Slides are available on my website.
`http://lukaseigentler.github.io`

Delayed loss of stability of periodic travelling waves affects wavelength changes of patterned ecosystems

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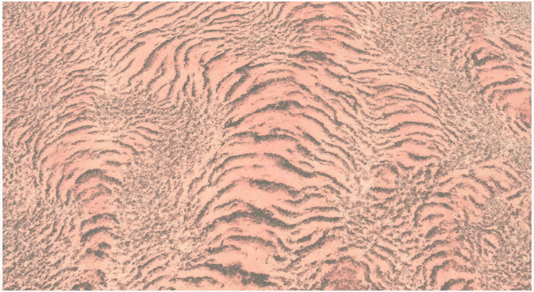
joint work with Mattia Sensi (Politecnico di Torino)

Stripe patterns

Banded vegetation patterns and intertidal mussel beds are classic examples of self-organisation principles in ecology.

Vegetation stripes in Ethiopia.



Intertidal mussel beds in the Wadden Sea.

- Parallel to topographic contours and shoreline.
- Caused by a scale-dependent feedback loop comprising long-range competition for a limiting resource and short-range facilitation.

Klausmeier model

One of the most basic phenomenological models for vegetation patterns is the extended Klausmeier reaction-advection-diffusion model.[1]

$$\frac{\partial u}{\partial t} = \overbrace{u^2 w}^{\text{plant growth}} - \overbrace{Bu}^{\text{plant loss}} + \overbrace{\frac{\partial^2 u}{\partial x^2}}^{\text{plant dispersal}},$$

$$\frac{\partial w}{\partial t} = \underbrace{A}_{\text{rainfall}} - \underbrace{w}_{\text{evaporation}} - \underbrace{u^2 w}_{\substack{\text{water uptake}\\ \text{by plants}}} + \underbrace{\nu \frac{\partial w}{\partial x}}_{\substack{\text{water flow}\\ \text{downhill}}} + \underbrace{d \frac{\partial^2 w}{\partial x^2}}_{\substack{\text{water}\\ \text{diffusion}}}.$$

[1] Klausmeier, C. A.: *Science* 284.5421 (1999).

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$$\begin{aligned}
\frac{\partial u}{\partial t} &= \overbrace{u^2 w}^{\text{plant growth}} - \overbrace{Bu}^{\text{plant loss}} + \overbrace{\frac{\partial^2 u}{\partial x^2}}^{\text{plant dispersal}}, \\
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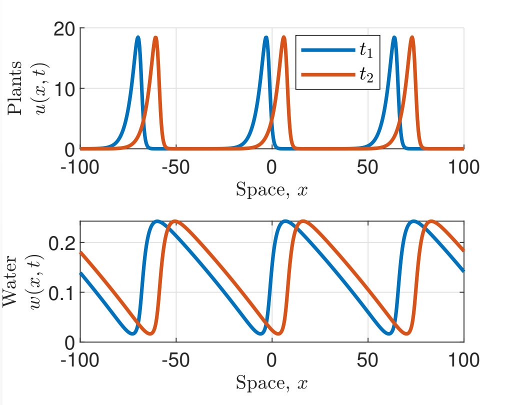
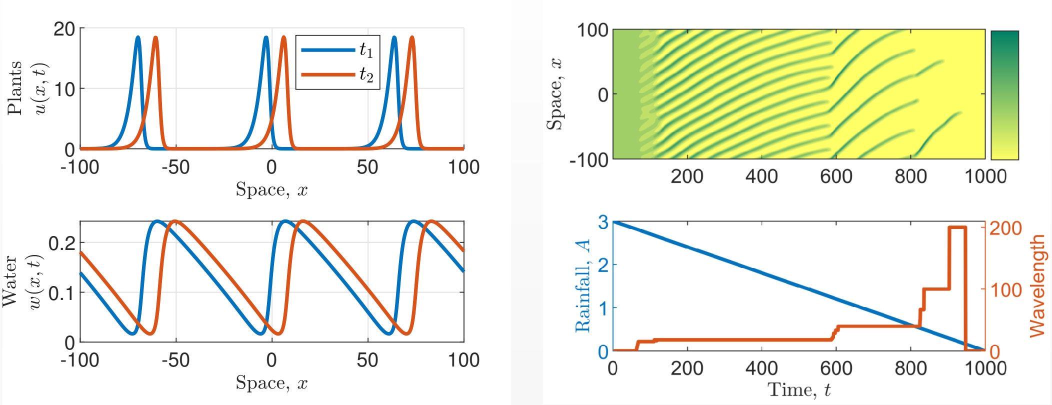
\end{aligned}$$

Periodic travelling waves



- Model represents vegetation patterns as periodic travelling waves (PTWs).

