For any worker type, it plots the average coworker type, $\hat{\mu}(x)$. Consider first

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the dotted line, which represents the case of perfect substitutability between coworker types. Then $\hat{\mu}(x)$ is the same for any $x$ – there is random matching. Next, consider the dashed-dotted line. It illustrates that when there are complementarities and no search frictions, pure assortative matching obtains. The remaining two lines represent cases with both complementarities and search frictions. Under weak complementarities, search frictions mean that worker types below the threshold $s^*$ are paired up with coworkers that are better than themselves by a margin of up to $s^*$ (solid line). The opposite applies to high types, that is if $x > 1 - s^*$. For types in between those two kink points, the average coworker type corresponds to their own type, as the matching set is symmetric. Now consider the dashed line, which represents the case with stronger complementarities and, hence, a lower value of $s^*$. For an intermediate worker, the matching set shrinks symmetrically in both directions. But for the highest types, stronger complementarities can only raise the minimum type with whom they get matched in equilibrium, so that the *average* coworker type is higher than under weak complementarities. The converse applies to low types.

In summary, when teams are formed in a search-frictional labor market, then a strengthening of complementarity implies that the equilibrium increasingly features firms with "superstar teams" composed of the best workers, on the one hand, and other firms with "laggard teams," on the other hand (cf. Andrews *et al.*, 2019; Autor *et al.*, 2020). This result underscores the importance of analyzing input complementarities and input choice jointly. Following Rosen (1981), greater *substitutability* may be associated with a "superstar" phenomenon, as it implies extra weight on, and hence reward for, the highest-quality input *within* a production unit of given input composition. The analysis here underlines a distinct mechanism when the choice of inputs is treated as endogenous and subject to frictions. Then stronger *complementarity* leads to some firms matching together the highest-quality inputs available, resulting in greater differences *between* firms.

## 3   Empirical evidence

Taking stock, according to the theoretical model greater worker-task specialization amplifies coworker complementarity in production (Corollary 1), which in turn fosters coworker sorting and between-firm inequality at the macro level (Corollary 2). In this section, I turn to the data. I first examine evidence on rising worker-task specialization (Section 3.1). Then, I describe how key model objects are measured in German matched employer-employee data (Section 3.2), provide evidence that coworker sorting and complementarity have increased (Section 3.3), and empirically test key model predictions by leveraging cross-sectional variation (Section 3.4).

## 3.1 Independent evidence for rising worker-task specialization

The primary technology parameter of interest is $\chi$, which controls the dispersion of worker-task specific efficiencies. Formally, $\chi$ thus describes a property of the joint distribution of worker-task-specific efficiencies, which does not have an immediately measurable counterpart in the data. In the quantitative application, I therefore indirectly estimate $\chi$ by exploiting the structure of the model. In this section, I examine plausible proxies for $\chi$ and argue that they, as well as related literature, point toward an increase in worker-task specialization over the past few decades.

My primary empirical proxy for $\chi$ is the degree to which tasks are abstract and non-routine, referred to as "task complexity" for short. Complex tasks require cognitive skills and cannot be performed by following explicitly programmed rules (cf. Autor *et al.*, 2003). Intuitively, there are several reasons why task complexity should positively correlate with $\chi$. Consider interpretations of dispersion in worker-task efficiencies through the lens of either "natural" or "acquired" comparative advantage (Grossman and Helpman, 1990). From the former vantage point, in routine jobs everyone is likely to be similarly productive almost by definition (Martellini and Menzio, 2021, p. 340), whereas non-routine jobs allow for more differentiation. If, instead, we conceive of task-specific productivities as acquired, dispersion will be pronounced if returns to specialization are high. That will be the case insofar as acquiring the knowledge needed to perform any individual task involves a fixed cost, such as time, or if there is significant scope for learning-by-doing (Rosen, 1983; Alon, 2018). In this respect, Caplin *et al.* (2022) find that the time needed to reach maximal productivity is highest for management occupations or those that require significant knowledge, which also tend to be those intensive in complex tasks.

A practical advantage of the task complexity proxy is that it can be measured both over time and cross-sectionally. I use micro-data files for a long-running, large-scale survey in Germany, which interviews employees about their job tasks. Following Spitz-Oener (2006) and others, I calculate the average share of abstract non-routine within each respondent's set of activities. This computation is performed at four time points as well as in various employee groups, categorized by education or occupation. Appendix D provides details.

Figure 3a depicts an upward trend in the average share of complex tasks reported by respondents since the mid-1980s. Rising from 0.252 in 1986 to 0.647 in 2018, the increase was particularly marked in the 1990s and early 2000s, affected all education groups, and primarily resulted from within-occupation changes rather than employment shifts across occupations. One influential technological driver that the literature highlights is automation, which displaced humans in routine tasks, leaving us to handle the more complex problems (Acemoglu and Restrepo, 2018).[25]

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[25]Worker-task specialization need not rise inexorably. For instance, Brynjolfsson *et al.* (2023) find that productivity gains from the roll-out of large language AI tools among customer support agents disproportionately accrued to
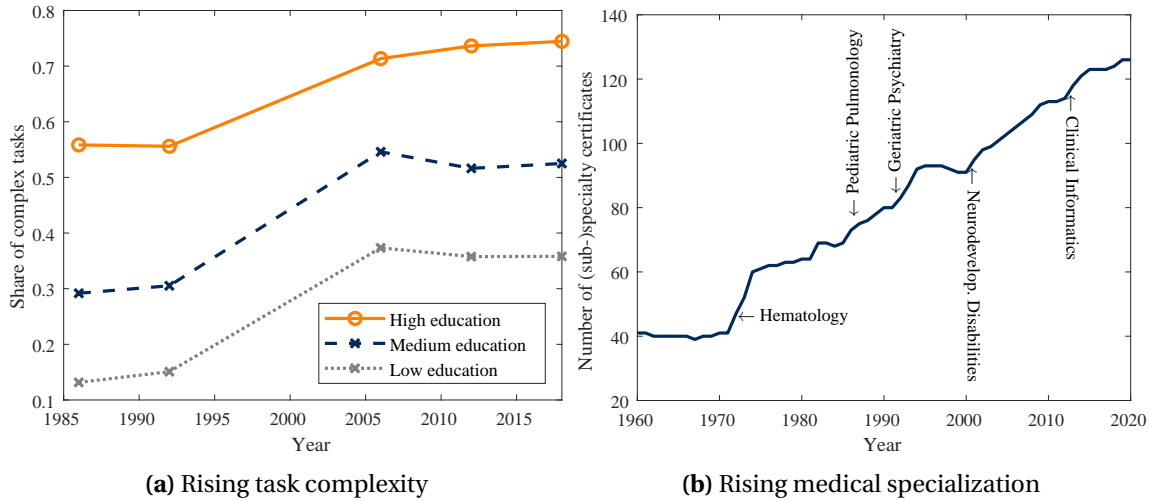
**(a)** Rising task complexity

**(b)** Rising medical specialization

**Figure 3:** Trends in task complexity & case study of medical specialization

*Notes.* Panel 3a is based on the German BIBB Employment Survey. It depicts the average share of complex, or abstract non-routine, tasks in individual workers' set of activities for four points in time and distinguishing between three education groups. The right-hand panel uses data from the American Board of Medical Specialities. For each year, it shows the number of unique speciality or sub-speciality certificates that have been approved and issued at least once by that year and which are are still being issued.

In the cross-section, the task complexity measure is positively correlated with an alternative proxy for $\chi$ that more immediately captures within-firm worker-task specialization. Bloesch *et al.* (2022) use U.S. vacancy data to construct an occupation-level measure of within-firm task differentiation across positions. The appeal of this measure is that it directly measures the importance of horizontal differentiation and comparative advantage across employees of the same firm. The downside is that data limitations mean it can be constructed only for recent years. Reassuringly, the two proxies yield very similar rankings of occupations, suggesting that they capture similar latent features.[26] Under either measure, managerial and professional jobs score the highest, followed by technicians. Routine service and manual jobs exhibit the lowest levels of task complexity and differentiation.

In addition, several other arguments that have been put forth in the literature likewise point to a rise in $\chi$ over recent decades. First, the cumulativeness of knowledge pushes individuals toward specialization (Jones, 2009; Neffke, 2019). As a concrete example, in the medical sector rapid advances in knowledge and technology have necessitated more narrowly focused expertise.[27]

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less-experienced and lower-skill workers, in contrast to evidence from studies of prior waves of computerization.

[26]At the 1-digit level of ISCO-08 occupations, the correlation between Bloesch *et al.*'s (2022) differentiation measure and the non-routine abstract score from Mihaylov and Tijdens (2019), which is likewise available for the ISCO classification, is 0.57. Many thanks to Justin Bloesch for kindly sharing their data.

[27]An increase in specialization is also consistent with the findings of Braxton *et al.* (2021), who document an increase in persistent earnings risk since the 1980s (in U.S. data), concentrated in occupations that involve more non-routine cognitive tasks.

Consistent with this notion, Figure 3b visualizes how the number of speciality and sub-speciality certificates issued by the American Board of Medical Specialities nearly doubled from 1980 to 2020.[28] Second, theories of knowledge-based hierarchies with endogenous skill acquisition suggest that declining within-firm communication costs likewise encourage a narrowing in individual expertise (Garicano, 2000). Bloom *et al.* (2014) find support for this hypothesis in data from U.S. and European manufacturing firms, considering intranets as a technology that improves communication. An example from a different industry illustrates the variety of concrete technological advances that can underpin a rise in $\chi$. In software development, version control systems like Git enable multiple developers to work on a shared codebase simultaneously. This encourages them to specialize in different areas, relying on teammates' expertise for other parts.

Finally, an increase in $\chi$, which per Proposition 1 reinforces the productivity advantages of working in teams, is also consistent with the rising importance of teamwork in the economy. Indeed, Jones (2021) channels Einstein's notion of "inevitable specialization" to explain the rise of teams: "[For] the greater the stock of knowledge in an area, the narrower the expertise of the individual investigator becomes, and the greater the role of teamwork in attacking broad problems." A crucial role for team interactions has been documented for scientific activities (Akcigit *et al.*, 2018). Pearce (2022) traces the rising importance of teams in innovation to increased production benefits from deeper and broader team expertise, consistent with the argument made here for the economy as a whole.[29]

## 3.2 Methodology

Turning to the measurement of coworker complementarities, I next explain how the structural model ties together complementarity in production and in wages, then describe the data and how I measure key model objects. At a high level, the joint distribution of worker types captures sorting, while that of wages is informative about complementarities (cf. Bonhomme *et al.*, 2019).

### 3.2.1 Measuring coworker complementarity: in theory

Considering the theoretical model, and treating each worker's quality type as known for the time being, how can we measure the degree of coworker complementarity in production, and

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[28]Furthermore, Grigsby (2023) develops a method to semi-parametrically estimate the economy-wide, multidimensional skill distribution given panel data on wages and occupation choices. Consistent with the hypothesis pursued here, he finds that U.S. workers have become more specialized since the 1980s, and that transferability of skills has declined amongst high-skill occupations, occupations employing manual labor, and occupations employing social skills.

[29]More generally, firms have reallocated skilled workers into flexible, team-based settings (Weidmann and Deming, 2021). Survey evidence documenting a rise in the role of teamwork is available, for instance, in the U.S. (Lazear and Shaw, 2007) and the UK (Wood and Bryson, 2009). Also see Bresnahan *et al.* (2002) and Bloom and Reenen (2011).

specifically $\frac{\partial^2 f(x,x')}{\partial x \partial x'}$? Except in circumscribed contexts (e.g., Mas and Moretti, 2009; Adhvaryu *et al.*, 2020) we do not have measures of individual *output*, while establishment or firm output is affected by many variables other than workforce quality. The theory indicates that a closely related moment, coworker complementarity in *wages*, is highly informative. After some algebra (described in Appendix A.2.3), we can write the implied wage of $x$ employed alongside $x'$ as

$$w(x|x') = \omega\big(f(x, x') - f(x')\big) + (1 - \omega)\rho V_u(x) - \omega(1 - \omega)\lambda_{v.u} \int \frac{d_u(\tilde{x}'')}{u} S(\tilde{x}''|x')^+ d\tilde{x}''.$$

Intuitively, the worker is rewarded with an $\omega$-share of the increase in production from $x$ being added to $x'$, plus a fraction of $x$'s outside option, minus compensation to the firm and its other employee for their foregone share of a surplus from eventually hiring a different worker. Differentiating twice with respect to $x$ and $x'$ respectively yields the following insight.

**Corollary 3** (Measuring complementarities in theory)**.** *The strength of coworker complementarities in production is proportional to the strength of coworker complementarities in wages:*

$$\frac{\partial^2 f(x, x')}{\partial x \partial x'} = \frac{1}{\omega} \frac{\partial^2 w(x|x')}{\partial x \partial x'}. \tag{35}$$

The intuition is as follows. In a situation of bilateral monopoly, as it arises in the presence of search frictions, a fraction of the surplus accrues to the worker. The *level* of the wage is influenced by the bargaining partners' respective outside options. When we consider the cross-partial derivative – focussing on differential wage responsiveness to changes in coworker talent –, these outside options drop out: that of employee $x$ is foregone irrespective of whether she is matched with a bad or a good coworker; those of the employer and coworker are foregone irrespective of whether $x$ or someone else is hired.[30] The curvature of surplus is shaped by the production function. If the surplus increases by some increment $\Delta$, the worker $x$ appropriates a fraction $\omega$ of that $\Delta$-increase, and the magnitude of the associated wage change is informative about output changes due to the interaction between $x$ and $x'$.

Corollary 3 indicates that we can use measurements of $\frac{\partial^2 w(x|x')}{\partial x \partial x'}$ to quantitatively discipline $\frac{\partial^2 f(x,x')}{\partial x \partial x'}$. One limitation of Corollary 3 that ought to be kept in mind is that the mapping between wage and production complementarities is contingent on a particular value of the bargaining power parameter $\omega$. I will treat that parameter as time-invariant going forward. If, however, workers' bargaining has declined over recent decades, as some studies argue is the case at least for the U.S. and France (Stansbury and Summers, 2020; Mengano, 2023), then any increase in

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[30]This argument extends to a setting with on-the-job search (OJS), where a worker's outside option in negotiation with a "poaching" employer may be tied to the surplus obtained with the current employer, as opposed to the value of unemployment. Appendix C.2.2 elaborates on the OJS case.

wage complementarities understates rising production complementarities.

### 3.2.2 Data & processing

To measure coworker sorting and wage complementarities, I use the Sample of Integrated Employer-Employee Data (SIEED) from the German Institute for Employment Research (IAB). The SIEED covers the entire workforce of a 1.5% sample of all establishments in Germany and tracks the complete employment biographies of these individuals, including when they are not employed at the sampled establishments. Employers in the dataset are identified as establishments, which are distinguished by industry, location and ownership.[31]

I convert the spell-level data into an annual panel, deflate wages to be measured in 2015 terms, and follow literature standards in imputing top-coded observation. I restrict the sample in several steps. First, I select full-time employed individuals at establishments in West Germany, aged 20-60, with real daily wages of at least 10 Euros. Then I restrict attention to the largest connected set, as we will rely on worker mobility for identification, and subsequently drop observations in establishment-year cells containing fewer than ten worker observations. The final sample (1985-2017) includes 17,126,027 observations for 1,982,239 unique individuals. Appendix B.1.1 provides further information on data processing and summary statistics.

Throughout, I use a measure of residualized wages. Since the model neither incorporates individual wage determinants such as life-cycle or tenure effects nor features aggregate productivity dynamics, I net out these factors. Specifically, I regress the log real daily wage of individual $i$ in year $t$, $\tilde{w}_{it}$, on an individual fixed effect and an index of time-varying characteristics, $X_{it}$, which includes year dummies, a cubic in age and a quadratic in job tenure.[32] As an input into the empirical analysis, I then use $w_{it} = \exp(\ln(\tilde{w}_{it}) - X'_{it}\hat{\beta})$. The variance of residual log wages accounts for around 81% of the variance in raw log wages.

### 3.2.3 Measuring types, sorting and complementarities: in practice

With the processed data at hand, I proceed in three main steps: estimating each worker's time-invariant quality type, constructing a representative coworker type, and measuring coworker wage complementarities. For the first step, I follow the literature and implement two alternative approaches. The first estimates two-way fixed effect wage regressions in the spirit of Abowd *et al.* (1999, AKM), using dimensionality reduction techniques to mitigate a well-known incidental parameter bias problem (Andrews *et al.*, 2008; Bonhomme *et al.*, 2019). I rank workers by their fixed effect to create an ordinal measure that corresponds to the quality $x \in [0, 1]$ in

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[31]For ease of language, in the remainder of the text I use the terms "establishment" and "firm" interchangeably.

[32]As the regression includes worker and year fixed effects, I exclude the linear age term in light of the age-year-cohort identification problem. As in Card *et al.* (2013), I normalize age around 40.

the theoretical model. The AKM approach is straightforward and popular, but the log-linear wage regression is inconsistent with the structural model (e.g., Eeckhout and Kircher, 2011; Bonhomme *et al.*, 2019). Therefore, I also implement the non-parametric ranking algorithm proposed in Hagedorn *et al.* (2017), which is fully consistent with the structural model.[33] The correlation between the resulting two rankings is high (0.86). I use the AKM-based measure as a baseline. Implementation details for both approaches and robustness checks are relegated to Appendices B.1.3 and B.2.6, respectively.

Three more comments are in order. First, I implement each approach separately for five sample periods that split 1985-2017 into intervals of roughly equal duration. Second, I bin the workers according to their decile rank. Worker $i$'s decile rank, denoted $\hat{x}_i$, is time-invariant for a given sample period. Binning aligns the empirical analysis with the structural model, which is solved by discretizing the (continuous) type space using ten equidistant grid points. It also reduces noise. Third, as a robustness check, I compute an alternative measure of worker types by ranking workers within their (KldB-1988 2-digit) occupation, as opposed to economy-wide. This specification ensures that a measured increase in coworker sorting, say, does not pick up shifts in occupational composition related to trends like outsourcing.

In step two, I construct an average coworker type for each worker-year. This step bridges a gap between data and model, hence I spend some time on explaining the choices made. In the theoretical model, each firm has one team which consists of only two workers. In practice, firms are typically larger and each person has more than one coworker. Hence, in the empirical work we need to take a stance on the relevant set of coworkers and how to aggregate them. In line with Herkenhoff *et al.* (2022), I collapse coworkers into a "representative coworker". Specifically, the baseline measure is the unweighted average quality (decile) among coworkers, who I take to be the set of employees in the same establishment-year cell. Defining the set of $i$'s coworkers in year $t$ as $S_{-it} = \{k : j(kt) = j(it), k \neq i\}$, where $j(it)$ is the identifier of $i$'s employer in period $t$, the average type of $i$'s coworkers in year $t$ is $\hat{x}_{-it} = \frac{1}{|S_{-it}|} \sum_{k \in S_{-it}} \hat{x}_k$.[34]

Taking an equally weighted average across multiple coworkers ignores a non-linearity in the aggregation that is implied by the structural model but which turns out to be quantitatively minor. The production function derived in Section 2.1 implies that the aggregation weight for low-quality coworkers ought to be greater relative to high-quality coworkers insofar as $\chi > 0$. Simply put, my baseline analysis of sorting and complementarity between individuals and

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[33]Appendix A.2.4 derives a Lemma confirming that the wage $w(x|x')$ is indeed monotonically increasing in $x$ and, hence, Hagedorn *et al.*'s (2017) approach extends to the present environment.

[34]The baseline specification corresponds to what Jarosch *et al.* (2021) refer to as "Team Definition 1". One might argue that the relevant coworkers are those who are not only in the same establishment but also the same two-digit occupation. I do not adopt this alternative specification, since Corollary 1 tells us that coworker complementarities will exist precisely when workers are *differentiated* in their task-specific productivities, which is more likely to obtain across different occupations who nevertheless contribute to the same final output.

their coworkers ignores what are effectively *complementarities between different coworkers*.[35] Appendix B.2.4 evaluates this concern in detail, reports robustness results using a weighted average among coworkers, and concludes that the baseline approach is indeed biased but the magnitude of this bias is very small. The main reason for this bias being small is that dispersion among coworkers is relatively limited in the data – which, of course, is another manifestation of coworker sorting, which the structural model predicts is high precisely when $\chi$ is large. I therefore adopt the linear aggregation method as a simpler baseline that is consistent with the existing literature.

In step three, I quantify the average cross-partial derivative $\frac{\partial^2 w(x|x')}{\partial x \partial x'}$, which measures how the slope of the wage function with respect to the average coworker type varies with the own type. This is accomplished by combining worker and coworker type measures with information from the joint wage distribution. There are two different routes.[36] One method is to construct a non-parametric wage function and numerically compute the average cross-partial derivative using finite-difference methods (see Appendix B.1.4). This method is straightforward and attractive given the minimal assumptions imposed on the shape of the wage function. We may be concerned, however, that it ignores threats to identification that may be salient in the data even as the theoretical model abstracts from them. Examples include time-varying shocks at the industry- or occupation level that correlate with coworker types and wages.

An alternative approach that addresses these concerns, at the cost of imposing somewhat more structure, is regression-based. Specifically, I estimate

$$\frac{w_{it}}{\overline{w}_t} = \beta_0 + \beta_x \hat{x}_i + \beta_{x'} \hat{x}_{-it} + \beta_c \left(\hat{x}_i \times \hat{x}_{-it}\right) + \psi_{j(it)} + \nu_{o(i)t} + \xi_{s(i)t} + \epsilon_{it}, \tag{36}$$

where $\psi_{j(it)}$ denotes employer fixed effects (FE), $\nu_{o(i)t}$ are occupation-year FEs, $\xi_{s(i)t}$ are industry-year FEs.[37] I estimate this regression using OLS. Weighting each observation by the inverse frequency at which the associated match between $\hat{x}_i$ and $\hat{x}_{-it}$ occurs ensures that each part of the underlying state space is equally explored, including those parts that are under-sampled in equilibrium. The coefficient of interest is $\beta_c$, which is informative about the following question: How does the effect of having a better coworker vary with your own type? If the true data-generating process for wages follows equation (36), then $\beta_c$ measures the coworker wage complementarities of interest; differentiating the wage with respect to $\hat{x}_i$ and $\hat{x}_{-it}$ yields $\beta_c$.

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[35] While my baseline approach is consistent with all other studies I am aware of (including Cornelissen *et al.*, 2017; Jarosch *et al.*, 2021; Herkenhoff *et al.*, 2022), ignoring nonlinear aggregation may give reason for more grave concern in the present context, which unlike the other papers squarely focusses on complementarities.

[36] Appendix B.1.4 shows that both the non-parametric and the regression method are capable of approximating the true average cross-partial well when applied to data generated from the structural model.

[37] I also include squared terms in $\hat{x}_i$ and $\hat{x}_{-it}$ to address the concern that the interaction term picks up convexity in the return to own or coworker talent. For sake of readability, I omit these terms in equation (36).

Since I treat $\hat{x}$ as continuous in the estimation, $\beta_c$ specifically indicates how much more the real wage of an individual $i$ rises, as a percentage of the average wage $\bar{w}_t$, with a one-decile increase in coworker talent compared to an individual $i'$ whose rank is one decile lower than that of $i$.

Estimating equation (36) builds on the peer effects literature and existing analyses of coworker wage effects in three ways.[38] First, identifying variation comes from both movers (changes in coworker quality for individuals who switch employer) and from stayers (changes in coworker quality induced by other employees joining or leaving the coworker group). Second, a rich set of fixed effects controls for unobserved time-invariant employer heterogeneity as well as shocks at the occupation-year or industry-year level. Controlling for the worker's own type furthermore accounts for the potential selection of high types into coworker groups with a high average type, that is, coworker sorting. Third, to address the reflection problem (Manski, 1993), I use a pre-determined, model-consistent measure of coworker quality and seek to identify so-called "exogenous peer effects," as opposed to simultaneously estimating types and peer effects.

Importantly, though, the aim here is to capture coworker wage *complementarity*, as opposed to coworker wage *effects* in a broader sense. Considering regression (36), this finds its expression in two forms. First, the main coefficient of interest applies to the interaction term, whereas peer effects studies typically aim to estimate an average treatment effect from coworker quality variation. Second, I deliberately use the *level* of the wage as the dependent variables (scaled by the average wage to aid both interpretation and comparability) rather than the *log* wage. This is important. If having a better coworker confers large production- and wage-benefits to a worker, but that increase is the same regardless of the beneficiary's own type, then this provides no incentive for positively assortative matching. What matters for such sorting is whether the benefit from greater coworker quality is differentially larger for high types. Moreover, it is the *absolute* gain that counts, not the gain *relative* to the individual's own average or prior wage.[39] To offer a simple example, suppose that there are low and high types. Under the allocation (low, low) each produces 1, under (high, high) each produces 10, and (low, high) yields 3 and 5. Then production is maximized under PAM (output equals 22) rather than mixing (16). This is the case even though the proportional gain from having a high-type coworker is greater for a low type than for a high type, since the absolute gain is greater for the latter.

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[38]For example, Cornelissen *et al.* (2017); Barth *et al.* (2018); Cardoso *et al.* (2018); Hong and Lattanzio (2022).

[39]A common peer effect specification where the dependent variable is in logs *imposes* a constant (semi)-elasticity. Hence, if having a better coworker is beneficial, this is necessarily true for all types, moreover the absolute gain is necessarily greater for high-wage types.

|        | Sorting |         | Complementarity |         |
|--------|---------|---------|-----------------|---------|
| Period | Spec. 1 | Spec. 2 | Spec. 1 | Spec. 2 |
| 1985-1992 | 0.427 | 0.423 | 0.0036 | 0.0022 |
| 1993-1997 | 0.458 | 0.443 | 0.0042 | 0.0031 |
| 1998-2003 | 0.495 | 0.452 | 0.0070 | 0.0045 |
| 2004-2009 | 0.547 | 0.470 | 0.0082 | 0.0054 |
| 2010-2017 | 0.617 | 0.519 | 0.0091 | 0.0058 |

**Table 1:** Coworker sorting & complementarity over time

*Notes.* The column labelled "Sorting" indicates the correlation between a worker's estimated type and that of their average coworker, separately for five sample periods. The "Complementarity" column indicates the point estimate of the regression coefficient $\beta_c$ in regression (36). Under "Spec. 1" workers are ranked economy wide, while under "Spec. 2" they are ranked within occupations.

## 3.3   Evolution of coworker sorting & complementarities in Germany

I now describe the estimated evolution of coworker sorting and wage complementarity. I report these moments separately for 5 sample periods, jointly spanning 1985-2017.

As the first main result, coworker sorting is positive and increasingly so time. The first two main columns of Table 1 report the correlation coefficient $\hat{\rho}_{xx} = \text{corr}(\hat{x}_i, \hat{x}_{-it})$. In the baseline, $\hat{\rho}_{xx}$ has increased from an average of 0.427 across the years 1985-1992 to 0.617 over 2010-2017, a 44.5% increase. If workers are ranked within occupation, instead, then the increase is substantially smaller (+0.096 instead of +0.19) but still represents a 22.7% rise. Part of the increase captured by the baseline specification thus picks up increased segregation along occupational lines, whereby higher-pay and lower-pay occupations cluster in distinct firms. That sorting is positive and rising when worker quality is measured conditional on occupation indicates, meanwhile, that the best workers *within* each occupation tend to collaborate.

