Emotional Inattention

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Abstract

A decision-maker allocates attention across additively separable dimensions (e.g., consumption dimensions, states, or time periods). In addition to being instrumentally valuable, attention generates utility, and so the decision-maker maximizes an attention-weighted objective function. Optimal attention to a dimension is increasing in its payoff and the instrumental value of attention. The attention-weighted objective generates behavioral phenomena such as belief distortions and time preferences, as well as predictions on when and in which form they occur. We discuss implications for policy interventions designed to increase overall utility or improve decisions.

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1 Introduction

Attention—"a state in which cognitive resources are focused on certain aspects of the environment rather than on others" (VandenBos, ed, 2015)—has (at least) two fundamental features. First, it allows individuals to better match their actions to the environment through, e.g., information acquisition and processing or changing a default. This feature is well recognized and studied by both psychologists (Desimone et al., 1995) and economists (Sims, 2003). Second, attention governs how individuals generate and regulate emotions.[1] Indeed, anticipatory utility or remembrance utility from non-contemporaneous consumption, which has been discussed as a key component of generating emotional utility (as in, e.g., Caplin and Leahy (2001) and Loewenstein (1987)), can only occur if the consumption is, in fact, 'anticipated' or 'remembered;' in other words, if the individual devotes attention to it. Despite widespread interest in this second feature among psychologists and cognitive scientists, it has received less study by economists.[2]

This paper combines these insights about the role of attention in order to make three main contributions. First, we formalize a model of attention allocation which captures both the instrumental and emotional roles that attention plays. Our formalization delivers two key insights. Because attention to a situation increases its emotional valence, individuals ignore low-payoff ones even when doing so comes at a cost, and may pay attention to others with a high payoff even when the are few material benefits ("attentional distortion"). Second, attention "is proactive in shap-

[1]See Dixon et al. (2017) and Gross (1998) for reviews.

[2]That is not to say it has been completely ignored. For example, Schelling (1988) highlights the role of the "mind" as a "pleasure machine or consuming organ, the generator of direct consumer satisfaction," in addition to the role of the "information processing and reasoning machine." Schelling also implicitly suggests that these roles should be considered jointly and writes: "Marvelous it is that the mind does all these things. Awkward it is that it seems to be the same mind from which we expect both the richest sensations and the most austere analyses." Our following analysis of the interaction of these roles hopes to alleviate this "awkwardness."

ing perceptual experience" (Todd et al., 2012), and so paying attention to a situation increases its weight in the emotional component of utility ("utility reweighting").

Our second contribution is using the model to provide a unifying explanation for several existing behavioral phenomena. Attentional distortion rationalizes observed attention patterns, often referred to as ostrich effects, in the domains of health and finance. It also results in a preference for variations in payoffs across situations as they allow individuals to hone in on good ones. As a result, a preferences for non-smooth consumption paths and additional risk emerge. Utility reweighting manifests as belief distortions in settings with uncertainty, while in a dynamic setting it generates time preferences.

Third, we use the model to make novel, testable predictions. For example, our model makes precise predictions about when we observe different forms of probability weighting (e.g., S- versus inverse S-shaped weighting), as well as when individuals act present or future focused. These results point towards future empirical exercises to more carefully understand the role of attention in decision-making.

To fix ideas, we begin by sketching out a simplified version of our model. A decision-maker (henceforth, DM—they) faces an environment with a number of consumption dimensions indexed by i. The DM devotes a unit of attention across these dimensions, with attention to dimension i denoted by α_i, and the vector of attention as α. The DM also takes an action x from a set $X(\alpha)$ which depends on the attention allocation. The DM's utility is the sum of material and emotional terms. They gain a material utility from each consumption dimension which equals a payoff, denoted by $V_i(x)$, associated with the dimension. In addition, they gain emotional utility in proportion to the payoff from a dimension, and the amount of attention devoted to it: $\alpha_i V_i(x)$. In spirit, this approach is similar to models of anticipatory utility (e.g., Loewenstein (1987); Caplin and Leahy (2001)), where the flow utility of emotions is a function of beliefs about future payoffs. Our innovation is to make this flow utility of emotions from a dimension a function of the attention paid to it. For this reason, we refer to this second component as attention utility.

When attention utility receives relative weight λ, total utility is:

$$\underbrace{\sum_i V_i(x)}_{\text{material utility}} + \lambda \underbrace{\sum_i \alpha_i V_i(x)}_{\text{attention utility}} . \tag{1}$$

Thus, attention is a measure on a set of consumption dimensions that determines: 1) which actions are available and 2) the weight a dimension takes—total weight on dimension i is $1 + \lambda\alpha_i$. Although the resulting objective is simple, it will allow us to capture a rich set of implications in a tractable way.[3]

In Section 2.1 we generalize Equation (1) allowing us to capture attention allocation across possible realizations of a random state and across different time periods. While these dimensions can be general, we maintain that the DM makes a single choice about how to allocate attention and what action to choose (i.e., a static decision problem). As in the simplified version, attention to a state or time period may be used to acquire information or reduce trembling.[4]

Given this framework, Section 2.2 derives general properties of the solution. We consider our model as entirely standard but with a twist—the DM maximizes their material utility (standard) as well as the value of their attention utility (twist). Unsurprisingly then, the DM responds to standard incentives: increasing the instrumental value of attention for a dimension increases attention to it. Unlike in the standard

[3]One could also allow for non-additivity across dimensions, the sub-utility functions, or for non-multiplicative interactions in attention utility. We suspect that most of our results would remain qualitatively similar.

[4]We show formally how our reduced-form formulation nests these canonical uses of attention in Appendix A.1.

model, however, a key determinant of attention are the payoff levels: ceteris paribus, the DM devotes more attention to dimensions with higher payoffs. The DM may thus avoid a low-payoff dimension, even though attending to it would increase its payoff while devoting excessive attention beyond the point where it is instrumentally valuable to others.

In the context of attention across consumption dimensions (Section 2.3), the aforementioned attentional patterns are in line with the well-documented ostrich phenomenon: individuals tend to be inattentive to situations with low payoffs. Such behavior has been extensively studied both in financial decision-making (starting with Karlsson et al. (2009)) and medical decision-making (e.g., Becker and Mainman (1975) and Oster et al. (2013)), as well as in the lab (Avoyan and Schotter, 2020), and we provide a new mechanism behind it.

Section 2.2 also shows that attention utility introduces a preference for varied payoffs across dimensions. The DM can exploit such variation and increase their attention utility by focusing on high-payoff dimensions. Thus, our model naturally generates "sparse" attention allocations (Gabaix, 2014).

In Section 2.4 we let different dimensions correspond to different states of the world. The attention-dependent weight of a state leads to an as-if subjective probability of that state occurring (although our DM understands the true probabilities perfectly). When attention is non-instrumental, the DM devotes all to high-payoff states disregarding low-payoff ones leading to optimism (Sharot, 2011). Our model also rationalizes individuals' preferences for positively-skewed payoffs: The DM gambles as to envision themselves winning the unlikely but high payoff.[5]

But the DM's optimism is not universal and can be mitigated or even turned

[5]See, e.g., Blume and Friend (1975) in the context of portfolio choice, or Garrett and Sobel (1999); Forrest et al. (2002) in the context of individuals playing lotto.

into pessimism when attention is instrumentally valuable. We next show how different environments lead to different kinds of probability weighting (i.e., the subjective probability of a state as a function of its objective probability). For instance, an inverse S-shaped probability weighting occurs in environments where a minimal amount of attention to each state is forced or necessary to ensure a good payoff. We thus add to the extensive literature on probability weighting by offering attention as a novel mechanism that predicts how environments affect the shape of the weighting function.

In Section 3 we extend our model to dynamic settings where the DM makes a choice about contemporaneous attention in multiple periods. The DM's behavior is the equilibrium of an intrapersonal game played between selves. In Section 3.1 different dimensions represent different time periods, i.e., there is only inter-period and no intra-period attention allocation. Our model leads to endogenous weights on time periods, i.e., time preferences. For instance, the DM may discount the future (i.e., they are present focused) if the payoff in the present is particularly high or attention to it is of particular instrumental value. Our model also rationalizes "memorable consumption"—episodes with high payoffs such as lavish vacations or weddings (Gilboa et al., 2016; Hai et al., 2020). The consumption path is optimally non-smooth allowing the DM to focus on high-payoff periods while ignoring others.

We then explore the implications of a future attention problem for actions today. In Section 3.2 the DM chooses a default action for a future consumption dimension. The DM chooses one that is optimal for when their future self is inattentive and the default binds. The optimal default may thus overweight low-payoff states as it is those lead to inattention.

Dynamically extending the model also allows us to study preferences over dynamic information acquisition (Section 3.3). Even without any instrumental value of information, we show that the DM has strict preferences over the resolution of uncertainty: 1) they acquire more information when the expected payoff is high, 2) attention as a hidden action implies a preference for early information acquisition, and 3) the DM exhibits strict preferences over the skeweness of information. These predictions are broadly consistent with the laboratory evidence in Masatlioglu et al. (2017); Nielsen (2020).

In Section 4 we consider interventions to improve payoffs. A payoff increase for a dimension is weighted by the attention devoted to that dimension, and so those for high-attention dimensions are most effective. We also show that policies designed to increase attention can be ineffective if they penalize bad outcomes. The DM may shy away from a problem if the possibility of a penalty reduces the expected payoff. Hence, negative commitment devices, those that penalize for deviation from the action committed to, may be not only be ineffective, but in fact counterproductive. Lastly, we consider the optimal way to mentally bundle different dimensions. Instead of considering dimensions i and i' as distinct, the DM may bundle them together as one larger dimension (with payoff $V_i + V_{i'}$). Given a set of small primitive dimensions, we characterizing the optimal bracketing, providing a microfoundation for one of the key premises of our model—the set of dimensions.

Section 5 discusses how our model relates to several other classes of models. We consider, in turn, models of rational inattention, Bayesian and non-Bayesian models with anticipatory utility, and other models of attention. Section 6 concludes with a discussion of some of the limitations of our approach and avenues for future research.

2 Model

We consider a decision-maker (henceforth, DM—they) and begin by introducing a general objective to formally express the two fundamental features of attention: attention determines which actions are available to the DM, and 2) generates attention utility. We then characterize the optimal attention allocation, in particular focusing on the role of attention utility, before exploring our model's implications in two simple environments: attention across consumption dimensions and uncertain states of the world (Sections 2.3 and 2.4).

2.1 Setup

The DM faces a finite number of dimensions $\mathcal{D}$ with generic dimension i.[6] A dimension can correspond to a dimension of consumption, a realization of an uncertain state or a time period, or a combination of these. Each dimension i is associated with a payoff, V_i. The DM chooses an (action, attention)-pair denoted by (x, α). The action determines the payoff in each dimension i as $V_i(x)$ where V_i is continuous in x. Attention is a measure on the set of dimensions with total measure of 1 (a normalization), i.e., $\alpha = (\alpha_i)_{i \in \mathcal{D}}$, where α_i denotes the attention devoted to dimension i with $\alpha_i \geq 0$ and $\sum_i \alpha_i = 1$.[7] As a notational convention, for any variable that is indexed by $i \in \mathcal{D}$, say b_i, we let $b_{-i} := (b_{i'})_{i' \in \mathcal{D} \setminus \{i\}}$.

Attention has two implications. First, the available actions depend on the attention allocation capturing its instrumental value. Given α, action x is chosen from a set $X(\alpha)$, where X is compact- and non-empty-valued and upper hemicontinuous. When

[6]We take here the dimensions as given. In practice, the boundaries between dimensions may not always be obvious. While we acknowledge this underspecification, we provide some guidance as to how our model can be applied in practice (Section 6) and study a meta optimization problem in which the DM chooses how to define a dimension (Section 4.3).

[7]Alternatively, one can impose an upper bound on the measure of attention (with no lower bound). Such a model is nested in ours by adding a trivial dimension with payoff 0 to $\mathcal{D}$.

$X(\alpha)$ is independent of α, attention has no instrumental value. This reduced-form formulation nests canonical settings with information acquisition, attention reducing "trembles," and recall of memories (Examples 1–3 in Appendix A.1).

Second, the DM values both material utility but also attention utility. The material utility from a dimension is due to actual consumption and equals the payoff associated with the dimension. Attention utility is jointly determined by the amount of attention devoted to a dimension and its payoff. The relative importance of attention utility to material utility is given by parameter λ. In some settings, attention utility can be interpreted as anticipatory utility (Loewenstein, 1987; Caplin and Leahy, 2001), but one that is only generated when the DM devotes attention to (future) consumption.

We view the first consequence of attention—its instrumental value—as relatively standard, and for $\lambda = 0$, it is the only consequence of attention. We thus refer to the case when $\lambda = 0$ as the "standard model" and the corresponding DM as the "standard DM." More generally, the DM's objective is the weighted sum of material utility and attention utility:

$$\underbrace{\sum_i \omega_i V_i(x)}_{\text{material utility}} + \lambda \underbrace{\sum_i (\alpha_i + \psi_i) V_i(x)}_{\text{attention utility}} . \tag{2}$$

Parameters ω_i and ψ_i are nonnegative and generalize (1) in two ways. First, we allow material utilities to be weighted differentially across dimensions (via ω_i's). When the dimensions are different states, these weights can capture the probabilities of each state, or when the dimensions are different time periods, they may capture some exogenous discounting of material utility from future consumption.

Second, we allow for some exogenous focus ψ_i devoted to dimension i. This allows us to model the fact that although attention regulates emotional reactions, it may not be able to fully control it.[8] This approach also allows us to capture the DM's concerns for (fixed) future attention utility in their objective.[9]

2.2 Optimal attention and action

We provide multiple comparative static results (Propositions 1–3) to understand how the DM optimal (action, attention)-pair depends on the environment. In turn, we consider the dependence on the payoff in a dimension, the relative weight on attention utility, λ, and parameters ω_i and ψ_i.

To strengthen some statements, we introduce the notion of a separable environment. The environment is separable if action x is a vector $x = (x_i)_{i\in\mathcal{D}}$, payoff $V_i(x_i, x_{-i})$ is independent of x_{-i} for all i and x_i, and $X(\alpha) = \Pi_{i\in\mathcal{D}} X_i(\alpha_i)$. In words, the DM takes separate actions for each dimension and whether a dimension-specific action is available depends only on the amount of attention devoted to the dimension. Note that maximizing (2) with respect to an (action, attention)-pair is then equivalent to maximizing $\sum_i(\omega_i + \lambda(\alpha_i + \psi_i))\hat{V}_i(\alpha_i)$ with respect to attention only, where $\hat{V}_i(\alpha_i) := \max_{x_i\in X_i(\alpha_i)} V_i(x_i, \cdot)$. We assume that X_i is monotone, i.e., $X_i(\alpha_i) \subseteq X_i(\alpha_i')$ for all $\alpha_i \leq \alpha_i'$, and so $\hat{V}_i$ is increasing.

We begin with varying payoff V_i. For each i, we fix some $\tilde{V}_i$ and define $V_i = \beta_i\tilde{V}_i + \gamma_i$, for scalars $\beta_i \geq 0$ and γ_i. Increasing γ_i increases the payoff level and

[8] A key tenet of our model is that some attention is voluntarily directed by the individual, a premise often referred to as "top-down attention." While involuntarily allocated attention (often referred to as "bottom-up attention") is also important, at least some is indeed directed (e.g., Corbetta and Shulman (2002); Buschman and Miller (2007); Bronchetti et al. (2020)). Our model can be viewed as determining the use of the residual stock of attention after involuntarily attentional allocations have been made. We return, in Section 6, to discussing how voluntary and involuntary attention may interact.

