A Multi-Commodity Partial Equilibrium Model of Imperfect Competition in Future Global Hydrogen Markets

Supplementary Material

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List of Acronyms

MCP . . . . . Mixed Complementarity Problem

QP . . . . . Quadratic Optimization Problem

1 Implementation and Validation

1.1 Overview and Dependencies

Implementation of the model relies upon several external packages. Reading and writing data is handled via the Julia `XLSX`[1] and `CSV`[2] packages.

Some of the data-related tasks are handled via the `DataFrames` package (Bouchet-Valat and Kamiński 2023), while plotting utilities are managed via `Makie`, `CairoMakie` and `GeoMakie` (Danisch and Krumbiegel 2021).

Model building is handled via the `JuMP` package (Lubin et al. 2023; Lubin and Dunning 2015; Dunning, Huchette, and Lubin 2017), and `MathOptInterface` (Legat et al. 2021), as well as the `Complementarity` package[3]. All packages are officially registered and hence easily available.

While the PATH solver (Dirkse and Ferris 1995) is automatically shipped as a dependency to the `Complementarity` package, an additional solver is required to solve the QP version.[4] Some open source options include e.g. OSQP (Stellato et al. 2020) or Ipopt (Wächter and Biegler 2005), while Gurobi provides a very performant commercial but free to academics option (Gurobi Optimization, LLC 2023).

1.2 Model Usage

1.2.1 Examples

Listings 1 and 2 depict exemplary uses of model codes for both the QP and MCP versions. When performance matters, it is recommended to utilize the QP version, as it is more performant in terms of both solution procedures, as well as model building and solution retrieval.[5]

[1] https://github.com/felipenoris/XLSX.jl.

[2] https://github.com/JuliaData/CSV.jl.

[3] https://github.com/chkwon/Complementarity.jl.

[4] Note, that despite being shipped with the `Complementarity` package, the PATH solver is limited in the number of variables if no license string is provided via the environment variable `PATH_LICENSE_STRING`. The corresponding environment variable may either be set on the machine, or via `ENV["PATH_LICENSE_STRING"] = "license_string"` from within Julia. At the time of writing, an official license string is available from https://pages.cs.wisc.edu/~ferris/path/LICENSE.

[5] This is due to more efficient implementations within the JuMP package, which is further developed than the extending Complementarity.jl package.

Listing 1: Exemplary Use of the QP Model Version

```
# The Package can be installed by downloading the package repository and typing:
# ]dev /path/to/the/dir
# in the REPL.
# To use the software package, type:
using GH2TMOD
# In addition, a solver that is capable of solving QPs is needed (such as Gurobi or Ipopt):
using Gurobi
# To interact with the solved model instance, JuMP is helpful:
using JuMP

# we can get the data by typing the following line:
# To get a different data file, use the keyword data = /path/to/some/datafile
data = get_GH2TMOD_data();
# data = get_GH2TMOD_data(data = "path/to/some/file");

# to store the data in a directory to be specified, use the following function:
write_csv("path/to/calculated/parameters",data)

# to build a JuMP Model with the data, use the following function:
model = build_optimization_model(data)

# to solve the JuMP Model with a solver of your choice, run the following function:
optimize!(model, Gurobi.Optimizer)

# you can also specify different solver options like this:
options = ("TimeLimit" => 100,"TimeLimit" => 100, "FeasibilityTol" => 1e-6)
optimize!(model, Gurobi.Optimizer, options)

# to write the results to a file, use the following function:
write_csv("path/to/resultsfile", model)

# you can further interact with the solved model like this:
prices = value.(model[:prices])

# to plot an overview of the model nodes, use this function:
plot_model_nodes()

# both arguments nodes_path and disputed_path can be specified to point to your own files:
plot_model_nodes(nodes_path = "path/to/nodes/file", disputed_path = "path/to/disputed/file")

# to plot results (e.g. prices) into the map, use the following function:
```

```
plot_model_nodes(prices[:, "CNG", "Block 1", "OnlySeason", 2020])

# This also works with custom model node files:
plot_model_nodes(prices[:, "CNG", "Block 1", "OnlySeason", 2020], nodes_path = "path/to/nodes/file", disputed_path = "path/to/disputed/file")

# To all plot_model_nodes, a path where to save the figure can be specified:
plot_model_nodes(prices[:, "CNG", "Block 1", "OnlySeason", 2020], save_path = "path/to/save/fig.pdf")

# for further examples, please check out the test sections or the implementation chapter...
```

Listing 2: Exemplary Use of the MCP Model Version

```
# Before using the package, a valid license string to the PATH solver needs to be specified:
ENV["PATH_LICENSE_STRING"] = "Copy_license_string_here"

# The Package can be installed by downloading the package repository and typing:
# ]dev /path/to/the/dir
# in the REPL.
# To use the software package, type:
using GH2TMOD

# To interact with the solved model instance, JuMP and Complementarity are helpful:
using JuMP, Complementarity

# we can get the data by typing the following line:
# To get a different data file, use the keyword data = /path/to/some/datafile
data = get_GH2TMOD_data();
# data = get_GH2TMOD_data(data = "path/to/some/file");

# to store the data in a directory to be specified, use the following function:
write_csv("path/to/calculated/parameters",data)

# to build a JuMP Model with the data, use the following function:
model = build_complementarity_model(data)

# to solve the JuMP Model with a solver of your choice, run the following function:
solve_mcp(model)
```

```julia

# you can also specify different solver options like this:
options = (output = "no", time_limit = 5)
solve_mcp(model; options)

# to write the results to a file, use the following function:
write_csv("path/to/resultsfile", model)

# you can further interact with the solved model like this:
prices = result_value.(model[:π_T_to_D])

# to plot an overview of the model nodes, use this function:
plot_model_nodes()

# both arguments nodes_path and disputed_path can be specified to point to your own files:
plot_model_nodes(nodes_path = "path/to/nodes/file", disputed_path = "path/to/disputed/file")

# to plot results (e.g. prices) into the map, use the following function:
plot_model_nodes(prices[:, "CNG", "Block 1", "OnlySeason", 2020])

# This also works with custom model node files:
plot_model_nodes(prices[:, "CNG", "Block 1", "OnlySeason", 2020], nodes_path = "path/to/nodes/file", disputed_path = "path/to/disputed/file")

# To all plot_model_nodes, a path where to save the figure can be specified:
plot_model_nodes(prices[:, "CNG", "Block 1", "OnlySeason", 2020], save_path = "path/to/save/fig.pdf")

# for further examples, please check out the test sections or the implementation chapter...
```

1.3 Documentation of User-Facing Functions

All user-facing functions are documented within the code using docstrings. This allows the user to interactively search the Julia documentation in help mode. Listings 3 to 10 in Section 3 provide an overview of callable function definitions.

1.4 Validation

In order to make sure that both mathematical derivations, as well as software implementation yield the desired solutions, the validation procedures carried out to verify model mechanics are twofold.

In the first part, 28 test cases are designed, each targeting specific parts of the model. Table 1 presents an overview of the test cases, while more detailed documentation, including input parameters, graphical depictions of the solution and testing criteria are shown in Sections 2.2.1 to 2.2.28.

Further, additional functionality tests are carried out. These relate to cost scaling of investment and operational cost, data exports and plotting utilities. Table 2 presents an overview of the detailed descriptions to be found in Sections 2.3.1 to 2.3.5.

Validation happens via an automated testing routine that can be run with the standard package testing procedures in Julia. To run the tests, one simply needs to activate the package in `Pkg`-Mode and type `test`.

Table 1: Overview of Test Cases

Test Case	**Purpose**	**Appendix**
Test Case 1	Tests inverse demand and cost data.	Section 2.2.1
Test Case 2	Tests multiple years with weights and includes conversion costs.	Section 2.2.2
Test Case 3	Tests exertion of market power with time step weights.	Section 2.2.3
Test Case 4	Tests discounting.	Section 2.2.4
Test Case 5	Tests storage and multiple time steps with more low demand weight.	Section 2.2.5
Test Case 6	Tests storage and multiple time steps with more high demand weight.	Section 2.2.6

Continued on next page...

Table 1: Overview of Test Cases (continued)

Test Case	**Purpose**	**Appendix**
Test Case 7	Tests same commodity from different origins.	Section 2.2.7
Test Case 8	Tests multiple demand commodities from same flow commodity.	Section 2.2.8
Test Case 9	Tests transport mechanisms including losses on arcs with time step weights.	Section 2.2.9
Test Case 10	Tests storage losses including multiple time steps.	Section 2.2.10
Test Case 11	Tests production capacity expansion, scaling of investment costs and asset lifetimes.	Section 2.2.11
Test Case 12	Tests conversion capacity expansion, scaling of investment costs and asset lifetimes.	Section 2.2.12
Test Case 13	Tests arc capacity expansion, scaling of investment costs and asset lifetimes.	Section 2.2.13
Test Case 14	Tests storage capacity expansion, scaling of investment costs and asset lifetimes.	Section 2.2.14
Test Case 15	Tests production capacity expansion limits.	Section 2.2.15
Test Case 16	Tests storage capacity expansion limits.	Section 2.2.16
Test Case 17	Tests trader sanctions.	Section 2.2.17
Test Case 18	Tests market power exertion in foreign markets including arc capacity expansion.	Section 2.2.18
Test Case 19	Tests restrictions by origin.	Section 2.2.19

Continued on next page...

Table 1: Overview of Test Cases (continued)

Test Case	**Purpose**	**Appendix**
Test Case 20	Tests repurposing of conversion facilities, including repurposing factors.	Section 2.2.20
Test Case 21	Tests repurposing of transportation arcs, including repurposing factors.	Section 2.2.21
Test Case 22	Tests repurposing of storage facilities, including repurposing factors.	Section 2.2.22
Test Case 23	Tests piece-wise linear operational cost for input procurement.	Section 2.2.23
Test Case 24	Tests demand from multiple blocks.	Section 2.2.24
Test Case 25	Tests input investment costs, availabilities and expansion limits.	Section 2.2.25
Test Case 26	Tests commodity adjacency structures for both pipeline and shipping.	Section 2.2.26
Test Case 27	Tests adjacency structures for arc capacity expansion.	Section 2.2.27
Test Case 28	Tests adjacency structures for arc capacity repurposing.	Section 2.2.28

Source: Own depiction.

Table 2: Overview of Functionality Tests

Functionality Test	**Purpose**	**Appendix**
Functionality Test 1	Tests correct scaling of operational and investment costs.	Section 2.3.1

Continued on next page...

Table 2: Overview of Functionality Tests (continued)

Functionality Test	**Purpose**	**Appendix**
Functionality Test 2	Tests input parameter exports for function `write_csv(datafile, data::ModelData)`.	Section 2.3.2
Functionality Test 3	Tests that model outputs for the QP implementation are stored correctly.	Section 2.3.3
Functionality Test 4	Tests that model outputs for the MCP implementation are stored correctly.	Section 2.3.4
Functionality Test 5	Tests that map plotting functionality works as expected.	Section 2.3.5

Source: Own depiction.

2 Detailed Description of Validation Procedures

2.1 General Remarks

The following sections provide a detailed account of the validation procedures. Two types of tests are carried out, the first one being fully scoped model tests, while the second tests specific functionalities. Section 2.2 contains 28 data sets with explanations regarding the desired solution. Every test has specific testing criteria that ensure the model works as desired with respect to specific functionalities. With some abuse of economic syntax, the calculations underlying each case in Section 2.2 use the notion of marginalized costs. Here, marginalized costs refer to the inclusion of investment costs into marginal costs. While this step is rarely ever plausible and hence likely counterintuitive to many economists, the specific design of test cases allows for this measure. We will see that this trick comes in handy when visualizing the results of individual test cases. Consider the following simple example: A good is to be provided in a certain period. Ex-ante, there do not exist any production capacities, and all investments have a lifetime such that they remain active only for a single period. It is only rational to invest as much capacity as is needed in the next time period. Hence, investment costs may also be viewed as a special type of operational cost in this case. Considering unit cost, an investment of

5 units of money to provide 5 units of commodity comes down to operational costs of 1 unit of money per 1 unit of commodity. This aggregate of operational and marginalized investment costs is henceforth termed marginalized costs.
Every test case can be run for both, the optimization model formulation and as an MCP. However, tests for the MCP formulation are only run for variable values (such as quantities or prices). Tests for e.g. the objective value are not run due to a lack of the underlying objective function.
Section 2.3 tests other specific functionalities such as plotting or reporting of results.

2.2 Test Cases

2.2.1 Test Case 1

Test Case 1 comprises a simple example. Table 3 depicts the model sets, while Table 4 gives relevant parameters. If the model is solved correctly, marginalized production costs should be equal to the price. Test Case 1 passes, if prices in the single node are equal to the intersection of marginalized provision cost and inverse demand, see also Figure 1. Marginalized provision cost at factor intensities of 1:1 are $MC = c^{I_l} + c^{I_q}Q + c^P$, while prices are $P(Q) = \alpha^D + \beta^D Q$. Hence, for the given data, prices should be $P = 1.5$, see also Table 5.

Table 3: Sets of Test Case 1

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU"}
$\mathcal{AC}$	{("DEU_to_DEU","CNG")}
$\mathcal{C}$	{"CNG"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Natural Gas"}
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"OnlyTimeStep"}

Continued on next page...

Table 3: Sets of Test Case 1 (continued)

Set Name	**Set Value**
$\mathcal{N}$	{"DEU"}
$\mathcal{O}$	{"FES"}
$\mathcal{P}$	{"P_DEU"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$
$\mathcal{S}$	$\emptyset$
$\mathcal{T}$	{"T_DEU"}
$\mathcal{V}$	$\emptyset$
$\mathcal{VT}$	$\emptyset$
$\mathcal{Y}$	{"2020"}

Source: Own depiction.

Table 4: Parameter Values of Test Case 1

Parameter	**y=2020**
$\frac{1}{\lVert\Delta\rVert_y}$	1
$\mathbb{1}^{NC}_{\text{"T_DEU","DEU","CNG"}}$	0
r_y	1
$d_{\text{"OnlyTimeStep"}}$	1
$c^{P}_{\text{"P_DEU","CNG","FES"},y}$	0.5
$c^{\Delta P}_{\text{"P_DEU","CNG","FES"},y}$	1
$fi^{P}_{\text{"CNG","Natural Gas","FES"}}$	1

Continued on next page...

Table 4: Parameter Values of Test Case 1 (continued)

Parameter	**y=2020**
$L^{P}_{\text{“CNG”,“FES”}}$	50
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10
$\Lambda^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	10
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0
$c^{\Delta^{I}}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”},y}$	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0
$L^{A}_{\text{“CNG”}}$	50
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.5
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	1.0
$av^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	1.0
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	2
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1

Source: Own depiction.

Table 5: Tested Results of Test Case 1

Expression	**Result y=2020**
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	1.5

Source: Own depiction.

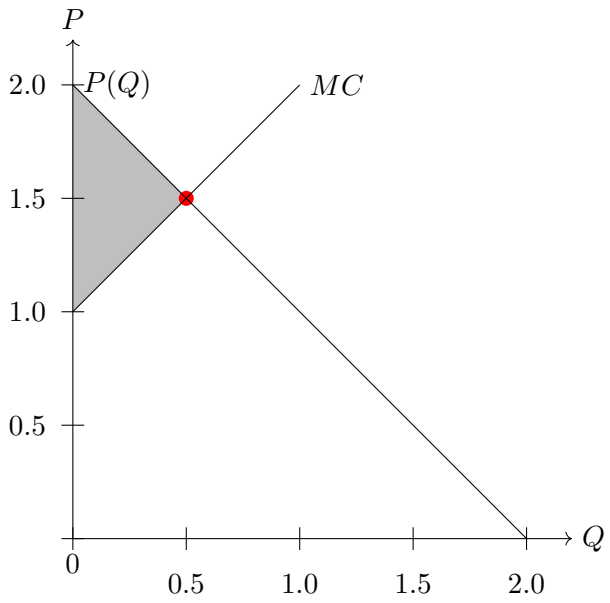

Figure 1: Inverse Demand and Marginalized Cost Curves for Test Case 1

Source: Own depiction.

2.2.2 Test Case 2

Test Case 2 adds multiple years and tests for prices in both, while also including conversion costs. As before, relevant sets and parameters can be seen from Tables 6 and 7, respectively. Marginalized costs for given factor intensities are $MC = c^{Pl} + c^{P} + c^{V} + \beta^{P} \cdot Q$, while prices are $P(Q) = \alpha^{D} + \beta^{D} Q$. Hence, as stated in Table 8 and illustrated by Figure 2, tests pass if prices in both years are $P = 1.25$ for “CNG” and $P = 0.0$ for “GH_2”.

Table 6: Sets of Test Case 2

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU"}
$\mathcal{AC}$	{("DEU_to_DEU","CNG"),("DEU_to_DEU","GH_2")}
$\mathcal{C}$	{"CNG","GH_2"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Electricity"}
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"OnlyTimeStep"}
$\mathcal{N}$	{"DEU"}
$\mathcal{O}$	{"RES"}
$\mathcal{P}$	{"P_DEU"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$
$\mathcal{S}$	$\emptyset$
$\mathcal{T}$	{"T_DEU"}
$\mathcal{V}$	{"V_DEU"}
$\mathcal{VT}$	{("GH_2","CNG")}
$\mathcal{Y}$	{"2020","2021"}

Source: Own depiction.

Table 7: Parameter Values of Test Case 2

Parameter	**y=2020**	**y=2021**
$\frac{1}{\lVert\Delta\rVert_y}$	1	1
$\mathbb{1}^{NC}_{\text{“T_DEU”,“DEU”,“CNG”}}$	0	0
$\mathbb{1}^{NC}_{\text{“T_DEU”,“DEU”,“GH}_2\text{”}}$	0	0
r_y	1	1
$d_{\text{“OnlyTimeStep”}}$	2	2
$c^{P}_{\text{“P_DEU”,“GH}_2\text{”,“RES”},y}$	0.1	0.1
$c^{P}_{\text{“P_DEU”,“CNG”,“RES”},y}$	0.0	0.0
$c^{\Delta P}_{\text{“P_DEU”,“GH}_2\text{”,“RES”},y}$	10	10
$c^{\Delta P}_{\text{“P_DEU”,“CNG”,“RES”},y}$	10	10
$fi^{P}_{\text{“GH}_2\text{”,“Electricity”,“RES”}}$	1	1
$fi^{P}_{\text{“CNG”,“Electricity”,“RES”}}$	1	1
$L^{P}_{\text{“GH}_2\text{”,“RES”}}$	50	50
$L^{P}_{\text{“CNG”,“RES”}}$	50	50
$\Lambda^{P}_{\text{“P_DEU”,“GH}_2\text{”,“RES”},y}$	10	10
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“RES”},y}$	0	0
$\Lambda^{I}_{\text{“P_DEU”,“Electricity”,“Block 1”},y}$	10	10
$\Omega^{I}_{\text{“P_DEU”,“Electricity”,“Block 1”},y}$	0	0
$c^{\Delta I}_{\text{“P_DEU”,“Electricity”,“Block 1”},y}$	0	0
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“GH}_2\text{”,“RES”},y}$	10	10
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“RES”},y}$	10	10
$\Omega^{P}_{\text{“P_DEU”,“GH}_2\text{”,“RES”},y}$	10	10

Continued on next page...

Table 7: Parameter Values of Test Case 2 (continued)

Parameter	**y=2020**	**y=2021**
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“RES”},y}$	10	10
$fi^{V}_{\text{“GH}_2\text{”,“CNG”}}$	1	1
$l^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”}}$	0	0
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0	0
$c^{V}_{\text{“V_DEU”,“GH}_2\text{”,“CNG”},y}$	0.1	0.1
$c^{\Delta V}_{\text{“V_DEU”,“GH}_2\text{”,“CNG”},y}$	10	10
$\Lambda^{V}_{\text{“V_DEU”,“GH}_2\text{”,“CNG”},y}$	10	10
$L^{V}_{\text{“GH}_2\text{”,“CNG”}}$	10	10
$c^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”},y}$	0	0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“GH}_2\text{”},y}$	0	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”},y}$	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0
$L^{A}_{\text{“GH}_2\text{”}}$	50	50
$L^{A}_{\text{“CNG”}}$	50	50
$c^{I_l}_{\text{“P_DEU”,“Electricity”,“Block 1”,“OnlyTimeStep”,y}}$	0.3	0.3
$c^{I_q}_{\text{“P_DEU”,“Electricity”,“Block 1”,“OnlyTimeStep”,y}}$	1	1
$av^{I}_{\text{“P_DEU”,“Electricity”,“Block 1”,“OnlyTimeStep”}}$	1	1
$\alpha^{D}_{\text{“DEU”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0	0

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Table 7: Parameter Values of Test Case 2 (continued)

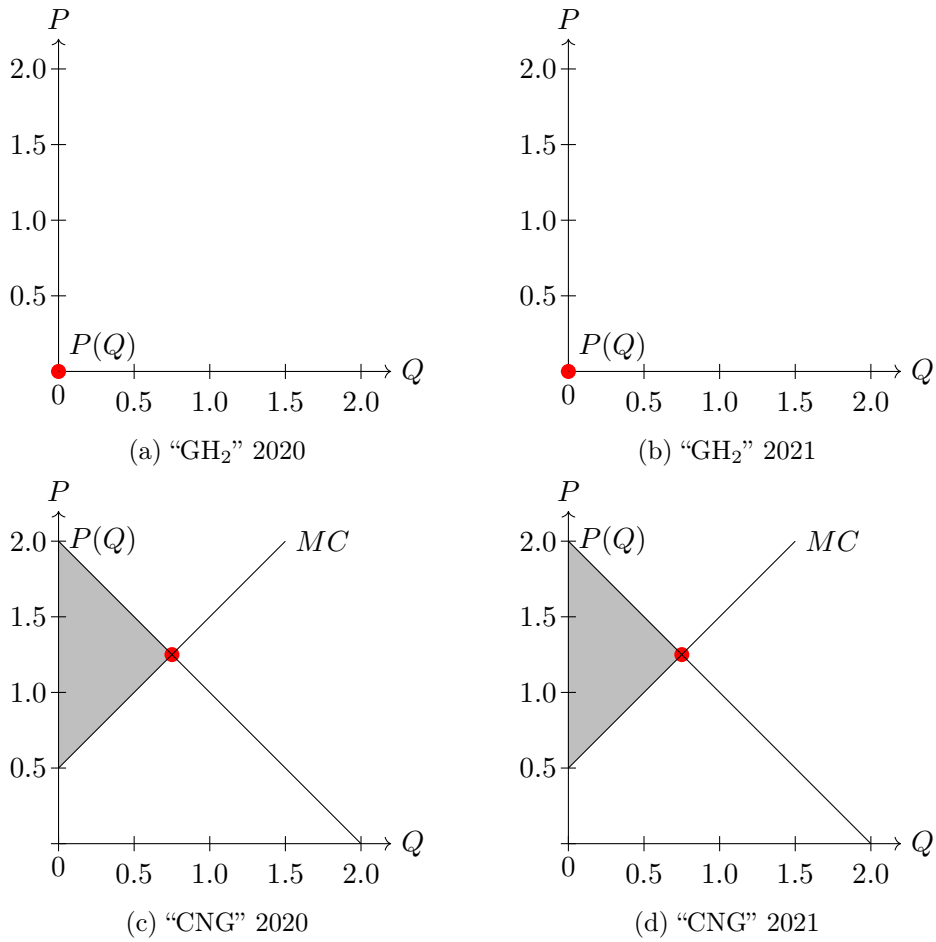
Parameter	**y=2020**	**y=2021**
$\beta^{D}_{\text{“DEU”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0	0
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	2	2
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1	-1

Source: Own depiction.

Table 8: Tested Results of Test Case 2

Expression	**Result y=2020**	**Result y=2021**
$\tilde{P}^{T\to D}_{\text{“DEU”,“GH2”,“Block 1”,“OnlyTimeStep”,y}}$	0.00	0.00
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	1.25	1.25

Source: Own depiction.



(a) "GH_2" 2020

(b) "GH_2" 2021

(c) "CNG" 2020

(d) "CNG" 2021

Figure 2: Inverse Demand and Marginalized Provision Cost Curves for Test Case 2

Source: Own depiction.

2.2.3 Test Case 3

Test Case 3 asserts exertion of market power in a simple Cournot setting with time step weights. Marginalized costs for given factor intensities are $MC = c^{I_l} + c^{I_q}Q + c^P$, while prices are $P(Q) = \alpha^D + \beta^D Q$. As there is only one trader, the model now describes a Cournot monopoly. It is well known from economic theory that in a profit-maximizing solution, marginalized costs are equal to marginal revenues ($MR = MC$). Figure 3 is a graphical representation of the optimal solution. Hence, this test passes if the equilibrium price is $P = 1.5$ (see Table 11).

Table 9: Sets of Test Case 3

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU"}
$\mathcal{AC}$	{("DEU_to_DEU","CNG")}
$\mathcal{C}$	{"CNG"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Natural Gas"}
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"OnlyTimeStep"}
$\mathcal{N}$	{"DEU"}
$\mathcal{O}$	{"FES"}
$\mathcal{P}$	{"P_DEU"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$
$\mathcal{S}$	$\emptyset$
$\mathcal{T}$	{"T_DEU"}
$\mathcal{V}$	$\emptyset$
$\mathcal{VT}$	$\emptyset$
$\mathcal{Y}$	{"2020"}

Source: Own depiction.

Table 10: Parameter Values of Test Case 3

Parameter	**y=2020**
$\frac{1}{\lVert\Delta\rVert_y}$	1
$\mathbb{1}^{NC}_{\text{“T_DEU”,“DEU”,“CNG”}}$	1
r_y	1
$d_{\text{“OnlyTimeStep”}}$	2
$c^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	0.5
$c^{\Delta P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	1
$fi^{P}_{\text{“CNG”,“Natural Gas”,“FES”}}$	1
$L^{P}_{\text{“CNG”,“FES”}}$	50
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10
$\Lambda^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	10
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0
$c^{\Delta I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”},y}$	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0
$L^{A}_{\text{“CNG”}}$	50
$c^{I_l}_{\text{“P_DEU”,“Electricity”,“Block 1”,“OnlyTimeStep”,y}}$	0.5
$c^{I_q}_{\text{“P_DEU”,“Electricity”,“Block 1”,“OnlyTimeStep”,y}}$	0

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Table 10: Parameter Values of Test Case 3 (continued)

Parameter	**y=2020**
$av^{I}_{\text{“P_DEU”,“Electricity”,“Block 1”,“OnlyTimeStep”}}$	1
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	2
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1

Source: Own depiction.

Table 11: Tested Results of Test Case 3

Expression	**Result y=2020**
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	1.5

Source: Own depiction.

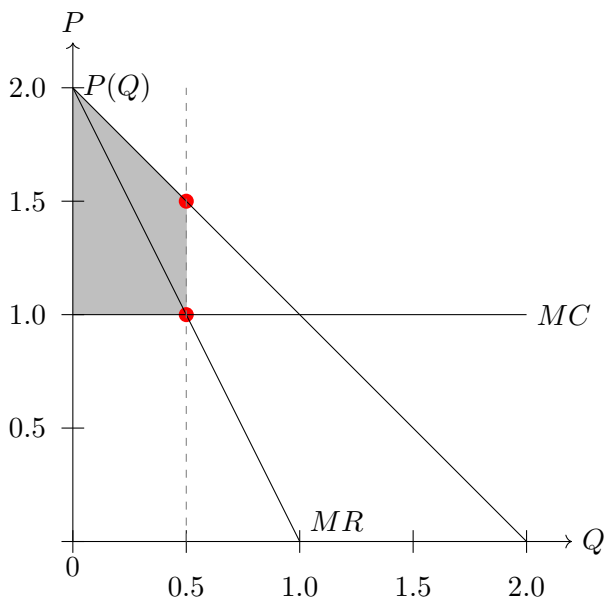

Figure 3: Profit Maximization in a Cournot Setting for Test Case 3

Source: Own depiction.

2.2.4 Test Case 4

Test Case 4 asserts that discounting works as intended. It depicts a perfectly competitive version of Test Case 3 over the course of two years, where the second period is discounted by 50%. Prices should be $P = 1.0$ in both years. Social welfare corresponds to the gray triangle in Figure 4, where the second period must be discounted accordingly. Hence, the total objective value must be 0.75. Both, prices and objective values, are tested against.

Table 12: Sets of Test Case 4

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU"}
$\mathcal{AC}$	{("DEU_to_DEU","CNG")}
$\mathcal{C}$	{"CNG"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Natural Gas"}
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"OnlyTimeStep"}
$\mathcal{N}$	{"DEU"}
$\mathcal{O}$	{"FES"}
$\mathcal{P}$	{"P_DEU"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$
$\mathcal{S}$	$\emptyset$
$\mathcal{T}$	{"T_DEU"}
$\mathcal{V}$	$\emptyset$

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Table 12: Sets of Test Case 4 (continued)

Set Name	**Set Value**
$\mathcal{VT}$	$\emptyset$
$\mathcal{Y}$	{“2020”,“2021”}

Source: Own depiction.

Table 13: Parameter Values of Test Case 4

Parameter	**y=2020**	**y=2021**
$\frac{1}{\lVert\Delta\rVert_y}$	1	1
$\mathbb{1}^{NC}_{\text{“T_DEU”,“DEU”,“CNG”}}$	0	0
r_y	1	0.5
$d_{\text{“OnlyTimeStep”}}$	1	1
$c^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	0.5	0.5
$c^{\Delta P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	1	1
$fi^{P}_{\text{“CNG”,“Natural Gas”,“FES”}}$	1	1
$L^{P}_{\text{“CNG”,“FES”}}$	50	50
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10	10
$\Lambda^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	10	10
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0	0
$c^{\Delta I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0	0
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”},y}$	10	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10	10
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0	0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0

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Table 13: Parameter Values of Test Case 4 (continued)

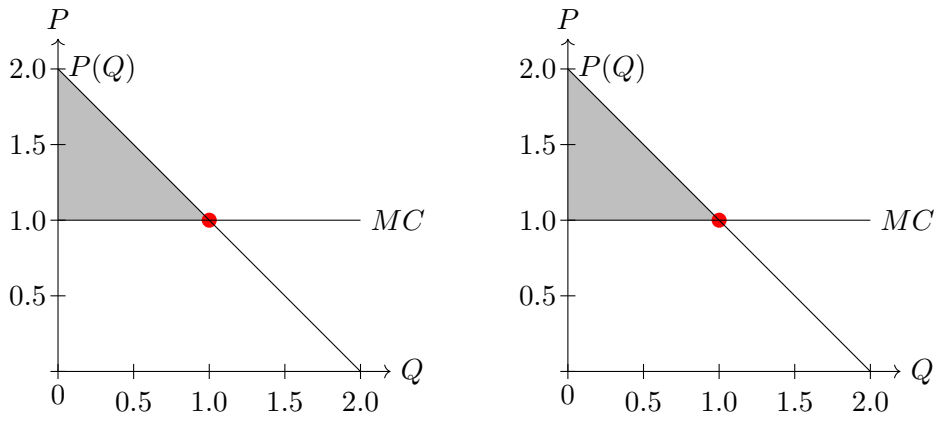
Parameter	**y=2020**	**y=2021**
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0
$L^{A}_{\text{“CNG”}}$	50	50
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.5	0.5
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0	0
$av^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	1	1
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	2	2
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1	-1

Source: Own depiction.

Table 14: Tested Results of Test Case 4

Expression	**Result y=2020**	**Result y=2021**
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,“2020”}}$	1.0	1.0
Objective Value	0.75	

Source: Own depiction.



(a) 2020 (b) 2021

Figure 4: Social Welfare in Both Years of Test Case 4

Source: Own depiction.

2.2.5 Test Case 5

Test Case 5 asserts that storages work as intended. For this, multiple time steps are introduced. During the low demand time step L, there is no demand for natural gas, but production is sufficiently cheaper than in the high demand time step H. In a loss-free system of perfect competition with factor intensities of 1:1, prices in the high-demand time step should be the sum of marginalized input, production, and storage costs of the marginal supplying time step. Tables 15 and 16 show the test data. Producing natural gas and selling "CNG" in time step "L" and storing it to time step "H", implies marginalized provision cost of $MC = 1$, much cheaper than producing in "H", which would lead to marginalized provision cost of $MC = 1.4$. Since no demand occurs in time step "L", this option remains feasible; Figure 5 depicts a graphical solution. Hence, the test is passed if $P_H = 1, P_L = 0$ and production is 0.0 and 0.5 during high and low demand time steps, respectively (see Table 17).[6]

[6]Note that the low demand time step has twice as many days, hence only half the demanded quantity must be injected into storage every day.

Table 15: Sets of Test Case 5

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU"}
$\mathcal{AC}$	{("DEU_to_DEU","CNG")}
$\mathcal{C}$	{"CNG"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Natural Gas"}
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"L","H"}
$\mathcal{N}$	{"DEU"}
$\mathcal{O}$	{"FES"}
$\mathcal{P}$	{"P_DEU"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$
$\mathcal{S}$	{"S_DEU"}
$\mathcal{T}$	{"T_DEU"}
$\mathcal{V}$	$\emptyset$
$\mathcal{VT}$	$\emptyset$
$\mathcal{Y}$	{"2020"}

Source: Own depiction.

