Stoikov Model High-Frequency Trading on Cryptocurrencies

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1 Introduction

In 2008, Marco Avellaneda and Sasha Stoikov introduced a high-frequency market making strategy which would consider their account inventory position. They derive a "reservation price" which is described as their indifference price given their inventory. Let s be the mid-price, q the number of units in the inventory, σ the volatility of s, T the final time, and γ the risk aversion parameter of the investor. Then the **Reservation Price** (or theoretical price) r_t at time t is defined as

$$r_t = s_t - q_t\gamma\sigma^2(T-t). \tag{1}$$

Let γ, σ, T, be defined as above and let k be an arbitrary parameter. The optimal **bid-ask spread** $\delta_t^a + \delta_t^b$ at time t, for this "inventory" strategy, is defined as

$$\delta_t^a + \delta_t^b = \gamma\sigma^2(T-t) + \frac{2}{\gamma}ln(1+\frac{\gamma}{k}). \tag{2}$$

2 Trading Strategy

We describe our market making trading strategy and some of its features before we implement it in Section 3. For our trading strategy, we extend the ideas of Avellaneda and Stoikov, adjusting it to fit different time series and adding extra features.

2.1 Theoretical Price and Spread

First, we need to define a theoretical price (or indifference price) to make our bid-ask quotes around. We draw inspiration from the reservation price defined by Avellaneda and Stoikov. However, the reservation price is a function of the inventory q_t which is quoted in absolute terms. It does not adjust for different values of scales of s_t.

For example, suppose we fix the inventory $q = 1$, $\gamma = 1$, $\sigma = 1$, $T = 1$, $t = 0$, and let $s_t = 100$. Then, the reservation price is

$$r_t = s_t - q\gamma\sigma^2(T-t) = 100 - 1 = 99, \tag{3}$$

and so an inventory position $q = 1$ results in a 1% decrease in the reservation price compared to the mid-price.

Now suppose the parameters are the same as above, $q = 1$, $\gamma = 1$, $\sigma = 1$, $T = 1$, $t = 0$, but let $s_t = 10$. Now the reservation price is

$$r_t = s_t - q\gamma\sigma^2(T-t) = 10 - 1 = 9, \tag{4}$$

which is a 10% decrease from the mid-price. Thus, the same inventory position yields a significantly different result on the reservation price.

2.1.1 Adjusted Theoretical Price

In order to solve the above issue with the reservation price, we treat the inventory position q as a percentage by multiplying it by $\frac{s_t}{100}$. We defined the **adjusted theoretical price** as

$$\tilde{r}(s,t) = s_t - \left(q\gamma\sigma^2(T-t)\right)\frac{s_t}{100}. \tag{5}$$

Skew

The adjusted theoretical price incorporates the idea of skew. The idea of a market maker is to remain market neutral and minimise the necessity of taking directional positions. Skewing your position is the method of adjusting the theoretical price to reduce inventory position. For example, when the market maker has accumulated a long position, the theoretical price is skewed down to reduce the ask price thus increasing the likelihood of selling some positions. Similarly, when the market maker has accumulated a short position, the theoretical price is skewed up to increase the bid price and thus increase the likelihood of buying back some positions. The second term in the adjusted theoretical price in Eq. 5 is

$$-\left(q\gamma\sigma^2(T-t)\right)\frac{s_t}{100}, \tag{6}$$

which is a function of the inventory q and thus incorporates the concept of skew into our adjusted theoretical pride $\tilde{r}_t$.

2.1.2 Adjusted Spread

Similarly, we define the **adjusted spread** as

$$\tilde{\delta}_t^a + \tilde{\delta}_t^b = \left(\gamma\sigma^2(T-t) + \frac{2}{\gamma}(1+\frac{\gamma}{k})\right)\frac{s_t}{100}, \tag{7}$$

where γ, σ, T, t, s_t and k are defined as before. We should note that the spread decreases as $t \to T$ to reduce inventory position towards the end of trading.

Volatility

Another concept in market making is to increase the spread when volatility increases to reduce risk. Our adjusted bid-ask spread integrates this concept from the first term

$$\gamma\sigma^2(T-t), \tag{8}$$

thus increasing the spread as volatility increases.