[9] Our model also nests the case where attention utility is independent of the amount of directed attention α_i as in anticipatory utility models like Loewenstein (1987); Caplin and Leahy (2001): let λ go to 0, and ψ_i go to infinity, keeping their product constant.

increasing β_i increases the payoff difference induced by different actions.

An increase in the payoff level of dimension i does not affect which (action, attention)-pair maximizes overall material utility and hence does not affect the standard (i.e., $\lambda = 0$) DM's solution. However, the attention utility from dimension i increases in proportion to the attention devoted it and so when the DM puts positive weight on attention utility (i.e., $\lambda > 0$), they devote more attention. If the environment is separable, this increase in attention, in turn, leads to a better action for that dimension.

An increase in the payoff difference from different actions increases the importance of taking an action suitable for dimension i. It may also move the payoff up or down (e.g., V_i increases everywhere if $\tilde{V}_i$ is nonnegative), inducing the DM to change their attention just as above. In the proposition below, we offset such level change and the DM always chooses an action better suited for i. If the environment is separable, the "more suitable" action can only be available if the DM increases their attention. Note that this comparative static does not rely on attention utility. It captures the standard intuition that the DM devotes attention where it is most instrumental.

For the rest of this section, we assume that the optimal solution is unique. We state and prove a general version of the propositions that does not assume a unique solution in Appendix C. All other results in the rest of the paper are also proved there.

Proposition 1. *Consider dimension $i \in \mathcal{D}$. Fix V_{-i} and consider changing parameters (γ_i, β_i) to (γ'_i, β'_i). Denote the optimal (action, attention)-pairs for each parameter set as (x, α) and (x', α'), respectively.*

- *If $\gamma'_i \geq \gamma_i$ and $\beta_i = \beta'_i$, then $\alpha'_i \geq \alpha_i$. If, in addition, the environment is separable, then $V_i(x) - V_i(x') \geq \gamma'_i - \gamma_i$.*

- *If $\beta_i' \geq \beta_i$ and $\gamma_i' = \gamma_i - (\beta_i' - \beta_i)\tilde{V}_i(x)$, then $V_i(x') \geq V_i(x)$. If, in addition, the environment is separable, then $\alpha_i' \geq \alpha_i$.*

Figure 1 illustrates Proposition 1. Panel (a) considers an increase of γ_i to γ_i' depicting the material utility (weighted by some ω_i; top figure) and attention utility (bottom figure) of dimension i as functions of attention. The optimal action x^* is held fixed (hence material utility is independent of α_i). This increase simply shifts the material utility up (top figure). However, the increase in attention utility is larger for higher α_i (bottom figure). Thus, the DM increases their attention in response (first part of Proposition 1).

Panel (b) considers an increase in β_i to β_i' with an offsetting change in γ_i to γ_i' and shows the material utility (top figure) and attention utility (bottom figure) of dimension i as functions of $\tilde{V}_i(x)$. Throughout, the optimal attention α_i^* is held fixed. This change then pivots the material utility around its initial optimal value (top figure). Already here, the DM benefits relatively more from increasing $\tilde{V}_i(x)$ than before. The same pivoting occurs for attention utility (bottom figure). Thus, the DM increases $\tilde{V}_i(x)$ in response to the change (second part of Proposition 1).

We next turn to the relative weight on attention utility λ and show three results. First, increasing λ decreases the relative importance of material utility and so it decreases. Second, parameterizing V_i again as $V_i = \tilde{V}_i + \gamma_i$, the DM's objective is evidently linear in payoff levels γ_i and hence the DM's value, i.e., (2) for optimal (action, attention)-pairs, is convex in γ_i. Note that in the absence of attention utility ($\lambda = 0$) it is only linear. Third, when the environment is separable and $\lambda > 0$, then increasing attention to dimension i increases both the payoff V_i but also the weight on dimension i; hence, accounting for attention utility makes the DM's objective more convex in attention. The proposition below provides the formal statements.

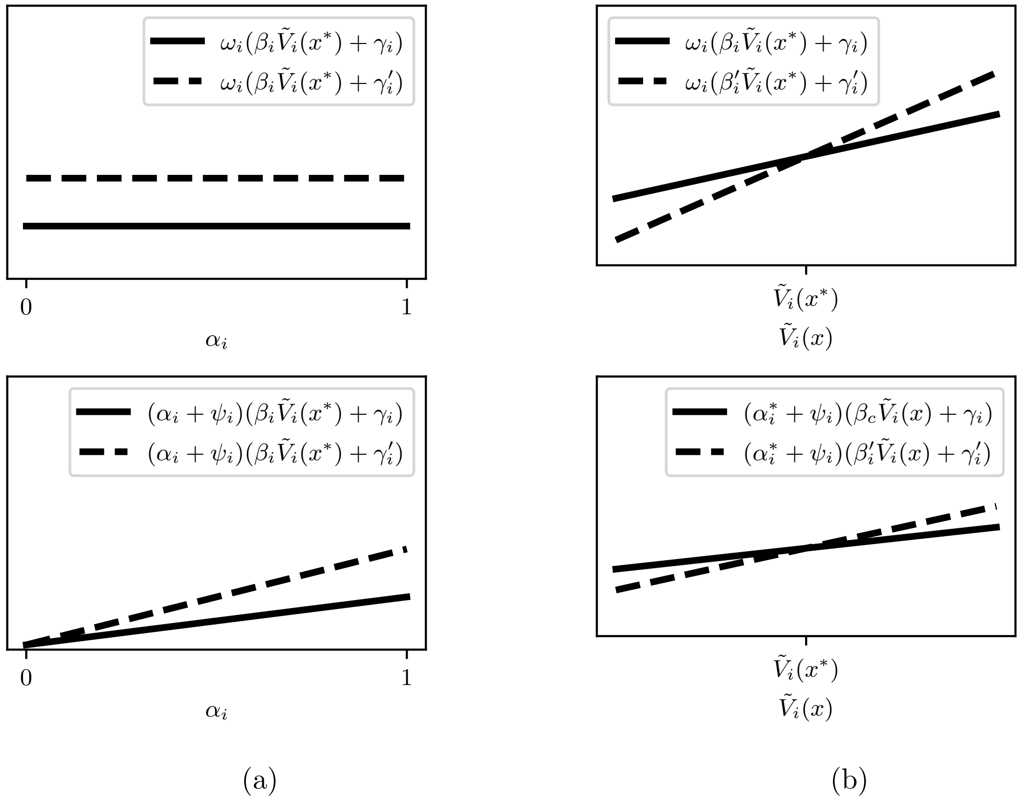


Figure 1: Panel (a) corresponds to an increase of γ_i to γ_i'. We hold the optimal action x^* fixed. The payoff V_i is shifted up and hence so is the material utility (top figure), independently of α_i. However, increasing α_i now increases the attention utility now by more than before (bottom figure). Panel (b) corresponds to an increase of β_i to β_i' with an offsetting change of γ_i to γ_i'. We hold the optimal attention α_i^* fixed. The material utility pivots around $\tilde{V}_i(x)$ (top figure) as does attention utility.

Proposition 2. *Consider a change of parameter λ to λ' with $\lambda' > \lambda$ and let x and x' denote the optimal action, respectively. We have:*

- $\sum_i \omega_i V_i(x) \geq \sum_i \omega_i V_i(x')$;
- *the DM's value is convex in $(\gamma_i)_{i\in\mathcal{D}}$;*
- *if the environment is separable, $\omega_i = 1$ and $\psi_i = 0$ for all $i \in \mathcal{D}$, and the objective given λ is convex in α, then it is also convex in α given λ'.*

The second and third part of the proposition imply that the DM has a preference for "extreme" payoffs so that they can hone in on high-payoff dimensions and ignore others. When the environment is separable, variation in payoffs is generated through extreme attention allocations and so the DM's attention is naturally "sparse" (Gabaix, 2014). One way of interpreting these results is that the DM has a preference for specialization—it is better to be outstanding in one area and relatively poor in many others, rather than doing mediocre in all. As we show, these results also lead to an additional preference for risk (Section 2.4), non-smooth consumption paths (Section 3.1), and intrinsic preferences for information (Section 3.3).

Lastly, we note the effects of ω_i, the weight on V_i in the material utility, and ψ_i, the involuntary focus on or future attention devoted to dimension i. Note that both ω_i and ψ_i play a similar role as β_i in Proposition 1. Thus, the following proposition follows straightforwardly and a formal proof is omitted.

Proposition 3. *Consider dimension $i \in \mathcal{D}$. Fix V_{-i} and consider changing parameters (ω_i, ψ_i) to (ω_i', ψ_i'), with $(\omega_i', \psi_i') \geq (\omega_i, \psi_i)$ elementwise. Denote the optimal (action, attention)-pairs for each parameter set as (x, α) and (x', α'), respectively. Then $V_i(x') \geq V_i(x)$. If, in addition, the environment is separable, then $\alpha_i' \geq \alpha_i$.*

If, e.g., ω_i represents the probability with which dimension i realizes, then the comparative static is again entirely standard. Less standard is that the DM increases the payoff of a dimension if there is some exogenous focus on that dimension.

We explore the implication of Proposition 3 and the other general comparative statics in more specific contexts next.

2.3 Attention across consumption dimensions

The first type of dimensions we consider are different dimensions of consumption. Those may be 'arranging a retirement home for a relative', 'vacation', 'personal health', 'financial situation', etc., each associated with some payoff and the DM's material utility is simply their sum (i.e., $\omega_i = 1$). In this context, Proposition 1 rationalizes the well-known ostrich phenomenon: (attentional) avoidance of low-payoff situations, and conversely excessive attention to high-payoff ones.[10] Evidence for such behavior has been extensively documented in the domains of finance and health.

For instance, retail investors' propensity to check their portfolios generally comoves with the market (Karlsson et al., 2009; Sicherman et al., 2015).[11] Such behavior is consistent with our model. Accessing one's portfolio requires attention to it. Our model suggests that doing so co-varies with the payoff associated with the portfolio. Reasonably, this payoff may be eventual consumption. A down market (for most investors) implies low future consumption, and by decreasing attention to their portfolio, investors can improve their attention utility.[12] Similar behavior has been documented in the domain of health.[13] Inattention to potential health issues to

[10]The term "ostrich effect" was coined in Galai and Sade (2006), where it describes individuals avoiding risky financial situations by pretending they do not exist—i.e., they bury their figurative heads in the sand like an ostrich. We use the term ostrich phenomenon to refer to attentional patterns due to changes in payoff levels, and not information. Some readers may be interested to know that, although the idea of ostriches burying their heads in the sand to avoid predators dates back to Roman times, they do not display this behavior. Instead, they put their heads into their nests (which are built on the ground) to check temperatures and rotate eggs.

[11]Gherzi et al. (2014) find increased monitoring following market downturns.

[12]While the propensity to check one's portfolio comoves with the market in both levels and changes, individuals may be avoiding payoffs that are low relative to some reference point (and not absolutely). Our model can be enriched to capture such behavior by supposing that attention utility is proportional to consumption payoffs relative to some reference point.

[13]For instance, researchers have noted low rates of testing for serious medical illnesses (Huntington's disease (Shouldson and Young, 2011; Oster et al., 2013); sexually transmitted diseases (Ganguly and Tasoff, 2017)). Our model predicts that an individual at risk of such a disease may have a low (expected) payoff related to the consumption problem 'health' and hence avoids any actions, such as taking a test, that require attention to it. Indeed, Ganguly and Tasoff (2017) document that the demand for medical testing for sexually transmitted diseases is decreasing as the expected health

manage one's emotions leads to worse material utility—the health outcomes—in line with Proposition 2.[14]

In principle, several factors can explain these attention patterns. For instance, the instrumental value of attention (e.g., via information) may vary with the market in a way that makes increased monitoring in up markets optimal. Similarly, variations pecuniary or cognitive (but non-emotional) costs of attention or the outside option of attention can also play a role. The variations in attention could also be explained by belief-based utility models, where utility is derived from anticipation (Caplin and Leahy, 2001; Brunnermeier and Parker, 2005) or from 'news' (Kőszegi and Rabin, 2009; Karlsson et al., 2009).

However, the ostrich phenomenon has also been documented in settings where all of these explanations are implausible suggesting an important role of direct emotional utility derived from attention. For instance, Avoyan and Schotter (2020) provide evidence in a stylized laboratory environment where experimental participants choose to allocate time (i.e., attention) between two games (i.e., consumption dimensions). In line with our model, they find that "as payoffs in a given game increase, subjects plan more attention to the game." Variations in non-emotional benefits or costs of attention are ruled out by design and belief-based utility models also seem implausible.[15]

outcome worsens. Individuals also often fail to follow medical recommendations, both with respect to information-generating activities (e.g., self-screening) but also with non-information-generating activities, such as taking medicine (Becker and Mainman, 1975; Sherbourne et al., 1992; Lindberg and Wellisch, 2001; DiMattero et al., 2007). For instance, DiMattero et al. (2007) find that, among individuals experiencing serious medical conditions, individuals with worse health status tend to adhere less to medical regimes.

[14]Low compliance with medical recommendations may negate the benefits from medical therapies or limited medical testing, e.g., in the context of Hunting's disease leads to worse decision about "childbearing, retirement, education, participation in clinical research" (Oster et al., 2013).

[15]For the results to be explained by belief-based utility models, the participants would need to consider the random payoffs from the game as separate and prefer the payoff variations induced by devoting more attention to the high-payoff game.

There is also strong evidence for an important role of attention utility in the context of retail investors. Sicherman et al. (2015) find a positive correlation between market returns and the frequency of investors logging in to their portfolio twice during a single weekend—when markets are closed and no new information can be revealed. Quispe-Torreblanca et al. (2020) find that individuals devote excessive attention to positive information that is already known.[16] The absence of new information from attention renders belief-based utility models mute and variations in non-emotional costs and benefits seem unlikely.[17]

2.4 Attention across states

We next consider attention allocation across possible realizations of an uncertain state. The attention-dependent weights on different states lead to as-if belief distortions (characterized by Propositions 1–3) and with them implications for the DM's attitude towards risk and probability weighting.

A state i is weighted in the DM's material utility by $\omega_i = p_i$, where p_i denotes the objective probability of state i occurring. For simplicity, we suppose that $\psi_i = 0$, i.e., there is no exogenous emotional utility flow outside the DM's control. The DM's objective is then to choose (x, α) with $x \in X(\alpha)$ to maximize $\sum_i p_i V_i(x) + \lambda \sum_i \alpha_i V_i(x)$, i.e., the expected material utility plus attention utility.

It is useful to divide the DM's objective by $1 + \lambda$ and denote the terms in front of V_i as $q_i := \frac{p_i + \lambda\alpha_i}{1+\lambda}$. Note that $q_i \in [0, 1]$ for all i and $\sum_i q_i = 1$, i.e., q_i describes a

[16] In a related setting, Olafsson and Pagel (2017) study individuals' attention to their financial accounts and finds increased attention after they are paid and decreased attention when the account balance becomes low, in particular, when it turns negative. Arguably individuals often know about their payment dates and amounts and their overdrawn status, so information avoidance may be an implausible motive.

[17] We note that the model in Karlsson et al. (2009) also has a direct emotional effect of attention what they call "impact effect." Even without further uncertainty after the initial news, this impact effect can still lead to the observed attentional patterns (in their model, set $\theta = 0$ and $SD(r_d) = 0$). But the impact effect is crucial, and not a feature of news utility models per se.

probability distribution. The DM, conditional on their attention allocation, behaves like a subjective expected payoff maximizer who assigns probability q_i to state i. Note that as attention to state i increases, so does the subjective probability q_i assigned to that state. Thus, Propositions 1–3 have direct implications for q_i.

