

# The Role of Referrals in Inequality, Immobility, and Inefficiency in Labor Markets

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## Abstract

We study labor markets in which firms can hire either via referrals or open applications. Referrals screen candidates and lead to better matches and increased productivity, but disadvantage workers who apply via open applications. We identify conditions under which distributing referrals more evenly across a population not only reduces inequality, but also increases productivity and also improves economic mobility across generations. We use the model to examine optimal policies, showing that one-time affirmative action policies have long-term impacts due to induced changes in future referrals. We also show how the possibility of firing workers improves hiring and lowers inequality.

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*I've never seen baseball advertise for a job, and I've never heard of whites applying for a job. I mean, there's an old boy network, and it's lily white.*

Frank Robinson

## 1 Introduction

The past few decades have seen inequality in countries like the US and UK rise to levels not seen since the Great Depression, and witnessed countries like China hit unprecedented levels of inequality. Inequality within a generation is accompanied by immobility across generations (a correlation between parent and child income), as made clear by Alan Krueger's 2012 famous demonstration of the "Great Gatsby Curve," which showed that countries with higher inequality also tend to have higher immobility.<sup>1</sup> Understanding the common sources of inequality and immobility is essential to designing policies that deal not only with their symptoms but also their causes.<sup>2</sup>

There are some forces underlying persistence in inequality and immobility – such as prejudice, racism, and cultural features like caste systems – that are based on biases in people's treatment and expectations of others. There are other forces that are structural – in the ways that markets work – and that can lead to persistent differences even in the absence of any behavioral biases. Both need to be understood and addressed to improve welfare. Here we study a central form of structural bias in markets that leads to persistent inequality, immobility and inefficiency in markets.

A key feature of labor markets is that many, if not most, employees – of all skill levels – are hired via referrals, and people applying without some connection are at a substantial disadvantage.<sup>3</sup> In this paper we develop a model in which firms hire both via referrals and open applications. This allows us to provide a detailed analysis of the role of referrals in driving inequality, the persistence of that inequality (immobility), and the impact on productivity. The model also enables us to study the dynamic impacts of key policies: the short and long-term effects of different forms of affirmative action, as well as how hiring from open applications is affected by the ease with which firms can later identify and fire workers who underperform.

The use of referrals in hiring is key to understanding all three outcomes: inequality, immobility and productivity. The effect of referrals on inequality is well known.<sup>4</sup> In particular,

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<sup>1</sup>See Corak (2016) for more background.

<sup>2</sup>See Chapter 6 in Jackson (2019) for additional discussion and references on inequality and its relationship to immobility.

<sup>3</sup>For background, see Granovetter (1974, 1995); Montgomery (1991); Ioannides and Datcher-Loury (2004); Topa (2011); Rubineau and Fernandez (2013).

<sup>4</sup>For some of the extensive evidence on this, see Munshi (2003); Calvo-Armengol and Jackson (2004); Arrow and Borzekowski (2004); Calvo-Armengol and Jackson (2007); Laschever (2013); Beaman (2012); Patacchini and Zenou (2012); Beaman, Keleher and Magruder (2018); Lalanne and Seabright (2016); Jackson

the reliance on referrals can result in inequality in employment and wages across groups. For instance, in our model, referred workers have an advantage of extra chances of being hired, and higher average wages conditional upon being hired, since their value is known and they are being selected for that higher value.<sup>5</sup> Inequality in employment and wages across groups then result from higher current employment among some ethnic, gender, age or other-based group, which gives them an advantage in getting referrals and hence jobs and higher wages.

Referrals also help us understand why inequality is so strongly tied with immobility. If one is born into a group that has poor employment, then it becomes hard to get referrals to attractive jobs. Thus, when there is high inequality across groups, referrals can make it naturally persistent. The effects of referrals on both inequality and immobility are driven by a concentration in referrals among a subgroup.

Referrals also serve a productive purpose. Namely, referrals provide information about the productivity of referred workers. Referrals thus help firms vet workers, and hence those hired via referrals are more productive (according to several measurements), on average, than those hired through a less informative open application process.<sup>6</sup>

The above ideas suggest that comparing an economy in which referrals are spread widely to one in which they are concentrated among a subset of a society should not only lead to increased inequality and immobility, but should also lower a society's productivity. Thus, one might expect inequality in employment and wages not only to be related to immobility, but also to lower productivity. These are the central questions that we explore in the first part of the paper.

In particular, to understand the impact of referrals, the key comparative static that we analyze is how things change when one holds constant the total number of referrals available in a population, but changes their distribution among that population. Comparing an economy in which connections to currently employed workers, and hence referrals, are spread evenly across the population, to one in which those connections and referrals are concentrated among a subgroup, is a general way of seeing how gaps in past employment across groups lead to future inequality, immobility, and also affect productivity.

There are two different ways in which referrals can be concentrated, and each has different

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(2019); Arbex, O'Dea and Wiczer (2019); Zeltzer (2020).

<sup>5</sup>For example, see Dustmann et al. (2016) for evidence on referred workers' wage advantage.

<sup>6</sup>For instance, referred workers tend to be more productive per unit of time, inventive, and stay longer in their positions than non-referred workers. For a wide variety of evidence consistent with these and other differences, and the fact that referrals provide information about the potential productivity of the worker, see Fernandez, Castilla and Moore (2000); Brown, Setren and Topa (2012); Fernandez and Galperin (2014); Burks, Cowgill, Hoffman and Housman (2015); Dustmann, Glitz, Schönberg and Brücker (2016); Pallais and Sands (2016); Bond and Fernandez (2019a); Benson, Board and Meyer-ter Vehn (2019). For more discussion of the many roles of referrals in different settings see Heath (forthcoming); Jackson (2019). We can also think of referrals as lowering search frictions, and we know from the search literature (e.g., see Rogerson, Shimer and Wright (2005) for a review), that decreasing frictions can raise overall productivity.

effects. To understand this difference it is important to note that more than one current employee might refer the same person. For instance, if whites have a higher employment rate than blacks, and tend to refer other whites, then as the current employment becomes more tilted towards whites, more whites will get referrals and fewer blacks will – with more whites getting multiple referrals. This corresponds to one way in which referrals can be concentrated, which is to have a smaller number of the next generation receive referrals. This necessarily means, since we are holding the total number of referrals constant, that people who receive at least one referral must be getting a higher number of referrals on average. In our model a worker’s productive value is the same across firms and so getting multiple referrals does not help in productively matching workers to firms. Thus, it is this sort of concentration – which we call “increase in the probability of no referrals” – that is the key to understanding the impact of changing the distribution of referrals on productivity.

To understand inequality in wages, one needs to understand the effects of multiple referrals. Having more than one referral can give an employee multiple offers and hence more bargaining power. Thus, determining the fraction of the population that gets higher wages as referrals are concentrated depends on the fraction of the population that gets multiple referrals.<sup>7</sup> Since how many people get any referral governs productivity and how many people get multiple referrals drives wage inequality, there is a close but imperfect relationship between how productivity and inequality are influenced by referrals.<sup>8</sup>

The second main part of our study is to examine some key policies in this sort of market, including different forms of affirmative action. In order to analyze affirmative action, we explicitly introduce labels on individuals in the population, dividing them into two groups – “blues” and “greens” – where the blues have a relative advantage in the starting employment rates and are relatively advantaged in referrals. The first class of policies we consider – affirmative action policies – boil down to influencing the relative fractions of greens and blues hired. Roughly, this can be done in two ways: encouraging the hiring of referred greens who normally would have been rejected, or else discouraging the hiring of some blues from referrals so that more greens from the pool have a chance to be hired.

We show two main results on the economic impact of affirmative action policies. First, while affirmative action is usually analyzed in terms of short-run implications, we show that

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<sup>7</sup>Increasing the probability of no referrals is neither necessary nor sufficient for having a larger fraction of the population get multiple referrals. For instance, suppose that the population of applicants consists of people who have zero, one, two, or three referrals. By redistributing the extra referrals from those who have three, to the rest of the population it is possible to both increase the number of people who have multiple referrals at the same time as the number of people who get any referrals. Thus, it is possible to have a larger of fraction of the population getting multiple referrals, while still having more of the population get referrals, and keeping the overall total constant.

<sup>8</sup>There are further distinctions that are of interest, between inequality in employment versus in wages, and we examine both. We cannot talk about inequality in employment until we introduce groups, which happens later in the paper.

affirmative action has long-lasting implications for inequality, immobility, and productivity. The basic intuition is that it helps even the distribution of referrals, and that this network effect then persists across generations. This offers an explanation as to why data show that affirmative action has a lasting effect, even after it is removed.<sup>9</sup> By bringing the balance of blue and green employment into better alignment with their prevalence in the population, the concentration of future referrals (in both senses) is reduced. This not only reduces current inequality, but also reduces immobility and bolsters future productivity. Second, discouraging the hiring of referred blues versus encouraging the hiring of referred greens involve different productivity impacts. Which policy involves less current loss of productivity depends on whether the imbalance is coming because high productivity blues are very attractive relative to the pool, or whether low productivity greens are very unattractive compared to the pool. When the pool has workers with high expected productivity, then it is better to discourage the hiring of referred blues and hire more greens from the pool, while if the expected productivity of workers in the pool is low, then it is better to encourage the hiring of (low-productivity) referred greens. Furthermore, subject to feasibility, it is always better to pursue one of these policies rather than a mixture of the two.

Thus our analysis provides new insights both about the longevity of affirmative action policies, and that the different ways in which it can be implemented have different short-term consequences.

Lastly, we consider one more type of policy that is a change in the structure of the model. In our basic model, once a worker is hired, then the firm retains with that worker for the next full period. In an extension, we consider what happens when firms can fire workers part way through a period. The ability to fire a worker improves the market by making it relatively more attractive for a firm to hire from the pool, since the firm can get rid of a low productivity worker should it end up hiring one, and get a second try at hiring. This improvement of the efficiency of hiring from the pool ends up improving productivity, and also lowering inequality and immobility.

Our analysis is nuanced and more complicated than initial intuitions would suggest because of a key equilibrium effect. There are two key mechanisms through which referrals impact outcomes. Referrals have a direct effect on who gets employment offers. But referrals also have an indirect countervailing effect, a *lemons effect*, which drives subtle comparative statics regarding the impacts of referrals. The lemons effect refers to the phenomenon that some people who apply for jobs via open applications had referrals but were not hired via those referrals – so they were already rejected by at least one firm. The fact that the open applications include previously-rejected job seekers lowers the expected productivity from

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<sup>9</sup>For recent empirical evidence of long-lasting effects of temporary policies, see Miller (2017). For more background on affirmative action, see Holzer and Neumark (2000).

hiring via open applications. A main complication in our analysis is that the lemons effect decreases as one concentrates referrals, since there are then fewer workers who are screened by hiring firms and rejected. This effect attenuates the relationship between productivity and the concentration of referrals. It also complicates the relationship between inequality and the concentration of referrals, and causes comparative statics on inequality to differ fundamentally from the comparative statics on productivity. All of our results, and their proofs, deal with this issue.

**More Discussion of the Relationship to the Literature** As discussed in the many references mentioned above, it is well-known that referrals play a prominent role in job markets and, when combined with homophily (the tendency of people to be connected to others to whom they are similar), can lead to inequality. Also, the fact that search processes can lead to lemons effects has also been noted before (Gibbons and Katz, 1991; Montgomery, 1991; Farber, 1999; Krueger, Cramer and Cho, 2014; Bond and Fernandez, 2019b). Our contributions are in the analysis of how productivity, inequality, and immobility are all inter-related because of their shared relationships with referrals; and in showing how affirmative action and market rigidities (ease of firing) affect these relationships.

The closest antecedents to our work in similarity to some aspects of our model are Montgomery (1991) and Calvo-Armengol and Jackson (2004, 2007). Montgomery (1991) examines a model in which people are either of high or low productivity and their productivities correlate with their friends' productivities. This gives firms a reason to hire via referrals and leads to higher wages for referred workers compared to those who are not referred and suffer from a lemons effect.<sup>10</sup> Montgomery establishes that referred workers get higher wages, and firms earn higher profits from referred workers. Also, he establishes that the lemons effect gets worse with an increase in the number of referrals and the correlation between referrer and referred values, as does the difference in wages between referred and unreferrals workers. So, his analysis shows that inequality in wages can result from inequality in referrals. In his model, however, there is always full employment and so there are no questions of productivity or unemployment. He also does not examine the dynamics and immobility, as they make no sense in his model which lacks different groups of workers. The full employment in his model also precludes any analysis of policy impacts on employment and productivity. Thus, none of our main questions can be asked in his model.

Calvo-Armengol and Jackson (2004, 2007) show how referrals can correlate employment between friends and examine dynamic incentives to invest and a resulting poverty trap. Thus, immobility in their model comes from an investment decision, rather than the dynamics of

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<sup>10</sup>Other previous research in which informational advantages lead to better outcomes for some workers include Waldman (1984); Milgrom and Oster (1987); Arrow and Borzekowski (2004).

the referral distribution. They also do not model firm behavior, and thus do not explore productivity or wage distributions.

Our contributions are to develop a dynamic model that combines referrals and open applications with less than full employment (within this industry) and with different groups of workers. This allows us to characterize how the distribution of referrals affects all three of inequality, immobility, and productivity; and to shed new light on the dynamic implications of some prominent policy interventions. To our knowledge, there are no antecedents to our comparative statics, including the distinction between the different ways in which referrals can be concentrated, as well as our results establishing the relationships between inequality, immobility, and productivity, and the policy implications. Importantly, our results on productivity imply that one should care about imbalances in people's access to referrals not just because of inequality and immobility (e.g., fairness), but also because such imbalances reduce efficiency. Furthermore, and perhaps most importantly, we show that one should care about affirmative action not just because it provides a fairer distribution of employment within a given period, but also because it improves mobility and productivity in future periods.

Finally, there is a literature on the impacts of a wide variety of labor market rigidities (e.g., Nickell (1997)), and our results on the effects of the ease of firing provide some new comparative statics and hypotheses in that direction, by showing how greater ease of firing lowers the lemons effect and reduces the advantage and impact of referrals.

## 2 A Model and Preliminaries

We consider a labor market with a unit mass of risk-neutral firms, each having one current but retiring employee. Each firm wishes to hire at most one next-generation worker to replace its retiring employee. There is a mass  $n \geq 1$  of risk-neutral agents seeking work at these firms; so there can be unemployment. We refer to these agents as "workers" regardless of their employment status and the currently employed agents as employees.

A generic worker  $i$  provides a productive value  $v_i$  to any firm that employs that worker. This value includes the worker's skill and talent, and whatever else makes the worker productive, and is the maximum amount that the firm would be willing to pay to hire the worker, if the firm had no other possibilities of filling the vacancy. The distribution of  $v_i$  is denoted by  $F$ , and is independent across workers.<sup>11</sup>  $F$  has a finite mean and we consider the nondegenerate case in which  $F$  has weight on more than one value, and so  $v_i$  has nonzero variance.

Firms hire workers either through referrals or via a pool of open applications – via

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<sup>11</sup>See Currarini et al. (2009) for background references and a discussion about the ways in which one can justify the independence here and the random matching with a continuum of people.

“referrals” or “the pool”. Referrals are generated through a network in which each firm’s current (retiring) employee refers a next-generation worker. For simplicity, we assume each employee refers exactly one worker. Nonetheless, some next-generation workers may receive multiple referrals as several current employees may each refer the same next-generation worker to their respective firms. We denote the distribution of referrals a worker receives by  $P$ , with  $P(k)$  being the probability that a generic worker gets exactly  $k$  referrals.

We allow for a variety of different referral processes, each captured via a different degree distribution. For example, if referrals are made uniformly at random across all workers then  $P$  is a Poisson distribution; i.e.,

$$P(k) = \frac{n^{-k} e^{-1/n}}{k!},$$

with  $1/n$  being the average number of referrals per worker. We assume throughout that  $1 > P(0) > 0$  where the first inequality rules out the trivial case in which no referral market exists (which is implied by the assumption that each employee refers a worker) and the latter has to be the case if e.g.  $n > 0$ .

Workers have a minimum wage that they must be paid,  $w_{min}$  (which is presumed to be at least their outside option value). This minimum wage can either be the worker’s outside option value, or else a minimum wage imposed by some government. We presume throughout that  $w_{min}$  lies below the max of the support of  $F$ , so that there is a positive mass of workers that firms find strictly worth hiring.

Each firm observes the value of its referred worker and then chooses whether to hire the worker. If a firm chooses not to hire its referred worker, it can go to an anonymous pool, consisting of all workers who either have no referrals or are not hired via any of their referrals, and may hire a worker picked at random from that pool.

Firms have a payoff of the expected value of the worker they hire (if they hire one) minus the wage. A worker’s payoff is the wage if they are hired and their outside option value otherwise (which is no more than  $w_{min}$  by the assumption above).

In summary, the timing of the game is as follows.

1. Hiring from Referrals:

- (a) Each firm receives a referral to a worker and chooses whether to make an offer to its referred worker and if so, at what wage.
- (b) Referred workers who receive at least one wage offer choose to accept one of them or to reject all of them. Any accepted offer is consummated and the firm and worker are matched.

2. Hiring From the Pool:

- (a) Workers and firms who are unmatched after the referral stage, go to the pool. Each such firm gets matched with one worker chosen uniformly at random from the pool (without replacement, so that no two firms are matched to the same worker from the pool). Firms choose whether to make an offer to its worker from the pool and if so, at what wage.
- (b) Workers from the pool who receive a wage offer accept or reject it. Any accepted offer is consummated and the firm and worker are matched. Remaining firms and workers are unmatched.