2.1.3 Bid-Ask Quotes

Our **bid** b_t^1 and **ask** a_t^1 is defined as

$$b_t^1 = \tilde{r}_t - \tilde{\delta}_t^b, \tag{9}$$

$$a_t^1 = \tilde{r}_t + \tilde{\delta}_t^a, \tag{10}$$

where $\tilde{r}_t$ is the adjusted theoretical price and $\tilde{\delta}_t^b$, $\tilde{\delta}_t^a$ the adjusted spread as defined above. We shall always quote a size of 1 unit simplicity.

2.2 Added Features

Level 2

The majority of our trades will be from our level 1 bid b_t^1 and ask a_t^1prices. However, in the case of a large market order taking significant depth in the orderbook, we add in a level 2 bid and ask prices to take advantage of the large price impact. We define the **level 2 bid** b_t^2 and **level 2 ask** a_t^2 price as

$$b_t^2 = \tilde{r}_t - 2\tilde{\delta}_t^b, \tag{11}$$

$$a_t^2 = \tilde{r}_t + 2\tilde{\delta}_t^a. \tag{12}$$

Inventory Limit

As mentioned before, market makers want to reduce their directional risk. In our trading strategy, we set a limit on our inventory position to a maximum of 20 positions long or 20 positions short. In reality, it would be ideal to hedge using another product, e.g., a future, to remain delta neutral throughout trading.

3 Model and Implementation

We implement and backtest our trading strategy using Python code.

Assumptions

To simplify our model, we make some assumptions and simplifications:

- Whenever we trade our order always gets filled (and never partially filled).
- We assume that there's no latency in quoting and cancelling orders.
- We assume a maker fee of 1bp.

Modelling Trades Intensity

It is rational to assume that the probability of trading reduces exponentially as the bid-ask prices become further away from the mid-price s_t. To model market trading intensity, we introduce the Poisson intensity parameter λ which Avellaneda and Stoikov derive as

$$\lambda(\delta) = e^{-k\delta} \tag{13}$$

where k is an arbitrary constant and the bid price distance δ_t^b and ask price distance δ_t^a is

$$\delta_t^b = \frac{s_t - b_t}{s_t}, \tag{14}$$

$$\delta_t^a = \frac{a_t - s_t}{s_t}. \tag{15}$$

We now consider the affect of time in the orderbook with the probability of trading. The longer our orders remain in the orderbook, without the mid-price changing, the more likely it is to be traded. Let dt be the time-increment of each tick and A to be the constant frequency of market order. Then we model the **probability** that market orders will hit or lift our limit orders as

$$P_t^a = \lambda(\delta_t^a)dt = Ae^{-k\delta_t^a}dt, \tag{16}$$

$$P_t^b = \lambda(\delta_t^b)dt = Ae^{-k\delta_t^b}dt. \tag{17}$$

Brownian Motion

We backtest our trading strategy using simulated Brownian motion data. To simulate 1 hours worth of data points in tick sizes of 1 second, we set $T = 1$, $dt = \frac{1}{3600}$ and σ as the hourly volatility. The stochastic differential equation for a Brownian motion S_t with zero drift is defined as

$$dS_t = \sigma dW_t, \tag{18}$$

where $W_t \sim \mathcal{N}(\mu, \sigma^2)$ is a standard Brownian motion. Therefore, we can simulate a Brownian motion using the iterative formula

$$S_t = S_{t-1} + \sigma W_t \sqrt{dt}, \tag{19}$$

where $S_0 = 100$.

Kraken Market Data

We also backtest our trading strategy using real-world market trade data from Kraken. We backtest on the first 3600 data points (approx. 1 hour). We let the data be the mid-prices and we calculate dt as the time between each trade.

Volatility Calculation

We calculate volatility for our theoretical price $\tilde{r}$ and bid-ask spread $\tilde{\delta}$. For each time iteration, we calculate the volatility σ^{seconds} by calculating the standard deviation of the log returns of the past 50 ticks (which we assume each tick to be 1 second). We then scale it by $\sqrt{3600}$ to obtain the hourly volatility σ^{hour}. The hourly volatility σ_t^{hour} at time iteration t is defined as

$$\sigma_t^{\text{hour}} = \sqrt{3600} * \sigma_t^{\text{seconds}}. \tag{20}$$

4 Backtests

To test the effectiveness of our trading strategy, we backtest it on simulated Brownian motion data points and real-world market trade data provided by Kraken. We backtest our trading strategy on 1 hours worth of data in increments of 1 second (3600 ticks).