Table 16: Parameter Values of Test Case 5

Parameter	**m=L**	**m=H**
$\frac{1}{\lVert\Delta\rVert_y}$	1	1
$\mathbb{1}^{NC}_{\text{“T_DEU”,“DEU”,“CNG”}}$	0	0
$r_{\text{“2020”}}$	1	1
d_m	4	2
$c^{P}_{\text{“P_DEU”,“CNG”,“FES”,“2020”}}$	0.4	0.4
$c^{\Delta P}_{\text{“P_DEU”,“CNG”,“FES”,“2020”}}$	1	1
$fi^{P}_{\text{“CNG”,“Natural Gas”,“FES”}}$	1	1
$L^{P}_{\text{“CNG”,“FES”}}$	50	50
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“FES”,“2020”}}$	10	10
$\Lambda^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2020”}}$	10	10
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2020”}}$	0	0
$c^{\Delta I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2020”}}$	0	0
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”,“2020”}}$	10	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”,“2020”}}$	10	10
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0	0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”,“2020”}}$	0	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”,“2020”}}$	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”,“2020”}}$	0	0
$L^{A}_{\text{“CNG”}}$	50	50
$l^{S}_{\text{“CNG”,m,“L”}}$	0	0
$l^{S}_{\text{“CNG”,m,“H”}}$	0	0

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Table 16: Parameter Values of Test Case 5 (continued)

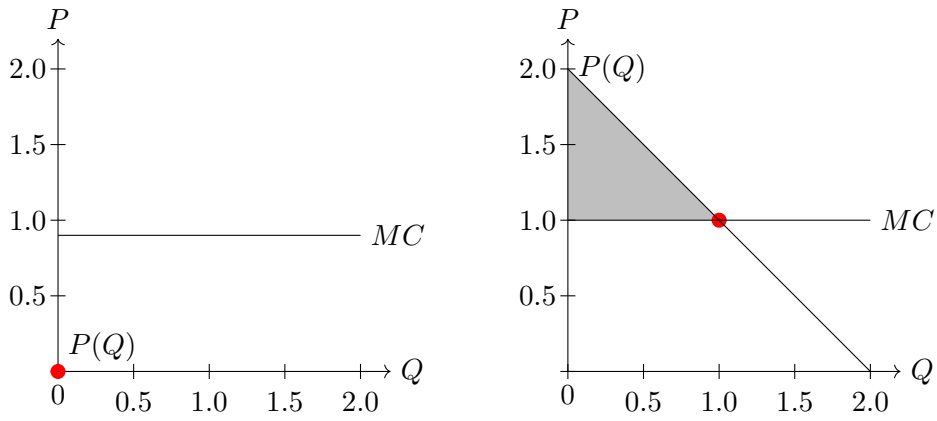
Parameter	**m=L**	**m=H**
$c^{S_{in}}_{\text{“S_DEU”,“CNG”,“2020”}}$	0.05	0.05
$c^{S_{out}}_{\text{“S_DEU”,“CNG”,“2020”}}$	0.05	0.05
$c^{\Delta S}_{\text{“S_DEU”,“CNG”,“2020”}}$	10	10
$\Lambda^{S}_{\text{“S_DEU”,“CNG”,“2020”}}$	10	10
$\Omega^{S}_{\text{“S_DEU”,“CNG”,“2020”}}$	10	10
$L^{S}_{\text{“CNG”}}$	10	10
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m,“2020”}}$	0.5	1.0
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m,“2020”}}$	0	0
$av^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m}}$	1	1
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2020”}}$	0	2
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2020”}}$	0	-1

Source: Own depiction.

Table 17: Tested Results of Test Case 5

Expression	**Result m=L**	**Result m=H**
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2020”}}$	0.0	1.0
$q^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m,“2020”}}$	0.5	0.0

Source: Own depiction.



(a) time step "L" (b) time step "H"

Figure 5: Marginalized Provision Costs and Prices for Different Time Steps in Test Case 5

Source: Own depiction.

2.2.6 Test Case 6

Test Case 6 works similarly to Test Case 5, but here the low demand time step has half as many days. Prices remain unchanged, however daily low-demand production quantity should be twice as high as consumption in the high-demand time step. The test is passed if $P_H = 1, P_L = 0$ and production is 0.0 and 2.0 during high and low demand time steps, respectively.

Table 18: Sets of Test Case 6

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU"}
$\mathcal{AC}$	{("DEU_to_DEU","CNG")}
$\mathcal{C}$	{"CNG"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Natural Gas"}

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Table 18: Sets of Test Case 6 (continued)

Set Name	**Set Value**
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"L","H"}
$\mathcal{N}$	{"DEU"}
$\mathcal{O}$	{"FES"}
$\mathcal{P}$	{"P_DEU"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$
$\mathcal{S}$	{"S_DEU"}
$\mathcal{T}$	{"T_DEU"}
$\mathcal{V}$	$\emptyset$
$\mathcal{VT}$	$\emptyset$
$\mathcal{Y}$	{"2020"}

Source: Own depiction.

Table 19: Parameter Values of Test Case 6

Parameter	**m=L**	**m=H**
$\frac{1}{\lVert\Delta\rVert_y}$	1	1
$\mathbb{1}^{NC}_{\text{"T_DEU","DEU","CNG"}}$	0	0
$r_{\text{"2020"}}$	1	1
d_m	2	4
$c^{P}_{\text{"P_DEU","CNG","FES","2020"}}$	0.4	0.4

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Table 19: Parameter Values of Test Case 6 (continued)

Parameter	**m=L**	**m=H**
$c^{\Delta P}_{\text{“P_DEU”,“CNG”,“FES”,“2020”}}$	1	1
$fi^{P}_{\text{“CNG”,“Natural Gas”,“FES”}}$	1	1
$L^{P}_{\text{“CNG”,“FES”}}$	50	50
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“FES”,“2020”}}$	10	10
$\Lambda^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2020”}}$	10	10
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2020”}}$	0	0
$c^{\Delta I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2020”}}$	0	0
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”,“2020”}}$	10	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”,“2020”}}$	10	10
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0	0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”,“2020”}}$	0	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”,“2020”}}$	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”,“2020”}}$	0	0
$L^{A}_{\text{“CNG”}}$	50	50
$l^{S}_{\text{“CNG”,m,“L”}}$	0	0
$l^{S}_{\text{“CNG”,m,“H”}}$	0	0
$c^{S_{in}}_{\text{“S_DEU”,“CNG”,“2020”}}$	0.05	0.05
$c^{S_{out}}_{\text{“S_DEU”,“CNG”,“2020”}}$	0.05	0.05
$c^{\Delta S}_{\text{“S_DEU”,“CNG”,“2020”}}$	10	10
$\Lambda^{S}_{\text{“S_DEU”,“CNG”,“2020”}}$	10	10

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Table 19: Parameter Values of Test Case 6 (continued)

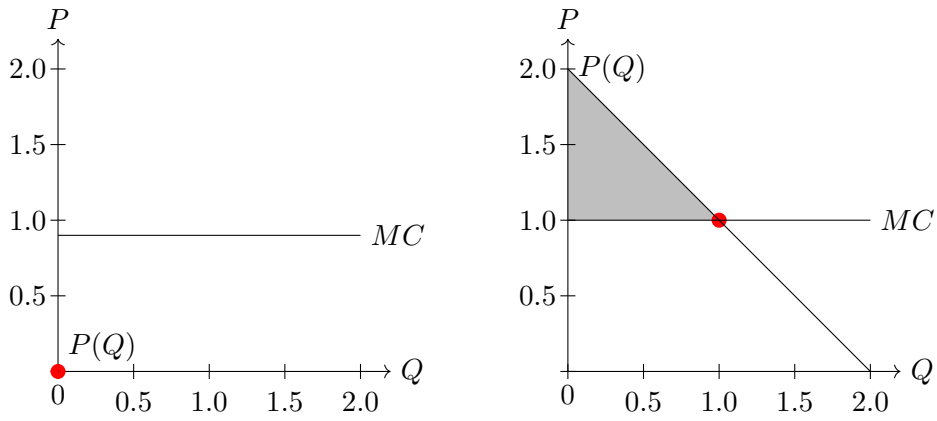
Parameter	**m=L**	**m=H**
$\Omega^{S}_{\text{“S_DEU”,“CNG”,“2020”}}$	10	10
$L^{S}_{\text{“CNG”}}$	10	10
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m,“2020”}}$	0.5	1.0
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m,“2020”}}$	0	0
$av^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m}}$	1	1
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2020”}}$	0	2
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2020”}}$	0	-1

Source: Own depiction.

Table 20: Tested Results of Test Case 6

Expression	**Result s=L**	**Result s=H**
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2020”}}$	0.0	1.0
$q^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m,“2020”}}$	2.0	0.0

Source: Own depiction.



(a) time step "L" (b) time step "H"

Figure 6: Marginalized Provision Costs and Prices for Different Time Steps in Test Case 6

Source: Own depiction.

2.2.7 Test Case 7

Test Case 7 assures that demand for a commodity can be met from different origins. Cost optimal allocation in this case requires that cheapest marginalized provision cost from each origin must be utilized. Marginalized provision cost and the resulting price for the data given in Tables 21 and 22 are indicated in Figure 7. Note that the sum of production quantities from each origin must equal total demand quantity. Tests are passed if $P = 0.5$ and produced quantities are 1 from each natural gas and renewable electricity (see also Figure 7). Note that conversion from renewable electricity to "CNG" happens at 2:1. Marginalized demand provision costs of renewables are hence $MC_{RES} = 2 \cdot (0.1 + 0.025Q) + 0.25$, while $MC_{FES} = 0.5$.

Table 21: Sets of Test Case 7

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU"}
$\mathcal{AC}$	{("DEU_to_DEU","CNG")}

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Table 21: Sets of Test Case 7 (continued)

Set Name	**Set Value**
$\mathcal{C}$	{"CNG"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Natural Gas","Renewable Electricity"}
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"OnlyTimeStep"}
$\mathcal{N}$	{"DEU"}
$\mathcal{O}$	{"FES","RES"}
$\mathcal{P}$	{"P_DEU"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$
$\mathcal{S}$	$\emptyset$
$\mathcal{T}$	{"T_DEU"}
$\mathcal{V}$	$\emptyset$
$\mathcal{VT}$	$\emptyset$
$\mathcal{Y}$	{"2020"}

Source: Own depiction.

Table 22: Parameter Values of Test Case 7

Parameter	**y=2020**
$\frac{1}{\lVert\Delta\rVert_y}$	1
$\mathbb{1}^{NC}_{\text{"T_DEU","DEU","CNG"}}$	0

Continued on next page...

Table 22: Parameter Values of Test Case 7 (continued)

Parameter	**y=2020**
r_y	1
$d_{\text{“OnlyTimeStep”}}$	1
$c^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	0.25
$c^{\Delta P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10
$fi^{P}_{\text{“CNG”,“Natural Gas”,“FES”}}$	1
$fi^{P}_{\text{“CNG”,“Renewable Electricity”,“FES”}}$	0
$L^{P}_{\text{“CNG”,“FES”}}$	50
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10
$c^{P}_{\text{“P_DEU”,“CNG”,“RES”},y}$	0.25
$c^{\Delta P}_{\text{“P_DEU”,“CNG”,“RES”},y}$	10
$fi^{P}_{\text{“CNG”,“Renewable Electricity”,“RES”}}$	2
$fi^{P}_{\text{“CNG”,“Natural Gas”,“RES”}}$	0
$L^{P}_{\text{“CNG”,“RES”}}$	50
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“RES”},y}$	10
$\Lambda^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	10
$\Lambda^{I}_{\text{“P_DEU”,“Renewable Electricity”,“Block 1”},y}$	10
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0
$\Omega^{I}_{\text{“P_DEU”,“Renewable Electricity”,“Block 1”},y}$	0
$c^{\Delta I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0
$c^{\Delta I}_{\text{“P_DEU”,“Renewable Electricity”,“Block 1”},y}$	0

Continued on next page...

Table 22: Parameter Values of Test Case 7 (continued)

Parameter	**y=2020**
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”},y}$	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“RES”},y}$	10
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0
$L^{A}_{\text{“CNG”}}$	50
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.25
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0
$av^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	1
$c^{I_l}_{\text{“P_DEU”,“Renewable Electricity”,“Block 1”,“OnlyTimeStep”,y}}$	0.1
$c^{I_q}_{\text{“P_DEU”,“Renewable Electricity”,“Block 1”,“OnlyTimeStep”,y}}$	0.025
$av^{I}_{\text{“P_DEU”,“Renewable Electricity”,“Block 1”,“OnlyTimeStep”}}$	1
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	2
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1

Source: Own depiction.

Table 23: Tested Results of Test Case 7

Expression	**Result y=2020**
$\tilde{P}^{T\rightarrow D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0.5
$q^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	1

Continued on next page...

Table 23: Tested Results of Test Case 7 (continued)

Expression	**Result y=2020**
$q^{I}_{\text{“P_DEU”,“Renewable Electricity”,“Block 1”,“OnlyTimeStep”,y}}$	1

Source: Own depiction.

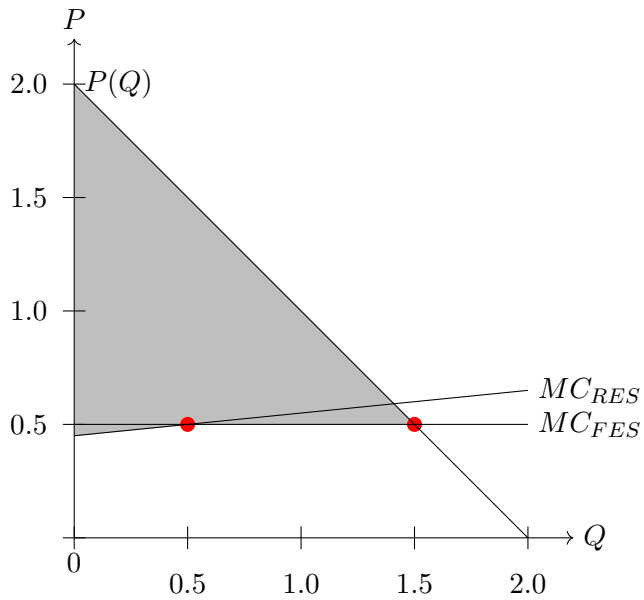


Figure 7: Marginalized Provision Costs and Prices in Test Case 7

Source: Own depiction.

2.2.8 Test Case 8

Test Case 8 asserts, that demand for multiple commodities can be satisfied by one input. Tables 24 and 25 depict the sets and parameters, respectively. For both natural gas and hydrogen, prices should be equal to marginalized cost of provision. $MC_{Hydrogen} = 2 \cdot 0.9 + 0.2$ and $MC_{NaturalGas} = 1 \cdot (0.9) + 0.1$. Hence, $P_{Hydrogen} = 2$ and $P_{NaturalGas} = 1$, with a total production quantity of 5 (4 of which are used in conversion to hydrogen and 1 of which goes to demand directly). Passing criteria can be seen in Table 26, while Figure 8 depicts a graphical solution.

Table 24: Sets of Test Case 8

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU"}
$\mathcal{AC}$	{("DEU_to_DEU","CNG"), ("DEU_to_DEU","GH_2") }
$\mathcal{C}$	{"CNG","GH_2"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Natural Gas"}
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"OnlyTimeStep"}
$\mathcal{N}$	{"DEU"}
$\mathcal{O}$	{"FES",}
$\mathcal{P}$	{"P_DEU"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$
$\mathcal{S}$	$\emptyset$
$\mathcal{T}$	{"T_DEU"}
$\mathcal{V}$	$\emptyset$
$\mathcal{VT}$	$\emptyset$
$\mathcal{Y}$	{"2020"}

Source: Own depiction.

Table 25: Parameter Values of Test Case 8

Parameter	**y=2020**
$\frac{1}{\lVert\Delta\rVert_y}$	1
$\mathbb{1}^{NC}_{\text{“T_DEU”,“DEU”,“CNG”}}$	0
r_y	1
$d_{\text{“OnlyTimeStep”}}$	1
$c^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	0.5
$c^{P}_{\text{“P_DEU”,“GH}_2\text{”,“FES”},y}$	1
$c^{\Delta P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10
$c^{\Delta P}_{\text{“P_DEU”,“GH}_2\text{”,“FES”},y}$	10
$fi^{P}_{\text{“CNG”,“Natural Gas”,“FES”}}$	1
$fi^{P}_{\text{“GH}_2\text{”,“Natural Gas”,“FES”}}$	2
$L^{P}_{\text{“CNG”,“FES”}}$	50
$L^{P}_{\text{“GH}_2\text{”,“FES”}}$	50
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10
$\Lambda^{P}_{\text{“P_DEU”,“GH}_2\text{”,“FES”},y}$	10
$\Lambda^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	10
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0
$c^{\Delta I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”},y}$	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10
$\Omega^{P}_{\text{“P_DEU”,“GH}_2\text{”,“FES”},y}$	10
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0

Continued on next page...

Table 25: Parameter Values of Test Case 8 (continued)

Parameter	**y=2020**
$l^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”}}$	0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0
$L^{A}_{\text{“CNG”}}$	50
$c^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”},y}$	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“GH}_2\text{”},y}$	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”},y}$	0
$L^{A}_{\text{“GH}_2\text{”}}$	50
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.5
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0
$av^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	0
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	2
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1
$\alpha^{D}_{\text{“DEU”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	4
$\beta^{D}_{\text{“DEU”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	-1

Source: Own depiction.

Table 26: Tested Results of Test Case 8

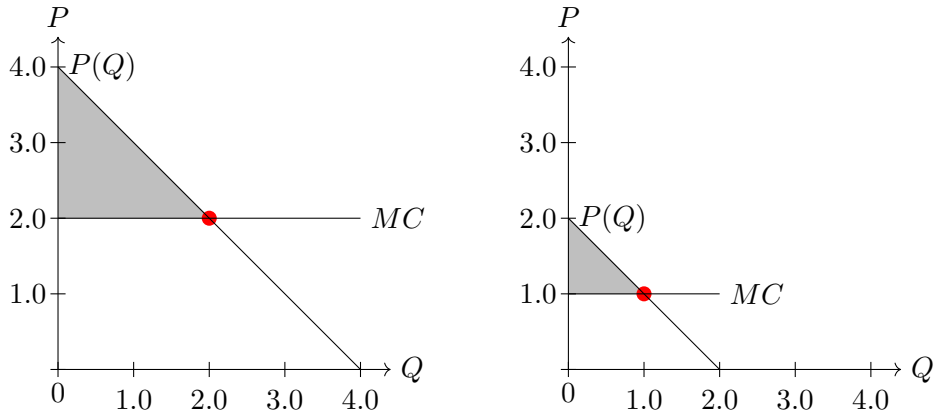
Expression	**Result y=2020**
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	1
$\tilde{P}^{T\to D}_{\text{“DEU”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	2

Continued on next page...

Table 26: Tested Results of Test Case 8 (continued)

Expression	**Result y=2020**
$q^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	5

Source: Own depiction.



(a) Hydrogen (b) Natural Gas

Figure 8: Marginalized Provision Costs and Prices in Test Case 8

Source: Own depiction.

2.2.9 Test Case 9

Test Case 9 is used to verify functionality and losses on transportation arcs with time step weights. Tables 27 and 28 depict the sets and parameters, respectively. Production costs in DEU are much too high. However, NLD has no own demand and can export to DEU with operational cost of 0.1 and losses of 10%. In addition to production and conversion costs of 1, 10% must be produced additionally, and operational costs of 0.1 come on top. Marginalized provision costs are hence 1.2. Tests are passed, if prices are 1.2 and 0 in DEU and NLD, respectively. Further, 0.88 must be produced in NLD, corresponding to a consumption quantity of 0.8 in DEU, as can be seen from Table 29. Figure 9 depicts a graphical solution. The implementation in the testing routine features two separate runs for shipping and pipelines, however, mathematical model data remain unchanged.

Table 27: Sets of Test Case 9

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU","DEU_to_NLD","NLD_to_DEU","NLD_to_NLD"}
$\mathcal{AC}$	{("DEU_to_DEU", "CNG"),("DEU_to_NLD", "CNG"), ("NLD_to_DEU", "CNG"),("NLD_to_NLD", "CNG")}
$\mathcal{C}$	{"CNG"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Natural Gas"}
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"OnlyTimeStep"}
$\mathcal{N}$	{"DEU","NLD"}
$\mathcal{O}$	{"FES"}
$\mathcal{P}$	{"P_DEU","P_NLD"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$
$\mathcal{S}$	$\emptyset$
$\mathcal{T}$	{"T_DEU","T_NLD"}
$\mathcal{V}$	$\emptyset$
$\mathcal{VT}$	$\emptyset$
$\mathcal{Y}$	{"2020"}

Source: Own depiction.

Table 28: Parameter Values of Test Case 9

Parameter	**y=2020**
$\frac{1}{\lVert\Delta\rVert_y}$	1
$\mathbb{1}^{NC}_{\text{“T_DEU”,“DEU”,“CNG”}}$	0
$\mathbb{1}^{NC}_{\text{“T_DEU”,“NLD”,“CNG”}}$	0
$\mathbb{1}^{NC}_{\text{“T_NLD”,“DEU”,“CNG”}}$	0
$\mathbb{1}^{NC}_{\text{“T_NLD”,“NLD”,“CNG”}}$	0
r_y	1
$d_{\text{“OnlyTimeStep”}}$	2
$c^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	0.5
$c^{\Delta P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	1
$c^{P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	0.5
$c^{\Delta P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	1
$fi^{P}_{\text{“CNG”,“Natural Gas”,“FES”}}$	1
$L^{P}_{\text{“CNG”,“FES”}}$	50
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10
$\Lambda^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	10
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0
$c^{\Delta I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”},y}$	10
$\Lambda^{T}_{\text{“T_DEU”,“NLD”,“CNG”,“FES”},y}$	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10
$\Lambda^{P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	10

Continued on next page...

Table 28: Parameter Values of Test Case 9 (continued)

Parameter	**y=2020**
$\Lambda^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”},y}$	10
$\Omega^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”},y}$	0
$c^{\Delta^{I}}_{\text{“P_NLD”,“Natural Gas”,“Block 1”},y}$	0
$\Lambda^{T}_{\text{“T_NLD”,“DEU”,“CNG”,“FES”},y}$	10
$\Lambda^{T}_{\text{“T_NLD”,“NLD”,“CNG”,“FES”},y}$	10
$\Omega^{P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	10
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0.0
$l^{A}_{\text{“DEU_to_NLD”,“CNG”}}$	0.0
$l^{A}_{\text{“NLD_to_DEU”,“CNG”}}$	0.1
$l^{A}_{\text{“NLD_to_NLD”,“CNG”}}$	0.0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0.0
$c^{A}_{\text{“DEU_to_NLD”,“CNG”},y}$	0.0
$c^{A}_{\text{“NLD_to_DEU”,“CNG”},y}$	0.1
$c^{A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0.0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0
$c^{\Delta A}_{\text{“DEU_to_NLD”,“CNG”},y}$	0
$c^{\Delta A}_{\text{“NLD_to_DEU”,“CNG”},y}$	0
$c^{\Delta A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0
$\Lambda^{A}_{\text{“DEU_to_NLD”,“CNG”},y}$	0

Continued on next page...

Table 28: Parameter Values of Test Case 9 (continued)

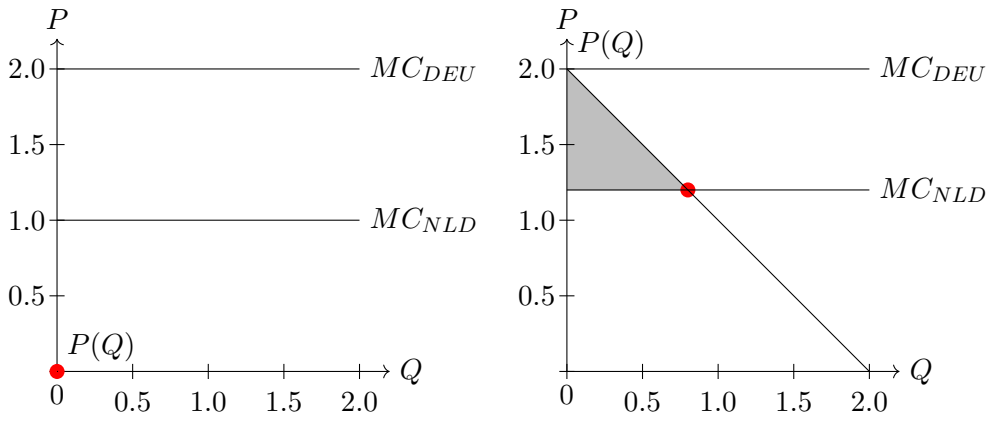
Parameter	**y=2020**
$\Lambda^{A}_{\text{“NLD_to_DEU”,“CNG”},y}$	10
$\Lambda^{A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0
$L^{A}_{\text{“CNG”}}$	50
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	2
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0
$av^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	1
$c^{I_l}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.5
$c^{I_q}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0
$av^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	1
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	2
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1
$\alpha^{D}_{\text{“NLD”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0
$\beta^{D}_{\text{“NLD”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1

Source: Own depiction.

Table 29: Tested Results of Test Case 9

Expression	**Result y=2020**
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	1.2
$\tilde{P}^{T\to D}_{\text{“NLD”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0.0
$q^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.0
$q^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.88

Source: Own depiction.



(a) Node "NLD" (b) Node "DEU"

Figure 9: Marginalized Provision Costs and Prices for Different Nodes in Test Case 9

Source: Own depiction.

2.2.10 Test Case 10

Test Case 10 is structurally similar to Test Case 5, but 10% losses between time steps are incurred in addition to 0.1 of extraction costs. Tables 30 and 31 depict the sets and parameters, respectively. Marginalized provision costs from low to high demand time step are hence $MC = 1.1$. Tests are passed, if prices are $P = 1.1$ and $P = 0$ in high and low demand time steps, as can be seen from Table 32. Figure 10 depicts a graphical solution.

Table 30: Sets of Test Case 10

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU"}
$\mathcal{AC}$	{("DEU_to_DEU", "CNG")}
$\mathcal{C}$	{"CNG"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Natural Gas"}

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Table 30: Sets of Test Case 10 (continued)

Set Name	**Set Value**
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"L","H"}
$\mathcal{N}$	{"DEU"}
$\mathcal{O}$	{"FES"}
$\mathcal{P}$	{"P_DEU"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$
$\mathcal{S}$	{"S_DEU"}
$\mathcal{T}$	{"T_DEU"}
$\mathcal{V}$	$\emptyset$
$\mathcal{VT}$	$\emptyset$
$\mathcal{Y}$	{"2020"}

Source: Own depiction.

Table 31: Parameter Values of Test Case 10

Parameter	**m=L**	**m=H**
$\frac{1}{\lVert\Delta\rVert_y}$	1	1
$\mathbb{1}^{NC}_{\text{"T_DEU","DEU","CNG"}}$	0	0
$r_{\text{"2020"}}$	1	1
d_m	2	1
$c^{P}_{\text{"P_DEU","CNG","FES","2020"}}$	0.4	0.4

Continued on next page...

Table 31: Parameter Values of Test Case 10 (continued)

Parameter	**m=L**	**m=H**
$c^{\Delta P}_{\text{“P_DEU”,“CNG”,“FES”,“2020”}}$	1	1
$fi^{P}_{\text{“CNG”,“Natural Gas”,“FES”}}$	1	1
$L^{P}_{\text{“CNG”,“FES”}}$	50	50
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“FES”,“2020”}}$	10	10
$\Lambda^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2020”}}$	10	10
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2020”}}$	0	0
$c^{\Delta I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2020”}}$	0	0
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”,“2020”}}$	10	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”,“2020”}}$	10	10
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0	0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”,“2020”}}$	0	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”,“2020”}}$	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”,“2020”}}$	0	0
$L^{A}_{\text{“CNG”}}$	50	50
$l^{S}_{\text{“CNG”,m,“L”}}$	0	0
$l^{S}_{\text{“CNG”,m,“H”}}$	0.1	0
$c^{S_{in}}_{\text{“S_DEU”,“CNG”,“2020”}}$	0	0
$c^{S}_{\text{“S_DEU”,“CNG”,“2020”}}$	0.1	0.1
$c^{\Delta S}_{\text{“S_DEU”,“CNG”,“2020”}}$	10	10
$\Lambda^{S}_{\text{“S_DEU”,“CNG”,“2020”}}$	10	10

Continued on next page...

Table 31: Parameter Values of Test Case 10 (continued)

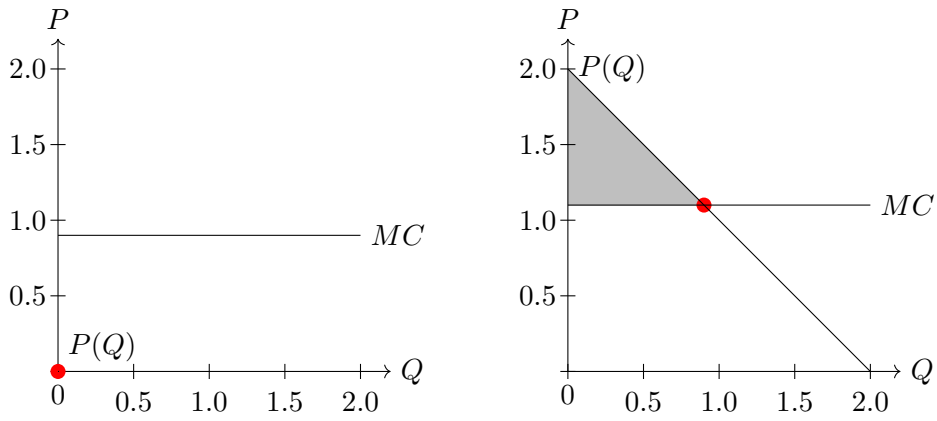
Parameter	**m=L**	**m=H**
$\Omega^{S}_{\text{“S_DEU”,“CNG”,“2020”}}$	10	10
$L^{S}_{\text{“CNG”}}$	10	10
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m,“2020”}}$	0.5	1
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m,“2020”}}$	0	0
$av^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m}}$	1	1
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2020”}}$	0	2
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2020”}}$	0	-1

Source: Own depiction.

Table 32: Tested Results of Test Case 10

Expression	**Result m=L**	**Result m=H**
$\tilde{P}^{T\rightarrow D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2020”}}$	0.0	1.1

Source: Own depiction.



(a) time step "L" (b) time step "H"

Figure 10: Marginalized Provision Costs and Prices for Different Time Steps in Test Case 10

Source: Own depiction.

2.2.11 Test Case 11

Test Case 11 tests production capacity expansion, including scaling of investment costs and lifetimes. Tables 33 and 34 depict the sets and parameters, respectively. Input, production and conversion costs during all periods amount to 0.6 for "CNG" and 0.9 for "GH_2". However, there exists no demand for "CNG" and no production capacity is available in any period. Capacity expansion comes at a unit cost of 0.5, which is rescaled to 0.1 per modeled year. Lifetimes of investments are assumed to be 5 years. In combination with the model time steps of five years, investment costs behave like marginalized costs and can be added on top. Prices in 2025 will be $P = 0.9 + 0.1$, since demand is able to justify an investment. However, invested capacities from 2020 should not carry over to 2030, and the inverse demand intercept of 0.99 does not justify additional investments. Hence, prices for "GH_2" should be $P = 0$, $P = 1$, $P = 0.99$, in 2020, 2025 and 2030 respectively, while there exists no demand for "CNG". In addition, production capacity of 1 should be invested in 2020 for "CNG", while no investments should be made in 2025 and 2030 as well as for "GH_2". Test Case 11 passes if all these criteria are met, as can be seen from Table 35. Figure 11 depicts a graphical solution.

Table 33: Sets of Test Case 11

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU"}
$\mathcal{AC}$	{("DEU_to_DEU", "CNG"), ("DEU_to_DEU", "GH_2") }
$\mathcal{C}$	{"CNG","GH_2"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Natural Gas"}
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"OnlyTimeStep"}
$\mathcal{N}$	{"DEU"}
$\mathcal{O}$	{"FES"}
$\mathcal{P}$	{"P_DEU"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$
$\mathcal{S}$	$\emptyset$
$\mathcal{T}$	{"T_DEU"}
$\mathcal{V}$	$\emptyset$
$\mathcal{VT}$	{("CNG","GH_2")}
$\mathcal{Y}$	{"2020","2025","2030"}

Source: Own depiction.