As the mass of workers (weakly) exceeds the mass of firms, some workers will necessarily be unemployed and the referral market helps in selecting more productive workers for employment.

## 2.1 Some Comments on the Model

In our model referrals have an extreme informational advantage over those from the pool, since a firm perfectly observes the value of a referred worker, but only knows the (equilibrium) expected value of the workers in the pool. What is essential for our results is that there is some informational advantage (for which there is ample evidence<sup>12</sup>) - current workers have some valuable information about how well their connections would perform on the job, and that information is less costly than learning the equivalent information about an applicant from the pool.

We have modeled the process of referrals and hiring from the pool as happening in direct succession and within a period. In practice, some firms (especially large ones) are constantly hiring, and exploit their referrals when available and hire from applications when they have openings that they are not filling otherwise. What is important is that some of the people who openly apply for positions have used connections to get interviews but failed to land a job, thus exhibiting a lemons effect, a fact well-documented by the duration-dependence literature.<sup>13</sup>

In the way we have described our model, each current worker refers a friend to its employer even if that worker is of a low value and will be rejected by the firm. A current worker may or may not wish to make such referrals (depending on whether this is perceived as a favor by a friend or a nuisance by the firm).<sup>14</sup> The model works with absolutely no changes in what follows if (some or all) current workers only refer friends whom they know will be hired in

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<sup>12</sup>For instance, see Rees (1966); Fernandez, Castilla and Moore (2000); Brown, Setren and Topa (2012); Pallais and Sands (2016)

<sup>13</sup>For instance, see the references in Ioannides and Datcher-Loury (2004); Eriksson and Rooth (2014).

<sup>14</sup>See Beaman and Magruder (2012); Beaman et al. (2018) for discussion of how workers choose whom to refer.

equilibrium. Having referrals be automatic simply eliminates an inconsequential from the equilibrium analysis.

## 2.2 Equilibrium Characterization

We examine (weak) perfect Bayesian equilibria of the game.

The basic equilibrium structure is easy to discern in this game, and can be seen from backward induction. Workers accept any wages at or above  $w_{min}$  but not below. The only possibility to have workers mixing at  $w_{min}$  if both firms and workers are indifferent.<sup>15</sup> In that (non-generic) case, the mixing becomes irrelevant since there is 0 net expected value in the relationship to either side.<sup>16</sup> Thus, equilibrium behavior in the pool stage is such that firms hire workers from the pool at a wage of  $w_{min}$  if the expected value of workers in the pool exceeds  $w_{min}$ , do not hire from the pool if the expected value is below  $w_{min}$ , and both sides can mix arbitrarily if the expected value of workers in the pool is exactly  $w_{min}$  (and this happens to be the workers' outside option value).

Then given the continua of firms and workers, taking the strategies of others as given, firms have a well-defined value from waiting and hiring (or not) from the pool - either something positive or 0. Thus, in the first period, a firm prefers to hire a referred worker if and only if that worker's value minus the wage exceeds the expected value from the second period potential pool hiring. If there is no competition for the referred worker, the worker's wage is  $w_{min}$ ; otherwise the competing firms bid.

The key to characterizing the equilibrium is thus a threshold such that firms attempt to hire a referred worker who has a value above that level, and do not hire workers below that level. That threshold corresponds to the value of waiting and possibly hiring from the pool. To characterize the threshold, the relevant function is the expected value of workers in the pool conditional on firms hiring referred workers who have values strictly above  $\tilde{v}$ , not hiring workers with values strictly below  $\tilde{v}$ , and hiring workers with values exactly equal to  $\tilde{v}$  with probability  $r$ :<sup>17</sup>

$$E_{\tilde{v},r}[v_i | i \in \text{pool}] := \frac{P(0)E[v_i] + (1 - P(0))(\Pr(v_i < \tilde{v})E[v_i | v_i < \tilde{v}] + \Pr(v_i = \tilde{v})(1 - r)\tilde{v})}{P(0) + (1 - P(0))(\Pr(v_i < \tilde{v}) + \Pr(v_i = \tilde{v})(1 - r))}. \quad (1)$$

So, all equilibria are equivalent to using a threshold  $\tilde{v}$  (and a number  $r$ ) that is based on a

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<sup>15</sup>Otherwise, there cannot exist an equilibrium in which workers mix since if workers mixed and firms were not indifferent, then firms would deviate and offer a higher wage.

<sup>16</sup>Mixing can matter in the relative employment rates of different groups of workers, and so we track mixing when we get to that analysis.

<sup>17</sup>Neither firms nor workers have to follow similar strategies at the cutoff,  $r$  simply represents the overall probability that a worker with exactly the threshold value is hired overall and includes the mixing of both firms and workers. For simplicity, we refer to  $r$  as the mixing parameter.

fixed point of (1).<sup>18</sup> Thus, we use the term “equilibrium threshold” to refer to a fixed point of (1).

**LEMMA 1.** *There is a unique threshold solving*

$$\tilde{v} = \max [w_{min}, E_{\tilde{v},r}[v_i | i \in \text{pool}]] , \quad (2)$$

*and that threshold value characterizes the following equilibrium behavior:*

1. *a referred worker  $i$  is hired if the worker’s value  $v_i > \tilde{v}$ , not hired if  $v_i < \tilde{v}$ , and hired with any arbitrary probability if  $v_i = \tilde{v}$ ;<sup>19</sup>*
2. *firms that are unsuccessful in hiring a referred worker hire from the pool if  $E_{\tilde{v},r}[v_i | i \in \text{pool}] > w_{min}$ , do not hire if  $E_{\tilde{v},r}[v_i | i \in \text{pool}] < w_{min}$ , and hire from the pool with any arbitrary probability if  $E_{\tilde{v},r}[v_i | i \in \text{pool}] = w_{min}$ .*

*The equilibrium wages of a hired worker are  $v_i - \tilde{v} + w_{min}$  if  $i$  has more than one referral, and  $w_{min}$  otherwise.*

Although we account for all cases in what follows, the reader may find it easier to concentrate on situations in which firms find it worthwhile to hire workers from the pool, since then  $\tilde{v} = E_{\tilde{v},r}[v_i | i \in \text{pool}]$  and the threshold is simply the expected productivity in the pool.

The remaining details behind the proof of Lemma 1 (beyond the discussion that precedes the lemma), including why the threshold is unique, as well as all other proofs, unless otherwise indicated, appear in Appendix A.

## 2.3 The Lemons Effect

In the previous section we argued that hiring on the referral market follows a simple cutoff rule: workers with referrals are hired if their productivity is above some cutoff, and rejected workers go to the pool. This lowers the value of workers in the pool. The impact of this selection on the pool productivity is what we term the lemons effect. This lemons effect plays a central role in our analysis, sometimes amplifying and sometimes mitigating the various effects that we study.

**LEMMA 2.** *In equilibrium there is a (strict) lemons effect:  $E_{\tilde{v},r}[v_i | i \in \text{pool}] < E[v_i]$ .*

The lemons effect is an important characteristic of labor markets with referrals: the unemployed pool of workers includes unlucky people who had no contacts, together with people

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<sup>18</sup>For discrete distributions  $F$ , there could be a multiplicity of thresholds that all correspond to the same decisions.

<sup>19</sup>Note that if  $(\tilde{v}, r)$  satisfies (2), then so does  $(\tilde{v}, r')$  for any  $r' \in [0, 1]$

who have been rejected. This inclusion of previously rejected workers with low productivity spoils the pool, making hiring from the pool less attractive. This in turn provides additional incentives for firms to hire referred workers. This feedback effect means that the hiring threshold for referrals,  $\tilde{v}$ , is less than the unconditional expected productivity value in the population.

The lemons effect gives a further advantage to workers who have a referral. Some referred workers have lower-than-average productivity but, due to the lemons effect, are better than the expected value in the pool. Firms therefore knowingly hire these below-average referred workers even though the pool also contains some above-average workers. Thus workers who have referrals not only have an additional chance to be hired compared to those who are only in the pool, but also benefit from the lemons effect which makes firms even more willing to hire via referrals.

### 3 The Impact of Referral Inequality

Next, we examine how the inequality in the distribution of referrals in a society impacts income inequality as well as overall productivity in the society.

To keep the analysis uncluttered, we consider a case throughout this section in which  $w_{min}$  is the value of workers' outside options: their productivity at home or their wage in some other form of employment if they are not hired on this market. This assumption allows us to assess the value of unhired workers and consider them in assessing the overall welfare/productivity/income of the society. We thus define productivity to be the total value of employed workers, plus the minimum wage for all the unemployed workers.<sup>20</sup>

#### 3.1 Defining Referral Inequality

We investigate how the economy changes as the distribution of referrals throughout the society become more unequal. That distribution of referrals is embodied in  $P$ .

Let  $P(2+) = \sum_{k \geq 2} P(k)$  denote the fraction of workers who get two or more referrals.

The key characteristics of  $P$  that affect the matching and wages are  $P(0)$  and  $P(2+)$ .  $1 - P(0)$  is the fraction of job-seekers who have referrals and hence are vetted and have a chance of being hired at the first stage. This is the key to understanding both the cutoff threshold and the equilibrium productivity.  $P(2+)$  is then key in determining the wage distribution. Once a person has two firms competing to hire them, having any additional

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<sup>20</sup>This assumption that unemployed workers have an outside option of the minimum wage, is generally inconsequential, except if reducing the number of referrals changes the lemons effect so much that it reverses hiring from the pool, and the outside option is substantially worse than the minimum wage.

bidders does not matter and so  $P(2+)$  is sufficient (together with the cutoff) to predicting the fraction of workers who will get above minimum wage.

Thus, for our analysis, everything can be characterized by changes in  $P(0)$  and  $P(2+)$ . These can be thought of as two ways of changing referral inequality: increasing the probability of no referral (increasing  $P(0)$ ); and increasing the probability of multiple referrals (increasing  $P(2+)$ ).

#### DEFINITION 1.

*Distribution  $P'$  increases the probability of no referral compared to distribution  $P$  if  $P'(0) \geq P(0)$ . Such an increase is strict if  $P'(0) > P(0)$ .*

*Distribution  $P'$  increases the probability of multiple referrals compared to distribution  $P$  if  $P'(2+) \geq P(2+)$ . Such an increase is strict if  $P'(2+) > P(2+)$ .*

Neither of these implies the other. It is possible to have  $P'$  increase the probability of no referral but decrease the probability of multiple referrals compared to  $P$ , or vice versa.

Even though these two types of changes in the distribution of referrals are logically distinct, many of the interesting comparisons are ones in which they happen together. In particular, an important comparative static is to think of holding the total number of referrals constant, but concentrating referrals among some subset of the population. As more referrals are taken from the rest of the population and added to this subgroup, this naturally leads to an increase in  $P(0)$  and an increase in  $P(2+)$ , so both increases happen together. Such a concentration of referrals can be caused by a variety of factors. One is that people can be more likely to refer their popular friends than a friend at random, as their more popular friend might be in a better position to return the favor some day. This causes the distribution of referrals to have a higher probability of no referral as well as multiple referrals. Referral inequality can also be induced by homophily. As we show in Section 4, if people belong to groups (e.g., ethnic or geographic), one group has an advantage in terms of its initial employment, and people tend to refer people of their own group, then this leads to referrals with a higher probability of no referrals and multiple referrals compared to a world that is group-blind (e.g., see Zeltzer (2020) for evidence of exactly this effect in the context of gender homophily in referring physicians for employment). As a third example, it could be that some groups get relatively more referrals per capita, than others (e.g., in some settings males are more strongly biased towards referring males than females are to referring females, as seen in Lalanne and Seabright (2016)). In that case, holding the overall mean constant and comparing this to a situation in which males and females are equally biased towards own type leads to a concentration of referrals among males (again, see Section 4), which overall leads to a referral distribution with a higher probability of no referrals and multiple referrals.

## 3.2 Concentrating Referrals Decreases Productivity

We first investigate how equilibrium productivity changes with the underlying distribution of referrals. In this subsection, we presume that firms hire when indifferent ( $r = 1$ ). The mixing cases do not change any of the productivity results, but comparing across distributions requires specifying the mixing for each distribution, which has no real consequence but complicates the proofs substantially.<sup>21</sup>

Equilibria have inefficiencies: firms hire referred workers with productivity below other workers in the pool. Nonetheless, one can show that an equilibrium is still *constrained* efficient: the threshold for hiring referred workers that maximizes total production is the unique equilibrium threshold. However, although each equilibrium uses the best threshold it can, the equilibrium varies with the distribution of referrals, and spreading referrals out increases productivity. We now formalize that statement.

To understand how productivity changes with changes in the referral distribution it is useful to note that a single referral is productivity enhancing: If a worker has high productivity (above the equilibrium threshold), then that worker is hired. If the referred worker has low productivity, then the firm has the option of taking a match from the pool. When some worker gets two or more referrals instead of one, then that does not improve the matching of that worker beyond the first referral: if they are low value then the referrals are all wasted, and if they are of high value then any referral past the first one is wasted. This then suggests that any change that increases the probability of no referral,  $P(0)$ , is detrimental to productivity: the total value of production in the society goes down as fewer high-value workers are hired overall in the economy.

There are some subtleties: An increase in  $P(0)$  to  $P'(0)$  implies that there are fewer lemons in the pool, making the pool more productive and raising the equilibrium threshold. The proof shows that this countervailing force is always overcome. The basic intuition is that having a higher probability of no referrals means that fewer workers get vetted overall, and more ultimately end up being hired without any vetting (lemons or otherwise). Ultimately, jobs that are not filled by a high-value worker end up being filled by someone of lower value or not filled at all (depending on the equilibrium, presuming that we are not changing from one type of equilibrium which hires from the pool to the other which does not), and replacing those with high value is good. The full proof takes care of all the possible cases, including in which we change whether workers are hired from the pool or not.

**PROPOSITION 1.** *Consider two referral distributions,  $P$  and  $P'$ , and let the unique equilibria be given by  $\tilde{v}$  and  $\tilde{v}'$ , respectively. If  $P'$  increases the probability of no referral compared to  $P$  (i.e.,  $P'(0) \geq P(0)$ ), then*

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<sup>21</sup>It can only be consequential when firms do not hire from the pool, as then it can make a difference in the value in the pool; but the pool value does not affect productivity in that case.

- $E_{\tilde{v}'}[v_i | i \in \text{pool}] \geq E_{\tilde{v}}[v_i | i \in \text{pool}]$ , and so there is a weak decrease in the lemons effect and a weak increase in the expected value of workers in the pool;
- and the total production in the economy associated with  $P'$  is less than or equal to that associated with  $P$ .

All comparisons are strict if there is a strict increase in the probability of no referral (i.e.,  $P'(0) > P(0)$ ).

Proposition 1 states that concentrating referrals among a smaller part of the population, leads to a decreased lemons effect and decreased productivity.

Increasing the probability of no referrals reduces the lemons effect since fewer people are rejected and pushed into the pool (even accounting for the raised threshold, as that raised threshold is itself a reflection of the extent of the lemons effect<sup>22</sup>). In fact, the lemons effect moves exactly with the total production in the economy: the average productivity of the workers in the pool equals the average productivity of the unemployed, and thus must counter-weight the average productivity of the employed since the overall average productivity in the society is fixed.<sup>23</sup>

We study how changes in the size of the population impact productivity in Section B.3 of the supplemental appendix. One can interpret changes in  $n$  as changes in the “tightness” of the labor market, and a larger  $n$  corresponds to a looser market, with more job searchers per vacancy - which spreads referrals out among a larger population. However, such an increase also corresponds to a greater fraction of the pool never having had a referral, which alleviates the lemons effect. Overall more people have been vetted, and so productivity increases because there are more option values, and it also increases because of the better values hired from the pool. Note that contrast in different comparative statics: as we increase  $n$  we get improvement in both the lemons effect and productivity, while when  $n$  is fixed and one spreads referrals more widely one still gets an improvement in productivity, but there there is a worsening of the lemons effect. The difference in the impact on the lemons effect comes from the fact that it is the fraction of the pool that have been rejected from referrals that drives the lemons effect, and that moves in opposite directions in these two comparative statics; yet both changes increase the number of people vetted which is the key to determining productivity.

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<sup>22</sup>If the lemons effect had actually gone up, then the threshold would have had to fall.

<sup>23</sup>This result relies on the assumption that the outside option of workers equals the minimum wage. If  $w_{min}$  is above outside option value to unemployed workers, then overall production could be higher under  $P'$  under some circumstances. That would require that there was no hiring from the pool under  $P$ , and that the difference between  $P'$  and  $P$  be large enough to ease the lemons sufficiently to make hiring under the pool attractive under  $P'$ , and that the gain in value from hiring those workers (the difference between the minimum wage and their outside values) is sufficiently large to offset the loss from reduced vetting. More generally, this means that the size of employment could be changing, and here productivity is including outside options and so increased productivity does not necessarily mean increased employment.

### 3.3 The Impact of Concentrating Referrals on Wage Inequality

Next, we explore how changes in the referral distribution impact wage inequality, which in this model is equivalent to income inequality (more comments on this below).

The first thing to note is the key factor in determining the wage distribution in our model is the number of people who get more than one referral: those are the people who have some competition for their services and earn above the minimum wage. Thus, rather than how many people get referrals, what is key here is what fraction of people get multiple referrals; i.e., the probability of multiple referrals. The probability of no referrals still determines the expected value from hiring from the pool, and thus impacts wages that people with multiple referrals obtain, so it is also involved but with a different (marginal) effect.

