

# 5-Factor Experiment

## Flavor Scores for Potatoes

A. Mackey and J. Stockman (1958). "Cooking Quality of Oregon-Grown Russet Potatoes," *American Potato Journal*, Vol.35, pp.395-407

# Data Description

- Response: Flavor Score (Mean of 5 judges/4 reps)
- Factors:
  - Grow Area (1=Southern OR, 2=Central OR)
  - 2-Week Holding Temp (1=75F, 2=40F)
  - Potato Size (1=Large, 2=Medium)
  - Storage Period (1=0months, 2=2m, 3=4m, 4=6m)
  - Cook Method  
(1=Boil, 2=Steam, 3=Mash, 4=Bake350, 5=Bake450)

$N=2 \times 2 \times 2 \times 4 \times 5 = 160$  sample measurements

# Factor Level (Marginal) Means (Overall=2.91125)

growarea	growarea
1	2
2.85375	2.96875

holdtemp	holdtemp
1	2
2.99375	2.82875

size	size
1	2
2.91	2.9125

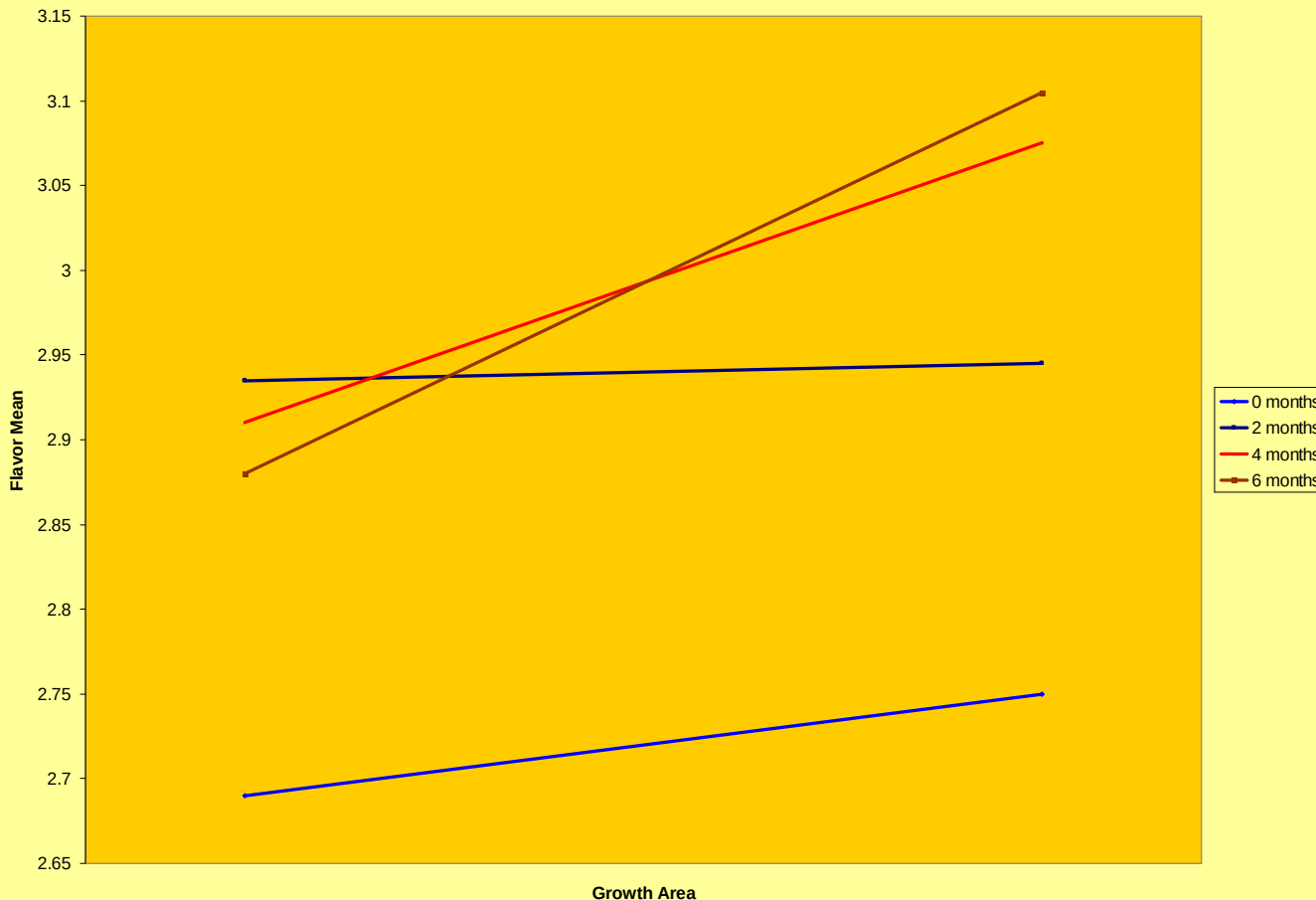
storperd	storperd	storperd	storperd
1	2	3	4
2.72	2.94	2.9925	2.9925