Proposition 1 implies that individuals devote more attention to high-payoff states and thus (at least in a separable environment) will take actions suited for those states relative to those with a low payoff—an ostrich phenomenon in environments with uncertainty. For instance, in the context of individuals devoting attention across future contingencies, they will know what to do with a financial windfall (as they have contemplated such contingency) but not which expenses to cut when they are laid off (as this has been ignored). Proposition 2 indicates that these plans lead to a lower material utility. Day dreaming, individuals devoting attention to a state with a very low probability or zero probability, is an example.[18]

Proposition 3 implies a standard intuition: increasing the objective probability of a state p_i leads to an action better suited for that state.[19]

The additional structure imposed by dimensions as states allows us to study further implications of our model. We begin by considering the DM's preference over lotteries (or acts) and suppose that there is no instrumental value of attention. Thus, let X be the set of available lotteries (that is independent of attention). Lottery $x \in X$ has monetary payoff x_i in state i. The DM is equipped with an increasing Bernoulli utility u and hence, the payoff in state i given lottery x is $V_i(x) = u(x_i)$. For parts of the following proposition, we consider binary lotteries, those where any

[18]Anecdotally, individuals engage in "Zillow surfing," a form of escapism where they browse through home buying sites and imagine themselves in different houses, possibly much more expensive ones compared to their current accommodation.

[19]The one nuance in applying Proposition 3 is that there is a constraint on the set of probabilities: increasing the probability of one state means reducing the probability of another. In order for the result to hold it must be the case that the probability shift to i comes from a "trivial" state—one where attention has no material benefit.

state either pays a low payoff $L(x)$ or a high payoff $H(x)$. Also let $X(\mu, L)$ denote the set of binary lotteries with mean μ and low payoff L and $Y(\mu, H)$ those with mean μ and high payoff H.

Proposition 4.

- *Let DM(λ) refer to the DM given λ. DM(λ) is more risk-averse than DM(λ') for any $\lambda' > \lambda$.*[20]

- *Suppose u is unbounded and $\lambda > 0$. For any μ, L and $x \in X(\mu, L)$, there exists a lottery $\hat{x} \in X(\mu, L)$ so that if a lottery $x' \in X(\mu, L)$ has high payoff $H(x') > H(\hat{x})$ then the DM's prefers x' to x.*

- *For any μ, L and $x, x' \in X(\mu, L)$ with $H(x) > H(x')$, the DM prefers x to x' if λ is large enough.*

- *For any μ and H, the DM's preferences over $Y(\mu, H)$ are independent of λ.*

Proposition 4 first states that the DM has an additional preference for risk. Intuitively, given a lottery x, the DM devotes attention to the high-payoff states—the "upside" of the lottery—resulting in those states having a higher subjective probability q_i. The second and third cases of the proposition state that the DM has a preference for positively skewed lotteries.[21] Intuitively, positive skew always increases attention utility since the DM devotes their attention exclusively to high-payoff states. If the high payoff is large enough (second case) or the DM puts enough weight on attention utility (third case), the DM prefers the more positively skewed lottery. The fourth case states that no equivalent preferences exist for negatively skewed lotteries.

[20]Let δ_y be the lottery with monetary payoff y in each state. Given two preference relations on the set of lotteries, $\succeq$ and $\succeq'$, $\succeq$ is more risk averse than $\succeq'$ if $x \succeq \delta_y \implies x \succeq' \delta_y$ for all lotteries x and payoff y.

[21]The skew of a lottery is defined as its third standardized moment; fixing a low outcome and a mean for a set of binary lotteries, comparing the skewness of two lotteries is equivalent to comparing their high payoffs.

An implication of Proposition 4 is that individuals appear more optimistic than objective probabilities justify. Such optimism has been documented in a wide range of circumstances as Sharot (2011) summarizes: "we underrate our chances of getting divorced, being in a car accident, or suffering from cancer. We also expect to live longer than objective measures warrant, overestimate our success in the job market, and believe that our children will be especially talented." In our model, the DM devotes little attention to low-payoff states, thus acting as if they "underrate" them, and, conversely, overweights the high-payoff ones. There is laboratory evidence consistent with optimism, e.g., Mayraz (2011).[22] Participants guess the realization of a random variable and some are rewarded for high and others for low realizations. Our model predicts that participants whose payoff is high for high realizations devote attention to those realizations, which then have a large weight in their objective, leading them to guess a high realization. Indeed, this type of "wishful thinking" is what Mayraz (2011) finds.

There is also extensive evidence for preferences for positively skewed lotteries.[23] Furthermore, consistent with our model, Jullien and Salanié (2000); Snowberg and Wolfers (2010) suggest that the preference for skewness is driven by subjective probabilities, as in our model, rather than the Bernoulli utility u. On the other hand, individuals buy insurance for their house, car, and health, and hence pay a premium to avoid negatively skewed lotteries.

We next study how our model leads to probability weighting and consider the mapping of objective probabilities p_i to subjective probabilities q_i which we denote

[22] See also Mijović-Prelec and Prelec (2010); Engelmann et al. (2019); Orhun et al. (2021) for related evidence in both monetary and non-monetary domains.

[23] Evidence comes from a variety of contexts: portfolio choice (Blume and Friend, 1975), betting on horses (Golec and Tamarkin, 1998; Jullien and Salanié, 2000; Snowberg and Wolfers, 2010), individuals playing lotto (Garrett and Sobel, 1999; Forrest et al., 2002), as well as in various laboratory settings (Ebert and Wiesen, 2011; Grossman and Eckel, 2015; Ebert, 2015; Åstebro et al., 2015; Dertwinkel-Kalt and Köster, 2020).

$q_i(p_i)$. We focus on separable environments with only two states $\mathcal{D} = \{i, i'\}$.

We first suppose that at least $\underline{\alpha}$ attention must be devoted to each state and further attention is of no instrumental use. The DM devotes all residual attention to the high-payoff state. When no attention is forced ($\underline{\alpha} = 0$), the DM always overweights the high-payoff state as in Proposition 4. Otherwise, the probability weighting function is compressed, similar to an inverse S-shape.

We then drop the hard constraint on attention and replace it with large and decreasing initial returns to attention. A similar compression of the probability weighting occurs. Additionally, the DM devotes no attention to zero-probability states and so probability weighting is inverse S-shaped. The following proposition summarizes.

Proposition 5. *Suppose $\mathcal{D} = \{i, i'\}$ and that the environment is separable.*

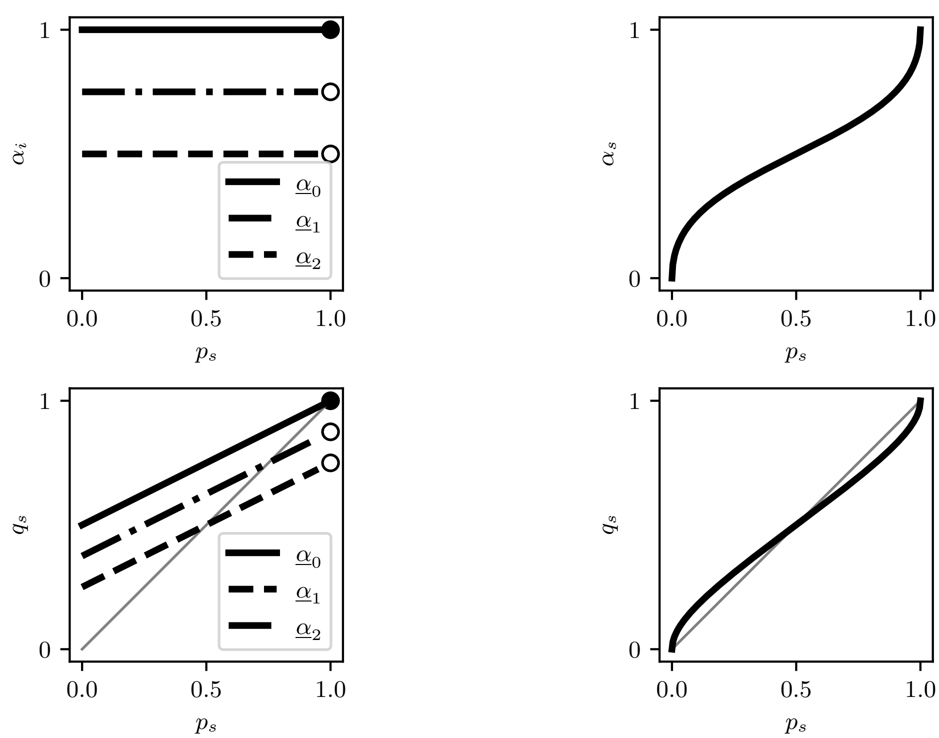
- *If $\hat{V}_i(\alpha_i) = -\infty$ for $\alpha_i < \underline{\alpha}$ and $\bar{V}$ otherwise, and $\hat{V}_{i'} = \hat{V}_i - \Delta$ with $\Delta > 0$, then $q_i(p_i) = \frac{p_i + \lambda(1-\underline{\alpha})}{1+\lambda}$ for $p_i < 1$ and $q_i(1) = 1$ and $q_{i'}(p_{i'}) = 1 - q_i(p_i)$.*
- *If $\hat{V}_i = \hat{V}_{i'} = \hat{V}$, $\hat{V}$ is continuously differentiable, $\lim_{a \to 0} a \frac{\partial}{\partial a}\hat{V}(a) = \infty$, and $\frac{\partial}{\partial a}\hat{V}(1) < \infty$, then, $q_i = q_{i'} = q$ and there exist some $\bar{p}$ with $0 < \bar{p} < 1/2$, such that*[24]

$$q(p) \begin{cases} = 0 & \text{if } p = 0 \\ > p & \text{if } 0 < p < \bar{p} \\ < p & \text{if } 1 - \bar{p} < p < 1 \\ = 1 & \text{if } p = 1. \end{cases}$$

Figure 2 provides an illustration of Proposition 5. In each panel, the top subfigure

[24] Although this result generates two classic features of inverse S-shaped probability weighting (underweighting of high probabilities and overweighting of low probabilities), the probability weighting need not be concave and then convex (as is often assumed). Intuitively, the instrumental value of attention needs to be small for high values of attention, i.e., $\hat{V}(1) - \hat{V}(1/2)$ small, to guarantee the inverse S-shape probability weighting everywhere.

shows optimal attention to state i as a function of the probability p_i, and the bottom subfigure shows the resulting probability weighting, $q_i(p_i)$. Panel (a) corresponds to the first case with forced attention, for three levels of $\underline{\alpha}$: $\underline{\alpha}_0, \underline{\alpha}_1, \underline{\alpha}_2$, with $\underline{\alpha}_0 < \underline{\alpha}_1 < \underline{\alpha}_2$. Panel (b) corresponds to the second case and we choose $\hat{V}(a) = -\frac{1}{a}$ as tractable functional form.[25]



(a) (b)

Figure 2: Panels (a) and (b) correspond to the two cases discussed in Proposition 5. The weight on attention utility is always $\lambda = 1$. For the first case, $\underline{\alpha}$ takes values $\underline{\alpha}_0 = 0, \underline{\alpha}_1 = 1/4$, and $\underline{\alpha}_2 = 1/2$. For the second case the payoff as a function of attention is given by $\hat{V}(a) = -\frac{1}{a}$.

Probability weighting has been extensively studied since first discussed in Kahneman (1979), and there is now a voluminous literature analyzing and empirically esti-

[25] One can easily show that the optimal attention is $\alpha_i = (p_i - \sqrt{p_i(1-p_i)})/(2p_i - 1)$ which is inverse S-shaped and hence so is q_i.

mating prospect-theory models which are typically descriptive in nature (see Wakker (2010); Barberis (2013) for two surveys). Our model provides attention as a mechanism leading to probability weighting. The classic finding is that individuals' probability weighting follows an inverse S-shape (Wu and Gonzalez, 1996)—which our model provides a microfoundation for under the conditions described in Proposition 5.[26] Our model can also provide conditions for other forms of probability weighting, e.g., an S-shaped probability weighting.[27]

3 Extension: Multi-period attention allocations

The previous section explored the implications of our model in static environments where attention is allocated once. In this section, we explicitly consider time as a type of dimension which naturally leads to repeated attentional choice. Doing so allows us to study attention allocation across time periods, which endogenizes time preferences, as well as implications of a future attention problem on actions today.

Because the DM cannot commit their future selves to a particular attention allocation, the solution to the DM's problem is the equilibrium of an intrapersonal game played between each period's self. As eluded to in the introduction, one can treat the results in Propositions 1–3 as defining the best response of the DM in a single period, given the choices of the DM in all other periods.

These intrapersonal games require us to make an assumption about what any given

[26]Our model has implications for probability weighting that distinguishes it from other models. For example, cumulative prospect theory (Tverseky and Kahneman, 1992) or rank dependent utility (Quiggin, 1982) predicts that the probability assigned to a state (q_i) depends only on the ranking of the states ($V_s > V_{i'}$ or $V_i < V_{i'}$) and the objective probabilities of each state occurring (p_i). In contrast, in our model, q_i additionally depends on the difference in payoffs, $V_i - V_{i'}$, as well as the instrumental value of attention.

[27]For instance, suppose that $\hat{V}_i = \hat{V}_{i'} = \hat{V}$ and that $\hat{V}$ is convex and not constant. The DM optimally devotes attention to the more likely state leading them to overweight high and underweight low probabilities.

time period's self believes about the strategies of other players. As a benchmark, we assume full sophistication. Thus, our results are driven solely by the role of attention in generating utility, rather than any kind of misprediction. Bronchetti et al. (2020) provides evidence of some albeit not full sophistication, while Falk and Zimmermann (2016)) provide evidence that individuals seem to alter their choices in anticipation of being able to redirect attention in the future.

3.1 Attention across time periods

We first focus on attention across time periods. The DM now faces a sequence of time periods $\mathcal{D} = \{1, \ldots, T\}$, with generic period t. For simplicity, we assume that there is no exogenous discounting, i.e., $\omega_t = 1$ for all t. However, the attention-dependent weights on the different periods (the dimensions) can be interpreted as endogenous time preferences.

In each period t, the DM chooses an (action, attention)-pair denoted by (x_t, α_t). The actions jointly determine the payoff in each period: given $x := (x_t)_{t=1}^T$, the payoff in period t is $V_t(x)$.[28] Attention is a measure on the set of time periods with total measure 1, i.e., $\alpha_t = (\alpha_{t \to t'})_{t' \in \mathcal{D}}$, where $\alpha_{t \to t'}$ denotes the attention (in period t) devoted to period t'; with $\alpha_{t \to t'} \geq 0$ and $\sum_{t'} \alpha_{t \to t'} = 1$; we also let $\alpha = (\alpha_t)_{t \in \mathcal{D}}$. We assume that the available actions in period t only depend on attention in period t, i.e., x_t must be in $X_t(\alpha_t)$ which is compact- and non-empty-valued and upper hemicontinuous.

In each period the DM receives a material utility and attention utility. We assume that the DM maximizes the sum of such utilities across periods and first consider the best response function in any period t holding fixed x_{-t}, α_{-t}: The DM chooses (x_t, α_t)

[28]One may make natural assumptions on future actions' impact on past payoff.

with $x_t \in X_t(\alpha_t)$ to maximize

$$\sum_{t'=t}^{T} \Bigg(\underbrace{V_t(x_t, x_{-t})}_{\text{material utility in } t'} + \underbrace{\lambda \sum_{t''=1}^{T} \alpha_{t' \to t''} V_{t''}(x_t, x_{-t})}_{\text{attention utility in } t'} \Bigg). \tag{3}$$

Notice that (3) can be written as (2) with $\psi_t = 0$ and $\psi_{t'} = \sum_{t''} \alpha_{t' \to t''}$ for $t' \neq t$ (and $\omega_{t'} = 1$ for all t').

