Mathematical model of snowflakes

Non-Riemannian

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1 AN OVERVIEW OF MATHEMATICAL MODEL

Axioms 1.1. *Let us have coordinates $\mathbb{G}$ in Euclidean space, which are a deformation of the coordinates $\mathbb{R}$ with the deformation gradient*

$$\mathbb{G} = \mathbb{R}\begin{bmatrix} 1 & 0 \\ \frac{1}{\sqrt{3}} & 1 \end{bmatrix} \tag{1}$$

then points ${}_{a^*}\gamma_M^n \in \mathbb{G}$, for $a, n \in \mathbb{Z}_0^+, M := \{1,2\}$, which I will denote as follows $\Gamma_{a^*}^m$, where $m \in \mathbb{Z}_0^+$, then $G_1^{a^*} :=\longleftrightarrow_{a^*} \gamma_1^n {}_{a^*}\gamma_1^k$, where $n \neq k$, these relationships apply

(i) ${}_{a^*}\gamma_1^n \in G_1^{a^*}, {}_{a^*}\gamma_2^k \in G_2^{a^*}, n \neq k,$ *kde* $\sphericalangle {}_{a^*}\gamma_1^n {}_{a^*}\gamma_2^k = \frac{\pi}{3}$

(ii) ${}_{a^*}\gamma_M^n \forall {}_{a^*}\gamma_M^k (S({}_{a^*}\gamma_M^n) = S({}_{a^*}\gamma_M^k)) \Rightarrow {}_{a^*}\gamma_M^n =_{a^*} \gamma_M^k$

(iii) $\forall {}_{a^*}\gamma_M^n ({}_{a^*}\gamma_M^n +_{a^*} \gamma_M^0 =_{a^*} \gamma_M^n,\ {}_{a^*}\gamma_1^0 + =_{a^*} \gamma_2^0,,$ *ale* $\forall n, k \neq 0, {}_{a^*} \gamma_1^n \neq_{a^*} \gamma_2^k$

(iv) $\forall {}_{a^*}\gamma_M^n \forall_M^k ({}_{a^*}\gamma_M^n + S({}_{a^*}\gamma_M^k) = S({}_{a^*}\gamma_M^n +_{a^*} \gamma_M^k) = S({}_{a^*}\gamma_M^{n+k})$

$\forall G_M^{n*} \| G_M, G_1 \cup G_2 \cong G_1^{n*} \cup G_2^{n*}, {}_{n^*} \gamma_M^k \in G_M^{n*}$, while $\Gamma^{\psi(n)} = \Gamma_{k*}^0$, then $\Gamma_{*a}^n =_{a^*} \gamma(x,y) = ({}_{a^*}\gamma_1^x, {}_{a^*}\gamma_2^y)$

Definition 1.1 (ψ function)**.** *Let function $\psi(x)$ is a surjective-noninjective function at x that satisfies the following relations*

(i) *for $x \in \mathbb{Z}^+$,*

$$x = 1 + \sum_{i<x} \psi(i) \tag{2}$$

(ii) *for $x \in \mathbb{Z}^+$,*

$$\psi(x) = \begin{cases} 1 & , x = 0 \\ n & , n \in \mathbb{Z}^+, 6(n-1)+1 \leq x \leq 6(n-1)+4 \\ n+1 & , n \in \mathbb{Z}^+, x = 6(n-1)+5 \\ n+2 & , n \in \mathbb{Z}^+, x = 6(n-1)+6 \end{cases} \tag{3}$$

Subsequently, it is important for the model to introduce the notion of oriented vectors.

$$\vec{e}_0 = \begin{pmatrix} \Gamma^1 \\ \Gamma^2 \end{pmatrix}, \vec{e}_x = \begin{pmatrix} \Gamma^{\psi(x)} \\ \Gamma^{\psi(x+1)} \end{pmatrix} \tag{4}$$

then I define a rotation for which $n \in \mathbb{Z}_0^+; k \in \mathbb{Z}^+; \Gamma^{\psi(n)} = \Gamma^0_{k^*}$, where h is fixed point

$$\begin{pmatrix} \Gamma^{\psi(n)} \\ \Gamma^{\psi(n+1)} \end{pmatrix} \times \frac{(n \mod 6)}{3} \pi \times h = \begin{pmatrix} \Gamma^0_{k^*} \\ \Gamma^{3h^2+[n \mod 6-3]h+1}_{k^*} \end{pmatrix} \tag{5}$$

so

$$\vec{e}_1 = \vec{e}_0 \times \frac{2}{3}\pi \times \psi(1) = \begin{pmatrix} \Gamma^2 \psi(1) \\ \Gamma^3 \psi(1) \end{pmatrix} \times \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}, n = 1 \tag{6}$$

$$\vec{e}_n = \vec{e}_{n-1} \times \frac{1}{3}\pi \times \psi(n) = \begin{pmatrix} \Gamma^{\psi(n-1)} \psi(n-1) \\ \Gamma^{\psi(n)} \psi(n-1) \end{pmatrix} \times \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}, n > 1 \tag{7}$$

then the relation holds

$$\begin{pmatrix} \Gamma^{\psi(n)} \\ \Gamma^{\psi(n+1)} \end{pmatrix} = \begin{pmatrix} \Gamma^0_{k^*} \\ \Gamma^{3\psi(n)^2+2\psi(n)+1}_{k^*} \end{pmatrix} \tag{8}$$

$\forall G_M^{n*} \| G_M, G_1 \cup G_2 \cong G_1^{n*} \cup G_2^{n*},_{n^*} \gamma_M^k \in G_M^{n*}$,
while $\Gamma^{\psi(n)} = \Gamma_{k*}^0$, then $\Gamma_{*a}^n =_{a^*} \gamma(x,y) = ({}_{a^*}\gamma_1^x,{}_{a^*}\gamma_2^y)$