cookmthd	cookmthd	cookmthd	cookmthd	cookmthd
1	2	3	4	5
2.9375	2.80625	2.96875	3.0375	2.80625

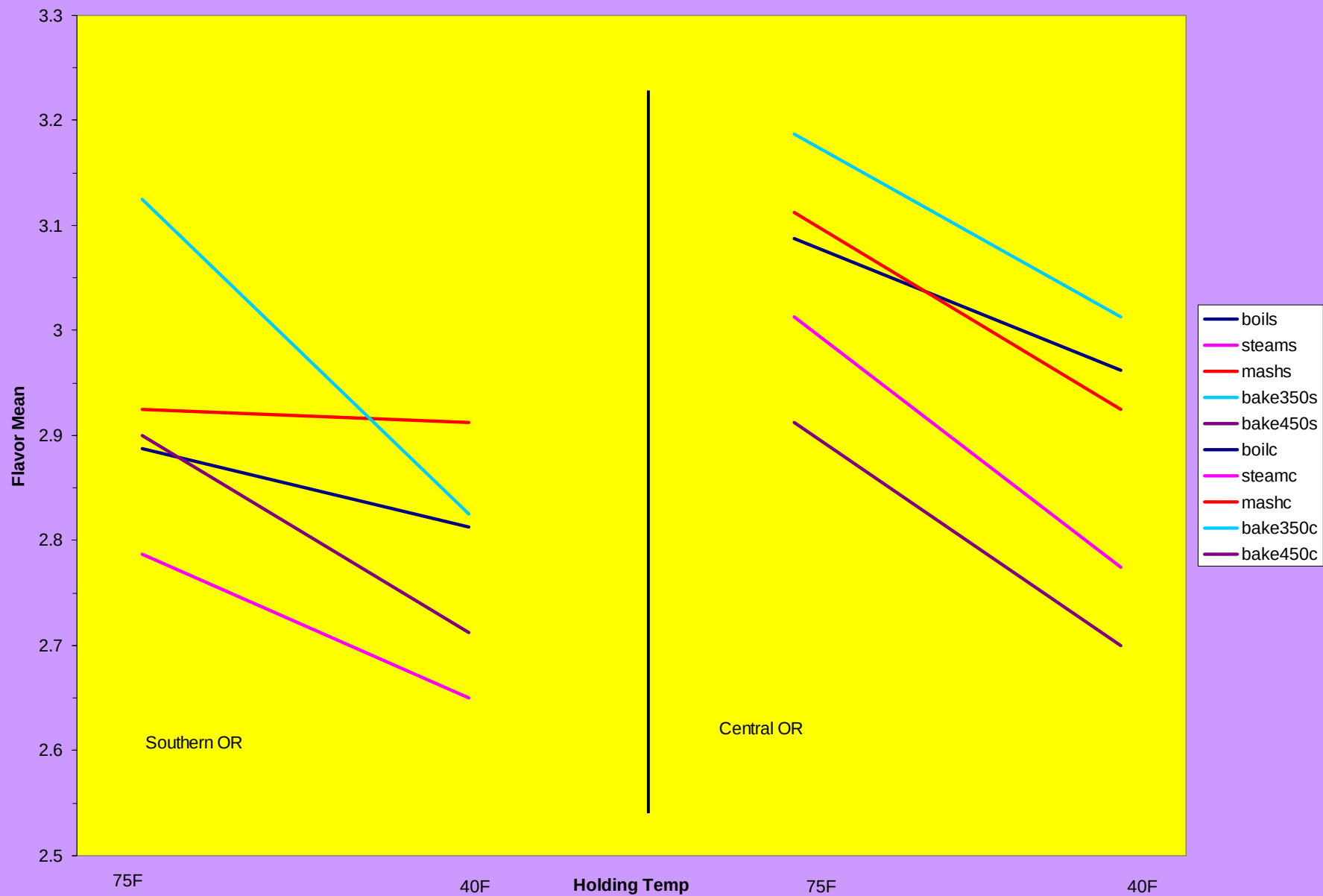
# 2-Factor Interaction: Grow area x Storage Period

