Praktikum Autonome Systeme

Automated Planning

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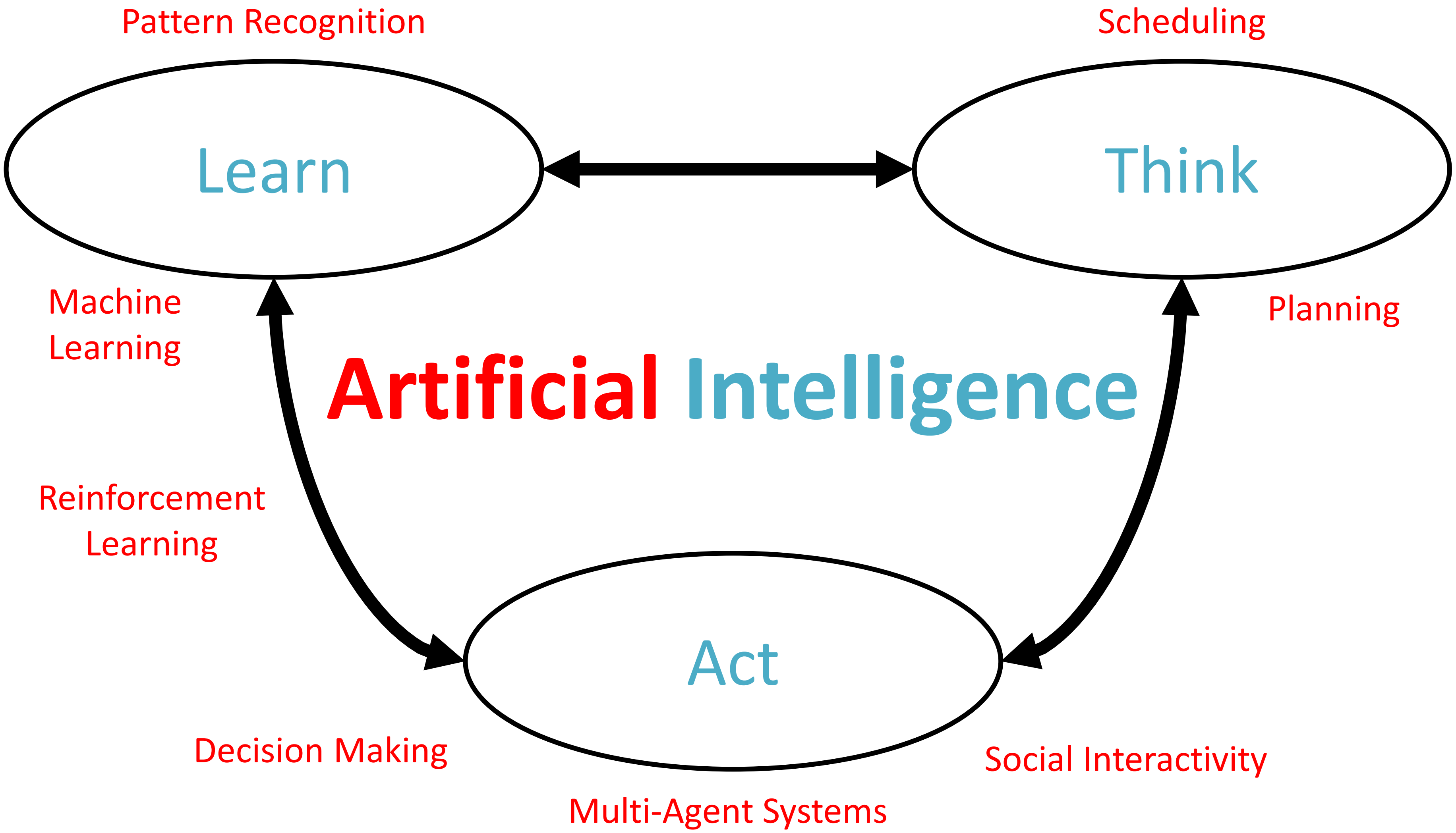
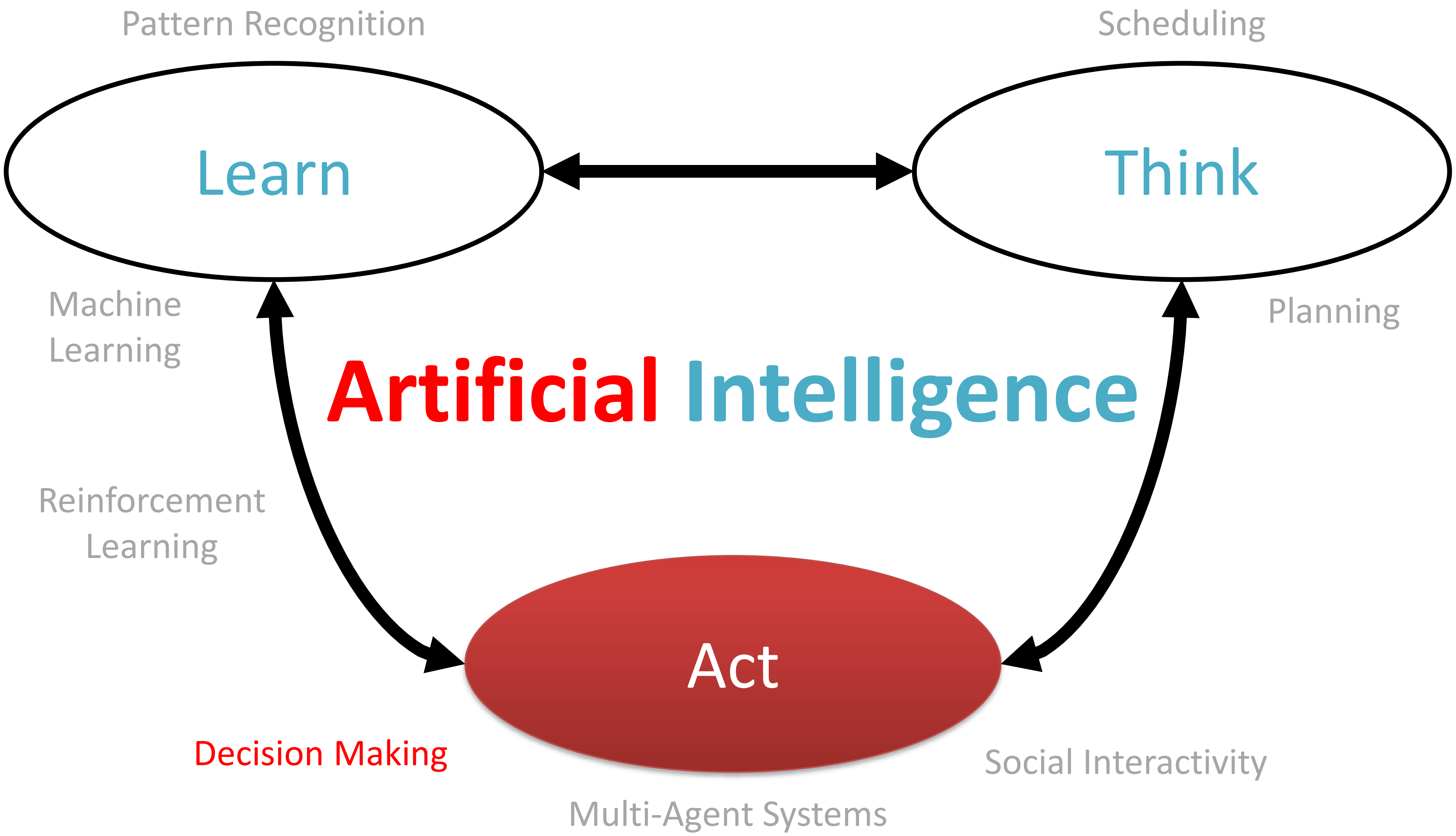
WiSe 2020/21

→ Recap: Decision Making


Pattern Recognition
Scheduling
Learn
Think
Machine Learning
Planning
Artificial Intelligence
Reinforcement Learning
Act
Decision Making
Social Interactivity
Multi-Agent Systems


Pattern Recognition
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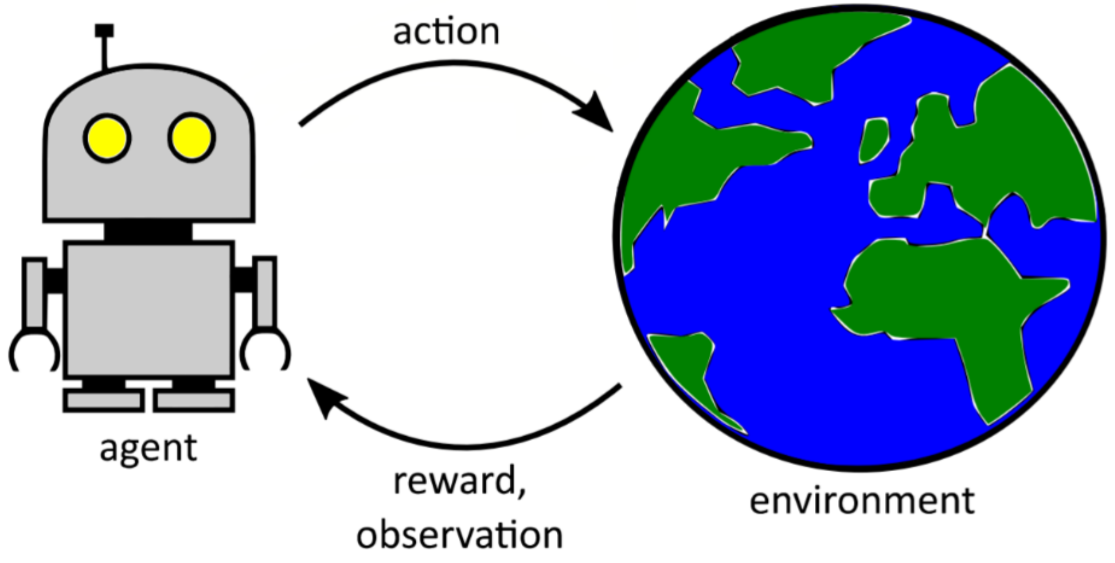
Decision Making