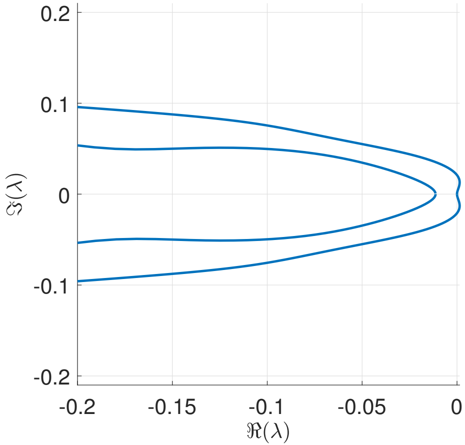
Periodic travelling waves



- Model represents vegetation patterns as periodic travelling waves (PTWs).
- Along rainfall gradient, transition from uniform vegetation to desert occurs via several pattern transitions.

Wavelength changes

- State-of-the-art: predict wavelength changes through PTW stability properties.
- PTW linear stability is determined by their essential spectra.



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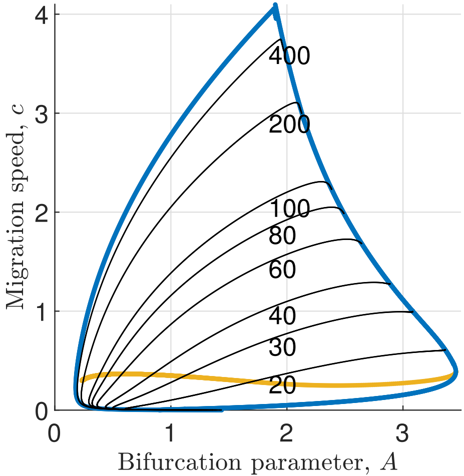
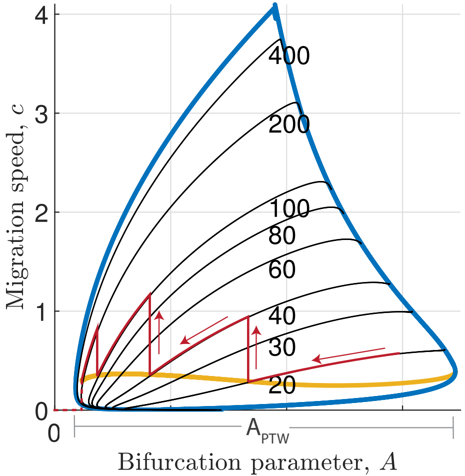
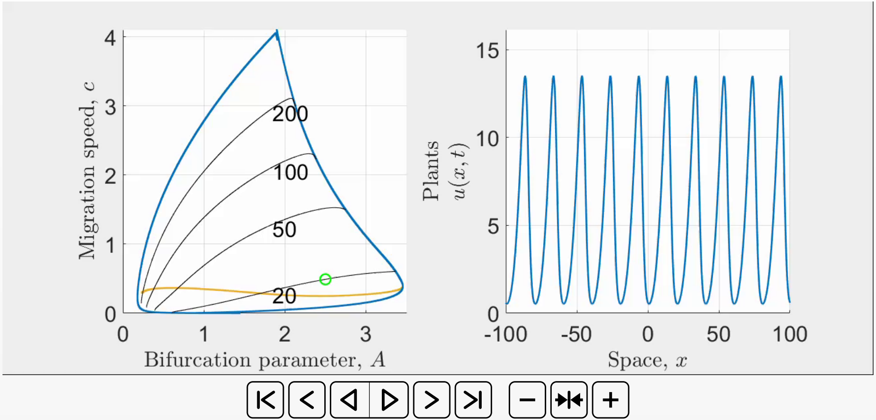


Wavelength changes

- State-of-the-art: predict wavelength changes through PTW stability properties.
- PTW linear stability is determined by their essential spectra.
- Wavelengths changes are typically predicted through the *Busse balloon*: parameter space of stable PTWs.
- Wavelengths are preserved, provided they remain stable.
- Upon destabilisation a wavelength change occurs.