Growarea	Storperd			
	0 months	2 months	4 months	6 months
1	2.69	2.935	2.91	2.88
2	2.75	2.945	3.075	3.105

Flavor vs Growarea by Storage Period



### 3-Way Interaction (GrowArea, HoldTemp, CookMthd)



## Statistical Model (Fixed Effects)

$$Y_{ijklm} = \mu + \alpha_i + \beta_j + \gamma_k + \delta_l + \tau_m + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\alpha\delta)_{il} + (\alpha\tau)_{im} + (\beta\gamma)_{jk} + (\beta\delta)_{jl} + (\beta\tau)_{jm} + (\gamma\delta)_{kl} + (\gamma\tau)_{km} + (\delta\tau)_{lm} + (\alpha\beta\gamma)_{ijk} + (\alpha\beta\delta)_{ijl} + (\alpha\beta\tau)_{ijm} + (\alpha\gamma\delta)_{ikl} + (\alpha\gamma\tau)_{ijm} + (\alpha\delta\tau)_{ilm} + (\beta\gamma\delta)_{jkl} + (\beta\gamma\tau)_{jkm} + (\beta\delta\tau)_{jlm} + (\gamma\delta\tau)_{klm} + \varepsilon_{ijklm}$$

$$i = 1, 2 \quad j = 1, 2 \quad k = 1, 2 \quad l = 1, 2, 3, 4 \quad m = 1, 2, 3, 4, 5$$

where:

$Y_{ijklm}$   $\equiv$  Flavorscore for Factor A@Level  $i$ , B@ $j$ , C@ $k$ , D@ $l$ , E@ $m$

$\mu$   $\equiv$  Overall Mean

$\alpha_i$   $\equiv$  Effect of Factor A (Growth Area) at level  $i$  (Sum to 0 over all levels)

$\beta_j$   $\equiv$  Effect of Factor B (Holding Temp) at level  $j$  (Sum to 0 over all levels)

$\gamma_k$   $\equiv$  Effect of Factor C (Potato Size) at level  $k$  (Sum to 0 over all levels)

$\delta_l$   $\equiv$  Effect of Factor D (Storage Period) at level  $l$  (Sum to 0 over all levels)

$\tau_m$   $\equiv$  Effect of Factor E (Cooking Method) at level  $m$

$(\alpha\beta)_{ij}, (\alpha\gamma)_{ik}, (\alpha\delta)_{il}, (\alpha\tau)_{im}, (\beta\gamma)_{jk}, (\beta\delta)_{jl}, (\beta\tau)_{jm}, (\gamma\delta)_{kl}, (\gamma\tau)_{km}, (\delta\tau)_{lm} \equiv$

Effects of 2 - Factor interactions among levels of the 5 main factors (sum to 0 over all subscripts)

$(\alpha\beta\gamma)_{ijk}, (\alpha\beta\delta)_{ijl}, (\alpha\beta\tau)_{ijm}, (\alpha\gamma\delta)_{ikl}, (\alpha\gamma\tau)_{ijm}, (\alpha\delta\tau)_{ilm}, (\beta\gamma\delta)_{jkl}, (\beta\gamma\tau)_{jkm}, (\beta\delta\tau)_{jlm}, (\gamma\delta\tau)_{klm} \equiv$

Effects of 3 - Factor interactions among levels of the 5 main factors (sum to 0 over all subscripts)