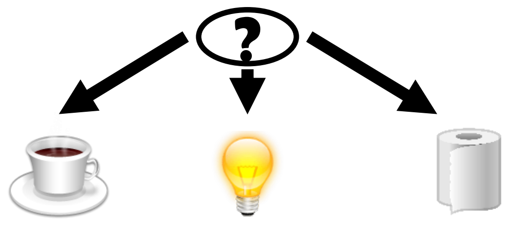
- **Goal:** Autonomously select actions to solve a (complex) task
  - time could be important (but not necessarily)
  - maximize the **expected reward** for each state



Multi-Armed Bandits

- **Multi-Armed Bandit**: situation, where you have to learn how to make a good (long-term) choice

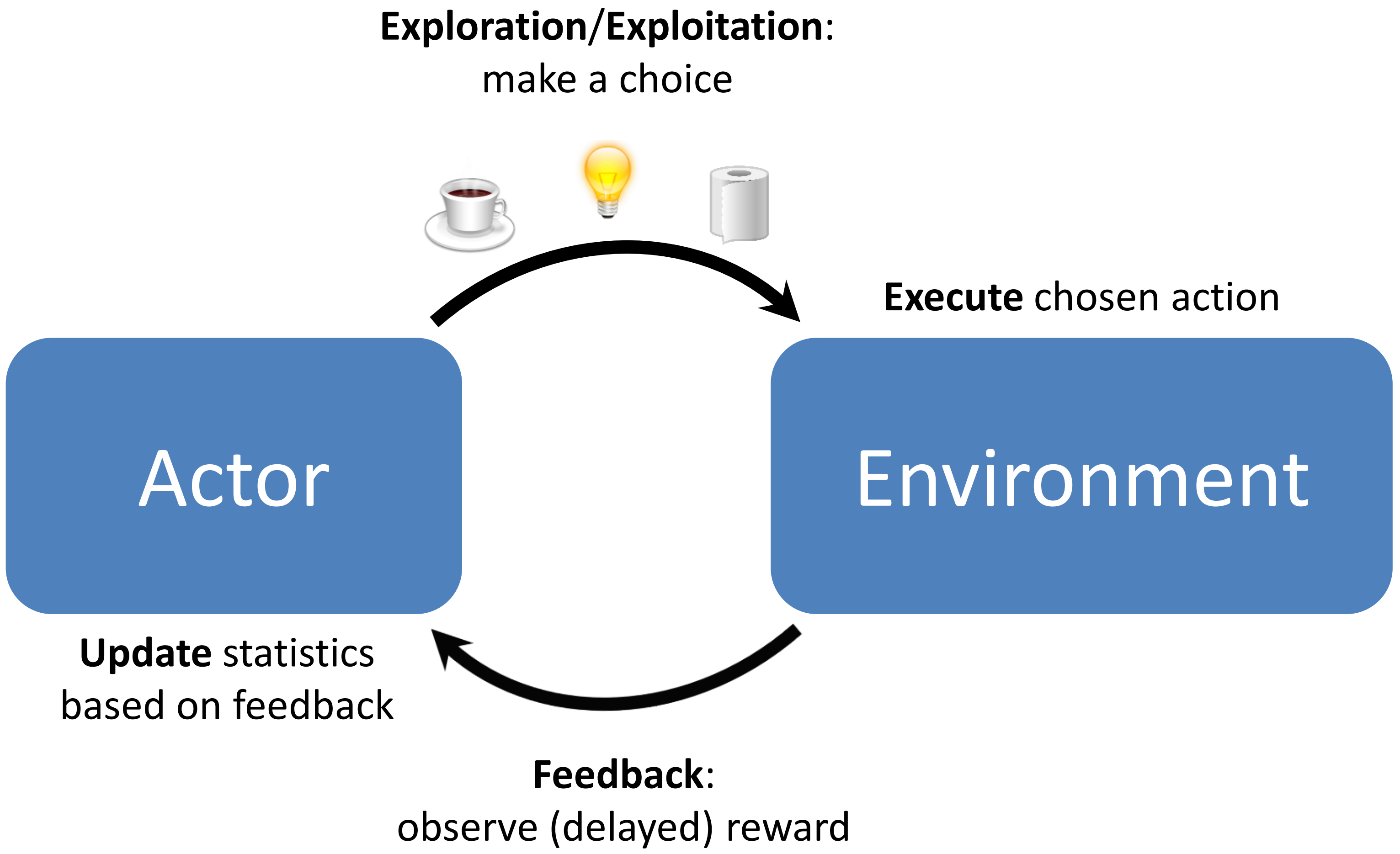
- **Explore** choices to gather information (= Exploration)
  - Example: random choice



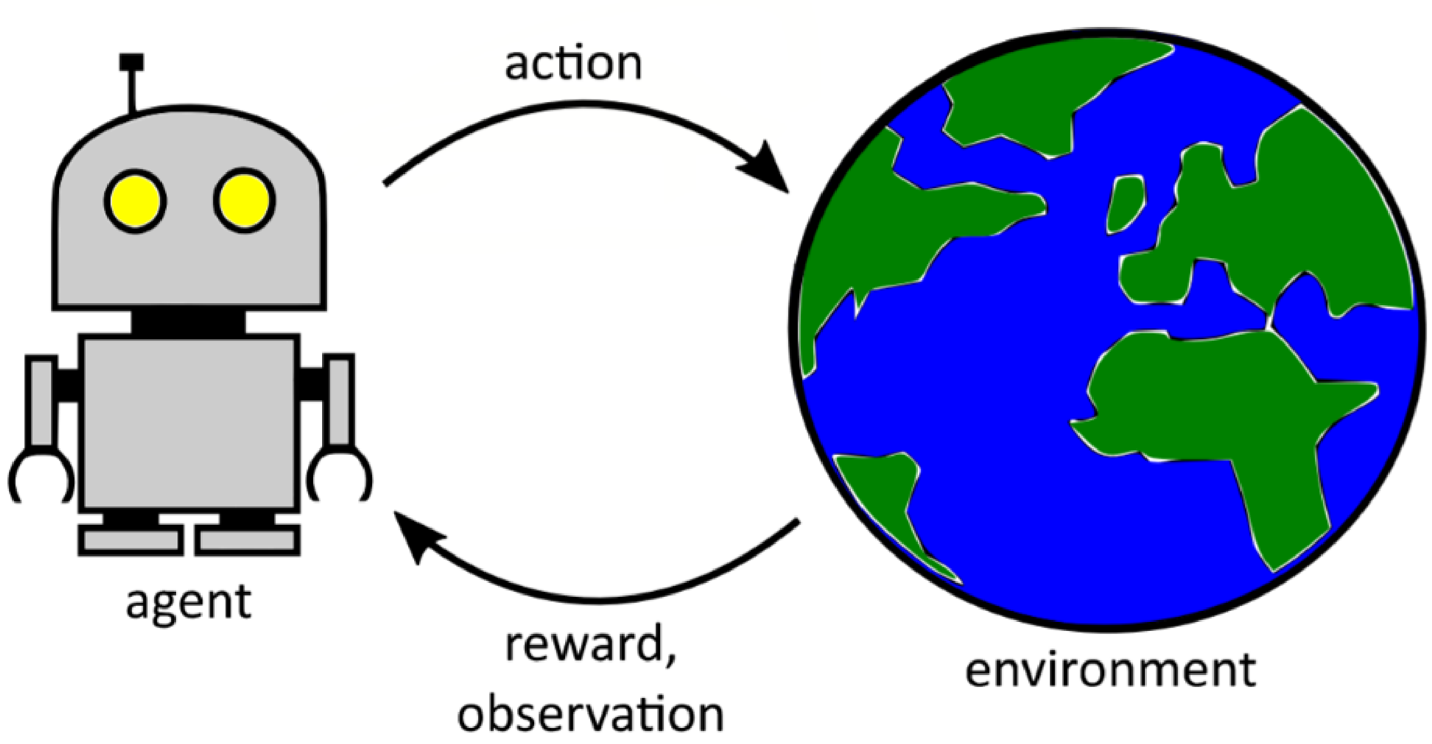
- **Prefer** promising choices (= Exploitation)
  - Example: greedy choice (e.g., using `argmax`)

- A good Multi-Armed Bandit solution should **always** balance between Exploration and Exploitation