To measure inequality, we use the Gini coefficient of wages (see Equation 3 for a formal definition in our setting). As comparing distributions is generally an incomplete exercise, using such a coefficient allows one to make comparisons of partially ordered distributions. However, even using this standard one-dimensional measure, and looking at simple settings, we will see that inequality can move in different ways from the same comparative statics, depending on the specifics of the environment.

One might expect that an increase in the probability of multiple referrals would be sufficient to increase inequality, but things are not so direct. First, even if few workers are high-wage earners, increasing the size of that group can actually increase or decrease inequality, depending on relative wages and the relative size of the group to begin with, as the Gini coefficient makes relative comparisons. In addition, increasing the probability of multiple referrals can alleviate the lemons effect if accompanied by a decrease in the probability of no referral. This tends to decrease the wages of the workers who have multiple referrals, lowering the high wage, which can then decrease inequality. Thus, there can be different sorts of countervailing effects.

To get a characterization of when inequality is raised by increasing the probability of multiple referrals, for this subsection only, we consider a situation in which productivity takes on two values  $v_H > v_L$ , with fraction  $f_H$  of the population being of the higher productivity. Combined with the assumption that  $w_{min}$  is the value of workers' outside options, this implies that there are just two levels of wages. All of the countervailing effects described above already exist with just two types, and this simplification makes it easier to see the intuition.

Let  $\pi_H = P(2+)f_H$  denote the fraction of workers who earn the high wage,  $w_H = v_H - \tilde{v} + w_{min}$  denote the equilibrium wage of a worker with high value, and  $\pi_L = (1 - P(2+)f_H)$  be the remaining fraction of workers, who all earn the minimum wage. Let  $W_L = \frac{w_{min}}{w_H}$  denote the low wage relative to the high wage. One can write the Gini coefficient, denoted

by *Gini*, of an economy as follows:<sup>24</sup>

$$Gini = \frac{\pi_H \pi_L (w_H - w_{min})}{\pi_H w_H + \pi_L w_{min}} = \frac{\pi_H \pi_L (1 - W_L)}{\pi_H + \pi_L W_L} = \frac{\pi_H (1 - \pi_H) (1 - W_L)}{\pi_H + (1 - \pi_H) W_L}. \quad (3)$$

Let us consider what happens when the fraction of people earning the high wage is increased ( $\pi_H$  is increased), which is a consequence of increasing the probability of multiple referrals. Straightforward calculations show that:

$$\frac{\partial Gini}{\partial \pi_H} = \frac{(1 - W_L)(W_L \pi_L^2 - \pi_H^2) - \frac{\partial W_L}{\partial \pi_H} \pi_H \pi_L}{(\pi_H + \pi_L W_L)^2}.$$

The change in the Gini coefficient consists of two parts: the effect of changing the fraction of people earning the high wage, and then the effect of changing the wage via the lemons effect.

The first part (ignoring the term including  $\frac{\partial W_L}{\partial \pi_H}$ ) is positive for low  $\pi_H$  and high  $W_L$ , but then becomes negative as  $\pi_H$  increases and  $W_L$  decreases. However, then the overall expression is impacted by the  $\frac{\partial W_L}{\partial \pi_H}$  term, which accounts for the lemons effect. The change in the lemons effect depends on both the probability of no referral as well as multiple referrals.

To fully sign the  $\frac{\partial W_L}{\partial \pi_H}$  term, we also need to know what happens to the probability of no referral. In particular, the change in  $W_L$  comes from the change in  $w_H$ , which is exactly governed by the probability of no referral. Knowing how  $P(2+)$  changes, tells us about  $\pi_H$ , but we need to know how  $P(0)$  changes to determine the change in  $w_H$ . If the change in the referral distribution is comprised of both an increase in the probability of no referral and the probability of multiple referrals, then the increase in  $\pi_H$  is accompanied by an increase in  $P(0)$  which, using Proposition 1, decreases the lemons effect and thus the high wage so that  $\frac{\partial W_L}{\partial \pi_H} > 0$ . This then counteracts the impact of the first term for low  $\pi_H$  and can reverse the change in the Gini coefficient.

These results are summarized in the following proposition, which should be clear from the above discussion, so we omit a formal proof. Let  $I_H = \pi_H w_H$  and  $I_L = \pi_L w_L$  denote the total income going to the high and low wage earners, respectively.

**PROPOSITION 2.** *The Gini coefficient decreases with an increase in the probability of no referral (an increase in  $P(0)$ ), holding  $P(2+)$  constant. And, holding  $P(0)$  constant, the Gini coefficient increases with an increase in the probability of multiple referrals (an increase in  $P(2+)$ ) if and only if  $I_L \pi_L > I_H \pi_H$ .*

The comparison  $I_L \pi_L > I_H \pi_H$  captures the relative total wages, population weighted,

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<sup>24</sup>The expression is based on the definition of the Gini coefficient as the mean absolute difference in wages, normalized by twice the average wage. This definition is equivalent to the more standard one in terms of the area underneath the Lorenz Curve (Sen, 1973).

which is important in determining whether the inequality is due to most of the society’s income coming from the low wage, which holds for low values of  $\pi_H$  and  $w_H$ , or the reverse, which determines whether things are getting more or less equal.

Proposition 2 implies that a mean-preserving spread in  $P$ , such that the probability of no referral goes down and the probability of multiple referrals goes up (presuming  $I_L\pi_L > I_H\pi_H$ ), pushes up the fraction of high wage earners and also increases their wage due to an increased lemons effect, thus increasing the Gini. If instead (again presuming that  $I_L\pi_L > I_H\pi_H$ ), we consider a first-order dominance shift in  $P$ , then there are countervailing forces: more people earn the higher wage, but that wage goes down due to an improved value of the pool. Either force can dominate depending on the particular parameters. Effects of referrals on inequality become less ambiguous when we consider group effects.

In Section B.4 of the supplemental appendix, we discuss what happens when firms’ profits are also included in the calculations of inequality.

## 4 Inequality Across Groups and Immobility

To this point, we have examined implications of the distribution of referrals, but without considering who got those referrals as a function of their demographics. Also, our discussion was focused on productivity and inequality, and we have not yet discussed immobility. We now examine both of these issues by explicitly modeling different groups of agents; e.g., ethnicity, gender, age, education, geography, etc. Coupled with homophily (tendencies to refer own group) or some other asymmetry (people being relatively biased towards referring some particular group), tracking groups shows us where concentration in referrals comes from. Moreover, tracking groups enables us to see how lower current employment among one group compared to another translates into lower employment and lower wages for the first group compared to the second in the next generation, hence tracking immobility.

### 4.1 Homophily in Referrals

For simplicity we consider two groups, but the results extend easily to more. We refer to one group as *blue* and the other as *green*, with respective masses  $n_b > 0$  and  $n_g > 0$  of workers per generation, such that  $n_b + n_g = n$ . Let  $e_b$  and  $e_g$  be the masses of employed blue and green workers at the beginning of the period respectively; i.e., the masses of the retiring employees or the “current employment” for short. There is a current employment bias towards blues if  $\frac{e_b}{e_g} > \frac{n_b}{n_g}$ .

To model homophily in referrals, we track group-dependent referral-bias parameters  $h_b \in [0, 1]$  and  $h_g \in [0, 1]$ . A fraction  $h_b$  of employed blue workers of the current generation refer

blue workers from the next generation and the remaining  $(1 - h_b)$  fraction of employed blue workers refer green workers; with  $h_g$  defined analogously. Levels of  $h_b > \frac{n_b}{n}$  and  $h_g > \frac{n_g}{n}$  indicate homophily; i.e., a bias towards referring one's own group.<sup>25</sup>

For the remainder of this paper, we assume that  $h_b \geq 1 - h_g$ . This rules out cases with reversals: greens referring blues more often than blues referring blues, and vice versa. This assumption implies the natural case: if a group's employment goes up, then it receives more referrals.<sup>26</sup>

To make the model concrete, suppose additionally that referrals to a group are made uniformly at random across workers from that group. So, if a worker decides to refer a blue, then that blue is picked uniformly at random from all blues. Thus, the distribution of the number of referrals received within a group is Poisson.

As homophily and current employment levels are adjusted, the distribution of referrals across groups, and hence the overall concentration of referrals, changes. For example, with full homophily ( $h_b = h_g = 1$ ), increased current employment inequality (moving  $\frac{e_b}{e_g}$  away from  $\frac{n_b}{n_g}$ ) results in an increase both in the probability of no referral as well as multiple referrals, which affects productivity, wage inequality, and the distribution of wages and employment between the two groups.

To extend our analysis to a dynamic version of the model, with overlapping generations, we simply have workers hired in period  $t$  refer workers in period  $t + 1$ . To keep the analysis uncluttered, we consider the case in which firms are myopic and maximize their profits in each time period separately, but we comment below on how things change if firms are forward looking.

Equilibrium can be analyzed as discussed in Section 2, since whether a worker is green or blue does not impact firm payoffs.

## 4.2 Inequality from Period to Period

Note that the economy has a Markov property: referrals and equilibrium actions depend only on current employment and not on how the system got there. Thus, to understand the full dynamics it is useful to first understand how employment levels from one period transition to the next.

In particular, a useful expression turns out to be what referrals would be if employment rates were equal to the relative population proportions. If there was no current employment

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<sup>25</sup>An alternative form of homophily is on values, which is of less direct interest here, but we explore in Section B.5 of the supplemental appendix.

<sup>26</sup>An absence of this assumption can lead to the non-existence of a steady-state and peculiar dynamics: start with high employment among blues who mostly refer greens, which then leads to high green employment who then mostly refer blues, continuing in a cycle. As we will see, even ruling this case out, it will still be possible to cycle, but those cycles will be more plausible ones.

bias, then the blue and green workers looking for jobs would receive a total of

$$R_b = \frac{h_b n_b + (1 - h_g) n_g}{n} \quad \text{and} \quad R_g = \frac{h_g n_g + (1 - h_b) n_b}{n} \quad (4)$$

referrals respectively. If  $\frac{R_b}{R_g} = \frac{n_b}{n_g}$ , then when employment matches population shares ( $\frac{e_b}{e_g} = \frac{n_b}{n_g}$ ), then there is a *balance* in referrals: each population would receive a number of referrals proportional to their representation in the population if employment reflected population shares.

If current employment rates are balanced ( $\frac{e_b}{e_g} = \frac{n_b}{n_g}$ ) and referrals are balanced ( $\frac{R_b}{R_g} = \frac{n_b}{n_g}$ ), then outcomes are equal for the two groups, regardless of the degree of homophily. On the other hand, should either of these conditions strictly favor one group, for instance blues ( $\frac{e_b}{e_g} \geq \frac{n_b}{n_g}$  and  $\frac{R_b}{R_g} \geq \frac{n_b}{n_g}$  with at least one strict inequality), then blue workers have a lower probability of no referrals and a higher probability of multiple referrals compared to green workers, resulting in better wages and higher employment rates.

Such an advantage also leads to a worse lemons effect for blues since *relatively* more of them are screened, and rejected, and so the average values of blues in the pool is worse than the average value of greens in the pool. As the expected productivity of employed and unemployed workers needs to average out, and unemployed blues have a lower value than unemployed greens, this then implies that employed blue workers have higher productivity than employed green workers. This is consistent with the observation that referral-hired workers tend to outperform other workers (again, see the discussion in the Introduction).

This discussion is summarized in the following proposition.

**PROPOSITION 3.** *If there is a (weak) employment bias ( $\frac{e_b}{e_g} \geq \frac{n_b}{n_g}$ ) and a (weak) referral imbalance ( $\frac{R_b}{R_g} \geq \frac{n_b}{n_g}$ ) in favor of blues, then*

- *the wage distribution of blue workers first-order stochastically dominates the wage distribution of green workers,*
- *and the employment rate of blue workers is (weakly) higher than the employment rate of green workers.*

*Furthermore, the average productivity of employed blue workers is at least as high that of employed green workers. Correspondingly, the average productivity of unemployed green workers is at least as high as that of blues. All comparisons are strict if at least one of the bias and imbalance conditions holds with strict inequality.*

We remark that the difference in average productivity of employed workers across types could perpetuate a biased perception of their respective abilities if observers do not understand the selection process. In particular, estimating productivity by looking at employed

populations (the way that productivity is usually measured) systematically overestimates blues' productivity and underestimates greens' productivity.

### 4.3 Productivity and Inequality

Proposition 1 stated that an increase in the probability of no referral increases total production. Thus, if employment gets more skewed away from being balanced, one group is getting relatively more of the referrals increasing both the probability of no referral as well as the probability of multiple referrals as the aggregate referral distribution is given by a mixture Poisson distribution. By Proposition 1, productivity is then lower when employment rates are further from balanced.

**COROLLARY 1.** *Suppose that referrals are made purely homophilous; i.e.  $h_b = h_g = 1$ . Employment ratios that are closer to being population-balanced strictly increase productivity: that is, if  $\frac{e'_b}{e'_g} > \frac{e_b}{e_g} \geq \frac{n_b}{n_g}$ , then the productivity in the equilibrium with current employment  $(e'_b, e'_g)$  is strictly less than the equilibrium productivity with current employment  $(e_b, e_g)$ .*

The proof of Corollary 1 appears in Section B.1 of the supplemental appendix.

This indicates that although an equilibrium is constrained efficient in terms of firms' hiring decisions given a certain mix of referrals, if that mix of referrals can be made more balanced, that would improve overall productivity. We examine such policy interventions in Section 5. However, before moving to policy interventions, we provide some background on when and how a group's relative advantage or disadvantage in employment persists over time.

### 4.4 Comparative Statics in Employment

As a next step in understanding dynamics and policy interventions, it is useful to understand how next period employment changes as we adjust current employment.

Proposition 3 might lead one to conjecture that increasing the level of green employment in one period would lead to an increase in the following period. This turns out to be false. To see why, consider an increase in the current employment of green workers,  $e_g$ , and suppose that groups are purely homophilous, so that  $h_b = h_g = 1$ . The probability of no referrals for green workers decreases while it increases for blues. If we held the equilibrium hiring threshold constant, this would increase the mass of green workers hired via referrals and decrease it for blue workers, and would increase green employment overall. Let us call this the *direct effect* of increasing current green employment. However, the threshold for hiring referrals is not fixed. As we increase current green employment to make it more balanced, the population-wide probability of no referral decreases. This exacerbates the lemons effect,

and thus reduces the referral hiring threshold. Thus, workers with referrals are now more likely to be hired. As there are relatively more blue than green workers with referrals, this *indirect effect* disproportionately benefits blue workers. Depending on the setting, this can overturn the direct effect; so that increasing current green employment leads to fewer greens being hired in the next period than would have been hired otherwise.

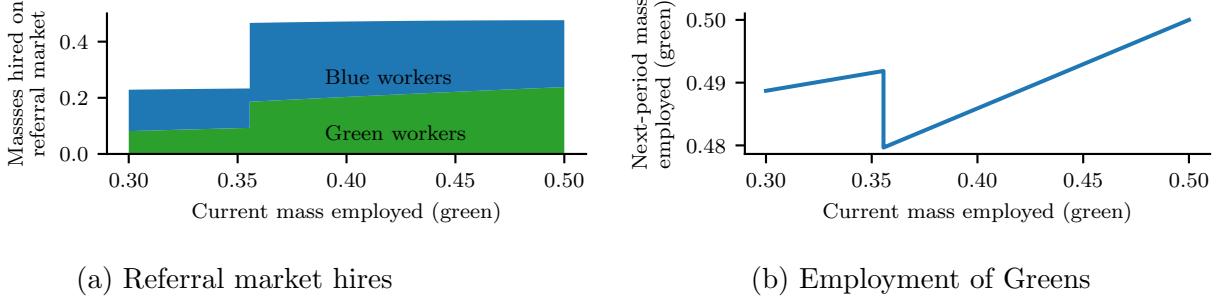


Figure 1: The parameter values are:  $n_b = n_g = .7$  so that  $n = 1.4$ ; and  $v_i$  takes on three values  $0, \frac{1}{3}$ , and  $1$ , with equal probability. We plot the mass of blue and green workers hired on the referral market, and the next-period employment mass of green workers, as a function of the current employment rate of green workers.

Figure 1 shows an example of this phenomenon. Increasing the current mass of employed green workers towards the unbiased employment rate of  $1/2$  increases the number of referrals received by green workers. The direct effect is then seen via the increase in green workers hired on the referral market. However, additionally the overall probability of no referrals decreases, which leads to more lemons in the pool, and so the hiring threshold on the referral market drops. In particular, at a current employment of greens below  $\approx 0.355$ , the threshold is such that referred workers with the middle value of  $\frac{1}{3}$  are not hired whereas they are once green employment rises above  $\approx 0.355$  (see Panel 1a). This change in the equilibrium threshold is the indirect effect: referred workers with the middle productivity value are hired for the higher current employment of green workers; but this disproportionately helps blue workers as they have more referrals. As a result, the next-period employment rate of green workers is lower when their current employment rate is just above  $\approx 0.355$  rather than just below  $\approx 0.355$  (see Panel 1b).<sup>27</sup>

<sup>27</sup>The observed nonmonotonicity may appear due to the discrete nature of our model, but it extends beyond that. There are two sources of discreteness. One is the discrete distribution. That is not essential to this example – a continuous approximation to this distribution leads to a similar result. The second is that the market operates in discrete time. With continuous time the dynamics depend on the relative slopes of the direct and indirect effects, which still go in opposite directions. Regardless, many labor markets are in fact best approximated by a discrete time model due to their seasonal nature.