Figure 4 provides a more disaggregated perspective. In a binscatter plot of the average coworker type for each worker type, the orange line represents 2017, while the blue line is based on 1990 data. This graph demonstrates how the slope of the "matching function" has steepened. It also verifies that the rise in $\hat{\rho}_{xx}$ does not pick up a idiosyncratic change at one or more isolated points in the type distribution.

The second main result is that coworker complementarity is positive and has strengthened over time, approximately doubling from the first to the final sample period. Figure 5 plots for each sample period the point estimate for the interaction coefficient of interest, $\hat{\beta}_c$, alongside 95% confidence intervals. The point estimate has risen from 0.0036 to 0.0091. To provide a sense of magnitudes, if $\beta_c$ is equal to 0.005, this means that the real hourly wage increase from a one decile improvement in the average coworker quality is 2.5% greater, as a percentage of the
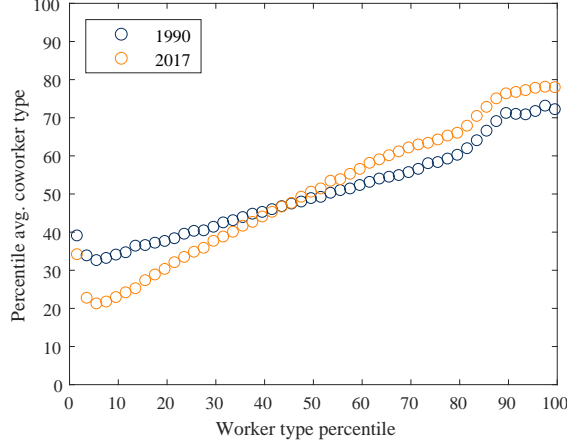
**Figure 4:** Trends in coworker sorting

*Notes.* This figure plots, for any percentile of the worker type distribution (i.e., before binning) the average quality of coworkers. For visual clarity, workers are grouped into 50 cells, then the coworker quality is computed for each cell.

average wage, for a worker who is themselves in the top decile as opposed to the fifth decile.

The same figure overlays the task complexity measure described in the preceding Section 3.1, indicated as circles. The evident co-movement of this proxy for $\chi$ and the estimated coworker complementarities is only indicative but does cohere well with the predictions of the model, specifically Corollary 1.

Table 1 also reports the point estimate $\hat{\beta}_c$ when workers are ranked within occupation. This alternative specification likewise indicates that complementarities have increased. Both starting and end point are slightly lower, but the change similarly amounts to an approximate doubling.

The finding of a rise in coworker complementarity is robust to a looming concern. The worry is that $\hat{\beta}_c$ may be biased because worker and coworker quality measures, which appear on the right-hand side of equation (36), are ultimately still a function of wages, including the contemporaneous wage which serves as the dependent variable. I address this concern through two different robustness exercises. First, Appendix B.2.7 considers education as a non-wage measure of worker types. In a regression of wages on fully interacted years of schooling and coworker years of schooling, the coefficient on the interaction term likewise indicates that complementarities are positive across all periods and have roughly doubled in magnitude from the first to the last sample period. Second, I re-run regression (36) using worker and coworker types estimated off of information on *past* wages. This approach, too, indicates that complementarities are approximately twice as strong in 2010-2010 than in 1993-1997.[40]

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[40]More precisely, I assign to each individual $i$ in periods $p \in \{2, 3, 4, 5\}$ the fixed effect estimated for $i$ in period $p - 1$, and then re-compute worker deciles and average coworker types, $\hat{x}_i^{p-1}$ and $\hat{x}_{-it}^{p-1} = (|S_{-it}|)^{-1} \sum_{k \in S} \hat{x}_k^{p-1}$, on that basis. The correlation between $\hat{x}_i$ and $\hat{x}_i^{p-1}$ is 0.85, consistent with the idea that unobserved individual characteristics captured by the fixed effect are very persistent. To compare like with like, I then re-estimate for each period $p \in \{2, 3, 4, 5\}$ the original wage complementarity regression using contemporaneous types as well as the
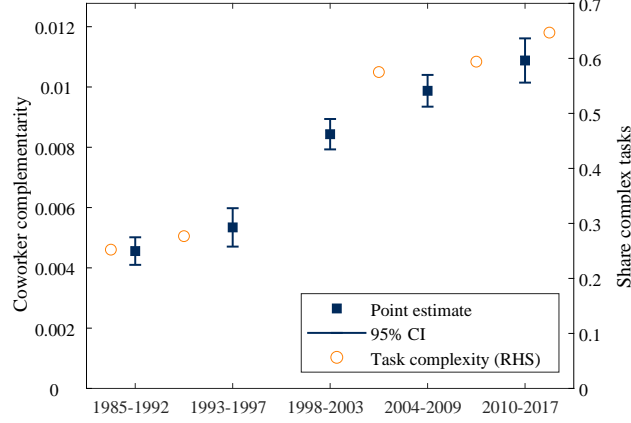
**Figure 5:** Trends in coworker complementarity

*Notes.* This figure reports the point estimate and confidence intervals for the coefficient $\beta_c$ in regression (36), separately for five sample periods. Standard errors are clustered at the employer-level. In addition, the share of complex tasks in workers' activities, depicted in circles, is reproduced from Figure 3a. The years of the survey waves and the sample split in the matched employer-employee data do not align perfectly, so the task measures are placed approximately at the mid-points of the closest sample period.

## 3.4  Validation of model mechanisms in the cross-section

Before quantifying the model, I further probe it by verifying if the following two predicted relationships are observed in the cross-section. First, coworker complementarities are stronger when worker-task specialization is more pronounced. Second, greater coworker complementarities are associated with more positively assortative coworker matching. In sum, I find that variation in task complexity, complementarities and sorting across occupations and industries lends support to the key model mechanisms.

For these analyses, I supplement the SIEED data with a second micro dataset, the Portuguese Quadros de Pessoal/Relatório Único (QP) matched employer-employee panel. All results reported for Portugal are drawn from joint work with Criscuolo and Gal (Criscuolo *et al.*, 2023).[41] What motivates this approach are some undesirable properties of the German micro data that limit further probing of the model: a non-negligible fraction of wage observations are imputed due to top coding, and the sample is large but does not cover the full population.[42] By contrast, the QP is an annual mandatory census of all employers in Portugal and wages are not top-coded. Appendix B.1.2 provides a more detailed description. I use the 2010-2017 subsample, which contains ISCO occupation codes and detailed industry codes at the NAICS 4-digit level. I then

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robustness exercise using lagged types in the same sample.

[41] I thank Chiara Criscuolo and Peter Gal, and the OECD's Directorate for Science, Technology, and Innovation (STI) more broadly, for facilitating these analyses and the usage of the results in this paper.

[42] Why not use Portugal as the primary data source? One reason is that I want to connect my analyses to previous results in the literature, which have often treated Germany, the largest European economy, as a reference case. Portugal has, furthermore, experienced several idiosyncratic macroeconomic shocks over the past three decades.
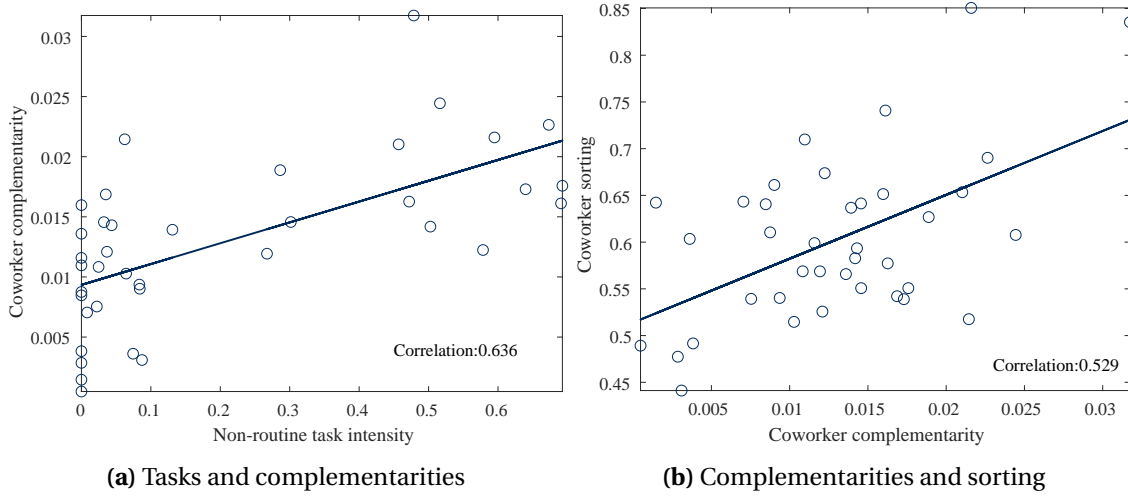
**(a)** Tasks and complementarities　　　**(b)** Complementarities and sorting

**Figure 6:** Occupation-level relationships

*Notes.* The left panel plots for each ISCO-08 2-digit occupation the occupation-specific point estimate for the coefficient on the interaction term, $\beta_c$, when estimating a version of equation (36) separately for each occupation against the non-routine task intensity of that occupation. The right panel plots the occupation-level coworker correlation coefficient, i.e., the correlation between a worker's type and the average coworker type against $\beta_c$, which is now shown on the horizontal axis. The linear regression line is fitted based on unweighted observations.

exploit variation across occupations and industries, while noting two caveats. First, the analysis merely establishes correlations that are consistent with the structural model, and I do not claim causality. Second, as the model does not explicitly incorporate occupations or industries, I effectively interpret each cell as a separate instance of the model.

OCCUPATIONS. I start by analyzing tasks, complementarities, and sorting at the occupation level. I classify each ISCO-08 2-digit occupation by its reliance on non-routine, abstract (NRA) tasks, relying on task indices from Mihaylov and Tijdens (2019).[43] Next, I construct sorting and complementarity measures separately for each occupation. For these measures to be independent across occupations, I use the within-occupation ranking of workers and, additionally, restrict the set of coworkers to those in the same employer-occupation-year cell ("Team definition 2" in Jarosch *et al.* (2021)). Coworker complementarity is computed by estimating regression (36) separately for each occupation.

The left-hand panel in Figure 6 plots the estimated, occupation-specific estimate of coworker wage complementarity against the NRA score. We observe a positive relationship, consistent with the theory. The estimated interaction coefficient is close to zero for those occupations

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[43]The index by Mihaylov and Tijdens (2019) is attractive because it is constructed from occupation-specific lists of tasks provided by ISCO-08. Therefore, no selection among many potential task scales is involved, which can be controversial, as discussed by Acemoglu and Autor (2011). Moreover, this occupational classification is consistent with the Portuguese classification for the period in question, whereas the task complexity measure discussed in Section 3.1 is based on an older, German classification, cross-walking which proved to me more than adventurous. I aggregate the measures to the 2-digit level.
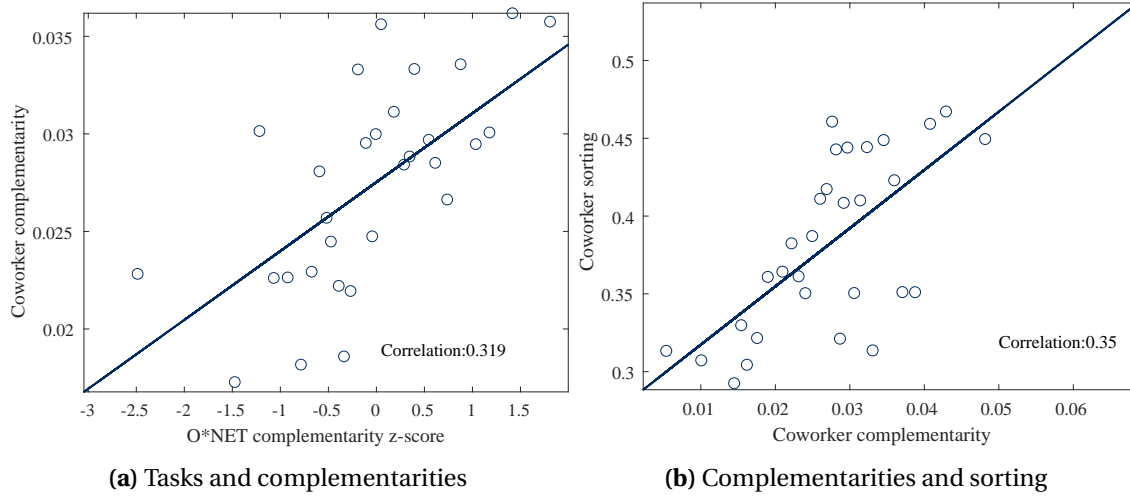
**(a)** Tasks and complementarities      **(b)** Complementarities and sorting

**Figure 7:** Industry-level relationships

*Notes.* The left panel plots the industry-level point estimate for the coefficient on the interaction term, $\beta_c$, when estimating a version of equation (36) separately for each industry, against an industry-level proxy for complementarity. This proxy is constructed from O*NET data following Bombardini *et al.* (2012). The right panel plots the industry-level coworker correlation coefficient against $\beta_c$, which is now shown on the horizontal axis. The unit of observation is a 4-digit NACE industry, with at least 5,000 person-year observations and where the industry-level proxy for complementarity is within two standard deviations. Observations are grouped into 30 bins. The linear regression line is fitted based on unweighted, non-binned observations.

with the lowest NRA score, whereas it is above 0.02 in occupations that primarily perform non-routine abstract tasks. Next, the right panel shows that workers in occupations featuring greater complementarity are also matched together in a more positively assortative pattern.

These findings are also consistent with previous literature studying broadly related phenomena. Neffke (2019) studies Swedish micro data and finds that coworker effects are important in many knowledge-intensive jobs (e.g., health care, engineering) and professional occupations (e.g., lawyers); and in skill-intensive (e.g., R&D) an crafts-based industries (e.g., construction). Jäger and Heining (2022) find that when a high-skilled or specialized worker dies, their coworkers in other occupations experience wage decreases.[44]

INDUSTRIES. Turning to an industry-level analysis, I classify each 4-digit NACE industry according to a proxy for complementarity proposed by Bombardini *et al.* (2012). The basic idea is that an industry is more likely to feature complementarity if many workers are in occupations for which the following four characteristics are important: teamwork, impact on coworker output, communication, and contact. I operationalize this measure using O*NET data alongside industry-level occupational employment weights from QP.[45] Similar to the approach adopted for

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[44]Appendix B.2.5 provides supplementary evidence using variation across seven different, vertical layers of the firm, grouped by similarity in the complexity of tasks and skills required, in the spirit of Caliendo *et al.* (2020).

[45]Clearly, we could characterize industries also on the basis of occupational task indices, as considered before. I adopt Bombardini *et al.*'s (2012) approach as a baseline in order to test the theory against a variety of proxies for $\chi$,

occupations, I also compute coworker complementarity and sorting separately for each industry.

The results cohere with the theoretical model predictions. The binscatter plots in Figure 7 shows that the higher an industry scores on to the proxy measure, the greater tends to be the estimated value of complementarity, which in turn is positively associated with coworker sorting. Appendix B.2.5 furthermore shows that measures of between-firm inequality in productivity and wages are increasing in the degree of coworker sorting.[46]

# 4 Quantitative analysis

Bringing theory and data together, this section provides a quantitative analysis. It is organized as follows. First, I discuss the calibration of the model. Second, I use the model to answer two main questions. To what extent can a rise in worker-task specialization and the implied strengthening of coworker complementarities explain a rising between-firm share of wage inequality? And what are the implications of (changing) complementarities and firm-level inequality for allocative efficiency? Finally, I perform robustness exercises.

## 4.1 Parameterization and quantitative properties

I start by describing how structural parameters are disciplined through a combination of micro- and macro-moments, including the complementarity estimates obtained in Section 3. I then discuss the baseline estimation results. Lastly, I re-estimate key parameters for an earlier period.

It is crucial to my overall approach that the key distributional moments of interest – the degree of coworker sorting and the between-firm share of wage inequality – are left untargeted. Instead, the point estimates of coworker wage complementarity at the micro-level serve to discipline the elasticity of complementarity, $\gamma$, and thus indirectly the worker-task specialization parameter $\chi$. The macro-distributional moments then serve as yardsticks to assess the model's performance to explain within- and between-firm heterogeneity in workforce quality composition and pay.

### 4.1.1 Methodology

Four parameters are externally calibrated, the job separation rate is set to directly target an empirical moment, the remaining five are structurally estimated by indirect inference. I set a unit interval of time to be one month, and since the model is solved in continuous time I can

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including ones used in the extant literature for related purposes.

[46]Reassuringly, the relationship between coworker sorting and the between-firm wage of the variance of log wages is almost linear with a slope equal to one. That is, the industries with the highest measure of coworker sorting (a correlation of around 0.55) have, on average, a between-firm share of the variance of log wages equal to 0.55; industries featuring a coworker sorting coefficient around 0.15 have a between-share of close to 0.15.

construct correctly time-aggregated measures at any desired frequency.[47] Empirical moments are constructed as averages across the years 2010-2017.

DISTRIBUTIONAL AND FUNCTIONAL FORM ASSUMPTIONS. The functional form of the production function is directly informed by the microfoundations set out in Section 2.1. For quantitative purposes, I introduce two generalizations. First, I introduce a constant factor, $a_0$, such that even the lowest-ranked employee produces a strictly positive amount of output. Including $a_0$ is motivated by the observation that in the model even the least productive worker searches for employment, which is likely not true in the data, where those individuals would be out of the labor force. One can interpret the intercept term $a_0$ as standing in for this selection margin. Hence, a firm with a single worker $x$ produces $f(x) = a_0 + a_1 x$. Second, the degree to which labor productivity is greater when working in teams than working alone is controlled by a parameter $a_2$, so that team output is[48]

$$f(x, x') = 2a_2\Big[a_0 + a_1\big(\frac{1}{2}(x)^{1-\gamma} + \frac{1}{2}(x')^{1-\gamma}\big)^{\frac{1}{1-\gamma}}\Big]$$

Next, the flow value of unemployment is proportional to output produced alone, $b(x) = \bar{b} \times f(x)$.

EXTERNALLY SET OR ESTIMATED OFFLINE. The following three parameters are externally calibrated. Regarding preferences and bargaining, I follow Herkenhoff *et al.* (2022). The discount rate $\rho$ is set to 0.008, consistent with an annual interest rate of 10%. Such a high rate is common in this type of model. A high discount factor effectively proxies for concavity in the utility function, which tractability requires us to abstract from. The bargaining parameter $\omega = 0.50$ implies equal sharing of surplus. Third, as explained in Section 2.1, the production parameter $a_2$ has to be greater than unity, else it would never be beneficial to work in teams, as complementarities mean that the least-capable team member disproportionately determines output. I set $a_2$ equal to 1.1, though the exact value turns out not to be crucial.[49]

The exogenous separation rate, $\delta$, directly targets the monthly job losing rate observed in the micro data, which on average across 2010-2017 equals 0.0085. I compute this moment by supplementing the SIEED with the Linked Employer Employee Data (LIAB), which unlike the SIEED contains information on non-employment spells (see Appendix C.1.1).

INTERNALLY ESTIMATED. The remaining five parameters, collected in $\psi = \{\chi, a_0, a_1, \bar{b}, \lambda_u\}$,

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[47] For example, if the empirical average probability that an employed worker loses their job during a month is $d$, then I recover the Poisson rate $\delta$ at which such a shock arrives as $d = 1 - e^{-\delta}$.

[48] I have experimented with a specification that allows $a_2$ to endogenously vary with $\chi$, as implied by Proposition 1. However, as the returns to scale are decreasing in team size, the assumption that teams are of maximum size two which was made for tractability reasons, could lead to an overstatement of this benefit. I therefore opt to instead treat $a_2$ as exogenous in order to cleanly isolate the effect of $\chi$ via complementarities.

[49] This means that the average labor productivity in a team with two workers $x$ and $x'$ is 10% greater than that of a worker producing alone if that worker's rank equalled the $\chi$-weighted power mean of $x$ and $x'$.

| Parameter | Description | Value | Source |
|-----------|-------------|-------|--------|
| $\rho$ | Discount rate | 0.008 | External |
| $\omega$ | Worker bargaining weight | 0.50 | External |
| $a_2$ | Team benefit | 1.1 | External |
| $\delta$ | Separation rate | 0.0085 | Offline estimation |
| $\gamma$ | Elasticity of complementarity | 0.837 | Internal estimation |
| $a_0$ | Production constant | 0.239 | Internal estimation |
| $a_1$ | Production quality scale | 1.557 | Internal estimation |
| $b_1$ | Unemployment flow scale | 0.664 | Internal estimation |
| $\lambda_u$ | Meeting rate | 0.230 | Internal estimation |

**Table 2:** Model parameters - baseline (2010s)

*Notes.* This table summarizes the baseline calibration of the model.

are estimated using standard indirect inference methods by matching moments. The estimated values of these parameters minimize the objective function

$$\mathcal{G}(\psi) = \sum_{j=1}^{5} \left( \frac{\hat{m}_j - m_j(\psi)}{\frac{1}{2}|\hat{m}_j| + \frac{1}{2}|m_j(\psi)|} \right)^2,$$

where $\hat{m}_j$ refers to the empirical moment and $m_j(\psi)$ denotes its model counterpart.

Before describing the moments used, two comments are in order. First, the model moments are obtained from the numerically computed stationary equilibrium, found using a fixed point algorithm after discretizing worker types into ten equally spaced grid points. I compute these moments in population, without creating a sample through simulation, so that for example the variance of log wages is recovered using equation (33).[50] Second, while the parameters in $\psi$ are jointly estimated, each is closely informed by one of these moments, as explained next. To validate my approach, and following Bilal *et al.* (2022), Appendix C.1.2 conducts two exercises that jointly support the notion that the vector $\psi$ and each of its elements are well-identified.

The most important moment is the coworker wage complementarity, which following Corollary 3 directly informs the strength of production complementarity. I estimate an analogous regression to equation (36) inside the model and target the empirically estimated point estimate of the interaction coefficient $\beta_c$, which for 2010-2017 is equal to 0.0091 (cf. Table 1). Next, the values of $a_0$ and $a_1$ are guided, respectively, by the average wage, which I normalize to unity, and the total variance of log wages, which is equal to 0.241. Note that both parameters raise the average wage but the dispersion of wages is decreasing in $a_0$ yet increasing in $a_1$. The parameter

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[50]It is straightforward to compute simulated moments, but for reasonable sample sizes there is vanishing sampling uncertainty around the population moments.

| Parameter | Targeted moment | Period 2 | | | Period 1 | | |
|---|---|---|---|---|---|---|---|
| | | Value | $m$ | $\hat{m}$ | Value | $m$ | $\hat{m}$ |
| $\gamma$ | $\hat{\beta}_c$ | 0.837 | 0.0091 | 0.0091 | 0.434 | 0.0035 | 0.0035 |
| $a_0$ | Avg. wage (norm.) | 0.239 | 1 | 1 | 0.378 | 1 | 1 |
| $a_1$ | Var. log wage | 1.557 | 0.241 | 0.241 | 1.216 | 0.143 | 0.143 |
| $b_1$ | Replacement rate | 0.664 | 0.63 | 0.63 | 0.740 | 0.69 | 0.69 |
| $\delta$ | Job loss rate | 0.008 | 0.008 | 0.008 | 0.007 | 0.007 | 0.007 |
| $\lambda_u$ | Job finding rate | 0.230 | 0.162 | 0.162 | 0.168 | 0.141 | 0.141 |

**Table 3:** Estimated parameters and targeted moments

*Notes.* This table lists for each of the estimated parameters, the targeted moment, the estimated value, and moment values in data ($\hat{m}$) and model ($m$). Each is shown separately for period 2 (2010-2017) and period 1 (1985-1992).

$b_1$ is informed by the ratio at which the flow value of unemployment replaces the (type-specific) average wage. Based on official administrative replacement rates in Germany and recent work by Koenig *et al.* (2021) that also accounts for non-monetary opportunity costs of employment, I target a ratio of 0.63. This value is somewhat lower than the replacement rate used by Jarosch (2023), for instance, since my calibration targets moments after the Hartz reforms of the labor market undertaken in the early to mid-2000s, which reduced the generosity of the unemployment insurance system. Finally, $\lambda_u$ targets a monthly job finding rate of unemployed workers equal to 16.2%, again computed from the LIAB.[51]

### 4.1.2 Results

Table 2 summarizes the baseline parameterization of the model, including the internally estimated parameters. The key parameter $\gamma$ in 2010 is inferred to be 0.84. This corresponds to $\chi$ equal to 5.15. Table 3 summarizes the model fit. It indicates that the model is capable of matching the targeted moments perfectly, $\psi$ being exact-identified.

I now turn to an evaluation of the model in terms of untargeted moments. In terms of matching patterns, the theoretical coworker correlation indicates substantial, positive coworker sorting, at a magnitude only slightly below what is observed in the data (0.53 vs. 0.62). Noting that the only moment which informs the degree of coworker sorting in the estimation is the micro estimate of coworker wage complementarity, I view this as a success for the model. Providing a more disaggregated picture, Figure 8 plots, for each type, the decile of the average coworker type, according to both model and data. The model successfully replicates the upward slope, but

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[51]This value is higher than that reported by, e.g., Jarosch (2023). This is unsurprising, as I compute moments in matched employer-employee data rather than a sample representative of both the employed and the non-employed. As the other moments likewise derive from employer-employee data, the estimation is internally consistent.
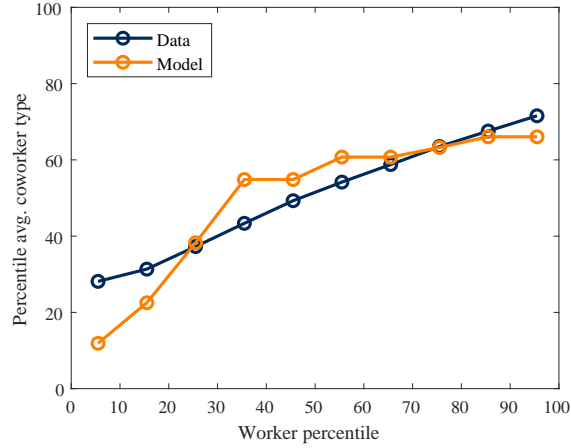
**Figure 8:** Average coworker type by worker type: data vs. model

*Notes.* This figure plots, for each type, the average coworker type, in both model and data.

slightly underestimates the quality of coworkers at both bottom and top. I show in Section 4.4 that incorporating on-the-job search improves the model fit in this respect.