Multi-Armed Bandits



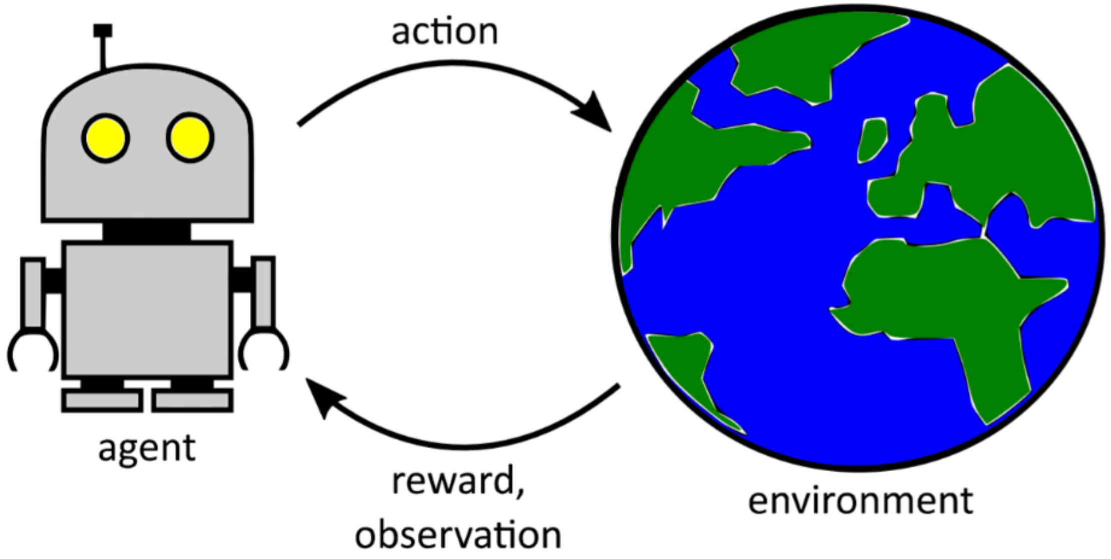
Multi-Armed Bandits



→ Sequential Decision Making

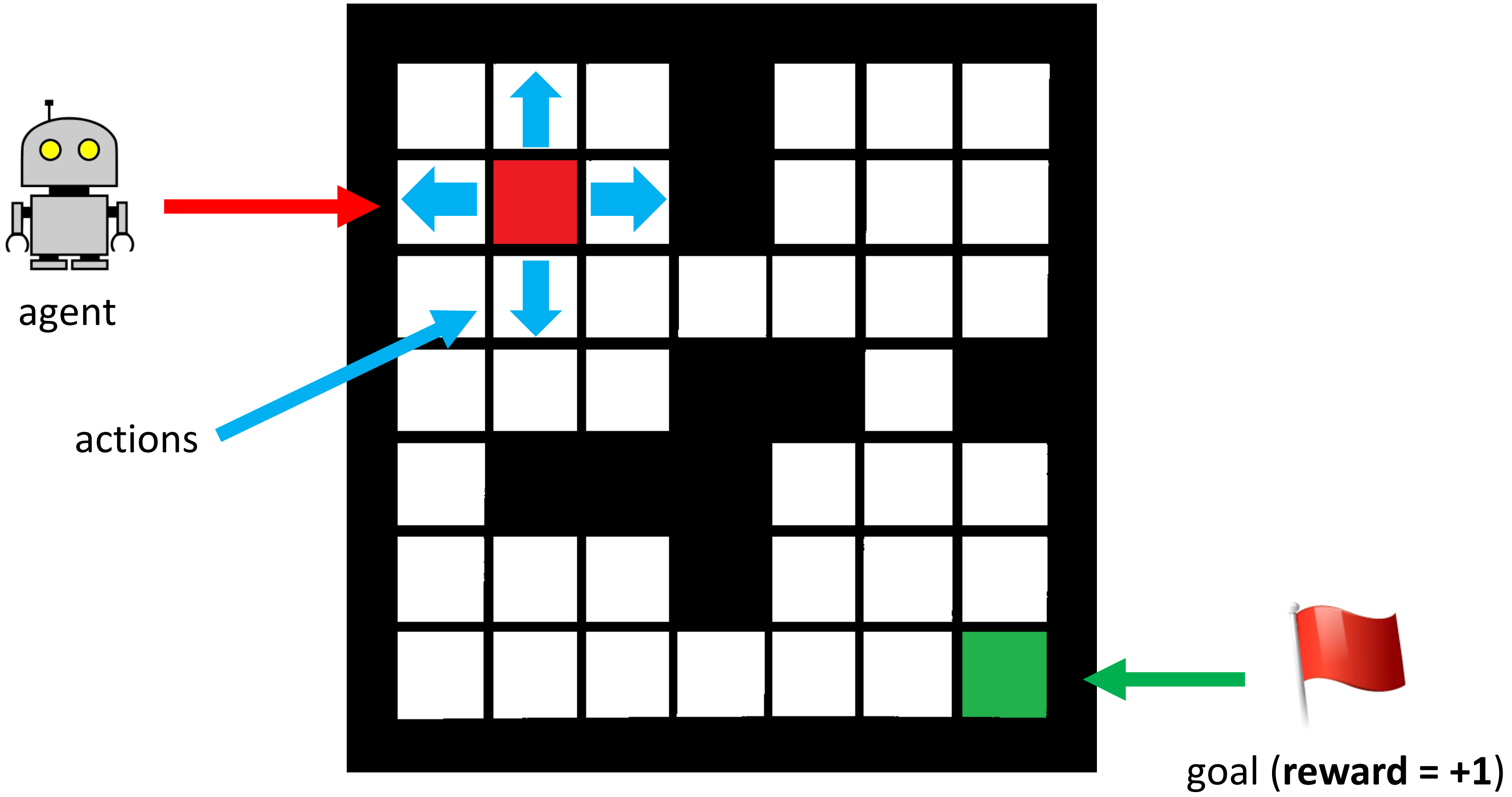
Sequential Decision Making

- **Goal:** Autonomously select actions to solve a (complex) task
  - time is important (actions might have **long term** consequences)
  - maximize the **expected cumulative reward** for each state



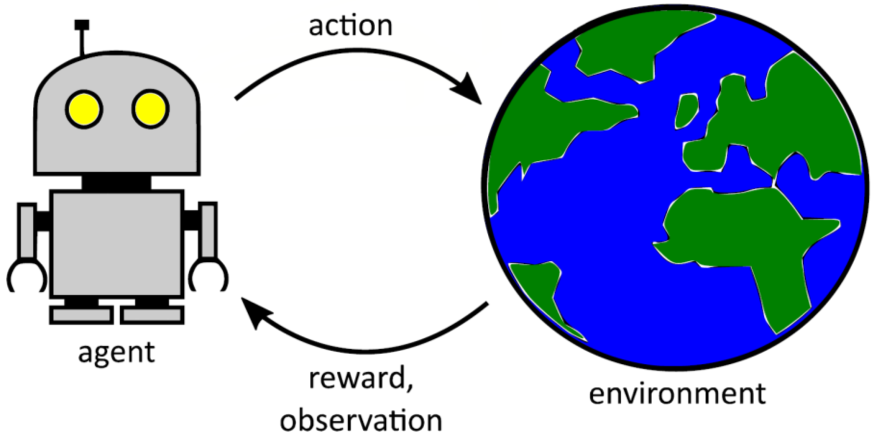
Sequential Decision Making Example

- **Rooms**: reach a goal as fast as possible



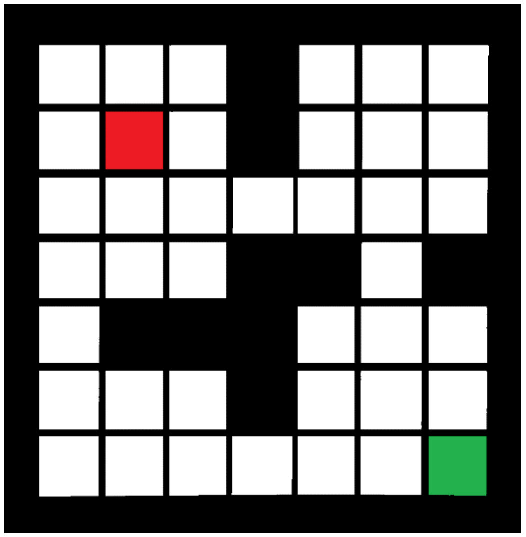
Markov Decision Processes

- A Markov Decision Process (MDP) is defined as $\mathrm{M} = \langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R} \rangle$:
  - $\mathcal{S}$ is a (finite) set of states
  - $\mathcal{A}$ is a (finite) set of actions
  - $\mathcal{P}(s_{t+1}|s_t, a_t) \in [0, 1]$ is the probability for reaching $s_{t+1} \in \mathcal{S}$ when executing $a_t \in \mathcal{A}$ in $s_t \in \mathcal{S}$
  - $\mathcal{R}(s_t, a_t) \in \mathbb{R}$ is a reward function



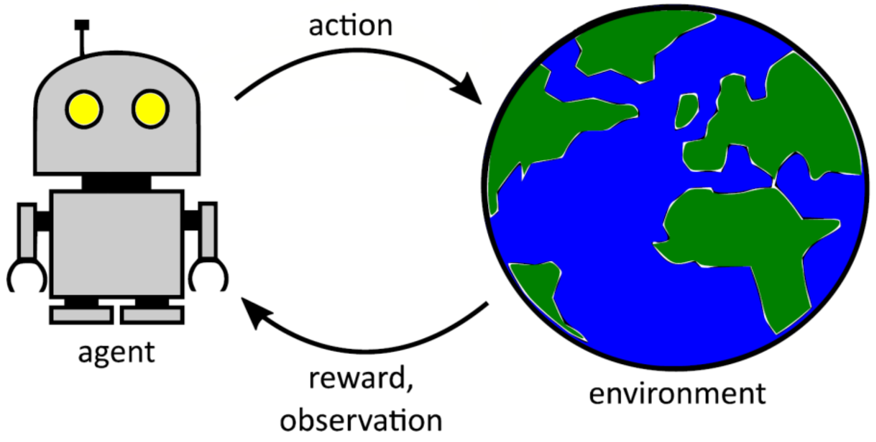
Rooms as MDP

- Define Rooms as MDP $\mathrm{M} = \langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R} \rangle$:
  - **States $\boldsymbol{\mathcal{S}}$**: position of the agent
  - **Actions $\boldsymbol{\mathcal{A}}$**: move north/south/west/east
  - **Transitions $\boldsymbol{\mathcal{P}}$**: deterministic movement. No transition if moving against wall.
  - **Rewards $\boldsymbol{\mathcal{R}}$**: +1 if goal is reached, 0 otherwise



Markov Decision Processes

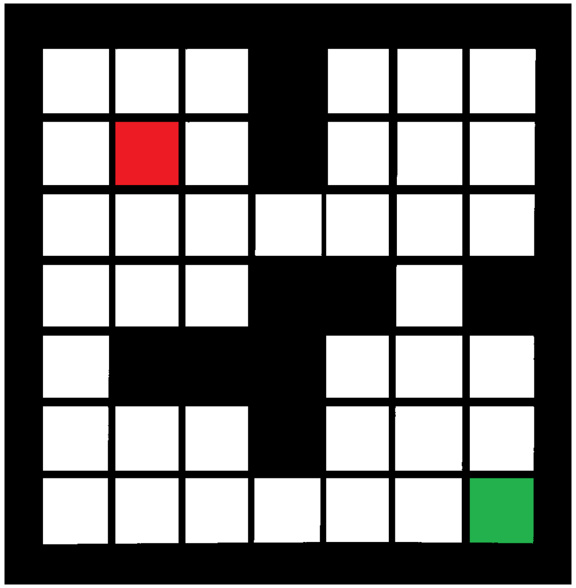
- MDPs formally describe environments for Sequential Decision Making
- All states $s_t \in \mathcal{S}$ are **Markov** such that
  $\mathbb{P}(s_{t+1}|s_t) = \mathbb{P}(s_{t+1}|s_1, \dots, s_t)$ (no history of past states required)
- Assumes **full observability** of the state
- States and actions may be **discrete** or **continuous**
- Many problems can be formulated as MDPs!
  - E.g., multi-armed bandits are MDPs with a single state



Policies

- A **policy** $\pi: \mathcal{S} \rightarrow \mathcal{A}$ represents the behavioral strategy of an agent
  - Policies may also be stochastic $\pi(a_t|s_t) \in [0,1]$

- Policy examples for Rooms:
  - π_0: maps each state $s_t \in \mathcal{S}$ to a random action $a_t \in \mathcal{A}$
  - π_1 : maps each state $s_t \in \mathcal{S}$ to action $a_t = MoveSouth \in \mathcal{A}$
  - π_2 : maps state $s_t \in \mathcal{S}$ to action $a_t = MoveSouth \in \mathcal{A}$ if t is odd and select a_t random otherwise .