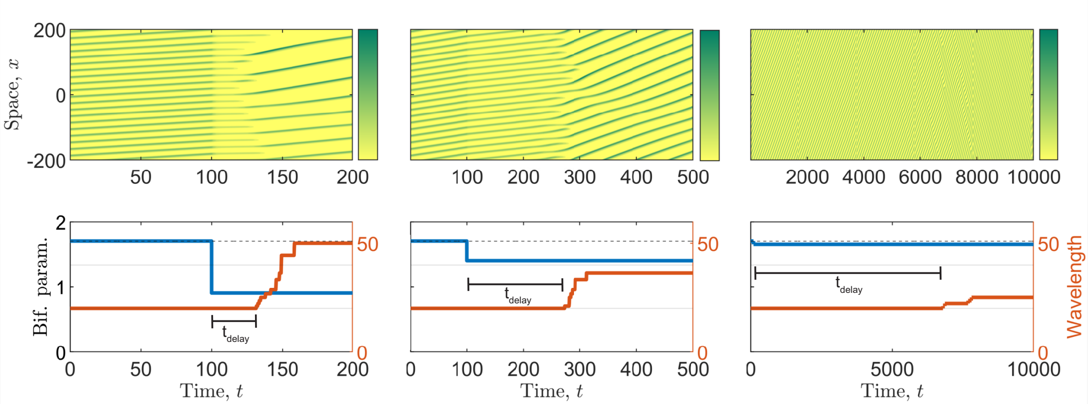
Wavelength changes



alternative video link.

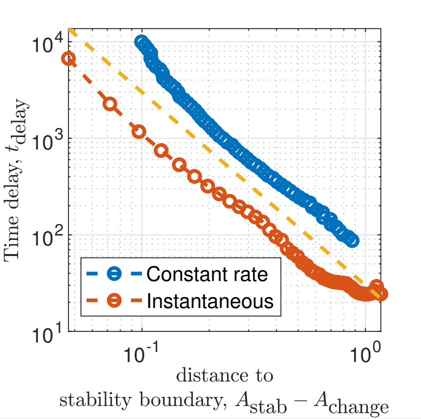
- Significant delays between crossing a stability boundary and observing wavelength changes occur.

Delays to wavelength changes



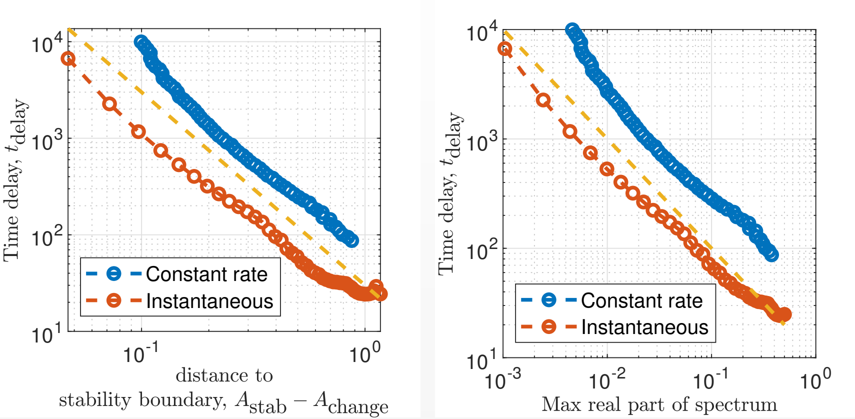
- Significant delays between crossing a stability boundary and observing wavelength changes occur.
- Order of magnitude differences in delay depending on parameter values.

Predicting delays



There are clear trends between delay and bifurcation parameter

Predicting delays



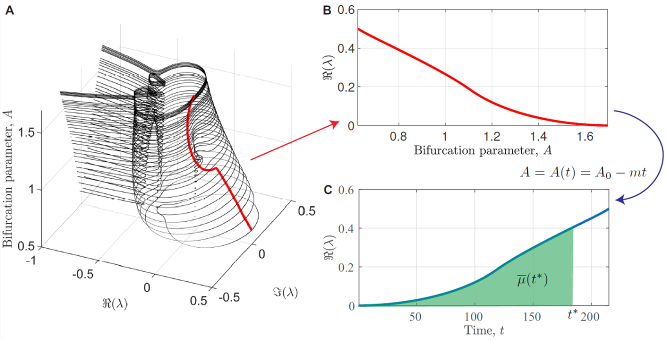
There are clear trends between delay and bifurcation parameter and delay and max real part of the spectrum. no predictive power

Predicting delays

Can predict the order of magnitude of the delay through the accumulated maximal instability[2]

$$\overline{\mu}(A(t)) = \int_{t_{\text{stab}}}^{t} \mu(\tau) d\tau, \quad t \geq t_{\text{stab}}.$$

t_{stab} is the time of the last crossing of the stability boundary.
$\mu(t)$ is the max real part of the spectrum at time t.



