Nobuyuki Otsu, 1979

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Známe

$$
\omega_0 = \sum_{i=1}^{k} p_i \tag{1}
$$

$$
\omega_1 = \sum_{i=k+1}^{L} p_i \tag{2}
$$

$$
\mu_0 = \frac{\sum_{i=1}^{k} i p_i}{\omega_0} \tag{3}
$$

$$
\mu_1 = \frac{\sum_{i=k+1}^{L} i p_i}{\omega_1} \tag{4}
$$

$$
\mu_T = \sum_{i=1}^{L} i p_i \tag{5}
$$

$$
\sigma_0^2 = \frac{\sum_{i=1}^{k} (i - \mu_0)^2 p_i}{\omega_0} \tag{6}
$$

$$
\sigma_1^2 = \frac{\sum_{i=k+1}^{L} (i - \mu_1)^2 p_i}{\omega_1} \tag{7}
$$

$$
\sigma_W^2 = \omega_0 \sigma_0^2 + \omega_1 \sigma_1^2 \tag{8}
$$

$$
\sigma_B^2 = \omega_0 \left(\mu_0 - \mu_T\right)^2 + \omega_1 \left(\mu_1 - \mu_T\right)^2 \tag{9}
$$

$$
\sigma_T^2 = \sum_{i=1}^{L} (i - \mu_T)^2 p_i \tag{10}
$$

$$
\mu_T = \omega_0 \mu_0 + \omega_1 \mu_1 \tag{11}
$$

$$
1 = \omega_0 + \omega_1 \tag{12}
$$

Výpočet 1

$$\sigma_T^2 \tag{13}$$

$$\sum_{i=1}^{L} (i - \mu_T)^2 p_i \tag{14}$$

$$\sum_{i=1}^{L} \left(i - \sum_{j=1}^{L} j p_j \right)^2 p_i \tag{15}$$

$$\sum_{i=1}^{L} \left(i^2 - 2i \sum_{j=1}^{L} j p_j + \left(\sum_{j=1}^{L} j p_j \right)^2 \right) p_i \tag{16}$$

$$\sum_{i=1}^{L} \left(i^2 p_i - \left(2i \sum_{j=1}^{L} j p_j \right) p_i + \left(\sum_{j=1}^{L} j p_j \right)^2 p_i \right) \tag{17}$$

$$\sum_{i=1}^{L} \left(i^2 p_i \right) - \sum_{i=1}^{L} \left(2i \left(\sum_{j=1}^{L} j p_j \right) p_i \right) + \sum_{i=1}^{L} \left(\left(\sum_{j=1}^{L} j p_j \right)^2 p_i \right) \tag{18}$$

$$\left(\sum_{i=1}^{L} i^2 p_i \right) - \left(\sum_{i=1}^{L} 2i p_i \right) \cdot \left(\sum_{j=1}^{L} j p_j \right) + \left(\sum_{i=1}^{L} \left(\sum_{j=1}^{L} j p_j \right)^2 p_i \right) \tag{19}$$

$$\left(\sum_{i=1}^{L} i^2 p_i \right) - \left(\sum_{i=1}^{L} 2i p_i \right) \cdot \left(\sum_{j=1}^{L} j p_j \right) + \left(\sum_{i=1}^{L} p_i \right) \cdot \left(\sum_{j=1}^{L} j p_j \right)^2 \tag{20}$$

$$\text{Protože}: \quad \left(\sum_{i=1}^{L} p_i \right) = 1, \quad (\text{Součet pravděpodobností} = 1) \tag{21}$$

$$\left(\sum_{i=1}^{L} i^2 p_i \right) - \left(\sum_{i=1}^{L} 2i p_i \right) \cdot \left(\sum_{j=1}^{L} j p_j \right) + \left(\sum_{j=1}^{L} j p_j \right)^2 \tag{22}$$