From period t's perspective, the weight on period t' is given by $1 + \lambda(\alpha_{t \to t'} + \psi_{t'})$. These weights across periods t' can be interpreted as discounting: Fixing (x, α) the DM behaves like a standard DM (with $\lambda = 0$) who discounts period t' (relative to period t) by $\delta_{t \to t'} \coloneqq \frac{1+\lambda(\alpha_{t \to t'} + \psi_{t'})}{1+\lambda\alpha_{t \to t}}$. For instance, as attention to the present period increases, the DM discounts future periods by more. Time preferences—whether the DM is present or future-focused—are endogenous and depend on circumstances.

Propositions 1–3 provide predictions on time preferences. In particular, Proposition 1 suggests that the DM may weight a period more, if its payoff level or the instrumental value of attention to that period increase. Magnitude dependent discounting is a well known empirical regularity (e.g., Green et al. (1997) is an early paper), although it has not been directly linked to attention. Proposition 3 implies that some (exogenous) flow emotional utility derived from period t' leads the DM to increase the payoff in period t'.

Holding (action, attention)-pairs in other periods fixed and looking at the DM's best response provides potentially useful intuition. Actual attentional choices are the result of an intrapersonal game where the DM predicts their optimal future behavior and how it depends on actions today. The set of solutions is found via backward induction.[29] Propositions 1–3 cease to hold due to coordination motives in the DM's

[29]Formally, let $\mathcal{H}_t \coloneqq (x_{t'}, \alpha_{t'})_{t'=1}^{t-1}$ denote the (action, attention)-pairs the DM chose up to (and excluding) period t. Let $\Gamma_t(\mathcal{H}_t)$ denote the set of credible (x, α) when the DM has chosen $\mathcal{H}_t$ so

problem. Example 4 in Appendix A.2 provides an example where increasing a future payoff leads to less attention to that period; Example 5 shows that varying λ can affect the material utility non-monotonically.

We next show how attention utility affects consumption paths. For tractability, we consider an environment where the payoff in a period can be written as the sum of attention devoted to that period; formally: suppose that x_t takes the form $x_t = (x_{t\to t'})_{t'=1}^{T}$, let $X(\alpha) = \{x : x_{t\to t'} \leq \alpha_{t\to t'} \forall t, t'\}$, and suppose $V_t(x) = V(\sum_{t'=1}^{t} x_{t'\to t})$ for some increasing and concave function V. Concavity of V implies that the standard DM (with $\lambda = 0$) prefers smooth payoffs across time periods. A DM who values attention utility, however, may prefer variations in payoffs across time in order to increase their attention utility.

To simplify the statement of the following proposition we also assume that V is satiated at exactly K, i.e., $V(K) = V(K')$ for all $K' \geq K$ and $V(K) > V(K')$ for all $K' < K$, and suppose K is a divisor of T.[30]

Proposition 6. *There exist $\underline{\lambda} > 0$ and $\bar{\lambda} < \infty$, such that*

- *if $\lambda < \underline{\lambda}$, in each period t, $\alpha_{t\to t} = 1$; and*
- *if $\lambda > \bar{\lambda}$, in each period t, $\alpha_{t\to K(t)} = 1$ where $K(t) \equiv \lceil \frac{t}{K} \rceil K$.*

When λ is small, the DM maximizes the material utility: Here, due to the concavity of V, this is achieved by devoting full attention to the present period t (in each period) resulting in a smooth consumption path. When λ is large, the DM allocates

far and now chooses (x_t, α_t), where credibility requires that the DM in each future period chooses their corresponding (action, attention)-pair optimally. Specifically, for $t < T$, $\Gamma_t(\mathcal{H}_t)$ is recursively defined as argmax of (3) over (x, α), with $(x, \alpha) \in \Gamma_{t+1}(\mathcal{H}_t, (x_t, \alpha_t))$ and $x \in X(\alpha)$; and $\Gamma_T(\mathcal{H}_T)$ as the argmax of (3) over (x, α), with $(x, \alpha) \in \{\mathcal{H}_T, (x_T, \alpha_T)\}$ and $x \in X(\alpha)$.

[30]If V was not strictly concave, then $\underline{\lambda}$ in the ensuing proposition could be 0. If K is not a divisor of T, then when $\lambda > \bar{\lambda}$ (again defined in the proposition) the last payoff in period T would be less than those in other high-payoff periods.

attention to create periods with particularly high payoffs which are subsequently exploited for attention utility. In between those periods, the payoff is low; however, the DM never devotes attention to these low-payoff periods and so their attention utility is unaffected. Our model thus rationalizes "memorable consumption" such as weddings, vacations, and other lavish celebrations (Gilboa et al., 2016; Hai et al., 2020). Hai et al. (2020) notes that the average expenditure on weddings is about USD 20,000 and that the average annual household income of a newly married couple is USD 55,000.

3.2 Implications for actions

We next explore today's choice of a default action for a future consumption dimension. After the choice of default, some payoff-relevant uncertainty is resolved. Anticipating their future attention, the DM chooses a default that is optimal for when they are inattentive and it binds. When those are the low-payoff states, the default is chosen pessimistically. However, when the default is impure and affects the future payoff irreversibly, the DM may act optimistically.

There are two periods—period 1 and period 2. The setup in period 2 is that of Section 2.3 with two consumption dimensions, $\mathcal{D} = \{c_{nt}, c_t\}$. The payoff from dimension c_{nt} (the non-trivial dimension) is parameterized by some state $s \in \mathcal{S}$, where $\mathcal{S}$ is finite, and the DM's default action x_1 chosen in the first period. c_t is trivial and its payoff V_{c_t} is constant.

Changing the action in period 2 from the default requires at least η attention. Formally, let $\alpha_2 := (\alpha_{2\to c_{nt}}, \alpha_{2\to c_t})$ denote the attention in period 2. For any state s, if $\alpha_{2\to c_{nt}} < \eta$, then $V_{c_{nt}}(x_1, x_2|s)$ (with $x_2 \in X_2(\alpha_2)$) is independent of x_2 (i.e., the default binds), and if $\alpha_{2\to c_{nt}} \geq \eta$, then $\max_{x_2 \in X_2(\alpha_2)} V_{c_{nt}}(x_1, x_2|s)$ is independent of x_1

for all x_1 (i.e., the default is irrelevant).

In period 2, the DM chooses (x_2, α_2) with $x_2 \in X_2(\alpha_2)$ to maximize

$$\underbrace{V_{c_{nt}}(x_1, x_2|s) + V_{c_t}}_{\text{material utility}} + \lambda(\underbrace{\alpha_{2\to c_{nt}} V_{c_{nt}}(x_1, x_2|s) + \alpha_{2\to c_t} V_{c_t}}_{\text{attention utility}}). \tag{4}$$

We let $U_2(x_1, s)$ denote the maximized value of (4) and the corresponding action and attention by $x_2(x_1, s)$ and $\alpha_2(x_1, s)$, respectively, where we suppose that the solution is unique to simplify notation.

Define $\underline{\mathcal{S}}$ as those states in which the DM is inattentive to the non-trivial dimension, i.e., $\underline{\mathcal{S}} := \{s : \alpha_{2\to c_{nt}}(x_1, s) < \eta\}$. If $x_1 \in \arg\max_{x_1 \in X_1} \sum_{s\in\mathcal{S}} p_s U_2(x_1, s)$ then $x_1 \in \arg\max_{x_1\in X_1} \sum_{s\in\underline{\mathcal{S}}} p_s U_2(x_1, s)$ (taking $\underline{\mathcal{S}}$ as fixed). In words, the optimal default for period-2 conditions only on states where it binds.

In period 1 when the default is chosen, the DM also values their current attention utility. We assume that they can devote attention to across the realizations of future consumption dimensions, but that this attention is non-instrumental (i.e., the set of available default actions $X_1(\alpha_1)$ is independent of attention α_1). Let $\alpha_{1\to(c_{nt},s)}$ denote the attention in period 1 devoted to the non-trivial consumption dimension in state s, and $\alpha_{1\to c_t}$ that to the trivial dimension. The DM's period-1 attention utility is $\sum_{s\in\mathcal{S}} \alpha_{1\to(c_{nt},s)} V_{c_{nt}}(x_1, x_2(x_1, s)|s) + \alpha_{1\to c_1} V_{c_1}$.

In period 1, the DM's objective is the sum period-1 attention utility and the expected period-2 utility, where the former has relative weight λ. However, the DM optimally devotes all period-1 attention to the highest payoff state, or the trivial consumption problem, whichever has a higher payoff, and so their period-1 attention utility is independent of the default chosen. The following proposition summarizes and characterizes the states when the default binds as the weight on attention utility increases.

Proposition 7. *Let* $\underset{\sim}{\mathcal{S}}(x_1) \coloneqq \{s : \alpha_{2\to c_{nt}}(x_1, s) < \eta\}$.

- *Suppose* $X_1(\alpha_1)$ *is independent of* α_1. *Then the optimal default action* x_1 *satisfies*

$$x_1 = \arg\max_{x_1' \in X_1} \sum_{s \in \underset{\sim}{\mathcal{S}}(x_1)} p_s V_{c_{nt}}(x_1', x_2(x_1', s)|s).$$

- *For any* x_1, *if* λ *is large enough, then*

$$\underset{\sim}{\mathcal{S}}(x_1) = \{s : \max_{(x_2,\alpha_2), x_2 \in X_2(\alpha_2)} V_{c_{nt}}(x_1, x_2|s) < V_t\}.$$

The choice of default is in part similar to how a default is chosen in models of, e.g., rational inattention. If changing the default is costly, the DM may keep it and the optimal default conditions on states where it binds. A key difference in our model is that the default is chosen asymmetrically as the second part of the proposition states. It identifies set $\underset{\sim}{\mathcal{S}}(x_1)$ for large λ as those states s for which the max $V_{c_{nt}}$ given s is strictly less than the payoff from the trivial dimension. In other words, the DM devotes attention to the non-trivial dimension if and only if it has a high payoff.

Our model thus predicts asymmetric reactions to changes in the environment. For instance, individuals may adjust their planned consumption only in response to positive income shocks ignoring negative ones. In anticipation, they plan for low consumption (the default) to insure against their inattention. Similarly, individuals may not want to revisit previously negotiated business or wage contracts in response to some bad events occurring. This may add to firm inertia in economic downturns.[31]

We lastly note that Proposition 7 relies on the default being a "pure default." We maintain the setting as described above with one modification: the payoff of

[31]More generally, our model suggests individuals fall back on heuristics and decision processes that can be implemented automatically ("System 1") when the situation at hand has a low payoff, whereas they engage with the situation and use System 2 when its payoff is high.

dimension c_{nt} is now given by $V_{c_{nt}}(x_1, x_2|s) = \tilde{V}_{c_{nt}}(x_1, x_2|s) + \beta F(x_1)$, where $\tilde{V}_{c_{nt}}$ has the "default property" from above, and $\beta \geq 0$. Thus, the previous setting is nested with $\beta = 0$.

Proposition 8. *Suppose $\beta > 0$ and that $X_1(\alpha_1)$ is independent of α_1 and finite. Let $(s, x_1) = \arg\max_{s' \in \mathcal{S}, x_1' \in X_1} V_{c_{nt}}(x_1', x_2(x_1', s')|s')$. If $V_{c_{nt}}(x_1, x_2(x_1, s)|s) > V_{c_t}$ and λ is large enough, then $x_1 = \arg\max_{x_1' \in X_1} F(x_1')$.*

For a large weight on attention utility λ, the DM's utility is primarily determined by their attention utility. Thus, they choose x_1 to maximize the payoff in the high-payoff states they devote attention to. For $\beta > 0$, this is done by maximizing F (Proposition 8). In contrast, when $\beta = 0$, x_1 does not affect the high-payoffs since the default does not bind. In this case, the DM chooses x_1 to maximize their material utility and chooses it optimally for the low-payoff states (Proposition 7). As the weight on attention utility increases, the DM As the weight on attention utility increases, the DM chooses an action that does well in the states they devote attention to. With $\beta > 0$, this is done by maximizing F. When $\beta = 0$, the payoffs are independent of the DM's action and so they choose x_1 to maximize the material utility.

3.3 Implications for information acquisition

Our model leads to intrinsic preferences over information as we explore next. We consider a setting in which there is no instrumental value of information. Although the standard DM (with $\lambda = 0$) is indifferent to information, our DM has strict preferences over whether and when they acquire information and what type. As we show, our model can rationalize many preferences relating to information observed in laboratory settings (Masatlioglu et al., 2017; Nielsen, 2020; Möbius et al., 2022).

There are again two periods, with two consumption dimensions, $\mathcal{D} = \{c_{nt}, c_t\}$ in period 2. The payoff from the trivial dimensions V_{c_t} is constant; the payoff from the non-trivial dimension is parameterized by the DM belief p, $V_{c_{nt}}(p) = pV_H + (1-p)V_L$, where $V_L < V_{c_t} < V_H$. The DM's belief in period t is denoted by p_t.

Attention is allocated across the two consumption dimensions. It also leads to information acquisition when devoted to the non-trivial dimension. Specifically, the DM's period-1 action x_1 is a distribution of posterior and we assume that more attention allows for more informative posteriors: $X_1(\alpha_{1\to c_{nt}}) = \{x_1 \in \Delta([0,1]) : \mathrm{Var}(x_1) \leq \beta\alpha_{1\to c_{nt}}, E[x_1] = p_1\}$. Parameter $\beta \geq 0$ governs how much information can be acquired. Period-2 attention may also lead to information, although optimal attention will not depend on it.

In period 2, the DM chooses $\alpha_2 \coloneqq (\alpha_{2\to c_{nt},c_t})$ to maximize

$$\underbrace{V_{c_{nt}}(p_2) + V_{c_t}}_{\text{material utility}} + \underbrace{\alpha 2 \to c_{nt} V_{c_{nt}} + \alpha 2 \to c_t V_{c_t}}_{\text{attention utility}}. \tag{5}$$

The DM devotes devotes full attention whichever dimension has the higher (expected) payoff, and so their period-2 attention utility is $\max\{V_{c_{nt}}(p_2), V_{c_t}\}$. Since $V_{c_{nt}}(p_2)$ is linear in p_2, the DM attention utility and hence total utility in period 2 is convex in p_2.

In period 1, the DM maximizes period-2 utility and their current attention utility given by $\alpha_{1\to c_{nt}} V_{c_{nt}}(p_1) + \alpha_{1\to c_t} V_{c_n}$. The following proposition characterizes the DM's optimal attention and information acquisition.

Proposition 9. *Let* $\bar{p} \coloneqq \frac{v - v_L}{v_H - v_L}$.

1. *In period 2 the DM optimally chooses* $\alpha_{2\to c_{nt}} = 0$ *if* $p_2 < \bar{p}$ *and* $\alpha_{2\to c_{nt}} = 1$ *otherwise.*

2. *There exists* $\tilde{p} \leq \bar{p}$ *such that in period 1 the DM optimally chooses* $\alpha_{1\to c_{nt}} = 0$

if $p_1 < \tilde{p}$, $\alpha_{1\to c_{nt}} > 0$ *if* $\tilde{p} \leq p_1 < \bar{p}$, *and* $\alpha_{1\to c_{nt}} = 1$ *otherwise. Furthermore,* $\tilde{p}$ *is decreasing in* β*.*

3. *For any* p_1*, if* $\alpha_{c_{nt}}\beta$ *is small enough, then if* $p_1 > \bar{p}$*,* x_1 *is negatively skewed, and if* $p_1 \in (\tilde{p}, \bar{p})$*,* x_1 *is positively skewed.*

The proposition highlights three preferences. First, the DM devotes attention and acquires information when the payoff is likely to be high (first case). This is an example of the ostrich phenomenon discussed in Section 2.3. Möbius et al. (2022) provide experimental evidence where participants' beliefs about a payoff-relevant state are exogenous varied. High beliefs increase the willingness-to-pay to learn about the state, as our model rationalizes.[32]

Second, the DM has a preference for early resolution of uncertainty. The available information in period 1, β, increases the DM utility as it relaxes a constraint. The DM's utility is independent of how much information is available in period 2 (it is not formally modeled for this reason). Thus, the DM prefers the same information to be received early. The reason is that it allows the DM to condition their period-2 attention on the information received which makes makes the period-2 utility convex in beliefs. It is this additional benefit of information that causes the DM to acquire more information early on holding their belief fixed ($\tilde{p} \leq \bar{p}$; second case). Experimental evidence has been consistent with a preference for early resolution (Masatlioglu et al., 2017; Nielsen, 2020).