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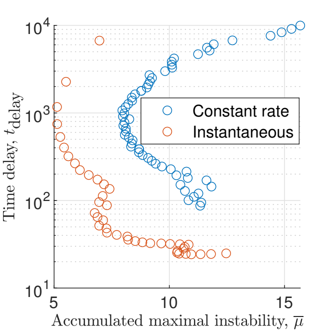
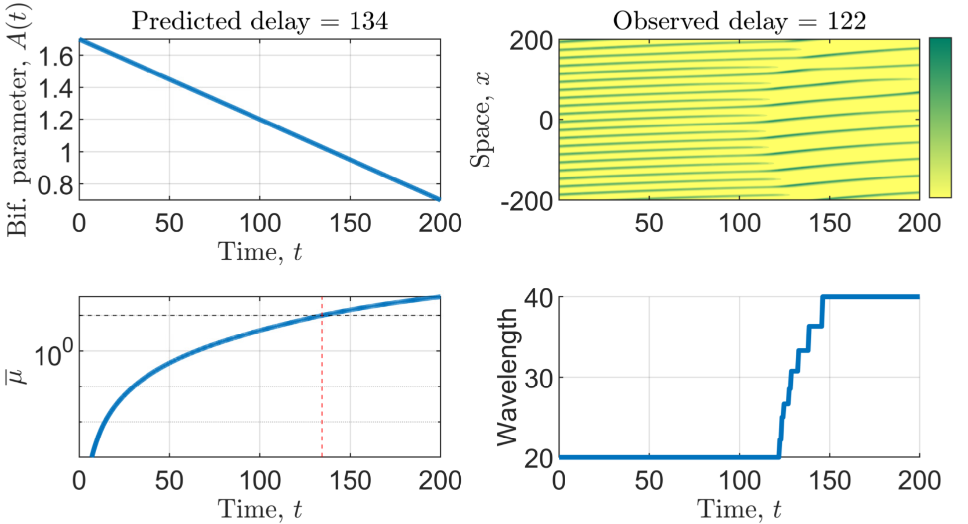


Wavelength change occurs when $\overline{\mu} \approx 10$

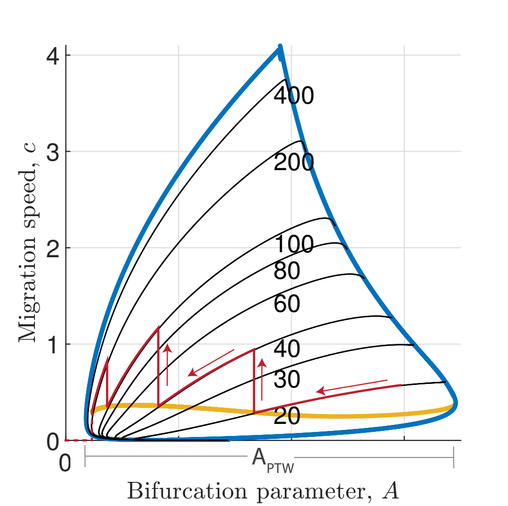
[2] EL and Sensi, M.: *arXiv preprint* (2023).

Delay prediction in practice



Conclusions

- Wavelength changes that occur after crossing a stability boundary are subject to a delay.



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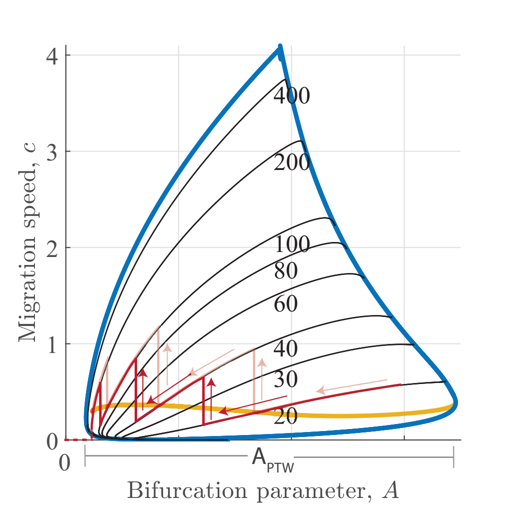
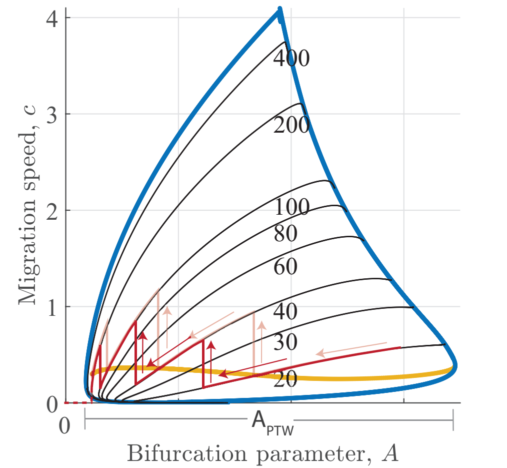


Conclusions

- Wavelength changes that occur after crossing a stability boundary are subject to a delay.
- Order of magnitude of the delay can be predicted by tracking the maximum real part of the spectrum of the destabilised pattern over time.



References

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[1] Eigentler, L. and Sensi, M.: 'Delayed loss of stability of periodic travelling waves: insights from the analysis of essential spectra'. *arXiv preprint* (2023).