$\varepsilon_{ijklm}$   $\equiv$  Random error term, Assumed independent,  $N(0, \sigma^2)$

# Formulas for Sums of Squares

$$i = 1, 2 \quad j = 1, 2 \quad k = 1, 2 \quad l = 1, 2, 3, 4 \quad m = 1, 2, 3, 4, 5 \Rightarrow a = 2, b = 2, c = 2, d = 4, e = 5$$

Main Effects (e.g. Factor A):

$$SSA = 2(2)(4)(5) \sum_{i=1}^2 (\bar{Y}_{i....} - \bar{Y}_{.....})^2 \quad df_A = a - 1 = 2 - 1$$

2-Factor Interactions (e.g. Interaction AB)

$$SSAB = 2(4)(5) \sum_{i=1}^2 \sum_{j=1}^2 (\bar{Y}_{ij...} - \bar{Y}_{i....} - \bar{Y}_{.j...} + \bar{Y}_{.....})^2 \quad df_{AB} = (a - 1)(b - 1) = 1(1) = 1$$

3-Factor Interactions (e.g. Interaction ABE):

$$SSABE = 2(4) \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^5 (\bar{Y}_{ij..k} - \bar{Y}_{ij...} - \bar{Y}_{i...k} - \bar{Y}_{.j..k} + \bar{Y}_{i....} + \bar{Y}_{.j...} + \bar{Y}_{....k} - \bar{Y}_{.....})^2$$

$$df_{ABE} = (a - 1)(b - 1)(e - 1) = 4$$

Error SS and df obtained by subtraction of

Total-(all Main Effects)-(all 2 Factor ints)-(All 3 Factor Ints)

# Analysis of Variance (Main Effects)

Source	df	SS	MS	F	Pr(>F)
ga	1	0.529	0.529	20.5727	3.15E-05
ht	1	1.089	1.089	42.3511	2.41E-08
sz	1	0.00025	0.00025	0.0097	0.92181
sp	3	2.02425	0.67475	26.2409	1.14E-10
cm	4	1.3435	0.33588	13.0621	1.52E-07
Residual	55	1.41425	0.02571		



# Analysis of Variance (2-Factor Interactions)

Source	df	SS	MS	F	Pr(>F)
ga:ht	1	0.02025	0.02025	0.7875	0.37872
ga:sz	1	0.049	0.049	1.9056	0.17304
ga:sp	3	0.2865	0.0955	3.714	0.01666
ga:cm	4	0.166	0.0415	1.6139	0.18379
ht:sz	1	0.016	0.016	0.6222	0.4336
ht:sp	3	0.9365	0.31217	12.1401	3.32E-06
ht:cm	4	0.1235	0.03087	1.2007	0.32094
sz:sp	3	0.94325	0.31442	12.2276	3.07E-06
sz:cm	4	0.076	0.019	0.7389	0.56951
sp:cm	12	1.447	0.12058	4.6895	3.32E-05
Residual	55	1.41425	0.02571		