Third, the DM prefers positively skewed information when their belief is low and negatively skewed information otherwise (third case). As explained before, early information is useful because it allows the DM to condition future attention on its

[32]An implication is that the DM is better informed about good states; if the initial news is good, they continue acquiring information and may become more certain of the good state, whereas they stop learning and remain pessimistic, but uncertain, about the state when the initial news is bad.

realization. But this requires sufficient movement in the DM's belief. When the prior is low, this means that good news need to be unlikely in order to change the DM's attention (and similarly for a high prior). The restriction to small amount of information acquisition is needed as, e.g., full information mechanically implies positive skew if $p_1 \leq \frac{1}{2}$ and negative skew otherwise.

4 Implications for (self-imposed) policies

In this section we study how a policymaker—a second party, say, a government or the DM themselves—should intervene to improve payoffs. Incorporating our model into policymaking is necessary to fully understand the behavioral changes a policy induces—which may be different from those if the DM was standard ($\lambda = 0$)—and its implications for the DM's overall utility. We consider three broad classes of policies: optimal resource allocation, incentivization of actions, and optimal construing of dimensions. For simplicity, we set $\omega_i = 1$ and $\psi_i = 0$.

4.1 Optimal resource allocation

Suppose first that the policymaker can increase the payoffs from the different dimensions by some total amount. Dimension i has payoff $V_i + \gamma_i$ after transfer $\gamma_i \geq 0$ to it, with total resource constraint $\sum_i \gamma_i \leq \gamma$. What is the optimal way—in terms of total utility—to increase the payoffs?

To a standard DM (with $\lambda = 0$), the choice of γ_i's does not matter. Their utility always increases by γ. In general, however, an increase in the payoff from dimension i is weighted by $1 + \lambda\alpha_i$. It follows that the policymaker should increase the payoffs of dimensions that receive the most attention. Intuitively, increasing the payoff associated with a something that an individual ignores does little to that individual's utility.

Suppose next that the policymaker can also devote attention similar to the DM in order to increase payoffs. The policymaker devotes attention r_i to dimension i and the payoff in dimension i is $\hat{V}_i(\alpha_i + r_i)$, i.e., the environment is separable and the two kinds of attention are perfect substitutes. For example, α_i and r_i could represent amounts of information, where the DM acquires the former and the latter is provided by the policymaker.

Suppose that all payoffs are continuously differentiable in attention. By the Envelope theorem, the DM's total utility differentiated with respect to r_i is

$$\frac{\partial}{\partial r_i}\hat{V}_i(\alpha_i + r_i)(1 + \lambda\alpha_i). \tag{6}$$

If the DM's optimal attention is interior, the first-order condition for α_i is given by $F(\alpha_i, r_i) \coloneqq \frac{\partial}{\partial \alpha_i}\hat{V}_i(\alpha_i + r_i)(1 + \lambda\alpha_i) + \lambda\hat{V}_i(\alpha_i + r_i) - \mu = 0$, where μ is the Lagrange multiplier on the constraint for the sum of attention. Using the fact that $\frac{\partial}{\partial r_i}\hat{V}_i(\alpha_i + r_i) = \frac{\partial}{\partial \alpha_i}\hat{V}_i(\alpha_i + r_i)$ and substituting it into (6) gives $-\lambda\hat{V}_i(\alpha_i + r_i) + \mu$.

The policymaker should then devote the marginal unit of attention to the dimension with the lowest payoff (if attention is interior) or to the problem that already receives full attention. At an optimum, the benefit from increasing attention equals its cost which for the DM decreases in the payoff. The policymaker does not bear this cost, and thus the benefit of their attention is largest for the problem with the highest cost, i.e., that with the lowest payoff. Interestingly, increasing r_i may not increase the payoff of dimension i due to an endogenous reduction in α_i.[33]

[33] Indeed, let $\alpha_i(r_i)$ denote the optimal level of the DM's attention as r_i varies. Implicitly differentiating the DM's first-order condition gives $\frac{\partial}{\partial r_i}\alpha_i(r_i) = -\frac{\frac{\partial}{\partial r_i}F(\alpha_i, r_i)}{\frac{\partial}{\partial \alpha_i}F(\alpha_i, r_i)} \leq -1$. Thus, the DM's attention to dimension i decreases by more than the increase in the policymaker's attention $r_i + \alpha_i$, and hence $\hat{V}_i$, decreases.

4.2 Providing incentives to induce better actions

We next consider two ways of inducing the DM to take better actions—increasing the rewards for "success" and increasing the penalty for "failure." For a standard DM (with $\lambda = 0$), their effects are similar, but when $\lambda > 0$, they may have very different consequences. A penalty decreases the expected payoff and can thus lead to lower attention and perversely worse actions.

Consider a separable environment. Attention α_i to dimension i, which may be interpreted as effort, leads to "success" with probability $p(\alpha_i)$, where p is increasing, continuously differentiable and bounded away from 0 and 1, and "failure" otherwise. The expected payoff of dimension i is $\hat{V}_i(\alpha_i) = p(\alpha_i)V_H + (1-p(\alpha_i))V_L$, with $V_H > V_L$.

The standard DM's ($\lambda = 0$) optimal attention is unchanged when V_H and V_L are increased by the same amount. They increase attention α_i in response to an increase in $V_H - V_L$, that is they respond by increasing attention to both "carrots" (an increase in V_H) and "sticks" (a decrease in V_L).

In contrast, when $\lambda > 0$, the DM increases α_i when both the low and high payoff are shifted up because this increases the expected payoff holding fixed the instrumental value of attention (see Proposition 1). They also increase attention in response to carrots: increasing V_H increases α_i. However, increasing the stick can decrease attention.

Proposition 10. *Consider the environment as introduced prior to this proposition and suppose the optimal α_i is unique.*

1. *Increasing V_H, V_L by the same amount increases α_i.*

2. *Increasing V_H increases α_i.*

3. *Decreasing V_L decreases α_i if $p(\alpha_i) + \alpha_i \frac{\partial}{\partial \alpha_i} p(\alpha_i) < 1$ everywhere and λ is large*

enough.

In the third part of Proposition 10, attention is not very effective in increasing p ($\frac{\partial}{\partial \alpha_i} p(\alpha_i)$ is low), and success is far from guaranteed ($p(\alpha_i)$ is also low). In these circumstances, the stick may induce the DM to shy away from dimension i so that they can decrease the weight of the associated payoff. An implication is that the DM may not demand commitment contracts that involve penalties (while those with rewards may be too expensive).

4.3 Optimal bracketing of dimensions

The last policy questions asks when different dimensions should be considered as separate and when they should be thought of as one dimension. For instance, the DM may be able to learn to associate one problem with another, either through some purely cognitive process or with the help of, say, physical cues that the policymaker installs. Such optimal bracketing serves as a microfoundation for the set of dimensions problems in Section 2.1, which can be viewed as the optimally bracketed set of smaller dimensions.

The setup is that of Section 2.1 where, in addition to choosing (x, α) with $x \in X(\alpha)$, the DM also chooses a bracketing $B \in \mathcal{P}(\mathcal{D})$. Let $B(i)$ be defined by $i \in B(i) \in B$. Whenever the DM devotes attention to i and all dimensions $i' \in B(i)$ "come to mind." As multiple dimensions come to mind, the DM's attention is diluted uniformly among them. Thus, given (x, α) and B, the DM utility is

$$\underbrace{\sum_i V_i(x)}_{\text{material utility}} + \lambda \underbrace{\sum_i \alpha_i \bar{V}_B(i)(x)}_{\text{attention utility}}, \tag{7}$$

where $\bar{V}_D(x) := \frac{\sum_{i \in D} V_i(x)}{|D|}$ for $D \subseteq \mathcal{D}$.

Note that the model in Section 2 is recovered when B consists of singleton sets and that a DM who uses one bracket, i.e., B is a singleton, is equivalent to the standard DM (with $\lambda = 0$).

Also let $\bar{\alpha}_D(x) := \frac{\sum_{i \in D} \alpha_i}{|D|}$ for $D \subseteq \mathcal{D}$.

Proposition 11. *Consider any (x, α) and B optimal given (x, α). Then $\bar{V}_D(X) > \bar{V}_{D'}(x)$ implies $\bar{\alpha}_D \geq \bar{\alpha}_{D'}$ for all $D, D' \in B$.*

In words, Proposition 11 states that the optimal the ordering of optimal brackets by attention is the same as ordering them by payoff. If this were not the case, the DM could combine a low-attention but high-payoff bracket with a high-attention but low-payoff bracket increasing their attention utility as the high payoffs take a larger weight.

5 Relation to existing models

In this section, we compare our model to related approaches.

Rational inattention: In models of rational inattention (Sims (2003); Mackowiak et al., 2022), attention serves an instrumental role as in ours. Additionally, attention is costly, e.g., because of deploying cognitive resources. While we do not model these costs explicitly other than via a total attention constraint, they can be captured in the functional form of the payoffs V_i. The key difference is thus that in our model, unlike in models of rational inattention, attention generates emotional utility, what we call attention utility.

Our model rationalizes behaviors that are at odds with rational inattention as monetary or cognitive (but non-emotional) costs do not seem sufficient in many important situations to justify individuals' behavior. For instance, genetic tests for

Hunting's disease cost no more than $300 (Oster et al., 2013). Furthermore, information avoidance (inattention) varies with the level of future payoffs (Karlsson et al. (2009); Sicherman et al. (2015) in the context of investors; and Ganguly and Tasoff (2017) in the context of health) with no (obvious) corresponding change in rational costs or benefits. Even more basic, there is no reason in models of rational inattention to devote attention to already known information (Quispe-Torreblanca et al., 2020). These examples suggest that there is a "cost" (or benefit) of attention missing from the consideration. In this project we model this cost (see also the discussion in Section 2.3).

Belief-based utility with Bayesian agents: There is by now an extensive literature in economics modeling agents who directly gain anticipatory utility from their (rational) beliefs (see Loewenstein (1987); Loewenstein and Elster (1992) for early contributions, and recent efforts of Caplin and Leahy (2001); Kőszegi (2010); Dillenberger and Raymond (2020)), or gain utility from changes in beliefs, or news utility (e.g., Kőszegi and Rabin (2009)). Broadly speaking both classes of models assume that some present utility may be generated via beliefs, or changes in beliefs, about future payoffs.

There are some similarities between models of anticipatory utility and our approach: In our model, the DM values material utility and attention utility. Attention utility, when stemming from a future problem, can be thought of as anticipatory utility. However, unlike in the aforementioned models, the DM only "receives" this anticipatory utility if they devote attention to its underlying payoff, not otherwise. The same applies to models where the DM receives gain utility from changes in their belief (e.g., Kőszegi and Rabin (2009)). There, the DM "receives" the gain utility regardless of their attention.

Just as models of rational inattention, models where attention is allocated to induce changes in anticipatory utility or gain utility (via information acquisition) rely on the presence of uncertainty. Such models thus also fail to make predictions in situations where information is unlikely to play a major role, such as in much of the evidence presented in Section 2.3.

Belief based utility with chosen beliefs: Our model, in Section 2.4, also relates to those where subjective beliefs are optimally chosen to increase anticipatory utility as in Bénabou and Tirole (2002); Brunnermeier and Parker (2005); Bracha and Brown (2012); Caplin and Leahy (2019)[34] While our model is, of course, conceptually very different (beliefs do not feature in Sections 2.3 and 3.1, and in Section 2.4, the DM chooses an attention allocation that leads to weights that we interpret a subjective belief), there are some similarities. Optimal attention and optimal beliefs are both determined by a tradeoff of "optimism" (here, devoting attention to high-payoff states) and the instrumental value of attention. Our model is not, however, observationally equivalent to models of chosen beliefs. For instance, the DM may, in fact, overweight a low-payoff state if states require some minimum amount of attention to ensure a good expected payoff (see Proposition 5 for an example).

Temporal discounting: There is a huge theoretical literature devoted to temporal discounting (see Frederick et al. (2002) for an overview). In our model, when attention is allocated across time, endogenous weights on periods appear, and the DM develops a preference for the timing of consumption. Our formulation is somewhat related to the ideas in Loewenstein (1987). There, as in our model, the DM may, e.g., negatively discount a high future payoff since it creates (high) anticipatory utility until it is realized.

[34]For a recent summary of the larger literature see Bénabou and Tirole (2016).

However, as for other models with anticipatory utility (see above), the weight of a future payoff in today's objective is fixed, in particular, independent of whether the DM devotes attention to it or not. It thus cannot capture our basic comparative static that discounting varies with the instrumental value of attention or the payoff level. One additional implication of this difference is that a non-smooth consumption path is generally not beneficial to a DM in Loewenstein (1987) whereas it is valued by ours since they ignore low-payoff periods and devote excessive attention to high-payoff ones.

Other Models of Attention:

A couple of other papers directly model the two fundamental features of attention in ways similar to ours. The model of Tasoff and Madarasz (2009) is perhaps closest. A DM faces a decision problem with multiple dimensions and receives anticipatory utility from each as a function of its payoff and the attention devoted to it. Attention to a dimension increases when its payoff changes because the DM chooses an action different from a default or receives payoff-relevant information. Similar to Proposition 1, the DM is more likely to take a non-default action or acquire information if the payoff is high. Such formulation is, in some sense, nested in ours: Let $x_d \in X(\alpha)$ for all α be a default action that is always available, and let acquiring information be encoded as some action x (providing payoff-relevant information for an underlying problem) and suppose x is only available for some attention allocations. A difference between the two formulations is that we allow for attention allocation with no instrumental consequence. More broadly, in our model, attention is chosen to enable non-default actions and information acquisition, whereas in theirs, the order is reversed.

Their subsequent focus is on how information provision (as requested by the DM or forced by an advertiser) can increase consumption, even when the DM learns their marginal payoff is less than what they expected (this follows from the increase in attention and hence the importance in the DM's objective; this intuition can be expressed in our framework as we show in Example 6 in Appendix A.3). Instead, our focus is on attention allocation across uncertain states and time and the behavioral phenomena our model rationalizes. We also consider dynamic extensions allowing us to explore additional questions such as preferences over dynamic information acquisition, and the implications for policymaking.

Another related model is that in Karlsson et al. (2009): The DM gains utility not from anticipatory emotions but rather as gain-loss utility from changes in expected future payoffs. Devoting attention to some initial news and acquiring further information increases the relative impact of gain-loss utility and speeds up the reference point adjustment. Under some conditions, the DM acquires additional information in response to positive initial news and not otherwise.

Our model is similar in that attention also increases the impact (or weight) of a payoff. We abstract away, however, from attention's effect on reference points and instead explicitly include actions whose availability depends on the attention allocation. We also construct our model more general, allowing us to consider different dimensions of attention allocation with different insights.

6 Conclusion

This paper has presented a model of attention allocation. Attention has two fundamental features: It helps the DM make better decisions, and it determines how payoffs are aggregated. We study our model in a variety of economic environments focusing on two key lessons. First, the DM may ignore a low-payoff situation (even if doing so is instrumentally harmful) to decrease its weight in their objective (and conversely devote excessive attention to high-payoff ones). Second, due attention reweighting the objective function, our model can lead to a variety of behavioral phenomena, where the exact form reflects the underlying economic environments.