Table 34: Parameter Values of Test Case 11

Parameter	**y=2020**	**y=2025**	**y=2030**
$\frac{1}{\lVert\Delta\rVert_y}$	0.2	0.2	0.2
$\mathbb{1}^{NC}_{\text{“T_DEU”,“DEU”,“CNG”}}$	0	0	0
r_y	1	1	1
$d_{\text{“OnlyTimeStep”}}$	1	1	1
$c^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	0.3	0.3	0.3
$c^{\Delta P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	0.5	0.5	0.5
$fi^{P}_{\text{“CNG”,“Natural Gas”,“FES”}}$	1	1	1
$L^{P}_{\text{“CNG”,“FES”}}$	50	50	50
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	0	0	0
$c^{P}_{\text{“P_DEU”,“GH}_2\text{”,“FES”},y}$	1.3	1.3	1.3
$c^{\Delta P}_{\text{“P_DEU”,“GH}_2\text{”,“FES”},y}$	0.5	0.5	0.5
$fi^{P}_{\text{“GH}_2\text{”,“Natural Gas”,“FES”}}$	1	1	1
$L^{P}_{\text{“GH}_2\text{”,“FES”}}$	50	50	50
$\Lambda^{P}_{\text{“P_DEU”,“GH}_2\text{”,“FES”},y}$	0	0	0
$\Lambda^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	10	10	10
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0	0	0
$c^{\Delta I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0	0	0
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”},y}$	10	10	10
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“GH}_2\text{”,“FES”},y}$	10	10	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10	10	10
$\Omega^{P}_{\text{“P_DEU”,“GH}_2\text{”,“FES”},y}$	10	10	10

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Table 34: Parameter Values of Test Case 11 (continued)

Parameter	**y=2020**	**y=2025**	**y=2030**
$fi^{V}_{\text{“CNG”,“GH}_2\text{”}}$	1	1	1
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0	0	0
$l^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”}}$	0	0	0
$c^{V}_{\text{“V_DEU”,“CNG”,“GH}_2\text{”},y}$	0.3	0.3	0.3
$c^{\Delta V}_{\text{“V_DEU”,“CNG”,“GH}_2\text{”},y}$	10	10	10
$\Lambda^{V}_{\text{“V_DEU”,“CNG”,“GH}_2\text{”},y}$	10	10	10
$L^{V}_{\text{“CNG”,“GH}_2\text{”}}$	5	5	5
$c^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0	0
$L^{A}_{\text{“CNG”}}$	50	50	50
$c^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”},y}$	0	0	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“GH}_2\text{”},y}$	0	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”},y}$	0	0	0
$L^{A}_{\text{“GH}_2\text{”}}$	50	50	50
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.3	0.3	0.3
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0	0	0
$av^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	1	1	1
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0	0	0
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	−1	−1	−1

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Table 34: Parameter Values of Test Case 11 (continued)

Parameter	**y=2020**	**y=2025**	**y=2030**
$\alpha^{D}_{\text{“DEU”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0	2	0.99
$\beta^{D}_{\text{“DEU”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	-1	-1	-1

Source: Own depiction.

Table 35: Tested Results of Test Case 11

Expression	**Result y=2020**	**Result y=2025**	**Result y=2030**
$\tilde{P}^{T\rightarrow D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0	0	0
$\tilde{P}^{T\rightarrow D}_{\text{“DEU”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0	1	0.99
$\Delta^{P}_{\text{“P_DEU”,“CNG”,“FES”,y}}$	1	0	0
$\Delta^{P}_{\text{“P_DEU”,“CNG”,“FES”,y}}$	0	0	0

Source: Own depiction.

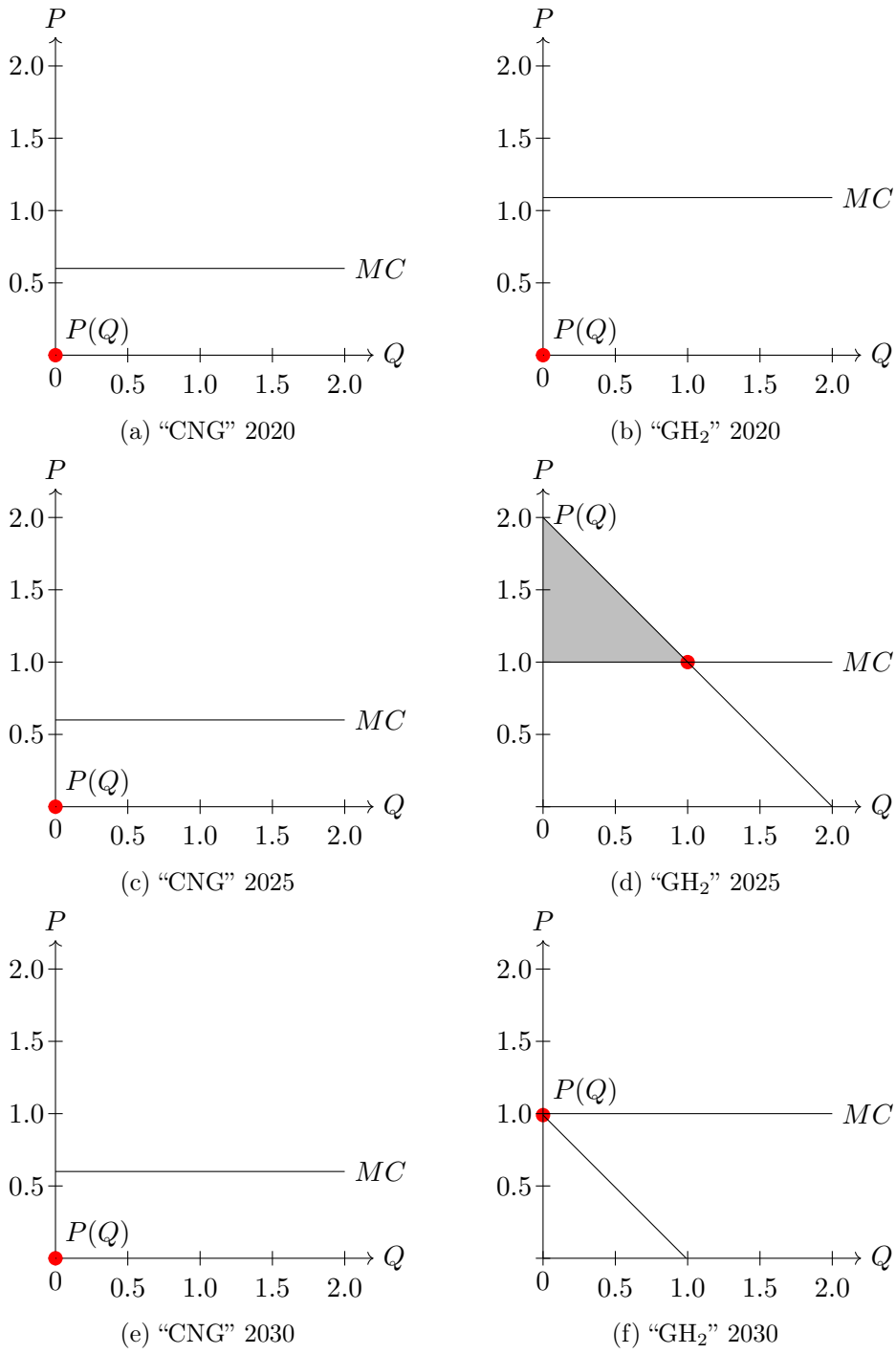

Figure 11: Marginalized Provision Costs and Prices for Different Time Steps in Test Case 11

Source: Own depiction.

2.2.12 Test Case 12

Test Case 12 is structurally similar to Test Case 11, but this time conversion capacities are non-existent instead of production capacities. Tables 36 and 37 depict the sets and parameters, respectively. The solution is the same as in Test Case 11, only that conversion capacity should be expanded instead of production capacity (at the same cost). Passing criteria can be seen in Table 38, while Figure 12 depicts the graphical solution.

Table 36: Sets of Test Case 12

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU"}
$\mathcal{AC}$	{("DEU_to_DEU", "CNG"), ("DEU_to_DEU", "GH_2") }
$\mathcal{C}$	{"CNG","GH_2"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Natural Gas"}
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"OnlyTimeStep"}
$\mathcal{N}$	{"DEU"}
$\mathcal{O}$	{"FES"}
$\mathcal{P}$	{"P_DEU"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$
$\mathcal{S}$	$\emptyset$
$\mathcal{T}$	{"T_DEU"}
$\mathcal{V}$	$\emptyset$

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Table 36: Sets of Test Case 12 (continued)

Set Name	**Set Value**
$\mathcal{VT}$	{("CNG","GH_2")}
$\mathcal{Y}$	{"2020","2025","2030"}

Source: Own depiction.

Table 37: Parameter Values of Test Case 12

Parameter	**y=2020**	**y=2025**	**y=2030**
$\frac{1}{\lVert\Delta\rVert_y}$	0.2	0.2	0.2
$\mathbb{1}^{NC}_{\text{"T_DEU","DEU","CNG"}}$	0	0	0
r_y	1	1	1
$d_{\text{"OnlyTimeStep"}}$	1	1	1
$c^{P}_{\text{"P_DEU","CNG","FES"},y}$	0.3	0.3	0.3
$c^{\Delta P}_{\text{"P_DEU","CNG","FES"},y}$	0.5	0.5	0.5
$fi^{P}_{\text{"CNG","Natural Gas","FES"}}$	1	1	1
$L^{P}_{\text{"CNG","FES"}}$	50	50	50
$\Lambda^{P}_{\text{"P_DEU","CNG","FES"},y}$	10	10	10
$c^{P}_{\text{"P_DEU","}GH_2\text{","FES"},y}$	1.3	1.3	1.3
$c^{\Delta P}_{\text{"P_DEU","}GH_2\text{","FES"},y}$	0.5	0.5	0.5
$fi^{P}_{\text{"}GH_2\text{","Natural Gas","FES"}}$	1	1	1
$L^{P}_{\text{"}GH_2\text{","FES"}}$	50	50	50
$\Lambda^{P}_{\text{"P_DEU","}GH_2\text{","FES"},y}$	10	10	10
$\Lambda^{I}_{\text{"P_DEU","Natural Gas","Block 1"},y}$	10	10	10
$\Omega^{I}_{\text{"P_DEU","Natural Gas","Block 1"},y}$	0	0	0

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Table 37: Parameter Values of Test Case 12 (continued)

Parameter	**y=2020**	**y=2025**	**y=2030**
$c^{\Delta I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0	0	0
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”},y}$	10	10	10
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“GH}_2\text{”,“FES”},y}$	10	10	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10	10	10
$\Omega^{P}_{\text{“P_DEU”,“GH}_2\text{”,“FES”},y}$	10	10	10
$fi^{V}_{\text{“CNG”,“GH}_2\text{”}}$	1	1	1
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0	0	0
$l^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”}}$	0	0	0
$c^{V}_{\text{“V_DEU”,“CNG”,“GH}_2\text{”},y}$	0.3	0.3	0.3
$c^{\Delta V}_{\text{“V_DEU”,“CNG”,“GH}_2\text{”},y}$	0.5	0.5	0.5
$\Lambda^{V}_{\text{“V_DEU”,“CNG”,“GH}_2\text{”},y}$	0	0	0
$L^{V}_{\text{“CNG”,“GH}_2\text{”}}$	5	5	5
$c^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0	0
$L^{A}_{\text{“CNG”}}$	50	50	50
$c^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”},y}$	0	0	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“GH}_2\text{”},y}$	0	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”},y}$	0	0	0
$L^{A}_{\text{“GH}_2\text{”}}$	50	50	50

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Table 37: Parameter Values of Test Case 12 (continued)

Parameter	**y=2020**	**y=2025**	**y=2030**
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.3	0.3	0.3
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0	0	0
$av^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	1	1	1
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0	0	0
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1	-1	-1
$\alpha^{D}_{\text{“DEU”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0	2	0.99
$\beta^{D}_{\text{“DEU”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	-1	-1	-1

Source: Own depiction.

Table 38: Tested Results of Test Case 12

Expression	**Result y=2020**	**Result y=2025**	**Result y=2030**
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0	0	0
$\tilde{P}^{T\to D}_{\text{“DEU”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0	1	0.99
$\Delta^{V}_{\text{“V_DEU”,“CNG”,“GH}_2\text{”},y}$	1	0	0

Source: Own depiction.

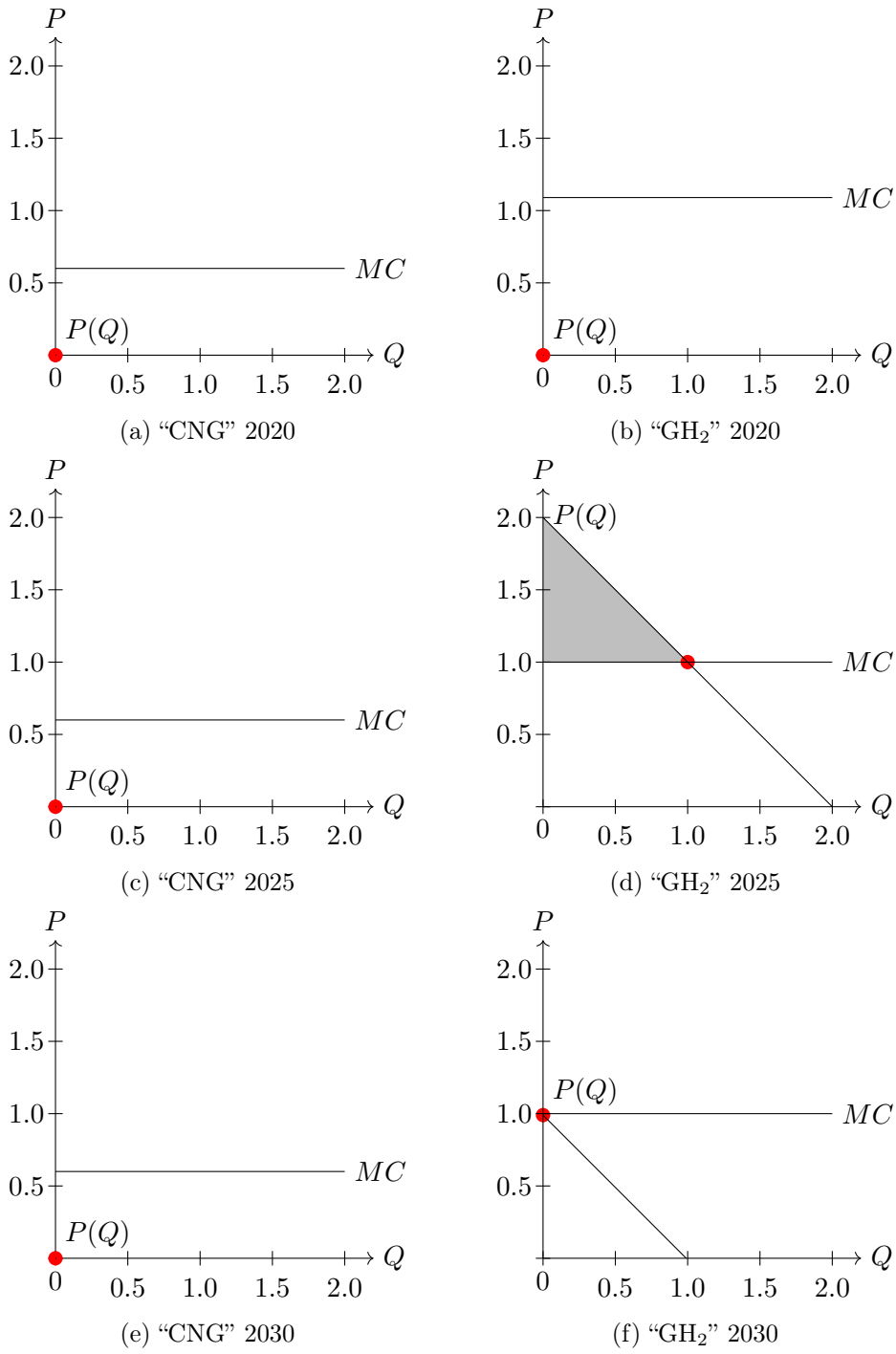

Figure 12: Marginalized Provision Costs and Prices in Test Case 12

Source: Own depiction.

2.2.13 Test Case 13

Test Case 13 is similar to Test Case 9, only that this time transport capacity is limited. Tables 39 and 40 depict the sets and parameters, respectively. Production costs in "DEU" are much too high. However, "NLD" has no own demand and can export to "DEU" with operational cost of 0.05 if capacity is invested. In addition to production and conversion costs of in total 0.9, rescaled investment costs of 0.05 must be added. Marginalized provision costs are hence 1.0. Tests are passed, if "DEU" prices are $P = 0$, $P = 1$, $P = 0.99$, in 2020, 2025 and 2030 respectively. In addition, capacity of 1 on arc "NLD_to_DEU" and reverse should be invested in 2020, while no investments should be made in 2025 and 2030. No other arc capacities should be expanded, and prices in "NLD" must be 0. Passing criteria can be seen in Table 41, while Figure 13 depicts a graphical solution.

Table 39: Sets of Test Case 13

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU","DEU_to_NLD","NLD_to_DEU","NLD_to_NLD"}
$\mathcal{AC}$	{("DEU_to_DEU", "CNG"),("DEU_to_NLD", "CNG"), ("NLD_to_DEU","CNG"),("NLD_to_NLD", "CNG")}
$\mathcal{C}$	{"CNG"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Natural Gas"}
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"OnlyTimeStep"}
$\mathcal{N}$	{"DEU","NLD"}
$\mathcal{O}$	{"FES"}
$\mathcal{P}$	{"P_DEU","P_NLD"}

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Table 39: Sets of Test Case 13 (continued)

Set Name	**Set Value**
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$
$\mathcal{S}$	$\emptyset$
$\mathcal{T}$	{“T_DEU”,“T_NLD”}
$\mathcal{V}$	$\emptyset$
$\mathcal{VT}$	$\emptyset$
$\mathcal{Y}$	{“2020”,“2025”,“2030”}

Source: Own depiction.

Table 40: Parameter Values of Test Case 13

Parameter	**y=2020**	**y=2025**	**y=2030**
$\frac{1}{\lVert\Delta\rVert_y}$	1	1	1
$\mathbb{1}^{NC}_{\text{“T_DEU”,“DEU”,“CNG”}}$	0	0	0
$\mathbb{1}^{NC}_{\text{“T_DEU”,“NLD”,“CNG”}}$	0	0	0
$\mathbb{1}^{NC}_{\text{“T_NLD”,“DEU”,“CNG”}}$	0	0	0
$\mathbb{1}^{NC}_{\text{“T_NLD”,“NLD”,“CNG”}}$	0	0	0
r_y	1	1	1
$d_{\text{“OnlyTimeStep”}}$	1	1	1
$c^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	0.4	0.4	0.4
$c^{\Delta P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	1	1	1
$c^{P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	0.5	0.5	0.5

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Table 40: Parameter Values of Test Case 13 (continued)

Parameter	**y=2020**	**y=2025**	**y=2030**
$c^{\Delta P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	1	1	1
$fi^{P}_{\text{“CNG”,“Natural Gas”,“FES”}}$	1	1	1
$L^{P}_{\text{“CNG”,“FES”}}$	50	50	50
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10	10	10
$\Lambda^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	10	10	10
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0	0	0
$c^{\Delta I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0	0	0
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”},y}$	10	10	10
$\Lambda^{T}_{\text{“T_DEU”,“NLD”,“CNG”,“FES”},y}$	10	10	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10	10	10
$\Lambda^{P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	10	10	10
$\Lambda^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”},y}$	10	10	10
$\Omega^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”},y}$	0	0	0
$c^{\Delta I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”},y}$	0	0	0
$\Lambda^{T}_{\text{“T_NLD”,“DEU”,“CNG”,“FES”},y}$	10	10	10
$\Lambda^{T}_{\text{“T_NLD”,“NLD”,“CNG”,“FES”},y}$	10	10	10
$\Omega^{P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	10	10	10
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0.0	0.0	0.0
$l^{A}_{\text{“DEU_to_NLD”,“CNG”}}$	0.0	0.0	0.0
$l^{A}_{\text{“NLD_to_DEU”,“CNG”}}$	0.1	0.1	0.1

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Table 40: Parameter Values of Test Case 13 (continued)

Parameter	**y=2020**	**y=2025**	**y=2030**
$l^{A}_{\text{“NLD_to_NLD”,“CNG”}}$	0.0	0.0	0.0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0.0	0.0	0.0
$c^{A}_{\text{“DEU_to_NLD”,“CNG”},y}$	0.0	0.0	0.0
$c^{A}_{\text{“NLD_to_DEU”,“CNG”},y}$	0.05	0.05	0.05
$c^{A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0.0	0.0	0.0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0.25	0.25	0.25
$c^{\Delta A}_{\text{“DEU_to_NLD”,“CNG”},y}$	0.25	0.25	0.25
$c^{\Delta A}_{\text{“NLD_to_DEU”,“CNG”},y}$	0.25	0.25	0.25
$c^{\Delta A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0.25	0.25	0.25
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0	0
$\Lambda^{A}_{\text{“DEU_to_NLD”,“CNG”},y}$	0	0	0
$\Lambda^{A}_{\text{“NLD_to_DEU”,“CNG”},y}$	0	0	0
$\Lambda^{A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0	0	0
$L^{A}_{\text{“CNG”}}$	50	50	50
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	2	2	2
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0	0	0
$av^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	1	1	1
$c^{I_l}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.5	0.5	0.5
$c^{I_q}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0	0	0
$av^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	1	1	1

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Table 40: Parameter Values of Test Case 13 (continued)

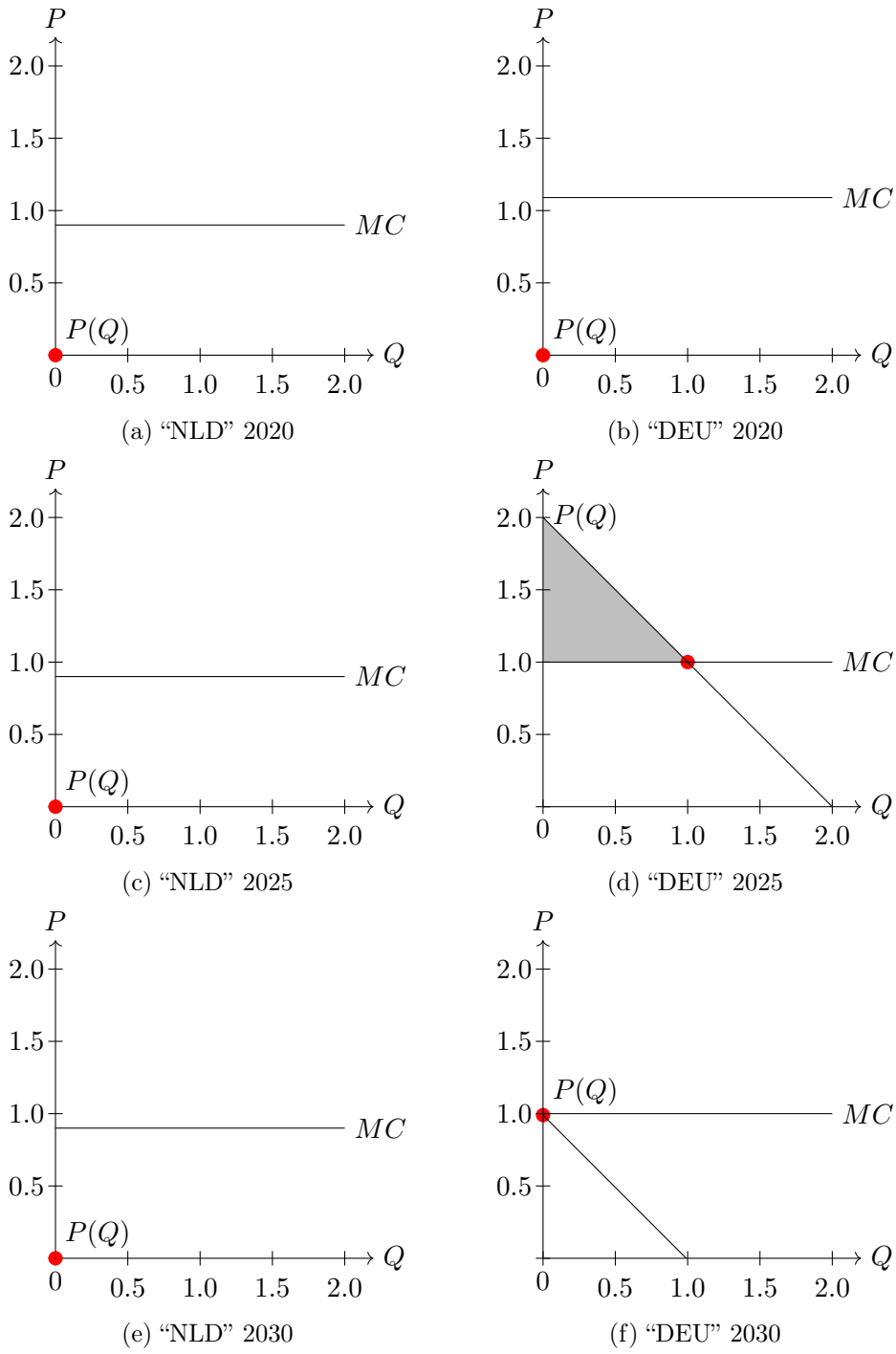
Parameter	**y=2020**	**y=2025**	**y=2030**
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	2	2	2
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	−1	−1	−1
$\alpha^{D}_{\text{“NLD”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0	0	0
$\beta^{D}_{\text{“NLD”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	−1	−1	−1

Source: Own depiction.

Table 41: Tested Results of Test Case 13

Expression	**Result y=2020**	**Result y=2025**	**Result y=2030**
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0	1	0.99
$\tilde{P}^{T\to D}_{\text{“NLD”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0.0	0.0	0.0
$\Delta^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0.0	0.0	0.0
$\Delta^{A}_{\text{“DEU_to_NLD”,“CNG”},y}$	1.0	0.0	0.0
$\Delta^{A}_{\text{“NLD_to_DEU”,“CNG”},y}$	1.0	0.0	0.0
$\Delta^{A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0.0	0.0	0.0

Source: Own depiction.



(a) "NLD" 2020

(b) "DEU" 2020

(c) "NLD" 2025

(d) "DEU" 2025

(e) "NLD" 2030

(f) "DEU" 2030

Figure 13: Marginalized Provision Costs and Prices in Test Case 13

Source: Own depiction.

2.2.14 Test Case 14

Test Case 14 tests capacity expansion, scaling of investment costs and lifetimes for storage. The set-out is similar to Test Case 5, only that there exist multiple years and no exogenous storage capacities. If provision costs (including investments) are sufficiently low in comparison to willingness to pay, investments are made to expand non-existing storage. Tables 42 and 43 depict the sets and parameters, respectively. Test Case 14 passes if capacity expansion of 1 only happens in 2020, and prices are 0 in all low-demand time steps, and 0, 1 and 0.99 in high-demand time steps of 2020, 2025 and 2030, respectively. Passing criteria can be seen in Table 44, while Figure 14 depicts a graphical solution.

Table 42: Sets of Test Case 14

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU"}
$\mathcal{AC}$	{("DEU_to_DEU", "CNG")}
$\mathcal{C}$	{"CNG"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Natural Gas"}
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"L","H"}
$\mathcal{N}$	{"DEU"}
$\mathcal{O}$	{"FES"}
$\mathcal{P}$	{"P_DEU"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$

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Table 42: Sets of Test Case 14 (continued)

Set Name	**Set Value**
$\mathcal{S}$	{"S_DEU"}
$\mathcal{T}$	{"T_DEU"}
$\mathcal{V}$	$\emptyset$
$\mathcal{VT}$	$\emptyset$
$\mathcal{Y}$	{"2020","2025","2030"}

Source: Own depiction.

Table 43: Parameter Values of Test Case 14

Parameter	**m=L**	**m=H**
$\frac{1}{\lVert\Delta\rVert_y}$	1	1
$\mathbb{1}^{NC}_{\text{"T_DEU","DEU","CNG"}}$	0	0
$r_{\text{"2020"}}$	1	1
$r_{\text{"2025"}}$	1	1
$r_{\text{"2030"}}$	1	1
d_m	2	1
$c^{P}_{\text{"P_DEU","CNG","FES","2020"}}$	0.4	0.4
$c^{P}_{\text{"P_DEU","CNG","FES","2025"}}$	0.4	0.4
$c^{P}_{\text{"P_DEU","CNG","FES","2030"}}$	0.4	0.4
$c^{\Delta P}_{\text{"P_DEU","CNG","FES","2020"}}$	1	1
$c^{\Delta P}_{\text{"P_DEU","CNG","FES","2025"}}$	1	1
$c^{\Delta P}_{\text{"P_DEU","CNG","FES","2030"}}$	1	1
$fi^{P}_{\text{"CNG","Natural Gas","FES"}}$	1	1

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Table 43: Parameter Values of Test Case 14 (continued)

Parameter	**m=L**	**m=H**
$L^{P}_{\text{“CNG”,“FES”}}$	50	50
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“FES”,“2020”}}$	10	10
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“FES”,“2025”}}$	10	10
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“FES”,“2030”}}$	10	10
$\Lambda^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2020”}}$	10	10
$\Lambda^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2025”}}$	10	10
$\Lambda^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2030”}}$	10	10
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2020”}}$	0	0
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2025”}}$	0	0
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2030”}}$	0	0
$c^{\Delta^{I}}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2020”}}$	0	0
$c^{\Delta^{I}}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2025”}}$	0	0
$c^{\Delta^{I}}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2030”}}$	0	0
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”,“2020”}}$	10	10
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”,“2025”}}$	10	10
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”,“2030”}}$	10	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”,“2020”}}$	10	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”,“2025”}}$	10	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”,“2030”}}$	10	10
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0	0

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Table 43: Parameter Values of Test Case 14 (continued)

Parameter	**m=L**	**m=H**
$c^{A}_{\text{“DEU_to_DEU”,“CNG”,“2020”}}$	0	0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”,“2025”}}$	0	0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”,“2030”}}$	0	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”,“2020”}}$	0	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”,“2025”}}$	0	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”,“2030”}}$	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”,“2020”}}$	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”,“2025”}}$	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”,“2030”}}$	0	0
$L^{A}_{\text{“CNG”}}$	50	50
$l^{S}_{\text{“CNG”,m,“L”}}$	0	0
$l^{S}_{\text{“CNG”,m,“H”}}$	0	0
$c^{S_{in}}_{\text{“S_DEU”,“CNG”,“2020”}}$	0.025	0.025
$c^{S_{in}}_{\text{“S_DEU”,“CNG”,“2025”}}$	0.025	0.025
$c^{S_{in}}_{\text{“S_DEU”,“CNG”,“2030”}}$	0.025	0.025
$c^{S_{out}}_{\text{“S_DEU”,“CNG”,“2020”}}$	0.025	0.025
$c^{S_{out}}_{\text{“S_DEU”,“CNG”,“2025”}}$	0.025	0.025
$c^{S_{out}}_{\text{“S_DEU”,“CNG”,“2030”}}$	0.025	0.025
$c^{\Delta S}_{\text{“S_DEU”,“CNG”,“2020”}}$	0.25	0.25
$c^{\Delta S}_{\text{“S_DEU”,“CNG”,“2025”}}$	0.25	0.25

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Table 43: Parameter Values of Test Case 14 (continued)

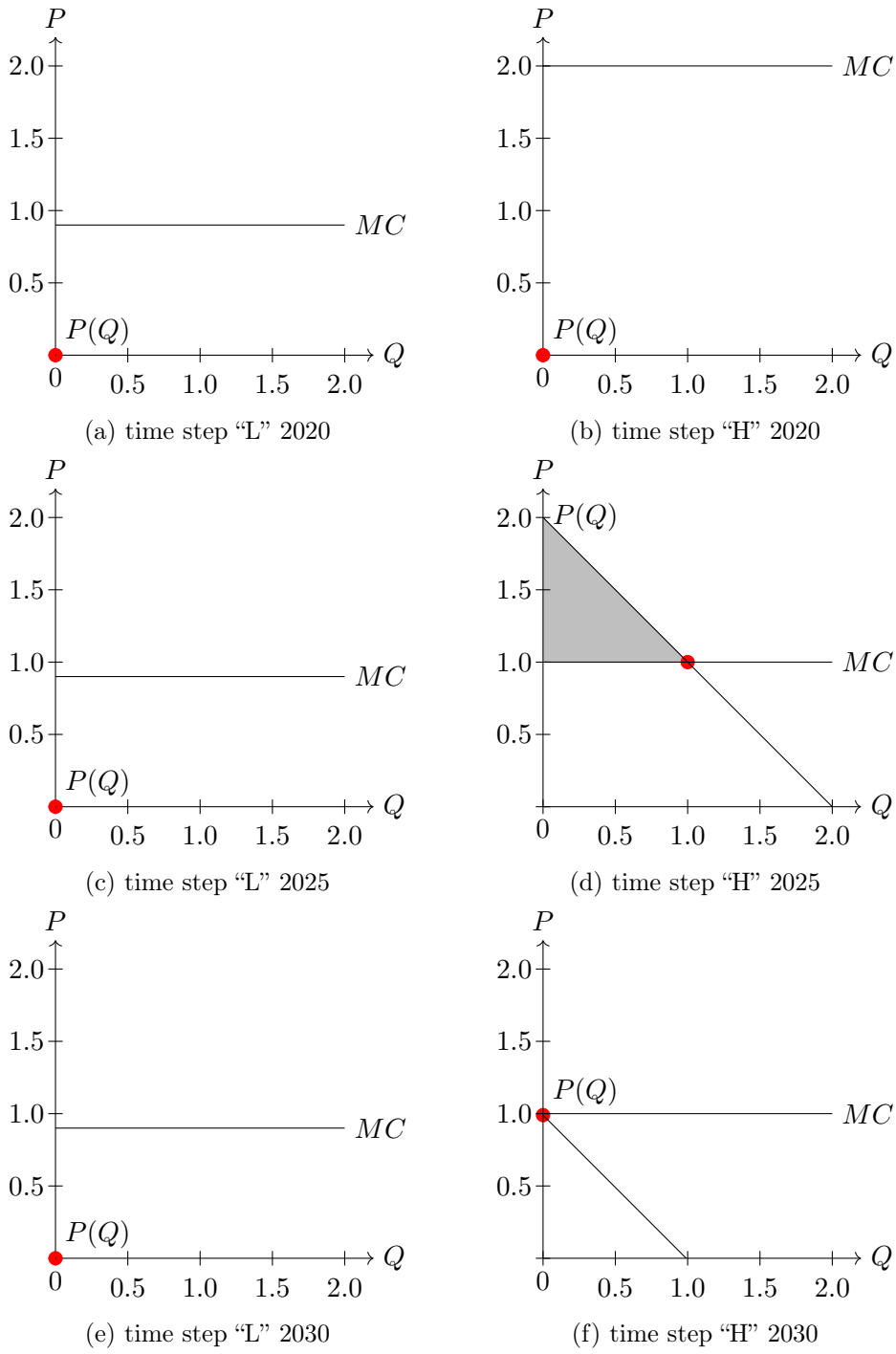
Parameter	**m=L**	**m=H**
$c^{\Delta S}_{\text{“S_DEU”,“CNG”,“2030”}}$	0.25	0.25
$\Lambda^{S}_{\text{“S_DEU”,“CNG”,“2020”}}$	0	0
$\Lambda^{S}_{\text{“S_DEU”,“CNG”,“2025”}}$	0	0
$\Lambda^{S}_{\text{“S_DEU”,“CNG”,“2030”}}$	0	0
$\Omega^{S}_{\text{“S_DEU”,“CNG”,“2020”}}$	10	10
$\Omega^{S}_{\text{“S_DEU”,“CNG”,“2025”}}$	10	10
$\Omega^{S}_{\text{“S_DEU”,“CNG”,“2030”}}$	10	10
$L^{S}_{\text{“CNG”}}$	5	5
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m,“2020”}}$	0.5	1.6
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m,“2025”}}$	0.5	1.6
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m,“2030”}}$	0.5	1.6
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m,“2020”}}$	0	0
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m,“2025”}}$	0	0
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m,“2030”}}$	0	0
$av^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m}}$	1	1
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2020”}}$	0	2
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2025”}}$	0	2
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2030”}}$	0	2
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2020”}}$	0	−1
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2025”}}$	0	−1
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2030”}}$	0	−1

Source: Own depiction.