## 4.5 Dynamics and Immobility Across Periods

We now examine the long-run dynamics of the employment levels of the groups.

In this subsection, we presume that the minimum wage is low enough so that firms find it worthwhile to hire from the pool in steady state.<sup>28</sup> Without this assumption, it is possible to get cycles in whether firms hire from the pool which can preclude convergence.<sup>29</sup>

If one group is more self-biased in giving referrals than the other group, then that group receives relatively more referrals, and they will dominate employment in the long-run. Convergence to balanced employment thus requires a balance in referral rates (recall the definitions in (4)).

**PROPOSITION 4.** *There exists a unique steady-state employment rate for each group. The steady-state employment rates are balanced ( $\frac{e_b}{e_g} = \frac{n_b}{n_g}$ ) if and only if there is referral balance ( $\frac{R_b}{R_g} = \frac{n_b}{n_g}$ ). If there is referral balance, then convergence to the steady state occurs from any starting employment levels.*<sup>30</sup>

Existence of a steady state follows from a standard fixed-point argument. Uniqueness is more subtle, and depends on bounding the slope of how next period employment of a group can grow as a function of increasing current employment of that group. If that “slope” is everywhere less than one, then there can be only one fixed point.<sup>31</sup> The idea is as follows. Adding one extra green today leads to at most one more green referral, and possibly a blue referral or a blue hire. So, the direct effect is at most one. When greens are in the minority, raising green employment increases the number of referrals and makes the lemons effect worse. This lowers the threshold and leads to relatively more blue hires, again working against green hires. Once greens are in the relative referral majority, then further increasing their numbers decreases the overall number of referrals and so actually decreases the lemons effect and raises the threshold – now disadvantaging the greens again since now they are getting more referrals. This establishes the uniqueness. Employment rates equal to population shares is a fixed point if referrals are balanced and not otherwise, and so uniqueness implies that this is the steady state if (and only if) balance holds. Convergence is again more subtle. Balanced referrals can be shown to imply a monotonicity, so that a group that is underemployed gains employment but never more than to a balanced level.

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<sup>28</sup>We identify a candidate steady state by presuming that firms hire from the pool and then checking whether the induced expected value in the pool justifies hiring; in other words, if the minimum wage is low enough relative to the expected value in the pool.

<sup>29</sup>See Section B.2 of the supplemental appendix for an example.

<sup>30</sup>If referral balance fails, then one can construct examples with non-convergence, see Section B.2 of the supplemental appendix.

<sup>31</sup>Generally, for any function  $f(x)$  to have two or more fixed points  $x < x'$  requires that  $f(x') - f(x) = x' - x$ , which cannot occur if the ‘slope’ is less than one. Slope is in quotes since we are not necessarily working with differentiable functions.

However, without referral balance, the indirect effect can dominate and lead to a situation where the change in employment overshoots the steady-state.

Proposition 4 shows that if referrals are balanced, then initial employment rates become irrelevant in the very long run. Of course, that is over generations, and so with high rates of homophily, inequality could persist for many generations.

There are (at least) two ways in which inequality may persist indefinitely. One is, as outlined in the proposition, that there is an imbalance in referral rates – for instance with one group being more homophilous in its referrals than another. A second, is that there may be additional incentives in the matching process that maintain long-run inequality in employment rates. For instance, if workers must make a costly investment – e.g., in education – to realize a productive value, then we may observe perpetual inequality. In particular, since Proposition 3 implies that the expected wage of blue workers of participating in the labor market is greater than that of green workers, if there is some cost to educating one's self before knowing whether one will have a referral, green workers may drop out while blue workers do not. Since poverty traps are analyzed elsewhere, we do not elaborate further here,<sup>32</sup> but we note that this model induces a poverty trap if workers have to invest in order to be productive.

## 5 Affirmative Action and Other Policies

A government can impose policies to alleviate the inequality and immobility that arise in referral networks. Moreover, as we have seen above, this not only improves ‘fairness’, but can also increase overall productivity. We now analyze some prominent policies in the context of the blue-green setting described above.

For simplicity, throughout this section, we specialize to the situation in which blues and greens are both completely homophilous, so that  $h_b = h_g = 1$ . We also suppose that each currently employed worker picks one person from the next generation of their group, uniformly at random, to refer; and presume that the minimum wage represents the value of a worker outside of employment – so the outside productive value. Lastly, we presume that firms hire when indifferent ( $r = 1$ ).<sup>33</sup> These restrictions simplify the exposition, but are not necessary for the results that follow.

Without loss of generality, suppose that blues are relatively advantaged in starting em-

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<sup>32</sup>For example, see Calvo-Armengol and Jackson (2004) for a discussion of a poverty trap in a job-referral setting.

<sup>33</sup>This assumption is only consequential when firms do not hire from the pool, as then the equilibrium threshold ( $w_{min}$ ) does not adjust to small changes in employment. As a result, firms could make more or fewer hires on the referral market. As long as there are no atoms in the distribution of productivities at exactly the min wage,  $\Pr(v_i = w_{min}) = 0$ , we do not have to worry about this issue.

ployment, so that  $\frac{e_b}{e_g} > \frac{n_b}{n_g}$ .

## 5.1 Affirmative Action's Dynamic Impact

Affirmative action policies change hiring thresholds with the intent of moving the employment distribution to be closer to balanced. This not only has immediate effects, but can also have longer term effects since referrals are also moved closer to being balanced. This points out an interesting aspect of affirmative action: it not only increases short-term equality of employment based on characteristics, but it also has long-term network effects that further reduce inequality and improve mobility and future productivity.

The complication, as we have seen above, is that a change in employment rates can also impact the equilibrium referral hiring threshold, which can result in an indirect effect that can be countervailing. In spite of that effect, there is much that we can deduce about the impact of such policies.

First, we show that (generically) a small one-period increase in the employment of the under-employed group increases their employment as well as overall production in all subsequent periods.

**PROPOSITION 5.** *Let  $F$  be any discrete distribution. For almost every  $e_g$ , there exists  $\varepsilon > 0$  so that an increase in  $e_g$  by up to  $\varepsilon$  leads to a strict increase in total production as well as green employment in all future periods (relative to what they would have been without the increase).*

Thus, affirmative action has long-run implications and even a one-time policy can have long lasting positive effects on both equality and total production.

We illustrate these dynamic effects in Figure 2, which pictures the impact of a one time increase in the employment of greens. We see three long-term effects. First, increasing current employment of greens increases their employment in all periods relative to what it would have been without. Second, the difference in average wages between blues and greens is decreased in all periods. Third, this also improves production in all subsequent periods, but has a cost in the current period from replacing referred blues with draws from the pool to get sufficient greens.

The above example works cleanly since it only involves two productivity levels. With more productivity levels, large changes in employment can lead to threshold reversals and so large affirmative action policies can involve some non-monotonicities in some future periods. In

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<sup>34</sup>As discussed below, this is the more efficient form of affirmative action in this case. Note that we consider a policy that induces enough firms who have blue referrals to go to the pool to effect this change, whether it be via regulation or a payment that compensates the firms forced to go to the pool when having a high value blue referral. We remark that blue workers with multiple referrals are less likely than ones with just one referral to be affected by this policy, since firms are independently chosen.

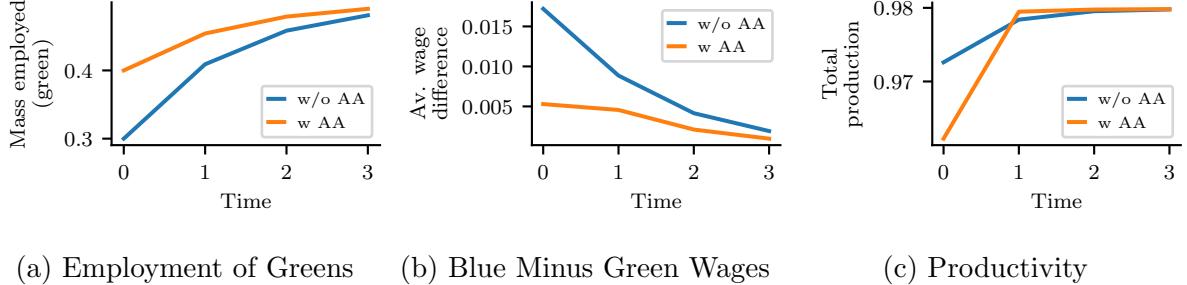


Figure 2: Blue is without affirmative action and orange is with affirmative action in the first period. The graphs above show the dynamic effects of changing green employment in from .3 to .4 by randomly choosing some firms with high value referred blues and having those firms to go to the pool.<sup>34</sup> The productivity has a .95 probability of being 1 and a .05 probability of being 0, the minimum wage is 0, and  $n_b = n_g = 1$ .

particular, as shown in Section 4.2, an increase in the current employment of a disadvantaged group does not necessarily increase their next-period employment rate compared to what it would have been, due to the indirect effect operating through the hiring threshold. As a result, employment rates in some future period can, under some particular situations, be further apart than they would have been without an affirmative action policy (furthermore implying a decrease in total production in that period relative to that without the policy). As shown in Proposition 4, there would still be eventual convergence, given referral balance, but it could be sped up in some periods and slowed down in others. Thus, care is warranted in designing such policies given the potential indirect effects due to changes in equilibrium lemons effects.

The long-term impact of affirmative action policies from a one-time intervention is important in light of the issues raised by Coate and Loury (1993). They show that a long-term affirmative action policy can make a group dependent upon the policy and reduce their skill acquisition. Our results showing that a one-time intervention can have a lasting impact implies that such policies can be designed to be temporary, thus avoiding the Coate and Loury (1993) conundrum.

## 5.2 Optimizing Affirmative Action

Next, we examine which of the two basic approaches to affirmative action – encouraging the hiring of greens versus discouraging the hiring of blues – is more effective.

More specifically, one can increase green employment by either encouraging the hiring of more greens from referrals (which decreases the number of blues hired from the pool), or discouraging the hiring of blues from referrals (and thus hiring relatively more greens from the pool). One can enact these by, for instance, paying a small subsidy for each green

worker who is hired or else taxing firms that hire a blue worker.<sup>35</sup> One can also think of a broader class of interventions that include quotas and caps for hires on the referral market. To capture the tradeoff between promoting green workers and demoting blue workers, we compare the effects of increasing the mass of green workers versus decreasing the mass of blue workers hired on the referral market.

Both types of policies distort the optimal hiring decision so that total production decreases in the period where the affirmative action is put in place, but have the longer term impact of increasing productivity in future periods. In general, one of the two policies will dominate the other, in the sense that it achieves the same desired goal at a lower reduction in current production.

We illustrate this trade-off in the simple case in which workers are either of high or low value,  $v_H$  or  $v_L$ . Let  $\tilde{v}$  be the expected value in the pool in the absence of any intervention. Without any intervention, high-valued referrals are hired, and otherwise the firm hires from the pool. Consider a small intervention – just changing a few hires so that we don't effect the pool value substantially. If one increases the number of green referrals hired, then that puts in a low valued worker in place of a random draw from the pool, so the lost productivity is  $\tilde{v} - v_L$ . If one decreases the number of blue referrals hired, then that puts in a random draw from the pool in place of a high valued worker, so the lost productivity is  $v_H - \tilde{v}$ . Which of these is better depends on whether the value of the pool is closer to the high value or the low value.

**PROPOSITION 6.** *One prefers to increase green employment by some  $\varepsilon$  close to 0 by increasing green referral hires if and only if*

$$\tilde{v} - v_L < v_H - \tilde{v}.$$

*Furthermore, as long as both policies can achieve the desired employment bias, using one of them as opposed to a combination of them is optimal.*

The proof of Proposition 6 appears in Section B.1 of the supplemental appendix.

### 5.3 Forward-Looking Firms

Our analysis has examined myopic firms. For most of our analysis myopic and forward-looking firms would act similarly, since the value of the future referrals only differ across workers once we get to our green-blue analysis. When greens are relatively under-employed, the referrals from the greens are more valuable than referrals from blues, since greens are more likely to refer other greens and those referred greens are less likely to have multiple referrals than a referred blue. So, a firm expects to hire referred greens at lower wages than

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<sup>35</sup>We presume that a firm can tell if a referred worker is blue or green.

referred blues. This provides a natural form of affirmative action: firms can have a (slight) preference for hiring the relatively disadvantaged group compared to the advantaged group, both at the referral stage and the pool stage.<sup>36</sup>

If the green/blue distinction can be observed by firms, then that may cause them to (slightly) prefer to hire greens. To illustrate this effect, we revisit the example from Figure 2, but instead of affirmative action, we consider what happens if firms can see whether a worker is blue or green, and can repeatedly search in the pool by paying a search cost each time. Firms' costs of repeated search are distributed uniformly on  $[0,1]$ .<sup>37</sup>

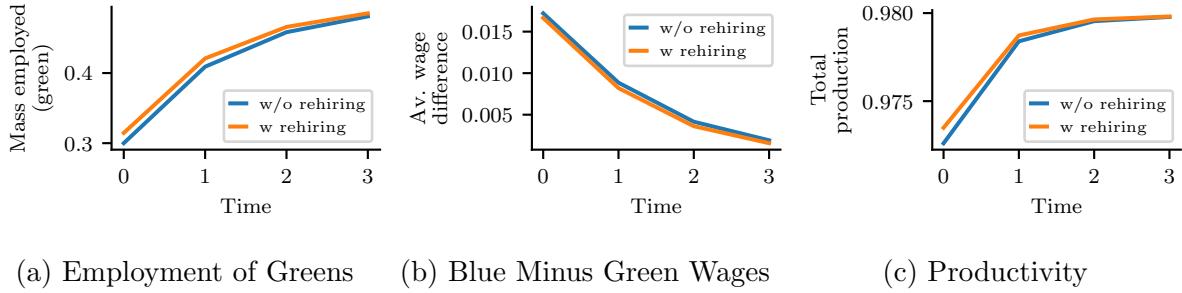


Figure 3: Blue is without affirmative action nor redraws from the pool. Orange is when firms can redraw from the pool. The parameters are as in Figure 2, with costs of redrawing from the pool uniformly distributed on  $[0,1]$ .

Without being able to distinguish blues and greens, the firms would never pay additional costs for more draws from the applicant pool; but if they can distinguish greens from blues, then for small enough costs they prefer to keep searching for a green. This means that more greens get hired from the pool. The effects of this policy are pictured in Figure 3. We see that employment and wages both increase for greens in all periods, as does overall productivity. This has similar directional effects as affirmative action, except that productivity increases in all periods rather than only in the future.

## 5.4 Firing Workers and Inequality

Referrals allow firms to learn about the type of a worker and are thus valuable to firms as well as the workers who are referred. Beyond affirmative action, there are other policies affecting what firms learn about workers that can also reduce inequality. In this section, we consider how the market changes when firms can fire a worker and replace them with another worker from an open application.

<sup>36</sup>A second effect is that greens in the pool also suffer from less of a lemons effect, which provides an extra reason for preferring to hire them.

<sup>37</sup>With a continuum of firms, some firms with tiny costs would keep drawing until they get a green, and so a simulation would never end. Figure 3 pictures a single simulation with 100,000,000 firms whose costs are equally spaced on  $[0,1]$ . The simulation ends when all firms made a hire.

In particular, let us suppose that after some time has elapsed, firms have learned the value of any worker they have hired (they already know the value of a referred worker if they kept her, so this applies mainly to a worker hired from the pool), and can then choose whether to fire that worker and hire a new one from the pool for the remaining time. This makes hiring from the pool more attractive, as there is less expected loss if the worker turns out to have low productivity.

Let  $\lambda$  be the fraction of time that is left in the period for which they would get the replacement worker, and  $1 - \lambda$  be the fraction of the period that has to elapse before a firm can fire a worker. In equilibrium, as we prove below, a firm would never fire a referred worker that they kept initially, so the decision will only be relevant for workers they hired from the initial pool.

The timing is as follows:

1. Firms get referrals from their current employees.
2. Firms can choose whether to (try to) hire a referred worker and make an offer.
3. Referred workers with offers can choose to accept any of those offers (in which case they exit with the firm) or to go to the pool, all other workers go to the pool.
4. Firms that did not hire a referred worker can choose whether to hire from the pool.
5. After  $1 - \lambda$  of the period has elapsed, firms can choose to fire their current worker.
6. All fired workers return to the pool, joining all other workers who are not employed.
7. Firms that have fired a worker can choose to hire from the new pool.

We refer to these pools as pool 1 and pool 2, respectively. We assume that the distribution of values is high enough so that firms prefer to hire from these pools than to have no worker at all in order to simplify the exposition; the full details are worked out in the appendix.

A firm's production is given by  $1 - \lambda$  of the value of its worker before any firing, and then  $\lambda$  of its worker that it has after any firing and rehiring decisions are made.<sup>38</sup> An equilibrium is now characterized by a pair of threshold strategies: referred workers are hired if their value is above  $\tilde{v}_1$  and otherwise firms hire from the first pool. Then at the second decision point

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<sup>38</sup>There is another closely related variant of the model for which the proposition below also holds, which is one in which some fraction  $\lambda$  of firms immediately learn the value of their worker (from the pool, and they already know the value from the referral) and can then immediately fire that worker and draw again from the pool. In terms of expected values, this variant of the model looks exactly the same from a firm's perspective, as they just have  $\lambda$  weight on an expectation of firing and rehiring, as opposed to a fraction of time. This second variant does have some differences in terms of values in the pools, as fewer firms have an opportunity to fire workers. We will point out when these differences arise; but the basic structure outlined in Lemma 3 is exactly the same.

firms fire workers, when given the opportunity, if the worker's value is below  $\tilde{v}_2$ . Again, firms can mix with some probability when indifferent.