4.1 Brownian Motion

We use the parameters $A = 140$ and $k = 1.5$ as set by Avellaneda and Stoikov and risk-aversion parameter $\gamma = 0.01$. The following figure shows the backtest results for a Brownian motion where $\sigma = 2$.

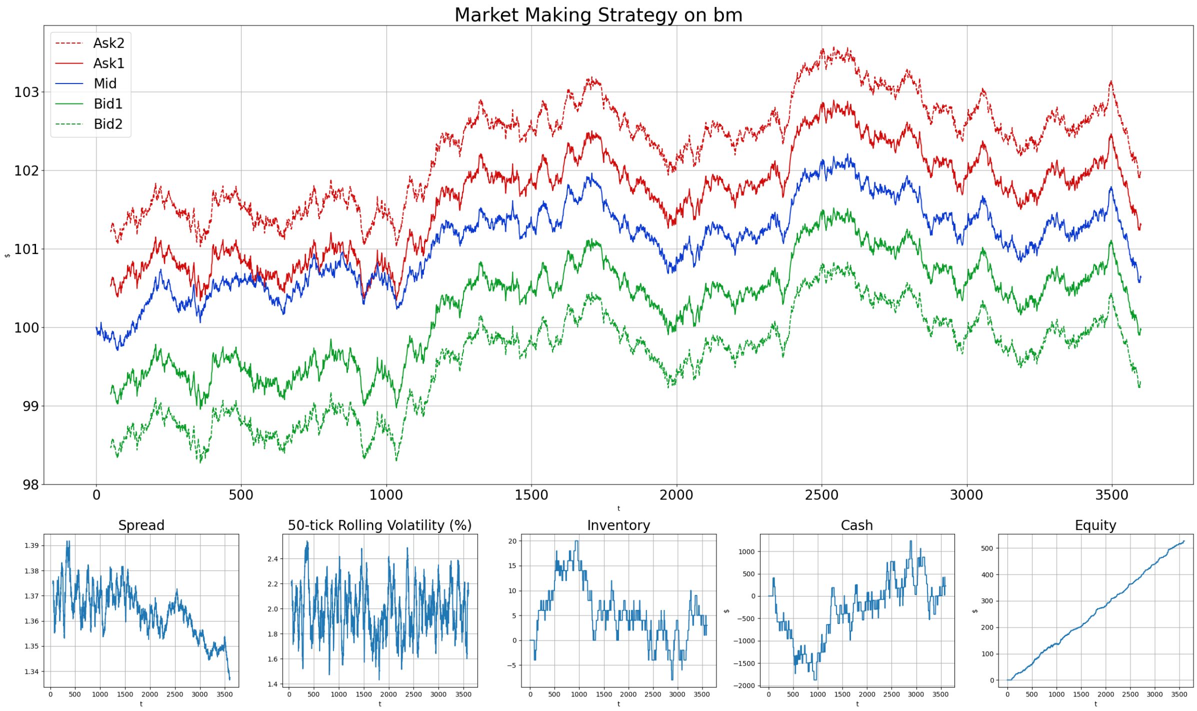

Figure 1: Backtest on Brownian Motion where $\sigma = 2$

From the equity graph, we can see that our trading strategy yields good results in the simulated environment. Furthermore, we do not take on too much directional risk as our inventory position does not exceed our limit of 20 and the inventory position mean-reverts towards 0.

Skew

Here we explore the effectiveness of skew incorporated into our theoretical price. We can see that at around $500 < t < 1000$ our trading strategy skews the theoretical price downwards when we accumulate around 15-20 long positions. The level 1 ask price is almost touching the mid-price, increasing the probability of being lifted and thus reducing our long position. The inventory positions reduces back down to 0 at around $t = 1300$ and the simulation ends with an inventory of only around 5 units.

4.1.1 Volatility and Final PnL

Here we explore the effect of volatility on our trading results. We backtest our trading strategy on 100 different Brownian motions for $\sigma = 1, 2, 5$ and explore the results of final PnL (Equity).

