**Temperature calculations**

To determine the 1-D effective viscosity and yield stress envelope, a temperature profile is required. The 1-D temperature profile is calculated by solving the 1-D heat equation (so far only with a radiogenic heat source) for variable thermal parameters with a proper conserving finite difference scheme. That is the heat flow is calculated on the centered and the remaining parameters on the regular grid points, respectively. The discretization scheme for variable thermal parameters is picked to solve for a temperature profile of a continental lithosphere with upper, lower crust, and mantle.

The 1-D heat equation is given by:

,

where *ρ*, *cp*, *T*, *t*, *k*, *H* are the density [kg/m3] the specific heat [J/kg/K], the temperature [K], the time [s], the thermal conductivity [W/m/K], and volumetric heat generation rate [W/m3], respectively.

Here, a proper conservative finite difference scheme means the heat flux is calculate on the centered grid points (A, B, etc.). The 1-D vertical heat flux is given by the Fourier’s law:

.

*Solving the equation*

Following the discretization shown in Figure 1 we need to solve the following equation (in an implicit finite difference formulation):

,

.

Sorting the variables (known variables on the right-hand side, unknown on the left-hand side):









with

 ,

and

.

*Thermal boundary conditions*

The thermal boundary conditions are defined as:

1. Constant temperature (*Dirichlet*)

The temperature at the top or bottom can just be set as constant to *Ttop* or *Tbot*, respectively.

1. Constant temperature gradient (*Neumann*)

The gradient of temperature (and thus the vertical heat flux) can be defined using so called ghost nodes at the top and the bottom of the profile. Therefore we define the condition at the top and bottom as:

,

Where *T0* and *Tnz+1* are the ghost nodes for temperature at the top and bottom, respectively. The constants *ctop*and *cbottom*are defined as

.

Using these conditions, we can define formulations for the temperature at the ghost nodes as:

.

Now one can solve equation for the top and the bottom using the formulations of the temperature at the ghost nodes with equation , which results in:

,

with

.

Similar for the bottom boundary with:

.



**Figure 1. Oceanic Lithosphere.** LEFT: Temperature profile [K] for an oceanic lithosphere of 60 Ma of age and constant thermal boundary conditions at the top and bottom. The blue line shows the initial temperature profile. The yellow dashed line shows the solution for a half-space cooling model. RIGHT: Heat flux [mW/m2] with depth. The parameters of this model are defined as the default values in the routine OceanicGeotherm.m.



**Figure 2. Oceanic Lithosphere II.** Same as Figure 1 but with constant heat flux boundary conditions qbottom =10 mW/m2 and qtop = 90 mW/m2.



**Figure 3. Continental Lithosphere.** LEFT: Temperature profile for a continental lithosphere of 1000 Ma of age with constant upper and lower thermal boundary conditions. The blue line shows the initial condition, the red line shows the solution of equation (1), the yellow dashed line shows the solution of the time-independent heat equation (1-D poisson equation), and the magenta dashed line shows the solution of a 2D, staggered finite difference code. MIDDLE: Heat flux with depth. RIGHT: Thermal parameter for the lithosphere setup: thermal conductivity [k], specific heat [cp], density [ρ], and volumetric heat generation rate [Q].



**Figure 4. Continental Lithosphere II.** Same as Figure 3 but with constant upper and lower heat flux boundary conditions, qtop = 40 mW/m2 and qbottom = 10 mW/m2.