Let  $E_{\tilde{v}_1, r_1}[v_i | i \in \text{pool 1}]$  be defined as in equation (1) and  $E_{\tilde{v}_1, \tilde{v}_2, r_1, r_2}[v_i | i \in \text{pool 2}]$  denote the expected value in pool 2 if the hiring threshold on the referral market is given by  $\tilde{v}_1$ , the firing threshold by  $\tilde{v}_2$ , firms hire referral workers at the margin with probability  $r_1$  and firms fire workers at the margin with probability  $1 - r_2$ . The threshold strategies then imply the following two equilibrium conditions:

$$\tilde{v}_1 = (1 - \lambda)E_{\tilde{v}_1, r_1}[v_i | i \in \text{pool 1}] + \lambda E_{\tilde{v}_1, r_1}[\max\{v_i, \tilde{v}_2\} | i \in \text{pool 1}]; \quad (5)$$

$$\tilde{v}_2 = E_{\tilde{v}_1, \tilde{v}_2, r_1, r_2}[v_i | i \in \text{pool 2}]. \quad (6)$$

Let  $\tilde{v}$  denote the hiring threshold for the model without firing (so that  $\lambda = 0$ ). For  $\lambda \in (0, 1]$ , not hiring a referred worker and instead going to pool 1, now features an *option value*: The firm randomly draws a worker but may get a second draw to replace the worker if the worker's value is too low; a draw from pool 2. As a result, firms have a higher threshold on the referral market compared to the base model. This lessens the lemons effect in the first pool and so the first pool value is actually higher than  $\tilde{v}$ . The second pool, however, has a worse lemons effect since it then includes all fired workers as well as rejected referral workers. The fact that the second threshold is less than  $\tilde{v}$  takes some proof, but is true. Essentially, the extra opportunity to fire improves the overall productivity, and so the workers left in the second period pool are worse than in the case where there is just one chance to hire.

**LEMMA 3.** *There are unique thresholds  $(\tilde{v}_1, \tilde{v}_2)$  and numbers  $(r_1, r_2)$  with  $r_2$  arbitrary satisfying equations (5) and (6). For  $\lambda > 0$ , they satisfy  $\tilde{v}_2 < \tilde{v} < \tilde{v}_1$ .*

Note that our base model is nested in this formulation with  $\lambda = 0$ .

The equilibrium thresholds, as before, are unique, so that we can perform comparative statics with respect to unique equilibrium quantities. Firms have a higher hiring threshold on the referral market when  $\lambda > 0$  and rely more on search via open applications. The additional weight on open applications in the hiring process increases the opportunities for disadvantaged groups – e.g., the green workers – and improves their employment prospects. The willingness of firms to engage in further search decreases the production in the first part of the period.

However, as the proposition below shows, overall production – the weighted production in the two parts of the period – increases with  $\lambda$ . Thus, efficiency and equality are always greater when there are opportunities for firms to fire their worker.

**PROPOSITION 7.** *For any  $\lambda > 0$ , the employment rate bias is less than, and total production is greater than, what it would have been without the opportunity to fire ( $\lambda = 0$ ). In terms*

*of timing within the period: (i) both the employment bias in favor of blue workers and productivity measured before the firing stage are decreasing in  $\lambda$ ; and (ii) the productivity at the end of the period is increasing in  $\lambda$  while the employment bias in favor of blue workers at the end of the period changes ambiguously.*

The proof of Proposition 7 appears in Section B.1 of the supplemental appendix.

Comparing productivity and equality between two different positive values of  $\lambda$  is nuanced. While efficiency unambiguously increases the greater the opportunities for firms to fire a worker, the employment rate bias at the end of the period may behave nonmonotonically in  $\lambda$ . More use of the first pool leads to a worse lemons effect in the second pool, which can disadvantage the greens. As a result, the overall implication for the employment rate bias measured at the end of the period is ambiguous.

## 6 Concluding Discussion

We have shown how referrals can lead to inequality, inefficient production, and persistent differences across groups. We have also shown that there is a relationship between inequality and inefficiency: concentrating referrals decreases productivity and increases inequality. However, we have also showed that the relationships between these are more subtle than one might superficially imagine, due to the lemons effect, in which hiring from a pool of workers to which a firm has no referral can be more or less attractive depending on the extent to which that pool contains workers that were rejected by other firms. A sufficiently large change in the lemons effect can reverse one or both of the relationships between the concentration of referrals and productivity and inequality.

As we have seen, inequality, inefficiency, and immobility can be counteracted by policies – for instance, affirmative action policies – that encourage hiring from the pool. These policies have long-run impacts even if enacted for short periods. We focused on whether the policy penalized the hiring of blues, or enhanced the hiring of greens; but there are many ways in which this can be done. For instance, requiring interviewing of qualified minorities for open positions as in many sports leagues for coaching positions, is a way to force firms to investigate workers from the pool. There are also other policies that have similar effects, for instance simply subsidizing internships of workers from the pool, or providing more information about non-referred workers, via various sorts of certification. In addition, even though full efficiency is not reached in the setting with firing, the ability to fire does improve overall productivity and reduce inequality; which provides insight into one advantage of flexible labor markets.

The model that we have presented here should be useful as a base for additional explorations. For instance, we have not modeled how firms choose how many people to hire.

Firms' decisions on hiring could react to the attractiveness of the pool and how much access they have to referrals,<sup>39</sup> and so wider-spread referrals could also lead to the creation of new openings, amplifying some of the effects we have shown. Another effect of referrals is on how long workers last at their current jobs, and allowing for reduced turnover due to referrals would also be interesting to explore, but would require changes to the model. One could also explore career choices using this model as a base. People would (inefficiently) be driven to choose professions of their family and friends, even if they might be more talented in another profession, because of the improved wage and employment prospects due to referral connections. Finally, firms sometimes hire referred workers as favors to their current employees, even when the referred workers are of low value. That effect would exacerbate the advantage to referrals, and lead to further increases in inequality and immobility across groups.

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<sup>39</sup>For an analysis of the size of firms in reaction to information about labor markets, see Chandrasekhar, Morten and Peter (2020).

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## A Proofs Omitted from the Text

*Proof of Lemma 1.* We first argue the existence of the fixed point. Define a correspondence  $G$  by

$$G(\tilde{v}) = \{\max [w_{\min}, \mathbb{E}_{\tilde{v},r}[v_i | i \in \text{pool}]] | r \in [0, 1]\}.$$

Varying over  $r$  has an impact on the expected value in the pool at atoms in the distribution (that differ from a fixed point). The existence of an equilibrium threshold follows from the fact that  $G$  is convex- and compact-valued and upper-hemicontinuous, and is bounded above (by  $\mathbb{E}[v_i]$  from Lemma 2) and below (by  $w_{\min}$ ). Thus, by Kakutani's Theorem there is a fixed point. The fact that firms can mix arbitrarily at a fixed point, comes from the fact that if the equilibrium threshold equals the expected value in the pool,  $\tilde{v} = \mathbb{E}_{\tilde{v},r}[v_i | i \in \text{pool}]$ , the value of the referred worker and the expected value in the pool are then exactly equal, and then keeping or putting such a worker into the pool does not change the average value in the pool. If the equilibrium threshold equals the minimum wage,  $\tilde{v} = w_{\min}$ , and the expected value in the pool is strictly less, then adding or removing the referred worker does not change the fact that the expected value in the pool is strictly less than the equilibrium threshold.

The uniqueness of the fixed point is shown as follows. Suppose to the contrary that there are two distinct equilibrium thresholds,  $\tilde{v} < \tilde{v}'$ . We provide the argument for the case in which  $\tilde{v}' > w_{\min}$  as otherwise it must be that  $\tilde{v} = \tilde{v}' = w_{\min}$ , since the correspondence is lower-bounded by  $w_{\min}$ .

The anonymous pool for threshold  $\tilde{v}'$  consists of workers without any referral, referred workers with values not exceeding  $\tilde{v}$ , and referred workers with value between  $\tilde{v}$  and  $\tilde{v}'$  (and possibly some at exactly  $\tilde{v}'$  depending on the mixing). The expected productivity of workers without referrals and referred workers with values not exceeding  $\tilde{v}$  is at most  $\tilde{v}$ , since  $\tilde{v}$  is an equilibrium threshold. Averaging the value of this group with the expected value of workers with referral and values between  $\tilde{v}$  and  $\tilde{v}'$  gives the expected productivity in the pool if the threshold is  $\tilde{v}'$ ; but this cannot result in an expected value of  $\tilde{v}'$  as  $\tilde{v} < \tilde{v}'$ .

The equilibrium behavior of firms and the wages of workers are immediate.  $\square$

*Proof of Lemma 2.* Clearly  $E_{\tilde{v},r}[v_i | i \in \text{pool}] \leq E[v_i]$ , since the pool is a convex combination of workers who have no referrals, who thus have the unconditional expected value, and workers who had referrals but had a value no higher than a threshold, and thus have expected values no higher than the unconditional expected value. If the threshold is equal to the minimum value of  $F$ , then the expected value in the pool is as well and we are done. Thus, since  $P(0) < 1$ , there are some referrals, and since  $F$  is nondegenerate it follows that there is a positive mass of workers with values above and a positive mass below the threshold (which is at most the expected value and strictly greater than the minimum value), and thus the expected value of rejected referred workers is strictly below the unconditional expected value, which then means that the expected value of the pool is also below.  $\square$

*Proof of Proposition 1.* Let  $G(\tilde{v}, P(0)) := E_{\tilde{v}}[v_i | i \in \text{pool}]$ ; i.e.,

$$\begin{aligned} G(\tilde{v}, P(0)) &:= \frac{P(0)E[v_i] + (1 - P(0))\Pr(v_i < \tilde{v})E[v_i | v_i < \tilde{v}]}{P(0) + (1 - P(0))\Pr(v_i < \tilde{v})}. \\ &= E[v_i] \frac{P(0) + (1 - P(0))\Pr(v_i < \tilde{v}) \frac{E[v_i | v_i < \tilde{v}]}{E[v_i]}}{P(0) + (1 - P(0))\Pr(v_i < \tilde{v})}. \end{aligned}$$

This expression is nondecreasing in  $\tilde{v}$ . For any  $\tilde{v}$ , as  $E_{\tilde{v},r}[v_i | i \in \text{pool}] < E[v_i]$  by Lemma 2, it is strictly increasing in  $P(0)$  as well. Therefore

$$G(\tilde{v}, P(0)) \leq G(\tilde{v}, P'(0)) \leq G(\tilde{v}', P'(0)),$$

where the first inequality is strict if  $P'(0) > P(0)$ . As we've assumed  $r = 1$ ,  $G(\tilde{v}, P(0)) = E_{\tilde{v}}[v_i | i \in \text{pool}]$  and  $G(\tilde{v}', P'(0)) = E_{\tilde{v}'}[v_i | i \in \text{pool}]$ , this proves the first claimed comparative static.

To prove the second claimed comparative static, note that by the equilibrium condition, employed workers always have higher average productivity than the productivity of unemployed workers in the broader economy (which is assumed to be equal to  $w_{min}$ ). Therefore, to compare the productivities in the two economies, it suffices to compare the mass and productivity of employed workers.

We consider three cases based on whether firms hire from the pool in equilibrium.

First suppose that firms hire from the pool in both economies. Then the mass of employed workers is the same in both economies, and thus the value of the unemployed is the same in both cases. Note that, by a simple accounting argument, the productivity of employed workers is  $nE[v]$  minus the value of those not hired from the pool ( $(n - 1)E_{\tilde{v}}[v_i | i \in \text{pool}]$ , or  $(n - 1)E_{\tilde{v}'}[v_i | i \in \text{pool}]$ ). Those not hired from the pool have value given by  $G$  above, and so the productivity moves in the opposite direction as  $G$  and the result follows from the first claim.

Next, suppose that they do not hire from the pool in either case. Then by the equilibrium condition, the hiring threshold in both economies is  $w_{min}$ . Therefore, the productivity of employed

workers in the two economies is the same (namely,  $E[v_i | v_i \geq w_{min}]$ , which is higher than the minimum wage). However, as there are weakly more referrals under  $P$ , there are weakly more employed workers and so productivity is weakly higher in that economy. The inequalities hold strictly if  $P'(0) > P(0)$  (as then there are strictly more referrals under  $P$ ).

Finally suppose firms hire from the pool in one economy and not the other. Therefore, by the equilibrium condition and the first comparative static, it must be that

$$E_{\tilde{v}}[v_i | i \in \text{pool}] < w_{min} \leq E_{\tilde{v}}[v_i | i \in \text{pool}].$$

Thus  $G(\tilde{v}', P'(0)) = \tilde{v}'$  and  $G(\tilde{v}, P(0)) < \tilde{v} = w_{min}$ . Consider  $\bar{P}$  so that  $G(\tilde{v}, \bar{P}(0)) = w_{min}$ .<sup>40</sup> Then  $G(\tilde{v}', P'(0)) \geq G(\tilde{v}, \bar{P}(0)) \geq G(\tilde{v}, P(0))$  and so, by the converse of the first comparative static, it must be that

$$P(0) \leq \bar{P}(0) \leq P'(0).$$

<sup>41</sup> Supposing firms do not hire workers from the pool given  $\bar{P}$ , we are in the first case and can conclude that productivity is higher given  $P$  than  $\bar{P}$  (and strictly so if  $P(0) < \bar{P}(0)$ ). Given that firms hire workers from the pool given  $\bar{P}$ , we are in the second case and can conclude that productivity is weakly higher given  $\bar{P}$  than  $P'$  (and strictly so if  $\bar{P}(0) < P'(0)$ ). Combining these two inequalities, it follows that productivity is higher in the economy with  $P$  than in the economy with  $P'$  (and strictly so if  $P'(0) > P(0)$ ).  $\square$

*Proof of Proposition 3.* Note that referral imbalance and employment bias imply that

$$\frac{n_b}{n_g} \leq \frac{r_b}{r_g} = \frac{h_b n_b + (1 - h_g) n_g}{h_g n_g + (1 - h_b) n_b} = \frac{h_b \frac{n_b}{n} + (1 - h_g) \frac{n_g}{n}}{h_g \frac{n_g}{n} + (1 - h_b) \frac{n_b}{n}} \leq \frac{h_b e_b + (1 - h_g) e_g}{h_g e_g + (1 - h_b) e_b}.$$

where the last inequality also uses the assumption that  $h_b \geq 1 - h_g$ . The last expression gives the ratio of the masses of referrals received. The inequality implies that blue workers receive more referrals per capita than green workers.

The first claimed comparative static now follows from the fact that the wage distribution of a worker with at least 2 referrals first-order stochastically dominates the wage distribution of a worker with exactly 1 referral which first-order stochastically dominates the wage distribution of a worker with no referrals; and, by our assumption that the distribution of referrals within a group is Poisson, the referral distribution of blue workers first-order stochastically dominates that of green workers. The second comparative static also follows as having more referrals per capita implies a higher the employment rate. As the inequality above becomes strict if either of the conditions is strict, so do the first-order dominance comparison of the wage distributions and the difference in employment rates.

We next show the average productivity of employed green workers is lower than that of employed blue workers. Workers with value less than  $\tilde{v}$  have the same probability of being hired regardless of their group as they are hired from the pool. Workers with value more than  $\tilde{v}$  are hired on the referral market if they have at least one referral. As blues have (weakly) more referrals than greens, again by our assumption that the distribution of referrals within a group is Poisson, this implies blue workers with value more than  $\tilde{v}$  are (weakly) more likely to be hired overall than green workers with value more than  $\tilde{v}$  (and strictly so if either of the conditions is strict). Together, these observations show the desired comparison of the productivity of employed green versus blue workers.

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<sup>40</sup>Such  $\bar{P}$  exists since  $G$  is continuous in  $P(0)$ .

<sup>41</sup>The converse follows immediately from the contrapositive and switching  $P(0)$  and  $P'(0)$ .