Our model, of course, has limits in terms of what it can explain. There are situations where individuals choose to engage with negative emotion-generating activities with low instrumental value. For instance, the premise of our model seems at odds with pessimists who constantly focus on the negative aspects of any situation and overweight those, or the fact that many people doom-scroll and look at Twitter feeds that induce negative feelings. Our framework still allows us to study the attention-weighted decision environment and the ensuing behavioral phenomena, regardless of what model of attention allocation (e.g., negativity bias or salience) produces them. For instance, a present focus or distorted subjective probabilities result from excessive attention to the present or a particular state—regardless of whether that attention is directed as in our model or simply because the present or the state are salient.

Of course, our approach, which assumes that the entire stock of attention is under the control of the DM (i.e., the top down approach), is likely not entirely true. Involuntarily allocated attention, as is highlighted by recent models on attribute-based choice, e.g., Bordalo et al. (2013); Kőszegi and Szeidl (2013); Bushong et al. (2021), can also play important role. That said, as Desimone et al. (1995) points out, the "attentional system, however, would be of little use if it were entirely dominated by bottom-up biases." Thus, we believe our model is a useful first step in understanding how an individual may utilize the remaining stock of attention after bottom-up allocations have been made.

Like many other models of attention, our model also suffers from a recursion problem (see Lipman (1991) for an discussion infinite regress issues in economic models). We suppose that the DM fully understands all parameters of the model, and is able to conduct the optimization procedure of allocating attention and taking an action without reweighting the payoffs. Although such an approach is tractable, it does beg the question of how the implications of the model might be changed if even the act of optimization itself—during which the DM arguably devotes attention to different payoffs—reweights the payoffs. One can embed higher-level learning by the DM about the parameters of the model and consider a multi-period model with consumption payoffs only in the final period; but we do not provide formal results.

Our model also requires carefully specifying the environment: a key component is a way of partitioning the environment into sets of consumption problems, states, and time periods. In many real economic environments natural partitions exist. However, in many situations it may not be as obvious what the correct sets are. Although Section 4.3 provides some guidance given a finest possible partition, there are also likely situations the environment is determined differently.

Thankfully, the novel primitives of our model, the set of dimensions and λ, can be identified from the data. The details would vary by the environment, but here we provide the intuition for a situation where the dimensions are states. We first can identify whether two states are considered jointly (they are in the same "bracket," Section 4.3) by reducing the payoff of one problem and increasing the other the same amount and seeing whether the (action, attention)-pair changes. If we can find some shift such that it does, then the two states are not part of the same bracket. Then choices over lotteries allow us to identify the degree of overweighting of the high payoff state(s) and thus λ.

Our paper also focused on the DM's problem. In Section B we consider what happens in strategic interactions where many agents gain attention utility. In a setup similar to that of Brunnermeier and Parker (2005) Section III, ex-ante identical agents are placed in an endowment economy and in equilibrium choose to hold idiosyncratic risk. Such risk-taking is optimal since it allows agents to increase their attention utility, and possibly through agents taking opposing gambles so that which states are the high-payoff states differs by agent. Similarly, ex-ante identical agents would "trade consumption payoffs" to create payoffs that vary across problems and agent groups. Thus, in strategic settings, agents may sort into ex-post different groups and naturally some sort of polarization occurs.

There are other strategic implication: agents may fail to readjust strategies in dynamic games, or firms may change the framing of their product in order to be more appealing to emotionally inattentive consumers. To the extent that firms may be able to exogenously shift the attention of consumers, and even change what are the relevant dimensions, opens up a new way to view framing effects in product markets.

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A Addition examples

A.1 Examples of canonical problems

In Examples 1–3, we consider a separable environment and a particular problem $i \in \mathcal{D}$. "Actions" and "payoffs" shall refer to those in the now explicitly modeled dimension i.

Example 1. *Dimension i is the reduced form of a canonical choice problem with imperfect information and information acquisition (using the framework of Matějka and McKay (2015)).*

The DM chooses an action j from set $A = \{1, \ldots, N\}$. The state of nature is a vector $v \in \mathbb{R}^N$ where v_j is the payoff of action $j \in A$. When the DM's belief is $B \in \Delta(\mathbb{R}^N)$, they receive payoff $v(B) \coloneqq \max_{j \in A} E_B[v_j]$. The DM is initially endowed with some belief $G \in \Delta(\mathbb{R}^N)$. They can receive signals $s \in \mathbb{R}^N$ on the state: They choose $F(s, v) \in \mathcal{F}(\alpha_i) \subseteq \Delta(\mathbb{R}^{2N})$, where $\mathcal{F}$ is compact-valued, increasing and non-empty, and for all α_i and $F \in \mathcal{F}(\alpha_i)$, F the law of iterated expectations holds, $\int_s F(ds, v) = G(v)$ for all $v \in \mathbb{R}^N$

The DM's payoff from dimension i given α_i is then

$$\hat{V}_i(\alpha_i) \coloneqq \max_{F \in \mathcal{F}(\alpha_c)} \int_v \int_s v(F(\cdot|s))F(ds|v)G(dv).$$

For an example of a particular $\mathcal{F}$, suppose that the information structure is fully flexible subject to a capacity constraint; i.e., let $\bar{\mathcal{F}} \coloneqq \{F \in \Delta(\mathbb{R}^{2N}) : \int_s F(ds, v) = G(v)$ for all $v \in \mathbb{R}^N\}$ (set of posterior distribution satisfying law of iterated expectations) and

$$\mathcal{F}(\alpha_c) = \{F \in \bar{\mathcal{F}} : \kappa(H(G) - E_{\mathbf{s}}[H(F(\cdot|\mathbf{s}))) \leq \alpha_i]\},$$

for some $\kappa \geq 0$ and where $H(B)$ denotes the entropy of belief B.[35]

Example 2. *Dimension i is the reduced form of a canonical choice problem with trembles.*

The DM chooses an element j from set $A = \{1, \ldots, N\}$. The vector $v \in \mathbb{R}^N$ where v_j is the payoff of element $j \in A$ is known. The DM's choice is random, they "tremble": They choose $B \in \mathbb{F}(\alpha_i) \subseteq \Delta(A)$, where $\mathcal{F}$ is compact-valued, increasing and non-empty. The DM's payof from dimension i given α_i is then

$$\hat{V}_i(\alpha_i) \coloneqq E_B[v_j].$$

For an example of a particular $\mathcal{F}$, consider

$$\mathcal{F}(\alpha_i) = \{B \in \Delta(A) : \kappa(H(\mathcal{U}) - H(B)) \leq \alpha_i\},$$

for some $\kappa \geq 0$, where $H(B)$ denotes the entropy of belief B (see footnote 35 for the definition) and $\mathcal{U}$ the uniform distribution on A; i.e., if the DM devotes no attention, they will make each choice with equal probability.

Example 3. *The setup is as in Example 1; we interpret a particular $\mathcal{F}$ as corresponding to the DM accessing information from their memory as we describe next.*

We follow memory recall models as discussed in Kahana (2012). Endow the DM with memory $M \in \mathbb{R}^{KN}$ which is a set of $|M|$ signal realization from some $F_1(s, v) \in \Delta(\mathbb{R}^{2N})$ with $\int_s F_1(ds, v) = G(v)$ for all $v \in \mathbb{R}^N$. F_1 corresponds to the distribution of individual memories (a signal) the DM has made. Given α_i, the DM can make up to $\lfloor \alpha_i \frac{1}{\kappa} \rfloor$ uniform draws with replacement from M. With K draws, prob-

[35]When the distribution of states is discrete, $H(B) = -\sum_k p_k \log(p_k)$, where p_k is the probability of state k; and for distribution that has a probability density function f, entropy is $-\int_v f(v) \log(f(v)) dv$.

ability of L distinct draws is $P(L|K) \coloneqq \binom{|M|}{L}\left(\frac{L}{|M|}\right)^K$. *Define* $F_L(s_1,\ldots,s_L,v) \coloneqq \Pi_{l=1,\ldots,L} F_1(s_l|v)G(v)$ *as joint distribution of L distinct memories and the state.*

Finally, let $\mathcal{F}$ be

$$\mathcal{F}(\alpha_i) = \{\sum_{L=1}^{M} P(L|K)F_L : K \in \mathbb{N}, K \leq \lfloor \alpha_i \frac{1}{\kappa} \rfloor\}.$$

As the DM devotes more attention to i, they make more draws from their memory; a form of information acquisition.

A.2 Examples for Section 3.1

Example 4. *There are three time periods, $T = 3$. The payoffs in periods 1 and 2 are constant and equal and denoted by $\bar{V}$. The payoff in period 3 is either high $\bar{V}_3$ or low $\underline{V}_3$ depending on the action the DM chooses in period 1 and 2. In each period $t \in \{1,2\}$, the available actions are*

$$X_t(\alpha_t) = \begin{cases} \{\underline{x}\} & \text{if } < \eta_t \\ \{\underline{x}, x^* *\} & \text{if } \alpha_{t\to 3} \geq \eta_t, \end{cases}$$

in particular, taking the action x^* requires attention devoted to period 3. The payoff in period 3 is high if the DM takes action x^* in at least one period, otherwise, it is low. We also force $\alpha_{2\to 3} \geq \underline{\alpha}_2$, with $0 < \underline{\alpha}_2 < \eta_2$ (formally, this is modeled by assuming any payoff is negative infinity if the DM's attention differs).

Suppose the payoff in period 3 is lower than that in periods 1 and 2, i.e., $\underline{V}_3 < \bar{V}_3 < \bar{V}$. We construct an example where the DM in period 1 prefers action x^* to be taken in period 2 over it being taken in period 1 over it never being taken. Initially, however, the DM in period 2 would not take x^* including if the DM in period 1 did *not take it and so the DM takes x^* (and devotes attention to period 3) in period 1. As the payoff in period 3 increases, this changes: the DM in period 2 now takes x^* and so the DM in period 1 does not, and hence reduces their attention to period 3. Let us derive the conditions.*

In period 3, the DM devotes all their attention to $\bar{V}$ (from either other period) and takes a degenerate action. If the DM took action x^* in period 1, then in period 2, they choose $\alpha_{2\to 2} = 1 - \underline{\alpha}_2$ and $\alpha_{2\to 3} = \underline{\alpha}_2$. Otherwise, they take action x^* (and $\alpha_{2\to 2} = 1 - \eta_2$ and $\alpha_{2\to 3} = \eta_2$) over $\underline{x}$ (and $\alpha_{2\to 2} = 1 - \underline{\alpha}_2$ and $\alpha_{2\to 3} = \underline{\alpha}_2$) if

$$(1 + \lambda(1 - \eta_2))\bar{V} + (1 + \lambda\eta_2)\bar{V}_3 \geq (1 + \lambda(1 - \underline{\alpha}_2))\bar{V} + (1 + \lambda\underline{\alpha}_2)\underline{V}_3. \tag{8}$$

In period 1, the DM prefers to take action $\underline{x}$ (and $\alpha_{1\to 1} = 1$) and the DM in period 2 taking action x^* (with aforementioned attention) over taking action x^* (and $\alpha_{1\to 1} = 1-\eta_1$ and $\alpha_{1\to 3} = \eta_1$) and the DM in period 2 taking $\underline{x}$ (with aforementioned attention) if

$$(1+\lambda(1+(1-\eta_2))\bar{V}+(1+\lambda\eta_2)\bar{V}_3 \geq (1+\lambda((1-\eta_1)+1)\bar{V}+(1+\lambda\eta_1)\bar{V}_3 \iff \eta_1 \geq \eta_2. \tag{9}$$

Still in period 1, the DM prefers taking action x^* (with aforementioned attention and action in period 2) over always taking action $\underline{x}$ (with no attention to period 3 in period 1 and minimal in period 2) if

$$(1 + \lambda(1 - \eta_1))\bar{V} + (1 + \lambda(\eta_1 + \underline{\alpha}_2))\bar{V}_3 \geq (1 + \lambda)\bar{V} + (1 + \lambda\underline{\alpha}_2)\underline{V}_3. \tag{10}$$

Since $\underline{V}_3 < \bar{V}_3 < \bar{V}$, there exists $\lambda > 0$ such that (10) *holds with equality. For such λ, since $\underline{\alpha}_2 > 0$, there exists $\eta_2 < \eta_1$ (i.e.,* (9) *holds) so that* (8) *does not hold. Furthermore, λ can be slightly decreased so that* (10) *now holds strictly and* (8) *still* *does not hold. Thus, for these parameter values, the unique solution is for the DM to take action x^* in period 1 and hence devote $\eta_1 > 0$ to period 3.*

Now, increase both $\underline{V}_3$ and $\bar{V}_3$ by γ. If γ is large enough (but still $\bar{V}_3 + \gamma < \bar{V}$), then (8) *holds (and* (9) *and* (10) *remain to hold), so that the unique solution is for the DM to take action x^* in period 2 only, i.e., the DM reduces their attention to period 3 in period 1.*

A non-monotonicity of the attention devoted to period 3 as a function of β_3 (as in the parameterization used for the comparative statics) can be constructed similarly, but is omitted.

Example 5. *Take the setting of Example 4. The construction proceeds almost identically.*

Since $\underline{V}_3 < \bar{V}_3 < \bar{V}$, there exists $\lambda > 0$ such that (10) *holds with equality. For such λ, since $\underline{\alpha}_2 > 0$, there exists $\eta_2 < \eta_1$ (i.e.,* (9) *holds) so that* (8) *does not hold. Furthermore, λ can be slightly decreased so that* (10) *now holds strictly and* (8) *still does not hold. Thus, for these parameter values, the unique solution is for the DM to take action x^* in period 1 and hence devote $\eta_1 > 0$ to period 3.*

Now decrease λ to something still strictly positive but so that (8) *holds. As before, the DM now takes action x^* in period 2. Of course, the unweighted consumption payoff is unchanged. However, all comparisons in our constructions are strict; thus, assuming that taking action x^* in period 2 only leads to a payoff of $\bar{V}_3 - \epsilon$ in period 3 does not change the construction for $\epsilon > 0$ small enough. In this case, decreasing λ leads to a decrease in the unweighted consumption payoff.*

A.3 Example of bad news about the quality of a product increasing consumption

Example 6. *This example builds on the ideas of Tasoff and Madarasz (2009).*

Consider the setup of Section 2.3 and suppose $\mathcal{D} = \{c, m\}$. Consumption dimension c corresponds to the DM purchasing a quantity of a consumption good at unit price 1. Their valuation of quantity k is $\theta u(k)$, where u is strictly concave and continuously differentiable, and $\theta \in \{\theta_L, \theta_H\}$ with $P(\theta = \theta_H) = p \in (0, 1)$. The DM has wealth 1 available and whatever amount they do not consume, $1 - k$, leads to payoff $1 - k$ as part of dimension m (the "money" problem).

We assume that $\lim_{k \to 0} \frac{\partial}{\partial k} u(k) = \infty$ and $\frac{\partial}{\partial k} u(1) = 0$ so that the DM always chooses an interior k.

The DM can learn the value of θ by choosing $\alpha_c = 1$ (formally, such attention allows for some action x that corresponds to learning the value of θ). Otherwise, they decide on k before knowing θ and receive the expected payoff from consumption. The DM will optimally either choose $\alpha_c = 1$ or $\alpha_c = 0$.

Suppose the DM learns the value of θ. Then they choose c to satisfy

$$(1 + \lambda)\theta u'(c) = 1.$$

If they do not learn θ, the DM chooses c to satisfy

$$E[\theta] u'(c) = 1 + \lambda.$$

(The values of $V_c(x), V_m(x)$ are the expected payoffs with the just derived optimal level of consumption.)