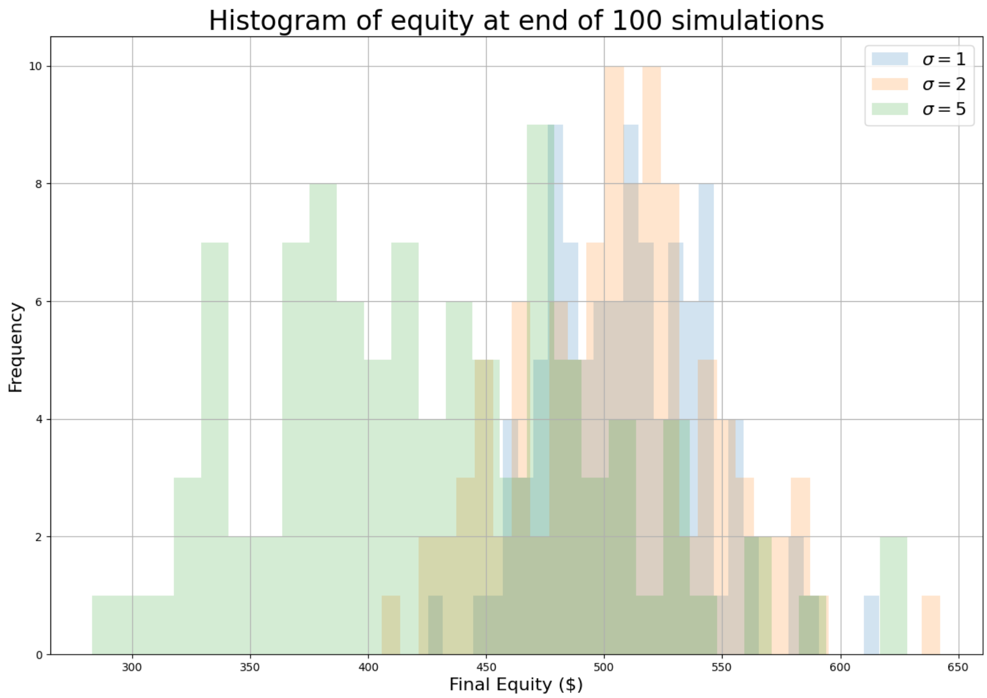

Figure 2: Histogram of Portfolio Equity at the End of Backtest for Different Values of σ

From the Figure 2, it is obvious that $\sigma = 5$ produces a final PnL which has a larger variance than for $\sigma = 1, 2$. Furthermore, the mean is significantly smaller than for $\sigma = 1, 2$. From these results, it seems that our trading strategy is more effective in less volatile environments and thus supports the idea of increasing the spread in high volatility to reduce the risk.

4.2 Kraken Market Trade Data

We use the parameters A = 140 and k = 5 and risk-aversion parameter $\gamma = 0.01$. In this section, we analyse the backtest results for Bitcoin (XBT/USD), Solana (SOL/USD) and Polygon (MATIC/USD). We also backtested our trading strategy on more additional coins and the results are displayed in the Appendix.