What about between- and within-firm wage inequality, the key untargeted moment(s) of interest? In the next section, we dissect the model-implied decomposition of wage dispersion into between- and within-firm components. In brief, this analysis reveals that the stationary equilibrium of the theoretical model fits the data very well along this dimension. That is despite assuming that there is no permanent, ex-ante heterogeneity across firms. Insofar as there are differences across firms in the stationary equilibrium, they emerge purely due to ex-post heterogeneity in firms' workforce composition; they do not reflect any intrinsic feature of firms. Furthermore, while I put less emphasis on the productivity dimension, it warrants highlighting that the estimated model generates substantial ex-post dispersion in firm productivity as well. As a reference point in terms of broad magnitudes, Syverson (2004) reports an average 90:10 labor productivity ratio of 4:1. The model-implied moment is a 90:10 labor productivity ratio of 3.14:1, thus going quite some way toward the data even without assuming ex-ante heterogeneity among firms. Overall, the model's predictions are borne out not only in the cross-section (cf. Section 3.4), but the calibrated version also quantitatively reproduces key features of the German economy in the 2010s quite well, in spite of its parsimonious structure.

### 4.1.3   Re-estimation for earlier period

In addition to the baseline estimation for the 2010s ("period 2"), I re-estimate the parameter vector $\psi$, as well as the separation rate, by targeting the same empirical moments as in the preceding step but measured over the years 1985-1992 ("period 1"). Two comments about the targeted moments for period 1 are warranted. First, regarding the replacement rate, I proceed

similarly to Jung *et al.* (2023) and summarize the multiple dimensions of the Hartz reforms as amounting to a reduction of the monetary replacement rate by around 10%. Hence, for the earlier period I target a higher overall replacement ratio equal to 0.72. Second, I deliberately re-estimate the labor market transition rates as well. This enables an evaluation of the role played by changes in search frictions in shaping the evolution of sorting and wage inequality.

The final main column in Table 3 indicates the estimated parameter values. As expected, the estimated elasticity of complementarity is lower, at 0.434, than the estimate obtained for the 2010s. In addition, the rise in $a_1$ and the decline in $a_0$ from period 1 to period 2 are consistent with technological change rendering match output more sensitive to team quality over time. One also observes an increase over time in both job arrival and job separation rates, perhaps reflecting the emergence of online job portals (Bhuller *et al.*, 2023).

## 4.2 A model-based perspective on firming-up inequality

Using the estimated model, we can evaluate the contribution of rising coworker complementarity to the "firming up" of inequality in Germany. Before that, it is useful to ascertain reduced-form facts. Figure 9a illustrates the empirically observed rise in firm-level wage inequality. It decomposes the variance of log (residual) wages into a between-employer and a within-employer component. In the SIEED data, the between-share rose from an average of 33.6% over the period 1985-1992 to 56.8% during 2010-2017. This trend is consistent with evidence documented in the literature, including Card *et al.* (2013) and Song *et al.* (2019).[52] To supplement this simple statistical analysis, I perform variance decompositions using the estimated fixed effects from two-way fixed effect AKM regressions, run separately for each sample period.[53] This exercise confirms that changes in workforce compositions across firms play a key role in the rise of between-firm inequality, as noted in Section 1. Increased worker-worker segregation and worker-firm sorting explain more than fourth fifths of the increase in the between-employer component, with changes in firm-specific pay premia accounting for the remainder.

Turning to the structural model, Figure 9b illustrates that the model successfully captures these trends in firm-level wage inequality. The figure depicts the model-implied decomposition of the variance of log wages in the stationary equilibrium, computed at the period-1 and period-2 parameters respectively.[54] It can be seen that the structural model qualitatively replicates

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[52]Appendix B.2.1 provides cross-country evidence and Appendix B.2.3 contains robustness checks for Germany.

[53]Appendix B.2.2 gives details. While these reduced-form regressions are not consistent with a structural model of the kind analyzed here, they are a popular statistical diagnosis tool.

[54]I stress that the model-based between-within decomposition includes a correction term that adjusts for a statistical upward-bias in the level of the between-share of wage inequality. This bias arises because in the theoretical model the team size is fixed to two, so that the law of large numbers has little bite within production units. Appendix C.1.3 explains the method I propose and adopt to adjust for this statistical bias. I apply the same correction factor
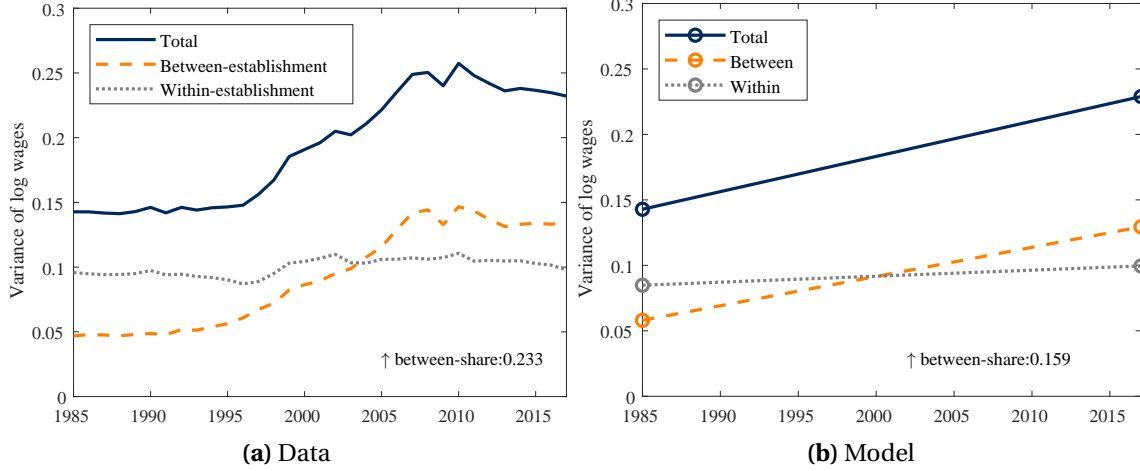
**Figure 9:** Decomposition of the variance of log wages: data vs. model

*Notes.* This figure shows the variance of log wages, decomposed into between- and within-employer components. The left panel reflects the data, the right panel indicates the model prediction. The empirical moments are computed on the basis of the same residualized wages used in the estimation of the model. The change in the between-share compares the average over 1985-1992 and 2010-2017. The model-generated between-within decomposition is corrected for a statistical bias, as described in Appendix C.1.3.

the empirically recorded increase in the share of wage inequality that is due to between-firm differences. In quantitative terms, the model captures 68% of the empirically observed increase in the between-firm share (0.159 vs. 0.233). This fit is noteworthy in light of the parsimonious model structure and since the estimation only targeted the *total* variance of log wages.

How much of the model predicted increase in the between-firm share of wage inequality is due to a strengthening in coworker complementarities? Ex ante, the answer is not obvious, since each of the internally estimated parameters changed from period to period and has an influence on the model-implied decomposition. Most immediately, a higher job arrival rate – effectively representing a reduction in search frictions – raises sorting, since workers can become more selective in which offers to accept. Moreover, as Song *et al.* (2019) flag in their empirical analysis, even with a stable distribution of worker types across firms, insofar as it exhibits positive sorting, technological change that amplifies the return to skill should mechanically lead to greater between-firm inequality: the remuneration of already highly-paid employees, who tend to cluster together, would rise relative to less well paid employees in other firms.[55]

To isolate the contribution of coworker complementarities, I perform counterfactual exercises inside the structural model. The specific question we confront the model with is: What would the between-firm share of the variance of log wages have been in the 2010s if $\chi$ had not increased

for both periods, so the predicted change is unaffected in any case.

[55]Through the lens of an AKM-framework, increased return to skill could magnify both the worker-firm sorting and the worker-worker segregation components. Song *et al.* (2019) present a calculation suggesting that, respectively, 9% and 35% of the increase in sorting and segregation they document for the U.S. are due to these mechanical effects.

43

|  | Δ model | Implied % Δ model due to Δ parameter |
|---|---|---|
| Model 1: baseline | 0.159 | - |
| Cf. a: fix period-1 complementarity | 0.065 | 59.23 |
| Cf. b: fix period-1 search frictions | 0.150 | 5.88 |
| Model 2: within-occ. ranking | 0.198 | - |
| Cf. a: fix period-1 complementarity | 0.076 | 61.47 |
| Cf. b: fix period-1 search frictions | 0.189 | 4.43 |
| Model 3: OJS | 0.096 | - |
| Cf. a: OJS, fix period-1 complementarity | 0.067 | 30.54 |
| Cf. b: OJS, fix period-1 search frictions | 0.067 | 30.45 |

**Table 4:** Counterfactuals: explaining the rise in the between-firm share of wage inequality

*Notes.* This table summarizes the counterfactual ("Cf.") exercises evaluating changes in the between-firm share of wage inequality. The second column indicates the change in the between-share from period 1 to period 2 for the model specified in the first column. The final column is computed as follows, using the example of "Cf. a:". Denoting by $m_j$ the model-implied between-share, the value is equal to $100 \times \left(1 - \frac{m_j(\gamma^{p1}, \psi^{p2}_{-1}) - m_j(\psi^{p1})}{m_j(\psi^{p2}) - m_j(\psi^{p1})}\right)$, where $\gamma^{pt}$ is the value of $\gamma$ estimated for period $pt$, $t \in \{1, 2\}$, and $\psi^{pt}_{-1}$ collects all other parameters.

at all? I then compute the implied difference in the between-share across the factual and the counterfactual scenarios.

The model indicates that increased coworker complementarity is an important driver of increased firm-level inequality. Table 4, specifically row 2, summarizes the analysis. According to the model, the between-share would have risen only by 7 percentage points had $\chi$ remained unchanged, instead of 15.9. Accordingly, increased coworker complementarities can account for about 59% of the rise in the between-firm share of wage inequality predicted by the model. This corresponds to 40% of the empirically observed change.

We can compare the influence of coworker complementarities to that of the estimated changes in labor market transition rates. Consider an experiment in which we counterfactually impose that both job arrival and separation rates, $\lambda_u$ and $\delta$, had remained at their period-1 levels. This exercise suggests that their joint increase can explain 6% of the model-predicted rise in the between-share.[56] As such, changes in the production technology represent a more important factor than changes in labor market 'technology', though both play a meaningful role.

The key mechanism behind the rise in the between-firm share of wage inequality in the structural model is more positively assortative matching, consistent with the empirical evidence on labor market segregation. The coworker correlation coefficient rises from 0.24 in period 1

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[56]An important caveat is that these rates are treated as exogenous in the structural model, but are, in fact, plausibly endogenous to technological changes. In particular, it is intuitive that an increase in production complementarities, by rendering mismatch more costly, itself would incentivize greater search effort. Such an analysis goes beyond this paper but may be interesting to consider in future research.
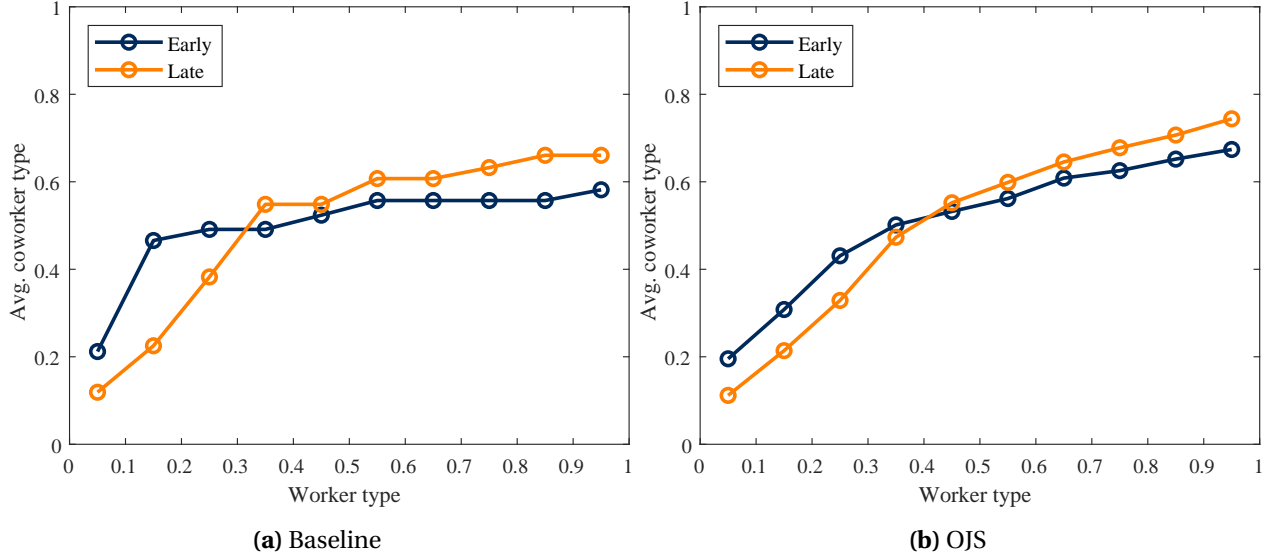
**Figure 10:** Model-implied coworker sorting

*Notes.* Both panels plot the average coworker type (vertical axis) for each worker type (horizontal axis), separately for the period-1 and period-2 parametrizations. The left panel is computed from the baseline model, the right panel considers the model extended to incorporate on-the-job search.

to 0.522 in period 2. In the model, stronger coworker complementarity signifies that coworker mismatch is more costly. Hence, the market price mechanism pushes towards greater sorting. Specifically, the wage that a firm with a low-quality worker can pay a high-quality worker lies farther below that which is affordable for a firm that employs another high-quality worker, since the high-quality worker will be considerably more productive when paired with a peer of similar talent. Figure 10a illustrates this shift as it occurs in the estimated model. The figure retraces the twist in the mapping between worker type and average coworker type that Figure 4 illustrated for the data: High types increasingly pair up amongst themselves, as do lower types.

As a side note, while the model is chiefly designed to explain the firm-level structure of wage inequality, it is perhaps useful to understand the relationship between coworker complementarities and individual-level inequality. In the model, an increase in $\chi$ does not necessarily translate into a higher overall variance of log wages. This statement may surprise in light of Kremer's (1993) influential O-ring theory, which may lead to the prior that greater task complexity – which is Kremer's (1993) label for the number of tasks involves – amplifies complementarities and sorting as well as leading to a "convexification" of the wage function. But notice that in Kremer (1993) input markets are frictionless. Hence, per discussion in Section 2.2, strictly positive complementarity of *any* degree suffices to yield pure PAM in equilibrium. Moreover, the prediction of a right-skewed earnings distribution depends crucially on the assumption that production is subject to increasing returns to team quality as a whole (this is already noted in Kremer (1993, Section III)). I deliberately considered the constant-returns case, instead, to isolate the role of

complementarities. As a result, the wage function is linear in a worker's own type absent search frictions; their presence render it concave if $\chi > 0$. Indeed, the quantitative model reveals that in a counterfactual scenario characterized by the period-2 parameterization but (lower) period-1 complementarities, the overall variance of low wages is actually slightly higher than in the factual period-2. The most important reason is that the increase in between-firm inequality is offset by a reduction in within-firm inequality.[57] In addition, within firms stronger complementarities reduce the contribution of higher-quality types to surplus (cf. Bombardini *et al.*, 2012).[58]

## 4.3   Coworker complementarities, sorting, and TFP

Beyond rationalizing patterns of firm-level wage inequality, the model shows how the interaction between worker-task specialization and labor market structure affects total factor productivity (TFP). Qualitatively, Section 2 clarified that specialization affects TFP through two channels. Specialization under the division of labor is a direct source of productivity gains, as is well understood. Yet, greater worker-task specialization also fosters complementarity, hence team productivity is disproportionately influenced by the least capable team member. Consequently, the interaction between specialization and search frictions, which generate within-firm worker quality heterogeneity ("coworker mismatch"), lowers allocative efficiency. In this section, I use the estimated model to quantify this mechanism. Specifically, I ask how large is the deviation from potential output due to coworker mismatch in the Germany economy in the 2010s, and how has it changed over time? Moreover, what does this mechanism tell us about the productivity implications of a rising between-firm share of inequality?

To quantify equilibrium mismatch costs, I compute the difference in labor productivity under the equilibrium allocation compared to a counterfactual one that exhibits perfectly assortative matching (PAM). This approach is motivated by the observation that for any value $\chi > 0$, TFP is maximized under PAM. Moreover, to isolate changes in coworker misallocation the distribution of worker types employed in teams is kept fixed. Counterfactual output per worker in the absence of frictions is, thus, equal to $\frac{1}{2} \int f(\phi(x), \phi(x)) dx$, $\phi(x)$ being the unconditional density of types in teams.

Turning to findings, in period 2 per capita output would be 2.17% higher absent coworker

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[57] In the model, higher overall wage dispersion in period 2 compared to period 1 is due to a lower value of $a_0$ and a higher value of $a_1$.

[58] Still, there are plausible channels through which strengthening complementarities could raise overall inequality, and which future research may examine. A first possibility is that fairness concerns or explicit policies generate within-firm pay compression for workers of heterogeneous quality. Increased coworker sorting due to coworker complementarity may then weaken the degree to which these forces dampen overall wage inequality. Secondly, coworker learning in interaction with sorting can dynamically foster lifetime inequality. Third, a version of the current model which in addition to complementarities and search frictions features increasing returns to workforce quality, as in Kremer (1993), could generate a positive relationship between complementarities and overall inequality.

quality mismatch. In period 1, when complementarities were weaker, the equilibrium costs of mismatch are slightly lower at 1.78%.

Through the lens of the microfounded production function, this increase in quality mismatch costs ultimately reflects more inefficient task assignment. A high-quality worker paired up with a low-quality worker has to take care of tasks that she is relatively less productive in. The associated reduction in overall output is greater when workers are more specialized. Figure 11a illustrates this point by plotting a simple measure of the strength of production complementarities against $\chi$. This measure compares the output produced by two teams, once under perfect assortative matching ("high-high" and "low-low") to that of the same types but mixing of types (twice "high-low"). Keeping all other parameters fixed, this ratio roughly doubles from 0.089 when $\chi$ is at its period-1 level to 0.19 in period 2.

But this result seems to raise a puzzle: Why have equilibrium coworker mismatch costs not risen to a similar extent? The answer is that a rise in coworker sorting over time signifies a diminishing distance between factual and efficient matching distribution. Consider the estimated production function and the unconditional distribution of types across teams in period 2 but suppose, counterfactually, that each worker type's conditional coworker type distribution is as in period 1.[59] This experiment indicates that absent any worker reallocation, productivity in period 2 would have been 2.58% lower. The model thus highlights that the rise in coworker sorting and ensuing, widened gaps between firms in average wages need not reflect worsening frictions such as product market power.[60] In this model, they are equilibrium outcomes that limit productivity losses due to greater complementarity, in effect keeping the economy closer to the productivity frontier.

Indeed, a labor market that efficiently allocates workers of heterogeneous quality into teams is more critical for the economy to perform at its productive potential if worker-task specialization is more pronounced. Figure 11b illustrates this point. It plots the percentage gap between realized and efficient allocation in terms of labor productivity against different values of $\chi$. The solid line indicates the value implied by the baseline estimation, that is holding all parameters other than $\chi$ fixed at their period-2 levels. Consistent with the preceding analyses, the gap increases little for higher values of $\chi$. The dotted line indicates the gap when, instead, a random allocation into teams is imposed (i.e., the joint distribution is just the product of the marginals). We now observe that misallocation costs rise rapidly, as stronger coworker complementarities now induce a greater gap to potential productivity without a mitigating effect due to endogenously rising

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[59]Counterfactual average output per worker is $\frac{1}{2} \int f(x, x') d\Phi^{p1}(x'|x) d\Phi^{p2}(x)$. This approach avoids confounding changes in the unconditional distribution of quality types with changes in conditional matching patterns.

[60]The model does not incorporate product market frictions, nor other features that render increased coworker sorting inefficient. The model shows, though, that such features are not necessarily required to explain the empirically observed trends. Hence, any policy response needs to carefully evaluate the underlying mechanism(s).
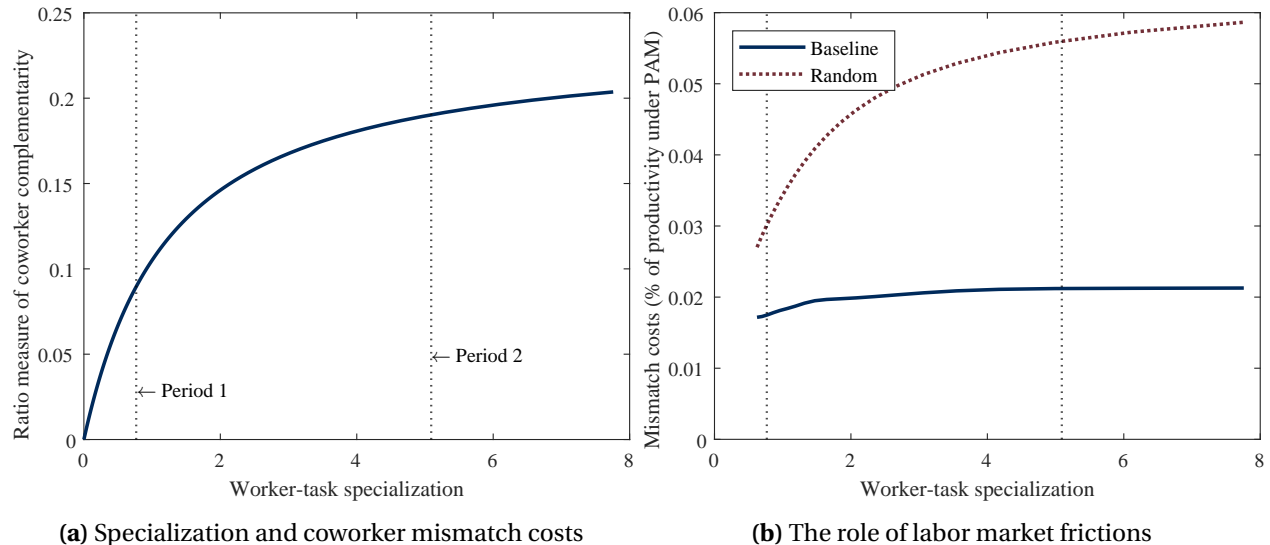
**(a)** Specialization and coworker mismatch costs      **(b)** The role of labor market frictions

**Figure 11:** Worker-task specialization, complementarity, and mismatch costs

*Notes.* This figure depicts the productivity implications of worker-task specialization ($\chi$), measured on the horizontal axis, through coworker complementarity. In the left-hand plot, the vertical axis indicates $\frac{f(x^{p80},x^{p80})+f(x^{p20},x^{p20})}{f(x^{p80},x^{p20})+f(x^{p80},x^{p20})} - 1$. The right hand panel plots, for any value of $\chi$, the cost of mismatch as a fraction of productivity under perfect sorting, under the realized allocation (solid line) and under random sorting (dotted line).

sorting. At $\chi^{p1}$, the gap is 3% (instead of 1.65% in the realized allocation), a value that rises to 5.42% at $\chi^{p2}$ (compared to 2.17%) . Simply put, the benefits accruing from specialization under the division of labor are limited not only by the extent of the market or the cost of coordination (Becker and Murphy, 1992), but also by frictions in the labor market.

## 4.4 Robustness exercises

The baseline analysis presents rising coworker complementarity as a quantitatively important driver of increased firm-level inequality. To probe the robustness of this finding, this section considers two alternative model specifications. Appendix C.2.1 reports the associated parameter estimates.

OUTSOURCING AND WITHIN-OCCUPATION WORKER RANKING. One concern is that the baseline measurement of complementarities and sorting may be confounded by outsourcing dynamics or related shifts in the boundary of firm. Indeed, Song *et al.* (2019, Section V.B.) hypothesize that outsourcing may be a factor behind increased between-firm inequality. To address this possibility, I consider an alternative specification of worker types, whereby workers are ranked within-occupation as opposed to economy-wide. An increase in sorting or complementarities estimated using this specification could not be driven by greater segregation of high- and low-wage occupations in different establishments. As discussed in Section 3, this empirical specification

does indeed yield slightly lower values of coworker complementarity in both periods, though the increase is similar to the baseline in proportional terms (see Table 1).

I re-estimate the model for both periods, targeting coworker complementarity moments based on within-occupation worker rankings, and repeat the counterfactual exercises. The second bloc of rows in Table 4 demonstrates that the important role of complementarities remains intact when controlling for changes in occupational composition across firms in this way. Indeed, the increase in complementarity is now found to explain 61% of the model-predicted increase in the between-employer share of wage inequality. This corresponds to 52% of the empirically observed rise, as this specification of the model predicts a somewhat larger increase in the overall between-share.

ON-THE-JOB SEARCH. Next, I study the implications of on-the-job search (OJS). There are two main reasons for incorporating OJS. First, E-E transitions represent an empirically salient feature of the labor market. Second, the relationship between OJS and coworker sorting is ex-ante ambiguous. On the one hand, when workers receive outside offers while employed, they will be less discriminating among offers they receive when unemployed, since accepting a job characterized by low match quality does not imply foregoing future job opportunities with better match quality. On the other hand, precisely this possibility of on-the-job switching could enhance coworker sorting.

I model OJS by supposing that employed workers receive outside offers from unmatched and one-worker firms at an exogenous Poisson rate $\lambda_e$. I furthermore assume that wages both off and on the job are continuously renegotiated under Nash bargaining, with unemployment serving as the outside option.[61] In the estimation, $\lambda_e$ is disciplined by EE rates taken from the matched employer-employee micro data. These indicate an increase in on-the-job mobility over time (the monthly rate is 0.0076 in period 1 and 0.0106 in period 2). I relegate details to Appendix C.2.2.

The estimated model with OJS generates a profile of coworker sorting that is remarkably close to what we observe in the data, including in terms of changes over time, as a comparison of Figures 4 and 10b reveals. Further, the quantitative model resolves that for any given degree of coworker complementarity, sorting is stronger with OJS than in its absence. This tells us that allowing for the possibility of climbing a (type-specific) "coworker ladder" is more than sufficient to offset any effect arising because sorting directly out of unemployment is now weaker. Meanwhile, the predicted increase in the between-share is smaller, reflecting a higher initial level of between-firm wage inequality. Turning to counterfactuals, when we repeat the previous exercises using the model with OJS, the rise in coworker complementarities explains about 30.5%

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[61]This wage specification is motivated by the empirical evidence in Di Addario *et al.* (2021), who document that where a worker is hired from tends to be relatively inconsequential for their wages in comparison to where they are currently employed, contrary to sequential auction models of labor market competition.

of the overall, model-predicted change; this contribution should be viewed as a lower bound, as Appendix C.2.2 explains. a similar amount is attributed to a reduction in search frictions, which now includes rising EE rates.

# 5   Conclusion

This paper developed, empirically tested, and quantified a model of the firm as a "team assembly technology." Conceptually, the model opens a small window into the black box of production, shedding light on the origins and macroeconomic implications of one cog in the machinery of firms: complementarity between heterogeneous coworkers' qualities. Through the development of a task-based microfoundation for coworker complementarity we could trace how changes in the nature of work alter who works with whom and shape the firm-level distribution of wages.

The analysis yielded three main insights. First, when production involves the division of labor among workers specialized in different tasks, their qualities are naturally complements; the more differentiated workers' task-specific skills are, the stronger is this complementarity. Second, in labor markets characterized by search frictions, coworker complementarity lowers equilibrium output below the efficient benchmark, and its strength determines between- and within-firm heterogeneity in worker talent and pay. Third, coworker complementarities have strengthened over time, paralleling an increase in task complexity, which accounts for a quantitatively significant part of the rise in the between-firm share of wage inequality.