$$\left(\sum_{i=1}^{L} i^2 p_i \right) - \left(\sum_{i=1}^{L} 2i p_i \right) \cdot \left(\sum_{j=1}^{L} j p_j \right) + \mu_T^2 \tag{23}$$

$$\left(\sum_{i=1}^{L} i^2 p_i \right) - \left(2 \sum_{i=1}^{L} i p_i \right) \cdot \left(\sum_{j=1}^{L} j p_j \right) + \mu_T^2 \tag{24}$$

$$\left(\sum_{i=1}^{L} i^2 p_i \right) - 2\mu_T \left(\sum_{j=1}^{L} j p_j \right) + \mu_T^2 \tag{25}$$

$$\left(\sum_{i=1}^{L} i^2 p_i \right) - 2\mu_T^2 + \mu_T^2 \tag{26}$$

$$\left(\sum_{i=1}^{L} i^2 p_i \right) - \mu_T^2 \tag{27}$$

Výpočet 2

$$\sigma_W^2 + \sigma_B^2 \quad (28)$$

$$\left(\omega_0\sigma_0^2 + \omega_1\sigma_1^2\right) + \sigma_B^2 \quad (29)$$

$$\left(\omega_0\sigma_0^2 + \omega_1\sigma_1^2\right) + \left(\omega_0\left(\mu_0 - \mu_T\right)^2 + \omega_1\left(\mu_1 - \mu_T\right)^2\right) \quad (30)$$

$$\left(\omega_0\sigma_0^2 + \omega_1\sigma_1^2\right) + \left(\omega_0\left(\mu_0^2 - 2\mu_0\mu_T + \mu_T^2\right) + \omega_1\left(\mu_1^2 - 2\mu_1\mu_T + \mu_T^2\right)\right) \quad (31)$$

$$\left(\omega_0\sigma_0^2 + \omega_1\sigma_1^2\right) + \left(\omega_0\left(\mu_0^2 - 2\mu_0\mu_T\right) + \omega_1\left(\mu_1^2 - 2\mu_1\mu_T\right) + \left(\omega_0\mu_T^2 + \omega_1\mu_T^2\right)\right) \quad (32)$$

$$\left(\omega_0\sigma_0^2 + \omega_1\sigma_1^2\right) + \left(\omega_0\left(\mu_0^2 - 2\mu_0\mu_T\right) + \omega_1\left(\mu_1^2 - 2\mu_1\mu_T\right) + \left(\mu_T^2\left(\omega_0 + \omega_1\right)\right)\right) \quad (33)$$

$$\left(\omega_0\sigma_0^2 + \omega_1\sigma_1^2\right) + \left(\omega_0\left(\mu_0^2 - 2\mu_0\mu_T\right) + \omega_1\left(\mu_1^2 - 2\mu_1\mu_T\right) + \mu_T^2\right) \quad (34)$$

$$\left(\omega_0\sigma_0^2 + \omega_1\sigma_1^2\right) + \left(\omega_0\mu_0^2 + \omega_1\mu_1^2 + \mu_T^2 - 2\mu_T\left(\omega_0\mu_0 + \omega_1\mu_1\right)\right) \quad (35)$$

$$\left(\omega_0\sigma_0^2 + \omega_1\sigma_1^2\right) + \left(\omega_0\mu_0^2 + \omega_1\mu_1^2 + \mu_T^2 - 2\mu_T^2\right) \quad (36)$$

$$\left(\omega_0\sigma_0^2 + \omega_1\sigma_1^2\right) + \left(\omega_0\mu_0^2 + \omega_1\mu_1^2 - \mu_T^2\right) \quad (37)$$

$$\left(\omega_0\sigma_0^2 + \omega_1\sigma_1^2\right) + \left(\left(\sum_{i=1}^{k} p_i\right)\mu_0^2 - \omega_1\mu_1^2 - \mu_T^2\right) \quad (38)$$