4.2.1 Bitcoin

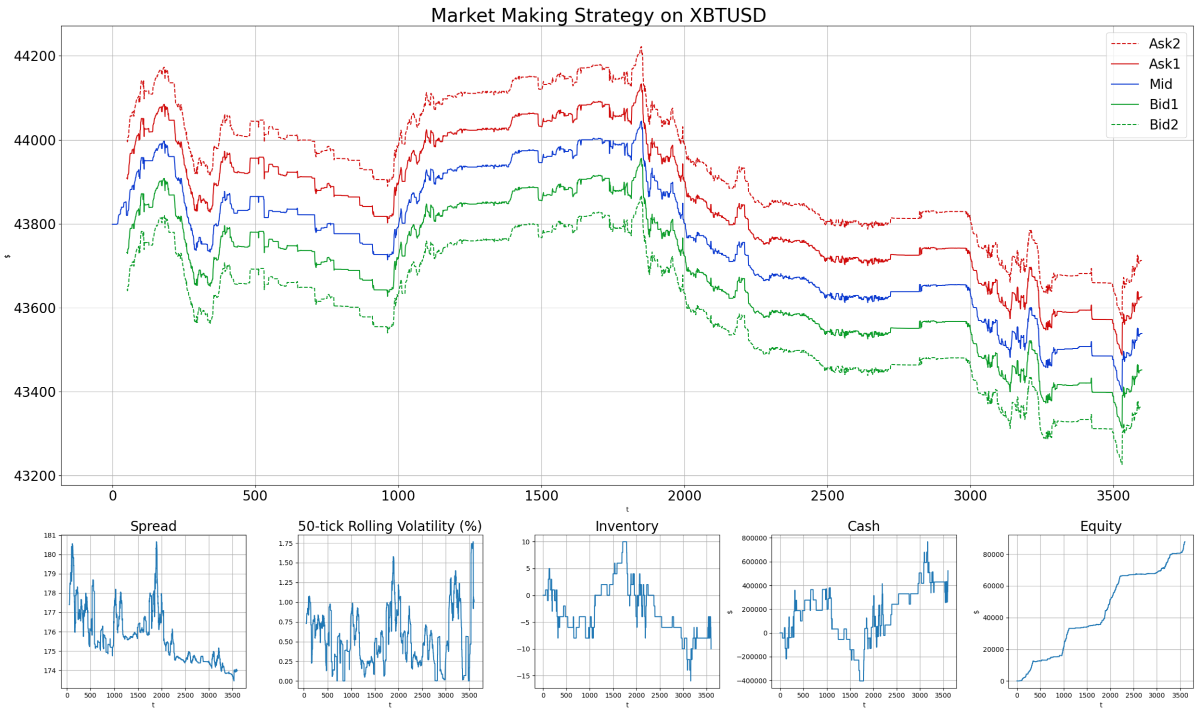

Figure 3: Backtest on XBT/USD

From Figure 2, we can see that backtesting our trading strategy on XBT/USD yields positive results similar to that of the simulated Brownian motion. We are quoting a bid-ask spread of around \$176 (40bps) which is a realistic amount.

Spread and Volatility

Here we explore the effectiveness of our spread in times of high volatility. Our trading logic should increase the spread in periods of high volatility to reduce our risk. We can see that at around $t = 1700$ that is a spike of high volatility and hence our trading strategy increases the spreaad to around \$180.

However, we should note that the trading strategy decreases the spread towards the end of the backtest to reduce inventory positions. This is why the spread deceases towards the end although the volatility spikes.

Trading Latency

Unlike in the Brownian motion backtest, the equity plot in Figure 2 shows periods where the equity is flat and does not increase. It is important to note that when we backtest market data (including XBT/USD), we calculate dt using the time difference between trades and it is not fixed at $dt = \frac{1}{3600}$ as for the Brownian motion. In the market data provide by Kraken, several of the trades occurred within the same second and hence we calculate $dt = 0$. From Eq. 16 and 17, we then model the probability of trading as zero hence no increase in PnL. This models the situation in the real markets where other market makers are quicker to submit their orders and thus taking the trading opportunity away from us.