The model also abstracts from a number of potentially important forces, addressing which may reveal interesting implications. First, the within-firm division of labor was assumed to be perfect, when in reality coordination frictions impose limits (Becker and Murphy, 1992). Indeed, some firms appear to be better in coordinating their workforce than others (Coraggio *et al.*, 2022; Kuhn *et al.*, 2022). An extension of the team production model incorporating this margin of coordination quality indicates that greater worker-task specialization turns this organizational capacity into a more significant determinant of productivity (Freund, 2023). This theoretical conjecture invites empirical and quantitative evaluation. Second, the model features a stylized institutional and policy environment. Future work could evaluate how coworker complementarities influence the desirability of policies that facilitate or restrict labor market reallocation. Third, the quantitative model cannot generate a plausible firm size distribution. Integrating firm dynamics, as in Bilal *et al.* (2022), with coworker interdependencies is a promising avenue toward a rich framework to study labor market allocation and wage dynamics.

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# Online Appendices

# A  Theory

## A.1  Team production

The derivations are lengthy but standard in the trade literature (e.g., Eaton *et al.*, 2016; Allen and Arko, 2019).

### A.1.1  Derivation of equation (7)

Start with equation (1), repeated here for convenience:

$$Y = \left( \int_0^1 q(\tau)^{\frac{\eta-1}{\eta}} d\tau \right)^{\frac{\eta}{\eta-1}},$$

multiply both sides by $\lambda^{\frac{1}{\eta-1}}$, and bring this term inside the integral on the left-hand side. Substituting for $\lambda^{\frac{1}{\eta-1}}$ on that left-hand side using (6), rearranged as

$$\lambda^{\frac{1}{\eta-1}} = \left( \frac{Q(\tau)}{Y} \right)^{\frac{1}{\eta-1}} \frac{\tilde{\lambda}(\tau)}{\lambda},$$

and simplifying, we obtain

$$\left( \int_0^1 \left( q(\tau)\tilde{\lambda} \right)^{\frac{\eta-1}{\eta}} dQ(\tau)^{\frac{1}{\eta}} \left( \frac{1}{Y} \right)^{\frac{1}{\eta-1}} \left( \frac{1}{\lambda} \right)^{\frac{\eta-1}{\eta}} d\tau \right)^{\frac{\eta}{\eta-1}} = Y \lambda^{\frac{1}{\eta-1}}. \tag{A.1}$$

Use $Q(\tau) = \tilde{\lambda}(\tau) q(\tau)$, bring the terms independent of $\tau$ outside the integral, and cancel exponents. Then

$$\int_{\mathcal{T}} Q(\tau) d\tau = \lambda Y. \tag{A.2}$$

### A.1.2  Proof of Lemma 1

PART (I). Step 1 is to derive the distribution of shadow costs of worker $i$ providing task $\tau$, $\lambda_i(\tau)$. Since the efficiency draws are independently and identically distributed, the probability of $i$ producing that task at a shadow price less than $p$ is the same for all $\tau \in \mathcal{T}$. Given the definition

$$G_i(p) := \Pr(\lambda_i(\tau) \le p), \tag{A.3}$$

the properties of the Fréchet distribution together with equation (10) imply

$$G_i(p) = \Pr\left\{ \frac{\lambda_i^L}{a_1 z_i(\tau)} \leq p \right\}$$

$$= \Pr\left\{ \frac{\lambda_i^L}{a_1 p} \leq z_i(\tau) \right\}$$

$$= 1 - \Pr\left\{ z_i(\tau) \leq \frac{\lambda_i^L}{a_1 p} \right\} \tag{A.4}$$

$$= 1 - \exp\left( -\left( \frac{\lambda_i^L}{\iota a_1 p x_i} \right)^{-\frac{1}{\chi}} \right). \tag{A.5}$$

For step 2, consider the probability that a task $\tau \in \mathcal{T}$ can be obtained for a shadow cost of less than $p$,

$$G(p) := \Pr\{\tilde{\lambda}(\tau) \leq p\}. \tag{A.6}$$

Using equation (9) and basic tools in probability:

$$G(p) = \Pr\left\{ \min_i \tilde{\lambda}_i(\tau) \leq p \right\}$$

$$= 1 - \Pr\left\{ \cap_{i \in \mathcal{S}} \left( \tilde{\lambda}_i(\tau) \geq p \right) \right\}$$

$$= 1 - \Pi_{i \in \mathcal{S}} \left( 1 - G_i(p) \right)$$

Intuitively, the lowest shadow price is weakly lower than $p$ unless the shadow price of producing that task is greater than $p$ for each worker, so that the distribution $G(p)$ is the complement of the probability that for every $i \in \mathcal{S}$ the shadow cost of providing the task is greater than $p$.

In step 3, substitute for $G_i(p)$ using equation (A.5)

$$G(p) = 1 - \Pi_{i \in \mathcal{S}} \exp\left( \left( \frac{\lambda_i^L}{\iota a_1 p x_i} \right)^{-\frac{1}{\chi}} \right)$$

$$= 1 - \exp\left( -(\iota p)^{\frac{1}{\chi}} \sum_{i \in \mathcal{S}} \left( \frac{\lambda_i^L}{a_1 x_i} \right)^{-\frac{1}{\chi}} \right) \tag{A.7}$$

$$= 1 - \exp(-(\iota p)^{\frac{1}{\chi}} \Phi), \tag{A.8}$$

where $\Phi = \sum_{i \in \mathcal{S}} \left( \frac{\lambda_i^L}{a_1 x_i} \right)^{-\frac{1}{\chi}}$

For the final step, consider equation (8), and substitute using equation (A.8):

$$\lambda^{1-\eta} = \int_0^\infty p^{1-\eta} dG(p)$$

$$= \int_0^\infty p^{1-\eta} \left( \frac{d}{dp} \left( 1 - \exp(-(\iota p)^{\frac{1}{\chi}} \Phi) \right) \right) dp,$$

$$= \frac{1}{\chi} \iota^{\frac{1}{\chi}} \Phi \int_0^\infty p^{\frac{1}{\chi} - \eta} \left( \exp(-(\iota p)^{\frac{1}{\chi}} \Phi) \right) dp.$$

Now use a change of variables, with $m = (\iota p)^{1/\chi} \Phi$, so that $p = \iota^{-1} \left( \frac{m}{\Phi} \right)^\chi$ and $dp = \iota^{-1} \chi \left( \frac{m}{\Phi} \right)^{1-\chi} \frac{1}{\Phi} dm$. Performing the integration by substitution,

$$\lambda^{1-\eta} = \frac{1}{\chi} \iota^{\frac{1}{\chi}} \Phi \int_0^\infty \left( \iota^{-1} \left( \frac{m}{\Phi} \right)^\chi \right)^{\frac{1}{\chi} - \eta} \exp(-m) \iota^{-1} \chi \left( \frac{m}{\Phi} \right)^{1-\chi} \frac{1}{\Phi} dm$$

$$= \iota^{\eta-1} \Phi^{-\chi(1-\eta)} \int_0^\infty m^{\chi(\eta-1)} \exp(-m) dm$$

$$= \iota^{\eta-1} \Phi^{-\chi(1-\eta)} \Gamma(1 + \chi(1-\eta))$$

where $\Gamma(\cdot)$ is the Gamma function, evaluated at the argument $1 + \chi(1-\eta)$. Convergence of this integral requires that $1 + \chi(1-\eta) > 0$. Intuitively, tasks may not be so substitutable that workers' time is concentrated on tasks that takes them close to zero time to perform, in which case final good production could be unbounded (cf. Alvarez and Lucas, 2007, Footnote 3). Finally, since $\iota = \Gamma(1 + \chi - \eta\chi)^{\frac{1}{1-\eta}}$, we obtain

$$\lambda = (\Phi_i)^{-\chi} = \left( \sum_{i \in \mathcal{S}} \left( \frac{a_1 x_i}{\lambda_i^L} \right)^{1/\chi} \right)^{-\chi},$$

as stated in equation (12) in the main text.

PART (II). What is the probability that worker $i$ has the lowest shadow cost of providing a task $\tau \in \mathcal{T}$? Because productivity draws are iid, and since tasks are on a continuum, by the law of large numbers this probability will be equal to the fraction of tasks that $i$ produces according to

the optimal plan. Define this share as

$$
\pi_i := \Pr\left\{\tilde{\lambda}_i(\tau) \leq \min_{k \in \mathcal{S} \backslash i} \tilde{\lambda}_k(\tau)\right\}
$$

$$
= \Pr\left\{\min_{k \in \mathcal{S} \backslash i} \geq p\right\} dG_i(p)
$$

$$
= \int_0^\infty \Pr\left\{\cap_{k \in \mathcal{S} \backslash i} \tilde{\lambda}_k(\tau) \geq p\right\} dG_i(p)
$$

$$
= \int_0^\infty \Pi_{k \in \mathcal{S} \backslash i}(1 - G_k(p)) dG_i(p)
$$

Now substitute for the distribution of shadow costs:

$$
\pi_i = \int_0^\infty \left[1 - \exp\left(-\left(\frac{\lambda_k^L}{\iota a_1 p x_k}\right)^{\frac{1}{\chi}}\right)\right] dp \left[\frac{d}{dp}\left(\left[1 - \exp\left(-\left(\frac{\lambda_i^L}{\iota a_1 p x_i}\right)^{-\frac{1}{\chi}}\right)\right]\right)\right]
$$

$$
= \left(\frac{\lambda_i^L}{\iota a_1 x_i}\right)^{-\frac{1}{\chi}} \int_0^\infty \left(\frac{1}{\chi} \iota^{\frac{1}{\chi}} p^{\frac{1}{\chi}-1}\right) \left(\exp(-(p\iota)^{\frac{1}{\chi}} \Phi_n)\right) dp
$$

$$
= \frac{(a_1 x_i)^{\frac{1}{\chi}} \left(\lambda_i^L\right)^{-\frac{1}{\chi}}}{\Phi_n} \left[-\exp(-(\iota p)^{\frac{1}{\chi}} \Phi_n)\right]_0^\infty
$$

$$
= \frac{(a_1 x_i)^{\frac{1}{\chi}} \left(\lambda_i^L\right)^{-\frac{1}{\chi}}}{\Phi_n}.
$$

Lastly, we need to show that $\pi_i$ is not only the fraction of *tasks* that $i$ produces; it is also the fraction of the *value* of tasks. This result obtains because under the Fréchet distribution, the distribution of shadow prices of tasks that worker $n$ actually uses from any team member will be the same. Here is a proof, adapted from Allen and Arko (2019). The probability that the shadow cost of $i$ providing a task $\tau$ is lower than $\tilde{p}$, conditional on that cost being the lowest for $i$ among all team members, is

$$
\Pr\left\{\tilde{\lambda}_i(\tau) \leq \tilde{p} | \tilde{\lambda}_i(\tau) \leq \min_{k \in \mathcal{S} \backslash i} \tilde{\lambda}_k(\tau)\right\} = \frac{1}{\pi_i} \int_0^{\tilde{p}} \Pr\left\{\min_{k \in \mathcal{S} \backslash i} p_k(\tau) \geq p\right\} dG_i(p),
$$

were the first term on the right-hand side is the inverse probability that $i$ has the lowest shadow cost of producing a given task, and the second term is the probability that the firm can be provided with a a task at a shadow cost lower than $\tilde{p}$ by a team member other than $i$.

Then using the same logic as in the derivation of $\pi_i$, we find that this is equal to

$$\Pr\left\{\tilde{\lambda}_i(\tau) \leq \tilde{p} \big| \tilde{\lambda}_i(\tau) \leq \min_{k \in \mathcal{S}\setminus i} \tilde{\lambda}_k(\tau)\right\} = \int_0^{\tilde{p}} \Pi_{k \in \mathcal{S}\setminus i}\left(1 - G_k(p)\right) dG_i(p),$$

$$= \frac{1}{\pi_i} \frac{(a_1 x_i)^{\frac{1}{\chi}} \left(\lambda_i^L\right)^{-\frac{1}{\chi}}}{\Phi}\left[-\exp(-(\iota p)^{\frac{1}{\chi}}\Phi)\right]_0^{\tilde{p}}$$

$$= \frac{1}{\pi_i} \frac{(a_1 x_i)^{\frac{1}{\chi}} \left(\lambda_i^L\right)^{-\frac{1}{\chi}}}{\Phi}\left[\left[-\exp(-(\iota\tilde{p})^{\frac{1}{\chi}}\Phi) - (-\exp(0))\right]\right]$$

$$= \frac{1}{\pi_i}\pi_i\left(1 - \exp(-(\iota\tilde{p})^{\frac{1}{\chi}}\Phi)\right)$$

$$= G(\tilde{p}),$$

which is independent of $i$.

Intuitively, the planner makes team members with an absolute advantage provide a greater range of tasks exactly up to the point where the distribution of shadow costs associated with producing tasks is the same as the overall distribution of shadow costs.

PART (III). The final result immediately follows since

$$Q_i = \pi_i\left(\int_{\mathcal{T}} \tilde{\lambda}(\tau)q(\tau)d\tau\right) = \pi_i \int_{\mathcal{T}} Q(\tau)d\tau = \pi_i Q.$$

### A.1.3  Proposition 1

When $\lambda = 1$, then part (i) of Lemma 1 implies that

$$1 = \sum_{i \in \mathcal{S}} \left(\frac{a_1 x_i}{\lambda_i^L}\right)^{\frac{1}{\chi}}, \tag{A.9}$$

Hence, from part (ii) of Lemma 1,

$$\pi_i = ((a_1 x_i)/\lambda_i^L)^{\frac{1}{\chi}}.$$

Next, if $y_i(\tau) > 0$, then after some straightforward manipulation the FOC w.r.t. $l_i(\tau)$ reads

$$\lambda_i(\tau)\frac{y_i(\tau)}{l_i(\tau)} = \lambda_i^L.$$

Integrating and using the time constraint (3) implies that

$$\int_{\mathcal{T}} \lambda_i(\tau) q_i(\tau) = \lambda_i^L.$$

Since $\tilde{\lambda}(\tau) = \lambda_i(\tau)$ when $y_i(\tau) > 0$, this also means that

$$Q_i = \lambda_i^L,$$

which says that the shadow value of worker $i$'s time is equal to the shadow value of all tasks produced by that worker. Hence, from part (iii) of Lemma 1, and given the normalization $\lambda = 1$,

$$\lambda_i^L = \pi_i Q,$$
$$\Leftrightarrow \lambda_i^L = \pi_i Y,$$
$$\Leftrightarrow \left( \frac{a_1 x_i}{\lambda_i^L} \right)^{\frac{1}{\chi}} = \lambda_i^L / Y.$$

Substituting this expression into equation (A.9) and rearranging for $Y$ yields

$$Y = n^{1+\chi} \left( \frac{1}{n} \sum_{i=1}^{n} (a_1 x_i)^{\frac{1}{1+\chi}} \right)^{1+\chi}$$

## A.2  Team hiring

### A.2.1  Population dynamics

For any type $x$, the measure of unemployment satisfies

$$\delta \left( d_{m.1}(x) + \int d_{m.2}(x, \tilde{x}') d\tilde{x}' \right) = d_u(x) \lambda_u \left( \int \frac{d_{f.0}}{v} h(x, \tilde{y}) + \int \frac{d_{m.2}(\tilde{x}')}{v} h(x|\tilde{x}') d\tilde{x}' \right). \tag{A.10}$$

The measure of exogenously separated workers of any type is equal to the measure of unemployed workers of that type finding new employment at either one-worker or two-worker firms.

For all $x$, the measure of one-worker matches follows

$$d_{m.1}(x) \left( \delta + \lambda_{v.u} \int \frac{d_u(\tilde{x}')}{u} h(\tilde{x}'|x) d\tilde{x}' \right) = d_u(x) \lambda_u \frac{d_{f.0}}{v} h(x) + \delta \int d_{m.2}(x, \tilde{x}') d\tilde{x}'. \tag{A.11}$$

Outflows from this state occur due to exogenous separation or because the one-worker firm meets and decides to hire a coworker of some type. Inflows occur when an unemployed worker

of type $x$ meets and gets hired by an unmatched firm *or* because a two-worker firm that has a type $x$ as one of its employees loses the coworker.

Finally, for all $(x, x')$,[A.1]

$$2\delta d_{m.2}(x, x') = d_u(x)\lambda_u \frac{d_{m.1}(x')}{v} h(x|x') + d_u(x')\lambda_u \frac{d_{m.1}(x)}{v} h(x'|x). \tag{A.12}$$

The economic intuition parallels the aforementioned reasoning.

## A.2.2 Derivation of surplus equations

### A.2.2.1 $S(x)$.
We start with the definition of $S(x)$, repeated here for convenience:

$$S(x) = \Omega_1(x) - V_u(x) - V_{f.0} \tag{A.13}$$

Consider equation (28). The term in $[\cdot]$ obviously corresponds to $S(x)$. Rearranging the surplus sharing equations furthermore implies that

$$\big(-\Omega_1(x) + V_{e.2}(x|x') + V_{f.1}(x, x')\big) = (1 - \omega)S(x'|x)$$

In words, the joint value of firm and $x$ from being with another worker $x'$ minus their joint outside option is equal to $(1 - \omega)$ of the surplus of adding $x'$.

Hence, we can write

$$\rho\Omega_1(x) = f_1(x) - \delta S(x) + \lambda_{v.u}(1 - \omega)\int \frac{d_u(\tilde{x}')}{u} S(\tilde{x}'|x)^+ d\tilde{x}'$$

Substituting this expression into equation (A.13) multiplied by $\rho$ yields

$$\big(\rho + \delta\big)S(x) = f_1(x) - \rho\big(V_u(x) + V_{f.0}\big) + \lambda_{v.u}(1 - \omega)\int \frac{d_u(\tilde{x}')}{u} S(\tilde{x}'|x)^+ d\tilde{x}'. \tag{A.14}$$

### A.2.2.2 $S(x|x')$.
The first equation we need is a relationship between the surplus and the total joint value. Start with the definition of $S(x|x')$, substitute using the sharing rule (20), and

---

[A.1] From an accounting perspective, one of the preceding three equations is redundant given because the adding-up constraint represented by equation (17) must hold and the distribution of worker types is exogenous. Similarly, $d_v(y)$ can be backed out from equation (18) given $d_f(y)$.

simplify:

$$S(x|x') = \Omega_2(x, x') - \left(V_{e.1}(x') + V_{f.1}(x')\right) - V_u(x)$$

$$= \Omega_2(x, x') - \left(V_u(x') + \omega S(x') + V_{f.0} + (1 - \omega)S(x')\right) - V_u(x)$$

$$= \Omega_2(x, x') - S(x') - \left(V_u(x) + V_u(x') + V_{f.0}\right)$$

Hence,

$$\Omega_2(x, x') = S(x|x') + S(x') + V_u(x) + V_u(x') + V_{f.0}. \tag{A.15}$$

and analogously

$$S(x'|x) = \Omega_2(x, x') - \left(S(x) + V_u(x) + V_u(x') + V_{f.0}\right). \tag{A.16}$$

Second, consider the HJB for $\Omega_2(x, x')$, equation (27). Substitute for the the terms in brackets using the definitions of surplus to get a second expression for surplus:

$$\rho\Omega_2(x, x') = f_2(x, x') + \delta\left[-S(x|x') - S(x'|x)\right]$$

$$\Leftrightarrow \delta S(x|x') = f_2(x, x') - \rho\Omega_2(x, x') - \delta S(x'|x)$$

Substituting for $S(x'|x)$ using equation (A.16) as well as for $\Omega_2(x, x')$ using equation (A.15), and collecting terms yields

$$\delta S(x|x') = f_2(x, x') - \rho\Omega_2(x, x') - \delta\left[\Omega_2(x, x') - \left(S(x) + V_u(x) + V_u(x') + V_{f.0}\right)\right]$$

$$= f_2(x, x') - (\rho + \delta)\Omega_2(x, x') + \delta\left[S(x) + V_u(x) + V_u(x') + V_{f.0}\right]$$

$$= f_2(x, x') - (\rho + \delta)\left(S(x|x') + S(x') + V_u(x) + V_u(x') + V_{f.0}\right)$$

$$+ \delta\left[S(x) + V_u(x) + V_u(x') + V_{f.0}\right]$$

Simplifying yields

$$S(x|x')(\rho + 2\delta) = f_2(x, x') - \rho\left(V_u(x) + V_u(x') + V_{f.0}\right) + \delta S(x) - (\rho + \delta)S(x'). \tag{A.17}$$

### A.2.3 Derivation of the wage function

The value of employment for worker $x$ when their coworker is of type $x'$ is

$$\rho V_{e.2}(x|x') = w(x|x') - 2\delta\omega S(x|x') + \delta\omega S(x).$$

Combining with the surplus sharing rule (24) yields

$$w(x|x') = \rho V_u(x) + (\rho + 2\delta)\omega S(x|x') - \delta\omega S(x) \tag{A.18}$$

We can simplify this expression further. First, substitute for $(\rho + 2\delta)S(x|x')$ from equation (A.17) to get

$$w(x|x') = \rho V_u(x) + \omega\left[f_2(x, x') - \rho\big(V_u(x) + V_u(x') + V_{f.0}\big) + \delta S(x) - (\rho + \delta)S(x')\right] - \delta\omega S(x),$$

$$= \omega f_2(x, x') + (1-\omega)\rho V_u(x) - \omega\rho\big(V_u(x') + V_{f.0}\big) - \omega(\rho + \delta)S(x'),$$

Next, substitute for $S(x')$ from equation (A.14) to obtain

$$w(x|x') = \omega f_2(x, x') + (1-\omega)\rho V_u(x) - \omega\rho\big(V_u(x') + V_{f.0}\big)$$
$$- \omega\left[f_1(x') - \rho\big(V_u(x') + V_{f.0}\big) + \lambda_{v.u}(1-\omega)\int \frac{d_u(\tilde{x}'')}{U}S(\tilde{x}''|x')^+ d\tilde{x}''\right]$$
$$= \omega\big(f_2(x, x') - f_1(x')\big) + (1-\omega)\rho V_u(x) - \omega(1-\omega)\lambda_{v.u}\int \frac{d_u(\tilde{x}'')}{U}S(\tilde{x}''|x')^+ d\tilde{x}''.$$
$$\tag{A.19}$$

### A.2.4 Monotonicity Lemma

The following lemma implies that Hagedorn *et al.*'s (2017) non-parametric ranking algorithm extends to the present environment.

**Lemma 2.** *Assume that $\frac{f_1(x)}{\partial x} > 0$, $\frac{\partial f_2(x,x')}{\partial x} > 0$, and $\omega > 0$. Then: (i) The value of unemployment $V_u(x)$ is monotonically increasing in $x$, and (ii) so is the wage function $w(x, (y, x'))$.*

The proof is lengthy but straightforward, involving substitution and differentiation of the surplus equations. It was included in a working paper version but is omitted here for sake of brevity.

## A.3 Closed-form results for a stylized version

The full, quantitative model does not admit closed-form characterization of matching decisions and the distribution of wages. The issue lies with the value of either side's outside option (waiting) which at once informs the matching decision and at the same time is endogenous to the matching decisions taken by all agents in the economy. After all, it is those decisions that pin down who is available for meetings if you do wait. This general-equilibrium interplay between individual decisions and distributional dynamics is characteristic of the class of mean-field games to which the present model belongs. To solve for matching decisions and distributions by hand, then, we need to sever their mutual interdependence. To do so, I adapt the model of Eeckhout and Kircher (2011).[A.2] I focus on matching of pairs of workers into a team, each managed by a firm, rather than matching between one worker and one firm. I show how to derive closed-form expressions for the degree of coworker sorting and firm-level inequality.

### A.3.1 Environment

As in the main model, there is unit mass of workers, with types uniformly distributed over $[0, 1]$. There is exactly half as many firms which, as in the main text, are ex-ante homogeneous with productivity normalized to unity.

Production with a single worker is normalized to be zero and $f_2(x, x') = x + x' - \gamma(x - x')^2$, where $\gamma > 0$ again indicates the strength of complementarities. This specification of team production is motivated by a second-order Taylor approximation to the micro-founded CES function in Proposition 1 around the mean worker type, i.e., around $\frac{x+x'}{2}$. Then $(x - x')^2$ is proportional to the variance of types and $\gamma$ measures the production loss due to coworker mismatch.

Consider a finite-horizon setup in which there are three stages $s \in \{0, 1, 2\}$. All the focus will be on the outcomes in $s = 1$; the other two stages merely serve to frame the decision problem in $s = 1$. Everyone starts out unmatched. In $s = 0$, each firm meets and matches with one worker. Our focus is on the following matching decision, to be taken in $s = 1$. Each worker-firm pair coming into $s = 1$, of which there is a mass $\frac{1}{2}$, is randomly paired with one of the remaining unmatched workers. *Either* they match, produce, and share the output. The agents are then idle in $s = 2$. *Or* they decide to *wait*, in which case all agents separate, each worker pays a *fixed search cost $c$*, and they can be active in stage $s = 2$. Specifically, in return for paying the search cost,

---

each worker in the final stage is paired with their optimal match. Finally, payoffs are determined according to the following, simplified protocol. Firms have no bargaining power and each worker receives their outside option plus half the surplus generated by the match.

Relative to the full model, we have complete destruction of matches after the production stage ($\delta = 1$); every firm is guaranteed to meet one worker in every period; and there is zero discounting. As in Atakan (2006), the search cost is *explicit* instead and, as such, *type-independent*. This contrasts with the full model, where search costs are *implicit*. In that case, more productive agents have greater opportunity costs of time due to discounting, hence search frictions disproportionately erode their value of search (Sandmann and Bonneton, 2022).

### A.3.2 Solving the model by backward induction

FRICTIONLESS MATCHING IN $s = 2$. Matching in the second stage is frictionless. As described e.g. in Boerma *et al.* (2021), the problem of a firm is to choose workers $x$ and $x'$ to maximize profits, taking the wage schedule for workers, $w : [0, 1] \rightarrow \mathbb{R}_+$, as given:

$$v = \max_{x,x'} \left( f_2(x, x') - w(x) - w(x') \right).$$

Any worker $x$ chooses an employer (among which they are indifferent) and coworker $x'$ to maximize their wage income, taking as given the firm value, $v$, and the coworker's wage schedule:

$$w(x) = \max_{x'} \left( f_2(x, x') - w(x') - v \right).$$

An equilibrium is a tuple $(\pi, w, v)$, with probability measure $\pi$ over workers and coworkers indicating the assignment, such that firms solve their profit maximization problem, each worker solves their respective problem, and payoffs are feasible with respect to production. The firm's first-order condition for a worker is $\frac{\partial f_2(x,x')}{\partial x} - \frac{\partial w(x)}{\partial x} = 0$. From the second-order conditions, per Becker (1973), we know that when the production function is supermodular in worker types, i.e., when $\gamma > 0$, then the optimal assignment among workers features positive assortative matching between coworkers. That is, we have a deterministic coupling, or matching function, $\mu : [0, 1] \rightarrow [0, 1]$ and, specifically, $\mu^*(x) = x$. The wage schedule is then derived by integrating over the first-order condition under the equilibrium assignment. Under our parametric assumptions on the production function and the bargaining process, this yields $w^*(x) = x$ for any $x$ and $v = 0$.