$$\left(\omega_0\sigma_0^2 + \omega_1\sigma_1^2\right) + \left(\left(\sum_{i=1}^{k} p_i\right)\mu_0^2 + \left(\sum_{i=k+1}^{L} p_i\right)\mu_1^2 - \mu_T^2\right) \quad (39)$$

$$\left(\omega_0\sigma_0^2 + \omega_1\sigma_1^2\right) + \left(\left(\sum_{i=1}^{k} \mu_0^2 p_i\right) + \left(\sum_{i=k+1}^{L} p_i\right)\mu_1^2 - \mu_T^2\right) \quad (40)$$

$$\left(\omega_0\sigma_0^2 + \omega_1\sigma_1^2\right) + \left(\left(\sum_{i=1}^{k} \mu_0^2 p_i\right) + \left(\sum_{i=k+1}^{L} \mu_1^2 p_i\right) - \mu_T^2\right) \quad (41)$$

$$\left(\omega_0\frac{\sum_{i=1}^{k}\left(i - \mu_0\right)^2 p_i}{\omega_0} + \omega_1\sigma_1^2\right) + \left(\left(\sum_{i=1}^{k} \mu_0^2 p_i\right) + \left(\sum_{i=k+1}^{L} \mu_1^2 p_i\right) - \mu_T^2\right) \quad (42)$$

$$\left(\left(\sum_{i=1}^{k}\left(i - \mu_0\right)^2 p_i\right) + \omega_1\sigma_1^2\right) + \left(\left(\sum_{i=1}^{k} \mu_0^2 p_i\right) + \left(\sum_{i=k+1}^{L} \mu_1^2 p_i\right) - \mu_T^2\right) \quad (43)$$

$$\left(\left(\sum_{i=1}^{k}\left(i - \mu_0\right)^2 p_i\right) + \omega_1\frac{\sum_{i=k+1}^{L}\left(i - \mu_1\right)^2 p_i}{\omega_1}\right) + \left(\left(\sum_{i=1}^{k} \mu_0^2 p_i\right) + \left(\sum_{i=k+1}^{L} \mu_1^2 p_i\right) - \mu_T^2\right) \quad (44)$$

$$\left(\sum_{i=1}^{k}\left(i - \mu_0\right)^2 p_i\right) + \left(\sum_{i=k+1}^{L}\left(i - \mu_1\right)^2 p_i\right) + \left(\sum_{i=1}^{k} \mu_0^2 p_i\right) + \left(\sum_{i=k+1}^{L} \mu_1^2 p_i\right) - \mu_T^2 \quad (45)$$

$$\left(\sum_{i=1}^{k}\left(\left(i - \mu_0\right)^2 + \mu_0^2\right) p_i\right) + \left(\sum_{i=k+1}^{L}\left(i - \mu_1\right)^2 p_i\right) + \left(\sum_{i=k+1}^{L} \mu_1^2 p_i\right) - \mu_T^2 \quad (46)$$

$$\left(\sum_{i=1}^{k}\left(\left(i - \mu_0\right)^2 + \mu_0^2\right) p_i\right) + \left(\sum_{i=k+1}^{L}\left(\left(i - \mu_1\right)^2 + \mu_1^2\right) p_i\right) - \mu_T^2 \quad (47)$$

$$\left(\sum_{i=1}^{k}\left(i^2 - 2i\mu_0 + 2\mu_0^2\right) p_i\right) + \left(\sum_{i=k+1}^{L}\left(\left(i - \mu_1\right)^2 + \mu_1^2\right) p_i\right) - \mu_T^2 \quad (48)$$

$$\left(\sum_{i=1}^{k}\left(i^2 - 2i\mu_0 + 2\mu_0^2\right) p_i\right) + \left(\sum_{i=k+1}^{L}\left(i^2 - 2i\mu_1 + 2\mu_1^2\right) p_i\right) - \mu_T^2 \quad (49)$$