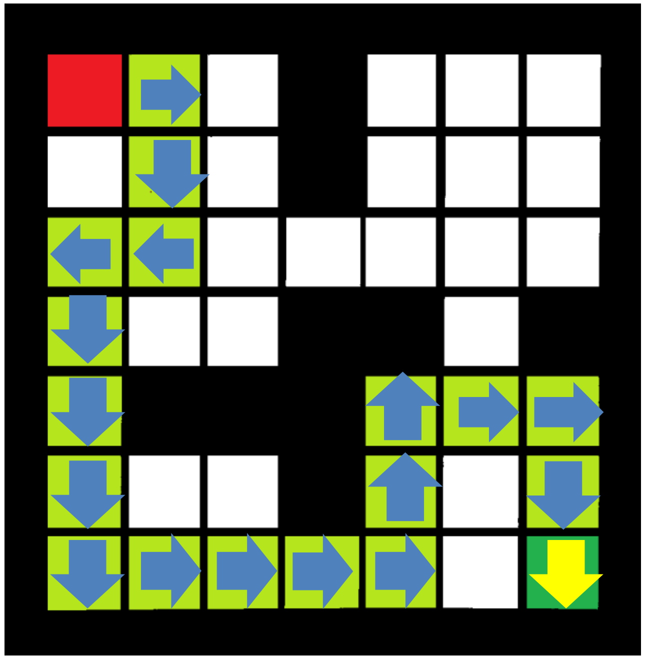
1. **How do we know which policy is better?**
2. **How can we improve a given policy?**

Returns

- The **return** of a state $s_t \in \mathcal{S}$ for a horizon h given a policy π is the cumulative (discounted) future reward (h may be infinite!):

$$G_t = \sum_{k=0}^{h-1} \gamma^k \mathcal{R}(s_{t+k}, \pi(s_{t+k})), \gamma \in [0,1]$$

- Rooms Example: $\gamma = 0.99$
  - The chosen paths needs **18 steps** to reach the goal
  - Thus, the return from the starting point is: $G_1 = r_1 + \gamma r_2 + \ \dots + \gamma^{17} r_{18} =$
    $= \gamma^{17} r_{18} = 0.99^{17} \sim 0.843$
- What would be the return G_1, if the goal isn‘t reached at all?
- What is the optimal value of G_1?



Value Functions

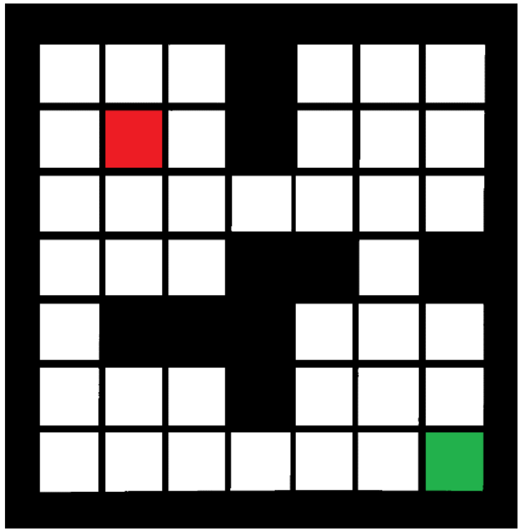
- The **value** of a state $s_t \in \mathcal{S}$ is the expected return of s_t for a horizon $h \in \mathbb{N}$ given a policy π:

$$V^{\pi}(s_t) = \mathbb{E}[G_t | s_t]$$

- The **action value** of a state $s_t \in \mathcal{S}$ and action $a_t \in \mathcal{A}$ is the expected return of executing a_t in s_t for a horizon $h \in \mathbb{N}$ given a policy π:

$$Q^{\pi}(s_t, a_t) = \mathbb{E}[G_t | s_t, a_t]$$

- Rooms Example:
  - V^{π} and/or Q^{π} can be estimated by **averaging** over **several returns** G_t observed by executing a (fixed) policy π
- **Value functions (V^{π} and/or Q^{π}) can be used to evaluate policies π**



Remark: Return / Value Estimation

- Estimating the return G_t or the value $Q^\pi(s_t, a_t)$ of state-action pair $\langle s_t, a_t \rangle$ always has the following form:

$$\mathcal{R}(s_t, a_t) + \gamma X$$

- X could be:
  - The successor return G_{t+1}
    - reward seqence must be known
  - The successor value $Q^\pi(s_{t+1}, a_{t+1})$
    - $\langle s_{t+1}, a_{t+1} \rangle$ and Q^π must be known
  - The expected successor value $\mathbb{E}[G_{t+1} | s_{t+1}, a_{t+1}]$
    - $\mathcal{P}(s_{t+1} | s_t, a_t)$, $\langle s_{t+1}, a_{t+1} \rangle$, and Q^π must be known

Monte Carlo Planning / Learning

Temporal-Difference Learning

Dynamic Programming

Optimal Policies and Value Functions

- **Goal:** Find an *optimal policy* π^* which maximizes the expected return $\mathbb{E}[G_t|s_t]$ for each state:

$$\pi^* = argmax_\pi V^\pi(s_t), \forall s_t \in \mathcal{S}$$

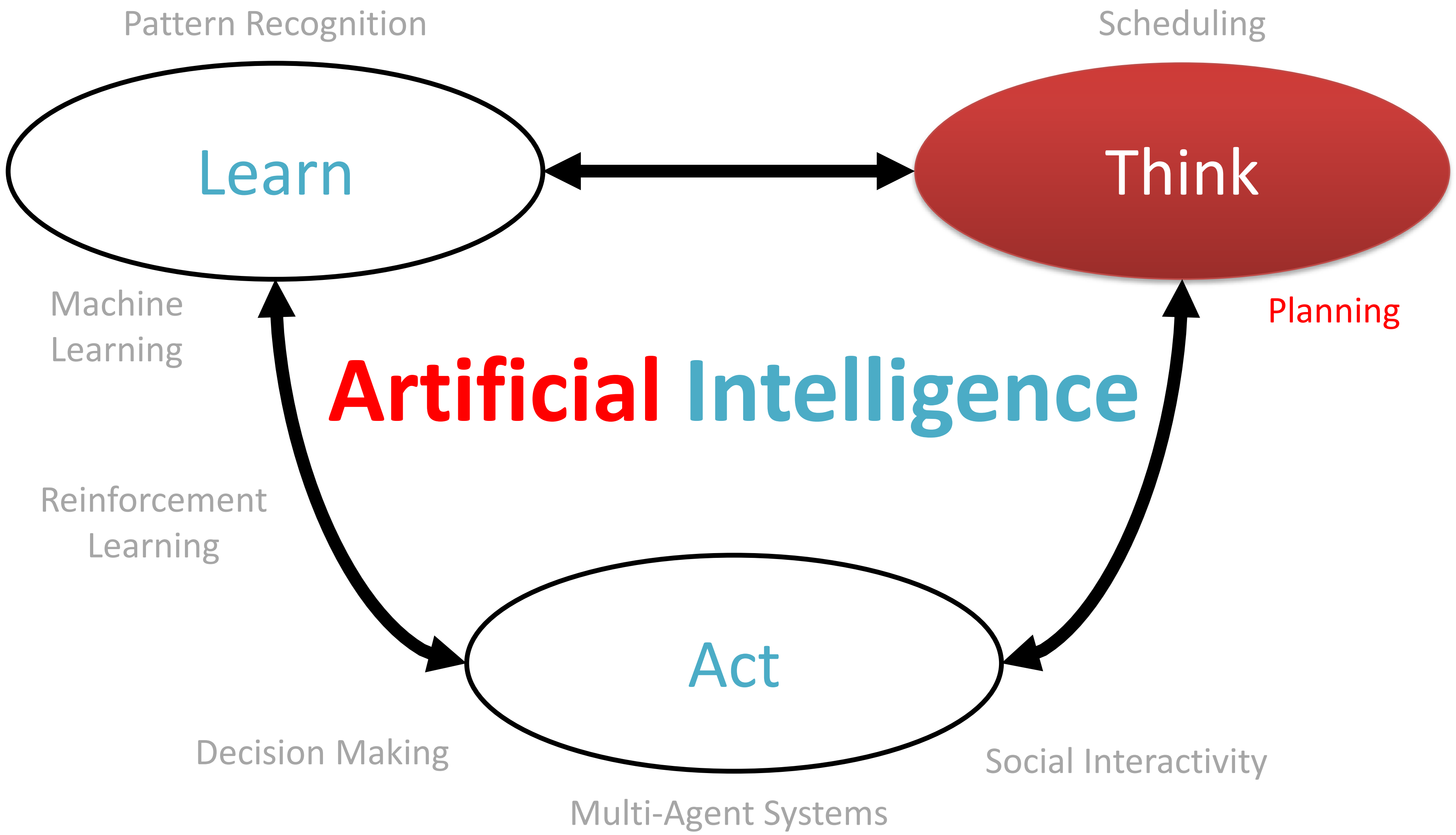
- The *optimal value function* is defined by:

$$V^*(s_t) = V^{\pi^*}(s_t) = max_\pi V^\pi(s_t)$$
$$Q^*(s_t, a_t) = Q^{\pi^*}(s_t, a_t) = max_\pi Q^\pi(s_t, a_t)$$

- When V^* or Q^* is known, π^*can be derived.