STAGE-1 MATCHING DECISION. Each firm with one worker, the latter being denoted $x'$, is randomly matched with an unmatched worker whose type is $x$. If they match, the team produces and the output value is shares. Then they are idle in the final stage. Else, the partners are all

unmatched, each worker pays a fixed search cost $c$, and all agents actively participate in the stage-2 matching process.

A firm with worker $x'$ that is randomly matched a worker $x$ decides to hire them if the joint value of production is (weakly) greater than the sum of the respective outside options, taking into account the cost of search.[A.3] Using the same notation as in the full model, this means that $h(x|x') = 1 \Leftrightarrow S(x|x') > 0$, where[A.4]

$$S(x|x') = f_2(x, x') - \left[ w^*(x) + w^*(x') + v^* - 2c \right].$$

Substituting for the last-period payoffs, i.e., $w^*(x) = x$ and $v^*(y) = 0$, and simplifying shows that a match is formed whenever $|x' - x| > s^*$, where the equilibrium threshold $s^*$ satisfies

$$s^* = \sqrt{2c/\gamma}. \tag{A.20}$$

Equivalently, the (symmetric) matching set is $M(x') = \{x \in \mathcal{X} : x' - s^* > x < x' + s^*\}$.[A.5]

It immediately follows that greater complementarities, corresponding to a higher value of $\gamma$, render the matching set narrower, whereas greater search costs, $c$, render the matching set wider.

STAGE-0 MATCHING DECISION. At the very beginning, each firm is randomly paired with one worker. It is optimal to match, since the opportunity cost of doing so is zero.[A.6]

### A.3.3 Characterization

Next, we can characterize the sorting patterns and wage-distributional outcomes implied by this matching rule in closed-form. To ensure maximum comparability with the full model, I take the following approach: I focus on the outcomes for workers that are part of matched formed in stage 1; and I re-weight these workers according to the (uniform) population distribution. That is, the marginal distribution of worker types participating in stage-1 matches is taken to be uniform. In this way, the results are not biased by the fact that stage-2 outcomes exhibit no mismatch by construction; nor is the workforce composition mechanically biased towards workers with intermediate quality levels who accept a wider range of partners than those at the tails of the

---

[A.3]I assume that when the parties are indifferent between producing now and waiting they opt for the former. This assumption does not, of course, affect the substantive results.

[A.4]As discussed in Footnote 7 of Eeckhout and Kircher (2011), for low types the surplus in the next period may not exceed the total waiting cost. In order to avoid keeping track of endogenous entry, I assume that people will search even if that is the case.

[A.5]This simple formulation also emerges as a special case in Eeckhout and Kircher (2011). The result also resembles circular production models à la Marimon and Zilibotti (1999).

[A.6]To resolve the indifference case, we could of course assume an infinitesimally small positive production value.

quality distribution .[A.7]

MATCHING PATTERNS. The key step affording analytical tractability is to observe that for any threshold level $s$, the distribution of coworker types conditional on type $x$, denoted $\Phi(x'|x)$, takes the form of a piecewise uniform distribution:

**Lemma 3** (Conditional type distribution). *Given a threshold distance $s$, the conditional distribution of coworkers for $x \in X$ is*

$$\Phi(x'|x) = \begin{cases} 0 & \textit{for } x' < \sup\{0, x - s\} \\ \frac{x - \sup\{0, x-s\}}{\inf\{x+s, 1\} - \sup\{0, x+s\}} & \textit{for } x' \in [\sup\{0, x - s, \}, \inf\{x + s, 1\}] \\ 1 & \textit{for } x' > \inf\{x + s, 1\} \end{cases}$$

*Proof.* The distribution of coworkers for a worker of type $x$ is

$$\Phi(x'|x) = \frac{\int_X \mathbf{1}\{\tilde{x} \in M(x)\} d\Phi(\tilde{x})}{\Phi(\overline{m}(x)) - \Phi(\underline{m}(x))}$$

where $\overline{m}(x)) = \sup M(x)$ and $\underline{m}(x)) = \inf M(x)$; given the assumption of uniform weights on types among the matched $\Phi(x)$ is the uniform cdf.

Assuming that potential coworkers are uniformly distributed, the equilibrium matching rule then implies that

$$x' \sim \begin{cases} U[0, x + s] & \text{for } x \in [0, s) \\ U[x - s, x + s] & \text{for } x \in [s, 1 - s] \\ U[x - s, 1] & \text{for } x \in [1 - s, 1] \end{cases}$$

Writing this out more carefully for three different segments, for $x \in [0, s)$:

$$\Phi(x'|x) = \begin{cases} 0 & \text{for} & x' = 0 \\ \frac{x'}{x+s} & \text{for} & x' \in [0, x + s) \\ 1 & \text{for} & x' > x + s. \end{cases}$$

---

[A.7]Concretely, since the matching set is $M(x) = \{x' \in X : x - s^* < x > x + s^*\}$, for any $s^* < 1$, workers outside the interval $[s^*, 1 - s^*]$ would be under-represented relative to those inside that interval.

Next, for $x \in [s, 1-s]$:

$$\Phi(x'|x) = \begin{cases} 0 & \text{for } x' < x - s \\ \frac{x'-(x-s)}{1-(x+s)} & \text{for } x' \in [x-s, x+s] \\ 1 & \text{for } x' > x+s. \end{cases}$$

Finally, for $x \in (1-s, 1]$:

$$\Phi(x'|x) = \begin{cases} 0 & \text{for } x' < x - s \\ \frac{x'-(x-s)}{1-(x-s)} & \text{for } x' \in (x-s, 1] \\ 1 & \text{for } x' = 1. \end{cases}$$

$\square$

Using Lemma 3, we can describe coworker sorting patterns in closed-form as follows:

**Proposition A.1** (Coworker sorting). *For a given threshold $s$,*

(i) *the coworker correlation is given by*

$$\rho_{xx} = (2s+1)(s^2-1)^2;$$

(ii) *the average coworker type is*

$$\hat{\mu}(x) = \hat{\mu}(x) = \begin{cases} \frac{x+s^*}{2} & \text{for} & x \in [0, s^*) \\ x & \text{for} & x \in [s^*, 1-s^*] \\ \frac{1+x-s^*}{2} & \text{for} & x \in (1-s^*, 1]. \end{cases}$$

*Proof.* Part (ii) immediately follows from 3. For part (ii), notice first that under the uniform distribution, the mean type is $\bar{x} = \frac{1}{2}$ and the variance is $\frac{1}{12}$. We need to derive the covariance term, which is defined as

$$\bar{c}_{xx} = \int_X \int_X (x-\bar{x})(x'-\bar{x}) d\Phi(x'|x) d\Phi(x).$$

**Step 1.** Substituting for the piece-wise conditional match distribution and the uniform

population weights, for a threshold $s$,

$$
\begin{aligned}
\bar{c}_{xx} &= \int_0^s \int_0^{x+s} (x - \tfrac{1}{2})(x' - \tfrac{1}{2}) \frac{1}{x+s} dx' dx \\
&+ \int_s^{1-s} \int_{x-s}^{x+s} (x - \tfrac{1}{2})(x' - \tfrac{1}{2}) \frac{1}{2s} dx' dx \\
&+ \int_{1-s}^1 \int_{x-s}^1 (x - \tfrac{1}{2})(x' - \tfrac{1}{2}) \frac{1}{1 - (x - s)} dx' dx
\end{aligned}
$$

**Step 2.** Integrate over $x'$.

The different components are:

$$
\int_0^{x+s} (x - \tfrac{1}{2})(x' - \tfrac{1}{2}) \frac{1}{x+s} dx' = \frac{1}{2}(x - \bar{x})(s + x - 2\bar{x})
$$

$$
\int_{x-s}^{x+s} (x - \tfrac{1}{2})(x' - \tfrac{1}{2}) \frac{1}{2s} dx' = (x - \bar{x})^2
$$

$$
\int_{x-s}^1 (x - \tfrac{1}{2})(x' - \tfrac{1}{2}) \frac{1}{1 - (x - s)} dx' = \frac{1}{2}(x - \bar{x})(1 - s + x - \bar{x})
$$

**Step 3.** Integrate over $x$.

$$
\begin{aligned}
\bar{c}_{xx} &= \int_0^s \frac{1}{2}(x - \bar{x})(s + x - 2\bar{x}) dx + \int_s^{1-s} (x - \bar{x})^2 dx + \int_{1-s}^1 \frac{1}{2}(x - \bar{x})(1 - s + x - \bar{x}) dx \\
&= \left[ \frac{5}{12} s^3 - \frac{5}{8} s^2 + \frac{1}{4} s \right] + \frac{(2s - 1)^3}{12} + \left[ \frac{5}{12} s^3 - \frac{5}{8} s^2 + \frac{1}{4} s \right] \\
&= \frac{1}{12} \left[ (2s + 1)(s - 1)^2 \right]
\end{aligned}
$$

Hence, dividing by the variance, the *correlation* is

$$
\rho_{xx} = (2s + 1)(s^2 - 1)^2.
$$

□

Part (i) of Proposition A.1, together with equation (A.20) summarizing the equilibrium matching decision, demonstrates that the coworker correlation coefficient, $\rho_{xx}$ is *increasing* in $\gamma$ – stronger complementarities give rise to more pronounced coworker sorting. In particular, $\rho_{xx} \to 1$ as $\gamma \to \infty$. Conversely, greater search costs dilute the incentive to wait for the best match and accordingly lower the degree of sorting observed in the economy. This result therefore sharply summarizes the tradeoff between complementarities and search costs in determining

coworker sorting patterns in the economy.[A.8]

Part (ii) of Proposition A.1 paints a more disaggregated picture of matching patterns that reveals *non-linearities* arising from *asymmetries* in the matching set. It defines for every worker type the average coworker type for a given threshold level $s$. Figure 2 in the main text graphically illustrates. The dashed-dotted and dotted lines describe matching patterns under, respectively, the deterministic coupling $\mu(x) = x$ that captures matching decisions under PAM in the friction-less economy (dashed 45-degree line); and the independent coupling that intuitively corresponds to a random matching process (dotted, horizontal line). The solid line describes $\hat{\mu}(x)$ for a low value of $\gamma$ and the dashed line illustrates $\hat{\mu}(x)$ for a higher value of $\gamma$.

We can make three observations. First, for "middle types" such as $x = \frac{1}{2}$, the average coworker type is invariant to changes in $\gamma$. Intuitively, stronger complementarities mean that such an agent is less likely to be in a team with a much better agents but the likelihood of being teamed up with a much worse agent shrinks in symmetric fashion. Second, the presence of search costs means that low types are, on average, paired up with agents better than them, whereas high types are typically paired up with coworkers *worse* than them. Lastly, a strengthening of complementarities *increases* the average coworker type for the *best*; and it *lowers* it for the worst. This force has thus the potential to engender polarizing dynamics wherein firms with "superstar teams" pull away while "laggards" fall behind.

WAGE DISTRIBUTION.

The production function, matching patterns, and wage sharing rule jointly determine the distribution of wages in the economy. Here we are particularly interested in the fraction of wage dispersion that occurs *between* firms. A tedious but otherwise straightforward sequence of integration and algebra steps yields the following result.

**Proposition A.2.** *Given a threshold distance $s$ and a value of $\gamma$, the between-firm share of the variance of wages is equal to*

$$\frac{-\frac{13\,\gamma^2\,s^5}{2400} + \frac{\gamma^2\,s^4}{80} + \frac{5\,s^3}{36} - \frac{s^2}{6} + \frac{1}{12}}{\frac{\gamma^2\,s^4}{45} - \frac{4897\,\gamma^2\,s^5}{10800} - \frac{\gamma^2\,s^6}{324} + \frac{19\,\gamma^2\,s^5\,\ln(2)}{30} + \frac{1}{12}}.$$

*Proof.* To find the between-team share of the variance of wages, first compute the total wage dispersion as the sum of the variance of wages between and within *types*. Then compute the variance of the average wage by team, and the between-share is the ratio of the latter over the

---

[A.8]In practice, the worker types $x$ are not observable, of course. Section **??** below defines an observable measure of types following Borovičková and Shimer (2020), denoted $\lambda(x)$. The corresponding coworker sorting correlation coefficient is virtually indistinguishable from $\rho_{xx}$ (in population, that is). It is also characterizable in closed form but less intuitively, being equal to $\rho_{\lambda\lambda} = \frac{10\gamma^2 s^6 - 3\gamma^2 s^5 + 3240 s^3 - 4860 s^2 + 1620}{-60\gamma^2 s^6 + 36\gamma^2 s^5 + 1620}$.

former. Many of the steps are straightforward but lengthy, following similar steps as in the preceding section, hence I only sketch the procedures and intermediate results.

**Total wage dispersion.** The average wage can be computed as

$$\bar{\lambda} = \int_0^s \lambda^b(x)dx + \int_s^{1-s} \lambda^m(x)dx + \int_{1-s}^1 \lambda^h(x)dx$$

where

$$\lambda(x) = \int_0^1 w(x, x')d\Phi(x'|x).$$

Performing the relevant integration steps, utilizing Lemma 3, and simplifying yields

$$\lambda(x) = \begin{cases} \lambda^b(x) := x - \frac{\gamma}{6}\left[(x-s)^2 + xs\right] & \text{for } x \in [0, s) \\ \lambda^m(x) := x - \frac{\gamma}{6}s^2 & \text{for } x \in [s, 1-s] \\ \lambda^h(x) := x - \frac{\gamma}{6}\left[1 + s - 2x - s^2 - x^2 - sx\right] & \text{for } x \in (1-s, 1]. \end{cases}$$

After some further integration and simplification, we obtain

$$\bar{\lambda} = \frac{1}{2} - \frac{1}{6}\gamma s^2 (1 - \frac{s}{3})$$

The between-type variance is equal to

$$\bar{\sigma}^2_{\text{between-type}} = \int_0^s \left(\lambda^b(x) - \bar{\lambda}\right)^2 dx + \int_s^{1-s} \left(\lambda^m(x) - \bar{\lambda}\right)^2 dx + \int_{1-s}^1 \left(\bar{\lambda}^h(x) - \bar{\lambda}\right)^2 dx,$$

which corresponds to the contribution of worker heterogeneity to total wage inequality.

After more integration and algebra, we find that this is equal to

$$\bar{\sigma}^2_\lambda = \frac{1}{12} - \frac{\gamma^2 s^5}{12}\left(\frac{s}{27} - \frac{1}{45}\right)$$

Conditional on a type $x$, the variance of wages is equal to

$$\sigma^2_{\lambda(x)} = \int_0^1 \left(w(x|x') - \lambda(x)\right)^2 d\Phi(x'|x).$$

Calculating this expression for the three segments of worker types – bottom, middle, and high –

we then obtain the average within-type wage variance as

$$\sigma^2_{\text{within }\lambda} = \int_0^s \sigma^2_{\lambda^b(x)} dx + \int_s^{1-s} \sigma^2_{\lambda^m(x)} dx + \int_{1-s}^1 \sigma^2_{\lambda^h(x)} dx.$$

Performing the usual piece-wise integration yields

$$\sigma^2_{\text{within-type}} = \frac{\gamma^2 s^4 \left(2280\, s\, \ln(2) - 1639\, s + 80\right)}{3600},$$

and, hence, the total wage variance is equal to

$$\sigma^2_w = \frac{1}{12} + \frac{\gamma^2 s^4}{45} - \frac{4897\, \gamma^2 s^5}{10800} - \frac{\gamma^2 s^6}{324} + \frac{19\, \gamma^2 s^5 \ln(2)}{30}.$$

**Between-share of wage inequality.** Finally, we compute the variance of the average wage in a firm, which given our assumptions on the firm earning zero return just corresponds to output per worker (i.e., productivity). That is,

$$
\begin{aligned}
\sigma^2_{w,\text{between-firm}} &= \int_0^s \left[ \int_0^{x+s} \left(\frac{f_2(x,x')}{2} - \bar{\lambda}\right)^2 \frac{1}{x+s} dx' \right] dx \\
&+ \int_s^{1-s} \left[ \int_{x-s}^{x+s} \left(\frac{f_2(x,x')}{2} - \bar{\lambda}\right)^2 \frac{1}{2s} dx' \right] dx \\
&+ \int_{1-s}^1 \left[ \int_{x-s}^1 \left(\frac{f_2(x,x')}{2} - \bar{\lambda}\right)^2 \frac{1}{1+s-x} dx' \right] dx \\
&= \frac{1}{12} - \frac{13\, \gamma^2 s^5}{2400} + \frac{\gamma^2 s^4}{80} + \frac{5\, s^3}{36} - \frac{s^2}{6},
\end{aligned}
$$

where the last equality follows after another sequence of integration and algebra. The result in Proposition A.2 then follows by taking the ratio $\frac{\sigma^2_{w,\text{between-firm}}}{\sigma^2_w}$. $\qquad\square$

Proposition A.2 makes precise the following two points. First, the between-firm variance share unambiguously increases with the strength of coworker complementarities, captured by $\gamma$. Second, for $s = 0$, which in particular obtains in equilibrium when search costs are absent, between-firm inequality accounts for all of the dispersion in wages.

# B  Empirics

## B.1  Methodology

### B.1.1 SIEED

This section provides further details on the Sample of Integrated Employer-Employee Data (SIEED 7518) and how I process the data. Access is provided by the Research Data Center of the German Federal Employment Agency at the Institute for Employment Research (IAB). A detailed description can be found in vom Berge *et al.* (2020). To ensure best practices, I extensively rely on publicly available code by Eberle and Schmucker (2017) and Dauth and Eppelsheimer (2020). Individual records originate in labour administration and social security data processing.[B.1]

The SIEED covers every worker at a random sample of establishments as well as, crucially, the complete employment biographies of each of these workers, even when not employed at the establishments in the sample. To maximize sample coverage, I do not restrict myself to the panel establishments, but instead require a minimum number of persons in every establishment-year cell (see below). Variables available include a worker's establishment and average daily wage alongside a rich set of other characteristics, including employment status, age, gender, tenure, occupation, and education, among others. Throughout, I use the KldB-1988 2-digit occupational classification, which the IAB reports in harmonized form throughout the entire sample period.

The employment biographies come in spell format. I transform the dataset into an annual panel. Where a worker holds multiple jobs in a year, I define the job with the highest daily wage as the main episode. Nominal values are deflated using the Consumer Price Index (2015 = 100).

My sample selection criteria are similar to other studies using this dataset or studying similar topics (e.g., Card *et al.*, 2013). In a first step, I select employees aged 20-60 with workplaces in West German states who are liable to social security and are not in part-time or marginal employment (i.e., I limit the sample to full-time employeese). I also drop jobs with real daily earnings of less than 10 Euros. I drop observations in select industries and create a consistent industry classification at the 2-digit level of the OECD STAN-A38 nomenclature.[B.2]

A well-known and non-trivial limitation of the German matched employer-employee data is that the earnings variable is top-coded at the so-called "contribution assessment limit" of the social security system ("Beitragsbemessungsgrenze"). To impute right-censored wages, I implement best practices, specifically following Card *et al.* (2013), who build on Gartner (2005) and Dustmann *et al.* (2009). This approach involves fitting a series of Tobit models to log daily wages, then imputing an uncensored value for each censored observation using the estimated parameters of these models and a random draw from the associated (censored) distribution. I fit

---

[B.1]Relative to the LIAB dataset, which is more commonly used, including in a previous version of this paper, the SIEED is a new product offered by the IAB. It does not include information from the IAB Establishment Panel survey but, crucially, comes with a much larger coverage.

[B.2]I drop Agriculture, forestry and fishing (1); Mining and quarrying (5); Utilities (35-36); as well as Activities of households as employers; undifferentiated activities of households for own use (97) and Activities of extraterritorial organizations and bodies (99). The selection aims to ensure consistency with analyses for the Portuguese case.

16 Tobit models (4 age groups, 4 education groups), after having restricted the sample per the above). I follow Card *et al.* (2013) in the specification of controls by including not only age, firm size, firm size squared and a dummy for firms with more than ten employees, but also the mean log wage of co-workers and fraction of co-workers with censored wages. Finally, following Dauth and Eppelsheimer (2020), whose publicly available code I heavily rely on, I limit imputed wages at 10 times the 99th percentile. In a second sample restriction step, I then drop establishment-year cells with fewer than ten full-time employees or worker-year observations and restrict attention to the largest connected set. The final sample (1985-2017) includes 17,126,027 person-year observations for 1,982,239 unique persons, whose average age is 38.49. The median unweighted establishment size is 29. The average real daily wage in 2010 is 106.48 Euros.

### B.1.2 Portuguese micro-data

The construction of the Portuguese dataset is described in detail in Criscuolo *et al.* (2023), from which the results relating to Portugal that are shown in this paper are excerpted.[B.3] Here I provide a brief summary. After an extensive data cleaning procedure to handle duplicate person and employer identifiers, we impose sample restrictions similar to those described in the preceding section. All observations retained relate to persons employed by third parties. Real hourly wages include base pay, regular benefits, and bonus pay. As wages are not top-coded, no imputation procedure is needed. I drop observations below the statutory minimum monthly pay, as in Cardoso *et al.* (2018). By merging in balance sheet and income statement information from the Informação Empresarial Simplificad (SCIE), we can construct a measure of value-added per worker to study dispersion in firm-level productivity. Given the sampling frame of the SCIE, I restrict myself to the non-financial corporate sector. While primarily focussing on the 2010-2017 sample, where longer periods are considered I use a combination of official and unofficial but widely used harmonization crosswalks for occupations and industries. The final sample (1986-2017) includes 21,200,256 person-year observations, of which 6,930,892 belong to the 2010-2017 sample. In 2017, there are 848,970 workers employed at 19,391 unique firms. Throughout, as a measure of a worker's type I use the AKM method. The same analysis but using the non-parametric ranking approach yields very similar results when comparing methods in a subsample containing only one gender (the full sample is too large for the non-parametric method to be feasible).

Regarding the industry-level proxy for coworker skill complementarity used in Section 3.4, I first construct the "CTIC" measures (Communication, Impact, Commmunication, Contact) for each O*NET SOC occupation based on the average score assigned by the O*NET experts on a

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Likert scale from one to five, and taking the mean across the four different dimensions. Then, after cross-walking to the ISCO-2d classification available in the QP data using publicly available crosswalks, I construct an employment-weighted mean score for each 4-digit industry. The reader is referred to Bombardini *et al.* (2012) for details on the questions selected from O*NET.

### B.1.3 Measuring worker types

As stated in Section 3.2, I implement two alternative approaches to estimate each worker's time-invariant quality type in the data. One ranks workers by their fixed effect from Abowd *et al.* (1999, AKM hereafter) style log-linear wage regressions. An alternative that is less common but more consistent with the structure of the model uses the non-parametric ranking algorithm proposed in Hagedorn *et al.* (2017). This section provides details on the implementation of both methods.

Under either method, given a individual's unique rank in the (sample-period specific) type distribution, I compute an individual's decile rank in the year-specific distribution. Hence, types are uniformly distributed in each year, even as the number of workers varies across years (different from the model). In any case, the binning procedure also implies that a worker's decile rank generally does not change across years. I omit a time subscript to underline that $\hat{x}_i$ denotes an individual characteristic that is time-invariant modulo sample composition changes.

**B.1.3.1  AKM models.**  Estimation of two-way fixed-effects regressions in the spirit of AM is a popular approach to account for unobservable worker and firm effects, is widely used in empirical studies of wage inequality and can, at a minimum, be viewed as a useful diagnostic tool. I estimate the AKM model separately for five overlapping sample periods. This estimation is implemented in Stata using the *reghdfe* package (Correia, 2017).

My implementation takes care to mitigate limited mobility bias, which is a form of incidental parameter bias arising from the fact that a large number of firm-specific parameters (i.e., fixed effects) that are solely identified from workers who move across employers. To this end I follow Bonhomme *et al.* (2019) and reduce the dimensionality of the estimation problem by clustering similar firms.[B.4] Clusters are found by solving a weighted k-means problem,

$$\min_{k(1),...,k(J),H_1,...,H_K} \sum_{j=1}^{J} n_j \int (\hat{F}_j(w) - H_{K_j}(w))^2 d\mu(w), \tag{B.1}$$

where $k(1), ..., k(J)$ constitutes a partition of firms into $K$ known classes; $\hat{F}_j$ is the empirical cdf

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[B.4]For Portugal, I have experimented with a variety of bias-correction methods. In particular, the method discussed here as well as the approaches of Andrews *et al.* (2008) and Kline *et al.* (2020) yield very similar findings. Moreover, the uncorrected version appears significantly biased, overestimating the contributions of firm-specific pay premia and underestimating the degree of worker-firm and worker-worker segregation.

of log-wages in firm $j$; $n_j$ is the average number of workers of firm $j$ over the sample period; and $H_1, ...., H_K$ are generic cdf's.

I use a baseline value of $K = 20$ but have experimented with $K = 10$ and $K = 100$ as well (the choice makes little practical difference, as reported also by Bonhomme *et al.* (2019)). I use firms' wage distributions over the entire sample period on a grid of 20 percentiles for clustering.

Different from Bonhomme *et al.* (2019), and in similarity to Palladino *et al.* (2021), I then stick to the two-way fixed effect regression approach rather than estimating a correlated randon effects model. That is, after imputing a cluster to each worker-year observation, I estimate the following regression, which uses cluster effects instead of firm effects:

$$\ln(w_{it}) = \alpha_i + \sum_{k=1}^{K} \psi_k \mathbf{1}(j(i,t) = k) + \epsilon_{it} \tag{B.2}$$

where $\mathbf{1}(J(i,t) = k)$ are dummies indicating which cluster $k$ firm the employer of $i$ in period $t$, $j(i,t)$ has been assigned to. I associate with each firm $j$ the fixed effect of the cluster to which $j$ belongs and denote it $\psi_j$. For the sake of exposition, I abstract from observables, but in practice I control for an index of time-varying characteristics that includes a cubic in age and a quadratic in job tenure, exactly as in the construction of the residualized wages described in Section 3.2.

**B.1.3.2 Non-parametric ranking algorithm.** As is well known (Eeckhout and Kircher, 2011; Lopes de Melo, 2018; Bonhomme *et al.*, 2019), the restrictions imposed by the AKM model are inconsistent with common structural models, including that laid out in Section 2. As an alternative way of estimating worker quality types, I therefore implement a version of the non-parametric ranking algorithm proposed in Hagedorn *et al.* (2017). For a lemma indicating the theoretical applicability of this method, see Appendix section A.2.4. Intuitively, since both production and the value of unemployment are increasing in worker productivity, wages within firms are also increasing in worker type. We can then infer a partial ranking of employees in a given firm from their wages (Anna and Bob both work at Zara in Cambridge; if Anna's wage is higher than Bob's, the algorithm interprets that as Anna being ranked above Bob). If no firm matches with all workers, we have to aggregate the partial within-firm rankings to a global one by exploiting the logic of transitivity and worker mobility across employers (Bob changes job to work for H&M and has a new coworker, Carl; if Bob is ranked above Carl, then we infer Anna's rank to be greater than Carl's.)