Table 44: Tested Results of Test Case 14

Expression	**Result m=L**	**Result m=H**
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2020”}}$	0.0	0.0
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2025”}}$	0.0	1.0
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2030”}}$	0.0	0.99
$\Delta^{S}_{\text{“S_DEU”,“CNG”,“2020”}}$	1	1
$\Delta^{S}_{\text{“S_DEU”,“CNG”,“2025”}}$	0	0
$\Delta^{S}_{\text{“S_DEU”,“CNG”,“2030”}}$	0	0

Source: Own depiction.



(a) time step "L" 2020

(b) time step "H" 2020

(c) time step "L" 2025

(d) time step "H" 2025

(e) time step "L" 2030

(f) time step "H" 2030

Figure 14: Marginalized Provision Costs and Prices in Test Case 14

Source: Own depiction.

2.2.15 Test Case 15

Test Case 15 is similar to Test Case 11, except that production capacity expansion is disallowed. Tables 45 and 46 depict the sets and parameters, respectively. Since no production is possible now, prices must be equal to inverse demand intercepts. The test is passed, if prices in 2020, 2025 and 2030 are 0, 2 and 0.99, respectively and no capacities are invested in. Passing criteria can be seen in Table 47, while Figure 15 depicts a graphical solution.

Table 45: Sets of Test Case 15

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU"}
$\mathcal{AC}$	{("DEU_to_DEU", "CNG"), ("DEU_to_DEU", "GH_2")}
$\mathcal{C}$	{"CNG","GH_2"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Natural Gas"}
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"OnlyTimeStep"}
$\mathcal{N}$	{"DEU"}
$\mathcal{O}$	{"FES"}
$\mathcal{P}$	{"P_DEU"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$
$\mathcal{S}$	$\emptyset$
$\mathcal{T}$	{"T_DEU"}

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Table 45: Sets of Test Case 15 (continued)

Set Name	**Set Value**
$\mathcal{V}$	$\emptyset$
$\mathcal{VT}$	{("CNG","GH_2")}
$\mathcal{Y}$	{"2020","2025","2030"}

Source: Own depiction.

Table 46: Parameter Values of Test Case 15

Parameter	**y=2020**	**y=2025**	**y=2030**
$\frac{1}{\lVert\Delta\rVert_y}$	0.2	0.2	0.2
$\mathbb{1}^{NC}_{\text{"T_DEU","DEU","CNG"}}$	0	0	0
r_y	1	1	1
$d_{\text{"OnlyTimeStep"}}$	1	1	1
$c^{P}_{\text{"P_DEU","CNG","FES"},y}$	0.3	0.3	0.3
$c^{\Delta P}_{\text{"P_DEU","CNG","FES"},y}$	0.5	0.5	0.5
$fi^{P}_{\text{"CNG","Natural Gas","FES"}}$	1	1	1
$L^{P}_{\text{"CNG","FES"}}$	50	50	50
$\Lambda^{P}_{\text{"P_DEU","CNG","FES"},y}$	0	0	0
$c^{P}_{\text{"P_DEU","}GH_2\text{","FES"},y}$	1.3	1.3	1.3
$c^{\Delta P}_{\text{"P_DEU","}GH_2\text{","FES"},y}$	0.5	0.5	0.5
$fi^{P}_{\text{"}GH_2\text{","Natural Gas","FES"}}$	1	1	1
$L^{P}_{\text{"}GH_2\text{","FES"}}$	50	50	50
$\Lambda^{P}_{\text{"P_DEU","}GH_2\text{","FES"},y}$	0	0	0
$\Lambda^{I}_{\text{"P_DEU","Natural Gas","Block 1"},y}$	10	10	10

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Table 46: Parameter Values of Test Case 15 (continued)

Parameter	**y=2020**	**y=2025**	**y=2030**
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0	0	0
$c^{\Delta I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0	0	0
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”},y}$	10	10	10
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“GH}_2\text{”,“FES”},y}$	10	10	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	0	0	0
$\Omega^{P}_{\text{“P_DEU”,“GH}_2\text{”,“FES”},y}$	0	0	0
$fi^{V}_{\text{“CNG”,“GH}_2\text{”}}$	1	1	1
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0	0	0
$l^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”}}$	0	0	0
$c^{V}_{\text{“V_DEU”,“CNG”,“GH}_2\text{”},y}$	0.3	0.3	0.3
$c^{\Delta V}_{\text{“V_DEU”,“CNG”,“GH}_2\text{”},y}$	10	10	10
$\Lambda^{V}_{\text{“V_DEU”,“CNG”,“GH}_2\text{”},y}$	10	10	10
$L^{V}_{\text{“CNG”,“GH}_2\text{”}}$	5	5	5
$c^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0	0
$L^{A}_{\text{“CNG”}}$	50	50	50
$c^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”},y}$	0	0	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“GH}_2\text{”},y}$	0	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”},y}$	0	0	0

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Table 46: Parameter Values of Test Case 15 (continued)

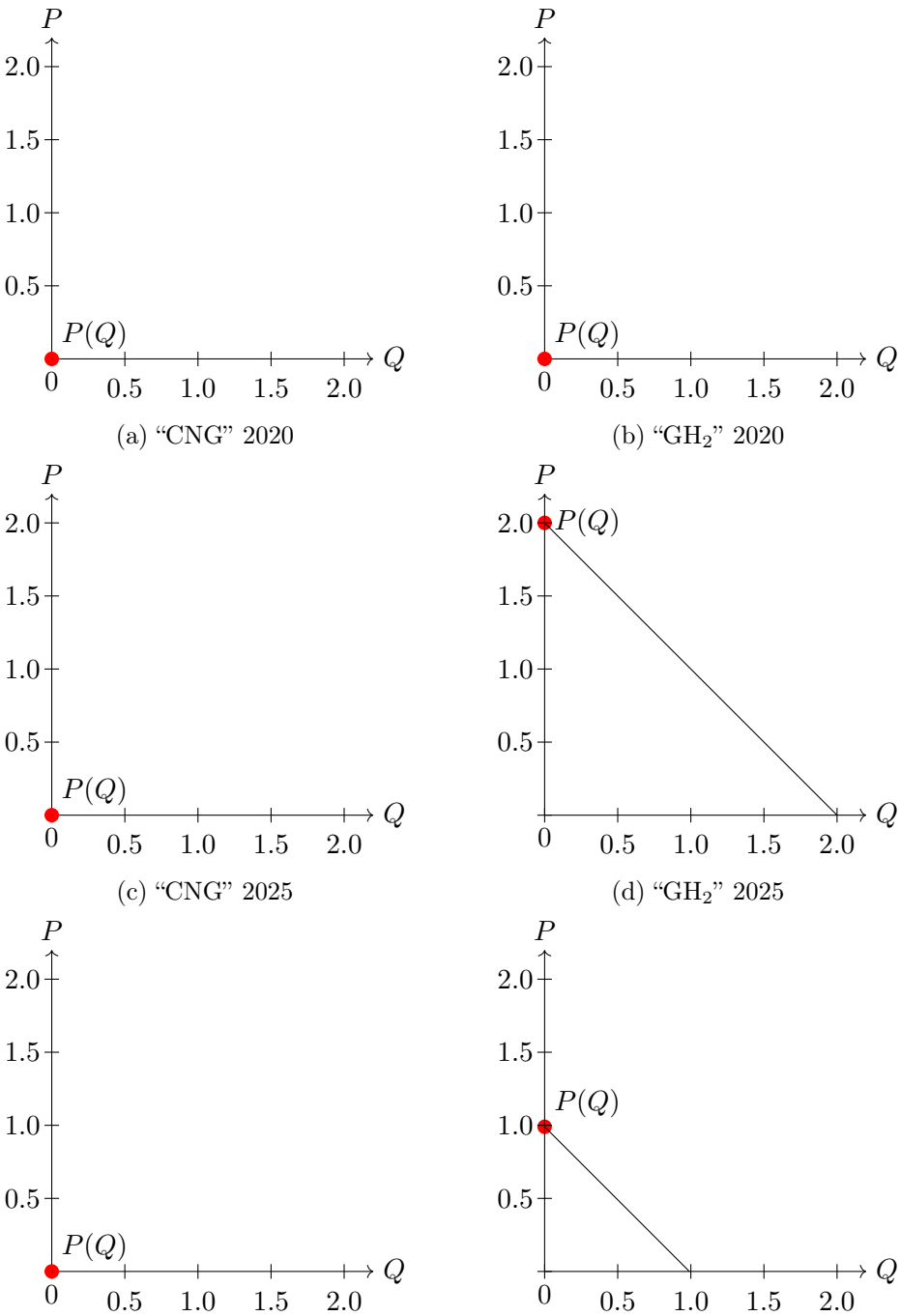
Parameter	**y=2020**	**y=2025**	**y=2030**
$L^{A}_{\text{“GH}_2\text{”}}$	50	50	50
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.3	0.3	0.3
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0	0	0
$av^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	1	1	1
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0	0	0
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1	-1	-1
$\alpha^{D}_{\text{“DEU”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0	2	0.99
$\beta^{D}_{\text{“DEU”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	-1	-1	-1

Source: Own depiction.

Table 47: Tested Results of Test Case 15

Expression	**Result y=2020**	**Result y=2025**	**Result y=2030**
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0	0	0
$\tilde{P}^{T\to D}_{\text{“DEU”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0	2	0.99
$\Delta^{P}_{\text{“P_DEU”,“CNG”,“FES”,y}}$	0	0	0
$\Delta^{P}_{\text{“P_DEU”,“CNG”,“FES”,y}}$	0	0	0

Source: Own depiction.



(a) "CNG" 2020

(b) "GH_2" 2020

(c) "CNG" 2025

(d) "GH_2" 2025

(e) "CNG" 2030

(f) "GH_2" 2030

Figure 15: Marginalized Provision Costs and Prices for Different Time Steps in Test Case 15

Source: Own depiction.

2.2.16 Test Case 16

Test Case 16 is similar to Test Case 14, except that storage capacity expansion is disallowed. Tables 48 and 49 depict the sets and parameters, respectively. Since no storage is possible now, prices must be equal to inverse demand intercepts. Tests are passed, if high demand prices in 2020, 2025 and 2030 are 0, 2 and 0.98, respectively and zero during low demand time steps. Further, no capacity expansions are allowed to happen for the test to pass. Passing criteria can be seen in Table 50, while Figure 16 depicts a graphical solution.

Table 48: Sets of Test Case 16

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU"}
$\mathcal{AC}$	{("DEU_to_DEU", "CNG")}
$\mathcal{C}$	{"CNG"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Natural Gas"}
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"L","H"}
$\mathcal{N}$	{"DEU"}
$\mathcal{O}$	{"FES"}
$\mathcal{P}$	{"P_DEU"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$
$\mathcal{S}$	{"S_DEU"}

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Table 48: Sets of Test Case 16 (continued)

Set Name	**Set Value**
$\mathcal{T}$	{"T_DEU"}
$\mathcal{V}$	$\emptyset$
$\mathcal{VT}$	$\emptyset$
$\mathcal{Y}$	{"2020","2025","2030"}

Source: Own depiction.

Table 49: Parameter Values of Test Case 16

Parameter	**m=L**	**m=H**
$\frac{1}{\lVert\Delta\rVert_y}$	1	1
$\mathbb{1}^{NC}_{\text{"T_DEU","DEU","CNG"}}$	0	0
$r_{\text{"2020"}}$	1	1
$r_{\text{"2025"}}$	1	1
$r_{\text{"2030"}}$	1	1
d_m	2	1
$c^{P}_{\text{"P_DEU","CNG","FES","2020"}}$	0.4	0.4
$c^{P}_{\text{"P_DEU","CNG","FES","2025"}}$	0.4	0.4
$c^{P}_{\text{"P_DEU","CNG","FES","2030"}}$	0.4	0.4
$c^{\Delta P}_{\text{"P_DEU","CNG","FES","2020"}}$	1	1
$c^{\Delta P}_{\text{"P_DEU","CNG","FES","2025"}}$	1	1
$c^{\Delta P}_{\text{"P_DEU","CNG","FES","2030"}}$	1	1
$fi^{P}_{\text{"CNG","Natural Gas","FES"}}$	1	1
$L^{P}_{\text{"CNG","FES"}}$	50	50

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Table 49: Parameter Values of Test Case 16 (continued)

Parameter	**m=L**	**m=H**
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“FES”,“2020”}}$	10	10
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“FES”,“2025”}}$	10	10
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“FES”,“2030”}}$	10	10
$\Lambda^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2020”}}$	10	10
$\Lambda^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2025”}}$	10	10
$\Lambda^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2030”}}$	10	10
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2020”}}$	0	0
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2025”}}$	0	0
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2030”}}$	0	0
$c^{\Delta^{I}}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2020”}}$	0	0
$c^{\Delta^{I}}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2025”}}$	0	0
$c^{\Delta^{I}}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“2030”}}$	0	0
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”,“2020”}}$	10	10
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”,“2025”}}$	10	10
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”,“2030”}}$	10	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”,“2020”}}$	10	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”,“2025”}}$	10	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”,“2030”}}$	10	10
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0	0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”,“2020”}}$	0	0

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Table 49: Parameter Values of Test Case 16 (continued)

Parameter	**m=L**	**m=H**
$c^{A}_{\text{“DEU_to_DEU”,“CNG”,“2025”}}$	0	0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”,“2030”}}$	0	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”,“2020”}}$	0	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”,“2025”}}$	0	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”,“2030”}}$	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”,“2020”}}$	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”,“2025”}}$	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”,“2030”}}$	0	0
$L^{A}_{\text{“CNG”}}$	50	50
$l^{S}_{\text{“CNG”,m,“L”}}$	0	0
$l^{S}_{\text{“CNG”,m,“H”}}$	0	0
$c^{S}_{\text{“S_DEU”,“CNG”,m,“H”,“2020”}}$	0.05	0.05
$c^{S}_{\text{“S_DEU”,“CNG”,m,“H”,“2025”}}$	0.05	0.05
$c^{S}_{\text{“S_DEU”,“CNG”,m,“H”,“2030”}}$	0.05	0.05
$c^{S}_{\text{“S_DEU”,“CNG”,m,“L”,“2020”}}$	0.05	0.05
$c^{S}_{\text{“S_DEU”,“CNG”,m,“L”,“2025”}}$	0.05	0.05
$c^{S}_{\text{“S_DEU”,“CNG”,m,“L”,“2030”}}$	0.05	0.05
$c^{\Delta S}_{\text{“S_DEU”,“CNG”,“2020”}}$	0.25	0.25
$c^{\Delta S}_{\text{“S_DEU”,“CNG”,“2025”}}$	0.25	0.25
$c^{\Delta S}_{\text{“S_DEU”,“CNG”,“2030”}}$	0.25	0.25

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Table 49: Parameter Values of Test Case 16 (continued)

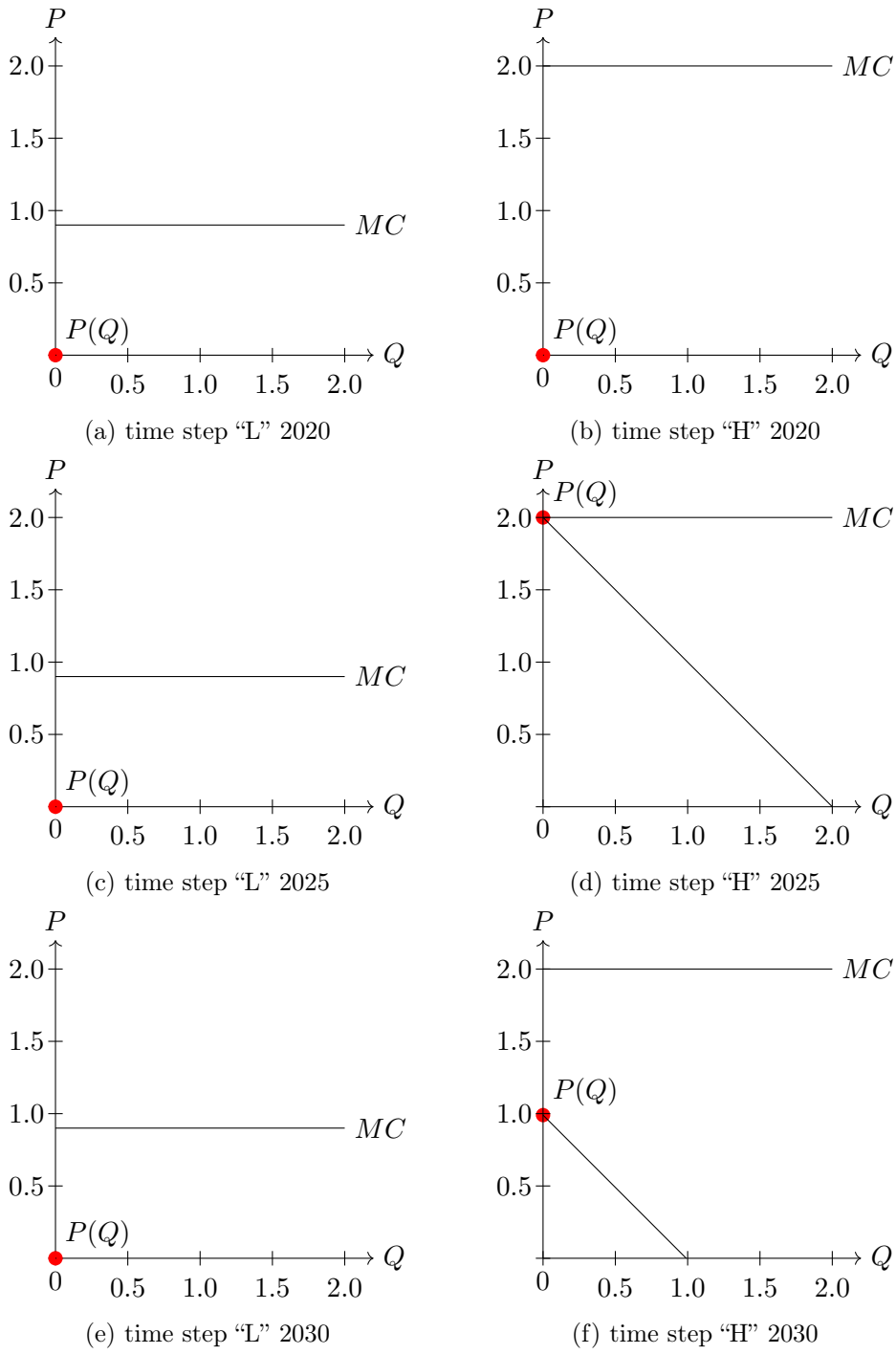
Parameter	**m=L**	**m=H**
$\Lambda^{S}_{\text{“S_DEU”,“CNG”,“2020”}}$	0	0
$\Lambda^{S}_{\text{“S_DEU”,“CNG”,“2025”}}$	0	0
$\Lambda^{S}_{\text{“S_DEU”,“CNG”,“2030”}}$	0	0
$\Omega^{S}_{\text{“S_DEU”,“CNG”,“2020”}}$	0	0
$\Omega^{S}_{\text{“S_DEU”,“CNG”,“2025”}}$	0	0
$\Omega^{S}_{\text{“S_DEU”,“CNG”,“2030”}}$	0	0
$L^{S}_{\text{“CNG”}}$	5	5
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m,“2020”}}$	0.5	1.6
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m,“2025”}}$	0.5	1.6
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m,“2030”}}$	0.5	1.6
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m,“2020”}}$	0	0
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m,“2025”}}$	0	0
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m,“2030”}}$	0	0
$av^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m}}$	1	1
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2020”}}$	0	2
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2025”}}$	0	2
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2030”}}$	0	2
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2020”}}$	0	-1
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2025”}}$	0	-1
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2030”}}$	0	-1

Source: Own depiction.

Table 50: Tested Results of Test Case 16

Expression	**Result m=L**	**Result m=H**
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2020”}}$	0.0	0.0
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2025”}}$	0.0	2.0
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,“Block 1”,m,“2030”}}$	0.0	0.99
$\Delta^{S}_{\text{“S_DEU”,“CNG”,“2020”}}$	0	0
$\Delta^{S}_{\text{“S_DEU”,“CNG”,“2025”}}$	0	0
$\Delta^{S}_{\text{“S_DEU”,“CNG”,“2030”}}$	0	0

Source: Own depiction.



(a) time step "L" 2020

(b) time step "H" 2020

(c) time step "L" 2025

(d) time step "H" 2025

(e) time step "L" 2030

(f) time step "H" 2030

Figure 16: Marginalized Provision Costs and Prices in Test Case 16

Source: Own depiction.

2.2.17 Test Case 17

Test Case 17 is similar to Test Case 13, only that "T_NLD" is not allowed to sell natural gas from "FES" in "DEU". Tables 51 and 52 depict the sets and parameters, respectively. For the test to pass, capacity expansions of all arcs must be zero, and prices must be equal to inverse demand intercepts, i.e. for "DEU" 0, 2 and 0.98 in 2020, 2025 and 2030, respectively, and 0 in "NLD" in all years. Passing criteria can be seen in Table 53, while Figure 17 depicts a graphical solution.

Table 51: Sets of Test Case 17

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU","DEU_to_NLD","NLD_to_DEU","NLD_to_NLD"}
$\mathcal{AC}$	{("DEU_to_DEU", "CNG"),("DEU_to_NLD", "CNG"), ("NLD_to_DEU", "CNG"),("NLD_to_NLD", "CNG")}
$\mathcal{C}$	{"CNG"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Natural Gas"}
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"OnlyTimeStep"}
$\mathcal{N}$	{"DEU","NLD"}
$\mathcal{O}$	{"FES"}
$\mathcal{P}$	{"P_DEU","P_NLD"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$
$\mathcal{S}$	$\emptyset$

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Table 51: Sets of Test Case 17 (continued)

Set Name	**Set Value**
$\mathcal{T}$	{“T_DEU”,“T_NLD”}
$\mathcal{V}$	$\emptyset$
$\mathcal{VT}$	$\emptyset$
$\mathcal{Y}$	{“2020”,“2025”,“2030”}

Source: Own depiction.

Table 52: Parameter Values of Test Case 17

Parameter	**y=2020**	**y=2025**	**y=2030**
$\frac{1}{\lVert\Delta\rVert_y}$	1	1	1
$\mathbb{1}^{NC}_{\text{“T_DEU”,“DEU”,“CNG”}}$	0	0	0
$\mathbb{1}^{NC}_{\text{“T_DEU”,“NLD”,“CNG”}}$	0	0	0
$\mathbb{1}^{NC}_{\text{“T_NLD”,“DEU”,“CNG”}}$	0	0	0
$\mathbb{1}^{NC}_{\text{“T_NLD”,“NLD”,“CNG”}}$	0	0	0
r_y	1	1	1
$d_{\text{“OnlyTimeStep”}}$	1	1	1
$c^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	0.4	0.4	0.4
$c^{\Delta P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	1	1	1
$c^{P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	0.5	0.5	0.5
$c^{\Delta P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	1	1	1
$fi^{P}_{\text{“CNG”,“Natural Gas”,“FES”}}$	1	1	1
$L^{P}_{\text{“CNG”,“FES”}}$	50	50	50
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10	10	10

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Table 52: Parameter Values of Test Case 17 (continued)

Parameter	**y=2020**	**y=2025**	**y=2030**
$\Lambda^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	10	10	10
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0	0	0
$c^{\Delta^{I}}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0	0	0
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”},y}$	10	10	10
$\Lambda^{T}_{\text{“T_DEU”,“NLD”,“CNG”,“FES”},y}$	10	10	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10	10	10
$\Lambda^{P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	10	10	10
$\Lambda^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”},y}$	10	10	10
$\Omega^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”},y}$	0	0	0
$c^{\Delta^{I}}_{\text{“P_NLD”,“Natural Gas”,“Block 1”},y}$	0	0	0
$\Lambda^{T}_{\text{“T_NLD”,“DEU”,“CNG”,“FES”},y}$	0	0	0
$\Lambda^{T}_{\text{“T_NLD”,“NLD”,“CNG”,“FES”},y}$	10	10	10
$\Omega^{P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	10	10	10
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0.0	0.0	0.0
$l^{A}_{\text{“DEU_to_NLD”,“CNG”}}$	0.0	0.0	0.0
$l^{A}_{\text{“NLD_to_DEU”,“CNG”}}$	0.1	0.1	0.1
$l^{A}_{\text{“NLD_to_NLD”,“CNG”}}$	0.0	0.0	0.0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0.0	0.0	0.0
$c^{A}_{\text{“DEU_to_NLD”,“CNG”},y}$	0.0	0.0	0.0
$c^{A}_{\text{“NLD_to_DEU”,“CNG”},y}$	0.05	0.05	0.05

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Table 52: Parameter Values of Test Case 17 (continued)

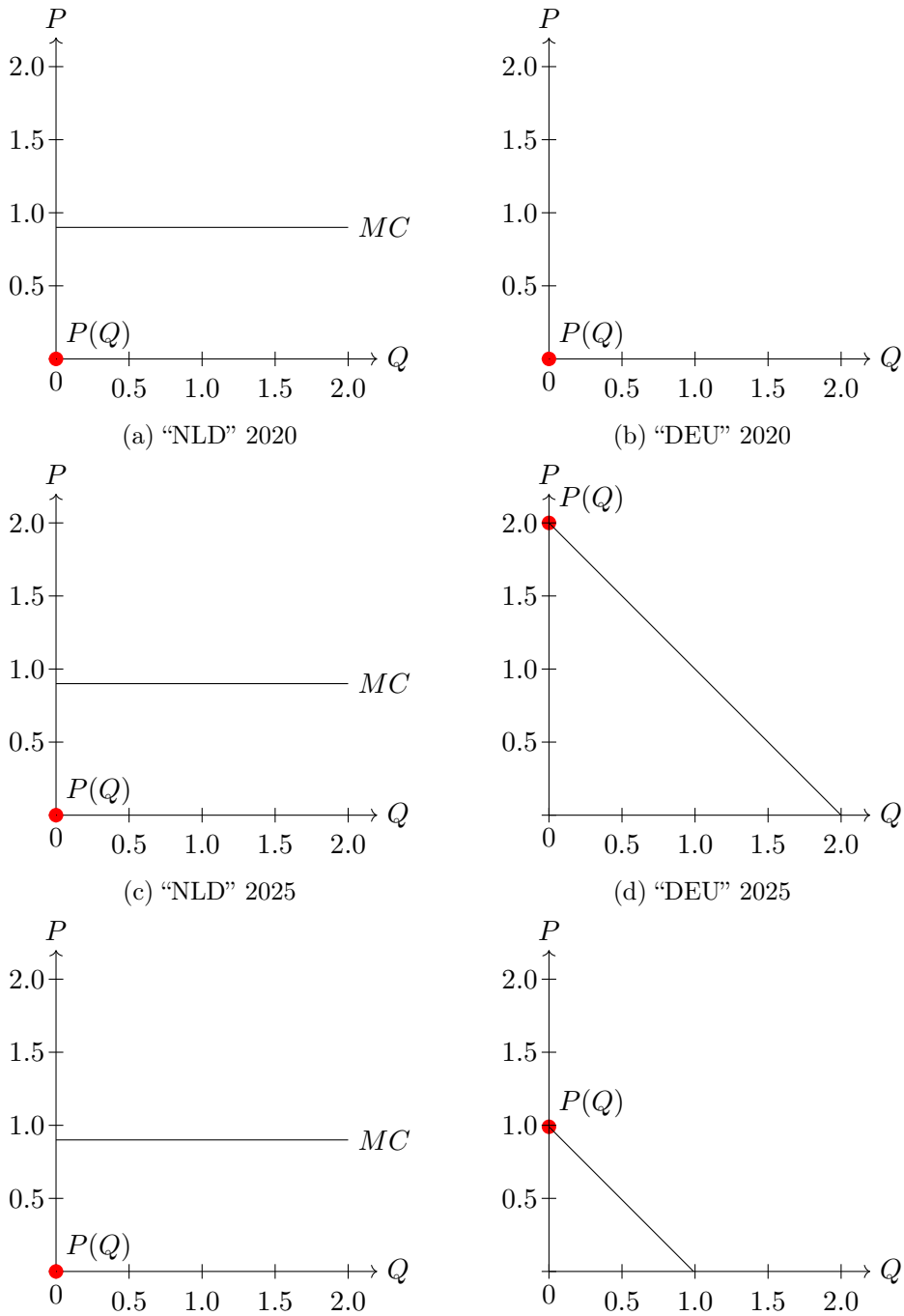
Parameter	**y=2020**	**y=2025**	**y=2030**
$c^{A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0.0	0.0	0.0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0.25	0.25	0.25
$c^{\Delta A}_{\text{“DEU_to_NLD”,“CNG”},y}$	0.25	0.25	0.25
$c^{\Delta A}_{\text{“NLD_to_DEU”,“CNG”},y}$	0.25	0.25	0.25
$c^{\Delta A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0.25	0.25	0.25
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0	0
$\Lambda^{A}_{\text{“DEU_to_NLD”,“CNG”},y}$	0	0	0
$\Lambda^{A}_{\text{“NLD_to_DEU”,“CNG”},y}$	0	0	0
$\Lambda^{A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0	0	0
$L^{A}_{\text{“CNG”}}$	50	50	50
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	2	2	2
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0	0	0
$av^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	1	1	1
$c^{I_l}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.5	0.5	0.5
$c^{I_q}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0	0	0
$av^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	1	1	1
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	2	2	2
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1	-1	-1
$\alpha^{D}_{\text{“NLD”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0	0	0
$\beta^{D}_{\text{“NLD”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1	-1	-1

Source: Own depiction.

Table 53: Tested Results of Test Case 17

Expression	**Result y=2020**	**Result y=2025**	**Result y=2030**
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0	2	0.99
$\tilde{P}^{T\to D}_{\text{“NLD”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0.0	0.0	0.0
$\Delta^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0.0	0.0	0.0
$\Delta^{A}_{\text{“DEU_to_NLD”,“CNG”},y}$	0.0	0.0	0.0
$\Delta^{A}_{\text{“NLD_to_DEU”,“CNG”},y}$	0.0	0.0	0.0
$\Delta^{A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0.0	0.0	0.0

Source: Own depiction.



(a) "NLD" 2020

(b) "DEU" 2020

(c) "NLD" 2025

(d) "DEU" 2025

(e) "NLD" 2030

(f) "DEU" 2030

Figure 17: Marginalized Provision Costs and Prices in Test Case 17

Source: Own depiction.

