**Effective Viscosity Calculations**

To determine the strength of the lithosphere, one needs to know its stress state and thus its effective viscosity (assuming a constant strain-rate!).

For the lithospheric mantle a composite rheology is assumed, governed by dislocation and diffusion creep. The strain rate for each deformation mechanism can be determined in deformation experiments and is generally defined as:

,

where *ε*, *Atr*, *COH*, *d*, *r*, *m*, *E*, *P*, *V*, *R*, *T*, *σ*, and *n* are the strain rate, the experimental exponential pre-factor, the water content, the grain-size, the water content exponent, the grain-size exponent, the activation energy, the pressure, the activation volume, the universal gas constant, the temperature, the differential stress (*σ1* – *σ2*), respectively. The index *i* stands for the different creep mechanism. This creep law describes the principal strain in one direction (usually vertical) for a triaxial compression experiment with a certain confining pressure (*σ*2= *σ*3).

For numerical purposes, one wants to express the strain rate and stress via the second invariants of the corresponding tensors (see Appendix A). Thus, the strain rate stress relationship looks like:

,

where *εII* and *τII*, are the second invariant of the strain rate and the stress tensor, respectively.

The viscosity is described by the constitutive equation:



We want to express the viscosity using a constant strain rate. Thus, we need to solve for the stress:



where .

Set equation into :



Assuming both deformation processes occur parallel, the total strain rate is given by:

.

Thus, the effective viscosity can be estimated by:

.

Grain Size …

**Appendix**

**A Second Invariant Expression**

We need to rewrite equation to use the second invariant of the strain rate and stress tensor. The invariant of a tensor *t* is given by (e.g., Gerya, 2010):

.

Within the creep law, total stress is used. Thus, we also need to express *σ* by the deviatoric stress:

,

where .

The second invariant of the deviatoric stress tensor can then be described by:

,

.

Equally, the second invariant of the strain rate is described by:

.

We can express the strain rate in the second direction with . Thus:



With equations and , the creep law is given by:

.

**B Steady State Grain Size**

Lkjasd

**C Numerical Solution**

Lkjsdf