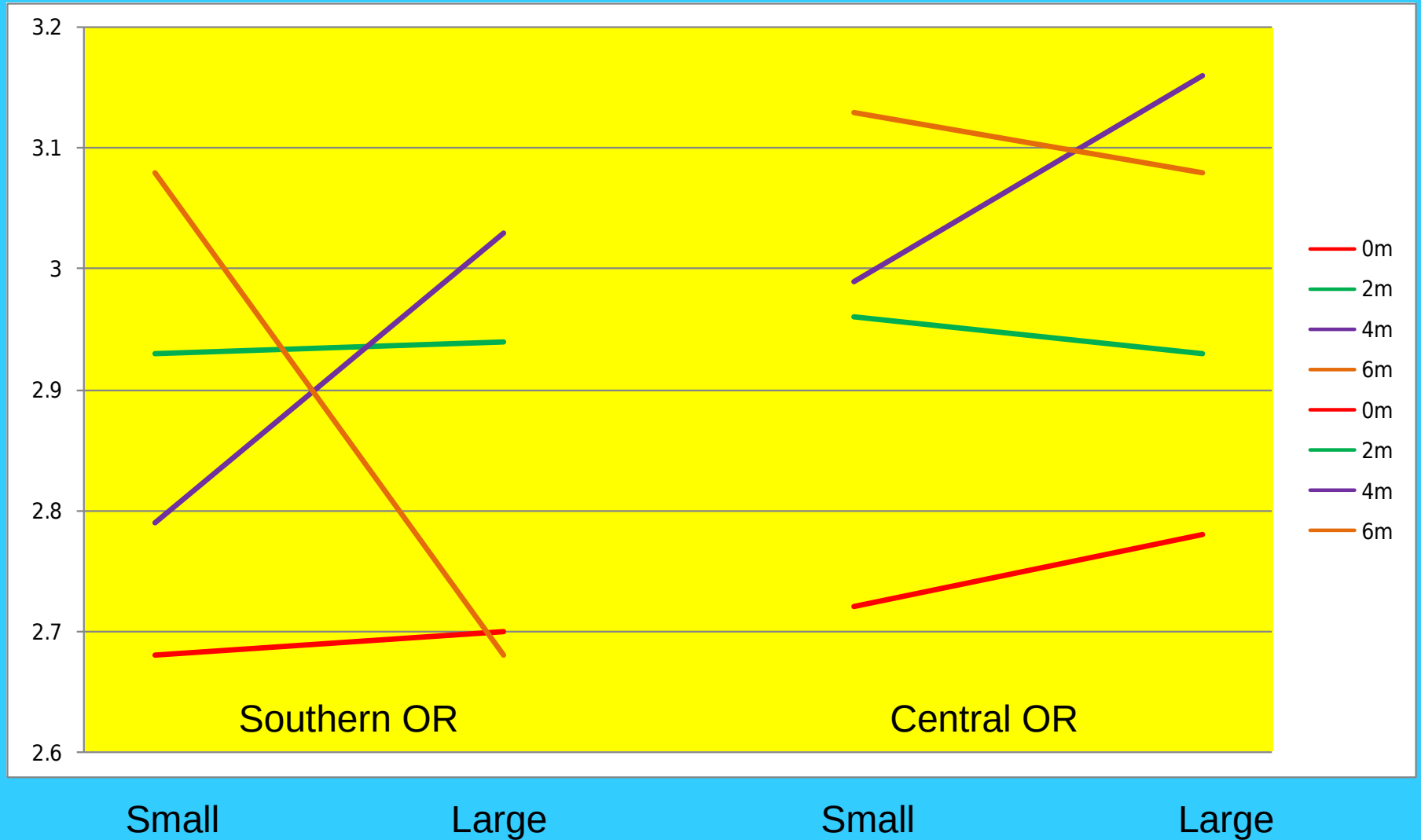
# Analysis Of Variance (3-Factor Interactions)

Source	df	SS	MS	F	Pr(>F)
ga:ht:sz	1	0.00025	0.00025	0.0097	0.92181
ga:ht:sp	3	0.02625	0.00875	0.3403	0.79626
ga:ht:cm	4	0.0985	0.02463	0.9577	0.4381
ga:sz:sp	3	0.2775	0.0925	3.5973	0.01907
ga:sz:cm	4	0.291	0.07275	2.8292	0.03316
ga:sp:cm	12	0.5685	0.04738	1.8424	0.06353
ht:sz:sp	3	0.0105	0.0035	0.1361	0.93806
ht:sz:cm	4	0.0965	0.02412	0.9382	0.44879
ht:sp:cm	12	0.526	0.04383	1.7047	0.09095
sz:sp:cm	12	0.3605	0.03004	1.1683	0.32833
Residual	55	1.41425	0.02571		

# Interpretations

- Two 3-Factor Interactions are Significant
  - Grow Area/Size/Storage Period
  - Grow Area/Storage Period/Cooking Method
- Four 2-Factor Interactions are Significant
  - Note though that with 3-Factor Interactions being significant, many of 2-Factor Interactions differ among levels of 3<sup>rd</sup> Factor
- Four Main Effects are Significant
  - Similar Caveats as for 2-Factor Interactions

# Grow Area/Size/Storage Period Interaction



# Grow Area/Storage Period/Cooking Method Interaction