Finally, we show that the average productivity of unemployed green workers is at least as high as that of blues. Note that the productivity of an unemployed green (resp. blue) worker is equal to the productivity of a green (resp. blue) worker in the pool. Let

$$\begin{aligned} \mathbb{E}_{\tilde{v},r}^g[v_i | i \in \text{pool} \cap \text{green}] &:= \frac{P_g(0)\mathbb{E}[v_i] + (1 - P_g(0))(\Pr(v_i < \tilde{v})\mathbb{E}[v_i | v_i < \tilde{v}] + \Pr(v_i = \tilde{v})(1 - r)\tilde{v})}{P_g(0) + (1 - P_g(0))(\Pr(v_i < \tilde{v}) + \Pr(v_i = \tilde{v})(1 - r))}; \\ \mathbb{E}_{\tilde{v},r}^b[v_i | i \in \text{pool} \cap \text{blue}] &:= \frac{P_b(0)\mathbb{E}[v_i] + (1 - P_b(0))(\Pr(v_i < \tilde{v})\mathbb{E}[v_i | v_i < \tilde{v}] + \Pr(v_i = \tilde{v})(1 - r)\tilde{v})}{P_b(0) + (1 - P_b(0))(\Pr(v_i < \tilde{v}) + \Pr(v_i = \tilde{v})(1 - r))}. \end{aligned}$$

be the productivity in the pool of greens and blues, where  $P_g(0)$  (resp.  $P_b(0)$ ) is the probability that a green (resp. blue) worker receives no referrals. As  $P_g(0) \geq P_b(0)$  by our assumption that the distribution of referrals within a group is Poisson, it follows that  $\mathbb{E}_{\tilde{v},r}^g[v_i | i \in \text{pool} \cap \text{green}] \geq \mathbb{E}_{\tilde{v},r}^b[v_i | i \in \text{pool} \cap \text{blue}]$ , proving the claim.  $\square$

*Proof of Proposition 4.* We first prove that there exists a unique steady state of the employment rates. Let

$$P_b^t(0 | e_b^t, e_g^t) := \exp\left(-\frac{1}{n_b}(h_b e_b^t + (1 - h_g)e_g^t)\right),$$

be the probability a blue worker receives no referrals given current employment masses  $e_b^t$  and  $e_g^t$  of blues and greens respectively. When it is clear from the context, we suppress the employment masses, writing  $P_b^t(0)$ . Define  $P_g^t(0)$  similarly. Then the difference in employment rates  $\Delta := \frac{e_b^{t+1}}{n_b} - \frac{e_g^{t+1}}{n_g}$ , in period  $t + 1$  is

$$\Delta = (P_b^t(0) \cdot Q^t + (1 - P_b^t(0)) \cdot R^t) - (P_g^t(0) \cdot Q^t + (1 - P_g^t(0)) \cdot R^t)$$

where  $Q^t$  is the probability of being hired in the pool and

$$R^t = (\Pr(v_i > \tilde{v}^t) + r \Pr(v_i = \tilde{v}^t) + Q^t(1 - r) \Pr(v_i = \tilde{v}^t) + Q^t \Pr(v_i < \tilde{v}^t))$$

is the probability of being hired given that you received a referral. Simplifying, we see

$$\Delta = (P_b^t(0) - P_g^t(0)) \cdot (Q^t - R^t). \quad (7)$$

Note that the right-hand side of (7) gives a convex- and compact-valued, upper hemicontinuous correspondence with respect to period  $t$  employment masses if one varies  $r$  whenever an atom at the referral threshold exists. Furthermore, if  $e_b^t = \min\{1, n_b\}$ , then mechanically it must be that  $e_b^{t+1} \leq e_b^t$  (and thus  $\Delta \leq \frac{e_b^t}{n_b} - \frac{e_g^t}{n_g}$ ). Similarly, if  $e_g^t = \min\{1, n_g\}$ , then  $e_g^{t+1} \leq e_g^t$  (and thus  $\Delta \geq \frac{e_b^t}{n_b} - \frac{e_g^t}{n_g}$ ). Thus, a fixed point of employment rates (or equivalently employment or referral masses) exists. By assumption, the induced expected value in the pool is above the minimum wage so that firms indeed hire from the pool.

Next, we show that the fixed point is unique. If  $h_b = 1 - h_g$ , then referrals per capita are always equal across groups and the unique fixed point is characterized by equal employment rates. Thus, assume that  $h_b > 1 - h_g$  for the remainder of the proof.

We make use of the following lemma.

LEMMA 4. Let  $\bar{e}_b$  and  $\bar{e}_g$  with  $\bar{e}_g = 1 - \bar{e}_b$  uniquely solve

$$\frac{1}{n_b}(h_b\bar{e}_b + (1 - h_g)\bar{e}_g) = \frac{1}{n_g}(h_g\bar{e}_g + (1 - h_b)\bar{e}_b),$$

i.e., the employment masses at which referral received per capita are equal across the two groups.

Take any two employment levels in period  $t$ ,  $e_b^t$  and  $\tilde{e}_b^t$ , and let the next period employment levels be denoted by  $e_b^{t+1}$  and  $\tilde{e}_b^{t+1}$  respectively; with  $e_g^t = 1 - e_b^t$ ,  $\tilde{e}_g^t = 1 - \tilde{e}_b^t$ ,  $e_g^{t+1} = 1 - e_b^{t+1}$ ,  $\tilde{e}_g^{t+1} = 1 - \tilde{e}_b^{t+1}$ . If  $\tilde{e}_b^t > e_b^t \geq \bar{e}_b$ , then

$$\left( \frac{\tilde{e}_b^{t+1}}{n_b} - \frac{\tilde{e}_g^{t+1}}{n_g} \right) - \left( \frac{e_b^{t+1}}{n_b} - \frac{e_g^{t+1}}{n_g} \right) < \left( \frac{\tilde{e}_b^t}{n_b} - \frac{\tilde{e}_g^t}{n_g} \right) - \left( \frac{e_b^t}{n_b} - \frac{e_g^t}{n_g} \right).$$

Similarly, if  $\tilde{e}_b^t < e_b^t \leq \bar{e}_b$ , then

$$\left( \frac{\tilde{e}_b^{t+1}}{n_b} - \frac{\tilde{e}_g^{t+1}}{n_g} \right) - \left( \frac{e_b^{t+1}}{n_b} - \frac{e_g^{t+1}}{n_g} \right) > \left( \frac{\tilde{e}_b^t}{n_b} - \frac{\tilde{e}_g^t}{n_g} \right) - \left( \frac{e_b^t}{n_b} - \frac{e_g^t}{n_g} \right).$$

The proof of Lemma 4 is given in Section B.1 of the supplemental appendix.

Let  $e_b$  be a fixed point and consider any employment level  $\hat{e}_b^t \neq e_b$ ; with  $\hat{e}_g^t = 1 - \hat{e}_b^t$ . If  $e_b, \hat{e}_b^t \geq \bar{e}_b$  or  $e_b, \hat{e}_b^t \leq \bar{e}_b$ , then Lemma 4 directly implies that  $\hat{e}_b^t$  is not a fixed point. To see this, e.g. consider the case  $e_b > \hat{e}_b^t \geq \bar{e}_b$ . By Lemma 4, letting  $e_b^t = \hat{e}_b^t$  and  $\tilde{e}_b^t = e_b$ , we have

$$\left( \frac{e_b}{n_b} - \frac{e_g}{n_g} \right) - \left( \frac{\hat{e}_b^{t+1}}{n_b} - \frac{\hat{e}_g^{t+1}}{n_g} \right) < \left( \frac{e_b}{n_b} - \frac{e_g}{n_g} \right) - \left( \frac{\hat{e}_b^t}{n_b} - \frac{\hat{e}_g^t}{n_g} \right),$$

as  $e_b$  is a fixed point so that  $\hat{e}_b^t$  is not be a fixed point.

Suppose now that  $e_b < \bar{e}_b \leq \hat{e}_b^t$ . By Lemma 4 letting  $\tilde{e}_b^t = e_b$  and  $e_b^t = \bar{e}_b$ , we have

$$\left( \frac{e_b}{n_b} - \frac{e_g}{n_g} \right) - 0 > \left( \frac{e_b}{n_b} - \frac{e_g}{n_g} \right) - \left( \frac{\bar{e}_b}{n_b} - \frac{\bar{e}_g}{n_g} \right),$$

as referrals received per capita are equal when the employment of blue workers is given by  $\bar{e}_b$ , and again as  $e_b$  is a fixed point. Again by Lemma 4, now letting  $e_b^t = \bar{e}_b$  and  $\tilde{e}_b^t = \hat{e}_b^t$ , we have

$$\left( \frac{\hat{e}_b^{t+1}}{n_b} - \frac{\hat{e}_g^{t+1}}{n_g} \right) - 0 \leq \left( \frac{\hat{e}_b^t}{n_b} - \frac{\hat{e}_g^t}{n_g} \right) - \left( \frac{\bar{e}_b}{n_b} - \frac{\bar{e}_g}{n_g} \right),$$

where equality holds if  $\hat{e}_b^t = \bar{e}_b$ . Combining the two previous inequality yields

$$\frac{\hat{e}_b^{t+1}}{n_b} - \frac{\hat{e}_g^{t+1}}{n_g} < \frac{\hat{e}_b^t}{n_b} - \frac{\hat{e}_g^t}{n_g},$$

so that  $\hat{e}_b^t$  is not a fixed point.

Lastly, the final case,  $\hat{e}_b^t \leq \bar{e}_b < e_b$ , is proven analogously to the previous case by interchanging the subscripts as the role of green and blue workers is symmetric, and is thus omitted.

Let us now show that referral imbalance goes hand in hand with long-run bias in employment rates. As before, let  $e_b, e_g$  be the fix point; further, let  $R_b(e_b, e_g)$  denote the associated mass of

referrals received; with  $e_g = 1 - e_b$  and  $R_g(e_b, e_g)$  defined analogously. We have

$$\frac{e_b}{e_g} = \frac{n_b}{n_g} \iff \frac{R_b(e_b, e_g)}{R_g(e_b, e_g)} = \frac{n_b}{n_g} \iff \frac{e_b h_b + e_g(1 - h_g)}{e_g h_g + e_b(1 - h_b)} = \frac{n_b}{n_g} \iff \frac{R_b}{R_g} = \frac{n_b}{n_g}.$$

As employment and referral are positively linked, we can further say that  $\frac{e_b}{e_g} \geq \frac{n_b}{n_g} \iff \frac{R_b}{R_g} \geq \frac{n_b}{n_g}$ .

Finally, we show that the system of employment rates converges to this fixed point if there is no referral imbalance, i.e.  $\frac{R_b}{R_g} = \frac{n_b}{n_g}$ . As there is no referral imbalance, the unique fixed point features equal employment rates.

First, by assumption, the expected value in the pool induced by the steady state of equal employment rates is above the minimum wage. As with referral balance, the probability no referrals is minimized with equal employment rates, so is the expected value in the pool. Hence, for any employment levels, firms will be hiring from the pool.

Consider any initial employment,  $e_b^t, e_g^t$ , and suppose without loss that  $\frac{e_b^t}{e_g^t} > \frac{n_b}{n_g}$ . Then

$$h_b e_b^t + (1 - h_g) e_g^t > R_b, \quad h_g e_g^t + (1 - h_b) e_b^t < R_g$$

implying

$$\frac{h_b e_b^t + (1 - h_g) e_g^t}{h_g e_g^t + (1 - h_b) e_b^t} > \frac{R_b}{R_g} = \frac{n_b}{n_g}.$$

Therefore blue workers receive relatively more referrals per capita than green workers. Consequently, blue workers have a lower probability of no referral and thus have a higher employment rate at the end of the period than greens. By Lemma 4, letting  $e_b^t = \bar{e}_b = \frac{n_b}{n}$  and  $\tilde{e}_b^t = e_b^t$ , we have

$$\frac{e_b^{t+1}}{n_b} - \frac{e_g^{t+1}}{n_g} < \frac{e_b^t}{n_b} - \frac{e_g^t}{n_g},$$

i.e. the employment of blue workers is decreasing whenever  $\frac{e_b^t}{e_g^t} > \frac{e_b}{e_g}$ . As the employment of blue workers is bounded below, because per capita it is always greater than that of green workers, it must converge, and therefore converges to the unique fixed point.

□

## B Online Supplementary Material

### B.1 Additional Proofs

*Proof of Proposition 5.* Recall  $h_g = h_b = 1$  and the initial employment in period one, i.e.,  $e_g^0 := e_g$  is biased (i.e.,  $\frac{e_g^0}{e_b^0} < \frac{n_g}{n_b}$ ). A small increase in  $e_g^0$  will not change the bias in employment. Thus, by Proposition 3 the initial employment in any period  $t$  is biased with and without the increase in  $e_g^0$  (i.e.,  $\frac{e_g^{t-1}}{e_b^{t-1}} < \frac{n_g}{n_b}$  for all finite  $t \geq 1$ ).

Therefore, if green employment strictly increase in all future periods, this implies a strict increases in total production as well by Corollary 1. The remainder of this proof is thus devoted to showing there is a strict increase in green employment in all future periods.

The proof consists of three steps. We first show that for all but finitely many levels of initial employment  $e_g^{t-1}$  of green workers, final employment  $e_g^t$  is strictly increasing and continuous in  $e_g^{t-1}$ . We then use this to argue that, over a finite number of periods  $1, \dots, T$ , for all but finitely many initial employment levels  $e_g^0$ , final employment  $e_g^t$  is strictly increasing and continuous in  $e_g^0$ . Finally, we argue that we can choose  $T$  large enough that the convergence in employment levels from period  $T$  onward is monotone.

**Step 1:** We first show that for all but finitely many initial employment levels  $e_g^{t-1}$  of green workers in period  $t$ , final employment  $e_g^t$  is strictly increasing and continuous in  $e_g^{t-1}$ . Let  $\tilde{v}(e_g)$  denote the equilibrium threshold if initial green employment is given by  $e_g$ .

We proceed in cases. First suppose that the employment level is such that firms weakly prefer not to hire from the pool, i.e., employment levels in the set  $B := \{e_g : \tilde{v}(e_g) = w_{min}\}$ . If firms do not hire from the pool, then employment  $e_g^t$  is strictly increasing and continuous in  $e_g^{t-1}$  as, in this case,

$$e_g^t = (1 - P_g(0|1 - e_g^{t-1}, e_g^{t-1}))n_g(\Pr(v_i > w_{min}) + r\Pr(v_i = w_{min}))$$

and by the properties of the Poisson,  $P_g(0|1 - e_g, e_g)$  is strictly decreasing and continuous in  $e_g$  for employment levels  $e_g$  biased towards blues; and  $r$  is constant ( $r = 1$ ) by assumption. If firms are indifferent about hiring from the pool, then the choice of mixing parameter  $r$  for hiring on the referral market does not impact the expected value in the pool. Furthermore, the expected value in the pool must be  $w_{min}$ . Therefore, we can solve Equation (1) for  $P(0)$ . As  $P(0)$  is strictly decreasing and continuous in  $e_g$ , this implies there is only one employment level  $e_g^{t-1}$  that leads to firms being indifferent about hiring from the pool (in which case  $e_g^t$  may decrease).

Next suppose that the employment level  $e_g^{t-1}$  is such that firms prefer to hire from the pool, i.e.,  $e_g^{t-1} \in C := \{e_g : \tilde{v}(e_g) > w_{min}\}$ . Note that  $B$  and  $C$  form a partition of all possible employment levels. Let  $A := \{e_g : \tilde{v}(e_g) \in \text{support}(F)\}$  and let  $D := C \cap A$ . We use the following fact.

**FACT 1.** *For  $e_g \in C$ ,  $\tilde{v}(e_g)$  is strictly decreasing in  $e_g$ .*

To see this, first note for  $e_g \in C$ ,  $\tilde{v}(e_g) = E_{\tilde{v},r}[v_i | i \in \text{pool}]$ . Furthermore, for such  $e_g$ , the choice of mixing parameter  $r$  does not impact the expected value in the pool. Thus by Proposition 1,  $\tilde{v}(e_g)$  is decreasing in  $P(0)$ . Furthermore, by the properties of the Poisson distribution,  $P(0)$  is strictly decreasing and continuous in  $e_g$  (i.e., as employment becomes more equal, and given that workers refer their own group, fewer workers have no referrals), implying the fact.

Fact 1, combined with the assumption that  $F$  has finite support, implies  $|D| < \infty$ .  $D$  can thus be enumerated as  $D = \{x_0, x_1, \dots, x_N\}$  with  $x_i < x_{i+1}$  for  $i = 0, \dots, N - 1$ . We claim that employment  $e_g^t$  is strictly increasing and continuous in  $e_g^{t-1}$  on each open interval  $(x_i, x_{i+1})$  for

$i = 0, 1, \dots, N - 1$ , and on  $[0, x_0)$  and  $(x_N, \min\{1, n_g\}]$ . To see this, note that  $E_{\tilde{v},r}[v_i | i \in \text{pool}]$  is constant in  $\tilde{v}$  for  $\tilde{v} \notin \text{support}(F)$ . Furthermore, as an increase in  $e_g^{t-1}$  continuously decreases  $P(0)$ , and  $e_g^{t-1} \in (x_i, x_{i+1})$  implies  $\tilde{v}(e_g^{t-1}) \notin \text{support}(F)$ , increasing  $e_g^{t-1}$  continuously decreases  $E_{\tilde{v},r}[v_i | i \in \text{pool}]$  so that the equilibrium threshold must also change continuously. As a result, we may ignore the change in the equilibrium threshold when assessing the induced change in  $e_g^t$ . Thus we can write  $e_g^t$  as a function of the probabilities of blues and greens being hired on the referral market and the hiring threshold  $\tilde{v}(e_g^{t-1})$ :

$$e_g^t = n_g (M_g^t + (1 - M_g^t)Q^t),$$

where  $M_g^t = (1 - P_g(0))(\Pr(v_i > \tilde{v}(e_g^{t-1})))$  is the probability a green worker is hired on the referral market (resp.  $M_b^t$ ) and  $Q^t = (1 - (M_g^t + M_b^t))/(n - (M_g^t + M_b^t))$  is the probability a worker is hired in the pool. As  $P_g(0)$  strictly decreases continuously and  $P_b(0)$  strictly increases continuously, and  $e_g^t$  is a continuous function of  $P_g(0)$  and  $P_b(0)$ , it must be that  $e_g^t$  indeed strictly increases and is continuous. Thus, for any time period  $t$ , for all but finitely many values of the initial employment,  $e_g^{t-1}$ , the resulting employment,  $e_g^t$ , is strictly increasing and continuous in  $e_g^{t-1}$ .