To handle potential inconsistencies across partial rankings, measurement error being one relevant source, Hagedorn *et al.* (2017, Online Appendix C) adapt the Kemeny-Young method, which minimizes the sum of Kendall-tau distances between two rankings, and propose a compu-

tationally feasible approximation to the NP-hard problem of uncovering the exact solution. In my implementation for convergence I require the correlation between the rankings obtained from consecutive iterations to be at least 0.995. The implementation in Octave, run on the FDZ-IAB servers, takes around three weeks when a ranking is created separately for each of five sample periods.

### B.1.4   Approximating the cross-partial wage derivative

The cross-partial derivative of the wage function, denoted $\frac{\partial^2 w(x|x')}{\partial x \partial x'}$, is a high-dimensional object. The main text, specifically Section 3.2.3, discussed two methods to approximate the average cross-partial: a regression-based approach and a non-parametric (NP) method. Here I first briefly summarize the practical implementation of the NP method, and then show that in simulated data generated by the structural model, both methods approximate the truth quite well.

   To non-parametrically approximate the average cross-partial derivative of the wage function, I proceed as follows:

   *(i)* Construct the non-parametric wage function, which given an approximation of the worker type space on $n_x$ grid points is a $n_x \times n_x$ matrix. Denote as $w_{ij}$ the average wage of a worker in decile $i$ of the type distribution whose average coworker is in decile $j$ of the coworker type distribution, $i = 1, ..., n_x$ and $j = 1, \ldots, n_x$.

   *(ii)* To prevent local nonlinearities in the conditional wage function from biasing the estimate, fit a polynomial separately for each worker type $x$. Denote the predicted values $\tilde{w}_{ij}$. (In that sense, the approach here is not completely non-parametric.)

   *(iii)* Given a matrix of these predicted values, numerically compute the cross-partial derivative based on the matrix $[\tilde{w}_{ij}]_{i=1,...,n_x}^{j=1,...,n_x}$ using finite difference methods. Specifically, using forward differences for for $i, j = 1$; using backward differences for $i, j = n_x$; and using central differences everywhere else.

   *(iv)* Finally, compute the (unweighted) average; $\overline{\frac{\partial^2 w(x|x')}{\partial x \partial x'}} = \sum_{i=1}^{n_x} \sum_{j=1}^{n_x} \left[ \widehat{\frac{\partial^2 w(x|x')}{\partial x \partial x'}} \right]_{ij}$.

   For validation purposes, I evaluate both the NP and the regression-based method in data generated from the structural model. In the latter model, I can compute the exact average cross-partial derivative of the wage function given the parametrized production function and the mapping between production cross-partial and wage cross-partial in Corollary 3. Figure B.1 shows that both methods slightly under-estimate the exact value for any given degree of complementarity, but replicate the positive relationship very well. The slight under-estimation
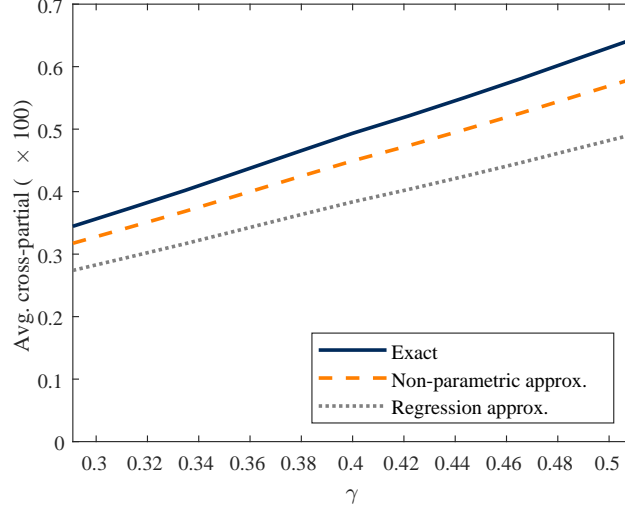
**Figure B.1:** Recovering the average cross-partial derivative

*Notes.* This figure shows the true average cross-partial (solid line) alongside approximations obtained using the non-parametric (dashed) and regression-based (dotted) methods, respectively. The model parameters are those indicated in Table 2, except that the elasticity of complementarity $\gamma$ is varied, as indicated along the horizontal axis.

comes from the fact that neither method fully captures that the cross-partial is increasing in the distance between $x$ and $x'$. For the approximation to work well, it turns out to be important to weight observations by the inverse probability that the worker-coworker combination associated with that observation occurs in the data. Else, the parts of the state space where matches are unlikely to occur is underweighted.

## B.2 Additional results

### B.2.1 Cross-country evidence on the "firming up" of between-firm inequality

Figure B.2 illustrates cross-country trends for the between-firm share of wage inequality, drawing on aggregated statistics kindly made available by Tomaskovic-Devey *et al.* (2020). While the levels cannot be straightforwardly compared across countries due to variation in the measure of earnings used (e.g., hourly vs. daily vs. monthly earnings), one can observe a consistent upward trend for almost all countries.

Interestingly, even in a country like France, where total wage inequality has broadly flatlined over the past few decades, the between-firm component tends to have increased due to rising sorting and segregation, whereas within-firm inequality has declined for a variety of reasons (Babet *et al.*, 2022).
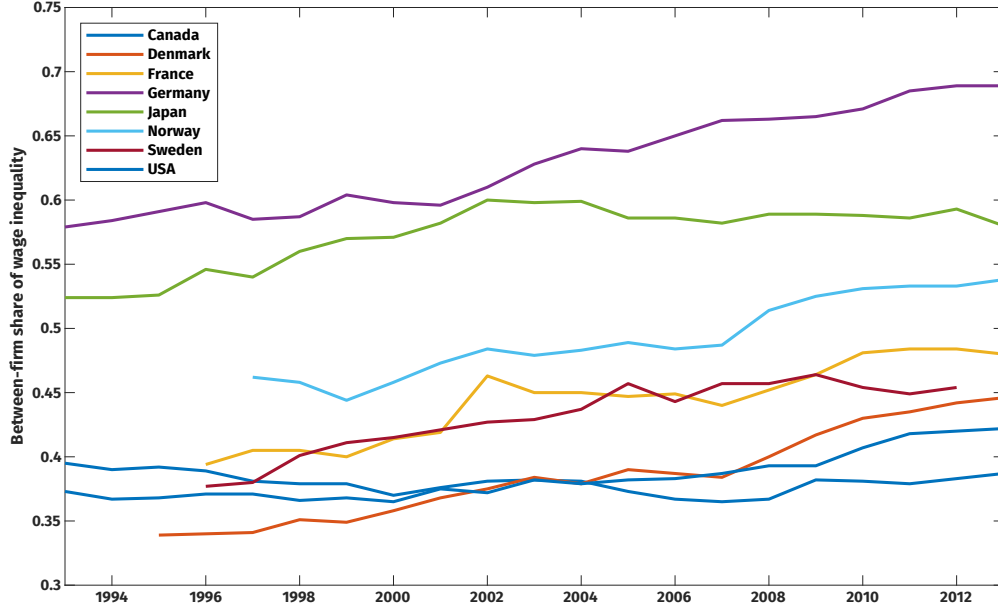
**Figure B.2:** Cross-country evidence on the between-firm share of wage inequality

*Notes.* This figure reports the evolution of the between-firm share of wage inequality for a set of OECD economies. Data source: Tomaskovic-Devey *et al.* (2020).

### B.2.2 AKM-based variance decompositions

The log-linear structure of the AKM model facilitates widely used diagnostic decompositions. Given estimated worker and employer fixed effects, $\alpha_i$ and $\psi_j$, we can decompose the variance of log wages as in Song *et al.* (2019):

$$\mathrm{Var}(w_{it}) = \underbrace{\mathrm{Var}(\alpha_i - \bar{\alpha}_j) + \mathrm{Var}(\epsilon_{it})}_{\text{within-component}} + \underbrace{\mathrm{Var}(\psi_j)) + 2\mathrm{Cov}(\bar{\alpha}_j, \psi_j) + \mathrm{Var}(\bar{\alpha}_j)}_{\text{between-components}}, \tag{B.3}$$

where $\bar{\alpha}_j$ is the average worker FE in firm $j$. A common interpretation then relates $\mathrm{Var}(\psi_k)$ to firm/cluster-specific "wage premia", $\mathrm{Cov}(\bar{\alpha}_j, \psi_j)$ to "worker-firm sorting", and $\mathrm{Var}(\bar{\alpha}_j)$ to "worker segregation."

Before presenting the empirical decompositions, I stress that through the lens of a structural model with coworker complementarities the three components just mentioned are not neatly separable. In particular, the AKM model will attribute correlations in output between workers at the same employer, insofar as they are reflected in wages (and conditional on worker types) to a common employer component, even if these correlations are, in reality, due to coworker complementarities. This has at least two implications. First, the "worker-firm sorting" component may partly pick up coworker sorting. Second, the common interpretation of the employer fixed effect

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as reflecting firm wage premia or wage-setting practices (as opposed to worker characteristics) is not necessarily warranted.

A natural follow-up question is whether these conjectures are borne out when estimating AKM regressions on panel data simulated from the structural model. Unfortunately, a quantitatively credible answer to this question requires a version of the model that allows for a firm size $n$ significantly above two. Otherwise, and imposing empirically consistent patterns of worker mobility, there is insufficient variation for an AKM-type model to stand a fair chance at identifying the employer fixed effects. Qualitatively, exercises confirm that estimating an AKM model on simulated panel data yields positive contributions of the variance of employer fixed effects and the covariance between worker and employer fixed effects. Solving a model with unrestricted within-firm coworker interactions in production and "large" firms remains an unresolved computational challenge (given the exploding state space), which these exercises suggests may be a promising avenue.

Turning to the empirical variance decompositions, the left panel in Figure B.3 depicts the results for the German economy on a period-by-period basis. We see that all three between-components have risen over time. Moreover, the Kremer-Maskin segregation index (Kremer and Maskin, 1996), which indicates the share of the variance of worker FEs that occurs between rather than within establishments – and, thus, represents an alternative measure of coworker sorting – has risen over time. The picture looks very similar if occupation and industry fixed effects are accounted for as well.[B.5] Overall, changes in workforce composition account for more than fourth fifths of the total increase in the between-employer component, with changes in what are commonly labelled "firm pay premia" accounting for the remainder.[B.6]

For comparison purposes, the right-hand panel reports the variance decomposition for the Portuguese economy. A key takeaway is that even though the between-firm share of the variance has remained roughly constant since the late 1990s, both $\mathrm{Cov}(\bar{\alpha}_j, \psi_j)$ and $\mathrm{Var}(\bar{\alpha}_j)$ have increased, as has the segregation index – similar to the German economy. Controlling for changes in the firm-specific pay premia, therefore, the sorting and segregation components would have pushed up between-firm inequality, in line with what we observe for Germany and many other advanced economies. Silva *et al.*'s (2022) analysis of the Portuguese economy come to a similar conclusion.

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[B.5]Interestingly, if one performs the decomposition separately by major sector groups, the upward trend is most pronounced for knowledge-intensive services.

[B.6]I thus find a slightly smaller contribution of firm pay premia than Card *et al.* (2013), my numbers being more similar to those by Song *et al.* (2019) for the U.S. One potential reason is that I use a dimension-reduction technique following Bonhomme *et al.* (2019) to mitigate limited-mobility bias. In addition, I do not include education as a control variable. My results remain similar when estimating a higher-dimensional model that also takes out occupation and industry fixed effects.
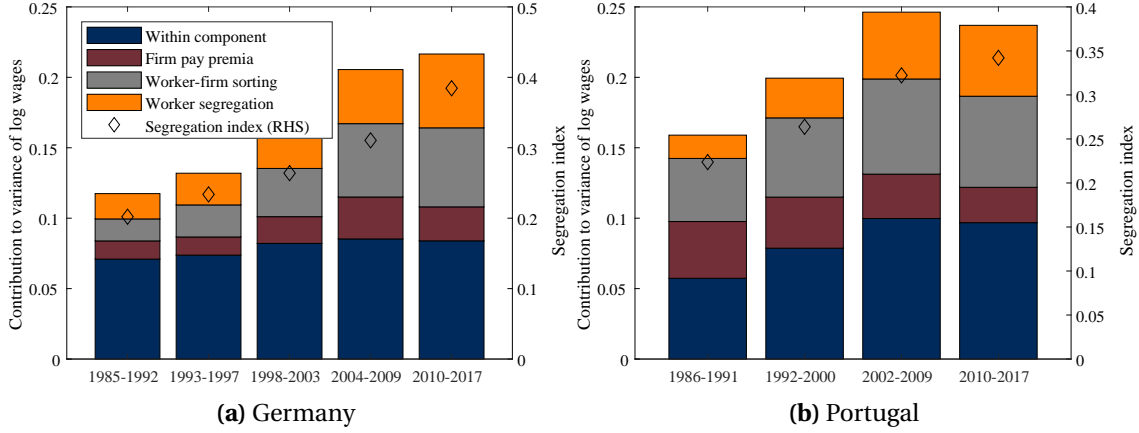
**Figure B.3:** AKM-based wage variance decomposition

*Notes.* These panels report the components of the variance of log wages, following equation (B.3), based on the estimation of the AKM model described in equation (B.2). The left panel is for Germany, the right panel for Portugal (where I only use four sample periods to better span missing years in 1990 and 2001).

### B.2.3   Robustness: rising between-share of inequality in Germany

This section examines how robust the increase in the between-establishment share of the variance of log wages in Germany is to alternative measures of individual wages. Figure B.4 provides a graphical summary and indicates that controlling for different sets of covariates does not alter the main conclusion of a rising between-employer component.

The solid line in Figure B.4, in either panel, depicts the standard between-within establishment decomposition of the variance of log raw wages, $\ln(\tilde{w})_{it}$. In the left panel, the dashed line in the same colors swaps in the baseline measure of residualized wages used in the main text (see Section 3.2 for a description). Next, the dotted, dashed-dotted, and dotted-crossed lines also remove, respectively, (2-digit) occupation FEs, (2-digit) industry FEs, or both. Throughout, by including the worker FEs in the "residual" component, it is ensured that we do not omit variation in time-invariant individual earnings that is also present in – indeed, central to – the structural model. For robustness, the right panel in the figure repeats the same exercise but without including these person FEs when residualizing wages.

Several observations stand out. First, and reassuringly, the rise in the between-establishment share of wage inequality is highly robust across all specifications. While the *level* of this share varies, the increase over time is quite uniform, ranging from 13 to 25 percentage points (1985-2017). Second, and unsurprisingly, both the level and the percentage point increase in the between-establishment share are smaller in magnitude when worker FEs are not accounted for. Third, turning to the role of occupations and industrie, when controlling for both occupation and industry FEs, the between share rises from 0.21 to 0.35, with most of the increase occurring up
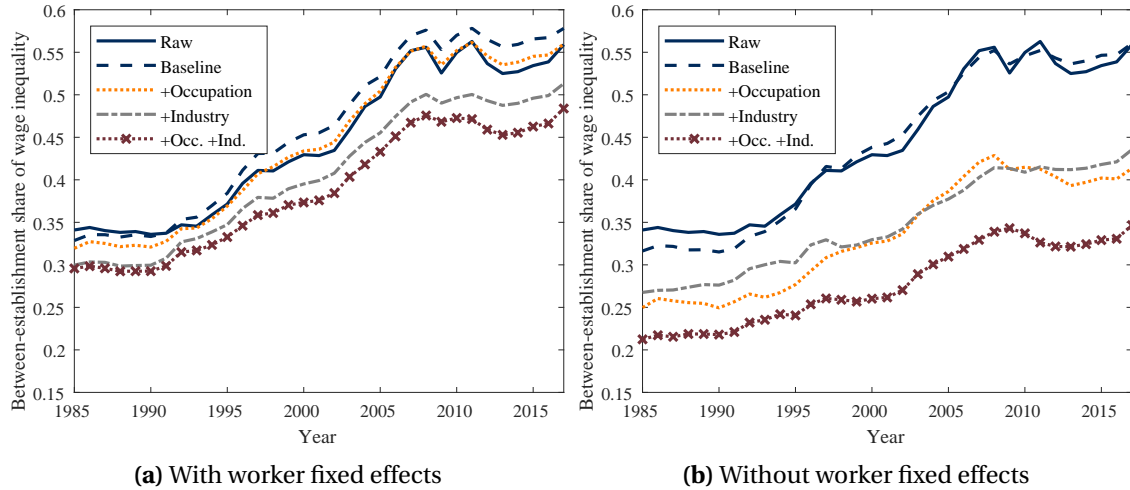
**(a)** With worker fixed effects  **(b)** Without worker fixed effects

**Figure B.4:** Rising between-establishment share of wage inequality in Germany

*Notes.* This figure depicts the share of the variance of log wages occurring between as opposed to within establishments for different measures of wages: raw wages, the baseline residuals used in the main text, and alternatives that also incorporate, respectively, occupation or industry fixed effects, or both. The relevant regressions are performed for the entire sample, then the decomposition is performed year-by-year.

until the Global Financial Crisis. That the increase in the between share is smaller, in percentage point terms, when taking out occupation and industry FEs is consistent with two points made in the literature. The first point is that between-firm inequality partly arises from between-industry differences in average pay (Haltiwanger and Spletzer, 2020). The second is that some of the rise in between-firm inequality is due to occupational outsourcing. Goldschmidt and Schmieder (2017), in particular, document that outsourcing of cleaning, security, and logistics services accounts for around 9% of the increase in German wage inequality since the 1980s.[B.7] Moreover, through the lens of an AKM model – considered in the preceding Appendix section – outsourcing manifests in a reduction in all three of the between components. In summary, additional controls can account for some but not the majority of the rise in the between-firm share of wage inequality.

### B.2.4 Non-linear coworker aggregation

The main text explains that for each person-year observation I construct an average coworker type by computing the unweighted arithmetic mean of all coworkers' types, and notes that this approach ignores a non-linearity in the aggregation that is implied by the structural model. Here I elaborate on this approach and discuss robustness results which suggest that the bias from ignoring this non-linearity is small.

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[B.7]Bilal and Lhuillier (2021) argue, in the French context, that outsourcing leads to rising labor market sorting but relatively stable wage inequality, due to an offsetting general-equilibrium mechanism associated with pro-competitive effects of contractors at the bottom of the job ladder.

|  | Unweighted | Weighted: (0.25) | Weighted (0.50) | Weighted (0.75) |
|---|---|---|---|---|
| Sorting | 0.613 | 0.617 | 0.616 | 0.614 |
| Complementarity | 0.00905 | 0.00896 | 0.00882 | 0.00870 |

**Table B.1:** Robustness: coworker sorting and complementarities under non-linear aggregation

*Notes.* This table summarizes estimates of coworker sorting and complementarities when the average coworker type is computed, respective, as the unweighted arithmetic mean or the weighted power mean, using three different weighting parameters $\gamma$. The "Sorting" indicates the correlation coefficient between own type and average coworker type, while "Complementarity" refers to the point estimate of the interaction coefficient in regression (36). The sample period is 2010-2017.

To recap, my baseline approach is to average over coworkers without weighting, that is I compute $\hat{x}_{-it} = \left( \frac{1}{|S_{-it}|} \sum_{k \in S_{-it}} \hat{x}_k \right)$, where $S_{-it}$ is the set of $i$'s coworkers in year $t$. However, the structural model and specifically the micro-foundation in Section 2.1 suggest that we ought to aggregate using a power mean that assigns disproportionate weight to low-type coworkers insofar as coworker complementarities are important. Denote this alternative by $\tilde{x}_{-it} = \left( \frac{1}{|S_{-it}|} (\hat{x}_k)^{1-\gamma} \right)^{\frac{1}{1-\gamma}}$. Some intuition for what is lost by ignoring this non-linear aggregation can be gained by taking a second-order Taylor approximation around $\hat{x}_{-it}$, which shows that $\tilde{x}_{-it} - \hat{x}_{-it} \approx -\frac{1}{2}\gamma \frac{\sigma_x^2}{\hat{x}_{-it}}$, where $\sigma_x^2$ is the variance of coworker types. This shows that the unweighted average is upward biased in proportion to the product of complementarities and the dispersion among coworkers. From the perspective of the structural model, this offers reassurance, since dispersion – which is another manifestation of imperfect coworker sorting — should be low precisely when $\gamma$ is high.

To evaluate the magnitude of bias empirically, I compare the estimates of coworker sorting and complementarities for the final sample period under different weighting schemes.[B.8] In addition to the baseline relying on linear aggregation, I recompute the coworker correlation and estimate regression (36) when computing a weighted average coworker, for each person-year, under three different values of $\gamma$: 0.25, 0.50 and 0.75. Table B.1 summarizes the results. It demonstrates that all four approaches yield very similar estimates. I have, in addition, performed Monte Carlo simulations using a statistical model of team formation, similar to that introduced in Section C.1.3, an exercise that likewise suggests a bias in the estimated complementarities that is positive but small.

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[B.8]Briefly, it is tempting to consider an iterative approach that alternates between, on the one hand, guessing a value of $\gamma$ and constructing a weighted average coworker based on that value, and etimating wage complementarities and inferring a new value of $\gamma$, on the other hand. While this is possible in principle, note that this inference ultimately relies on the entire structure of the model, since $\gamma$ is not uniquely pinned down by a given estimate of wage complementarities. Apart from the computational load, it is not possible to estimate the structural model directly on the servers of the IAB, where the micro data are stored.
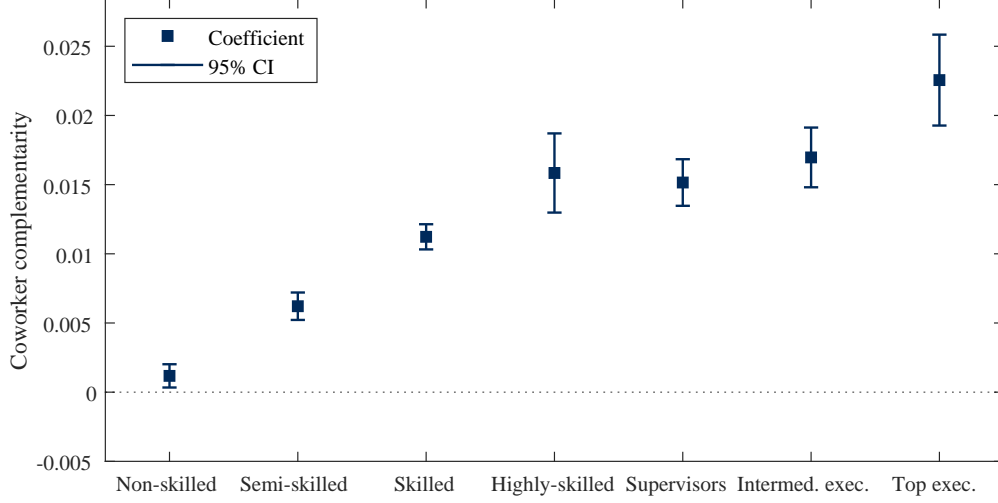
**Figure B.5:** Complementarities across different hierarchical layers

*Notes.* This figure reports the point estimate for the coefficient $\beta_c$, alongside confidence intervals, when regression (36) is estimated separately for workers belonging to different hierarchy levels. To avoid overlap across hierarchy levels, the set of employee $i$'s coworkers here is restricted to those employees in the same employer-year-layer cell.

### B.2.5 Additional cross-sectional validation results

This section reports additional, empirical analysis using the Portuguese micro-data.

First, the richness of the Portuguese data facilitates an alternative operationalization of who an employees' coworkers are: those who work at the same hierarchical level of a firm. From 2010 onward, we can use a variable that assigns workers according to a consistent definition into seven different, vertically differentiated layers, grouped by similarity in the complexity of tasks and skills required. These range from top executives to non-skilled.[B.9] If we interpret these hierarchical layers as an ordinal proxy for $\chi$, in the spirit of Garicano (2000), then we expect coworker wage complementarity to be greater in higher layers.

To test this hypothesis, I run a variation of the regression specification in (36), for each $l = 1, \ldots, 7$.[B.10] Figure B.5 shows the estimated value of $\beta_c$ alongside 95% confidence bands for each layer. The point estimates are almost monotonically increasing in the hierarchical layer, ranging from effectively zero for those classified as non-skilled to above 0.02 for top executives. This is not due to the fact that the left-hand side of the regression is the wage in levels and higher-

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[B.9] Per Decree-Law 380/80, firms should indicate for each employee the qualification level indicated in the relevant Collective Agreement. If this is not available, firms should select the qualification level of the worker. Table B-1 in Mion and Opromolla (2014) provides details on tasks and skills. Also see Caliendo *et al.* (2020). I exclude apprentices/interns/trainees.

[B.10] I deliberately estimate the regression separately for each layer, as opposed to pooling samples and including a double interaction between the interaction term and $l$, to avoid restricting potential variation in $\beta_1$ and $\beta_2$ across $l$. The downside is that, as a result, we do not exploit variation from switchers across layers within firms.

**(a)** Between-firm share of wage dispersion $\qquad$ **(b)** Productivity dispersion
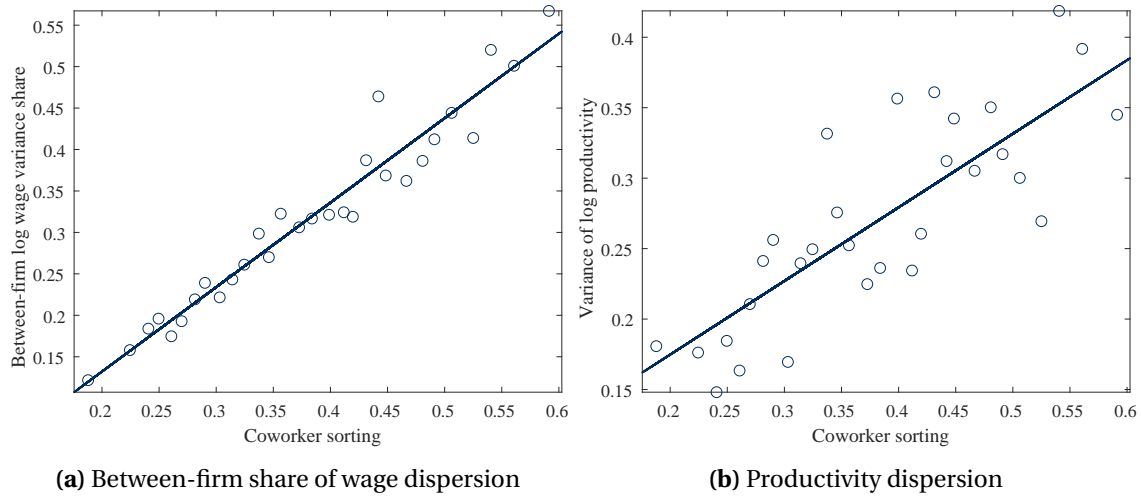
**Figure B.6:** Coworker sorting is associated with greater between-firm inequality

*Notes.* This figure plots the variance of log wages (left panel) and the between-firm share of the variance of log wages (right panel) against the coworker sorting correlation. The unit of observation is a 4-digit NACE industry, with at least 5,000 person-year observations and where the industry-level proxy for complementarity is within two standard deviations. Observations are grouped into 30 bins. The linear regression line is fitted based on unweighted, non-binned observations.

layer workers earn more, so that $\beta_c$ is mechanically increasing in the layer, as the dependent variable is the wage level divided by the layer-specific average wage.