Thus, if $1 + \lambda > \frac{E[\theta]}{\theta_L}$, the DM consumes more of the good if they receive the *information and learn it is of low value compared to when they do not receive the information.*

B Attention utility in a strategic environment

In this section we extend the environment of Section 2.4 to allow for strategic interaction. We use the a setup similar to that of Brunnermeier and Parker (2005) Section III, suitably adjusted to our model. That is, there is a unit mass of agents with the same continuously differentiable and increasing Bernoulli utility function u situated in an exchange economy with no aggregate risk. Each agent is initially endowed with one unit of a safe and can purchase a risky asset that is in zero net supply. The price of the risky asset is P and determined in equilibrium. The risky asset's net return is random and denoted by x^r with payoff x_i^r in state i, where $x_s^r \neq x_{s'}^r$ for all $s \neq s'$. We denote a generic state by s and agent by i to minimize confusion.

Agent i acquiring an amount ξ^i of the risky assets leads to monetary payoff of $c_s^i = 1 - \xi^i + \xi^i \frac{1+x_s^r}{P}$ in state s. Thus, each agent i takes the price of the risky asset P as given and chooses a lottery x^i from the set $X(P) = \{\delta_1 - \xi^i + \xi^i \frac{1+x^r}{P} : c_s^i \geq 0, \forall s \in \mathcal{D}\}$. An equilibrium is a price P and choice of lottery x^i and attention allocation α^i for each agent i (with amount ξ^i purchased of the risky asset) such that each agent maximizes their objective

$$\sum_s p_s u(c_s) \tag{11}$$

and $\int_i \xi^i = 0$.

Proposition 12. *An equilibrium exists. If $\lambda > 0$, for $|\mathcal{D}| \geq 2$, agents have heterogeneous subjective beliefs q_s such that there exists a subset $\mathcal{I}$ some agents hold the risky asset and some agents short the risky asset.*

Proof of Proposition 12. If $|\mathcal{D}| = 1$, then it must be that $P = 1 + x_s^r$, and each agent maximizes their objective, e.g., with $\xi^i = 0$.

Suppose $|\mathcal{D}| > 1$. Let $\bar{s} = \arg\max_{s\in\mathcal{D}} x_s^r$ and $\underline{s} = \arg\min_{s\in\mathcal{D}} x_s^r$. First note that $\xi^i \neq 0$ as thus it must be that $\alpha_{\bar{s}}^i = 1$ or $\alpha_{\underline{s}}^i = 1$. To see this, suppose $\xi^i = 0$. Given $\xi^i = 0$, both $\alpha_{\bar{s}}^i = 1$ and $\alpha_{\underline{s}}^i = 1$ are optimal, but $\xi^i = 0$ cannot be optimal for both $\alpha_{\bar{s}}^i = 1$ and $\alpha_{\underline{s}}^i = 1$, a contradiction. Conditional on $\alpha_{\bar{s}}^i = 1$ ($\alpha_{\underline{s}}^i = 1$), the payoff (11) is continuously decreasing (increasing) in P. Furthermore, for large enough P, the payoff given $\alpha_{\bar{s}}^i = 1$ is less than that given $\alpha_{\underline{s}}^i = 1$ with optimal ξ^i. Thus, there exists a unique P^* such that each agent i is indifferent between $\alpha_{\bar{s}}^i = 1$ and $\alpha_{\underline{s}}^i = 1$. Furthermore, at P^*, it must be that $\alpha_{\bar{s}}^i = 1$ implies be that $\xi^i > 0$ and $\alpha_{\underline{s}}^i = 1$ implies be that $\xi^i < 0$ (otherwise, the associated payoffs cannot be the same). But then $\int_i \xi^i = 0$ is easily achieved by assigning the appropriate masses of agents to either $\alpha_{\bar{s}}^i = 1$ or $\alpha_{\underline{s}}^i = 1$ with the corresponding optimal ξ^i. The heterogeneity in attention implies the heterogeneity in subjective beliefs. □

C Proofs

C.1 Proposition 1

We first state a version of Proposition 1 that does not rely on uniqueness of the solutions which we subsequently prove.

Proposition 1*. *Take consumption problem $i \in \mathcal{D}$. Fix V_{-i}, and let $\Gamma(\gamma_i, \beta_i)$ denote the set of optimal (action, attention)-pairs.*

- *If $\lambda > 0$: If $\gamma_i' > \gamma_i$ then $\min_{(x,\alpha)\in\Gamma(\gamma_i',\beta_i)} \alpha_i \geq \max_{(x,\alpha)\in\Gamma(\gamma_i,\beta_i)} \alpha_i$. If, in addition, the environment is separable, then $\min_{(x,\alpha)\in\Gamma(\gamma_i',\beta_i)} V_i(x) - \max_{(x,\alpha)\in\Gamma(\gamma_i,\beta_i)} V_i(x) \geq \gamma_i' - \gamma_i$.*

- *If for β_i and γ_i, $\max_{(x,\alpha)\in\Gamma(\gamma_i,\beta_i)} V_i(x) = \min_{(x,\alpha)\in\Gamma(\gamma_i,\beta_i)} V_i(x)$, then for any $\beta_i' > \beta_i$ and $\gamma_i' = \gamma_i - (\beta_i' - \beta_i)\tilde{V}_i(x)$, where $(x,\alpha) \in \Gamma(\gamma_i, \beta_i)$, we have $\min_{(x,\alpha)\in\Gamma(\gamma_i',\beta_i')} V_i(x) \geq \max_{(x,\alpha)\in\Gamma(\gamma_i,\beta_i)} V_i(x)$. If, in addition, the environment is separable, then $\min_{(x,\alpha)\in\Gamma(\beta_i',\gamma_i')} \alpha_i \geq \max_{(x,\alpha)\in\Gamma(\beta_i,\gamma_i)} \alpha_i$.*

It is immediate that Proposition 1* implies Proposition 1*.

Proof of Proposition 1.* Take any γ_i', γ_i with $\gamma_i' > \gamma_i$ and β_i. Let (x, α) and (α', a') denote a solution given γ_i and γ_i', respectively. Optimality of (x, α) and (α', a') implies

$$
\begin{aligned}
&\underbrace{\sum_{i'\in\mathcal{D}\setminus\{i\}} (\omega_{i'} + \lambda(\alpha_{i'} + \psi_{i'}))V_{i'}(x)}_{:=\kappa_0} + (\omega_i + \lambda(\alpha_i + \psi_i))(\beta_i \tilde{V}_i(x) + \gamma_i) \\
\geq &\underbrace{\sum_{i'\in\mathcal{D}\setminus\{i\}} (\omega_{i'} + \lambda(\alpha'_{i'} + \psi_{i'}))V_{i'}(x')}_{:=\kappa_1} + (\omega_i + \lambda(\alpha'_i + \psi_i))(\beta_i \tilde{V}_i(x') + \gamma_i) \quad \text{and} \\
&\underbrace{\sum_{i'\in\mathcal{D}\setminus\{i\}} (\omega_{i'} + \lambda(\alpha'_{i'} + \psi_{i'}))V_{i'}(x')}_{=\kappa_1} + (\omega_i + \lambda(\alpha'_i + \psi_i))(\beta_i \tilde{V}_i(x') + \gamma_i') \\
\geq &\underbrace{\sum_{i'\in\mathcal{D}\setminus\{i\}} (\omega_{i'} + \lambda(\alpha_{\prime} + \psi_{i'}))V_{i'}(x)}_{=\kappa_0} + (\omega_i + \lambda(\alpha_i + \psi_i))(\beta_i \tilde{V}_i(x) + \gamma_i').
\end{aligned}
$$

Combining the above, gives

$$
\begin{aligned}
&-\Big((\omega_i + \lambda(\alpha_i + \psi_i))(\beta_i \tilde{V}_i(x) + \gamma_i') - (\omega_i + \lambda(\alpha'_i + \psi_i))(\beta_i \tilde{V}_i(x') + \gamma_i')\Big) \\
\geq &\kappa_0 - \kappa_1 \\
\geq &-\Big((\omega_i + \lambda(\alpha_i + \psi_i))(\beta_i \tilde{V}_i(x) + \gamma_i) - (\omega_i + \lambda(\alpha'_i + \psi_i))(\beta_i \tilde{V}_i(x') + \gamma_i)\Big).
\end{aligned}
$$

The outer inequality implies

$$-\lambda(\alpha_i - \alpha_i')(\gamma_i' - \gamma_i) \geq 0,$$

and thus, it must be that $\alpha_i' \geq \alpha_i$ as $\lambda > 0$.

If the environment is separable, then $\tilde{V}_i$ is increasing in the amount of attention α_i devoted to dimension i, and the result follows.

Take any $\beta_i, \beta_i' \geq 0$ with $\beta_i' > \beta_i$ and γ_i and suppose that $\max_{(x,\alpha)\in\Gamma^*(\gamma_i,\beta_i)} V_i(x) = \min_{(a,\alpha)\in\Gamma(\gamma_i,\beta_i)} V_i(x)$. Let $\gamma_i^{'} = \gamma_i - (\beta_i^{'} - \beta_i)\tilde{V}_i(x)$, where $(x,\alpha) \in \Gamma(\gamma_i, \beta_i)$. Let (x, α) and (x', α') denote a solution given (β_i, γ_i) and (β_i', γ_i'), respectively. Optimality of (x, α) and (x', α') implies

$$\begin{aligned}
&\underbrace{\sum_{i'\in\mathcal{D}\setminus\{i\}} (\omega_{i'} + \lambda(\alpha_{i'} + \psi_{i'}))V_{i'}(x)}_{:=\kappa_2} + (\omega_i + \lambda(\alpha_i + \psi_i))(\beta_i\tilde{V}_i(x) + \gamma_i) \\
\geq &\underbrace{\sum_{i'\in\mathcal{D}\setminus\{i\}} (\omega_{i'} + \lambda(\alpha_{i'}' + \psi_{i'}))V_{i'}(x')}_{:=\kappa_3} + (\omega_i + \lambda(\alpha_i' + \psi_i))(\beta_i\tilde{V}_i(x') + \gamma_i) \\
&\underbrace{\sum_{i'\in\mathcal{D}\setminus\{i\}} (\omega_{i'} + \lambda(\alpha_{i'}' + \psi_{i'}))V_{i'}(x')}_{=\kappa_3} + (\omega_i + \lambda(\alpha_i' + \psi_i))(\beta_i'\tilde{V}_i(x') + \gamma_i') \quad \text{and} \\
\geq &\underbrace{\sum_{i'\in\mathcal{D}\setminus\{i\}} (\omega_{i'} + \lambda(\alpha_{i'} + \psi_{i'}))V_{i'}(x)}_{=\kappa_2} + (\omega_i + \lambda(\alpha_i + \psi_i))(\beta_i'\tilde{V}_i(x) + \gamma_i').
\end{aligned}$$

Combining the above and substituting for γ_i' gives

$$
\begin{aligned}
&-\Big((\omega_i+\lambda(\alpha_i+\psi_i))(\beta_i\tilde{V}_i(x)+\gamma_i)-(\omega_i+\lambda(\alpha_i'+\psi_i))(\beta_i'\tilde{V}_i(x')-(\beta_i'-\beta_i)\tilde{V}_i(x))\Big)\\
\geq&\kappa_2-\kappa_3\\
\geq&-\Big((\omega_i+\lambda(\alpha_i+\psi_i))(\beta_i\tilde{V}_i(x)+\gamma_i)-(\omega_i+\lambda(\alpha_i'+\psi_i))(\beta_i\tilde{V}_i(x')+\gamma_i)\Big).
\end{aligned}
$$

The outer inequality implies

$$
-(\omega_i+\lambda(\alpha_i'+\psi_i))(\tilde{V}_i(x)-\tilde{V}_i(x'))(\beta_i'-\beta_i)\geq 0,
$$

and thus, it must be that $\tilde{V}_i(x')\geq\tilde{V}_i(x)$.

If the environment is separable, then $\tilde{V}_i$ is increasing in the amount of attention α_i devoted to dimension i, and the result follows. $\square$

C.2 Proof of Proposition 2

Proof of Proposition 2. Take any $\lambda',\lambda\geq 0$ with $\lambda'>\lambda$. Let (x,α) and (x',α') denote a solution given λ and λ', respectively. Optimality of (x,α) and (x',α') implies

$$
\begin{aligned}
\sum_i\omega_iV_i(x)+\lambda\sum_i(\alpha_i+\psi_i)V_i(x)&\geq\sum_i\omega_iV_i(x')+\lambda\sum_i(\alpha_i'+\psi_i)V_i(x'),\quad\text{and}\\
\sum_i\omega_iV_i(x')+\lambda'\sum_i(\alpha_i'+\psi_i)V_i(x')&\geq\sum_i\omega_iV_i(x)+\lambda'\sum_i(\alpha_i+\psi_i)V_i(x).
\end{aligned}
$$

Combining the above, gives

$$-\lambda'\left(\sum_i(\alpha_i+\psi_i)V_i(x)-\sum_i(\alpha_i'+\psi_i)V_i(x')\right)\geq\sum_i\omega_iV_i(x)-\sum_i\omega_iV_i(x')$$
$$\geq-\lambda\left(\sum_i(\alpha_i+\psi_i)V_i(x)-\sum_i(\alpha_i'+\psi_i)V_i(x')\right).$$

If the expression in the middle is strictly negative, so must be the right one; but then it is strictly larger than the left one as $\lambda'>\lambda$. Thus, the first claim follows.

Now consider two sets of payoff levels, $(\gamma_i)_{i\in\mathcal{D}}$ and $(\gamma_i')_{i\in\mathcal{D}}$, and scalar $\chi\in[0,1]$. Then

$$\begin{aligned}&\max_{\alpha,x\in X(\alpha)}\sum_i(\omega_i+\lambda(\alpha_i+\psi_i))(\beta_i\tilde{V}_i(x)+\chi\gamma_i+(1-\chi)\gamma_i')\\ =&\max_{\alpha,x\in X(\alpha)}\left(\chi\sum_i(\omega_i+\lambda(\alpha_i+\psi_i))(\beta_i\tilde{V}_i(x)+\gamma_i)+(1-\chi)\sum_i(\omega_i+\lambda(\alpha_i+\psi_i))(\beta_i\tilde{V}_i(x)+\gamma_i')\right)\\ \geq&\chi\max_{\alpha,x\in X(\alpha)}\sum_i(\omega_i+\lambda(\alpha_i+\psi_i))(\beta_i\tilde{V}_i(x)+\gamma_i)+(1-\chi)\max_{\alpha,x\in X(\alpha)}\sum_i(\omega_i+\lambda(\alpha_i+\psi_i))(\beta_i\tilde{V}_i(x)+\gamma_i'),\end{aligned}$$

and so the second claim follows.