4.2.2 Solana

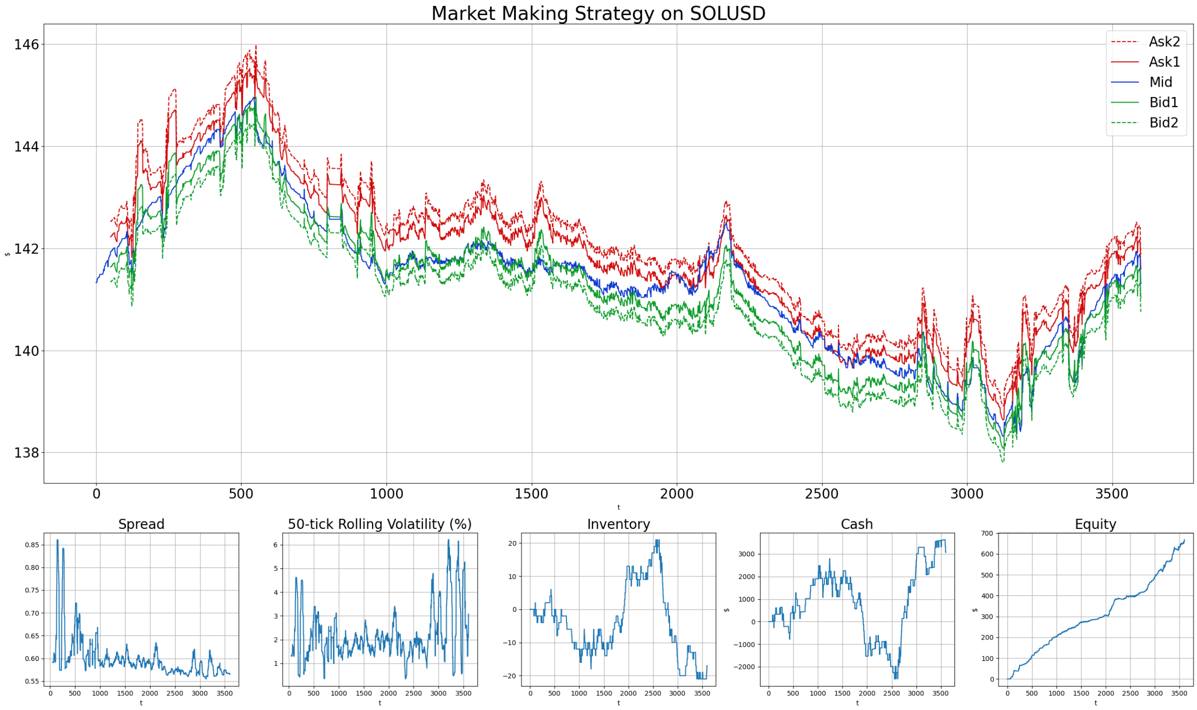

Figure 4: Backtest on SOL/USD

The backtest of SOL/USD also produces positive results. We should note that there is one period which the equity seems to flatten out which we shall discuss in Section 4.2.4. The trading strategy quotes a bid-ask spread of around \$0.6 (42bps). Also note that the hourly volatility of SOL/USD is higher than XBT/USD. Volatility of SOL/USD averages around 2% but spikes up to around 6% but XBT/USD averages around 0.5% and spikes up to 2%.

4.2.3 Polygon

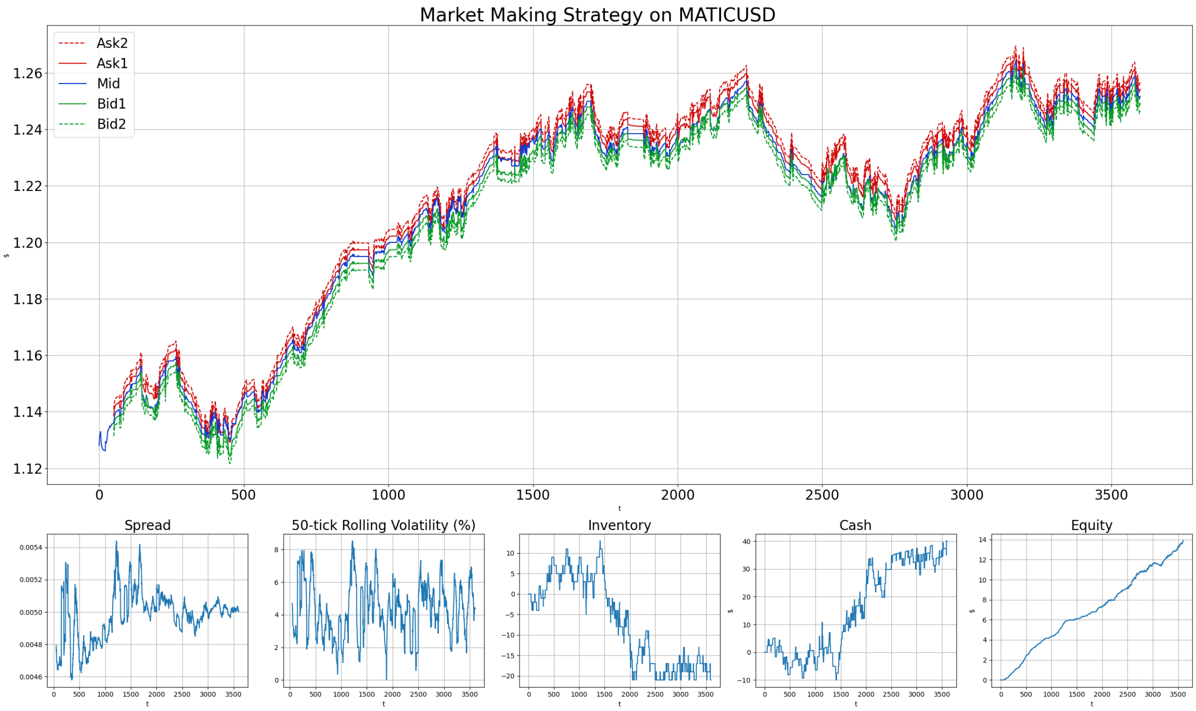

Figure 5: Backtest on MATIC/USD

The backest on MATIC/USD also yields positive results. The trading strategy quotes a bid-ask spread of around \$0.0050 (43bps) which is larger than XBT/USD and SOL/USD. It makes sense that our strategy quotes the largest bid-ask spread for MATIC/USD as the hourly volatility of MATIC/USD is larger than both other coins: averaging around 4% which then peaks at around 8%.