2.2.18 Test Case 18

Test Case 18 is similar to Test Case 13, only that "T_NLD" is able to exercise market power in "DEU". Tables 54 and 55 depict the sets and parameters, respectively. As the trader acts as a Cournot monopolist, the necessary capacity expansion in 2020 reduces to 0.5. The test is passed, if only "NLD_to_DEU" and reverse is expanded by 0.5 in 2020, while no other expansions take place. In addition, prices in "DEU" must be 0, 1.5 and 0.98 in 2020, 2025 and 2030, respectively, while those in "NLD" must be equal to zero. Passing criteria can be seen in Table 56, while Figure 18 depicts a graphical solution.

Table 54: Sets of Test Case 18

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU","DEU_to_NLD","NLD_to_DEU","NLD_to_NLD"}
$\mathcal{AC}$	{("DEU_to_DEU", "CNG"),("DEU_to_NLD", "CNG"), ("NLD_to_DEU", "CNG"),("NLD_to_NLD", "CNG")}
$\mathcal{C}$	{"CNG"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Natural Gas"}
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"OnlyTimeStep"}
$\mathcal{N}$	{"DEU","NLD"}
$\mathcal{O}$	{"FES"}
$\mathcal{P}$	{"P_DEU","P_NLD"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$

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Table 54: Sets of Test Case 18 (continued)

Set Name	**Set Value**
$\mathcal{S}$	$\emptyset$
$\mathcal{T}$	{“T_DEU”,“T_NLD”}
$\mathcal{V}$	$\emptyset$
$\mathcal{VT}$	$\emptyset$
$\mathcal{Y}$	{“2020”,“2025”,“2030”}

Source: Own depiction.

Table 55: Parameter Values of Test Case 18

Parameter	**y=2020**	**y=2025**	**y=2030**
$\frac{1}{\lVert\Delta\rVert_y}$	1	1	1
$\mathbb{1}^{NC}_{\text{“T_DEU”,“DEU”,“CNG”}}$	0	0	0
$\mathbb{1}^{NC}_{\text{“T_DEU”,“NLD”,“CNG”}}$	1	1	1
$\mathbb{1}^{NC}_{\text{“T_NLD”,“DEU”,“CNG”}}$	1	1	1
$\mathbb{1}^{NC}_{\text{“T_NLD”,“NLD”,“CNG”}}$	0	0	0
r_y	1	1	1
$d_{\text{“OnlyTimeStep”}}$	1	1	1
$c^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	0.4	0.4	0.4
$c^{\Delta P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	1	1	1
$c^{P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	0.5	0.5	0.5
$c^{\Delta P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	1	1	1
$fi^{P}_{\text{“CNG”,“Natural Gas”,“FES”}}$	1	1	1
$L^{P}_{\text{“CNG”,“FES”}}$	50	50	50

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Table 55: Parameter Values of Test Case 18 (continued)

Parameter	**y=2020**	**y=2025**	**y=2030**
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10	10	10
$\Lambda^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	10	10	10
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0	0	0
$c^{\Delta^{I}}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0	0	0
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”},y}$	10	10	10
$\Lambda^{T}_{\text{“T_DEU”,“NLD”,“CNG”,“FES”},y}$	10	10	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10	10	10
$\Lambda^{P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	10	10	10
$\Lambda^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”},y}$	10	10	10
$\Omega^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”},y}$	0	0	0
$c^{\Delta^{I}}_{\text{“P_NLD”,“Natural Gas”,“Block 1”},y}$	0	0	0
$\Lambda^{T}_{\text{“T_NLD”,“DEU”,“CNG”,“FES”},y}$	10	10	10
$\Lambda^{T}_{\text{“T_NLD”,“NLD”,“CNG”,“FES”},y}$	10	10	10
$\Omega^{P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	10	10	10
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0.0	0.0	0.0
$l^{A}_{\text{“DEU_to_NLD”,“CNG”}}$	0.0	0.0	0.0
$l^{A}_{\text{“NLD_to_DEU”,“CNG”}}$	0.1	0.1	0.1
$l^{A}_{\text{“NLD_to_NLD”,“CNG”}}$	0.0	0.0	0.0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0.0	0.0	0.0
$c^{A}_{\text{“DEU_to_NLD”,“CNG”},y}$	0.0	0.0	0.0

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Table 55: Parameter Values of Test Case 18 (continued)

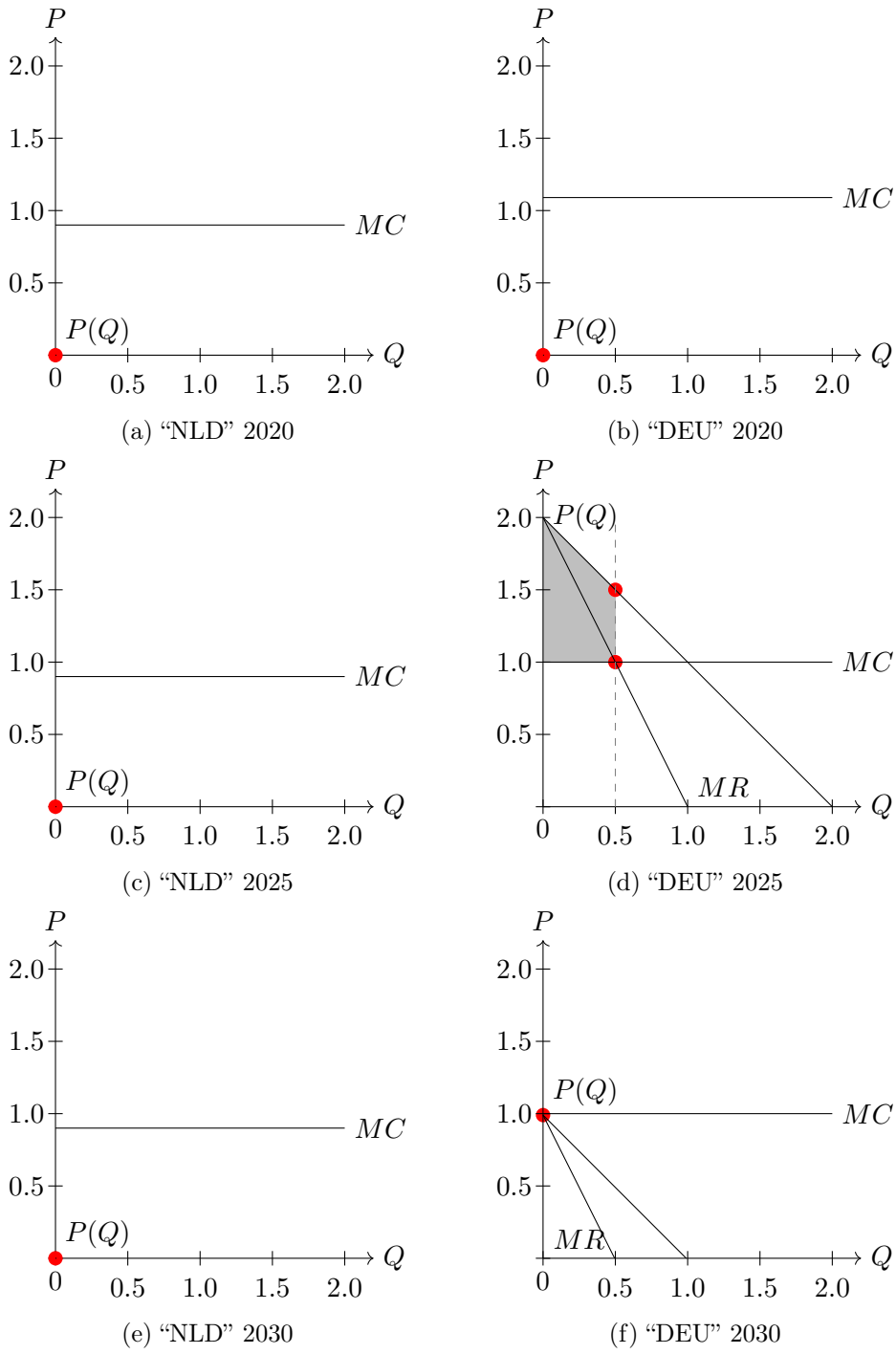
Parameter	**y=2020**	**y=2025**	**y=2030**
$c^{A}_{\text{“NLD_to_DEU”,“CNG”},y}$	0.05	0.05	0.05
$c^{A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0.0	0.0	0.0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0.25	0.25	0.25
$c^{\Delta A}_{\text{“DEU_to_NLD”,“CNG”},y}$	0.25	0.25	0.25
$c^{\Delta A}_{\text{“NLD_to_DEU”,“CNG”},y}$	0.25	0.25	0.25
$c^{\Delta A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0.25	0.25	0.25
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0	0
$\Lambda^{A}_{\text{“DEU_to_NLD”,“CNG”},y}$	0	0	0
$\Lambda^{A}_{\text{“NLD_to_DEU”,“CNG”},y}$	0	0	0
$\Lambda^{A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0	0	0
$L^{A}_{\text{“CNG”}}$	50	50	50
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	2	2	2
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0	0	0
$av^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	1	1	1
$c^{I_l}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.5	0.5	0.5
$c^{I_q}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0	0	0
$av^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	1	1	1
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	2	2	2
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1	-1	-1
$\alpha^{D}_{\text{“NLD”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0	0	0
$\beta^{D}_{\text{“NLD”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1	-1	-1

Source: Own depiction.

Table 56: Tested Results of Test Case 18

Expression	**Result y=2020**	**Result y=2025**	**Result y=2030**
$\tilde{P}^{T\rightarrow D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0	1.5	0.99
$\tilde{P}^{T\rightarrow D}_{\text{“NLD”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0.0	0.0	0.0
$\Delta^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0.0	0.0	0.0
$\Delta^{A}_{\text{“DEU_to_NLD”,“CNG”},y}$	0.5	0.0	0.0
$\Delta^{A}_{\text{“NLD_to_DEU”,“CNG”},y}$	0.5	0.0	0.0
$\Delta^{A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0.0	0.0	0.0

Source: Own depiction.



(a) "NLD" 2020

(b) "DEU" 2020

(c) "NLD" 2025

(d) "DEU" 2025

(e) "NLD" 2030

(f) "DEU" 2030

Figure 18: Marginalized Provision Costs, Revenues and Prices for Test Case 18

Source: Own depiction.

2.2.19 Test Case 19

Test Case 19 is structurally similar to Test Case 7 and asserts, that trade restrictions by origin work as intended. As can be seen from Tables 57 and 58, trade of cheaper "CNG" from renewable electricity is restricted. The test is passed, if prices remain the same, but only fossil natural gas is produced. This is because fossil "CNG" had been price setting at 0.5 in Test Case 7 already. Passing criteria can be seen in Table 59, while Figure 19 depicts a graphical solution.

Table 57: Sets of Test Case 19

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU"}
$\mathcal{AC}$	{("DEU_to_DEU", "CNG")}
$\mathcal{C}$	{"CNG"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Natural Gas","Renewable Electricity"}
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"OnlyTimeStep"}
$\mathcal{N}$	{"DEU"}
$\mathcal{O}$	{"FES","RES"}
$\mathcal{P}$	{"P_DEU"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$
$\mathcal{S}$	$\emptyset$
$\mathcal{T}$	{"T_DEU"}

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Table 57: Sets of Test Case 19 (continued)

Set Name	**Set Value**
$\mathcal{V}$	$\emptyset$
$\mathcal{VT}$	$\emptyset$
$\mathcal{Y}$	{"2020"}

Source: Own depiction.

Table 58: Parameter Values of Test Case 19

Parameter	**y=2020**
$\frac{1}{\lVert\Delta\rVert_y}$	1
$\mathbb{1}^{NC}_{\text{"T_DEU","DEU","CNG"}}$	0
r_y	1
$d_{\text{"OnlyTimeStep"}}$	1
$c^{P}_{\text{"P_DEU","CNG","FES"},y}$	0.25
$c^{\Delta P}_{\text{"P_DEU","CNG","FES"},y}$	10
$fi^{P}_{\text{"CNG","Natural Gas","FES"}}$	1
$fi^{P}_{\text{"CNG","Renewable Electricity","FES"}}$	0
$L^{P}_{\text{"CNG","FES"}}$	50
$\Lambda^{P}_{\text{"P_DEU","CNG","FES"},y}$	10
$c^{P}_{\text{"P_DEU","CNG","RES"},y}$	0.25
$c^{\Delta P}_{\text{"P_DEU","CNG","RES"},y}$	10
$fi^{P}_{\text{"CNG","Renewable Electricity","RES"}}$	2
$fi^{P}_{\text{"CNG","Natural Gas","RES"}}$	0
$L^{P}_{\text{"CNG","RES"}}$	50

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Table 58: Parameter Values of Test Case 19 (continued)

Parameter	**y=2020**
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“RES”},y}$	10
$\Lambda^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	10
$\Lambda^{I}_{\text{“P_DEU”,“Renewable Electricity”,“Block 1”},y}$	10
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0
$\Omega^{I}_{\text{“P_DEU”,“Renewable Electricity”,“Block 1”},y}$	0
$c^{\Delta I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0
$c^{\Delta I}_{\text{“P_DEU”,“Renewable Electricity”,“Block 1”},y}$	0
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”},y}$	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“RES”},y}$	10
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0
$L^{A}_{\text{“CNG”}}$	50
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”},\text{y}}$	0.25
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”},\text{y}}$	0
$av^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	1
$c^{I_l}_{\text{“P_DEU”,“Renewable Electricity”,“Block 1”,“OnlyTimeStep”},\text{y}}$	0.1
$c^{I_q}_{\text{“P_DEU”,“Renewable Electricity”,“Block 1”,“OnlyTimeStep”},\text{y}}$	0.025

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Table 58: Parameter Values of Test Case 19 (continued)

Parameter	**y=2020**
$av^{I}_{\text{“P_DEU”,“Renewable Electricity”,“Block 1”,“OnlyTimeStep”}}$	1
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	2
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1

Source: Own depiction.

Table 59: Tested Results of Test Case 19

Expression	**Result y=2020**
$\tilde{P}^{T\rightarrow D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0.5
$q^{I}_{\text{“P_DEU”,“Natural Gas”,“OnlyTimeStep”,y}}$	1
$q^{I}_{\text{“P_DEU”,“Renewable Electricity”,“OnlyTimeStep”,y}}$	1

Source: Own depiction.

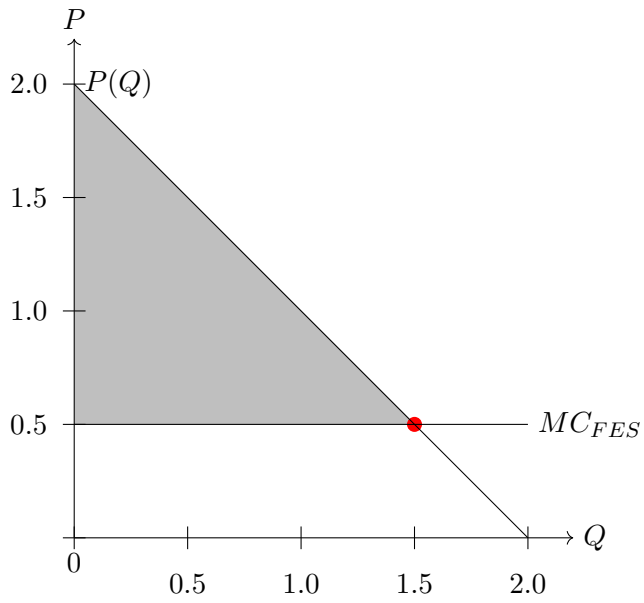

Figure 19: Marginalized Provision Costs and Prices in Test Case 19
Source: Own depiction.

2.2.20 Test Case 20

Test Case 20 is based on Test Case 2 and checks for repurposing of conversion facilities, including repurposing factors. Relevant sets and parameters can be seen from Tables 60 and 61, respectively. Tests are listed in Table 62 and illustrated by Figure 20.

Table 60: Sets of Test Case 20

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU"}
$\mathcal{AC}$	{("DEU_to_DEU", "CNG"), ("DEU_to_DEU", "LH_2"), ("DEU_to_DEU", "NH_3")}
$\mathcal{C}$	{"CNG","LH_2","NH_3"}
$\mathcal{DSB}$	{"Block 1"}

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Table 60: Sets of Test Case 20 (continued)

Set Name	**Set Value**
$\mathcal{I}$	{"Electricity"}
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"OnlyTimeStep"}
$\mathcal{N}$	{"DEU"}
$\mathcal{O}$	{"RES"}
$\mathcal{P}$	{"P_DEU"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$
$\mathcal{S}$	$\emptyset$
$\mathcal{T}$	{"T_DEU"}
$\mathcal{V}$	{"V_DEU"}
$\mathcal{VT}$	{("GH_2","CNG"),("GH_2","NH_3")}
$\mathcal{Y}$	{"2020","2021"}

Source: Own depiction.

Table 61: Parameter Values of Test Case 20

Parameter	**y=2020**	**y=2021**
$\frac{1}{\lVert\Delta\rVert_y}$	1	1
$\mathbb{1}^{NC}_{\text{"T_DEU","DEU","CNG"}}$	0	0
$\mathbb{1}^{NC}_{\text{"T_DEU","DEU","}GH_2\text{"}}$	0	0
$\mathbb{1}^{NC}_{\text{"T_DEU","DEU","}NH_3\text{"}}$	0	0

Continued on next page...

Table 61: Parameter Values of Test Case 20 (continued)

Parameter	**y=2020**	**y=2021**
r_y	1	1
$d_{\text{“OnlyTimeStep”}}$	1	1
$c^{P}_{\text{“P_DEU”,“GH}_2\text{”,“RES”},y}$	0.0	0.0
$c^{P}_{\text{“P_DEU”,“NH}_3\text{”,“RES”},y}$	0.1	0.1
$c^{P}_{\text{“P_DEU”,“CNG”,“RES”},y}$	0.0	0.0
$c^{\Delta P}_{\text{“P_DEU”,“GH}_2\text{”,“RES”},y}$	10	10
$c^{\Delta P}_{\text{“P_DEU”,“NH}_3\text{”,“RES”},y}$	10	10
$c^{\Delta P}_{\text{“P_DEU”,“CNG”,“RES”},y}$	10	10
$fi^{P}_{\text{“GH}_2\text{”,“Electricity”,“RES”}}$	1	1
$fi^{P}_{\text{“NH}_3\text{”,“Electricity”,“RES”}}$	1	1
$fi^{P}_{\text{“CNG”,“Electricity”,“RES”}}$	1	1
$L^{P}_{\text{“GH}_2\text{”,“RES”}}$	50	50
$L^{P}_{\text{“NH}_3\text{”,“RES”}}$	50	50
$L^{P}_{\text{“CNG”,“RES”}}$	50	50
$\Lambda^{P}_{\text{“P_DEU”,“GH}_2\text{”,“RES”},y}$	10	10
$\Lambda^{P}_{\text{“P_DEU”,“NH}_3\text{”,“RES”},y}$	0	0
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“RES”},y}$	0	0
$\Lambda^{I}_{\text{“P_DEU”,“Electricity”,“Block 1”},y}$	10	10
$\Omega^{I}_{\text{“P_DEU”,“Electricity”,“Block 1”},y}$	0	0
$c^{\Delta I}_{\text{“P_DEU”,“Electricity”,“Block 1”},y}$	0	0

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Table 61: Parameter Values of Test Case 20 (continued)

Parameter	**y=2020**	**y=2021**
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“GH}_2\text{”,“RES”},y}$	10	10
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“NH}_3\text{”,“RES”},y}$	10	10
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“RES”},y}$	10	10
$\Omega^{P}_{\text{“P_DEU”,“GH}_2\text{”,“RES”},y}$	10	10
$\Omega^{P}_{\text{“P_DEU”,“NH}_3\text{”,“RES”},y}$	10	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“RES”},y}$	10	10
$fi^{V}_{\text{“GH}_2\text{”,“CNG”}}$	1	1
$fi^{V}_{\text{“GH}_2\text{”,“NH}_3\text{”}}$	1	1
$l^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”}}$	0	0
$l^{A}_{\text{“DEU_to_DEU”,“NH}_3\text{”}}$	0	0
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0	0
$c^{V}_{\text{“V_DEU”,“GH}_2\text{”,“CNG”},y}$	0.1	0.1
$c^{V}_{\text{“V_DEU”,“GH}_2\text{”,“NH}_3\text{”},y}$	0.05	0.05
$c^{\Delta V}_{\text{“V_DEU”,“GH}_2\text{”,“CNG”},y}$	10	10
$c^{\Delta V}_{\text{“V_DEU”,“GH}_2\text{”,“NH}_3\text{”},y}$	10	10
$c^{\Delta^{RV}}_{\text{“V_DEU”,“GH}_2\text{”,“CNG”,“GH}_2\text{”,“NH}_3\text{”},y}$	0.025	0.025
$f^{RV}_{\text{“GH}_2\text{”,“CNG”,“GH}_2\text{”,“NH}_3\text{”}}$	0.5	0.5
$\Lambda^{V}_{\text{“V_DEU”,“GH}_2\text{”,“CNG”},y}$	10	10
$\Lambda^{V}_{\text{“V_DEU”,“GH}_2\text{”,“NH}_3\text{”},y}$	0	0
$L^{V}_{\text{“GH}_2\text{”,“CNG”}}$	10	10

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Table 61: Parameter Values of Test Case 20 (continued)

Parameter	**y=2020**	**y=2021**
$L^{V}_{\text{“GH}_2\text{”,“NH}_3\text{”}}$	10	10
$c^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”},y}$	0	0
$c^{A}_{\text{“DEU_to_DEU”,“NH}_3\text{”},y}$	0	0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“GH}_2\text{”},y}$	0	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“NH}_3\text{”},y}$	0	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”},y}$	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“NH}_3\text{”},y}$	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0
$L^{A}_{\text{“GH}_2\text{”}}$	50	50
$L^{A}_{\text{“NH}_3\text{”}}$	50	50
$L^{A}_{\text{“CNG”}}$	50	50
$c^{I_l}_{\text{“P_DEU”,“Electricity”,“Block 1”,“OnlyTimeStep”,y}}$	0.3	0.3
$c^{I_q}_{\text{“P_DEU”,“Electricity”,“Block 1”,“OnlyTimeStep”,y}}$	1	1
$av^{I}_{\text{“P_DEU”,“Electricity”,“Block 1”,“OnlyTimeStep”}}$	1	1
$\alpha^{D}_{\text{“DEU”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0	0
$\beta^{D}_{\text{“DEU”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0	0
$\alpha^{D}_{\text{“DEU”,“NH}_3\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0	2
$\beta^{D}_{\text{“DEU”,“NH}_3\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0	−1

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Table 61: Parameter Values of Test Case 20 (continued)

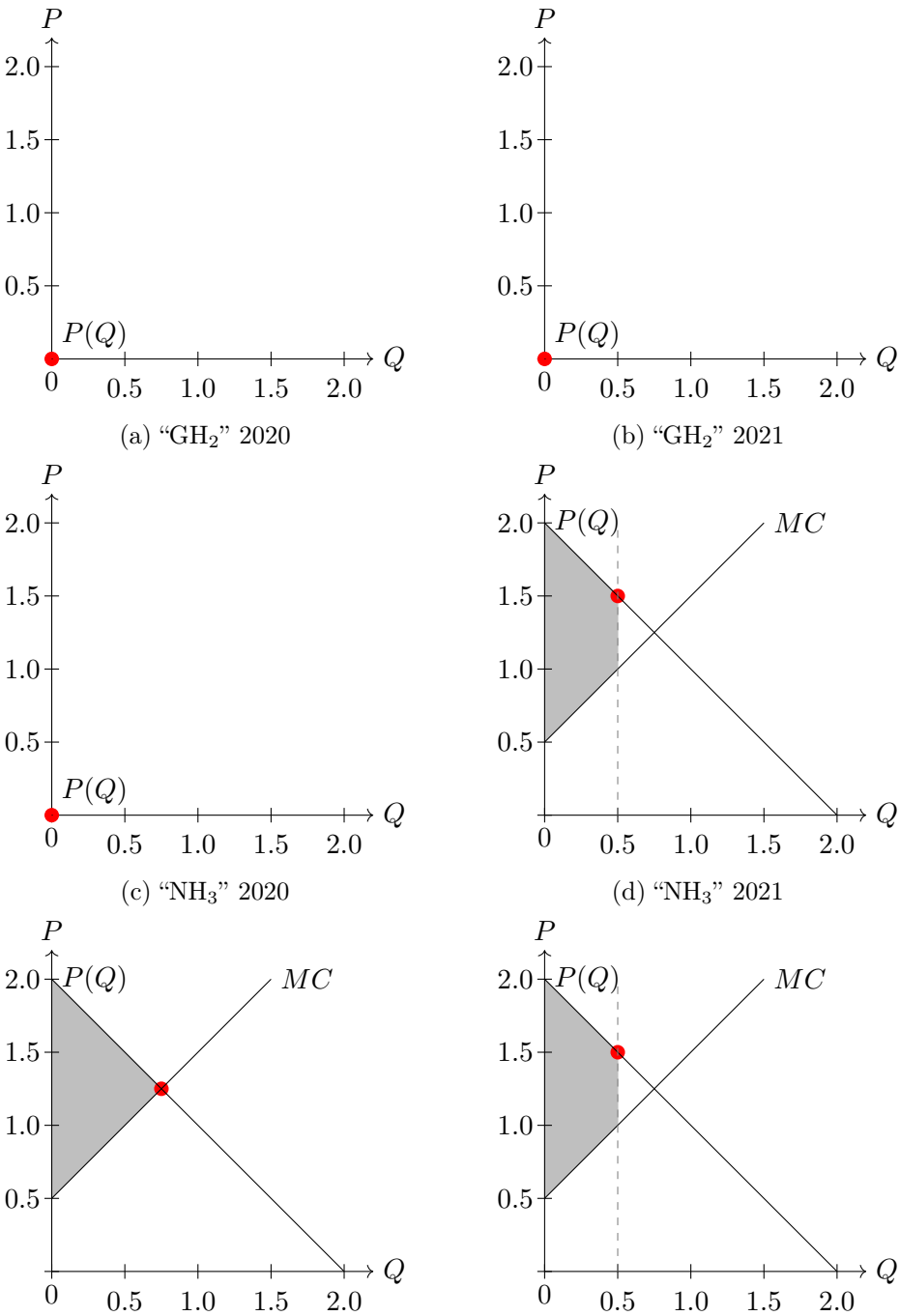
Parameter	**y=2020**	**y=2021**
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	2	0
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1	0

Source: Own depiction.

Table 62: Tested Results of Test Case 20

Expression	**Result y=2020**	**Result y=2021**
$\tilde{P}^{T\to D}_{\text{“DEU”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0.00	0.00
$\tilde{P}^{T\to D}_{\text{“DEU”,“NH}_3\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0.00	1.50
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	1.25	1.50

Source: Own depiction.



(a) "GH_2" 2020

(b) "GH_2" 2021

(c) "NH_3" 2020

(d) "NH_3" 2021

(e) "CNG" 2020

(f) "CNG" 2021

Figure 20: Marginalized Provision Costs and Prices for Test Case 20

Source: Own depiction.

2.2.21 Test Case 21

Test Case 21 asserts that repurposing of arc capacities works as intended, including repurposing factors. Tables 63 and 64 describe an extended version of Test Case 9 with two commodities and years instead of one. Table 65 shows the tested criteria, while Figure 21 illustrates a graphical solution.

Table 63: Sets of Test Case 21

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU","DEU_to_NLD","NLD_to_DEU","NLD_to_NLD"}
$\mathcal{AC}$	{ ("DEU_to_DEU", "CNG"), ("DEU_to_NLD", "CNG"), ("NLD_to_DEU", "CNG"), ("NLD_to_NLD", "CNG"), ("DEU_to_DEU", "GH_2"), ("DEU_to_NLD", "GH_2"), ("NLD_to_DEU", "GH_2"), ("NLD_to_NLD", "GH_2")}
$\mathcal{C}$	{"CNG","GH_2"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Natural Gas"}
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"OnlyTimeStep"}
$\mathcal{N}$	{"DEU","NLD"}
$\mathcal{O}$	{"FES"}
$\mathcal{P}$	{"P_DEU","P_NLD"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$
$\mathcal{S}$	$\emptyset$

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Table 63: Sets of Test Case 21 (continued)

Set Name	**Set Value**
$\mathcal{T}$	{“T_DEU”,“T_NLD”}
$\mathcal{V}$	$\emptyset$
$\mathcal{VT}$	$\emptyset$
$\mathcal{Y}$	{“2020”,“2021”}

Source: Own depiction.

Table 64: Parameter Values of Test Case 21

Parameter	**y=2020**	**y=2021**
$\frac{1}{\lVert\Delta\rVert_y}$	1	
$\mathbb{1}^{NC}_{\text{“T_DEU”,“DEU”,“CNG”}}$	0	0
$\mathbb{1}^{NC}_{\text{“T_DEU”,“NLD”,“CNG”}}$	0	0
$\mathbb{1}^{NC}_{\text{“T_NLD”,“DEU”,“CNG”}}$	0	0
$\mathbb{1}^{NC}_{\text{“T_NLD”,“NLD”,“CNG”}}$	0	0
$\mathbb{1}^{NC}_{\text{“T_DEU”,“DEU”,“GH}_2\text{”}}$	0	0
$\mathbb{1}^{NC}_{\text{“T_DEU”,“NLD”,“GH}_2\text{”}}$	0	0
$\mathbb{1}^{NC}_{\text{“T_NLD”,“DEU”,“GH}_2\text{”}}$	0	0
$\mathbb{1}^{NC}_{\text{“T_NLD”,“NLD”,“GH}_2\text{”}}$	0	0
r_y	1	1
$d_{\text{“OnlyTimeStep”}}$	1	1
$c^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	0.5	0.5
$c^{\Delta P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	1	1
$c^{P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	0.5	0.5

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Table 64: Parameter Values of Test Case 21 (continued)

Parameter	**y=2020**	**y=2021**
$c^{\Delta P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	1	1
$fi^{P}_{\text{“CNG”,“Natural Gas”,“FES”}}$	1	1
$L^{P}_{\text{“CNG”,“FES”}}$	50	50
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10	10
$c^{P}_{\text{“P_DEU”,“GH}_2\text{”,“FES”},y}$	0.4	0.4
$c^{\Delta P}_{\text{“P_DEU”,“GH}_2\text{”,“FES”},y}$	1	1
$c^{P}_{\text{“P_NLD”,“GH}_2\text{”,“FES”},y}$	0.5	0.5
$c^{\Delta P}_{\text{“P_NLD”,“GH}_2\text{”,“FES”},y}$	1	1
$fi^{P}_{\text{“GH}_2\text{”,“Natural Gas”,“FES”}}$	1	1
$L^{P}_{\text{“GH}_2\text{”,“FES”}}$	50	50
$\Lambda^{P}_{\text{“P_DEU”,“GH}_2\text{”,“FES”},y}$	10	10
$\Lambda^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	10	10
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0	0
$c^{\Delta I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0	0
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”},y}$	10	10
$\Lambda^{T}_{\text{“T_DEU”,“NLD”,“CNG”,“FES”},y}$	10	10
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“GH}_2\text{”,“FES”},y}$	10	10
$\Lambda^{T}_{\text{“T_DEU”,“NLD”,“GH}_2\text{”,“FES”},y}$	10	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10	10
$\Omega^{P}_{\text{“P_DEU”,“GH}_2\text{”,“FES”},y}$	10	10

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Table 64: Parameter Values of Test Case 21 (continued)

Parameter	**y=2020**	**y=2021**
$\Lambda^{P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	10	10
$\Lambda^{P}_{\text{“P_NLD”,“GH}_2\text{”,“FES”},y}$	10	10
$\Lambda^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”},y}$	10	10
$\Omega^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”},y}$	0	0
$c^{\Delta^{I}}_{\text{“P_NLD”,“Natural Gas”,“Block 1”},y}$	0	0
$\Lambda^{T}_{\text{“T_NLD”,“DEU”,“CNG”,“FES”},y}$	10	10
$\Lambda^{T}_{\text{“T_NLD”,“DEU”,“GH}_2\text{”,“FES”},y}$	10	10
$\Lambda^{T}_{\text{“T_NLD”,“NLD”,“CNG”,“FES”},y}$	10	10
$\Lambda^{T}_{\text{“T_NLD”,“NLD”,“GH}_2\text{”,“FES”},y}$	10	10
$\Omega^{P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	10	10
$\Omega^{P}_{\text{“P_NLD”,“GH}_2\text{”,“FES”},y}$	10	10
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0.0	0.0
$l^{A}_{\text{“DEU_to_NLD”,“CNG”}}$	0.0	0.0
$l^{A}_{\text{“NLD_to_DEU”,“CNG”}}$	0.1	0.1
$l^{A}_{\text{“NLD_to_NLD”,“CNG”}}$	0.0	0.0
$l^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”}}$	0.0	0.0
$l^{A}_{\text{“DEU_to_NLD”,“GH}_2\text{”}}$	0.0	0.0
$l^{A}_{\text{“NLD_to_DEU”,“GH}_2\text{”}}$	0.1	0.1
$l^{A}_{\text{“NLD_to_NLD”,“GH}_2\text{”}}$	0.0	0.0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0.0	0.0

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Table 64: Parameter Values of Test Case 21 (continued)

Parameter	**y=2020**	**y=2021**
$c^{A}_{\text{“DEU_to_NLD”,“CNG”},y}$	0.0	0.0
$c^{A}_{\text{“NLD_to_DEU”,“CNG”},y}$	0.1	0.1
$c^{A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0.0	0.0
$c^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”},y}$	0.0	0.0
$c^{A}_{\text{“DEU_to_NLD”,“GH}_2\text{”},y}$	0.0	0.0
$c^{A}_{\text{“NLD_to_DEU”,“GH}_2\text{”},y}$	0.1	0.1
$c^{A}_{\text{“NLD_to_NLD”,“GH}_2\text{”},y}$	0.0	0.0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0
$c^{\Delta A}_{\text{“DEU_to_NLD”,“CNG”},y}$	10	10
$c^{\Delta A}_{\text{“NLD_to_DEU”,“CNG”},y}$	10	10
$c^{\Delta A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“GH}_2\text{”},y}$	0	0
$c^{\Delta A}_{\text{“DEU_to_NLD”,“GH}_2\text{”},y}$	10	10
$c^{\Delta A}_{\text{“NLD_to_DEU”,“GH}_2\text{”},y}$	10	10
$c^{\Delta A}_{\text{“NLD_to_NLD”,“GH}_2\text{”},y}$	0	0
$c^{\Delta^{RA}}_{\text{“DEU_to_DEU”,“CNG”,“GH}_2\text{”},y}$	0	0
$c^{\Delta^{RA}}_{\text{“DEU_to_NLD”,“CNG”,“GH}_2\text{”},y}$	0	0
$c^{\Delta^{RA}}_{\text{“NLD_to_DEU”,“CNG”,“GH}_2\text{”},y}$	0.055	0.055
$c^{\Delta^{RA}}_{\text{“NLD_to_NLD”,“CNG”,“GH}_2\text{”},y}$	0	0
$f^{RA}_{\text{“CNG”,“GH}_2\text{”}}$	0.5	0.5

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Table 64: Parameter Values of Test Case 21 (continued)

Parameter	**y=2020**	**y=2021**
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0
$\Lambda^{A}_{\text{“DEU_to_NLD”,“CNG”},y}$	10	10
$\Lambda^{A}_{\text{“NLD_to_DEU”,“CNG”},y}$	10	10
$\Lambda^{A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”},y}$	0	0
$\Lambda^{A}_{\text{“DEU_to_NLD”,“GH}_2\text{”},y}$	0	0
$\Lambda^{A}_{\text{“NLD_to_DEU”,“GH}_2\text{”},y}$	0	0
$\Lambda^{A}_{\text{“NLD_to_NLD”,“GH}_2\text{”},y}$	0	0
$L^{A}_{\text{“CNG”}}$	50	50
$L^{A}_{\text{“GH}_2\text{”}}$	50	50
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	2	2
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0	0
$av^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	1	1
$c^{I_l}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.5	0.5
$c^{I_q}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0	0
$av^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	1	1
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	2	0
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1	0
$\alpha^{D}_{\text{“NLD”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0	0
$\beta^{D}_{\text{“NLD”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1	0

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Table 64: Parameter Values of Test Case 21 (continued)

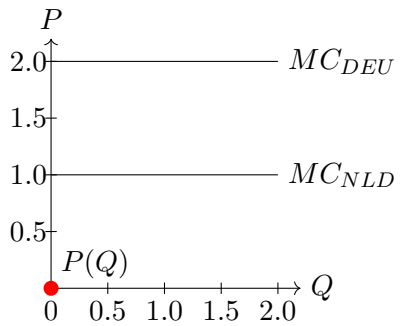
Parameter	**y=2020**	**y=2021**
$\alpha^{D}_{\text{“DEU”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0	2
$\beta^{D}_{\text{“DEU”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0	−1
$\alpha^{D}_{\text{“NLD”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0	0
$\beta^{D}_{\text{“NLD”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0	−1

Source: Own depiction.