Výpočet 3

$$\sigma_W^2 + \sigma_B^2 = \sigma_T^2 \quad (50)$$

$$\left(\sum_{i=1}^{k}\left(i^2 - 2i\mu_0 + 2\mu_0^2\right)p_i\right) + \left(\sum_{i=k+1}^{L}\left(i^2 - 2i\mu_1 + 2\mu_1^2\right)p_i\right) - \mu_T^2 = \left(\sum_{i=1}^{L}i^2 p_i\right) - \mu_T^2 \quad (51)$$

$$\left(\sum_{i=1}^{k}\left(-2i\mu_0 + 2\mu_0^2\right)p_i\right) + \left(\sum_{i=k+1}^{L}\left(-2i\mu_1 + 2\mu_1^2\right)p_i\right) - \mu_T^2 = -\mu_T^2 \quad (52)$$

$$\left(2\sum_{i=1}^{k}\left(-i\mu_0 + \mu_0^2\right)p_i\right) + \left(2\sum_{i=k+1}^{L}\left(-i\mu_1 + \mu_1^2\right)p_i\right) = 0 \quad (53)$$

$$\left(\sum_{i=1}^{k}\left(-i\mu_0 + \mu_0^2\right)p_i\right) + \left(\sum_{i=k+1}^{L}\left(-i\mu_1 + \mu_1^2\right)p_i\right) = 0 \quad (54)$$

$$\left(\sum_{i=1}^{k}-i\mu_0 p_i + \mu_0^2 p_i\right) + \left(\sum_{i=k+1}^{L}\left(-i\mu_1 + \mu_1^2\right)p_i\right) = 0 \quad (55)$$

$$\left(\sum_{i=1}^{k}-i\mu_0 p_i + \mu_0^2 p_i\right) + \left(\sum_{i=k+1}^{L}-i\mu_1 p_i + \mu_1^2 p_i\right) = 0 \quad (56)$$

$$\left(\sum_{i=1}^{k}-i\mu_0 p_i\right) + \left(\sum_{i=1}^{k}\mu_0^2 p_i\right) + \left(\sum_{i=k+1}^{L}-i\mu_1 p_i + \mu_1^2 p_i\right) = 0 \quad (57)$$

$$\left(\sum_{i=1}^{k}\left(-i p_i\right)\mu_0\right) + \left(\sum_{i=1}^{k}\mu_0^2 p_i\right) + \left(\sum_{i=k+1}^{L}-i\mu_1 p_i + \mu_1^2 p_i\right) = 0 \quad (58)$$

$$-\omega_0\mu_0^2 + \left(\sum_{i=1}^{k}\mu_0^2 p_i\right) + \left(\sum_{i=k+1}^{L}-i\mu_1 p_i + \mu_1^2 p_i\right) = 0 \quad (59)$$

$$-\left(\omega_0\mu_0^2 + \omega_1\mu_1^2\right) + \left(\sum_{i=1}^{k}\mu_0^2 p_i\right) + \left(\sum_{i=k+1}^{L}\mu_1^2 p_i\right) = 0 \quad (60)$$

$$-\left(\omega_0\mu_0^2 + \omega_1\mu_1^2\right) + \left(\sum_{i=1}^{k}p_i\right)\mu_0^2 + \left(\sum_{i=k+1}^{L}\mu_1^2 p_i\right) = 0 \quad (61)$$

$$-\left(\omega_0\mu_0^2 + \omega_1\mu_1^2\right) + \left(\sum_{i=1}^{k}p_i\right)\mu_0^2 + \left(\sum_{i=k+1}^{L}p_i\right)\mu_1^2 = 0 \quad (62)$$

$$-\left(\omega_0\mu_0^2 + \omega_1\mu_1^2\right) + \omega_0\mu_0^2 + \left(\sum_{i=k+1}^{L}p_i\right)\mu_1^2 = 0 \quad (63)$$

$$-\left(\omega_0\mu_0^2 + \omega_1\mu_1^2\right) + \omega_0\mu_0^2 + \omega_1\mu_1^2 = 0 \quad (64)$$

$$0 = 0 \quad (65)$$