**Step 2:** We next show for all  $T < \infty$ , there exists a finite set of initial employment levels  $E^0$  such that, if  $e_g^0 \notin E^0$ , then  $e_g^T$  is continuous and strictly increasing in  $e_g^0$ .

Let  $E$  denote the values of  $e_g^{t-1}$  for which  $e_g^t$  is not strictly increasing and continuous in  $e_g^{t-1}$  at  $e_g^{t-1}$ . By Step 1,  $|E| < \infty$ . Let  $\rho(e_g)$  denote the final green employment when the initial employment is  $e_g$ . Set  $E^T = \emptyset$  and for  $t = 1, \dots, T$  define

$$E^{t-1} := E \cup \rho^{-1}(E^t).$$

By Step 1, the preimage for any finite set of values of  $e_g^t$  is finite, as final employment is strictly increasing in initial employment on each interval in  $D$  and there are finitely many intervals. Thus, as  $E$  is finite, all sets  $E^t$  for  $t = 0, \dots, T$  are finite; in particular,  $E^0$  is finite. We argue by induction that  $e_g^t = \rho^t(e_g)$  is continuous and strictly increasing in  $e_g$  at all points  $e_g \notin E^0$  (and hence at all but finitely many points). Suppose  $e_g^{t-1}$  is continuous and strictly increasing in  $e_g^0$ . Note by construction  $e_g^{t-1} \notin E^{t-1}$ . As  $E \subset E^{t-1}$ , and  $e_g^t = \rho(e_g^{t-1})$ , Step 1 implies  $e_g^t$  is continuous and increasing at  $e_g^{t-1}$  and hence, by the inductive hypothesis, at  $e_g^0$ .

**Step 3:** We now extend the previous claim to the infinite. Note that by our assumption that  $h_b = h_g = 1$ , there is referral balance. Therefore, by Proposition 4, the employment rates converge. This implies that  $P(0)$  converges and as  $\tilde{v}$  is monotone in  $P(0)$  (by Proposition 1) and bounded, so does  $\tilde{v}$ . Again by Proposition 4, there is a unique steady state, and for all  $e_g^0$ , employment rates converge so that the limit must be the unique steady state. With referral balance, this steady state must feature proportional employment masses, i.e.  $\frac{e_b}{e_g} = \frac{n_b}{n_g}$ . The limit point of the equilibrium threshold is thus given by  $\tilde{v}\left(\frac{n_g}{n}\right)$ . As the equilibrium threshold converges, there exists some  $T < \infty$ , so that  $[\tilde{v}(e_g^T), \tilde{v}\left(\frac{n_g}{n}\right)] \cap \text{support}(F) = \emptyset$ . Then,  $e_g^t$  is strictly increasing in  $e_g^{t-1}$  for all  $t \geq T$ . Using the previous steps, we know that there exists a small enough increase in  $e_g^0$  so that  $e_g^t$  increases in all periods up to  $T$ . Combining these two results, we conclude that there exists a small enough increase in  $e_g^0$  so that  $e_g^t$  increases in all future periods.  $\square$

We make use of the following lemma in the remaining proofs.

**LEMMA 5.** *Let  $\tilde{v}$  denote the unique equilibrium threshold and  $v$  a generic hiring threshold. Then  $\min_r E_{v,r}[v_i | i \in \text{pool}] \geq \max_r E_{v',r}[v_i | i \in \text{pool}]$  if  $v < v' < \tilde{v}$  and  $\max_r E_{v,r}[v_i | i \in \text{pool}] \geq$*

$\min_r \mathbb{E}_{v',r}[v_i | i \in \text{pool}]$  if  $\tilde{v} < v < v'$ . In particular, if  $F$  is continuous, then (1) is decreasing in  $v$  for  $v < \tilde{v}$  and increasing otherwise.

As total production is inversely related to the average value in the pool, it is first increasing and then decreasing in the hiring threshold with the same caveat at mass points of the distribution.

*Proof.* As an immediate corollary of Lemma 1, for all threshold  $v$ ,

$$\mathbb{E}_{v,r}[v_i | i \in \text{pool}] > v \iff v < \tilde{v}.$$

Thus, if  $v < v' < \tilde{v}$ , then the anonymous pool with threshold  $v'$  consists of the union of marginal workers added to the pool through increases in the threshold each of which with values lower than the expected value in the pool when they were added. Thus, the result follows with (1) increasing for values larger than  $\tilde{v}$  show analogously.  $\square$

*Proof of Lemma 3.* Existence of equilibrium thresholds is proven analogously as in Lemma 1. Similarly, equations (5) and (6) are simply optimality conditions that need to be satisfied for firms to adhere to the equilibrium thresholds thus showing equilibrium.

Next, to order the thresholds, suppose that  $\tilde{v}_2 \geq \tilde{v}_1$ . Pool 2 consists of workers from pool 1 with some workers with values above  $\tilde{v}_2$  removed and (referral) workers with values between  $\tilde{v}_1$  and  $\tilde{v}_2$  added. The first change implies a decrease in the average value in the pool, whereas the second an increase. However, adding a mass of workers with values between  $\tilde{v}_1$  and  $\tilde{v}_2$  cannot result in the average value in the pool to increase to  $\tilde{v}_2$ . The nondegeneracy of  $F$  then implies that  $\tilde{v}_1$  must in fact be strictly greater than  $\tilde{v}_2$ .

Suppose that  $\tilde{v}_1 \leq \tilde{v}$ . As when the referral threshold equals  $\tilde{v}$ , the expected value in pool 1 equals  $\tilde{v}$  and is otherwise greater by Lemma 5, it is clear that (5) cannot be satisfied; again, with strictness coming from the nondegeneracy of  $F$ .

Suppose that  $\tilde{v}_2 \geq \tilde{v}$ . In equilibrium, total production is given by

$$n(1 - P(0))\mathbb{E}[\max\{v_i, \tilde{v}_1\}] + (1 - n(1 - P(0)))\tilde{v}_1. \quad (8)$$

Similarly, total production for  $\lambda = 0$  is given by

$$n(1 - P(0))\mathbb{E}[\max\{v_i, \tilde{v}\}] + (1 - n(1 - P(0)))\tilde{v},$$

and thus clearly total production is greater in the former case  $\tilde{v}_1 > \tilde{v}$ .

However, production before firing must be lower as a referral threshold of  $\tilde{v}$  maximizes (immediate) production by Lemma 5. Furthermore, as  $\tilde{v}_2 \geq \tilde{v}$ , production after firing is also lower when firing is permitted by a simple accounting exercise as in Lemma 1; a contradiction. Thus,  $\tilde{v}_2 < \tilde{v} < \tilde{v}_1$ .

To show uniqueness for  $\tilde{v}_1$ , suppose the contrary and consider two hiring thresholds  $\tilde{v}_1$  and  $\tilde{v}'_1$  and suppose without loss that  $\tilde{v} < \tilde{v}_1 < \tilde{v}'_1$ . Total production must be higher under  $\tilde{v}'_1$  than under  $\tilde{v}_1$  as is evident from (8). By Lemma 5, production before firing is lower under  $\tilde{v}'_1$  than under  $\tilde{v}_1$  as  $\tilde{v} < \tilde{v}_1 < \tilde{v}'_1$ . Hence, it must be that  $\tilde{v}'_2 < \tilde{v}_2$  for total production to be the same under the two thresholds. But then (5) cannot be satisfied for both  $\tilde{v}_1$  and  $\tilde{v}_2$  and  $\tilde{v}'_1$  and  $\tilde{v}'_2$  as the both terms on the right-hand side are larger given  $\tilde{v}_1$  and  $\tilde{v}_2$  than given  $\tilde{v}'_1$  and  $\tilde{v}'_2$  whereas  $\tilde{v}_1 < \tilde{v}'_1$  by assumption.

Lastly, let us show that  $\tilde{v}_2$  is unique. By the uniqueness of  $\tilde{v}_1$ , it must be that there are  $r_1, r'_1$  with  $r_1 r'_1$  (without loss) so that  $\tilde{v}_1$  with  $r_1$  and some  $\tilde{v}_2$  as well as  $\tilde{v}_1$  with  $r'_1$  and some  $\tilde{v}'_2$  with  $\tilde{v}_2 \neq \tilde{v}'_2$  satisfy (5) and (6). As  $\tilde{v}_1 > \tilde{v}$ , it must be that production before the firing is higher given

$\tilde{v}_1$  and  $r_1$  than  $\tilde{v}_1$  and  $r'_1$  and

$$E_{\tilde{v}_1, \tilde{v}_2, r_1, r_2}[v_i | i \in \text{pool 1}] < E_{\tilde{v}_1, \tilde{v}'_2, r'_1, r'_2}[v_i | i \in \text{pool 1}]$$

by an argument similar to that of Lemma 5. As total production in equilibrium is determined by  $\tilde{v}_1$  according to (8), it must be the same for both sets of thresholds. Hence, it must be that  $\tilde{v}_2 < \tilde{v}'_2$ , so that the comparison of production reverses after firing. However, then equilibrium condition (5) cannot be satisfied for both sets of thresholds as both terms on the right-hand side of (5) are larger for  $\tilde{v}_1$  with  $r'_1$  and  $\tilde{v}'_2$  than with  $r_1$  and  $\tilde{v}_2$  whereas the left-hand side,  $\tilde{v}_1$ , is constant.  $\square$

*Proof of Corollary 1.* As  $h_b = h_g = 1$ ,  $e_b$  and  $e_g$  are also the masses of referrals blue and green workers receive respectively. In this case,  $P(0)$  is given by

$$\frac{n_b}{n} e^{-\frac{e_b}{n_b}} + \frac{n_g}{n} e^{-\frac{e_g}{n_g}}.$$

Using  $e_g = 1 - e_b$ , we differentiate the above with respect to  $e_b$  to get

$$-\frac{1}{n} e^{-\frac{e_b}{n_b}} + \frac{1}{n} e^{-\frac{e_g}{n_g}},$$

which is strictly positive whenever  $\frac{e_b}{n_b} > \frac{e_g}{n_g}$ . Note that

$$\frac{e_b}{e_g} > \frac{e'_b}{e'_g} \geq \frac{n_b}{n_g} \implies \frac{e_b}{n_b} > \frac{e'_b}{n_b} \geq \frac{e'_g}{n_g} > \frac{e_g}{n_g}.$$

Thus, it must be that  $P(0)$  is strictly higher in the equilibrium starting with  $(e_b, e_g)$  than in the equilibrium starting with  $(e'_b, e'_g)$ . The conclusion follows by Proposition 1.  $\square$

*Proof of Lemma 4.* Suppose  $\tilde{e}_b^t > e_b^t \geq \bar{e}_b$ . Consider the first factor of (7). Differentiating the probabilities of no referrals of blue and green workers with respect to employment masses gives

$$\begin{aligned} P_b^t(0|\tilde{e}_b^t, \tilde{e}_g^t) - P_b(0|e_b^t, e_g^t) &= P_b^t(0|\tilde{e}_b^t, 1 - \tilde{e}_b^t) - P_b(0|e_b^t, 1 - e_b^t) \\ &= \int_{e_b^t}^{\tilde{e}_b^t} -\frac{h_b + h_g - 1}{n_b} \exp\left(-\frac{1}{n_b}(h_b e_b + (1 - h_g)(1 - e_b))\right) de_b \\ &> \int_{e_b^t}^{\tilde{e}_b^t} -\frac{1}{n_b} de_b = -\left(\frac{\tilde{e}_b^t}{n_b} - \frac{e_b^t}{n_b}\right). \end{aligned}$$

Similarly, we have

$$\begin{aligned} P_g^t(0|\tilde{e}_b^t, \tilde{e}_g^t) - P_g(0|e_b^t, e_g^t) &= P_g^t(0|\tilde{e}_b^t, 1 - \tilde{e}_b^t) - P_g(0|e_b^t, 1 - e_b^t) \\ &= \int_{e_b^t}^{\tilde{e}_b^t} \frac{h_b + h_g - 1}{n_g} \exp\left(-\frac{1}{n_g}(h_g(1 - e_b) + (1 - h_b)e_b)\right) de_b \\ &< \int_{e_b^t}^{\tilde{e}_b^t} \frac{1}{n_g} de_b = \left(\frac{\tilde{e}_b^t}{n_g} - \frac{e_b^t}{n_g}\right) = -\left(\frac{\tilde{e}_g^t}{n_g} - \frac{e_g^t}{n_g}\right). \end{aligned}$$

Consider the second factor of (7). As  $\tilde{e}_b^t > e_b^t \geq \bar{e}_b$  and there is a positive relationship between a

group's number of referrals received and its employment rate ( $h_b > 1 - h_g$ ), we have

$$\begin{aligned}
& \frac{n_b}{n} P_b^t(0|\tilde{e}_b^t, \tilde{e}_g^t) + \frac{n_g}{n} P_g^t(0|\tilde{e}_b^t, \tilde{e}_g^t) + \left( \frac{n_b}{n} P_b(0|e_b^t, e_g^t) + \frac{n_g}{n} P_g(0|e_b^t, e_g^t) \right) \\
&= \frac{n_b}{n} (P_b^t(0|\tilde{e}_b^t, \tilde{e}_g^t) - P_b^t(0|e_b^t, e_g^t)) - \frac{n_g}{n} (P_g^t(0|\tilde{e}_b^t, \tilde{e}_g^t) - P_g(0|e_b^t, e_g^t)) \\
&= \int_{e_b^t}^{\tilde{e}_b^t} -\frac{h_b + h_g - 1}{n_b} P_b^t(0|e_b, 1 - e_b) + \frac{h_b + h_g - 1}{n} P_g^t(0|e_b, 1 - e_b) de_b \\
&= \int_{e_b^t}^{\tilde{e}_b^t} \frac{h_b + h_g - 1}{n_b} (P_g^t(0|e_b, 1 - e_b) - P_g^t(0|e_b, 1 - e_b)) de_b > 0,
\end{aligned}$$

that is the aggregate probability of no referral is larger given  $\tilde{e}_b^t$  than given  $e_b^t$ . As a result, fewer workers are vetted, the lemons effect decreases and the equilibrium threshold increases (Proposition 1). Both the increase in the probability of no referrals and in the equilibrium threshold imply that fewer workers are hired on the referral market given  $\tilde{e}_b^t$  than given  $e_b^t$ ; furthermore implying that  $\tilde{Q}^t > Q^t$ , i.e., the probability of being hired from the pool is larger given  $\tilde{e}_b^t$  than given  $e_b^t$ .

$$Q^t - R^t = - (1 - (1 - r) \Pr(v_i = \tilde{v}^t) - \Pr(v_i < \tilde{v}^t)) (1 - Q^t),$$

and similarly for  $\tilde{Q}^t - \tilde{R}^t$ , it must be that the second factor of (7), when moving from current employment  $e_b^t$  to  $\tilde{e}_b^t$ , increases while still remaining negative. That is,  $Q^t - R^t < \tilde{Q}^t - \tilde{R}^t \leq 0$ . Let  $P_b^t$  and  $\tilde{P}_b^t$  denote  $P^t(0|e_b^t, e_g^t)$  and  $P^t(0|\tilde{e}_b^t, \tilde{e}_g^t)$ ; with analogous notation for green workers. Putting this together, we see

$$\begin{aligned}
& \left( \frac{\tilde{e}_b^{t+1}}{n_b} - \frac{\tilde{e}_g^{t+1}}{n_g} \right) - \left( \frac{e_b^t}{n_b} - \frac{e_g^t}{n_g} \right) = (\tilde{P}_b^t - \tilde{P}_g^t)(\tilde{Q}^t - \tilde{R}^t) - (P_b^t - P_g^t)(Q^t - R^t) \\
&\leq (\tilde{P}_b^t - \tilde{P}_g^t)(Q^t - R^t) - (P_b^t - P_g^t)(Q^t - R^t) \\
&= -(\tilde{P}_b^t - P_b^t)(-(Q^t - R^t)) + (\tilde{P}_g^t - P_g^t)(-(Q^t - R^t)) \\
&< \left( \frac{\tilde{e}_b^t}{n_b} - \frac{e_b^t}{n_b} \right) - \left( \frac{\tilde{e}_g^t}{n_g} - \frac{e_g^t}{n_g} \right) \\
&= \left( \frac{\tilde{e}_b^t}{n_b} - \frac{\tilde{e}_g^t}{n_g} \right) - \left( \frac{e_b^t}{n_b} - \frac{e_g^t}{n_g} \right),
\end{aligned}$$

as required.

Suppose now that  $\tilde{e}_b^t < e_b^t \leq \bar{e}_b$ . Then  $\tilde{e}_g^t > e_g^t \geq \bar{e}_g$ . Interchanging the subscripts and applying the previous part of this proof gives

$$\left( \frac{\tilde{e}_g^{t+1}}{n_g} - \frac{\tilde{e}_b^{t+1}}{n_b} \right) - \left( \frac{e_g^{t+1}}{n_g} - \frac{e_b^{t+1}}{n_b} \right) < \left( \frac{\tilde{e}_g^t}{n_g} - \frac{\tilde{e}_b^t}{n_b} \right) - \left( \frac{e_g^t}{n_g} - \frac{e_b^t}{n_b} \right),$$

which is a simple rearrangement of the claimed inequality.  $\square$

*Proof of Proposition 7.* First, we show that  $\tilde{v}_1$  is strictly increasing in  $\lambda$ . Fixing  $\tilde{v}_1$ , and thus  $\tilde{v}_2$ , the right-hand side of (5) strictly increases in  $\lambda$  (as  $\tilde{v}_2$  is bounded away from the lowest possible value as  $P(0) > 0$ ). Thus, it must be that  $\tilde{v}_1$  strictly increases with  $\lambda$  as  $\tilde{v}_1$  is unique by Lemma 3. In equilibrium, total production is given by (8) and in particular strictly increasing in  $\tilde{v}_1$ , so that

is also strictly increasing in  $\lambda$ .