How to find an **optimal** policy or the **optimal** value function?

→ Automated Planning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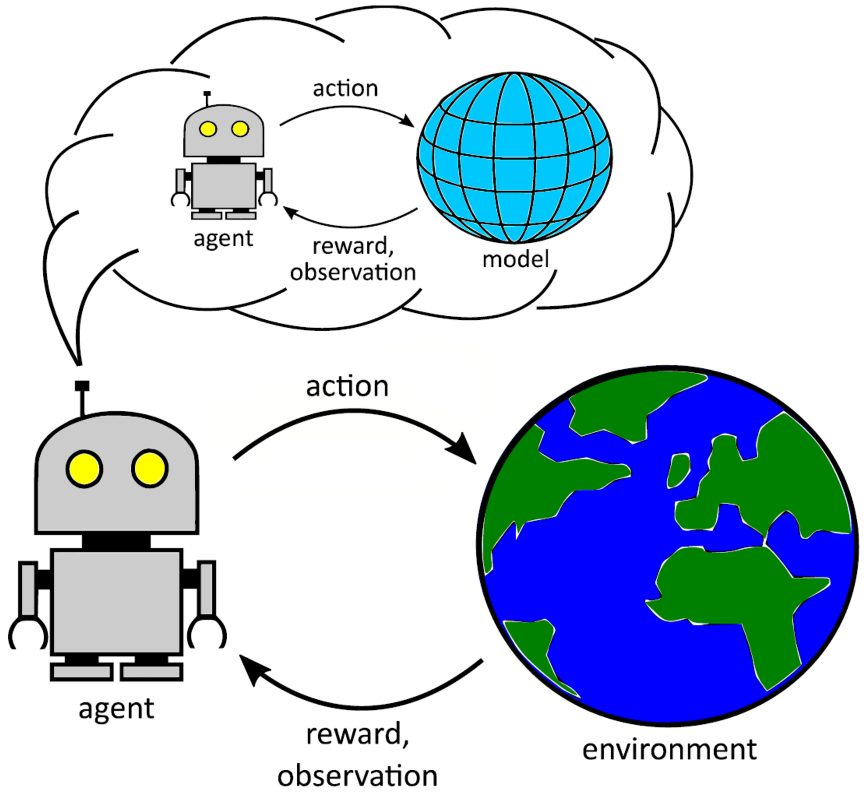
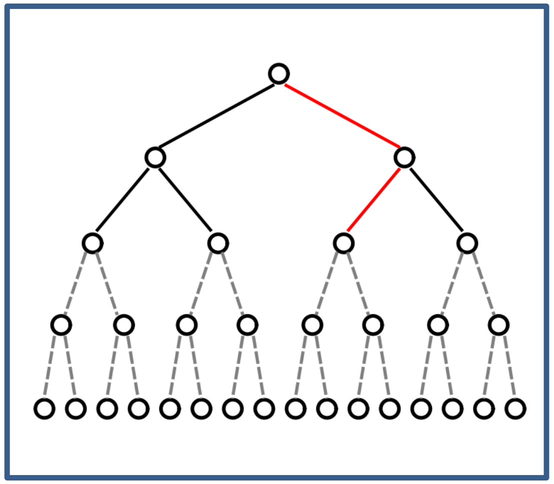
Automated Planning

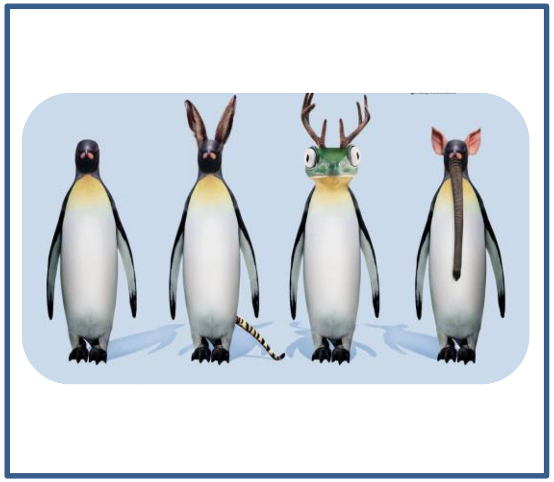
- **Goal:** Find (near-)**optimal policies** π^* to solve complex problems
- Use (heuristic) lookahead search on a **given model** $\widehat{M} \approx M$ of the problem



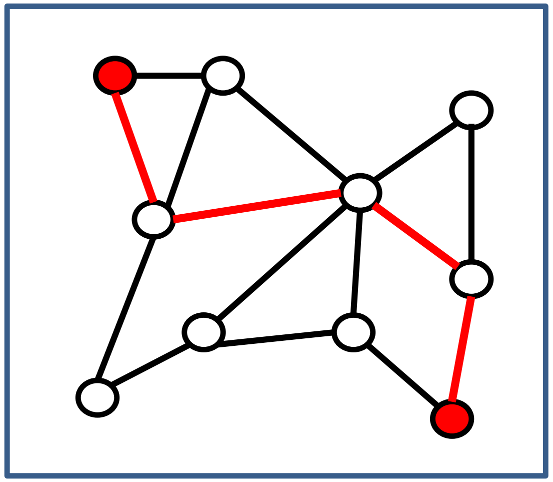
Planning Approaches (Examples)



Tree Search



Evolutionary Computation



Dynamic Programming

→ Dynamic Programming

Dynamic Programming

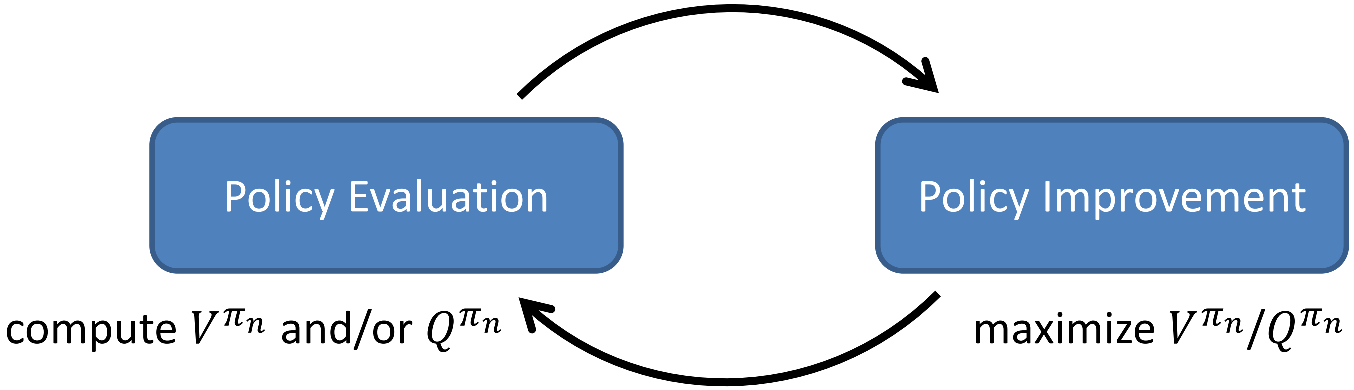
- **Dynamic** refers to sequential / temporal component of a problem
- **Programming** refers to optimization of a program

- We want to solve Markov Decision Processes (MDPs):
  - MDPs are **sequential** decision making problems
  - To find a solution, we need to optimize a **program** (policy π)

Policy Iteration

- Dynamic Programming approach to find an optimal policy π^*

- Starts with a (random) guess π_0
- Consists of two alternating steps given π_n:


Policy Evaluation
Policy Improvement
compute V^{π_n} and/or Q^{π_n}
maximize V^{π_n}/Q^{π_n}

- Terminates when $\pi_{i+1} = \pi_i$ or when a time budget runs out

- **Policy Iteration forms the basis for most Planning and Reinforcement Learning algorithms!**

Value Iteration

- Dynamic Programming approach to find the optimal value function V^*

- Starts with a (random) guess V^0
- Iteratively updates the value estimate $V^n(s_t)$ for **each state** $s_t \in \mathcal{S}$

$$V^{n+1}(s_t) = \underbrace{\max_{a_t \in \mathcal{A}}}_{\text{Policy Improvement}} \{ \underbrace{\mathcal{R}(s_t, a_t) + \gamma \sum_{s_{t+1} \in \mathcal{S}} \mathcal{P}(s_{t+1} | s_t, a_t) V^n(s_{t+1})}_{\text{Policy Evaluation}} \}$$

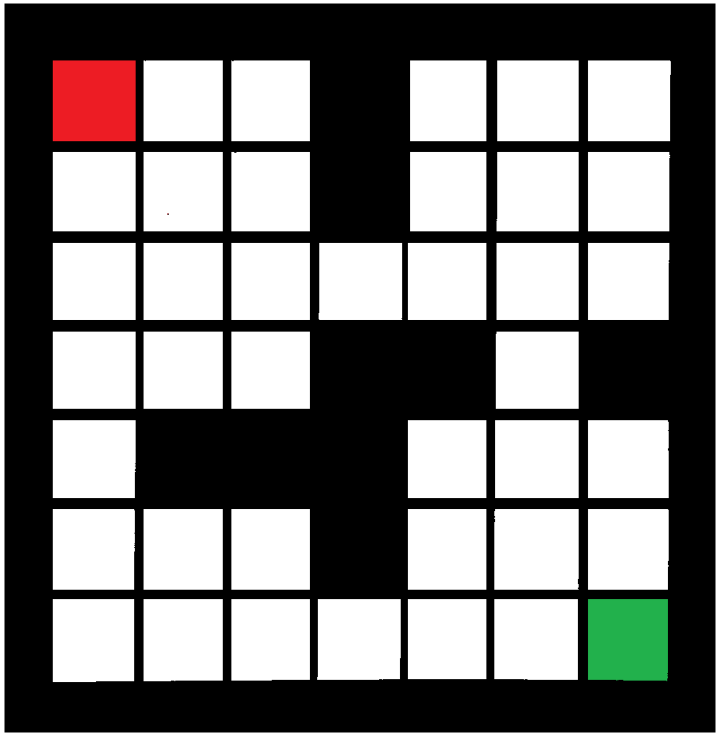
- Terminates when $V^{n+1} = V^n$ or when a time budget runs out
- The optimal action-value function Q^* is computed analogously