4.2.4 Market Competition

We can infer how competitive a market is by how many $dt = 0$ we calculate (the amount of trades occuring within the same second). The larger the number of $dt = 0$ values, the more volume in that market (as there are more trades occuring within the same second) and hence more competitive. In the periods where our equity graph remains flat, we can assume they are periods where $dt = 0$.

From CoinMarketCap we can see that XBT/USD has a largest trading volume, followed by SOL/USD and then MATIC/USD. Therefore, it makes sense that XBT/USD had 4 periods where the equity remained flat and MATIC/USD only had one short period. We were able to trade MATIC/USD more consistently compared to SOL/USD and XBT/USD and thus the PnL was consistently smoother (as there is less competition). This highlights the importance of knowing market competition and low-latency in market making strategies.

5 Conclusion and Improvements

The outcome of our market making trading strategy produces positive results. We produce consistently good results for the simulated Brownian motion data and for the XBT/USD, SOL/USD and MATIC/USD data.

We incorporated many market making concepts such as skew, spread volatility, levels, using a formalised approach which can be applied to other asset pairs. We also deduced that our trading strategy produces smaller variance returns for less volatile Brownian motion simulations. Furthermore, we attempt to model market competition using the time difference of trades dt to make our backtest more realistic.

However, our backtest does not account for latency and the real world isn't going to provide us with such a consistent, uniform environment, therefore we should be cautious about applying our trading strategy to the real world without more research and development.

Next Steps

To conclude, we discuss some ways which could improve our trading strategy.

First, we did not utilise the quote size data given by Kraken, nor did we adjust our on quote sizes. We can improve our trading strategy by setting our quote size as a function of the inventory or volatility.

In this project we calculate the volatility using a 50-tick lookback period. Volatility was a significant part of the theoretical price and spread calculations and thus the calculated volatility has a significant effect on the results. We could improve our trading strategy using a volatility forecasting model such as the GARCH which may more accurately predict volatility and thus yield better results.

6 References

- Avellaneda, Marco, and Sasha Stoikov. "High-frequency trading in a limit order book." Quantitative Finance 8.3 (2008): 217-224.
- Bouchaud, Jean-Philippe, Marc Mézard, and Marc Potters. "Statistical properties of stock order books: empirical results and models." Quantitative finance 2.4 (2002): 251.
- https://support.kraken.com/hc/en-us/articles/360047543791-Downloadable-historical-market-data-time-and-sales-