Next, we show that  $\tilde{v}_2$  is strictly decreasing in  $\lambda$ . Let  $\lambda' > \lambda$  and denote the equilibrium thresholds by  $(\tilde{v}'_1, \tilde{v}'_2)$  and  $(\tilde{v}_1, \tilde{v}_2)$  respectively. We know that  $\tilde{v}'_1 > \tilde{v}_1$ . To reach a contradiction suppose that  $\tilde{v}'_2 \geq \tilde{v}_2$ .

We study how production after firing differs given  $\lambda$  and  $\lambda'$  with  $\lambda, \lambda' > 0$ . To this end, we partition the set of workers into several subsets whose aggregate composition of workers stays constant given  $\lambda$  and  $\lambda'$ . We then track the extend to which each of these sets of workers is employed. To be clear, the realization of randomization may imply that individual workers belong to different sets depending on whether we consider  $\lambda$  or  $\lambda'$ . However, workers can always be grouped into such subsets so that their composition in terms of values stays constant.

Under  $\lambda'$ , as  $\tilde{v}'_1 > \tilde{v}_1$ , an additional set of workers enter the pool. In Figure 4, the thick solid rectangle represents workers who enter the pool under both  $\lambda$  and  $\lambda'$  whereas the thick dashed rectangle represents workers who only enter the pool under  $\lambda'$ , i.e. those with values between  $\tilde{v}_1$  and  $\tilde{v}'_1$ . Due to uniform sampling from pool 1, it must be that the same fraction of workers in both rectangles is hired from the aforementioned sets. We graphically represent this fact by placing the rectangles side-by-side and, given  $\lambda'$ , horizontally slice through them (the top dashed lines) with the top area representing the set of unemployed workers and the bottom area the set of workers hired from pool 1. The second dashed line (the bottom dashed line) represents the random hiring from pool 1 given  $\lambda$ . Note that the ordering of the dashed lines is deliberate and follows from the fact that an equal increase in firms hiring from pool 1 and workers entering pool 1 implies a higher fraction of workers from pool 1 hired. Furthermore, the size of the set of unemployed before firing needs to be constant, it equals  $n - 1$ , given  $\lambda$  or  $\lambda'$ . Thus, in Figure 4, the size of area  $C$  equals that of area  $D$  (all labels in Figure 4 correspond to the smallest rectangle containing them). A final set of workers consists of those who are hired on the referral market under both  $\lambda$  and  $\lambda'$ . Their contribution to total production after firing is obviously constant under  $\lambda$  and  $\lambda'$ .

Consider the firms hiring workers in  $A$  from pool 1. The distribution of values in set  $A$  is constant by construction; and, by assumption, a firm at first hiring such worker can replace the worker with a worker with value  $\tilde{v}'_2 \geq \tilde{v}_2$  under  $\lambda'$  and with a worker with value  $\tilde{v}_2$  under  $\lambda$ . Thus, average production after firing of firms hiring workers in  $A$  is larger given  $\lambda'$ .

Consider the firms hiring workers in  $B$  from pool 1. All such workers would have been hired on the referral market under  $\lambda$  and are drawn from pool 1 under  $\lambda'$ . Note that no such worker is fired (see Lemma 3). Thus, their contribution to total production after firing is constant under  $\lambda$  and  $\lambda'$ .

Finally, let us compare how firms fare that under  $\lambda$  hire workers in  $C$  from pool 1 and under  $\lambda'$  workers in  $D$  from pool 1. We have

$$E[v_i | i \in D] \leq \tilde{v}'_1 < E_{\tilde{v}'_1, r'_1} [\max\{v_i, \tilde{v}'_2\} | i \in \text{pool 1}], \quad (9)$$

where the first inequality follows as workers in  $D$  have values between  $\tilde{v}_1$  and  $\tilde{v}'_1$  and the second from (5). Note the last expectation in the equation above is taken over workers in pool 1 given  $\lambda'$ . As pool 1 given  $\lambda'$  is the union of pool 1 given  $\lambda$  and sets  $B$  and  $D$ , the following equation must hold

$$\begin{aligned} E_{\tilde{v}'_1, r'_1} [\max\{v_i, \tilde{v}'_2\} | i \in \text{pool 1}] &= \frac{|B| + |D|}{|A| + |B| + |C| + |D| + |E|} E[v_i | i \in B \cup D] \\ &\quad + \frac{|A| + |C| + |E|}{|A| + |B| + |C| + |D| + |E|} E_{\tilde{v}_1, r_1} [v_i | i \in \text{pool 1}], \end{aligned} \quad (10)$$

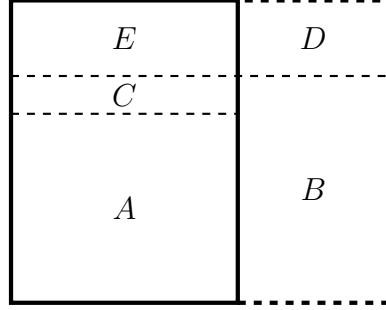


Figure 4: Pool 1

where the first expectation is taken over workers in pool 1 given  $\lambda'$  and the last over workers in pool 1 given  $\lambda$ . As  $E[v_i|i \in D] = E[v_i|i \in B \cup D]$ , (9) and (10) imply

$$E[v_i|i \in D] < E_{\tilde{v}_1, r_1}[\max v_i, \tilde{v}_2|i \in \text{pool 1}],$$

once more, with the expectation taken over workers in pool 1 given  $\lambda$ . The left-hand side is the average production after firing of the firms hiring workers in  $D$  under  $\lambda$  while the right-hand side is the average production after firing of firms hiring workers in  $C$  under  $\lambda'$ . Thus, the production after firing the of the latter equally sized mass of firms is larger.

We have then considered the set of all firms and production after firing is strictly greater under  $\lambda'$  than under  $\lambda$ . As production is inversely related to the average value of the unemployed workers, it must be that  $\tilde{v}'_2 < \tilde{v}_2$ ; contrary to our assumption. Thus, it must be  $\tilde{v}'_2 < \tilde{v}_2$ .

The employment rate of green workers before the firing stage is given by

$$\frac{R_g}{n_g} + \left(1 - \frac{R_g}{n_g}\right) \frac{1 - (R_b + R_g)}{n - (R_b + R_g)}, \quad (11)$$

where  $R_b$  and  $R_g$  are the masses of blue and green workers that are hired on the referral market, respectively. As blue workers have a lower probability of no referrals, the increase in  $\tilde{v}_1$  associated with an increase in  $\lambda$  implies that a larger decrease in  $\frac{R_b}{n_b}$  than in  $\frac{R_g}{n_g}$ . As  $\frac{R_b}{n_b} > \frac{R_g}{n_g}$ , it follows that the employment rate of green workers before the firing stage, given in (11), is increasing in  $\lambda$ .

However, the employment rate of green workers, and thus of blue workers, at the end of the period may increase or decrease in  $\lambda$ . As noted in the discussing leading up the this proposition, there are two competing effects on the employment rate of green workers at the end of the period: in essence, the reliance on pool 1 increases, while firms are more reluctant to hire from pool 2. As the first effect occurs chronologically before the second, we are able to sign the change in green employment before the firing stage (see above). To sketch examples proving the claimed ambiguity, note that the former effect relies on there being a mass of workers (on the referral market) with values near  $\tilde{v}_1$ , whereas the latter effect relies on there being a mass of workers (in pool 1) with values near  $\tilde{v}_2$ . By appropriately placing mass points in the value distribution  $F$ , either effect can dominate.

Finally, total production is increasing in  $\lambda$  as  $\tilde{v}_1$  is and by (8); production before firing is decreasing in  $\lambda$  as  $\tilde{v}_1$  is and by Lemma 5; and production after firing is increasing in  $\lambda$  as  $\tilde{v}_2$  is decreasing in  $\lambda$  and by the usual accounting exercise.

If  $\lambda' > \lambda = 0$ , it is clear from the proof so far that production is greater given  $\lambda'$  than given  $\lambda$  (also  $\tilde{v}'_1 > \tilde{v}_1$ ). Thus, we know that the employment rate of green workers at before the firing stage is greater given  $\lambda'$  than given  $\lambda$  which equals the employment rate of green workers at the end of the

period as no firing takes place. The firing stage will only further increase the employment rate of green workers. In particular, as  $\tilde{v}_1 > \tilde{v} > \tilde{v}_2$ , the masses of fired workers by group are proportional to the population sizes whereas the additional draw from pool 2 is still more likely to be a green worker.  $\square$

## B.2 Omitted Examples

**Example for footnote 29.** Consider  $h_b = h_g \geq 1/2$  and  $n_b = n_g > 1/2$  and suppose firms do not hire from the pool when initial employment is  $e_b = e_g = 1/2$ . Then, only workers from the referral market are hired and the the employment masses, while equal, are less than 1/2. This reduces the number of referrals and thus increases the probability of no referrals which drives up the expected value in the pool. As a result, in the next period, firms may hire from the pool resulting in a unit-mass of employed workers equal by group; i.e. a cycle.

**Example for footnote 30.** Let  $h_b = 1; h_g = .5$  and  $n_b = n_g > 1/2$  so that referrals lean towards blues. Let  $F$  be a three value distribution with  $v_H > v_M > v_L$  and small masses at the extremes. Start with equal employment rates and pick values  $v_H, v_L$  such that  $v_M$  is slightly above the hiring threshold. Blue employment increases as blues receive more referrals than greens. This leads to more concentration, an increase in  $P(0)$  and hence an increase in the hiring threshold. so that the hiring threshold is now above  $v_M$ . Then only most hiring comes from the pool and employment rates are mostly equal, and then the cycle repeats.

## B.3 Impact of an Increase in the Size of the Population

Our analysis has kept the population size,  $n$ , fixed. One can interpret  $n$  as the ‘tightness’ of the labor market. Small  $n$  generally means that there is relatively small unemployment, while a large  $n$  means that there are few vacancies compared to job searchers (1 to  $n$ ). An increase in  $n$  alleviates the lemons effect, as more of the workers in the pool had no referrals. This makes the pool more attractive and increases total production. Note that in this case, we see an improvement in the lemons effect corresponding to an increase in production, while when we did comparative statics in  $P$  holding  $n$  fixed we saw the opposite.

We illustrate this effect in the case of a Poisson distribution of referrals, so that we can trace how  $P(0)$  changes with  $n$ , as this is an important factor. Consider the case in which each current employee makes a referral uniform at random so that the referral distribution to the next generation is a Poisson distribution with mean  $1/n$ . To simplify the analysis, we suppose that firms hire from the pool and consider outside options of  $w_{min}$ .

As  $n$  increases, the probability of no referral,  $P(0)$ , increases as well as the same amount of referrals are now spread out to more workers. However, the mass of workers who receive at least one referral,  $n(1 - P(0))$ , increases in  $n$  since

$$\frac{\partial}{\partial n} n(1 - P(0)) = \frac{\partial}{\partial n} n(1 - P(0)) = 1 - \frac{n+1}{n} e^{-\frac{1}{n}} > 0.$$

By arguments similar to those in the proof of Proposition 1, the increase in the probability of no referrals implies an increase in the expected value in the pool; i.e., a decrease in the lemons effect. The effect on overall production is twofold. First, some firms do not hire their referred worker anymore as a result of the increase in  $n$  due to an increased threshold given that the pool is more attractive. Those firms’ average productions increase as they hire a worker from the improved

pool. Second, firms who hired from the pool initially now find a better pool so that their average production increases as well.

An overall intuition is that there are more workers who receive at least one referral, which implies that more workers with higher values are vetted, again increasing production. With the Poisson distribution, we also see an overall drop in the fraction of workers with two or more referrals, and so firms also pay lower average wages. Thus, the aforementioned effects resulting from an increase in  $n$  increase productivity and decrease average wages.

## B.4 Allocating Firms' Profits in the Gini Comparative Statics

Our analysis of the Gini coefficient focused on wages. Firms in this economy are earning profits since they can hire workers at  $w_{min}$  and earn a higher expected value. Accounting for who gets those profits as income can affect the inequality calculations.

Firms earn the same profit from workers they compete (those with high values and  $P(2+)$ ) as hiring a worker from the pool. This means that expected profits take a simple form (here presuming that the expected value from the pool exceeds the minimum wage): Profits are:

$$\Pi = nP(1)f_H v_H + (1 - P(1)f_H)\tilde{v} - w_{min}. \quad (12)$$

So, firms' profits depend entirely on how many people get just one referral, as well as what the cutoff is and hence the lemons effect. Thus, firms' profits can be fully characterized if we know  $P(0)$  and  $P(2+)$  (and hence  $P(1)$ ).

To provide some simple intuition, let us consider a distribution for which workers receive either 0, 1 or  $k$  referrals. Together with the assumption that the total number of referrals is constant equaling 1, this implies that  $P$  is completely characterized by one parameter:  $P(k)$ . In that case, simple but tedious calculations show that  $\frac{\partial \Pi}{\partial P(k)} = -\frac{f_L f_H (v_H - v_L)(f_L + (n-1)k)}{(P(0) + (1-P(0))f_L)^2} < 0$ . This is not obvious, since once again the lemons effect has a counteracting force, but in this situation the derivative can be unambiguously signed.<sup>42</sup>

This means that if profits are distributed uniformly across all workers, then concentrating referrals (an increase in  $P(k)$  which now corresponds to both an increase in the probability of no referral and multiple referrals) decreases profits, and so decreases income uniformly across all workers which increases the Gini coefficient and thus inequality.<sup>43</sup> Thus, compared to Proposition 2, this is another pressure increasing inequality. If profits instead go to some special class of citizens who are owners of the firms, then it depends on who they are, and so then the further effects are ambiguous.

## B.5 Correlated Values

Homophily may not only occur along group dimensions, so that blues tend to refer blues and greens tend to refer greens, but homophily could also occur along productivity dimensions; i.e., so that workers tend to refer workers who have a similar value to their own.

We study the effects of such homophily by means of an example. We consider a two-value value

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<sup>42</sup>The weaker assumption that an increase in the probability of multiple referrals implies an increase in the probability of no referral is not sufficient to sign the effect on profit.

<sup>43</sup>An easy way to see this is to note that the Gini decreases in  $W_L$  (see equation (3)) and so lowering the level of all incomes decreases  $W_L$ , thereby increasing the Gini.

distribution with equal amounts of high- and low-value workers: Let  $F$  be given by

$$v_i = \begin{cases} v_H & \text{wp } 1/2 \\ v_L & \text{wp } 1/2, \end{cases} \quad (13)$$

with  $v_H > v_L$ .

Let  $\alpha \geq 1/2$  denote the “inbreeding-bias”: the probability that the connection of some worker is of the same productivity type as that worker. For  $\alpha = 1/2$ , then values are uncorrelated and so there is no inbreeding-bias.

For simplicity, let  $n = 2$ ; i.e.  $n_H = n_L = 1$ , where  $n_H, n_L$  are the masses of high- and low-value workers, respectively; and, we work with a referral distribution that has weight only on  $P(0)$  and  $P(1)$ .

On the referral market, firms will always hire a high-value worker and reject a low-value worker. Denoting the employment of high-value workers by  $e_H$ , the probability that a high-value worker receives a referral is

$$P_H(1|e_H, 1 - e_H) := e_H\alpha + (1 - e_H)(1 - \alpha).$$

The steady state employment rate (or mass) of high-value workers,  $e_H$ , solves

$$P_H(1|e_H, 1 - e_H) + (1 - P_H(1|e_H, 1 - e_H)) \frac{1 - P_H(1|e_H, 1 - e_H)}{n - P_H(1|e_H, 1 - e_H)} = e_H.$$

Solving the above expression for  $e_H$  yields

$$e_H = \frac{1 + \alpha - \sqrt{(1 - \alpha)(5 - \alpha)}}{2(2\alpha - 1)}.$$

This is increasing in  $\alpha$  as

$$\frac{\partial}{\partial \alpha} e_H = \frac{7 - 5\alpha - 3\sqrt{(1 - \alpha)(5 - \alpha)}}{2(2\alpha - 1)^2 \sqrt{(1 - \alpha)(5 - \alpha)}},$$

which is positive since  $5\alpha + 3\sqrt{(1 - \alpha)(5 - \alpha)}$  is maximized for  $\alpha = 1/2$ , for which it equals 7. For  $\alpha > 1/2$ , the above expression is thus strictly positive.

We depict the mass of employed high-value workers in steady state as a function of the degree of value-homophily in Figure 5. We remark that value-homophily increases productivity, since in

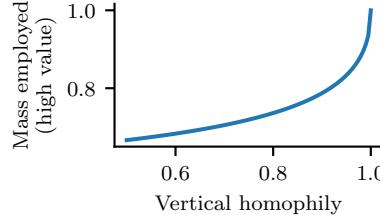


Figure 5: The parameter values are:  $n_H = n_L = 1$  so that  $n = 2$ . We plot the mass of high-value workers employed in steady state as a function of the degree of value-homophily,  $\alpha$ .

in this example the mass of high valued workers translates directly into the overall productivity.