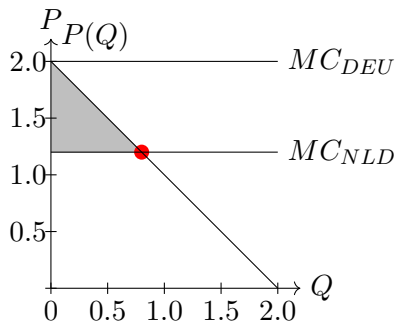
Table 65: Tested Results of Test Case 21

Expression	**Result y=2020**	**Result y=2021**
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	1.2	0
$\tilde{P}^{T\to D}_{\text{“NLD”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0.0	0.0
$\tilde{P}^{T\to D}_{\text{“DEU”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0	1.2
$\tilde{P}^{T\to D}_{\text{“NLD”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0.0	0.0
$q^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.0	0.0
$q^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.88	0.88

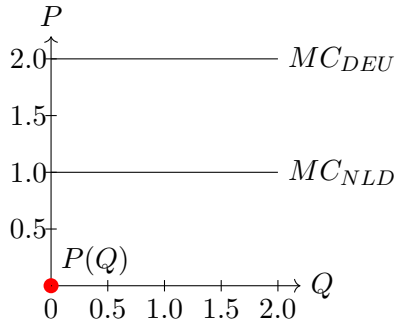
Source: Own depiction.



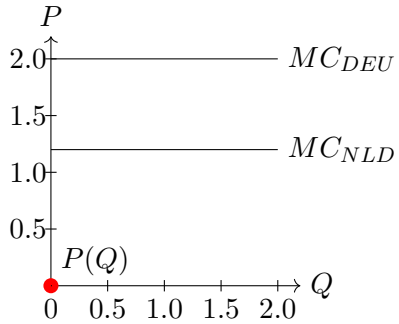
(a) Node "NLD" in 2020 for "CNG"



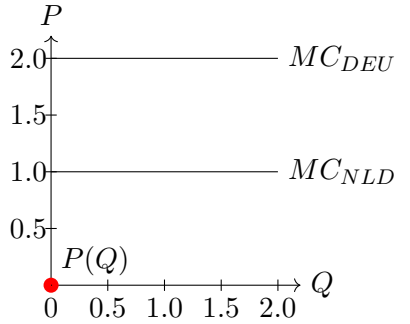
(b) Node "DEU" in 2020 for "CNG"



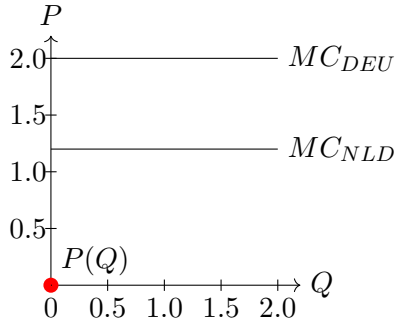
(c) Node "NLD" in 2021 for "CNG"



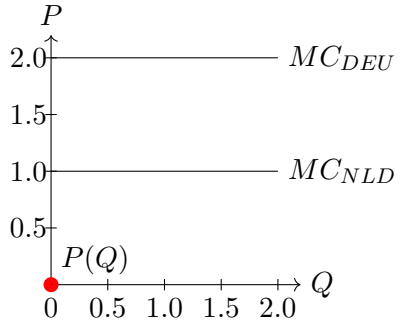
(d) Node "DEU" in 2021 for "CNG"



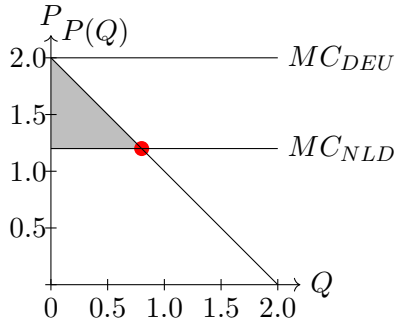
(e) Node "NLD" in 2020 for "GH_2"



(f) Node "DEU" in 2020 for "GH_2"



(g) Node "NLD" in 2021 for "GH_2"



(h) Node "DEU" in 2021 for "GH_2"

Figure 21: Marginalized Provision Costs and Prices for Different Nodes in Test Case 21
Source: Own depiction.

2.2.22 Test Case 22

Test Case 22 asserts that repurposing of storage capacities works as intended, including repurposing factors. Tables 66 and 67 describe an extended version of Test Case 5 with two commodities and years instead of one. Table 68 shows the tested criteria, while Figure 22 illustrates a graphical solution.

Table 66: Sets of Test Case 22

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU"}
$\mathcal{AC}$	{("DEU_to_DEU", "CNG"), ("DEU_to_DEU", "GH_2")}
$\mathcal{C}$	{"CNG","GH_2"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Natural Gas"}
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"L","H"}
$\mathcal{N}$	{"DEU"}
$\mathcal{O}$	{"FES"}
$\mathcal{P}$	{"P_DEU"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	{("CNG","GH_2")}
$\mathcal{RV}$	$\emptyset$
$\mathcal{S}$	{"S_DEU"}
$\mathcal{T}$	{"T_DEU"}
$\mathcal{V}$	$\emptyset$
$\mathcal{VT}$	$\emptyset$

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Table 66: Sets of Test Case 22 (continued)

Set Name	**Set Value**
$\mathcal{Y}$	{“2020”,“2021”}

Source: Own depiction.

Table 67: Parameter Values of Test Case 22

Parameter	**y=2020 m=L**	**y=2020 m=H**	**y=2021 m=L**	**y=2021 m=H**
$\frac{1}{\lVert\Delta\rVert_y}$	1	1	1	1
$\mathbb{1}^{NC}_{\text{“T_DEU”,“DEU”,“CNG”}}$	0	0	0	0
$\mathbb{1}^{NC}_{\text{“T_DEU”,“DEU”,“GH}_2\text{”}}$	0	0	0	0
r_{y}	1	1	1	1
d_m	2	1	2	1
$c^{P}_{\text{“P_DEU”,“CNG”,“FES”,y}}$	0.4	0.4	0.4	0.4
$c^{\Delta P}_{\text{“P_DEU”,“CNG”,“FES”,y}}$	1	1	1	1
$fi^{P}_{\text{“CNG”,“Natural Gas”,“FES”}}$	1	1	1	1
$L^{P}_{\text{“CNG”,“FES”}}$	50	50	50	50
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“FES”,y}}$	10	10	10	10
$c^{P}_{\text{“P_DEU”,“GH}_2\text{”,“FES”,y}}$	0.3	0.3	0.3	0.3
$c^{\Delta P}_{\text{“P_DEU”,“GH}_2\text{”,“FES”,y}}$	1	1	1	1
$fi^{P}_{\text{“GH}_2\text{”,“Natural Gas”,“FES”}}$	1	1	1	1
$L^{P}_{\text{“GH}_2\text{”,“FES”}}$	50	50	50	50
$\Lambda^{P}_{\text{“P_DEU”,“GH}_2\text{”,“FES”,y}}$	10	10	10	10
$\Lambda^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,y}}$	10	10	10	10

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Table 67: Parameter Values of Test Case 22 (continued)

Parameter	**y=2020 m=L**	**y=2020 m=H**	**y=2021 m=L**	**y=2021 m=H**
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,y}}$	0	0	0	0
$c^{\Delta I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,y}}$	0	0	0	0
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”,y}}$	10	10	10	10
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“GH}_2\text{”,“FES”,y}}$	10	10	10	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”,y}}$	10	10	10	10
$\Omega^{P}_{\text{“P_DEU”,“GH}_2\text{”,“FES”,y}}$	10	10	10	10
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0	0	0	0
$l^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”}}$	0	0	0	0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”,y}}$	0	0	0	0
$c^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”,y}}$	0	0	0	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”,y}}$	0	0	0	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“GH}_2\text{”,y}}$	0	0	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”,y}}$	0	0	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”,y}}$	0	0	0	0
$L^{A}_{\text{“CNG”}}$	50	50	50	50
$L^{A}_{\text{“GH}_2\text{”}}$	50	50	50	50
$l^{S}_{\text{“CNG”,m,“L”}}$	0	0	0	0
$l^{S}_{\text{“CNG”,m,“H”}}$	0	0	0	0
$c^{S_{in}}_{\text{“S_DEU”,“CNG”,y}}$	0.05	0.05	0.05	0.05
$c^{S_{out}}_{\text{“S_DEU”,“CNG”,y}}$	0.05	0.05	0.05	0.05

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Table 67: Parameter Values of Test Case 22 (continued)

Parameter	**y=2020 m=L**	**y=2020 m=H**	**y=2021 m=L**	**y=2021 m=H**
$c^{\Delta S}_{\text{“S_DEU”,“CNG”,y}}$	10	10	10	10
$l^{S}_{\text{“GH}_2\text{”,m,“L”}}$	0	0	0	0
$l^{S}_{\text{“GH}_2\text{”,m,“H”}}$	0	0	0	0
$c^{S}_{\text{“S_DEU”,“GH}_2\text{”,m,“H”,y}}$	0.1	0.1	0.1	0.1
$c^{S}_{\text{“S_DEU”,“GH}_2\text{”,m,“L”,y}}$	0.1	0.1	0.1	0.1
$c^{\Delta S}_{\text{“S_DEU”,“GH}_2\text{”,y}}$	10	10	10	10
$c^{\Delta^{RS}}_{\text{“S_DEU”,“CNG”,“GH}_2\text{”,y}}$	0.05	0.05	0.025	0.025
$f^{RS}_{\text{“CNG”,“GH}_2\text{”}}$	0.5	0.5	0.5	0.5
$\Lambda^{S}_{\text{“S_DEU”,“CNG”,y}}$	10	10	10	10
$\Lambda^{S}_{\text{“S_DEU”,“GH}_2\text{”,y}}$	10	10	10	10
$\Omega^{S}_{\text{“S_DEU”,“CNG”,y}}$	10	10	10	10
$\Omega^{S}_{\text{“S_DEU”,“GH}_2\text{”,y}}$	10	10	10	10
$L^{S}_{\text{“CNG”}}$	10	10	10	10
$L^{S}_{\text{“GH}_2\text{”}}$	10	10	10	10
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m,y}}$	0.5	1	0.5	1
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m,y}}$	0	0	0	0
$av^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m}}$	1	1	1	1
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,m,y}}$	0	2	0	0
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,m,y}}$	0	−1	0	0
$\alpha^{D}_{\text{“DEU”,“GH}_2\text{”,“Block 1”,m,y}}$	0	0	0	2

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Table 67: Parameter Values of Test Case 22 (continued)

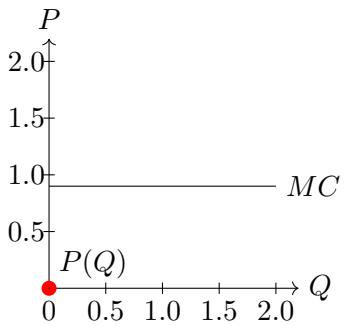
Parameter	**y=2020 m=L**	**y=2020 m=H**	**y=2021 m=L**	**y=2021 m=H**
$\beta^{D}_{\text{“DEU”,“GH}_2\text{”,“Block 1”,m,y}}$	0	0	0	−1

Source: Own depiction.

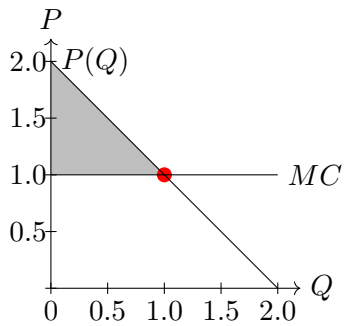
Table 68: Tested Results of Test Case 22

Parameter	**y=2020 m=L**	**y=2020 m=H**	**y=2021 m=L**	**y=2021 m=H**
$\tilde{P}^{T\rightarrow D}_{\text{“DEU”,“CNG”,“Block 1”,m,y}}$	0.0	1.0	0.0	0.0
$\tilde{P}^{T\rightarrow D}_{\text{“DEU”,“CNG”,“Block 1”,m,y}}$	0.0	0.0	0.0	1.0
$q^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,m,y}}$	0.5	0.0	0.5	0.0

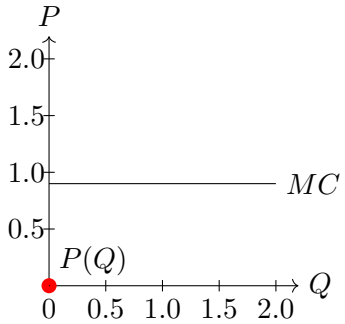
Source: Own depiction.



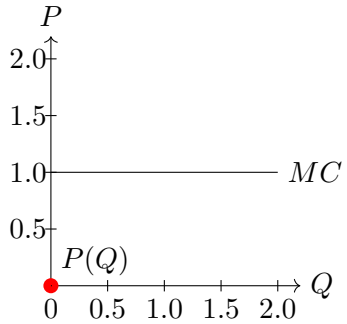
(a) time step "L" for "CNG" in 2020



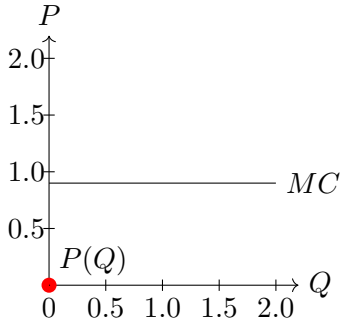
(b) time step "H" for "CNG" in 2020



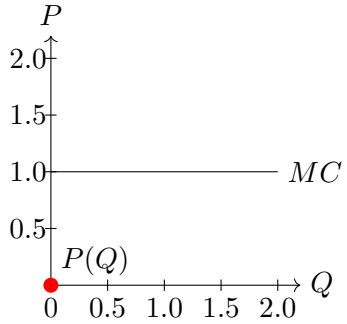
(c) time step "L" for "CNG" in 2021



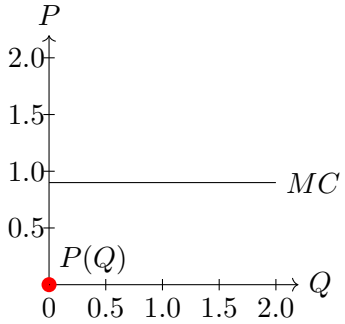
(d) time step "H" for "CNG" in 2021



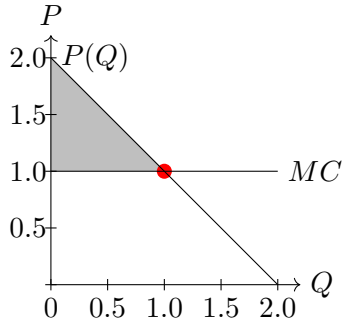
(e) time step "L" for "GH_2" in 2020



(f) time step "H" for "GH_2" in 2020



(g) time step "L" for "GH_2" in 2021



(h) time step "H" for "GH_2" in 2021

Figure 22: Illustration for Different Years, Commodities and Time Steps in Test Case 22
Source: Own depiction.

2.2.23 Test Case 23

Test Case 23 asserts that piece-wise quadratic cost functions for input procurement work as desired. It is structurally similar to Test Case 3, only that the first block of input procurement only goes to 25% of the total capacity of 1. After a quantity of 0.2, an increase in marginalized cost slope can be identified. Figure 23 is a graphical representation of the optimal solution. Hence, this test passes if the equilibrium price is $P = 1.5$, with an objective value of 0.45 and corresponding social welfare of 0.575 (see the grey area in Table 71).

Table 69: Sets of Test Case 23

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU"}
$\mathcal{AC}$	{("DEU_to_DEU", "CNG")}
$\mathcal{C}$	{"CNG"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Natural Gas"}
$\mathcal{IOB}$	{"Block 1","Block 2"}
$\mathcal{M}$	{"OnlyTimeStep"}
$\mathcal{N}$	{"DEU"}
$\mathcal{O}$	{"FES"}
$\mathcal{P}$	{"P_DEU"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$
$\mathcal{S}$	$\emptyset$

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Table 69: Sets of Test Case 23 (continued)

Set Name	**Set Value**
$\mathcal{T}$	{"T_DEU"}
$\mathcal{V}$	$\emptyset$
$\mathcal{VT}$	$\emptyset$
$\mathcal{Y}$	{"2020"}

Source: Own depiction.

Table 70: Parameter Values of Test Case 23

Parameter	**y=2020**
$\frac{1}{\lVert\Delta\rVert_y}$	1
$\mathbb{1}^{NC}_{\text{"T_DEU","DEU","CNG"}}$	1
r_y	1
$d_{\text{"OnlyTimeStep"}}$	1
$c^{P}_{\text{"P_DEU","CNG","FES"},y}$	0.3
$c^{\Delta P}_{\text{"P_DEU","CNG","FES"},y}$	1
$fi^{P}_{\text{"CNG","Natural Gas","FES"}}$	1
$L^{P}_{\text{"CNG","FES"}}$	50
$\Lambda^{P}_{\text{"P_DEU","CNG","FES"},y}$	10
$\Lambda^{I}_{\text{"P_DEU","Natural Gas","Block 1"},y}$	0.25
$\Lambda^{I}_{\text{"P_DEU","Natural Gas","Block 2"},y}$	0.75
$\Omega^{I}_{\text{"P_DEU","Natural Gas","Block 1"},y}$	0
$\Omega^{I}_{\text{"P_DEU","Natural Gas","Block 2"},y}$	0
$c^{\Delta^{I}}_{\text{"P_DEU","Natural Gas","Block 1"},y}$	0

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Table 70: Parameter Values of Test Case 23 (continued)

Parameter	**y=2020**
$c^{\Delta^I}_{\text{“P_DEU”,“Natural Gas”,“Block 2”},y}$	0
$\Lambda^T_{\text{“T_DEU”,“DEU”,“CNG”,“FES”},y}$	10
$\Omega^P_{\text{“P_DEU”,“CNG”,“FES”},y}$	10
$l^A_{\text{“DEU_to_DEU”,“CNG”}}$	0
$c^A_{\text{“DEU_to_DEU”,“CNG”},y}$	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0
$\Lambda^A_{\text{“DEU_to_DEU”,“CNG”},y}$	0
$L^A_{\text{“CNG”}}$	50
$c^{I_l}_{\text{“P_DEU”,“Electricity”,“Block 1”,“OnlyTimeStep”,y}}$	0.1
$c^{I_q}_{\text{“P_DEU”,“Electricity”,“Block 1”,“OnlyTimeStep”,y}}$	0.4
$av^I_{\text{“P_DEU”,“Electricity”,“Block 1”,“OnlyTimeStep”}}$	1
$c^{I_l}_{\text{“P_DEU”,“Electricity”,“Block 2”,“OnlyTimeStep”,y}}$	0.2
$c^{I_q}_{\text{“P_DEU”,“Electricity”,“Block 2”,“OnlyTimeStep”,y}}$	2.0
$av^I_{\text{“P_DEU”,“Electricity”,“Block 2”,“OnlyTimeStep”}}$	1
$\alpha^D_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	2
$\beta^D_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1

Source: Own depiction.

Table 71: Tested Results of Test Case 23

Expression	**Result y=2020**
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	1.5
Objective Value	0.45

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Table 71: Tested Results of Test Case 23 (continued)

Expression	**Result y=2020**
$\tilde{SWF}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0.575

Source: Own depiction.

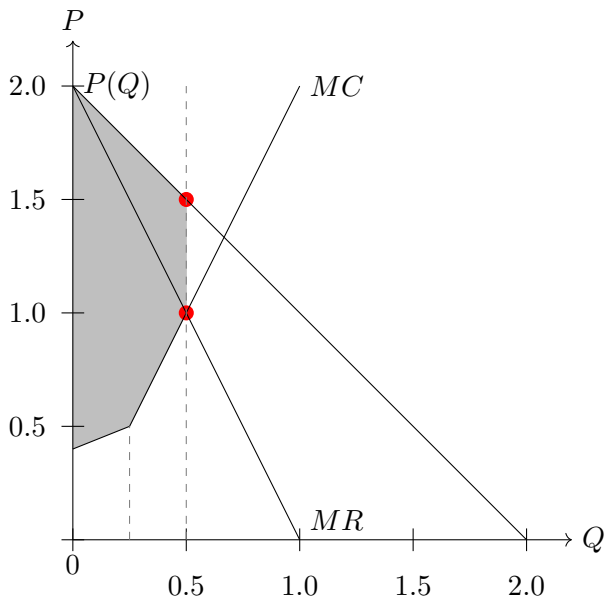

Figure 23: Profit Maximization in a Cournot Setting for Test Case 23
Source: Own depiction.

2.2.24 Test Case 24

Test Case 24 asserts that supplying to multiple demand blocks of the same commodity works as expected. Table 72 depicts the model sets, while Table 73 gives relevant parameters. Test Case 24 passes, if prices in the single node are equal to the intersection of marginalized provision cost and inverse demand, see Figure 24. For the given data, prices should be $P = 1.0$, see Table 74.

Table 72: Sets of Test Case 24

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU"}
$\mathcal{AC}$	{("DEU_to_DEU", "CNG")}
$\mathcal{C}$	{"CNG"}
$\mathcal{DSB}$	{"Block 1","Block 2"}
$\mathcal{I}$	{"Natural Gas"}
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"OnlyTimeStep"}
$\mathcal{N}$	{"DEU"}
$\mathcal{O}$	{"FES"}
$\mathcal{P}$	{"P_DEU"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$
$\mathcal{S}$	$\emptyset$
$\mathcal{T}$	{"T_DEU"}
$\mathcal{V}$	$\emptyset$
$\mathcal{VT}$	$\emptyset$
$\mathcal{Y}$	{"2020"}

Source: Own depiction.

Table 73: Parameter Values of Test Case 24

Parameter	**y=2020**
$\frac{1}{\lVert\Delta\rVert_y}$	1
$\mathbb{1}^{NC}_{\text{“T_DEU”,“DEU”,“CNG”}}$	0
r_y	1
$d_{\text{“OnlyTimeStep”}}$	1
$c^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	0.5
$c^{\Delta P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	1
$fi^{P}_{\text{“CNG”,“Natural Gas”,“FES”}}$	1
$L^{P}_{\text{“CNG”,“FES”}}$	50
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10
$\Lambda^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	10
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0
$c^{\Delta I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”},y}$	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0
$L^{A}_{\text{“CNG”}}$	50
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.5
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.0

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Table 73: Parameter Values of Test Case 24 (continued)

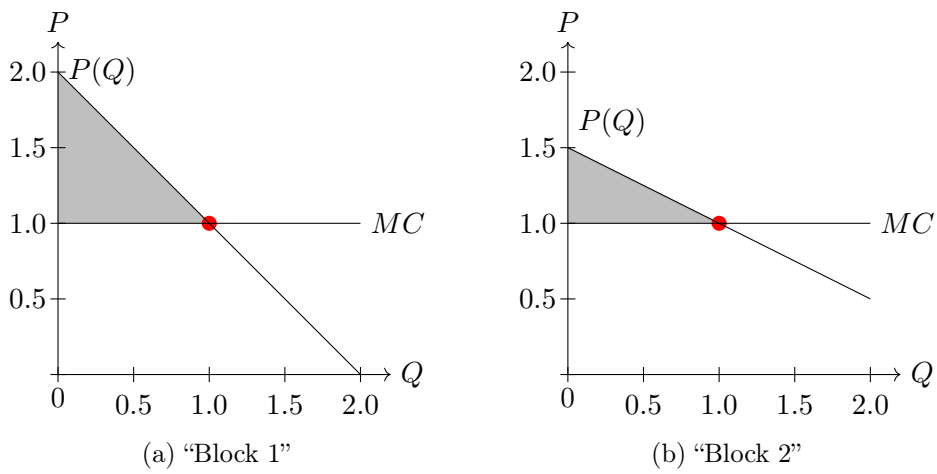
Parameter	**y=2020**
$av^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	1
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	2
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 2”,“OnlyTimeStep”,y}}$	1.5
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 2”,“OnlyTimeStep”,y}}$	-0.5

Source: Own depiction.

Table 74: Tested Results of Test Case 24

Expression	**b=“Block 1”**	**b=“Block 2”**
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,b,“OnlyTimeStep”,“2020”}}$	1.0	1.0
Objective Value	0.75	

Source: Own depiction.



(a) “Block 1” (b) “Block 2”

Figure 24: Inverse Demand and Marginalized Cost Curves for Test Case 24

Source: Own depiction.

2.2.25 Test Case 25

Test Case 25 asserts that input capacity expansion, availability of input capacity and input expansion limits work as desired. In 2020, no input capacity is available. With the corresponding investments and availabilities, a price of 1.5 materializes in 2025. No consumption should happen in 2030 and 2035 because investments are not lucrative to carry over to 2030 and because the aggregate active investments in 2035 should be 0, respectively. Figure 25 is a graphical representation of the optimal solution. Hence, this test passes if the equilibrium price is $P = 1.5$ in 2025 and equal to the intercept of the inverse demand function in all other periods, with an objective value of 0.375 and corresponding social welfare of 0.5 (see the sum of grey areas in Table 77).

Table 75: Sets of Test Case 25

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU"}
$\mathcal{AC}$	{("DEU_to_DEU", "CNG")}
$\mathcal{C}$	{"CNG"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Natural Gas"}
$\mathcal{IOB}$	{"Block 1","Block 2"}
$\mathcal{M}$	{"OnlyTimeStep"}
$\mathcal{N}$	{"DEU"}
$\mathcal{O}$	{"FES"}
$\mathcal{P}$	{"P_DEU"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$

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Table 75: Sets of Test Case 25 (continued)

Set Name	**Set Value**
$\mathcal{S}$	$\emptyset$
$\mathcal{T}$	{"T_DEU"}
$\mathcal{V}$	$\emptyset$
$\mathcal{VT}$	$\emptyset$
$\mathcal{Y}$	{"2020","2025","2030","2035"}

Source: Own depiction.

Table 76: Parameter Values of Test Case 25

Parameter	**y=2020**	**y=2025**	**y=2030**	**y=2035**
$\frac{1}{\lVert\Delta\rVert_y}$	5	5	5	5
$\mathbb{1}^{NC}_{\text{"T_DEU","DEU","CNG"}}$	1	1	1	1
r_y	1	1	1	1
$d_{\text{"OnlyTimeStep"}}$	1	1	1	1
$c^{P}_{\text{"P_DEU","CNG","FES"},y}$	0.1	0.1	0.1	0.1
$c^{\Delta P}_{\text{"P_DEU","CNG","FES"},y}$	1	1	1	1
$fi^{P}_{\text{"CNG","Natural Gas","FES"}}$	1	1	1	1
$L^{P}_{\text{"CNG","FES"}}$	50	50	50	50
$\Lambda^{P}_{\text{"P_DEU","CNG","FES"},y}$	10	10	10	10
$\Lambda^{I}_{\text{"P_DEU","Natural Gas","Block 1"},y}$	0	0	0	0
$\Lambda^{I}_{\text{"P_DEU","Natural Gas","Block 2"},y}$	0	0	0	0
$\Lambda^{T}_{\text{"T_DEU","DEU","CNG","FES"},y}$	10	10	10	10
$\Omega^{P}_{\text{"P_DEU","CNG","FES"},y}$	10	10	10	10

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Table 76: Parameter Values of Test Case 25 (continued)

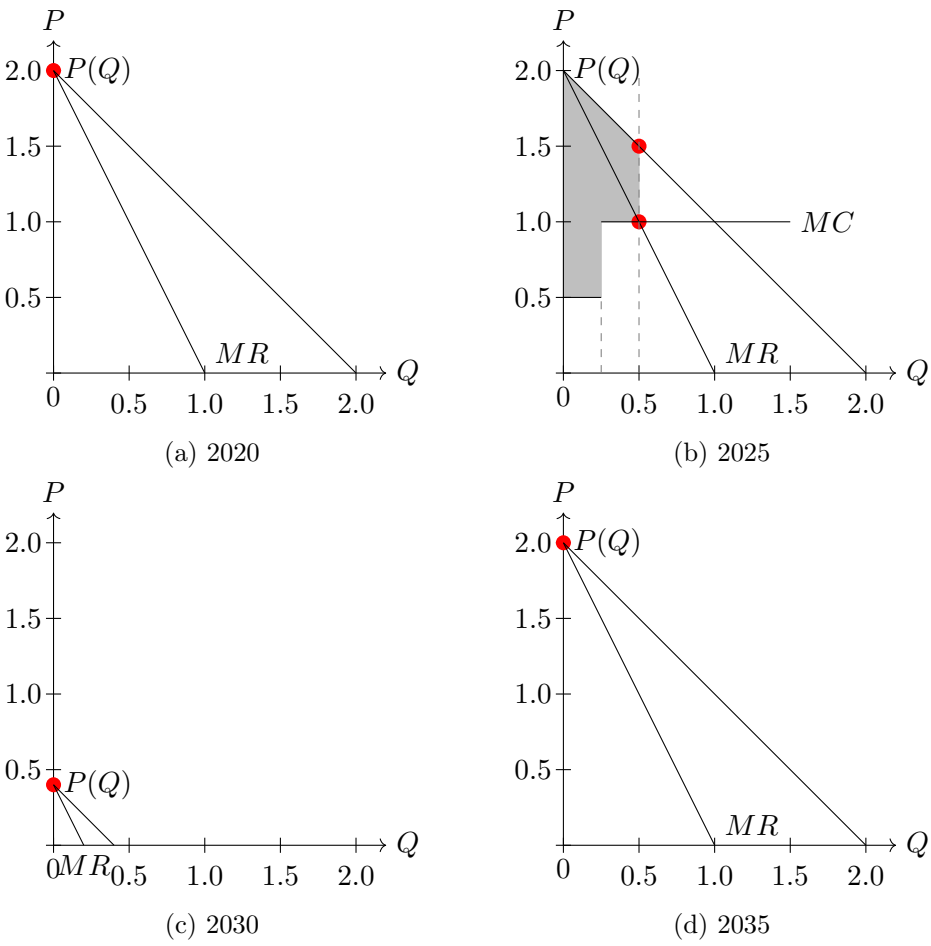
Parameter	**y=2020**	**y=2025**	**y=2030**	**y=2035**
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0	0	0	0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0	0	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0	0	0
$L^{A}_{\text{“CNG”}}$	50	50	50	50
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.1	0.1	0.1	0.1
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0	0	0	0
$av^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	1	1	1	1
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 2”,“OnlyTimeStep”,y}}$	0.2	0.2	0.2	0.2
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 2”,“OnlyTimeStep”,y}}$	0	0	0	0
$av^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 2”,“OnlyTimeStep”}}$	0.5	0.5	0.5	0.5
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,y}}$	0.25	0.25	0.25	0.0
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 2”,y}}$	1	1	1	0
$c^{\Delta^{I}}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,y}}$	1.5	1.5	1.5	1.5
$c^{\Delta^{I}}_{\text{“P_DEU”,“Natural Gas”,“Block 2”,y}}$	1.75	1.75	1.75	1.75
$L^{I}_{\text{“Natural Gas”}}$	5	5	5	5
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	2	2	0.4	2
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1	-1	-1	-1

Source: Own depiction.