- V^* and/or Q^* can be used to derive an optimal policy π^*

- **Do you see the link to Policy Iteration?**

Value Iteration - Example

- Optimal „Value Map“ in Rooms ($\gamma = 0.99$): for each state $s_t \in \mathcal{S}$
  - $V^{n+1}(s_t) = \max_{a_t \in \mathcal{A}} \{\mathcal{R}(s_t, a_t) + \gamma \sum_{s_{t+1} \in \mathcal{S}} \mathcal{P}(s_{t+1} | s_t, a_t) V^n(s_{t+1})\}$



Remember:

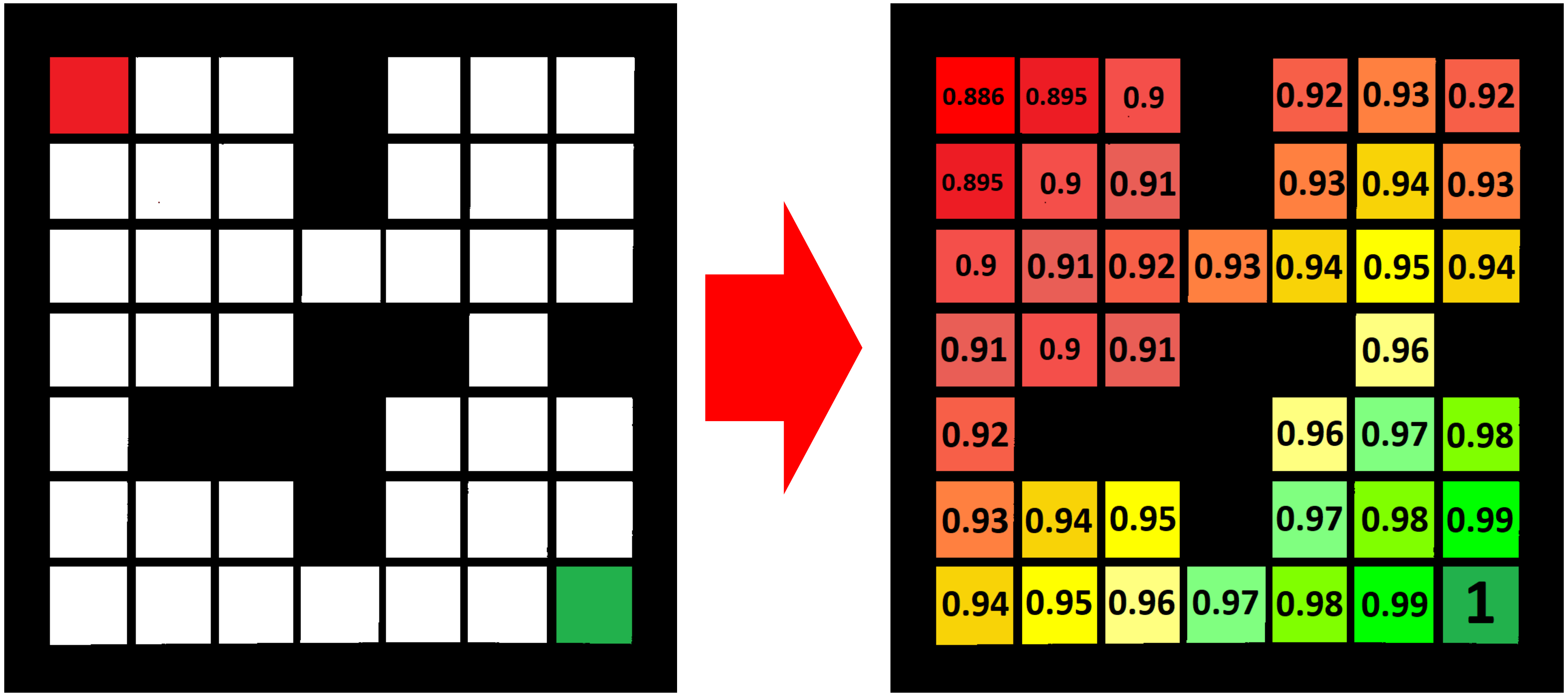
$$\mathcal{R}(s_t, a_t) + \gamma X$$

In this case

$$X = \sum_{s_{t+1} \in \mathcal{S}} \mathcal{P}(s_{t+1} | s_t, a_t) V^n(s_{t+1})$$

Value Iteration - Example

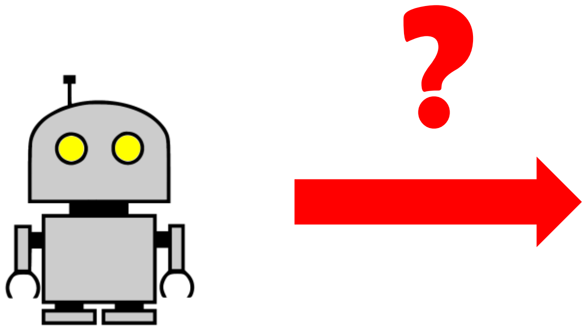
- Optimal „Value Map“ in Rooms ($\gamma = 0.99$): for each state $s_t \in \mathcal{S}$
  - $V^{n+1}(s_t) = \max_{a_t \in \mathcal{A}} \{\mathcal{R}(s_t, a_t) + \gamma \sum_{s_{t+1} \in \mathcal{S}} \mathcal{P}(s_{t+1} | s_t, a_t) V^n(s_{t+1})\}$

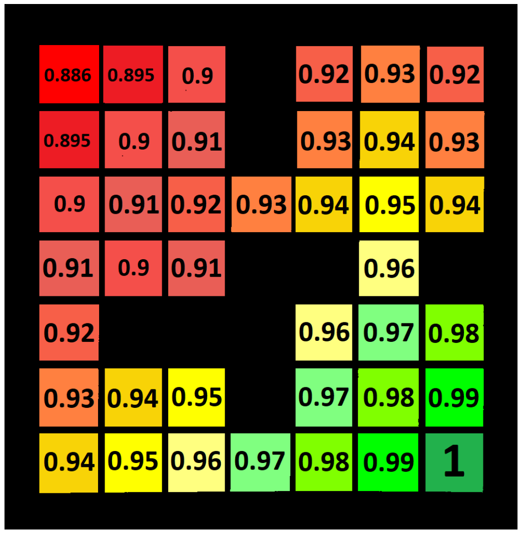


0.886	0.895	0.9		0.92	0.93	0.92
0.895	0.9	0.91		0.93	0.94	0.93
0.9	0.91	0.92	0.93	0.94	0.95	0.94
0.91	0.9	0.91			0.96	
0.92				0.96	0.97	0.98
0.93	0.94	0.95		0.97	0.98	0.99
0.94	0.95	0.96	0.97	0.98	0.99	1

Advantages and Disadvantages of DP

- General approach (does not require **explicit** domain knowledge)
- Converges to optimal solution
- Does not require exploration-exploitation (**all states** are visited anyway)
- Computational costs
- Memory costs
- Availability of an explicit model $\mathrm{M} = \langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R} \rangle$





Intermediate Summary

- What we know so far:
  - Markov Decision Processes (MDPs)
  - Policies and Value Functions
  - Optimally solve MDPs with Dynamic Programming

- What we don‘t know (yet):
  - How to find solutions in a more scalable way?
  - How to react to unexpected events?
  - How to find solutions without a model?

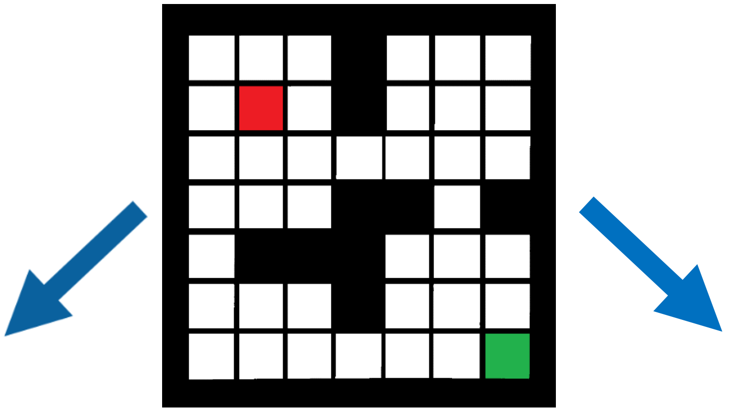
→ Monte Carlo Planning

Global Planning and Local Planning

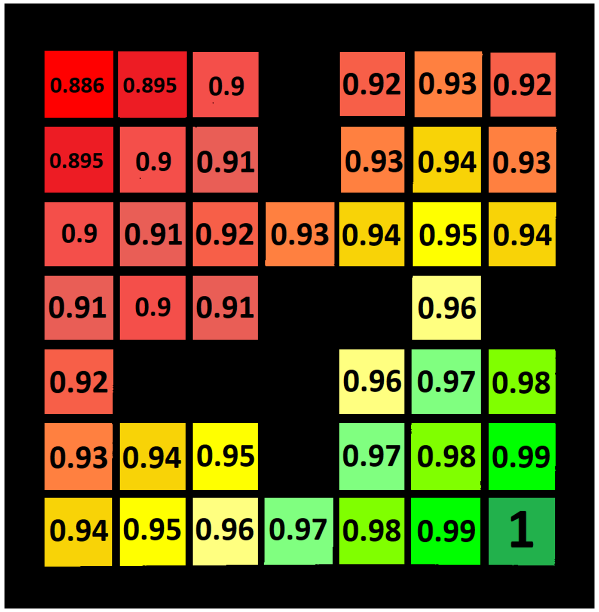
- Global Planning
  - considers the **entire state space** $\mathcal{S}$ to approximate π^*
  - produces for each state $s_t \in \mathcal{S}$ a mapping to actions $a_t \in \mathcal{A}$
  - typically performed **offline** (before deploying the agent)
  - *Example:* Dynamic Programming (Policy and Value Iteration)