Now suppose the environment is separable, $\omega_i=1$ and $\psi_i=0$ for all $i\in\mathcal{D}$, and that the objective given λ is convex in α. We show that if $\sum_i\hat{V}_i(\alpha_i)$ is convex in α_i, then so is $\sum_i \alpha_i \hat{V}_i(\alpha_i)$. Take any $\chi[0,1]$ and α_i, α_i' with $\alpha_i < \alpha_i'$. Then

$$\begin{aligned}
&\chi \sum_i \alpha_i \hat{V}_i(\alpha_i) + (1-\chi) \sum_i \alpha_i' \hat{V}_i(\alpha_i') \\
=&\sum_i \alpha_i (\chi \hat{V}_i(\alpha_i) + (1-\chi)\hat{V}_i(\alpha_i')) + \sum_i (\alpha_i' - \alpha_i)(1-\chi)\hat{V}_i(\alpha_i') \\
\geq &\sum_i \alpha_i \hat{V}_i(\chi\alpha_i + (1-\chi)\alpha_i') + \sum_i (\alpha_i' - \alpha_i)(1-\chi)\hat{V}_i(\alpha_i') \\
=&\chi \sum_i \alpha_i \hat{V}_i(\chi\alpha_i + (1-\chi)\alpha_i') + (1-\chi)\sum_i (\alpha_i \hat{V}_i(\chi\alpha_i + (1-\chi)\alpha_i') + (\alpha_i' - \alpha_i)\hat{V}_i(\alpha_i')) \\
\geq &\chi \sum_i \alpha_i \hat{V}_i(\chi\alpha_i + (1-\chi)\alpha_i') + (1-\chi)\sum_i (\alpha_i \hat{V}_i(\chi\alpha_i + (1-\chi)\alpha_i') + (\alpha_i' - \alpha_i)\hat{V}_i(\chi\alpha_i + (1-\chi)\alpha_i')) \\
=&\chi \sum_i \alpha_i \hat{V}_i(\chi\alpha_i + (1-\chi)\alpha_i') + (1-\chi)\sum_i \alpha_i' \hat{V}_i(\chi\alpha_i + (1-\chi)\alpha_i'),
\end{aligned}$$

where the first inequality follows by assumption, and the second as $\hat{V}_i$ is increasing. Thus, since $\sum_i \hat{V}_i(\alpha_i) + \lambda \sum_i \alpha_i \hat{V}_i(\alpha_i)$ is a linear combination of $\sum_i \hat{V}_i(\alpha_i)$ and $\sum_i \alpha_i \hat{V}_i(\alpha_i)$ with the relative weight on the latter increasing in λ, the third claim follows. □

C.3 Proof of Proposition 4

Proof of Proposition 4. Take any λ, λ' with $\lambda' > \lambda$, lottery x, and suppose that the DM(λ) prefers x to δ_y for arbitrary payoff y, i.e.,

$$\frac{p_{\bar{D}_x} + \lambda}{1+\lambda} u(H(x)) + \sum_{i \in \underline{D}} \frac{p_i}{1+\lambda} u(x_i) \geq u(y),$$

where the DM optimally devotes full attention to the states with the highest payoff, $\bar{D}(x) := \{i \in \mathcal{D} : x_i = H(x)\}$, and $\underline{D}(x) = \mathcal{D} \setminus \bar{D}$. We rewrite the above as the expected material utility plus attention utility, each divided by $1 + \lambda$, i.e.,

$$\frac{1}{1+\lambda}\sum_i p_i u(x_i) + \frac{\lambda}{1+\lambda}u(H(x)).$$

As $u(H(x)) \geq \sum_i u(x_i)$, the above is increasing in λ and so DM(λ') also prefers x to δ_y.

For the second claim, suppose $\lambda > 0$ and take any μ, L and $x \in X(\mu, L)$. Consider lottery x'; we can bound the DM's utility from x' as

$$\frac{p_{\bar{D}(x)} + \lambda}{1+\lambda}u(H(x')) + \frac{p_{\underline{D}(x)}}{1+\lambda}u(L) \geq \frac{\lambda}{1+\lambda}u(H(x')) + \frac{1}{1+\lambda}u(L)).$$

Since u is unbounded and $\lambda > 0$, the above goes to infinity as $H(x')$ goes to infinity. Thus, there exists some lottery $\hat{x} \in X(\mu, L)$ such that for all x' with $H(x') > H(\hat{x})$, the DM prefers x' to x.

For the third claim, take any μ, L and $x, x' \in X(\mu, L)$ with $H(x) > H(x')$. The DM's payoff from x is

$$\frac{p_{\bar{D}(x)} + \lambda}{1+\lambda}u(H(x)) + \frac{p_{\underline{D}(x)}}{1+\lambda}u(L(x)),$$

and similarly for lottery x'. The above converges to $u(H(x))$ as λ goes to infinity. As $u(H(x)) > u(H(x'))$, the claim follows.

Lastly, take any μ, H and $x, x' \in Y(\mu, H)$. For any λ, the DM devotes full attention to the states with the high payoff. Thus, they prefer x to x' if and only if

$$\begin{aligned} \frac{p_{\bar{D}(x)} + \lambda}{1+\lambda}u(H) + \frac{p_{\underline{D}(x)}}{1+\lambda}u(L(x)) \geq & \frac{p_{\bar{D}(x')} + \lambda}{1+\lambda}u(H) + \frac{p_{\underline{D}(x')}}{1+\lambda}u(L(x')) \\ p_{\bar{D}(x)}u(H) + p_{\underline{D}(x)}u(L(x)) \geq & p_{\bar{D}(x')}u(H) + p_{\underline{D}(x')}u(L(x')); \end{aligned}$$

i.e., independently of λ. $\square$

C.4 Proof of Proposition 5

Proof of Proposition 5. The first case is proved in the text leading up to the proposition.

Thus, suppose that $\hat{V}_i = \hat{V}_{i'} = \hat{V}$, with $\hat{V}$ continuously differentiable, $\lim_{a\to 0} a\frac{\partial}{\partial a}\hat{V}(a) = \infty$, and $\frac{\partial}{\partial a}\hat{V}(1) < \infty$. $q_i = q_{i'}$ since the labels, i, i', can be exchanged in the DM's objective. For $p_i = 0$, since $\hat{V}$ is increasing and not constant (by the limit condition), the DM optimally devotes full attention to state i'. Hence, $q(p) = 0$. The DM's overall payoff is given by

$$\frac{p_i + \lambda\alpha_i}{1+\lambda}\hat{V}(\alpha_i) + \frac{1 - p_i + \lambda(1-\alpha_i)}{1+\lambda}\hat{V}(1-\alpha_i).$$

Differentiating the above gives

$$\frac{(p_i + \lambda\alpha_i)\frac{\partial}{\partial a}\hat{V}(\alpha_i) - ((1-p_i) + \lambda(1-\alpha_i))\frac{\partial}{\partial a}\hat{V}(1-\alpha_i)}{1+\lambda} + \frac{\lambda(\hat{V}(\alpha_i) - \hat{V}(1-\alpha_i))}{1+\lambda}.$$

For any $p_i > 0$, the DM chooses $\alpha_i > 0$ since $\frac{\partial}{\partial a}\hat{V}(0) = \infty$ and so the above is strictly positive. Also note that the above is decreasing in p_i and for $p_i = 0$ and as $\alpha_i \to 0$ it tends to infinity. Thus, there exists $\bar{\alpha} > 0$ such that the above is strictly positive for all $\alpha_i \in (0, \bar{\alpha})$ for any p_i and so the DM chooses $\alpha_i \geq \bar{\alpha}$ for all p_i. Thus, for $0 < p_i < \bar{\alpha}$, we have $\alpha_i > p_i$ and thus $q_i(p_i) > p_i$. (If $q_i(p_i)$ is a set, then the comparison applies to each element of $q_i(p_i)$.) The remaining comparisons follow from the symmetry of q_s. $\square$

C.5 Proof of Proposition 6

Proof of Proposition 6. Notice that when $\lambda = 0$, the DM, in each period t, maximizes the sum of payoffs. Since V is strictly concave, by Jensen's inequality, this sum is uniquely maximized when $\sum_{t''=1}^{t} x_{t''\to t'} = 1$ for all t'. If in each previous period, the DM only devoted attention to that period, then for $t' = t$, this sum equals $\alpha_{t\to t}$; hence, the unique attention allocation achieving this optimum is $\alpha_{t\to t}$ for all periods t. λ changes the overall utility continuously; hence, for λ small enough, the above still maximizes the DM's overall utility in each period. Furthermore, this attention allocation is implementable in equilibrium. Hence, the first claim follows.

Normalizing (3) by $1 + \lambda$, when $\lambda = \infty$, the DM's overall utility in each period t is given by

$$\sum_{t'=t}^{T} \sum_{t''=t}^{T} \alpha_{t''\to t'} V_{t'}(\sum_{t''=1}^{t} x_{t''\to t'}).$$

This expression is maximized when the DM, in each period t'', devotes attention to a period t', with $\sum_{t''=1}^{t} x_{t''\to t'} \geq K$. The material utility given one of these optimal attention allocations for $\lambda = \infty$ is maximized when this inequality holds with equality; the unique such attention allocation is the one mentioned in the proposition statement. Thus, as λ changes the overall utility continuously, increasing the weight on material utility, the claim follows. □

C.6 Proof of Proposition 7

Proof of Proposition 7. The first claim is obvious.

For the second claim, fix x_1 and consider a realized s. Clearly, if $\max_{x_2 \in X_2(\alpha_2)} V_{c_{nt}}(x_1, x_2|s) \geq V_{c_t}$, solving (4) gives $\alpha_{2\to c_{nt}} = 1$ (for any λ). If $\max_{x_2 \in X_2(\alpha_2)} V_{c_{nt}}(x_1, x_2|s) < V_{c_t}$, then (4) for $\alpha_{2\to c_{nt}} = 0$ (and some finite $V_{c_{nt}}(x_1, x_2|s)$) is larger than (4) for any $\alpha_{2\to c_{nt}} \geq \eta$ for $\max_{x_2 \in X_2(\alpha_2)} V_{c_{nt}}(x_1, x_2|s)$ when λ is large enough. Since $\mathcal{D}$ is finite, taking the max λ implies the result. $\square$

C.7 Proof of Proposition 8

Proof of Proposition 8. Consider $x_1' \notin \arg\max_{x_1' \in X_1} F(x_1')$. For λ large enough, by Proposition 7, and since $\max_{(x_2,\alpha_2),x_2 \in X_2(\alpha_2)} \tilde{V}_{c_{nt}}(x_1', x_2|s) + \beta F(x_1') < \max_{(x_2,\alpha_2),x_2 \in X_2(\alpha_2)} \tilde{V}_{c_{nt}}(x_1, x_2|s) + \beta F(x_1)$, we have $B(x_1') \subseteq B(x_1) \neq$ (where non-emptiness follows from $V_{c_{nt}}(x_1, x_2(x_1, s)) > V_{c_t}$) and hence, again for λ large, the DM's period-2 utility (i.e., (4)) is strictly larger with x_1 than with x_1'.

The DM's objective in period 1 can be written as the sum of (4) and their period-1 attention utility. But the latter is also increasing when choosing x_1 instead of x_1', for large enough λ, and so the result follows. $\square$

C.8 Proof of Proposition 9

Proof of Proposition 9. The first claim follows immediately from the DM's period-2 objective(5).

For the second claim, if $p_1 \geq \bar{p}$, then $V_{c_{nt}} \geq V_{c_t}$, and so devoting full attention $\alpha_{1 \to c_{nt}}$ maximizes the DM's period-1 attention utility. Furthermore, period-2 utility is increasing in $\alpha_{1 \to c_{nt}}$ (for optimally chosen x_1) as it is convex in p_2. Hence, the DM devotes full attention.

Note that the DM's utility is continuous in p_1, and that hey not devote attention to c_{nt} in either period if $p_1 = 0$. Thus, there exists some $\tilde{p} \leq \bar{p}$ such that it is optimal to devote attention to c_{nt} when $p_1 = \tilde{p}$, but not for any $p_1 < \tilde{p}$. Take any $p_1 < p_1' \leq \bar{p}$. What remains to show is that if it is optimal for the DM to devote some attention when their prior is p_1, then it is optimal to devote some attention when their prior is p_1'. Note that it is without loss to assume the DM acquires a binary signal.[36] Let $p_H > p_1$ and $p_L < p_1$ be the posteriors that result from the acquired information given prior p_1. Let the DM with prior p_1' choose a distribution of posteriors with mass points at p_H, p_L and p_1' such that the probability of the high posterior occurring is held constant. Note that this distribution has a lower variance than that given p_1. It leads to the same increase in expected period-2 attention utility. Lastly, since $V_{c_{nt}}(p_1) < V_{c_{nt}(p_1')}$, period-1 attention to c_{nt} leads to a smaller decrease in period-1 attention utility given p_1' than p_1. Combining all these observations, it optimal to acquire some information given p_1'. $\tilde{p}$ is decreasing in β since the DM's objective has increasing differences in $(\alpha_{1\to c_{nt}}, \beta)$

We next prove the skewness result. Suppose that $p_1 > \bar{p}$. Then, $\alpha_{1\to c_{nt}} = 1$. It is uniquely optimal to acquire a binary signal. Let p_L and p_H denote the posteriors. The variance of such posteriors is given by $P(p_2 = p_L)P(p_2 = p_H)(p_H - p_L)^2$. It must be that $p_L < \bar{p}$ and $p_H > \bar{p}$ for x_1 optimal. Hence, $P(p_2 = p_L)P(p_2 = p_H) \leq \frac{\beta}{(p_1-\bar{p})^2}$. For β small enough, it must be that either the high or low posterior are likely. The low posterior cannot be likely since it is bounded away from p_1 (as $p_L < \bar{p} < p_1$) and so it must be that the high posterior is likely. Thus, $P(p_2 = p_H) > 1/2$ and so the distribution of posteriors is negatively skewed.

Now suppose that $p_1 \in (\tilde{p}, \bar{p})$. For optimal x_1, it must be again be that $p_H > \bar{p}$. Thus, it must be that $P(p_2 = p_L)P(p_2 = p_H) \leq \frac{\beta}{(\bar{p}-p_1)^2}$. If the right-hand side is small enough, it must be that either the high posterior or the low posterior are likely. But the high posterior cannot be likely since it is bounded away from p_1 (as $p_1 < \bar{p} < p_H$). Thus, $P(p_2 = p_L) > 1/2$ and so the distribution of posteriors is positively skewed.

□

[36] Any posterior distribution can be replaced with a binary distribution with values $E[p|p_2 \geq \bar{p}]$ and $E[p|p_2 < \bar{p}]$, with probability $P(p_2 \geq \bar{p})$ and $P(p_2 < \bar{p})$, respectively. This distribution of posteriors has a lower variance than the original distribution and leads to the period-2 utility.

C.9 Proof of Proposition 10

Proof of Proposition 10. The first claim follows from Proposition 1, the second from, e.g., Topkis's theorem since the DM's objective has increasing differences in α_i and V_H. For the third, note that the cross-partial derivative of the DM's objective with respect to α_i and V_L is given by

$$\lambda(1 - p(\alpha_i)) - (1 + \lambda\alpha_i)\frac{\partial}{\partial\alpha_i}p(\alpha_c).$$

If $p(\alpha_i)+\alpha_i\frac{\partial}{\partial\alpha_i}p(\alpha_i) < 1$ everywhere, then the above becomes positive for large enough λ and the claim follows from Topkis's theorem.[37] $\square$

C.10 Proof of Proposition 11

Proof of Proposition 11. Fix any (x, α) with $x \in X(\alpha)$. Take any $D, D' \in B$ and consider $B' \coloneqq (B \cup \{D \cup D'\}) \setminus \{D, D'\}$. Evaluate (7) at (x, α) and B' and subtract its value given (x, α) and B; after some simplifications, we have

$$-\frac{|D||D'|\lambda}{|D| + |D'|}(\bar{\alpha}_D - \bar{\alpha}_{D'})(\bar{V}_D(x) - \bar{V}_{D'}(x)).$$

Optimality the implies that the above is non-positive, i.e., if $\bar{V}_D(X) > \bar{V}_{D'}(x)$, then $\bar{\alpha}_D \geq \bar{\alpha}_{D'}$. $\square$

[37]E.g., take $\lambda > \frac{\max_{\alpha_i} \frac{\partial}{\partial\alpha_i}p(\alpha_i)}{\min_{\alpha_i}(1-p(\alpha_i)+\alpha_i\frac{\partial}{\partial\alpha_i}(\alpha_i)}$