Table 77: Tested Results of Test Case 25

Expression	**y=2020**	**y=2025**	**y=2030**	**y=2035**
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	2	1.5	0.4	2
Objective Value		0.375		

Source: Own depiction.



(a) 2020

(b) 2025

(c) 2030

(d) 2035

Figure 25: Profit Maximization in a Cournot Setting for Test Case 25

Source: Own depiction.

2.2.26 Test Case 26

Test Case 26 is used to assert correct working of adjacency structures for shipping and pipeline. All parameters are the same as in Test Case 9, however commodity-specific adjacency prohibits flows (since $\mathcal{AC}$ is empty). Tables 78 and 79 depict the sets and parameters, respectively. Tests are depicted in Table 80, while Figure 26 depicts a graphical solution. The implementation in the testing routine features two separate runs for shipping and pipelines, however, mathematical model data remain unchanged.

Table 78: Sets of Test Case 26

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU","DEU_to_NLD","NLD_to_DEU","NLD_to_NLD"}
$\mathcal{AC}$	$\emptyset$
$\mathcal{C}$	{"CNG"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Natural Gas"}
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"OnlyTimeStep"}
$\mathcal{N}$	{"DEU","NLD"}
$\mathcal{O}$	{"FES"}
$\mathcal{P}$	{"P_DEU","P_NLD"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$
$\mathcal{S}$	$\emptyset$
$\mathcal{T}$	{"T_DEU","T_NLD"}

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Table 78: Sets of Test Case 26 (continued)

Set Name	**Set Value**
$\mathcal{V}$	$\emptyset$
$\mathcal{VT}$	$\emptyset$
$\mathcal{Y}$	{"2020"}

Source: Own depiction.

Table 79: Parameter Values of Test Case 26

Parameter	**y=2020**
$\frac{1}{\lVert\Delta\rVert_y}$	1
$\mathbb{1}^{NC}_{\text{"T_DEU","DEU","CNG"}}$	0
$\mathbb{1}^{NC}_{\text{"T_DEU","NLD","CNG"}}$	0
$\mathbb{1}^{NC}_{\text{"T_NLD","DEU","CNG"}}$	0
$\mathbb{1}^{NC}_{\text{"T_NLD","NLD","CNG"}}$	0
r_y	1
$d_{\text{"OnlyTimeStep"}}$	1
$c^{P}_{\text{"P_DEU","CNG","FES"},y}$	0.5
$c^{\Delta P}_{\text{"P_DEU","CNG","FES"},y}$	1
$c^{P}_{\text{"P_NLD","CNG","FES"},y}$	0.5
$c^{\Delta P}_{\text{"P_NLD","CNG","FES"},y}$	1
$fi^{P}_{\text{"CNG","Natural Gas","FES"}}$	1
$L^{P}_{\text{"CNG","FES"}}$	50
$\Lambda^{P}_{\text{"P_DEU","CNG","FES"},y}$	10
$\Lambda^{I}_{\text{"P_DEU","Natural Gas","Block 1"},y}$	10

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Table 79: Parameter Values of Test Case 26 (continued)

Parameter	**y=2020**
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0
$c^{\Delta^{I}}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”},y}$	10
$\Lambda^{T}_{\text{“T_DEU”,“NLD”,“CNG”,“FES”},y}$	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10
$\Lambda^{P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	10
$\Lambda^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”},y}$	10
$\Omega^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”},y}$	0
$c^{\Delta^{I}}_{\text{“P_NLD”,“Natural Gas”,“Block 1”},y}$	0
$\Lambda^{T}_{\text{“T_NLD”,“DEU”,“CNG”,“FES”},y}$	10
$\Lambda^{T}_{\text{“T_NLD”,“NLD”,“CNG”,“FES”},y}$	10
$\Omega^{P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	10
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0.0
$l^{A}_{\text{“DEU_to_NLD”,“CNG”}}$	0.0
$l^{A}_{\text{“NLD_to_DEU”,“CNG”}}$	0.1
$l^{A}_{\text{“NLD_to_NLD”,“CNG”}}$	0.0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0.0
$c^{A}_{\text{“DEU_to_NLD”,“CNG”},y}$	0.0
$c^{A}_{\text{“NLD_to_DEU”,“CNG”},y}$	0.1
$c^{A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0.0

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Table 79: Parameter Values of Test Case 26 (continued)

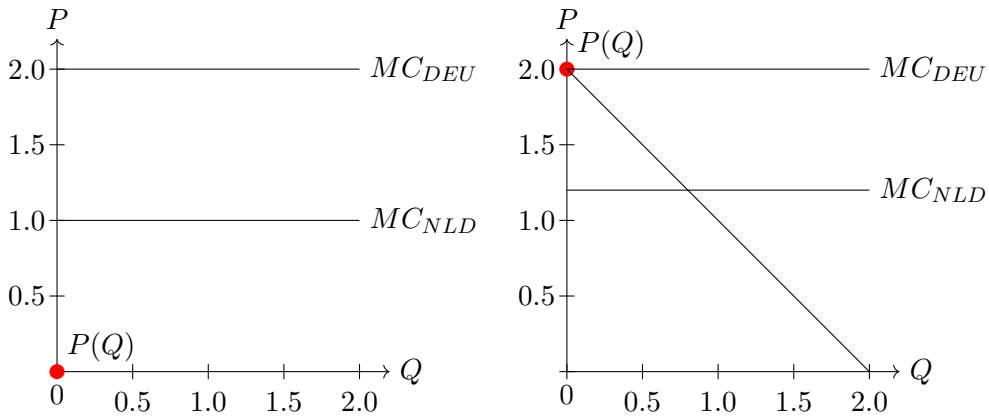
Parameter	**y=2020**
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0
$c^{\Delta A}_{\text{“DEU_to_NLD”,“CNG”},y}$	0
$c^{\Delta A}_{\text{“NLD_to_DEU”,“CNG”},y}$	0
$c^{\Delta A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0
$\Lambda^{A}_{\text{“DEU_to_NLD”,“CNG”},y}$	0
$\Lambda^{A}_{\text{“NLD_to_DEU”,“CNG”},y}$	10
$\Lambda^{A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0
$L^{A}_{\text{“CNG”}}$	50
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	2
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0
$av^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	1
$c^{I_l}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.5
$c^{I_q}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0
$av^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	1
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	2
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1
$\alpha^{D}_{\text{“NLD”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0
$\beta^{D}_{\text{“NLD”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1

Source: Own depiction.

Table 80: Tested Results of Test Case 26

Expression	**Result y=2020**
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	2.0
$\tilde{P}^{T\to D}_{\text{“NLD”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0.0
$q^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.0
$q^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.0

Source: Own depiction.



(a) Node “NLD” (b) Node “DEU”

Figure 26: Marginalized Provision Costs and Prices for Different Nodes in Test Case 26

Source: Own depiction. Note: Marginalized costs are depicted as if adjacency was given.

2.2.27 Test Case 27

Test Case 27 is used to assert correct working of adjacency structures for arc capacity expansion. All parameters are the same as in Test Case 13, however commodity-specific adjacency prohibits flows (since $\mathcal{AC}$ is empty). Tables 81 and 82 depict the sets and parameters, respectively. Tests are depicted in Table 83, while Figure 27 depicts a graphical solution.

Table 81: Sets of Test Case 27

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU","DEU_to_NLD","NLD_to_DEU","NLD_to_NLD"}
$\mathcal{AC}$	$\emptyset$
$\mathcal{C}$	{"CNG"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Natural Gas"}
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"OnlyTimeStep"}
$\mathcal{N}$	{"DEU","NLD"}
$\mathcal{O}$	{"FES"}
$\mathcal{P}$	{"P_DEU","P_NLD"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$
$\mathcal{S}$	$\emptyset$
$\mathcal{T}$	{"T_DEU","T_NLD"}
$\mathcal{V}$	$\emptyset$
$\mathcal{VT}$	$\emptyset$
$\mathcal{Y}$	{"2020","2025","2030"}

Source: Own depiction.

Table 82: Parameter Values of Test Case 27

Parameter	**y=2020**	**y=2025**	**y=2030**
$\frac{1}{\lVert\Delta\rVert_y}$	1	1	1
$\mathbb{1}^{NC}_{\text{“T_DEU”,“DEU”,“CNG”}}$	0	0	0
$\mathbb{1}^{NC}_{\text{“T_DEU”,“NLD”,“CNG”}}$	0	0	0
$\mathbb{1}^{NC}_{\text{“T_NLD”,“DEU”,“CNG”}}$	0	0	0
$\mathbb{1}^{NC}_{\text{“T_NLD”,“NLD”,“CNG”}}$	0	0	0
r_y	1	1	1
$d_{\text{“OnlyTimeStep”}}$	1	1	1
$c^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	0.4	0.4	0.4
$c^{\Delta P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	1	1	1
$c^{P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	0.5	0.5	0.5
$c^{\Delta P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	1	1	1
$fi^{P}_{\text{“CNG”,“Natural Gas”,“FES”}}$	1	1	1
$L^{P}_{\text{“CNG”,“FES”}}$	50	50	50
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10	10	10
$\Lambda^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	10	10	10
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0	0	0
$c^{\Delta I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0	0	0
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”},y}$	10	10	10
$\Lambda^{T}_{\text{“T_DEU”,“NLD”,“CNG”,“FES”},y}$	10	10	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10	10	10
$\Lambda^{P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	10	10	10

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Table 82: Parameter Values of Test Case 27 (continued)

Parameter	**y=2020**	**y=2025**	**y=2030**
$\Lambda^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”},y}$	10	10	10
$\Omega^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”},y}$	0	0	0
$c^{\Delta^{I}}_{\text{“P_NLD”,“Natural Gas”,“Block 1”},y}$	0	0	0
$\Lambda^{T}_{\text{“T_NLD”,“DEU”,“CNG”,“FES”},y}$	10	10	10
$\Lambda^{T}_{\text{“T_NLD”,“NLD”,“CNG”,“FES”},y}$	10	10	10
$\Omega^{P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	10	10	10
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0.0	0.0	0.0
$l^{A}_{\text{“DEU_to_NLD”,“CNG”}}$	0.0	0.0	0.0
$l^{A}_{\text{“NLD_to_DEU”,“CNG”}}$	0.1	0.1	0.1
$l^{A}_{\text{“NLD_to_NLD”,“CNG”}}$	0.0	0.0	0.0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0.0	0.0	0.0
$c^{A}_{\text{“DEU_to_NLD”,“CNG”},y}$	0.0	0.0	0.0
$c^{A}_{\text{“NLD_to_DEU”,“CNG”},y}$	0.05	0.05	0.05
$c^{A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0.0	0.0	0.0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0.25	0.25	0.25
$c^{\Delta A}_{\text{“DEU_to_NLD”,“CNG”},y}$	0.25	0.25	0.25
$c^{\Delta A}_{\text{“NLD_to_DEU”,“CNG”},y}$	0.25	0.25	0.25
$c^{\Delta A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0.25	0.25	0.25
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0	0
$\Lambda^{A}_{\text{“DEU_to_NLD”,“CNG”},y}$	0	0	0

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Table 82: Parameter Values of Test Case 27 (continued)

Parameter	**y=2020**	**y=2025**	**y=2030**
$\Lambda^{A}_{\text{“NLD_to_DEU”,“CNG”},y}$	0	0	0
$\Lambda^{A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0	0	0
$L^{A}_{\text{“CNG”}}$	50	50	50
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	2	2	2
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0	0	0
$av^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	1	1	1
$c^{I_l}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.5	0.5	0.5
$c^{I_q}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0	0	0
$av^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	1	1	1
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	2	2	2
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1	-1	-1
$\alpha^{D}_{\text{“NLD”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0	0	0
$\beta^{D}_{\text{“NLD”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1	-1	-1

Source: Own depiction.

Table 83: Tested Results of Test Case 27

Expression	**Result y=2020**	**Result y=2025**	**Result y=2030**
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0	2.0	0.99
$\tilde{P}^{T\to D}_{\text{“NLD”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0.0	0.0	0.0
$\Delta^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	$N.E.$	$N.E.$	$N.E.$
$\Delta^{A}_{\text{“DEU_to_NLD”,“CNG”},y}$	$N.E.$	$N.E.$	$N.E.$
$\Delta^{A}_{\text{“NLD_to_DEU”,“CNG”},y}$	$N.E.$	$N.E.$	$N.E.$

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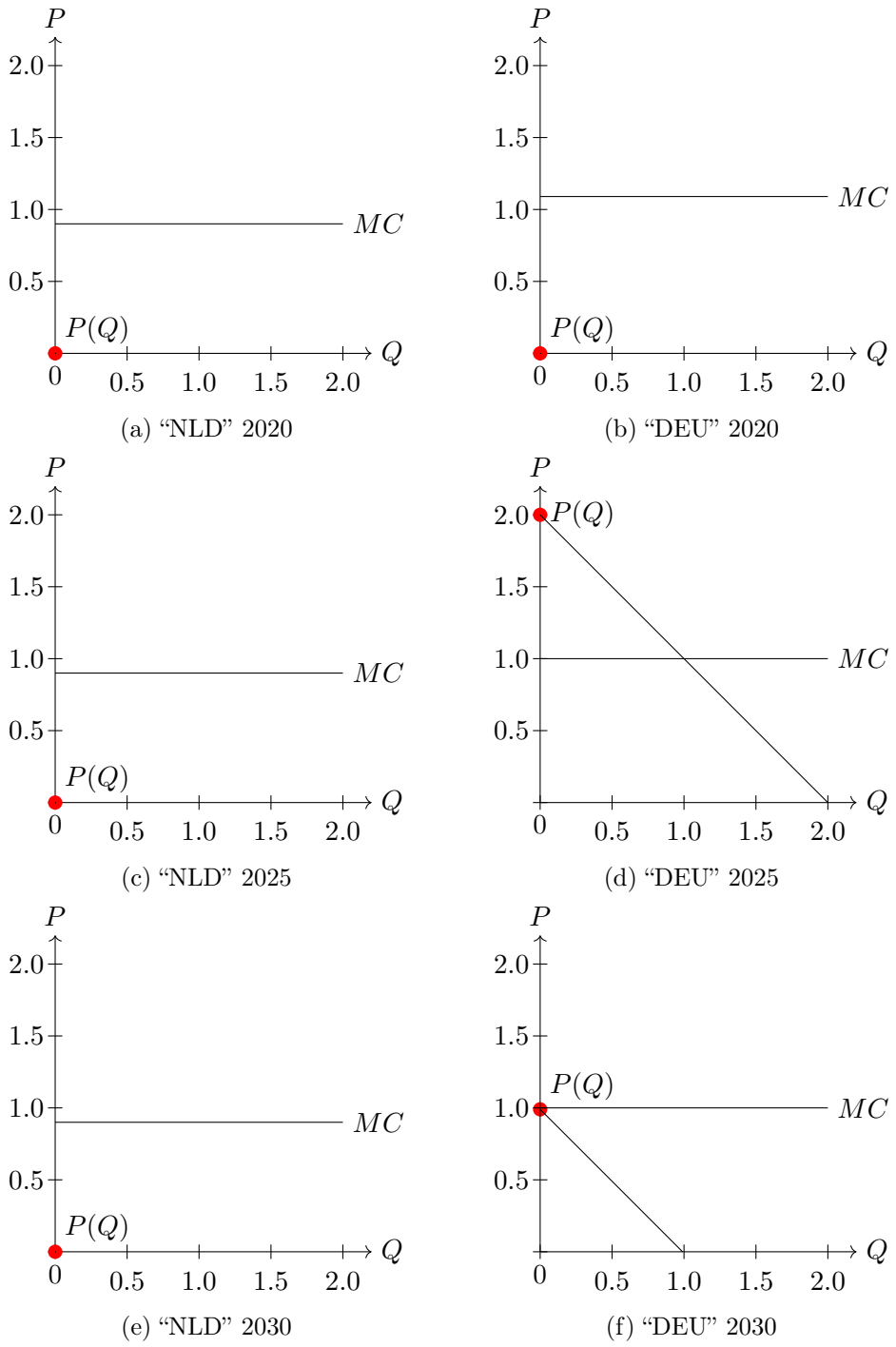
Table 83: Tested Results of Test Case 27 (continued)

Expression	**Result y=2020**	**Result y=2025**	**Result y=2030**
$\Delta^{A}_{\text{“NLD_to_NLD”,“CNG”},y}$	*N.E.*	*N.E.*	*N.E.*

Source: Own depiction.



(a) "NLD" 2020

(b) "DEU" 2020

(c) "NLD" 2025

(d) "DEU" 2025

(e) "NLD" 2030

(f) "DEU" 2030

Figure 27: Marginalized Provision Costs and Prices in Test Case 27

Source: Own depiction. Note: Marginalized costs are depicted as if adjacency was given.

2.2.28 Test Case 28

Test Case 28 is used to assert correct working of adjacency structures for arc capacity repurposing. All parameters are the same as in Test Case 21, however commodity-specific adjacency prohibits flows (since $\mathcal{AC}$ is empty). Tables 84 and 85 depict the sets and parameters, respectively. Tests are depicted in Table 86, while Figure 28 depicts a graphical solution.

Table 84: Sets of Test Case 28

Set Name	**Set Value**
$\mathcal{A}$	{"DEU_to_DEU","DEU_to_NLD","NLD_to_DEU","NLD_to_NLD"}
$\mathcal{AC}$	$\emptyset$
$\mathcal{C}$	{"CNG","GH_2"}
$\mathcal{DSB}$	{"Block 1"}
$\mathcal{I}$	{"Natural Gas"}
$\mathcal{IOB}$	{"Block 1"}
$\mathcal{M}$	{"OnlyTimeStep"}
$\mathcal{N}$	{"DEU","NLD"}
$\mathcal{O}$	{"FES"}
$\mathcal{P}$	{"P_DEU","P_NLD"}
$\mathcal{RA}$	$\emptyset$
$\mathcal{RS}$	$\emptyset$
$\mathcal{RV}$	$\emptyset$
$\mathcal{S}$	$\emptyset$
$\mathcal{T}$	{"T_DEU","T_NLD"}
$\mathcal{V}$	$\emptyset$

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Table 84: Sets of Test Case 28 (continued)

Set Name	**Set Value**
$\mathcal{VT}$	$\emptyset$
$\mathcal{Y}$	{“2020”,“2021”}

Source: Own depiction.

Table 85: Parameter Values of Test Case 28

Parameter	**y=2020**	**y=2021**
$\frac{1}{\lVert\Delta\rVert_y}$	1	
$\mathbb{1}^{NC}_{\text{“T_DEU”,“DEU”,“CNG”}}$	0	0
$\mathbb{1}^{NC}_{\text{“T_DEU”,“NLD”,“CNG”}}$	0	0
$\mathbb{1}^{NC}_{\text{“T_NLD”,“DEU”,“CNG”}}$	0	0
$\mathbb{1}^{NC}_{\text{“T_NLD”,“NLD”,“CNG”}}$	0	0
$\mathbb{1}^{NC}_{\text{“T_DEU”,“DEU”,“GH}_2\text{”}}$	0	0
$\mathbb{1}^{NC}_{\text{“T_DEU”,“NLD”,“GH}_2\text{”}}$	0	0
$\mathbb{1}^{NC}_{\text{“T_NLD”,“DEU”,“GH}_2\text{”}}$	0	0
$\mathbb{1}^{NC}_{\text{“T_NLD”,“NLD”,“GH}_2\text{”}}$	0	0
r_y	1	1
$d_{\text{“OnlyTimeStep”}}$	1	1
$c^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	0.5	0.5
$c^{\Delta P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	1	1
$c^{P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	0.5	0.5
$c^{\Delta P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	1	1
$fi^{P}_{\text{“CNG”,“Natural Gas”,“FES”}}$	1	1

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Table 85: Parameter Values of Test Case 28 (continued)

Parameter	**y=2020**	**y=2021**
$L^{P}_{\text{“CNG”,“FES”}}$	50	50
$\Lambda^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10	10
$c^{P}_{\text{“P_DEU”,“GH}_2\text{”,“FES”},y}$	0.4	0.4
$c^{\Delta P}_{\text{“P_DEU”,“GH}_2\text{”,“FES”},y}$	1	1
$c^{P}_{\text{“P_NLD”,“GH}_2\text{”,“FES”},y}$	0.5	0.5
$c^{\Delta P}_{\text{“P_NLD”,“GH}_2\text{”,“FES”},y}$	1	1
$fi^{P}_{\text{“GH}_2\text{”,“Natural Gas”,“FES”}}$	1	1
$L^{P}_{\text{“GH}_2\text{”,“FES”}}$	50	50
$\Lambda^{P}_{\text{“P_DEU”,“GH}_2\text{”,“FES”},y}$	10	10
$\Lambda^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	10	10
$\Omega^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0	0
$c^{\Delta^{I}}_{\text{“P_DEU”,“Natural Gas”,“Block 1”},y}$	0	0
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“CNG”,“FES”},y}$	10	10
$\Lambda^{T}_{\text{“T_DEU”,“NLD”,“CNG”,“FES”},y}$	10	10
$\Lambda^{T}_{\text{“T_DEU”,“DEU”,“GH}_2\text{”,“FES”},y}$	10	10
$\Lambda^{T}_{\text{“T_DEU”,“NLD”,“GH}_2\text{”,“FES”},y}$	10	10
$\Omega^{P}_{\text{“P_DEU”,“CNG”,“FES”},y}$	10	10
$\Omega^{P}_{\text{“P_DEU”,“GH}_2\text{”,“FES”},y}$	10	10
$\Lambda^{P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	10	10
$\Lambda^{P}_{\text{“P_NLD”,“GH}_2\text{”,“FES”},y}$	10	10

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Table 85: Parameter Values of Test Case 28 (continued)

Parameter	**y=2020**	**y=2021**
$\Lambda^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”},y}$	10	10
$\Omega^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”},y}$	0	0
$c^{\Delta^{I}}_{\text{“P_NLD”,“Natural Gas”,“Block 1”},y}$	0	0
$\Lambda^{T}_{\text{“T_NLD”,“DEU”,“CNG”,“FES”},y}$	10	10
$\Lambda^{T}_{\text{“T_NLD”,“DEU”,“GH}_2\text{”,“FES”},y}$	10	10
$\Lambda^{T}_{\text{“T_NLD”,“NLD”,“CNG”,“FES”},y}$	10	10
$\Lambda^{T}_{\text{“T_NLD”,“NLD”,“GH}_2\text{”,“FES”},y}$	10	10
$\Omega^{P}_{\text{“P_NLD”,“CNG”,“FES”},y}$	10	10
$\Omega^{P}_{\text{“P_NLD”,“GH}_2\text{”,“FES”},y}$	10	10
$l^{A}_{\text{“DEU_to_DEU”,“CNG”}}$	0.0	0.0
$l^{A}_{\text{“DEU_to_NLD”,“CNG”}}$	0.0	0.0
$l^{A}_{\text{“NLD_to_DEU”,“CNG”}}$	0.1	0.1
$l^{A}_{\text{“NLD_to_NLD”,“CNG”}}$	0.0	0.0
$l^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”}}$	0.0	0.0
$l^{A}_{\text{“DEU_to_NLD”,“GH}_2\text{”}}$	0.0	0.0
$l^{A}_{\text{“NLD_to_DEU”,“GH}_2\text{”}}$	0.1	0.1
$l^{A}_{\text{“NLD_to_NLD”,“GH}_2\text{”}}$	0.0	0.0
$c^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0.0	0.0
$c^{A}_{\text{“DEU_to_NLD”,“CNG”},y}$	0.0	0.0
$c^{A}_{\text{“NLD_to_DEU”,“CNG”},y}$	0.1	0.1

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Table 85: Parameter Values of Test Case 28 (continued)

Parameter	**y=2020**	**y=2021**
$c^{A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0.0	0.0
$c^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”},y}$	0.0	0.0
$c^{A}_{\text{“DEU_to_NLD”,“GH}_2\text{”},y}$	0.0	0.0
$c^{A}_{\text{“NLD_to_DEU”,“GH}_2\text{”},y}$	0.1	0.1
$c^{A}_{\text{“NLD_to_NLD”,“GH}_2\text{”},y}$	0.0	0.0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0
$c^{\Delta A}_{\text{“DEU_to_NLD”,“CNG”},y}$	10	10
$c^{\Delta A}_{\text{“NLD_to_DEU”,“CNG”},y}$	10	10
$c^{\Delta A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0	0
$c^{\Delta A}_{\text{“DEU_to_DEU”,“GH}_2\text{”},y}$	0	0
$c^{\Delta A}_{\text{“DEU_to_NLD”,“GH}_2\text{”},y}$	10	10
$c^{\Delta A}_{\text{“NLD_to_DEU”,“GH}_2\text{”},y}$	10	10
$c^{\Delta A}_{\text{“NLD_to_NLD”,“GH}_2\text{”},y}$	0	0
$c^{\Delta^{RA}}_{\text{“DEU_to_DEU”,“CNG”,“GH}_2\text{”},y}$	0	0
$c^{\Delta^{RA}}_{\text{“DEU_to_NLD”,“CNG”,“GH}_2\text{”},y}$	0	0
$c^{\Delta^{RA}}_{\text{“NLD_to_DEU”,“CNG”,“GH}_2\text{”},y}$	0.055	0.055
$c^{\Delta^{RA}}_{\text{“NLD_to_NLD”,“CNG”,“GH}_2\text{”},y}$	0	0
$f^{RA}_{\text{“CNG”,“GH}_2\text{”}}$	0.5	0.5
$\Lambda^{A}_{\text{“DEU_to_DEU”,“CNG”},y}$	0	0
$\Lambda^{A}_{\text{“DEU_to_NLD”,“CNG”},y}$	10	10

Continued on next page...

Table 85: Parameter Values of Test Case 28 (continued)

Parameter	**y=2020**	**y=2021**
$\Lambda^{A}_{\text{“NLD_to_DEU”,“CNG”},y}$	10	10
$\Lambda^{A}_{\text{“NLD_to_NLD”,“CNG”},y}$	0	0
$\Lambda^{A}_{\text{“DEU_to_DEU”,“GH}_2\text{”},y}$	0	0
$\Lambda^{A}_{\text{“DEU_to_NLD”,“GH}_2\text{”},y}$	0	0
$\Lambda^{A}_{\text{“NLD_to_DEU”,“GH}_2\text{”},y}$	0	0
$\Lambda^{A}_{\text{“NLD_to_NLD”,“GH}_2\text{”},y}$	0	0
$L^{A}_{\text{“CNG”}}$	50	50
$L^{A}_{\text{“GH}_2\text{”}}$	50	50
$c^{I_l}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	2	2
$c^{I_q}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0	0
$av^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	1	1
$c^{I_l}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.5	0.5
$c^{I_q}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0	0
$av^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”}}$	1	1
$\alpha^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	2	0
$\beta^{D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1	0
$\alpha^{D}_{\text{“NLD”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0	0
$\beta^{D}_{\text{“NLD”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	-1	0
$\alpha^{D}_{\text{“DEU”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0	2
$\beta^{D}_{\text{“DEU”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0	-1

Continued on next page...

Table 85: Parameter Values of Test Case 28 (continued)

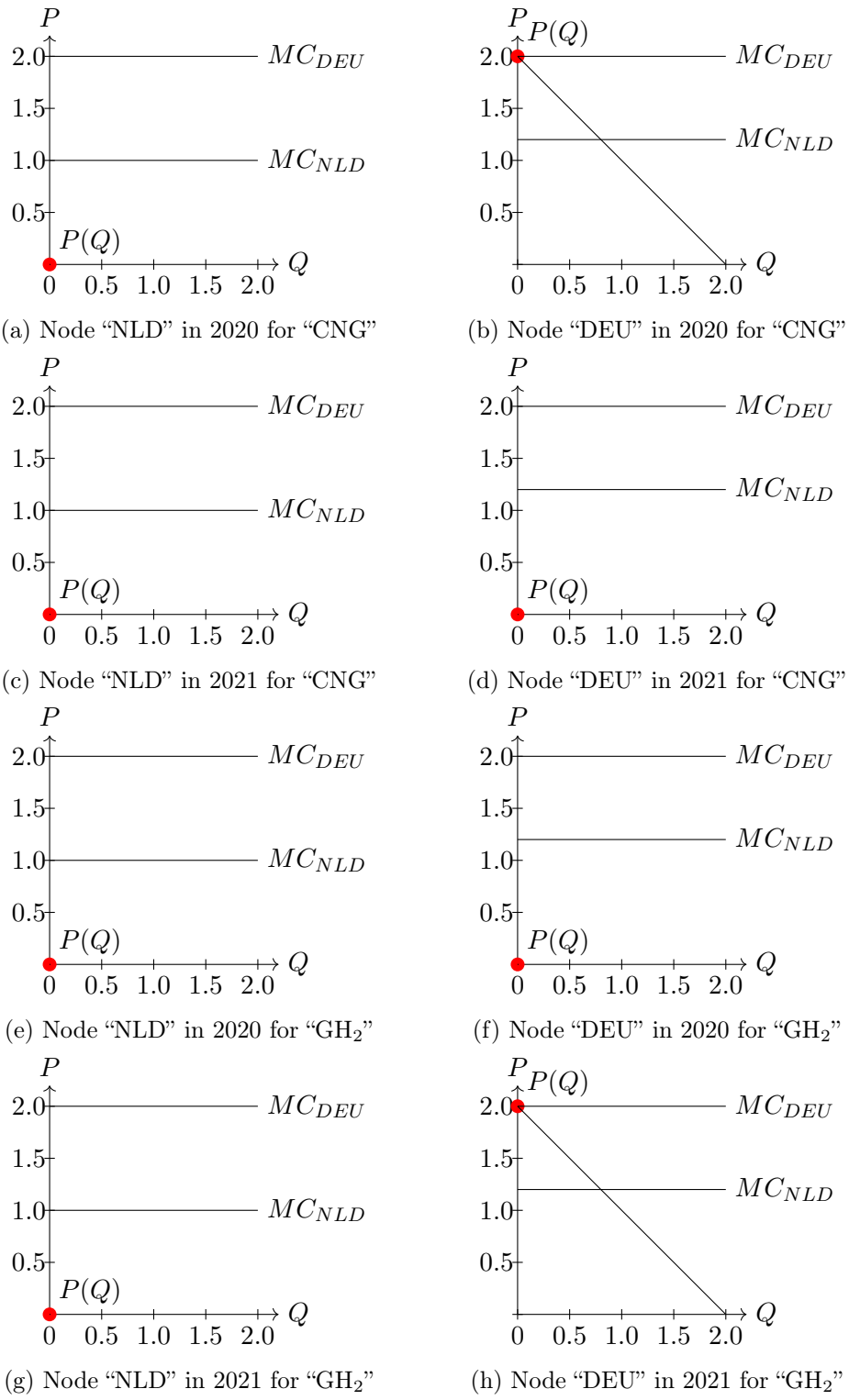
Parameter	**y=2020**	**y=2021**
$\alpha^{D}_{\text{“NLD”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0	0
$\beta^{D}_{\text{“NLD”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0	−1

Source: Own depiction.

Table 86: Tested Results of Test Case 28

Expression	**Result y=2020**	**Result y=2021**
$\tilde{P}^{T\to D}_{\text{“DEU”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	2.0	0
$\tilde{P}^{T\to D}_{\text{“NLD”,“CNG”,“Block 1”,“OnlyTimeStep”,y}}$	0.0	0.0
$\tilde{P}^{T\to D}_{\text{“DEU”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0.0	2.0
$\tilde{P}^{T\to D}_{\text{“NLD”,“GH}_2\text{”,“Block 1”,“OnlyTimeStep”,y}}$	0.0	0.0
$q^{I}_{\text{“P_DEU”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.0	0.0
$q^{I}_{\text{“P_NLD”,“Natural Gas”,“Block 1”,“OnlyTimeStep”,y}}$	0.0	0.0

Source: Own depiction.