- Local Planning
  - only considers the **current state** $s_t \in \mathcal{S}$ (and possible future states) to approximate $\pi^*(s_t)$
  - recommends an action $a_t \in \mathcal{A}$ for current state $s_t \in \mathcal{S}$
  - can be performed **online** (interleaving planning and execution)
  - *Example:* Monte Carlo Tree Search

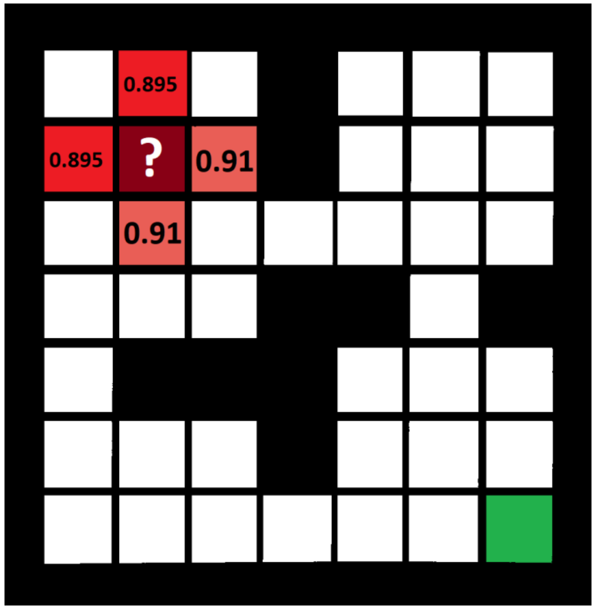
Global Planning vs. Local Planning



Global Planning



Local Planning



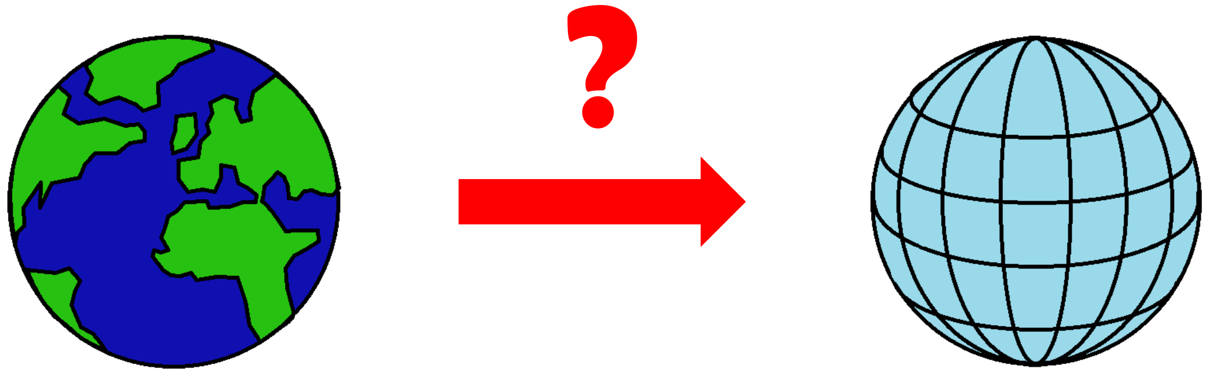
Monte Carlo Planning

- Dynamic Programming always assumes **full knowledge** of the underlying MDP $\mathrm{M} = \langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R} \rangle$
  - Most real-world applications have extremely large state spaces
  - Especially $\mathcal{P}(s_{t+1}|s_t, a_t)$ is hard to pre-specify in practice!

- Monte Carlo Planning only requires a generative model as **blackbox simulator** $\widehat{M} \approx \mathrm{M}$
  - Given some state $s_t \in \mathcal{S}$ and action $a_t \in \mathcal{A}$, the generative model provides a sample $s_{t+1} \in \mathcal{S}$ and $r_t = \mathcal{R}(s_t, a_t)$
  - Can be used to approximate V^* or Q^* via statistical sampling
  - Requires minimal domain knowledge ($\widehat{M}$ can be easily replaced)

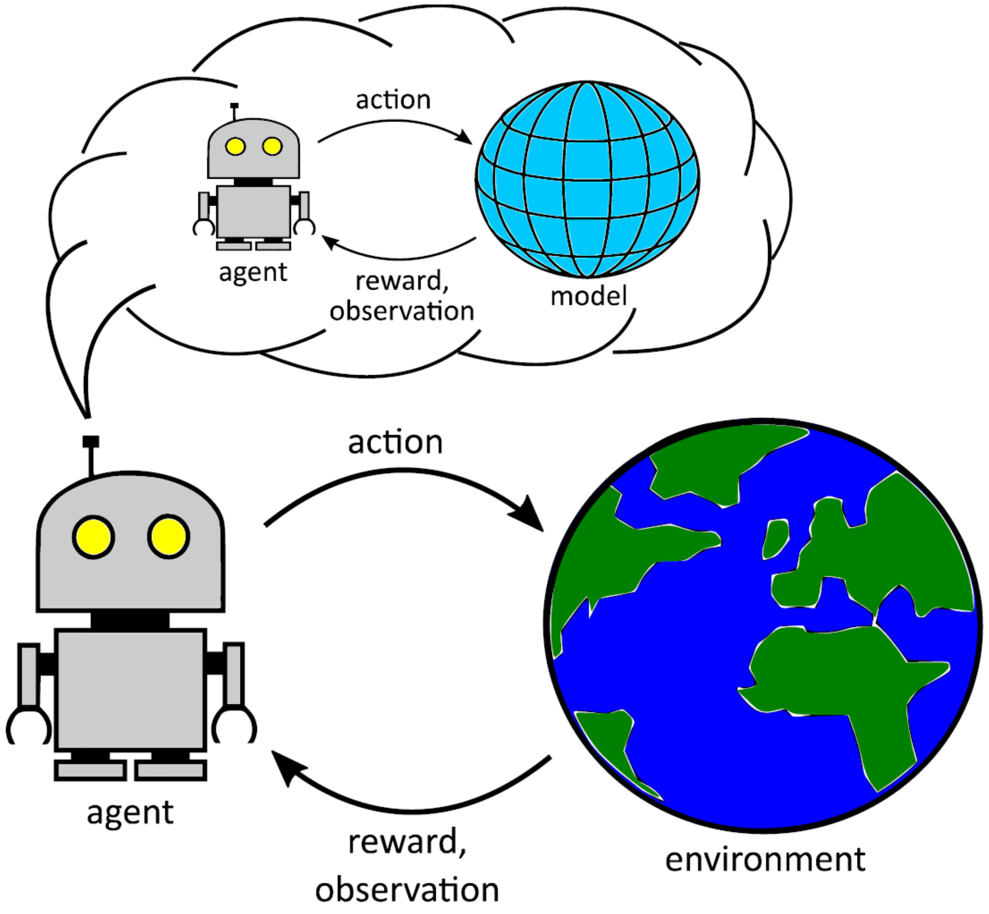
Explicit Model vs. Generative Model

- Generative model can be easier implemented than explicit probability distributions!



Real Environment Environment Model

Planning with Generative Model



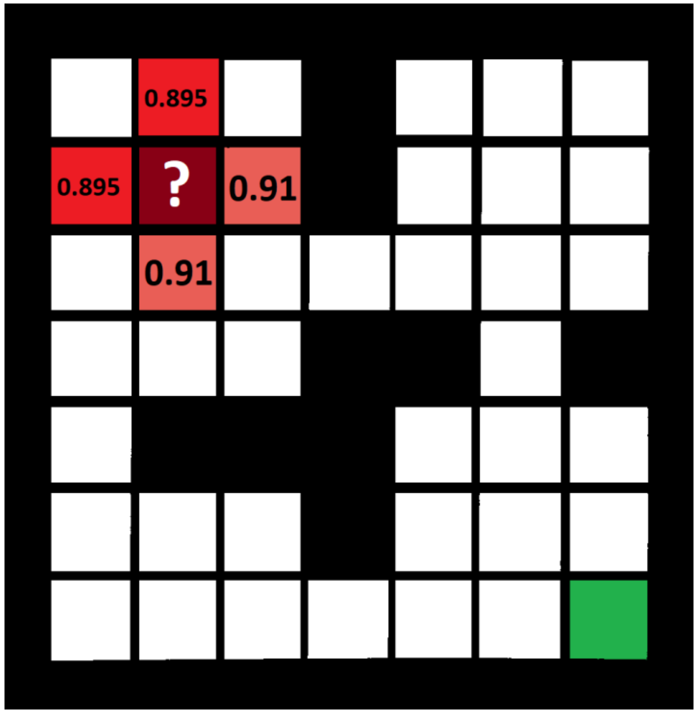
Monte Carlo Rollouts (MCR)

- **Goal:** Given a state $s_t \in \mathcal{S}$ and a policy $\pi_{rollout}$, we want to find the action $a_t \in \mathcal{S}$ which maximizes $Q^{\pi_{rollout}}(s_t, a_t) = \mathbb{E}[G_t | s_t, a_t]$
- **Approach**: Given a computation budget of K simulations and a horizon h
  - Sample K action sequences (= plans) of length h from $\pi_{rollout}$
  - Simulate all plans with generative model $\widehat{\mathrm{M}}$ and compute the return G_t for each plan
  - Update estimate of $Q^{\pi_{rollout}}(s_t, a_t) = \mathbb{E}[G_t | s_t, a_t]$*
  - **Finally:** Select action $a_t \in \mathcal{S}$ with highest $Q^{\pi_{rollout}}(s_t, a_t)$

*only estimate $Q^{\pi_{rollout}}(s_t, a_t)$ of the first action a_t in each plan!