Next, Figure B.6 expands on the cross-industry findings from Section 3.4. It shows in industries which feature strong coworker sorting a greater share of the variance of log wages occurs between as opposed to within firms. This is unsurprising but reassuring. In addition, firm-level productivity is more dispersed in these industries.

These results echo a literature that has highlighted the important role of differences in workforce composition across firms in explaining between-firm wage inequality. In addition to the aforementioned Card *et al.* (2016) and Song *et al.* (2019), Håkanson *et al.* (2021) use direct measures of workers skills from military enlistment tests in Sweden and find that between-firm skill inequality is a key driver behind between-firm wage inequality. In addition, Sorkin and Wallskog (2021) find for the U.S., that productivity dispersion, between-firm earnings inequality, and coworker sorting are successively higher for more recent cohorts of firms in the U.S (also see Berlingieri *et al.* (2017)).

Next, Figure B.7 shows that the industry-level measure of the importance of teamwork, impact on coworker output, communication, and contact following Bombardini *et al.* (2012), discussed in Section 3.4, is positively associated with coworker sorting, the between-share of wage inequality, overall between-firm wage inequality, and productivity dispersion.
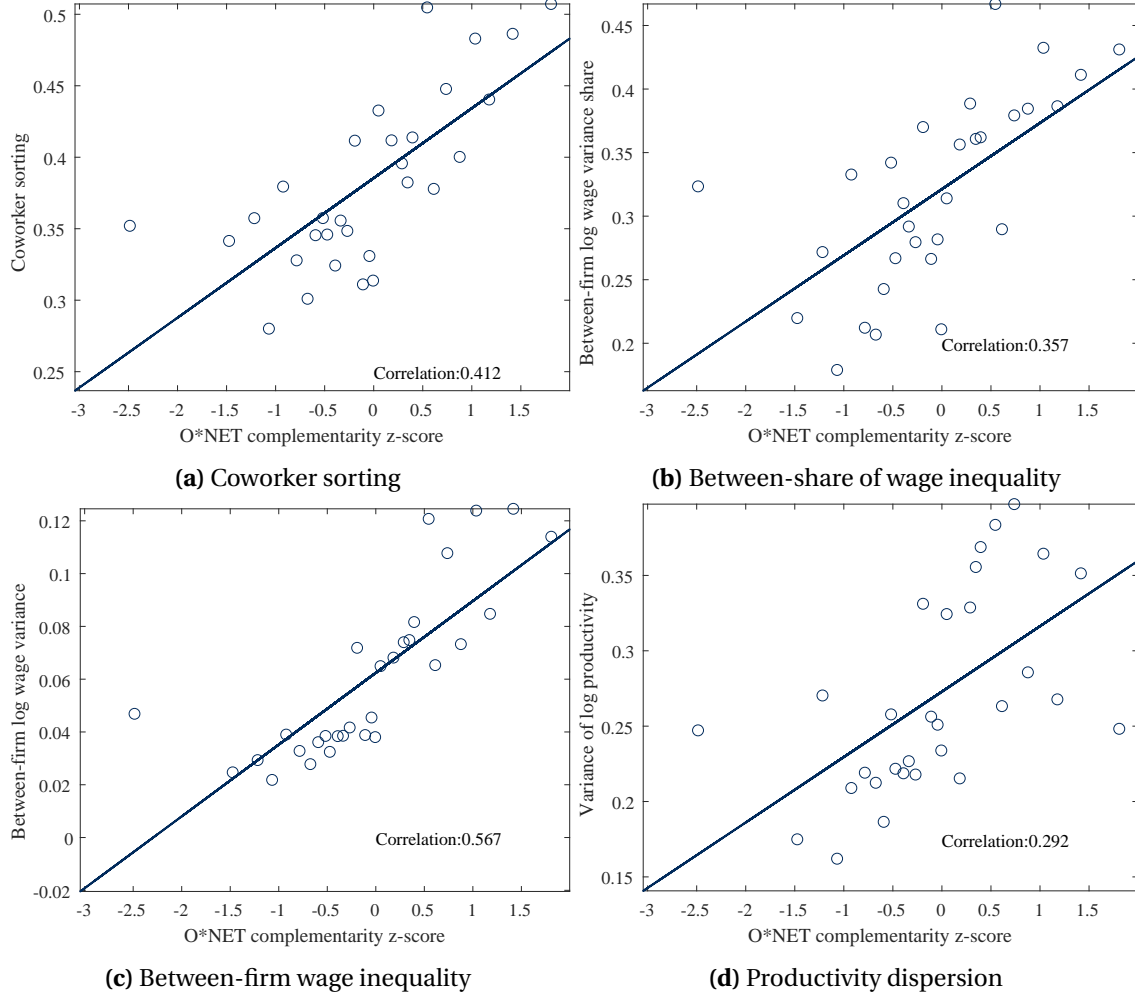
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**(a)** Coworker sorting

**(b)** Between-share of wage inequality

**(c)** Between-firm wage inequality

**(d)** Productivity dispersion

**Figure B.7:** Bombardini et al. (2012) proxy for complementarity and industry-level outcomes

*Notes.* Each panel plots the moment indicated in the subtitle of the respective plot against the industry-level proxy for complementarity constructed from O*NET data following Bombardini *et al.* (2012). The unit of observation is a 4-digit NACE industry, with at least 5,000 person-year observations and where the industry-level proxy for complementarity is within two standard deviations. Industry-level observations are grouped into 30 bins. Industries are binned into 30 cells. The linear regression line is fitted based on unweighted, non-binned observations.

### B.2.6   Results using non-parametric ranking algorithm

Table B.2 replicates Table 1 in the main text but the empirical moments are constructed off of worker types that are estimated using the non-parametric method described in Appendix B.1.3.2, instead of AKM regressions. In comparison, the levels of sorting are slightly higher and those of complementarities lower, respectively, but the trends are qualitatively the same.

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|        | Sorting | | Complementarities | |
|--------|---------|---------|---------|---------|
| Period | Spec. 1 | Spec. 2 | Spec. 1 | Spec. 2 |
| 1985-1992 | 0.47 | 0.38 | 0.001 | 0.000 |
| 1993-1997 | 0.56 | 0.46 | 0.002 | 0.001 |
| 1998-2003 | 0.60 | 0.48 | 0.004 | 0.002 |
| 2004-2009 | 0.65 | 0.50 | 0.005 | 0.002 |
| 2010-2017 | 0.68 | 0.51 | 0.005 | 0.004 |

**Table B.2:** Coworker sorting & complementarities: results based on non-parametric ranking method

*Notes.* This table indicates, under the column "Sorting" the correlation between a worker's estimated type and that of their average coworker, separately for five sample periods. The column "Complementarities" indicates the point estimate of the regression coefficient $\beta_c$ in regression (36). Under "Specification 1" workers are ranked economy wide, while under "Specification 2" they are ranked within two-digit occupations. Worker rankings are based on the non-parametric method described in Appendix B.1.3.2.

### B.2.7 Years of education as a quality measure

As discussed in Section 3, one concern regarding the evidence for coworker complementarity is that the dependent variable in regression (36) is the period-$t$ wage but, in addition, the independent variables of interest (own type and average coworker type) are likewise a function of wages (in all years $t$ belonging to the sample period in question). While this approach is theory-consistent, with identification coming from variation in wages over time while the regressors are invariant across years within sample periods, we may still be worried about a confounding effect. This section shows that when worker quality is proxied by education – similar to Nix (2020) – and thus a non-wage characteristic, instead, the main results remain robust.

Specifically, I adapt equation (36) by regression the wage $w_{it}$ of worker $i$ in period $t$ on their years of education, denoted $s_i$, their coworkers' average years of education, denoted $s_{-it}$, and the interaction of these two terms:

$$\frac{w_{it}}{\bar{w}_t} = \beta_0 + \beta_s \hat{s}_i + \beta_{s'} s_{-it} + \beta_c \left( s_i \times s_{-it} \right) + \psi_{j(it)} + \nu_{o(i)t} + \xi_{s(i)t} + \epsilon_{it}, \tag{B.4}$$

where $s_{-it} = \frac{1}{|S_{-it}|} \sum_{k \in S_{-it}} \hat{s}_k$. To implement this regression, I follow Card *et al.* (2013) in estimating the years of education based on the level of training indicated in the SIEED (on which see Fitzenberger *et al.*, 2006). Notice that the interpretation of the regression coefficients changes accordingly compared to the baseline, in which worker quality is measured in decile ranks. As before, I estimate equation (B.4) separately for 5 sample periods.

Table B.3 summarizes the estimation results. It shows, firstly, that the regression coefficient of interest, $\beta_c$, is positive across all periods, and, secondly, points to an increase over time in line

with the baseline estimates. The magnitude of the increase in $\beta_3$ is similar in proportional terms to the estimates reported in the main text, rising by about 74% from the first to the final sample period.

| | Dependent variable: wage | | | | |
|---|---|---|---|---|---|
| | '85-'92 | '93-'97 | '98-'03 | '04-'09 | '10-'17 |
| Own schooling | -0.0382 | -0.0359 | -0.0871 | -0.1099 | -0.1428 |
| | 0.0113 | 0.0102 | 0.0109 | 0.0100 | 0.0116 |
| Coworker schooling | -0.0737 | -0.0604 | -0.1203 | -0.1384 | -0.1726 |
| | 0.0114 | 0.0111 | 0.0107 | 0.0102 | 0.0114 |
| Interaction | 0.0063 | 0.0060 | 0.0099 | 0.0112 | 0.0129 |
| | 0.0008 | 0.0007 | 0.0008 | 0.0007 | 0.0009 |
| Fixed effects | Yes | Yes | Yes | Yes | Yes |
| Obs. (100,000s) | 3,613 | 2,508 | 2,694 | 3,836 | 4,376 |
| $R^2$ | 0.5033 | 0.5451 | 0.5746 | 0.6330 | 0.6425 |

**Table B.3:** Regression evidence on coworker complementarity using years of schooling

*Notes.* Dependent variable is the wage level over the year-specific average wage. Independent variables are a constant, years of schooling, coworker years of schooling, and the interaction between those two terms. All regressions include industry-year, occupation-year and employer fixed effects. Employer-clustered standard errors are given in parentheses. Observations are unweighted. The sample is unchanged from the main text, except that 96,517 observations with missing years of schooling are dropped. Observation count rounded to 100,000s.

# C  Quantitative analysis

## C.1  Methodology

### C.1.1  Labor market transition rates from the LIAB

This section describes how the empirical labor market transition rates that discipline the job arrival and destruction rates in the quantitative model are computed. As a data source, I supplement the SIEED with the Linked Employer Employee Data longitudinal model (LIAB LM7519), which contains information also on non-employment spells. I proceed in four main steps. First, I convert the spell-level data into a monthly panel. Second, I restrict the sample to approximate the selection criteria used in the other empirical analysis but without being limited to full-time employees. Specifically, I select individuals aged 20-60 who only ever worked for establishments in West Germany. Third, in the construction of transition rates I largely follow Jarosch (2023). Employment refers to full-time employment subject to Social Security. The job finding rate is computed as the rate at which currently non-employed workers who are receiving unemploy-

ment insurance (UI) transition into employment. For the job destruction rate, I compute the frequency with which a worker is employed in one month but not in the month thereafter. Note that here I do not condition on receiving UI after separation, as the model does not distinguish between unemployment and non-employment. I instead define the job finding rate based on un-employment to employment transitions, since the model does assume search effort conditional on non-employment. Finally, for job-to-job transitions I compute the rate at which currently employed workers are employed at another establishment the following month. In step four, I compute averages of these different transition rates across months and for different sample periods.

### C.1.2    Validation of identification approach

To validate the identification of the vector of jointly estimated parameters, $\psi$, I conduct two exercises, following Bilal *et al.* (2022). First, to support the argument laid out in the main text that each element of $\psi$ is closely linked to a particular moment, Figure C.1 plots the relevant moment against the respective parameter. As required for local identification, the relationships are monotonic and exhibit significant amount of variation. For the second exercise, let a given parameter $\psi_i$ vary around the estimated value $\psi_i^*$ and plot the distance function $\mathcal{G}(\psi_i, \psi_{-i}^*)$, where the remaining parameters are allowed to adjust to minimize $\mathcal{G}$. Figure C.2 indicates that $\mathcal{G}(\psi_i, \psi_{-i}^*)$ has a steep U-shape, suggesting that $\psi$ is indeed well-identified.

### C.1.3    Between-share adjustment procedure

Since in the structural model the number of workers in each production unit is two, the between-unit share of the variance of wages – or, for that matter, that of types – will be greater than zero even under random matching. More generally, for any degree of coworker sorting less than unity, i.e. $\rho_{xx} < 1$, the *level* of the between-share in a model with team size $n = 2$ will be biased upward relative to the case of $n > 2$ and, in particular, $n \to \infty$. These observations are simply an outgrowth of the law of large numbers not applying within production units. As such, it is a statistical phenomenon rather than an economically interesting mechanism. Furthermore, the upward bias is *greater* when the coworker correlation is *lower*. One can immediately verify this result intuitively by noting that for $\rho_{xx} = 1$, *all* dispersion is *across* and *none* within units, regardless of the value of $n$. Figure C.3 illustrates these ideas graphically (its construction is described below).

This statistical bias has two implications. First, without any further adjustment, the *level* of the between-share predicted by the estimated model, which assumes $n = 2$, will be excessively high relative to the real world, in which $m > 2$. Second, insofar as the estimated model predicts
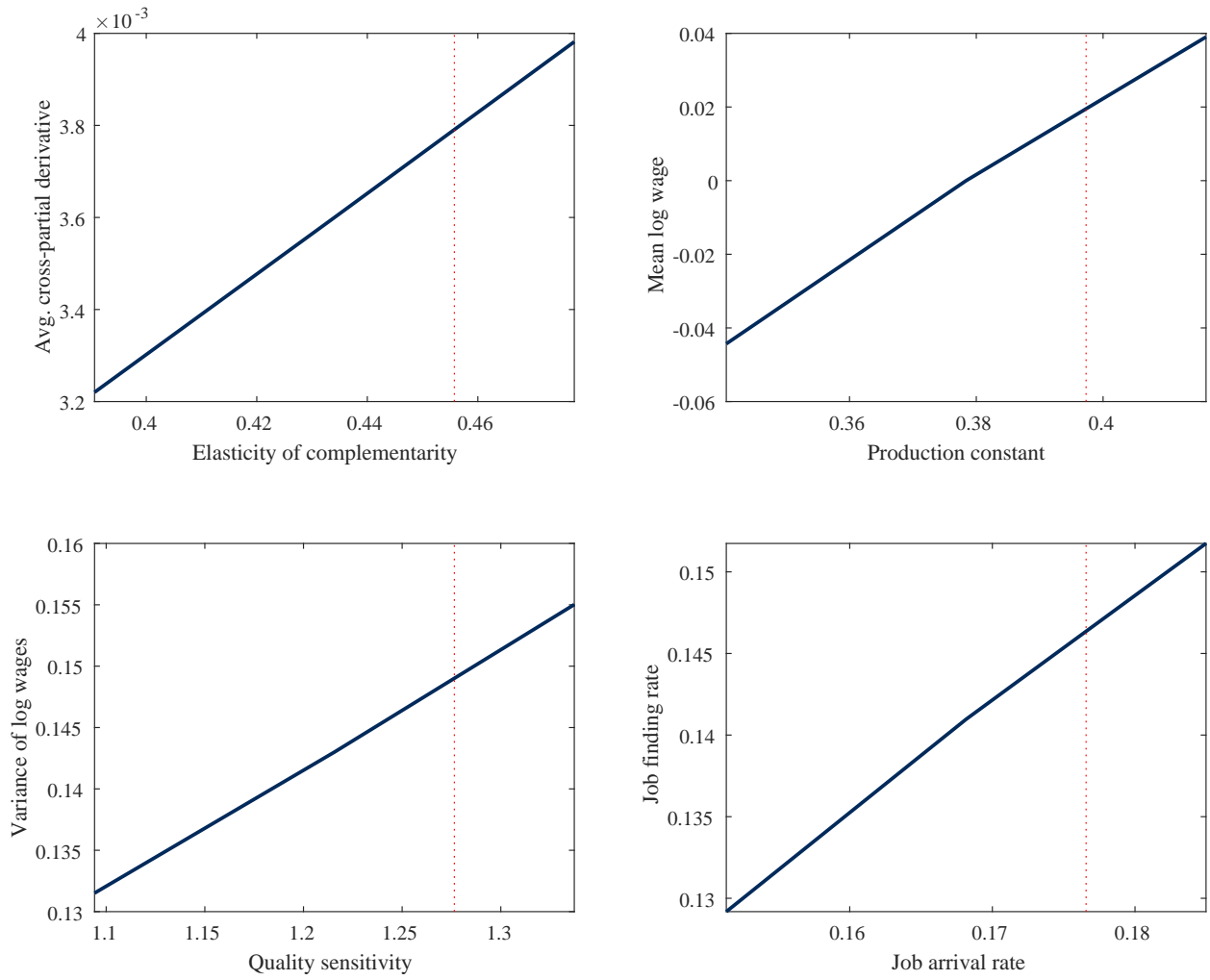
**Figure C.1:** Validation of identification method: moment against parameter

*Notes.* This figure plots the targeted moment against the relevant parameter, holding constant all other parameters.
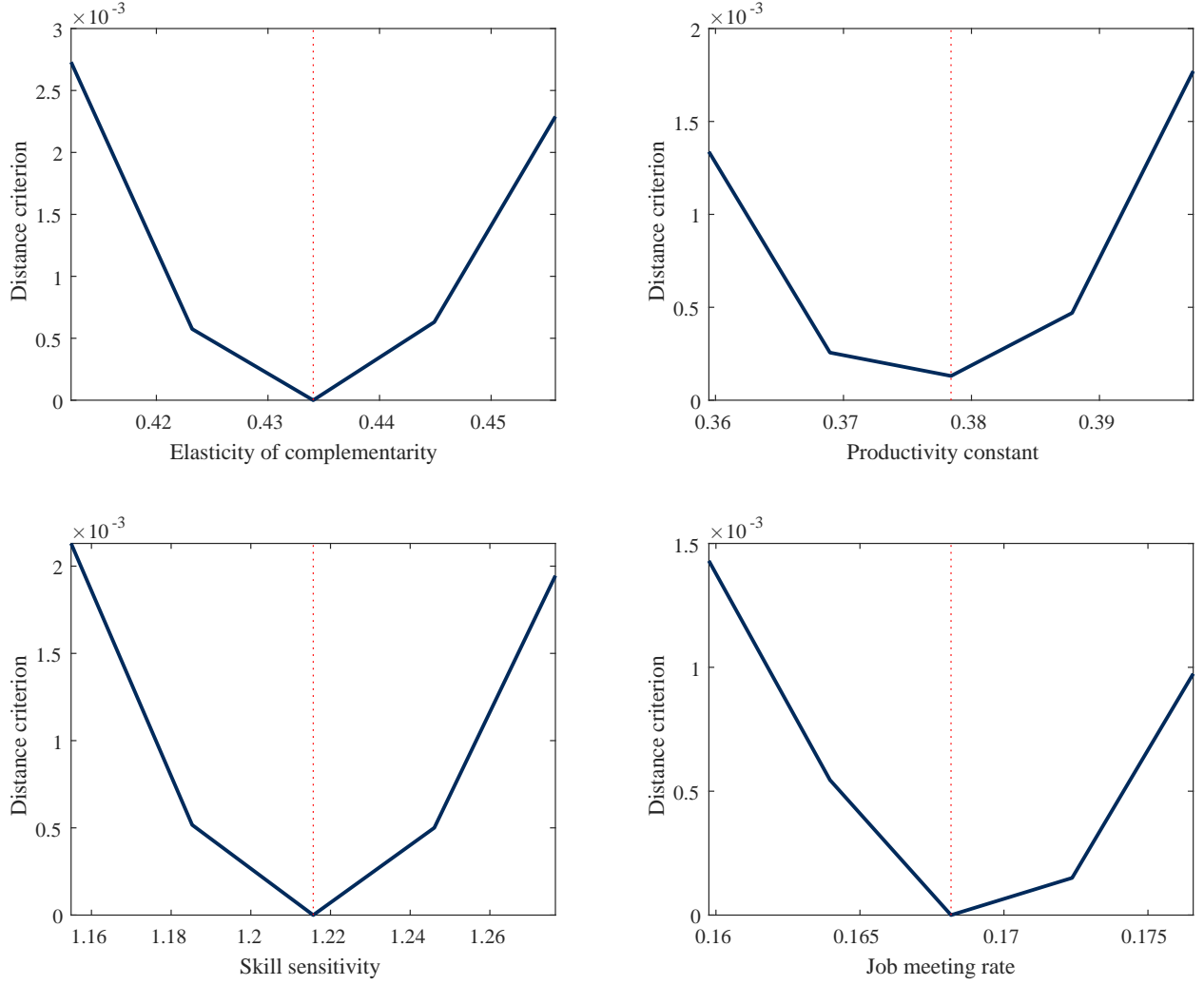
**Figure C.2:** Validation of identification method: distance criterion

*Notes.* This figure plots the distance function $\mathcal{G}(\psi_i, \psi_{-i}^*)$ when varying a given parameter $\psi_i$ around the estimated value $\psi_i^*$. The remaining parameters are allowed to adjust to minimize $\mathcal{G}$.

**(a)** Coworker sorting
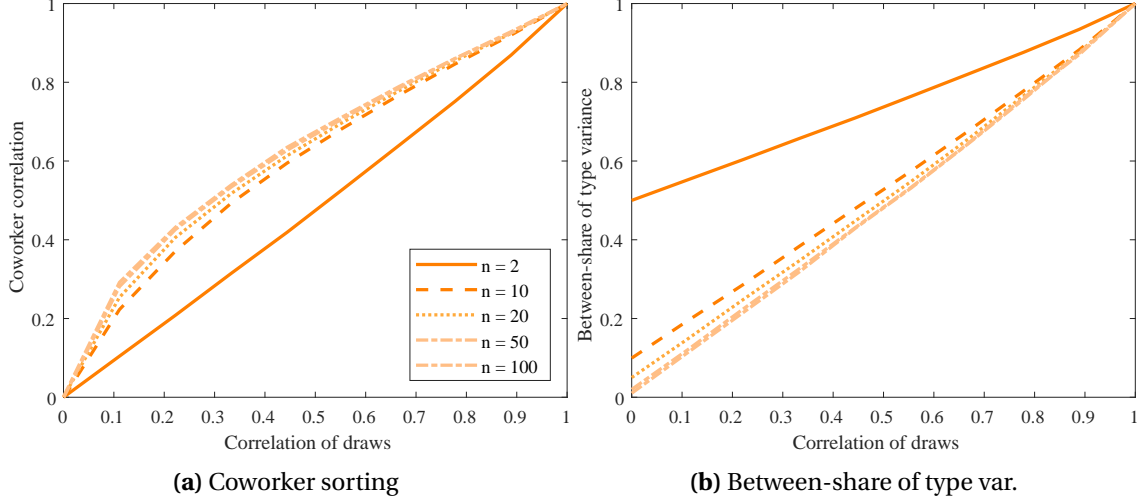
**(b)** Between-share of type var.

**Figure C.3:** Graphical illustration of the adjustment method

*Notes.* This figure illustrates how the coworker correlation coefficient (left panel) and the between-firm share of the variance of types (right panel) vary with the correlation parameter in the Gaussian copula. The different lines represent different "team sizes," that is, varying lengths $n$ of the vector $X$. Workers are binned into ten deciles. The results are based on one million draws.

greater stronger coworker sorting than the earlier period, the predicted increase in the between-share of wage inequality is a *lower bound*, because the statistical upward bias in the later period will be smaller than in the later period. This section proposes an approach to correct the level of the between-share, but all results for *changes over time* that are reported in the main text are a conservative estimate, insofar as I apply the same correction factor to both periods.

The adjustment method I propose is based on a *statistical* model that can flexibly accommodate different degrees of coworker sorting as well as team sizes. Consider a random vector $X = (X_1, X_2, \ldots, X_n)'$ whose distribution is described by a Gaussian copula over the unit hypercube $[0, 1]^n$, with an $n \times n$ correlation matrix $\Sigma(\rho^c)$, which contains ones on the diagonal, while the off-diagonal elements are all equal to a parameter $\rho_c$. Formally, the Gaussian copula with parameter matrix $\Sigma(\rho_c)$ is $C_\Sigma^{\text{Gauss}}(x) = \Phi_R(\Phi^{-1}(x_1), \ldots, \Phi^{-1}(x_n))$, where $\Phi^{-1}$ is the inverse cdf of a standard normal and $\Phi_R$ is the joint cdf of a multivariate normal distribution with mean vector zero and covariance matrix equal to $\Sigma(\rho_c)$. To map this onto our empirical context, $n$ may be interpreted as the average team size. Each vector of observations drawn from the distributions of $X$, $x_j = (x_{1j}, x_{2j}, \ldots, x_{nj})'$, describes the types of workers in that team, indexed by $j$.

This setup affords us with a closed-form description of the population between-team share of the variance of types as a function of $n$ and $\rho_c$. Since the marginals of the Gaussian copula are simply continuous uniforms defined over the unit interval, the variance of the union of all draws is just $\frac{1}{12}$. Furthermore, the mean of the elements of $X$ is itself a random variable, $\bar{X}$. That is, for some realization $x_j$, we can define $\bar{x}_j = \frac{1}{n} \sum_{i=1}^n x_{ij}$. Since the elements of $X$

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all have the same variance and we specified their correlation profile, the variance of $\bar{X}$ will be $\frac{1}{n^2}\left(\frac{n}{12} + n(n-1)(\frac{\rho_c}{12})\right)$. Taking the ratio, we find that the between share, as a function of $\rho_c$ and $n$, is equal to $\sigma^2_{x,\text{between-share}}(\rho_c, m) = \frac{1}{m}\left(1 + (m-1)\rho_c\right)$.

In the main text, and letting the empirical average size be $\hat{n}$, the adjusted results for the between-share are therefore obtained by subtracting the following correction factor: correction-factor = $\frac{1}{2}\left(1 + \rho_c\right) - \frac{1}{\hat{n}}\left(1 + (\hat{n}-1)\rho_c\right)$,

The value of $\rho_c$ I feed into this formula is the average coworker correlation value in the period-1 sample.[C.1] The implied value of the correction factor is $\approx 0.25$.

A concern with this approach is that $\rho_c$ is not the same measure as the coworker correlation, $\rho_{xx}$, that we typically consider in both the empirical analysis and the structural model. To compare the two measures, suppose we draw $M$ samples (i.e., distinct teams) from $X$, so that the total number of observations is $M \times n$. Each individual observation is indexed by $i = 1, \ldots, M \times n$, and the sample to which $i$ belongs is $j(i)$. Then we can define the leave-out-mean $\bar{x}_{-i,j} = \frac{1}{n-1}\sum_{k \neq i} x_{k,j(i)}$. As in our main analysis, the coworker correlation is $\rho_{xx} = \text{corr}(x_i, \bar{x}_{-i})$, where the $j$ indexed is suppressed to emphasize that we are considering a worker-weighted statistic. Figure C.3 confirms that $\rho_{xx}$ and $\rho_{cc}$ track each other quite closely, even though for larger values of $n$ and intermediate values of $\rho_c$ we find that $\rho_{xx} > \rho_{cc}$.[C.2]

Another second concern is that the adjustment approach pertains, strictly speaking, to the between-unit share of the variance of *types*, as opposed to that of *wages*. However, given a distribution of workers across production units derived from the statistical model, we can impute wages based on the wage function derived from the structural model, and then repeat the variance decomposition for wages. Simulations confirm that the two adjustment factors obtained when looking at types and wages, respectively, are very similar to one another. Overall, the proposed adjustment approach therefore seems to accomplish the desired goal.