(a) Node "NLD" in 2020 for "CNG"

(b) Node "DEU" in 2020 for "CNG"

(c) Node "NLD" in 2021 for "CNG"

(d) Node "DEU" in 2021 for "CNG"

(e) Node "NLD" in 2020 for "GH_2"

(f) Node "DEU" in 2020 for "GH_2"

(g) Node "NLD" in 2021 for "GH_2"

(h) Node "DEU" in 2021 for "GH_2"

Figure 28: Marginalized Provision Costs and Prices for Different Nodes in Test Case 28

Source: Own depiction. Note: Marginalized costs are depicted as if adjacency was given.

2.3 Additional Functionality Testing

2.3.1 Functionality Test 1

Functionality Test 1 tests correct scaling of operational and investment costs. Parameter values are checked against for two nodes, where one is half as expensive as the other. In addition, country-specific exertion of market power is tested, as well as shipping-related costs. The implementation is trivial, please see the corresponding code. Table 87 depicts an overview of tested parameters.

Table 87: Tests of Functionality Test 1

Parameter	**Type of Check**
$\mathbb{1}^{NC}$	Specific Market Power
c^{P}	Node specific scaling
$c^{\Delta P}$	Node specific scaling
c^{V}	Node specific scaling
$c^{\Delta V}$	Node specific scaling
$c^{\Delta^{RV}}$	Node specific scaling
c^{A}	Node specific scaling
$c^{\Delta A}$	Node specific scaling
$c^{\Delta^{RA}}$	Node specific scaling
$c^{S_{in}}$	Node specific scaling
$c^{S_{out}}$	Node specific scaling
$c^{\Delta S}$	Node specific scaling
$c^{\Delta^{RS}}$	Node specific scaling

Source: Own depiction.

2.3.2 Functionality Test 2

Functionality Test 2 validates `write_csv(datafile, data::ModelData)`. This function writes model input data to csv when called. Table 88 depicts the list of files whose existence is expected.

Table 88: Tests of Functionality Test 2

Parameter	File Type
$fi^{V}_{cc'}$	.csv
c^{A}_{acy}	.csv
c^{P}_{pcoy}	.csv
$c^{I_l}_{pibmy}$	.csv
$c^{I_q}_{pibmy}$	.csv
av^{I}_{pibm}	.csv
$c^{S_{in}}_{scy}$	.csv
$c^{S_{out}}_{scy}$	.csv
$c^{V}_{vcc'y}$	.csv
$c^{\Delta A}_{acy}$	.csv
$c^{\Delta P}_{pcoy}$	.csv
$c^{\Delta^{RA}}_{arcy}$	.csv
$c^{\Delta^{RS}}_{srcy}$	.csv
$c^{\Delta^{RV}}_{vrr'cc'y}$	.csv
$c^{\Delta S}_{scy}$	.csv
$c^{\Delta V}_{vcc'y}$	.csv
$c^{\Delta^{I}}_{piby}$	.csv

Continued on next page...

Table 88: Tests of Functionality Test 2 (continued)

Parameter	**File Type**
fi^{P}_{cio}	.csv
L^{A}_{c}	.csv
l^{A}_{ac}	.csv
L^{I}_{i}	.csv
L^{P}_{co}	.csv
Λ^{S}_{scy}	.csv
$l^{S}_{cmm^{+}(m)}$	.csv
$L^{V}_{cc'}$	.csv
$\mathbb{1}^{NC}_{tnc}$	.csv
α^{D}_{ncbmy}	.csv
β^{D}_{ncbmy}	.csv
Λ^{A}_{acy}	.csv
Λ^{I}_{piby}	.csv
Λ^{P}_{pcoy}	.csv
Λ^{S}_{scy}	.csv
Λ^{T}_{tncoy}	.csv
$\Lambda^{V}_{vcc'y}$	.csv
Ω^{I}_{piby}	.csv
Ω^{P}_{pcoy}	.csv
Ω^{S}_{scy}	.csv

Source: Own depiction.

2.3.3 Functionality Test 3

Functionality Test 3 makes sure that model outputs can be written accordingly. The tests are split into two accounts, one for variable results (Table 89) and one for calculated expressions (Table 90).

Table 89: Variable Tests of Functionality Test 3

Variable	File Type
q^{I}_{pibmy}	.csv
$q^{P\rightarrow T}_{pcomy}$	.csv
Δ^{I}_{piby}	.csv
Δ^{P}_{pcoy}	.csv
$q^{T\rightarrow D}_{tncbomy}$	.csv
$q^{V}_{vtcdomy}$	.csv
$\Delta^{V}_{vcc'y}$	.csv
$\Delta^{RV}_{vrr'cc'y}$	.csv
q^{T}_{tacomy}	.csv
Δ^{A}_{acy}	.csv
Δ^{RA}_{arcy}	.csv
q^{S}_{stcomy}	.csv
$q^{S_{in}}_{stcomy}$	.csv
$q^{S_{out}}_{stcomy}$	.csv
Δ^{S}_{scy}	.csv
Δ^{RS}_{srcy}	.csv

Source: Own depiction.

Table 90: Expression Tests of Functionality Test 3

Expression	**File Type**
Conversion Cost	.csv
Conversion Expansion Cost	.csv
Demand	.csv
Gross Surplus	.csv
Input Cost	.csv
Input Expansion Cost	.csv
Market Power Adjustment	.csv
Prices	.csv
Production Cost	.csv
Production Expansion Cost	.csv
Storage Cost	.csv
Storage Expansion Cost	.csv
Transport Cost	.csv
Transport Expansion Cost	.csv

Source: Own depiction.

2.3.4 Functionality Test 4

Functionality Test 4 makes sure that model outputs can be written accordingly. Table 91 depicts the tested files for all variables.

Table 91: Variable Tests of Functionality Test 4

Variable	**File Type**
q^{I}_{pibmy}	.csv

Continued on next page...

Table 91: Variable Tests of Functionality Test 4 (continued)

Variable	File Type
$q^{P\rightarrow T}_{pcomy}$	.csv
Δ^{I}_{piby}	.csv
Δ^{P}_{pcoy}	.csv
ϕ^{P}_{pimy}	.csv
λ^{I}_{piby}	.csv
λ^{P}_{pcomy}	.csv
ω^{P}_{pcoy}	.csv
ω^{I}_{piby}	.csv
$q^{T\rightarrow D}_{tncbomy}$	.csv
$q^{T\leftarrow P}_{tncomy}$	.csv
$q^{T\rightarrow V}_{tncomy}$	.csv
$q^{T\leftarrow V}_{tncomy}$	.csv
$q^{T\rightarrow S}_{tncomy}$	.csv
$q^{T\leftarrow S}_{tncomy}$	.csv
q^{T}_{tacomy}	.csv
ϕ^{T}_{tncomy}	.csv
λ^{T}_{tncoy}	.csv
$q^{V}_{vtcc'omy}$	.csv
$\Delta^{V}_{vcc'y}$	.csv
$\Delta^{RV}_{vrr'cc'y}$	.csv
$\lambda^{V}_{vcc'my}$	.csv

Continued on next page...

Table 91: Variable Tests of Functionality Test 4 (continued)

Variable	File Type
q^{A}_{acmy}	.csv
Δ^{A}_{acy}	.csv
Δ^{RA}_{arcy}	.csv
λ^{A}_{acmy}	.csv
q^{S}_{stcomy}	.csv
$q^{S_{in}}_{stcomy}$	.csv
$q^{S_{out}}_{stcomy}$	.csv
Δ^{S}_{scy}	.csv
Δ^{RS}_{srcy}	.csv
λ^{S}_{scmy}	.csv
ϕ^{S}_{stcomy}	.csv
ω^{S}_{scy}	.csv
π^{P}_{ncomy}	.csv
$\pi^{T\rightarrow D}_{ncbmy}$	.csv
$\pi^{T\rightarrow V}_{tncomy}$	.csv
$\pi^{V\rightarrow T}_{tncomy}$	.csv
π^{A}_{acmy}	.csv
$\pi^{T\rightarrow S}_{tncom_{l}y}$	.csv
$\pi^{S\rightarrow T}_{tncom_{h}y}$	.csv

Source: Own depiction.

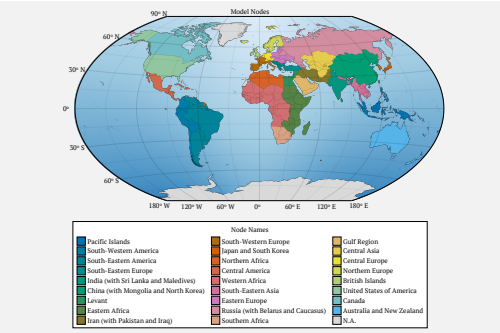
2.3.5 Functionality Test 5

Functionality Test 5 asserts that map plots from randomized data work as desired. Figure 29 shows how sample map plots should look like for different randomized inputs from Table 92.

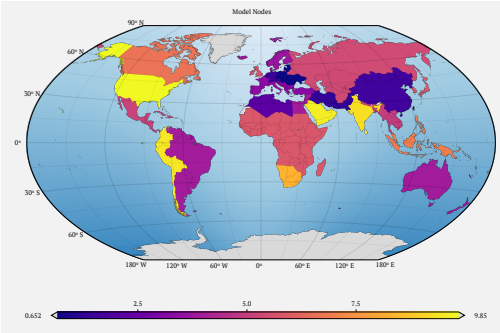
Table 92: Tests of Functionality Test 5

Plotting Function	**Reference**
`plot_model_nodes()`	Figure 29a
`plot_model_nodes(container::DenseAxisArray{T,N,Ax,L})`	Figure 29b
`plot_model_nodes(container::SparseAxisArray{T,N,K})`	Figure 29c
`plot_model_nodes(arcs::Vector{TransportArc})`	Figure 29d
`plot_model_nodes(container::DenseAxisArray{T,N,Ax,L},` `arcs::Vector{TransportArc})`	Figure 29e

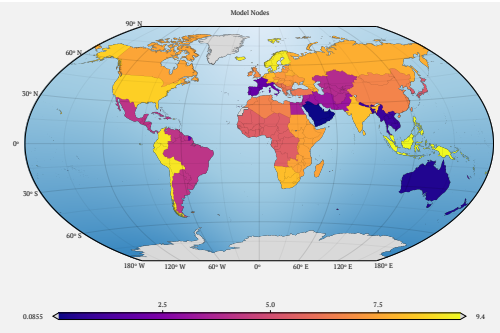
Source: Own depiction.



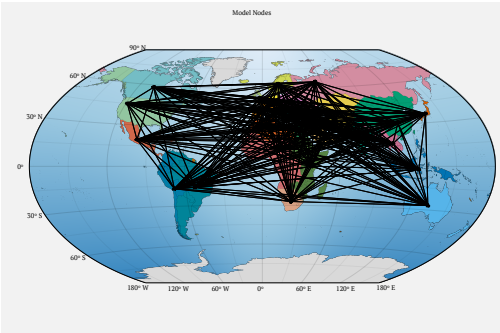
(a) Sample Output of Model Node Plots.



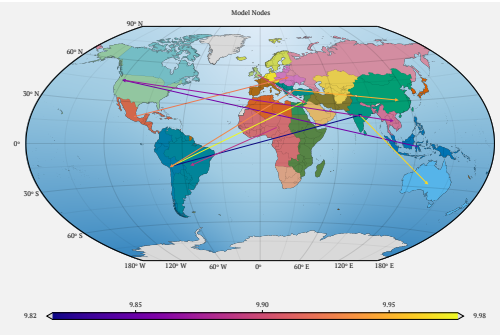
(b) Sample Output with DenseAxisArray as Argument.



(c) Sample Output with SparseAxisArray as Argument.



(d) Sample Output with Transport Arcs as argument.



(e) Sample Output with DenseAxisArray and Transport Arcs and as Argument.

Figure 29: Reference Plots for Functionality Test 5

Source: Own depiction.

3 Code Documentation

3.1 Detailed Documentation Outputs

Listings 3 to 10 provide an overview of callable function definitions.

Listing 3: Documentation and Help Mode Output for get_GH2TMOD_data

```
julia> ?
help?> get_GH2TMOD_data
search: get_GH2TMOD_data

  get_GH2TMOD_data(;data = joinpath(dirname(dirname(pathof(GH2TMOD))), "
      sample_data"), logfile = "")

  Get the model data

  A separate datafile and a file where to write the log records can be specified
       via
  keyword arguments.

  Examples
  ==========

  julia> using GH2TMOD
  julia> data = get_GH2TMOD_data();
```

Listing 4: Documentation and Help Mode Output for get_name

```
julia> ?
help?> get_name
search: get_name get_zero_subnormals

  get_name(v::AbstractSet)

  Get the name (string) of an underlying struct, such as a model node.

  Use broadcasting to retrieve a vector of names from a vector of objects.

  Examples
  ==========

```

```
  julia> using GH2TMOD
  julia> data = get_GH2TMOD_data();
  julia> get_name.(data.nodes)
  1-element Vector{String}:
   "DEU"
```

Listing 5: Documentation and Help Mode Output for build_optimization_model

```
julia> ?
help?> build_optimization_model
search: build_optimization_model

  build_optimization_model(data::ModelData, ub = 1e+6, logfile = "")

  Build a JuMP optimization model (QP) from the underlying model data.

  A logfile and upper bounds on variables can be specified.

  Examples
  ==========

  julia> using GH2TMOD
  julia> data = get_GH2TMOD_data();
  julia> build_optimization_model(data)
  A JuMP Model
  Maximization problem with:
  Variables: 11
  Objective function type: JuMP.QuadExpr
  `JuMP.AffExpr`-in-`MathOptInterface.EqualTo{Float64}`: 3 constraints
  `JuMP.AffExpr`-in-`MathOptInterface.LessThan{Float64}`: 8 constraints
  `JuMP.VariableRef`-in-`MathOptInterface.GreaterThan{Float64}`: 11 constraints
  `JuMP.VariableRef`-in-`MathOptInterface.LessThan{Float64}`: 11 constraints
  Model mode: AUTOMATIC
  CachingOptimizer state: NO_OPTIMIZER
  Solver name: No optimizer attached.
  Names registered in the model: arc_capacity, conversion_capacity,
      conversion_cost, conversion_expansion_cost, conversion_repurposing_cost,
      demand, equal_arc_expansion, equal_arc_repurposing, gross_surplus,
      input_block_limit, input_cost, input_expansion_cost,
      input_expansion_limit, market_power_adjustment, prices,
      producer_mass_balance, production_capacity, production_cost,
      production_expansion_cost, production_expansion_limit, q_I, q_P_T, q_S,
      q_S_in, q_S_out, q_T, q_T_D, q_V, storage_capacities, storage_cost,
```

```
        storage_expansion_cost, storage_expansion_limit, storage_mass_balance,
        storage_repurposing_cost, trade_sanctions, trader_mass_balance,
        transport_cost, transport_expansion_cost, transport_repurposing_cost,
        yearly_arc_flows, yearly_demand, yearly_input_procurement,
        yearly_production, Δ_A, Δ_I, Δ_P, Δ_RA, Δ_RS, Δ_RV, Δ_S, Δ_V
```

Listing 6: Documentation and Help Mode Output for build_complementarity_model

```
julia> ?
help?> build_complementarity_model
search: build_complementarity_model

  build_complementarity_model(data::ModelData, ub = 1e+6, logfile = "")

  Build a JuMP model with complementarity extension from the underlying model
      data.

  A logfile and upper bounds on variables can be specified.

  Examples
  ==========

  julia> using GH2TMOD
  julia> data = get_GH2TMOD_data();
  julia> build_complementarity_model(data)
  A JuMP Model
  Feasibility problem with:
  Variables: 35
  `JuMP.VariableRef`-in-`MathOptInterface.GreaterThan{Float64}`: 34 constraints
  `JuMP.VariableRef`-in-`MathOptInterface.LessThan{Float64}`: 34 constraints
  Model mode: AUTOMATIC
  CachingOptimizer state: NO_OPTIMIZER
  Solver name: No optimizer attached.
  Names registered in the model: arc_capacity, conversion_capacity,
      conversion_cost, conversion_expansion_cost, conversion_repurposing_cost,
      demand, equal_arc_expansion, equal_arc_repurposing, gross_surplus,
      input_block_limit, input_cost, input_expansion_cost,
      input_expansion_limit, market_power_adjustment, prices,
      producer_mass_balance, production_capacity, production_cost,
      production_expansion_cost, production_expansion_limit, q_I, q_P_T, q_S,
      q_S_in, q_S_out, q_T, q_T_D, q_V, storage_capacities, storage_cost,
      storage_expansion_cost, storage_expansion_limit, storage_mass_balance,
      storage_repurposing_cost, trade_sanctions, trader_mass_balance,
```

```
        transport_cost, transport_expansion_cost, transport_repurposing_cost,
        yearly_arc_flows, yearly_demand, yearly_input_procurement,
        yearly_production, Δ_A, Δ_I, Δ_P, Δ_RA, Δ_RS, Δ_RV, Δ_S, Δ_V
```

Listing 7: Documentation and Help Mode Output for optimize!

```
julia> ?
help?> optimize!
search: optimize! build_optimization_model

  optimize!(
      model::GenericModel;
      ignore_optimize_hook = (model.optimize_hook === nothing),
      _differentiation_backend::MOI.Nonlinear.AbstractAutomaticDifferentiation =
          MOI.Nonlinear.SparseReverseMode(),
      kwargs...,
  )

  Optimize the model.

  If an optimizer has not been set yet (see set_optimizer), a NoOptimizer error
      is
  thrown.

  If ignore_optimize_hook == true, the optimize hook is ignored and the model is
       solved
  as if the hook was not set. Keyword arguments kwargs are passed to the
      optimize_hook.
  An error is thrown if optimize_hook is nothing and keyword arguments are
      provided.

  Experimental features
  =====================

  These features may change or be removed in any future version of JuMP.

  Pass _differentiation_backend to set the
  MOI.Nonlinear.AbstractAutomaticDifferentiation backend used to compute
      derivatives of
  nonlinear programs.

  If you require only :ExprGraph, it is more efficient to pass _
      differentiation_backend
```

```
= MOI.Nonlinear.ExprGraphOnly().

---------------------------------------------------------------------------------------------------------

optimize!(model::Model, optimizer; options..., logfile = "")

Solve a JuMP Model with the given optimizer.

A logfile can be specified, and solver specific options can be passed.

Examples
==========

julia> using GH2TMOD
julia> using Ipopt
julia> data = get_GH2TMOD_data();
julia> model = build_optimization_model(data);
julia> optimize!(model, Ipopt.Optimizer, "max_wall_time" => 1e-5)
******************************************************************************
This program contains Ipopt, a library for large-scale nonlinear optimization.
 Ipopt is released as open source code under the Eclipse Public License (EPL).
         For more information visit https://github.com/coin-or/Ipopt
******************************************************************************

This is Ipopt version 3.14.13, running with linear solver MUMPS 5.6.0.

Number of nonzeros in equality constraint Jacobian...:        6
Number of nonzeros in inequality constraint Jacobian.:        7
Number of nonzeros in Lagrangian Hessian.............:        2

Total number of variables............................:       11
                     variables with only lower bounds:        0
                variables with lower and upper bounds:       11
                     variables with only upper bounds:        0
Total number of equality constraints.................:        3
Total number of inequality constraints...............:        8
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        0
        inequality constraints with only upper bounds:        8

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0 -9.0099910e-02 1.00e-02 1.00e+01  -1.0 0.00e+00    -  0.00e+00 0.00e+00   0

```

```
Number of Iterations....: 0

                                   (scaled)                 (unscaled)
Objective...............:   9.0099909800000114e-02   -9.0099909800000114e-02
Dual infeasibility......:   1.0000000000000000e+01    1.0000000000000000e+01
Constraint violation....:   9.9999800000000003e-03    9.9999800000000003e-03
Variable bound violation:   0.0000000000000000e+00    0.0000000000000000e+00
Complementarity.........:   9.9999999010000995e+05    9.9999999010000995e+05
Overall NLP error.......:   9.9999999010000995e+05    9.9999999010000995e+05


Number of objective function evaluations             = 1
Number of objective gradient evaluations             = 1
Number of equality constraint evaluations            = 1
Number of inequality constraint evaluations          = 1
Number of equality constraint Jacobian evaluations   = 1
Number of inequality constraint Jacobian evaluations = 1
Number of Lagrangian Hessian evaluations             = 0
Total seconds in IPOPT                               = 0.000

EXIT: Maximum wallclock time exceeded.
┌ Status: Optimizing Model
│   Progress:                                                    Solution Status
    TIME_LIMIT.
│   Time elapsed:
                                                                                0.14s
└ @ GH2TMOD /Users/lb/Desktop/git/GH2TMOD/src/optimization_model.jl:999
┌ Status: Optimizing Model
│   Progress:
    Returning Model.
│   Time elapsed:
                                                                                0.14s
└ @ GH2TMOD /Users/lb/Desktop/git/GH2TMOD/src/optimization_model.jl:1002
A JuMP Model
Maximization problem with:
Variables: 11
Objective function type: JuMP.QuadExpr
`JuMP.AffExpr`-in-`MathOptInterface.EqualTo{Float64}`: 3 constraints
`JuMP.AffExpr`-in-`MathOptInterface.LessThan{Float64}`: 8 constraints
`JuMP.VariableRef`-in-`MathOptInterface.GreaterThan{Float64}`: 11 constraints
`JuMP.VariableRef`-in-`MathOptInterface.LessThan{Float64}`: 11 constraints
Model mode: AUTOMATIC
CachingOptimizer state: ATTACHED_OPTIMIZER
Solver name: Ipopt
Names registered in the model: arc_capacity, conversion_capacity,
    conversion_cost, conversion_expansion_cost, conversion_repurposing_cost,
```

```
        demand, equal_arc_expansion, equal_arc_repurposing, gross_surplus,
        input_block_limit, input_cost, input_expansion_cost,
        input_expansion_limit, market_power_adjustment, prices,
        producer_mass_balance, production_capacity, production_cost,
        production_expansion_cost, production_expansion_limit, q_I, q_P_T, q_S,
        q_S_in, q_S_out, q_T, q_T_D, q_V, storage_capacities, storage_cost,
        storage_expansion_cost, storage_expansion_limit, storage_mass_balance,
        storage_repurposing_cost, trade_sanctions, trader_mass_balance,
        transport_cost, transport_expansion_cost, transport_repurposing_cost,
        yearly_arc_flows, yearly_demand, yearly_input_procurement,
        yearly_production, Δ_A, Δ_I, Δ_P, Δ_RA, Δ_RS, Δ_RV, Δ_S, Δ_V
```

Listing 8: Documentation and Help Mode Output for solve_mcp

```
julia> ?
help?> solve_mcp
search: solve_mcp

  solve_mcp(model::Model; logfile = "", kwargs...)

  Solve a JuMP model with complementarity extension using the PATH solver.

  A logfile can be specified, and solver specific options can be passed.

  Examples
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  julia> using GH2TMOD
  julia> data = get_GH2TMOD_data();
  julia> model = build_complementarity_model(data);
  julia> solve_mcp(model; output = "no", time_limit = 5)
  Reading options file /var/folders/c7/86nwrvmd1c325s4lzwg77jz40000gn/T/
      jl_CJTzvb
  Read of options file complete.

  Path 5.1.99 (Mon Jul 24 14:47:28 2023)
  Written by Todd Munson, Steven Dirkse, Youngdae Kim, and Michael Ferris
  :Solved
```

Listing 9: Documentation and Help Mode Output for plot_model_nodes

```
help?> plot_model_nodes
search: plot_model_nodes

  plot_model_nodes(; nodes_path = joinpath(@__DIR__, "maps", "
      WB_countries_Admin0_modified.geojson"), disputed_path = joinpath(@__DIR__
      , "maps", "WB_Admin0_disputed_areas_modified.geojson"), colorscheme = :
      seaborn_colorblind6, unassigned_color = :gray85, strokecolor = :black,
      strokewidth = 0.1, resolution = (1200, 800), backgroundcolor = RGBf(0.95,
       0.95, 0.95), rasterize = 10, map_title = "Model Nodes", legend_title = "
      Node Names", save_path = "",)

  Plot an overview of the model nodes.

  Only active, if no positional argument is passed.

  Keyword Arguments
  ==================

    - nodes_path = joinpath(@__DIR__, "maps",
      "WB_countries_Admin0_modified.geojson"): Path where to find model nodes
      geojson file.

    - disputed_path = joinpath(@__DIR__, "maps",
      "WB_Admin0_disputed_areas_modified.geojson"): Path where to find disputed
      areas geojson file.

    - colorscheme = :seaborn_colorblind6: Colorscheme for the model regions.

    - unassigned_color = :gray85: Colorscheme for unassigned and disputed
       regions.

    - strokecolor = :black: Strokecolor around regions.

    - strokewidth = 0.1: Strokewidth around regions.

    - resolution = (1200, 800): Picture resolution.

    - backgroundcolor = RGBf(0.95, 0.95, 0.95): Backgroundcolor.

    - rasterize = 10: Rasterization to reduce output file size, lesser implies
      smaller files.

    - map_title = "Model Nodes": Specification of the title.

    - legend_title = "Node Names": Title of the legend.

```

```
  - save_path = "": Where to save the resulting graphic. Ending of filename
    automatically implies file format.

Examples
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```

```
  - backgroundcolor = RGBf(0.95, 0.95, 0.95): Backgroundcolor.

  - rasterize = 10: Rasterization to reduce output file size, lesser implies
    smaller files.

  - map_title = "Model Nodes": Specification of the title.

  - save_path = "": Where to save the resulting graphic. Ending of filename
    automatically implies file format.

Examples
==========



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plot_model_nodes(container::JuMP.Containers.SparseAxisArray{T,N,K}; nodes_path
     = joinpath(@__DIR__, "maps", "WB_countries_Admin0_modified.geojson"),
    disputed_path = joinpath(@__DIR__, "maps", "
    WB_Admin0_disputed_areas_modified.geojson"), colorscheme = :plasma,
    unassigned_color = :gray85, strokecolor = :black, strokewidth = 0.1,
    resolution = (1200, 800), backgroundcolor = RGBf(0.95, 0.95, 0.95),
    rasterize = 10, map_title = "Model Nodes", save_path = "")

Plot the values inside container into the model node map.

Separate methods implemented for DenseAxisArray and SparseAxisArray.

Keyword Arguments
===================

  - nodes_path = joinpath(@__DIR__, "maps",
    "WB_countries_Admin0_modified.geojson"): Path where to find model nodes
    geojson file.

  - disputed_path = joinpath(@__DIR__, "maps",
    "WB_Admin0_disputed_areas_modified.geojson"): Path where to find disputed
    areas geojson file.

  - colorscheme = :plasma: Colorscheme for the model regions.

  - unassigned_color = :gray85: Colorscheme for unassigned and disputed
     regions.

```

```
115     -  strokecolor = :black: Strokecolor around regions.
116
117     -  strokewidth = 0.1: Strokewidth around regions.
118
119     -  resolution = (1200, 800): Picture resolution.
120
121     -  backgroundcolor = RGBf(0.95, 0.95, 0.95): Backgroundcolor.
122
123     -  rasterize = 10: Rasterization to reduce output file size, lesser implies
124        smaller files.
125
126     -  map_title = "Model Nodes": Specification of the title.
127
128     -  save_path = "": Where to save the resulting graphic. Ending of filename
129        automatically implies file format.
130
131   Examples
132   ==========
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136   ____________________________________________________________________________
        ________
137
138   plot_model_nodes(arcs::Vector{TransportArc}; nodes_path = joinpath(@__DIR__, "
        maps", "WB_countries_Admin0_modified.geojson"), disputed_path = joinpath(
        @__DIR__, "maps", "WB_Admin0_disputed_areas_modified.geojson"),
        centroids_path = joinpath(@__DIR__, "maps", "centroids.csv"), colorscheme
         = :seaborn_colorblind6, unassigned_color = :gray85, strokecolor = :black
        , strokewidth = 0.1, linewidth = 0.2, linecolor = :black, resolution = (
        1200, 800), backgroundcolor = RGBf(0.95, 0.95, 0.95), rasterize = 10,
        map_title = "Model Nodes", save_path = "")
139
140   Plot the passed arcs into the model node map.
141
142   Separate method implemented for also passing arc values.
143
144   Keyword Arguments
145   ===================
146
147     -  nodes_path = joinpath(@__DIR__, "maps",
148        "WB_countries_Admin0_modified.geojson"): Path where to find model nodes
149        geojson file.
150
151     -  disputed_path = joinpath(@__DIR__, "maps",
152        "WB_Admin0_disputed_areas_modified.geojson"): Path where to find disputed
```

```
153         areas geojson file.
154
155     -   centroids_path = joinpath(@__DIR__, "maps", "centroids.csv"): Path where
             to
156         find model region centroids file.
157
158     -   colorscheme = :seaborn_colorblind6: Colorscheme for the model regions.
159
160     -   unassigned_color = :gray85: Colorscheme for unassigned and disputed
             regions.
161
162     -   strokecolor = :black: Strokecolor around regions.
163
164     -   strokewidth = 0.1: Strokewidth around regions.
165
166     -   linewidth = 0.2: Arc linewidth connecting nodes.
167
168     -   linecolor = :black: Color of arcs connecting nodes.
169
170     -   resolution = (1200, 800): Picture resolution.
171
172     -   backgroundcolor = RGBf(0.95, 0.95, 0.95): Backgroundcolor.
173
174     -   rasterize = 10: Rasterization to reduce output file size, lesser implies
175         smaller files.
176
177     -   map_title = "Model Nodes": Specification of the title.
178
179     -   save_path = "": Where to save the resulting graphic. Ending of filename
180         automatically implies file format.
181
182   Examples
183   ==========
184
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186
187   ------------------------------------------------------------------------------
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188
189   plot_model_nodes(container::JuMP.Containers.DenseAxisArray{T,N,Ax,L}, arcs::
          Vector{TransportArc}; nodes_path = joinpath(@__DIR__, "maps", "
          WB_countries_Admin0_modified.geojson"), disputed_path = joinpath(@__DIR__
          , "maps", "WB_Admin0_disputed_areas_modified.geojson"), centroids_path =
          joinpath(@__DIR__, "maps", "centroids.csv"), colorscheme = :
          seaborn_colorblind6, unassigned_color = :gray85, strokecolor = :black,
          strokewidth = 0.1, linewidth = 1, linecolorscheme = :plasma, resolution =
```

```
        (1200, 800), backgroundcolor = RGBf(0.95, 0.95, 0.95), rasterize = 10,
        largestvalues = 10, map_title = "Model Nodes", save_path = "") where {T,N
        ,Ax,L<:NTuple{N,JuMP.Containers._AxisLookup}}

Plot the passed arcs with coloring corresponding to given values in the
    container into
the model node map.

Separate method implemented for just plotting normal values.

Keyword Arguments
===================

  - nodes_path = joinpath(@__DIR__, "maps",
    "WB_countries_Admin0_modified.geojson"): Path where to find model nodes
    geojson file.

  - disputed_path = joinpath(@__DIR__, "maps",
    "WB_Admin0_disputed_areas_modified.geojson"): Path where to find disputed
    areas geojson file.

  - centroids_path = joinpath(@__DIR__, "maps", "centroids.csv"): Path where
     to
    find model region centroids file.

  - colorscheme = :seaborn_colorblind6: Colorscheme for the model regions.

  - unassigned_color = :gray85: Colorscheme for unassigned and disputed
     regions.

  - strokecolor = :black: Strokecolor around regions.

  - strokewidth = 0.1: Strokewidth around regions.

  - linewidth = 1: Arc linewidth connecting nodes.

  - linecolorscheme = :plasma: Colorscheme for coloring arcs connecting
     regions
    based on container values.

  - resolution = (1200, 800): Picture resolution.

  - backgroundcolor = RGBf(0.95, 0.95, 0.95): Backgroundcolor.

  - rasterize = 10: Rasterization to reduce output file size, lesser implies
    smaller files.
```

```

    - largestvalues = 10: The n largest arc values and equal to select.

    - minval = 1: The smallest value to include in the arc plot.

    - map_title = "Model Nodes": Specification of the title.

    - save_path = "": Where to save the resulting graphic. Ending of filename
      automatically implies file format.

  Examples
  ==========
```

Listing 10: Documentation and Help Mode Output for write_csv

```
julia> ?
help?> write_csv
search: write_csv

  write_csv(datadir, data::ModelData; logfile = "")

  Write model data to a specific directory.

  A logfile can be specified.

  Examples
  ==========

  julia> using GH2TMOD
  julia> data = get_GH2TMOD_data();
  julia> write_csv("path/to/my/dir",data);

  ------------------------------------------------------------------------------
      --------

  write_csv(resultsfile, model::Model; logfile = "")

  Write model results to a specific directory.

  A logfile can be specified.

  Examples
  ==========
```

```
julia> data = get_GH2TMOD_data();
julia> write_csv("path/to/my/dir",data);
julia> model = build_optimization_model(data);
julia> optimize!(model, Ipopt.Optimizer);
julia> write_csv("path/to/my/dir",model);
```

References

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