Why do Monte Carlo Rollouts work?

- MCR estimates value function $Q^{\pi_{rollout}}(s_t, a_t)$ of $\pi_{rollout}$ via **sampling**
- Final decision is a **maximization** of $Q^{\pi_{rollout}}(s_t, a_t)$



- MCR makes always decisions with **the same or better** quality than $\pi_{rollout}$
- Thus, decision quality depends on $\pi_{rollout}$ and the simulation model

Monte Carlo Tree Search (MCTS)

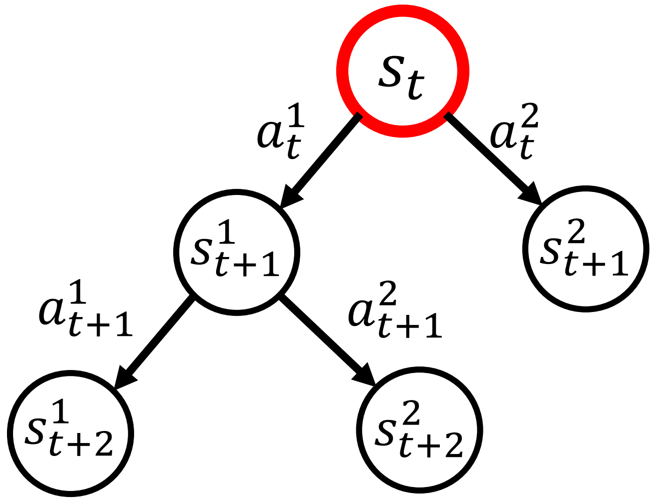
- Current state-of-the-art algorithm for Monte Carlo Planning. Used for:
  - board games like Go, Chess, Shogi
  - combinatorial optimization problems like Rubix Cube

- **Approach**: Incrementally construct and traverse a search tree given a computation budget of K simulations and a horizon h
  - nodes represent states $s_t \in \mathcal{S}$ (and actions $a_t \in \mathcal{A}$)
  - search tree is used to „learn“ $\hat{Q} \approx Q^*$ via blackbox simulation

Monte Carlo Tree Search Phases

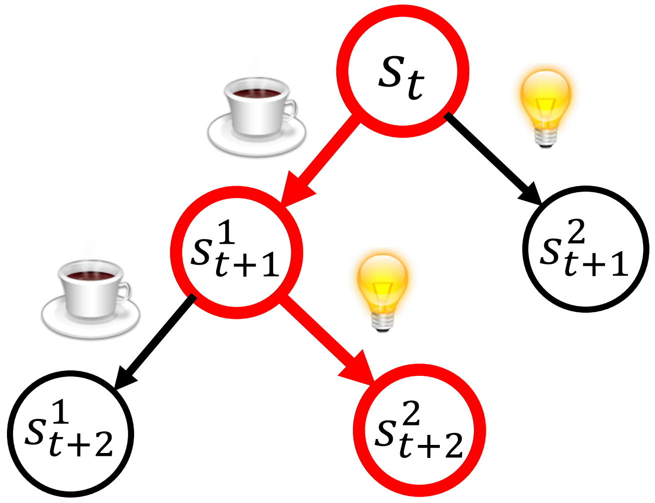
- Selection
- Expansion
- Evaluation/Simulation
- Backup

current state in „real world“



Monte Carlo Tree Search - Selection

- **Selection**
- Expansion
- Evaluation/Simulation
- Backup



Example Selection Strategies:

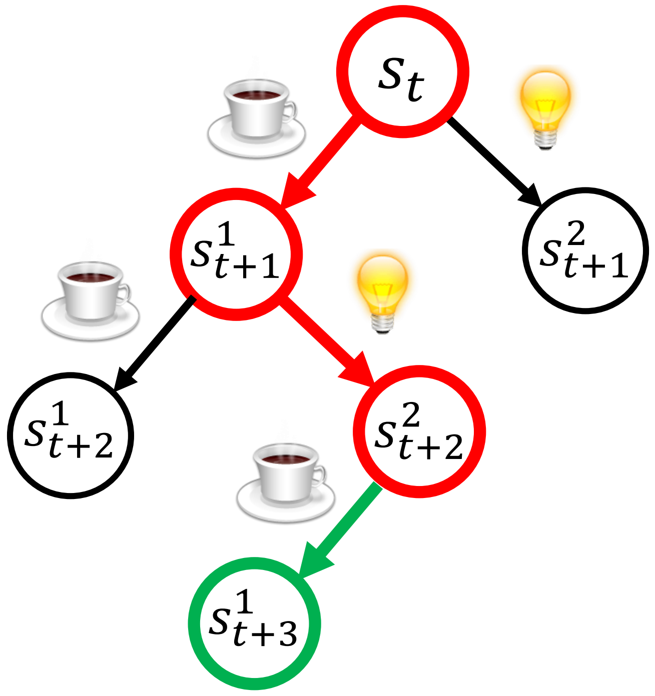
- Random
- Greedy
- ϵ-Greedy
- UCB1

} **Multi-Armed Bandits**

➡ Exploration-Exploitation!!!

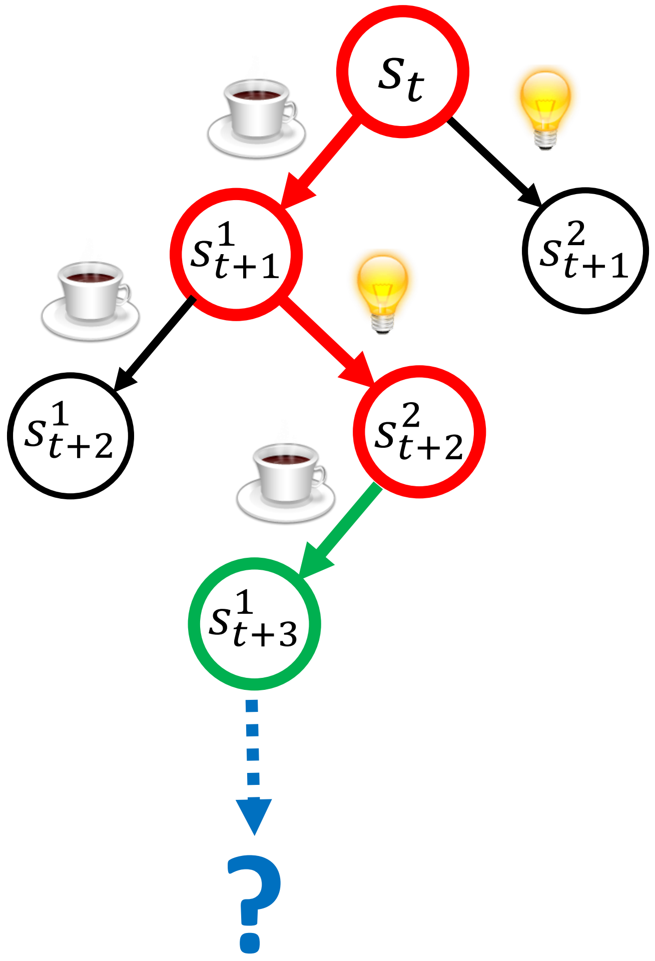
Monte Carlo Tree Search - Expansion

- Selection
- **Expansion**
- Evaluation/Simulation
- Backup



Monte Carlo Tree Search - Expansion

- Selection
- Expansion
- **Evaluation/Simulation**
- Backup

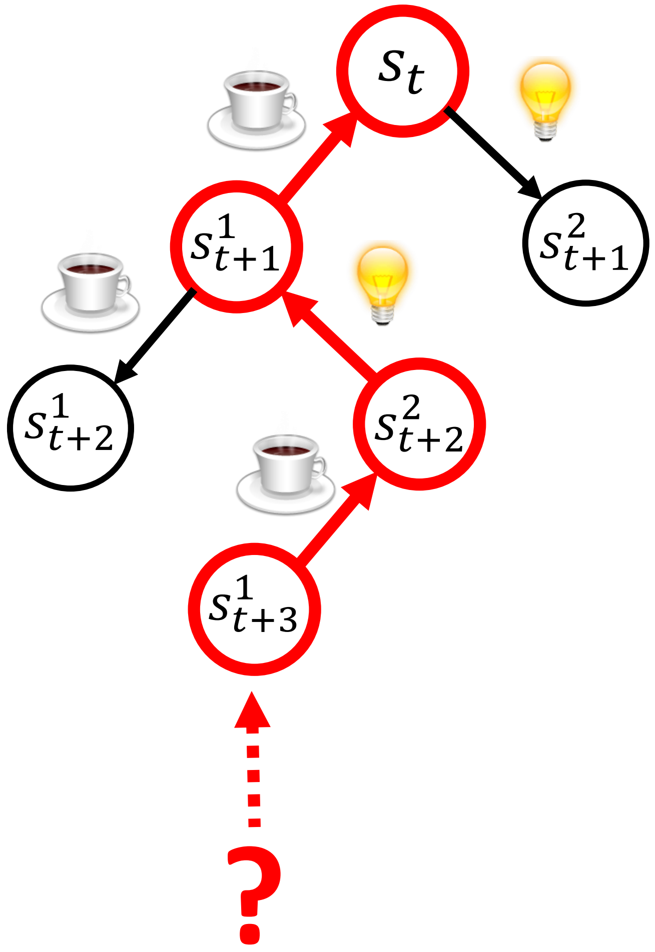


Example Evaluation Strategies:

- Rollouts (e.g., Random)
- Value Function $V^\pi(s_t)$ (e.g., Reinforcement Learning)

Monte Carlo Tree Search - Backup

- Selection
- Expansion
- Evaluation/Simulation
- **Backup**



Remember:

$$\mathcal{R}(s_t, a_t) + \gamma X$$

In this case $X = G_{t+1}$
(return from next state s_{t+1})

Summary

- What we know so far:
  - Markov Decision Processes (MDPs)
  - Policies and Value Functions
  - Optimally solve MDPs with Dynamic Programming
  - Approximately solve MDPs with Monte Carlo Search

- What we don‘t know (yet):
  - How to find solutions without a model?

Thank you!