---

[C.1]I choose to base the correction on the earlier sample, yielding a bigger downward adjustment, to avoid overstating the degree of between-firm inequality that the model can generate without assuming ex-ante firm heterogeneity. In addition, I use $\hat{n} = 15$. (The exact value of $\hat{n}$ does not matter much, since for reasonable values of $\hat{n}$ the implied correction factors are very close to each other. The magnitude of the bias rapidly diminishes as $\hat{n}$ grows, as is evident from the above formula.)

[C.2]To match the structural model as closely as possible, I binned the draws in the same way as I did in the structural model and for empirical analyses. Of course, the statistical environment makes it possible to examine the implications of such binning and, reassuringly, it makes little difference to the results.

| Parameter | Baseline | Within-occ ranking | OJS | OJS: constant EE rates |
|-----------|----------|-------------------|-----|------------------------|
| $\rho$ | | | 0.008 | |
| $\omega$ | | | 0.5 | |
| $a_2$ | | | 1.1 | |
| $\delta$ | | | 0.0071 \| 0.0085 | |
| $\gamma$ | 0.434 \| 0.836 | 0.277 \| 0.537 | 0.233 \| 0.315 | 0.215 \| 0.728 |
| $a_0$ | 0.378 \| 0.238 | 0.397 \| 0.239 | 0.400 \| 0.274 | 0.397 \| 0.268 |
| $a_1$ | 1.216 \| 1.559 | 1.194 \| 1.566 | 1.216 \| 1.559 | 1.204 \| 1.550 |
| $\bar{b}$ | 0.739 \| 0.664 | 0.742 \| 0.669 | 0.720 \| 0.654 | 0.721 \| 0.648 |
| $\lambda_u$ | 0.168 \| 0.230 | 0.155 \| 0.214 | 0.152 \| 0.179 | 0.152 \| 0.189 |
| $\lambda_e$ | 0 | 0 | 0.054 \| 0.103 | 0.057 \| 0.063 |

**Table C.1:** Model parameters: baseline and robustness

*Notes.* This table summarizes the parameterization of the model under alternative specifications. The value on the left side of | is for period 1, that on the right side for period 2. Discount rate, bargaining weight, team benefit, and separation rates are identical across specifications.

## C.2   Additional results and robustness checks

### C.2.1   Re-estimation of model under alternative specifications

Table C.1 summarizes the parameterization of the model under the three different specifications considered in Section 4.4. Note that four parameters, namely $\rho$, $\omega$, $a_2$ and $\delta$ are the same across specifications.

### C.2.2   Version with on-the-job search

SETUP. This section briefly summarizes the version of the model with on-the-job search (OJS). In terms of setup, an employed worker may now meet vacancies at a Poisson rate $\lambda_e$. The unconditional rate at which a vacancy, in turn, meets an employed worker of any type is denote $\lambda_{v.e} = \frac{\lambda_e e}{v}$. The equations become a bit lengthy, so I restrict myself to giving two examples, with a complete description available upon request.

For example, the joint value of a firm and a worker of type $x$ is now

$$\rho\Omega_1(x) = f_1(x) - \delta S(x)$$

$$+ (1-\omega)\left[\underbrace{\left(\lambda_{v.u}\int \frac{d_u(\tilde{x}')}{u}S(\tilde{x}'|x)^+ d\tilde{x}'\right)}_{\text{meet unemployed}}\right.$$

$$\left.+\underbrace{\left(\lambda_{v.e}\left(\int \frac{d_{m.1}(\tilde{x}')}{e}\{S(\tilde{x}'|x)-S(\tilde{x}')\}^+ d\tilde{x}' + \int\int \frac{d_{m.2}(\tilde{x}',\tilde{x}'')}{e}\{S(\tilde{x}'|x)-S(\tilde{x}'|\tilde{x}'')\}^+ d\tilde{x}'d\tilde{x}''\right)\right)}_{\text{meet employed}}\right]$$

$$+\omega\lambda_e\left(\int \frac{d_{f.0}}{v}\times \underbrace{\{S(x)-S(x)\}^+}_{x \text{ joins different 0-worker firm}} + \int\int \frac{d_{m.1}(\tilde{x}')}{v}\times \underbrace{\{S(x|\tilde{x}')-S(x)\}^+}_{x \text{ joins different 1-worker firm}} d\tilde{x}'\right).$$

Turning to flows, in the stationary equilibrium the 'density' of matches consisting of a firm with two workers of types $x$ and $x'$ satisfies the following balance condition, where for instance $h_{2.1}(x,x''|x') = \mathbf{1}\{S(x|x')-S(x|x'') > 0\}$ indicates whether a worker $x$ in a two-worker firm with coworker $x''$ would move to an employer that currently has one employee of type $x'$.

$$d_u(x)\lambda_u\frac{d_{m.1}(x')}{v}h_{u.1}(x|x')$$

$$+ d_{m.1}(x)\left\{\lambda_e\frac{d_{m.1}(x')}{v}h_{1.1}(x|x') + \lambda_{v.u}\frac{d_u(x')}{u}h_{u.1}(x'|x) + \lambda_{v.e}\left(\frac{d_{m.1}(x')}{e}h_{1.1}(x'|x)\right.\right.$$

$$\left.\left.+\int \frac{d_{m.2}(x',\tilde{x}'')}{e}h_{2.1}(x',\tilde{x}''|x)d\tilde{x}''\right)\right\}$$

$$+\int d_{m.2}(x,\tilde{x}'')\lambda_e\frac{d_{m.1}(x')}{v}h_{2.1}(x,\tilde{x}''|x')d\tilde{x}''$$

$$= d_{m.2}(x,x')\left\{2\delta + \lambda_e\left(\int \frac{d_{f.0}}{v}h_{2.0}(x,x') + \int\int \frac{d_{m.1}(\tilde{x}'')}{v}h_{2.1}(x,x'|\tilde{x}'')d\tilde{x}''\right)\right.$$

$$\left.+\lambda_e\left(\int \frac{d_{f.0}}{v}h_{2.0}(x',x) + \int\int \frac{d_{m.1}(\tilde{x}'')}{v}h_{2.1}(x',x|\tilde{x}'')d\tilde{x}''\right)\right\}.$$

IMPLICATIONS FOR MATCHING PATTERNS. Figure 10 seems to indicate that the version with OJS does a better job at capturing empirical coworker sorting patterns. Here I offer a brief explanation. In essence, with OJS matching patterns better reflect the heterogeneity in surplus across matches. Absent OJS, match outcomes do not fully reflect preferences over these outcomes. Specifically, when the match outcome is a binary comparison of the values from being unmatched or matched, then these outcomes do not express information about who among the accepted matching partners I would rather match with; and who among the rejected

ones I like even less than the other. By contrast, with OJS, preferences over match outcomes are also encoded in job-to-job transitions, which are determined by a comparison of match surpluses at alternative employers, respectively with different coworkers. These moves thus contain additional information about heterogeneity in surplus among the partners inside the acceptance set coming out of unemployment. In that sense, the stationary match distribution better reflects preferences over match outcomes.

TRADE-OFFS AND FUTURE RESEARCH. While incorporating OJS improves the model's capacity to fit empirical coworker sorting patterns, this comes at a twofold cost. First, unlike the baseline model, the version with OJS predicts a *level* of the between-firm share of wage inequality that exceeds what is observed in the data (even after correcting for a mechanical upward bias discussed in Appendix C.1.3). While this does not impair the model's ability to speak to *changes* over time, it does highlight a shortcoming of the model. I conjecture that this limitation partly reflects that the model does not incorporate a vertical dimension of firm organization as in Garicano and Rossi-Hansberg (2006), which would plausibly generate more within-firm wage dispersion. Future work that takes serious within-firm worker heterogeneity both across and within layers of the firm, perhaps by modelling the interaction between multiple teams, might improve on this dimension.

The second issue relates to the mapping between production and wage complementarities introduced in Corollary 3. While "origin" effects that differentiate workers' in terms of their bargaining position (e.g., unemployment vs. jobs yielding varying levels of surplus) leave this mapping unaffected – at least under the bargaining protocol assumed here – a different issues arises relating to anticipation effects. Specifically, the interaction between own type and coworker type influences the wage not only *directly* through the production value but also *indirectly*. The indirect effect arises because the coworker match quality influences the probability that either team member switches jobs conditional on receiving an outside offer. I next show this more carefully. Under the assumption that wages are continuously renegotiated under Nash bargaining, with unemployment as a worker's outside option, it is still possible to characterize the wage of a worker of type $x$ employed at a firm with another worker of type $x'$ in closed form:

$$
\begin{aligned}
w(x|x') = {}& \rho V_u(x) + (\rho + 2\delta)\omega S(x|x') \\
& - \omega S(x) \times \left( \delta + \lambda_e \int \frac{d_{f.0}}{v} h_{2.0}(x', x) + \lambda_e \int \int \frac{d_{m.1}(\tilde{x}'')}{v} h_{2.1}(x', x|\tilde{x}'')d\tilde{x}'' \right) \\
& - \omega \lambda_e \int \frac{d_{m.1}(\tilde{x}'')}{v} h_{2.1}(x, x'|\tilde{x}'')S(x|\tilde{x}'')d\tilde{x}''
\end{aligned}
$$

Here, the term $h_{2.1}(x', x|\tilde{x}'')$ is directly shaped by $S(x|x')$ and similarly for $h_{2.1}(x, x'|\tilde{x}'')$. Hence, $\frac{\partial^2 w(x|x')}{\partial x \partial x'}$ moves more than proportionately with $\frac{\partial^2 f(x|x')}{\partial x \partial x'}$ insofar as $\lambda_e > 0$. What underlies this

effect is the assumption that wage-setting is very forward-looking, with wages fully reflecting already today the expected change in future values of employer and worker(s) due to future events such as exogenous job separations or endogenous job moves. The reasoning in Hall and Milgrom (2008), among other things, leads me to doubt the empirical force of such forward-looking effects. But introducing a "partially myopic", alternative wage sharing rule à la Hall and Milgrom (2008) is difficult to reconcile with match formation being privately efficient and based on joint surplus maximization. I view this as a potentially important albeit challenging direction for future theoretical work, alongside an exploration of alternative bargaining protocols (e.g., Cahuc *et al.*, 2006).

Of course, in the structural estimation it is still possible to treat $\frac{\partial^2 w(x|x')}{\partial x \partial x'}$ as moment that is highly informative about the strength of production complementarities. That is indeed how I proceed in the re-estimation of an extended model that includes OJS. Since, however, the estimation targets an *increase* in EE rates from 0.0076 to 0.0106, matching which requires a rise in $\lambda_e$, this procedure yields a smaller increase in the estimated strength of *production* complementarities than the models without OJS do. The final column in Table C.1 shows that if, instead, the EE rate is kept constant, then the model with OJS predicts an increase in $\gamma$ that is very similar to the baseline model. Under this calibration, the model predicts 12.5 percentage point increase in the between share from period 1 to period 2, of which only 2.1 points would have occurred had the estimated value of $\gamma$ remained at the period-1 level. Thus, in this model complementarities rationalize around 45% of the empirically observed rise in the between-firm share of wage inequality, a value that is very similar to the baseline model.

### C.2.3 Extension: $n_{\max} = 3$

In the baseline model, firms can have zero, one or, at most, two employees. The associated assumption of extreme decreasing returns to scale setting in at $n_{\max} = 2$ facilitates a transparent discussion of the key link between complementarities and matching decisions, but it is primarily imposed for reasons of tractability. In particular, as the number of potential employees increases, the potential combinations of worker types and, hence, the state space explodes.

This section shows that the key model properties are robust to allowing at least for $n_{\max} = 3$.[C.3] In particular, Figure C.4 plots the average coworker quality for different types of workers, under alternative model specifications. Depicted in solid grey is the baseline model; in dashed orange the model with $n_{\max} = 3$, considering teams of size 2 or 3; and in dashed-dotted light orange the model with $n_{\max} = 3$ but considering only teams of size 2. As can be seen, the equilibrium sorting implications are very similar when raising the maximum team size.

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[C.3]The equations describing the stationary equilibrium become messier but are conceptually straightforward extensions of the $n_{\max} = 2$ case.
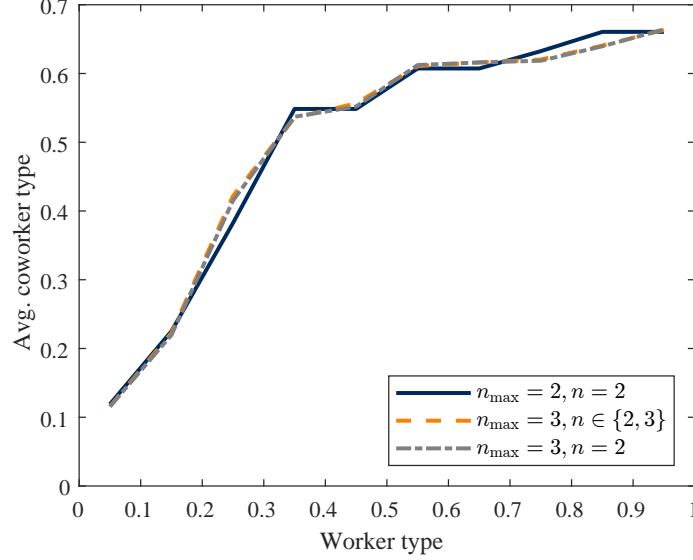
**Figure C.4:** Equilibrium coworker sorting patterns when $n_{\max} = 3$

*Notes.* This figure plots the average coworker type (vertical axis) for different worker types (horizontal axis), as implied by the equilibrium matching patterns emerging from the baseline model with maximum team size 2 ($n_{\max} = 2$) and the alternative model with maximum team size 3 ($n_{\max} = 3$).

# D   The task content of production in Germany

To document patterns in the task content of production I draw on the Employment Surveys (ES) carried out by the German Federal Institute for Vocational Training (Bundesinstitut fuer Berufsbildung, BIBB; Hall and et al. (2018)).[D.1] Following the influential methodology introduced in Autor *et al.* (2003) and first applied to the ES by Spitz-Oener (2006), I use the tasks that employees report to have performed to measure variations in the nature of work.

The BIBB surveys have several attractive features: they provide detailed information on tasks performed at work; the survey has been run, in repeated waves, since 1985 (1985/86, 1991/92, 1998/99, 2006, 2012, and 2018), facilitating time series analyses; each wave has a large sample size between 20,000 to over 30,000 respondents per wave, facilitating between-group comparisons; responses are at the *worker-level* and consistent occupation codes can be used across multiple waves, making it possible to capture changes in nature of work not only associated with employment shifts across occupations but also within-occupation (on the importance of which see, e.g., Spitz-Oener (2006); Atalay *et al.* (2020)); and a supplemental survey in 2012 allows enriching binary task indicators with information on the actual shares of time spent by employees in different occupations on various tasks.

The analysis uncovers several key findings, which I summarize next before discussing details:

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[D.1] I thank the Research Data Center of the BIBB for providing access to scientific use files as well as guidance. All remaining errors are my own.

*(i)* The aggregate usage share of complex tasks in workers' activities has monotonically risen since 1985/86; the 1990s saw a particularly sharp increase.

*(ii)* This trend is prevalent across different levels of education. It is not driven by occupational employment effects either, instead the majority of the increase occurs within-occupation.

*(iii)* In the cross-section, the task portfolio of more educated individuals tends to be disproportionately skewed toward complex tasks compared to less educated individuals. The ranking of different occupations is intuitive and likewise reveals large variation in task shares.

*(iv)* Cross-sectional differences are robust to using measures of time spent on different tasks.

Altogether, these results provide reassurance that the time trend is robust and not merely the result of composition effects, and that there is substantial cross-sectional variation that can be exploited as done in the main text.

## D.1 Methodology

SAMPLE RESTRICTIONS. As detailed in Rohrbach-Schmidt and Tiemann (2013) and Hall and Rohrbach-Schmidt (2020), time comparisons with the BIBB/IAB surveys require a standardization of the sample basis. To that end, I follow the steps detailed in those reports and focus on employees from West Germany, aged 15 to 65, who belonged to the labor force (defined as having a paid employment situation) with a regular working time of at least ten hours per week.[D.2] The final sample comprises 91,152 worker-year observations.

In addition, I entirely omit the 1998/99 wave from my analysis, because the number of activities queried in that wave is substantially lower than in the other surveys. While doing so reduces the overall sample size, this choice avoids bias to the results due to the limited comparability in tasks. For example, none of the activities "accommodating", "caring", "storing", "protecting", "programming" and "cleaning" were queried in the 1998/99 survey.[D.3]

TASK CLASSIFICATION. As the careful discussion in Rohrbach-Schmidt and Tiemann (2013) makes clear, comparisons over task intensities using the BIBB ES over time need to be implemented carefully and account for variation over time in what tasks are queried and whether their content has changed in meaning. Writing in the context of typical studies that compare task items in the categories non-routine analytical, non-routine interactive, non-routine manual,

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[D.2]In addition, I drop observations who report having performed none of the activities queried in at least two waves. Given the extensive use of occupational codes, I also drop any occupations with fewer than thirty observations across all waves.

[D.3]I thank Daniela Rohrbach-Schmidt for her generous advice on how to handle the older waves and for sharing programs illustrating how the scientific use files can be rendered maximally consistent with the original data.

routine-cognitive and routine-manual, the authors highlight in particular that routine-cognitive tasks are difficult to classify (e.g. "measuring" may be routine-cognitive or routine-manual; also see the findings of Antonczyk *et al.* (2009) in comparison to those by Spitz-Oener (2006)).[D.4] Given my focus on non-routine or complex tasks, these classification problems are less severe though, as "these items are regularly observable throughout the cross-sections, their content did not change significantly from year to year, and measurement validity is comparatively strong," as Rohrbach-Schmidt and Tiemann (2013) note when suggesting to researchers to focus on the increase in these tasks.

| Task classification | Task name | Description |
|---|---|---|
| Complex/abstract | investigating | Gathering information, investigating, documenting |
| | organizing | Organizing, making plans, working out operations, decision making |
| | researching | Researching, evaluating, developing, constructing |
| | programming | Working with computers, programming |
| | teaching | Teaching, training, educating |
| | consulting | Consulting, advising |
| | promoting | Promoting, marketing, public relations |
| Other | repairing, buying, accommodating, caring, cleaning, protecting, measuring, operating, manufacturing, storing, writing, calculating | |

**Table D.1:** Classification of tasks in the BIBB Employment Surveys

*Notes.* This table summarizes the classification of tasks into two groups: "Complex/abstract" and "Other."

As summarized in Table D.1, I therefore collect task items in the index of abstract tasks — guided by the classification of non-routine tasks in Spitz-Oener (2006), Rohrbach-Schmidt and Tiemann (2013) and Atalay *et al.* (2020) – and compare those with all other tasks, i.e., those that are broadly categorized as routine or manual.[D.5]

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[D.4]Autor and Handel (2013) also treat the "physical" dimension of tasks as a combined measure of physical and routine tasks. Meanwhile, Acemoglu and Autor (2011) subsume non-routine analytical and non-routine interactive into "abstract", while routine-cognitive and routine-manual tasks are subsumed into "routine".

[D.5]I do not use the task items "managing", "applying law" and "negotiating", because they are only measured in the early waves. Moreover, I associate buying/selling with "other", since even though they may be hard to automate (even that seems questionable in light of self-checkouts and e-commerce), they are arguably not among the most complex activities. This decision makes no practical difference to the results.

TASK INDEX. Given this classification, I then define an index capturing the usage of abstract/-complex tasks for worker $i$ in period $t$, following Antonczyk *et al.* (2009):

$$T_{it}^{\text{abstract}} = \frac{\text{number of activities performed by } i \text{ in task category "abstract" in sample year } t}{\text{total number of activities performed by } i \text{ in sample year } t}$$

To illustrate, if worker $i$ performs five distinct activities in sample period $t$ and two of those belong to the category of abstract/complex tasks, then the complexity index for her work is 0.4.

OCCUPATIONAL CLASSIFICATION. To ensure a consistent classification of occupations when using information from multiple waves, I use the German Classification of Occupations 1988 (KldB88). As the oldest classification available in the two most recent waves (2012 and 2018) is the KldB92 classification (Hall and Rohrbach-Schmidt, 2020, cf. Table 9), in processing these two waves I rely on a conversion table KldB92→KldB92; the conversion quality is high as the two classifications are very similar.[D.6]

## D.2 Results

PATTERNS OVER TIME. The first column in Table D.2 indicates that the aggregate usage share of complex tasks in workers' activities has monotonically increased from 1986 to 2018, with the increase being particularly pronounced in the first half of the time period.

The second to fourth columns decompose the period-by-period change in the importance of complex tasks into two components: a "between" component that captures shifts in occupational employment shares and a "within" component that measures changes in the task content within occupations. Formally, as in Atalay *et al.* (2020), I decompose changes in the usage of abstract tasks between periods $t$ and $t-1$ according to the equation

$$\Delta \bar{T}_t^{\text{complex}} = \sum_o \omega_{o,t-1}(\bar{T}_{t,o}^{\text{abstract}} - \bar{T}_{t-1,o}^{\text{complex}}) + \sum_o (\omega_{o,t} - \omega_{o,t-1})\bar{T}_{t,o}^{\text{abstract}}$$

where $\bar{T}_{t,o}^{\text{complex}}$ measures the average usage of complex tasks by members of occupation $o$ in period $t$ and $\omega_{o,t}$ is the period-$t$ employment share of occupation $o$.

Consistent with the findings of Atalay *et al.* (2020) for the US, this decomposition reveals that about three quarters of the increase in complex tasks over the sample period have occurred *within* occupations.

Education offers an alternative lens through which to view the changing task content. As shown already in Figure 3a in the main text, the share of complex tasks in the portfolio of

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[D.6]This crosswalk is based on the Klassifikationsserver der Statistischen Ämter des Bundes und der Länder, current occupations coded in the 2006 wave in which both KldB88 and KldB92 are available as well as personal judgements. I thank Anett Friedrich for creating and sharing the crosswalk.

|              | Total | Between | Within | Within-share |
|--------------|-------|---------|--------|--------------|
| 1986 level   | 0.252 |         |        |              |
| 1986-1992    | 0.025 | 0.002   | 0.022  | 0.906        |
| 1992-2006    | 0.298 | 0.057   | 0.241  | 0.809        |
| 2006-2012    | 0.019 | 0.002   | 0.017  | 0.890        |
| 2012-2018    | 0.053 | 0.028   | 0.025  | 0.476        |
| Total change | 0.395 | 0.089   | 0.306  | 0.775        |

**Table D.2:** The evolving task content of production in Germany

*Notes.* This table reports the within-between occupation decomposition of the change in the share of complex tasks over time. The "Total" column aggregates across all individuals. The decomposition is performed at the level of KldB-1988 2-digit occupations.

university-educated individuals is substantially greater than that of persons with less formal education. The *increase* over time takes place across the board, however.

CROSS-SECTIONAL PATTERNS. The share of complex tasks also varies substantially by occupation. I compute the average complex-task shares at the ISCO-08 2-digit level in the waves 2012 and 2018. Table D.3 lists the bottom 5 and top 5 occupations. In addition, I also show the non-routine abstract score from Mihaylov and Tijdens (2019) used in Section 3.4. The comparison reveals large and consistent variation in task shares across occupations according to either measure.

TIME USAGE. One concern is that the analysis only considered whether a given task represents an important activity in the respondent's job, as opposed to measuring how important that activity is relative to others. To address this concern, I draw on a supplemental survey from 2012 that precisely details the amount of time a subset of workers spent on the different tasks on a given day. Figure D.1 charts the shares of time spent on the seven abstract/complex activities for different occupational groups. Specifically, I rank occupations according to their task index and group them into 4 equally sized groups. Drawing on the supplemental survey, I then compute the average share of time members of these occupational groups spent on the various tasks. Figure D.1 shows that at least in more recent periods, each occupational group spends some time on such tasks as organizing or using the computer ("programming"). Crucially, the fraction of time spent on each of these tasks is multiples greater for the top quartile than for the bottom quartile. No one single task drives the overall increase in the complexity index.

| ISCO-08 2-digit occupation | $\bar{T}_o^{complex}$ | MT-NRA |
|---|---|---|
| Business and administration professionals | 0.84 | 0.47 |
| Legal, social and cultural professionals | 0.83 | 0.67 |
| Business and administration associate professionals | 0.82 | 0.29 |
| Teaching professionals | 0.81 | 0.57 |
| Administrative and commercial managers | 0.81 | 0.58 |
| ... | ... | ... |
| Drivers and mobile plant operators | 0.2 | 0 |
| Agricultural, forestry and fishery labourers | 0.14 | 0 |
| Food preparation assistants | 0.14 | 0 |
| Market-oriented skilled forestry, fishery and hunting workers | 0.12 | 0 |
| Cleaners and helpers | 0.12 | 0 |

**Table D.3:** Top- and bottom-5 occupations in terms of task complexity

*Notes.* This table reports the top-5 and bottom-5 ISCO-08 2-digit occupations when ranked by $\bar{T}_o^{complex}$ in pooled 2012 and 2018 waves. The column "MT-NRA" shows the non-routine abstract score taken from Mihaylov and Tijdens (2019) after collapsing to the ISCO-08-2d level using occupational employment shares (computed using the Portuguese data, in lne with the application in Section 3.4.
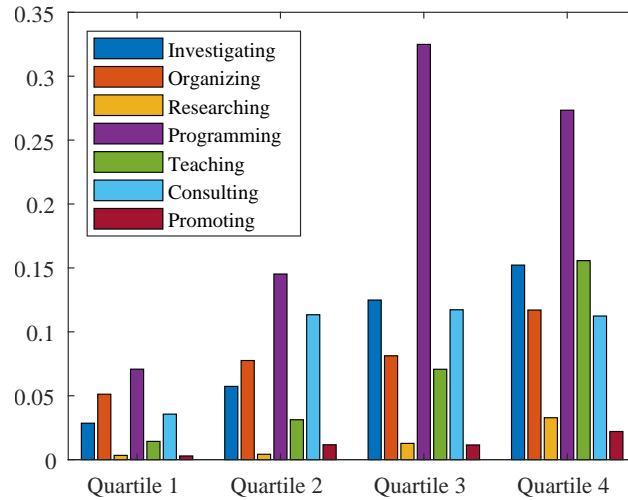


**Figure D.1:** Allocation of time to complex tasks by occupational groups

*Notes.* This table reports the share of time spent on complex tasks by different occupational groups. Occupations are first ranked according to their task complexity index and grouped into four groups of approximately equal size. Then the average share of time members of these occupational groups spend on the various tasks labelled as "complex" in Table D.1 is computed.