7 Appendix

7.1 Brownian Motion

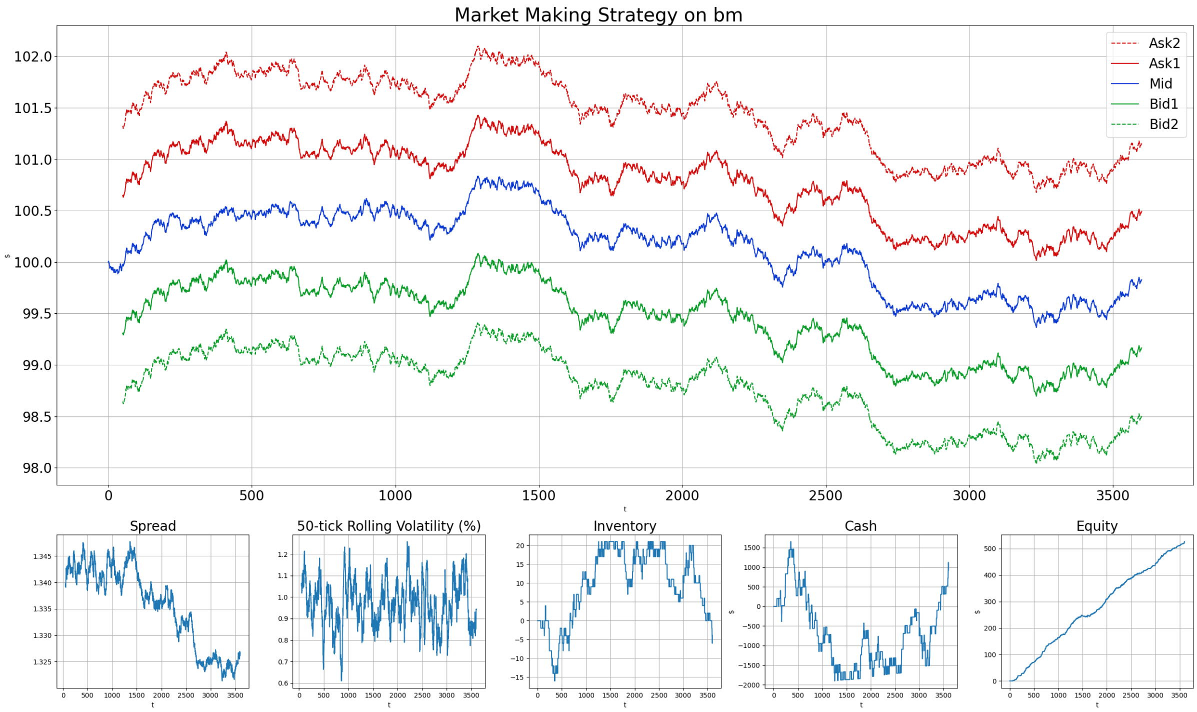

Figure 6: Backtest on Brownian Motion where $\sigma = 1$

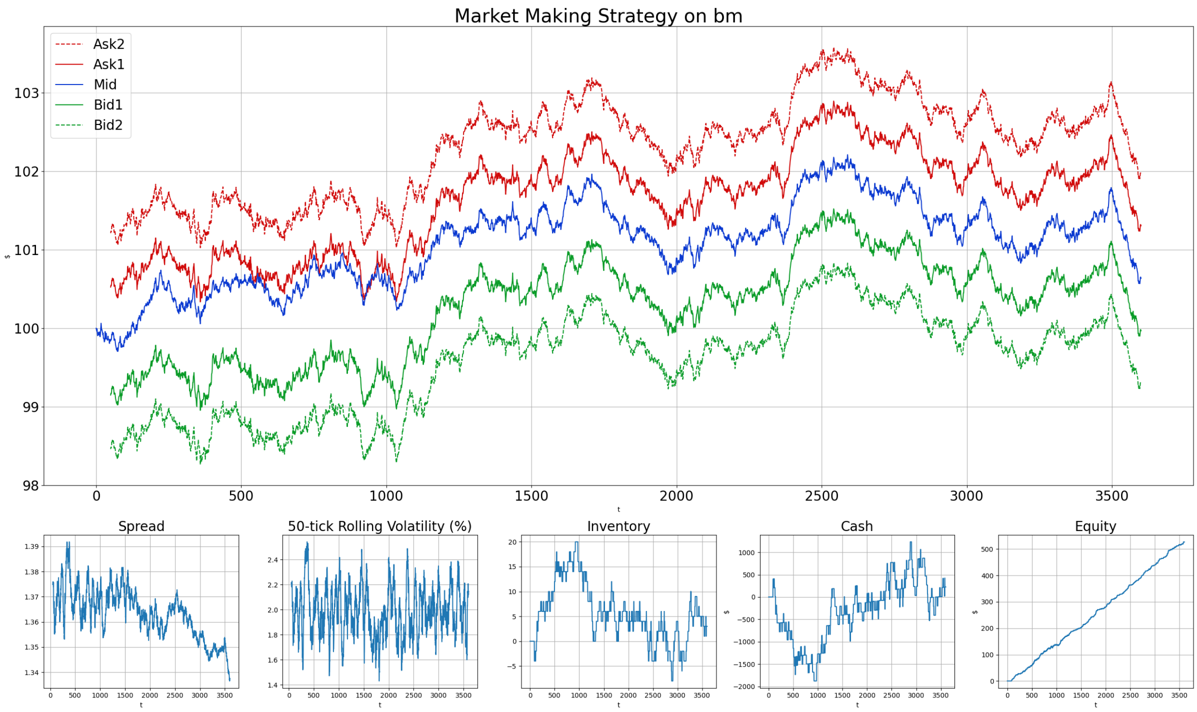

Figure 7: Backtest on Brownian Motion where $\sigma = 2$

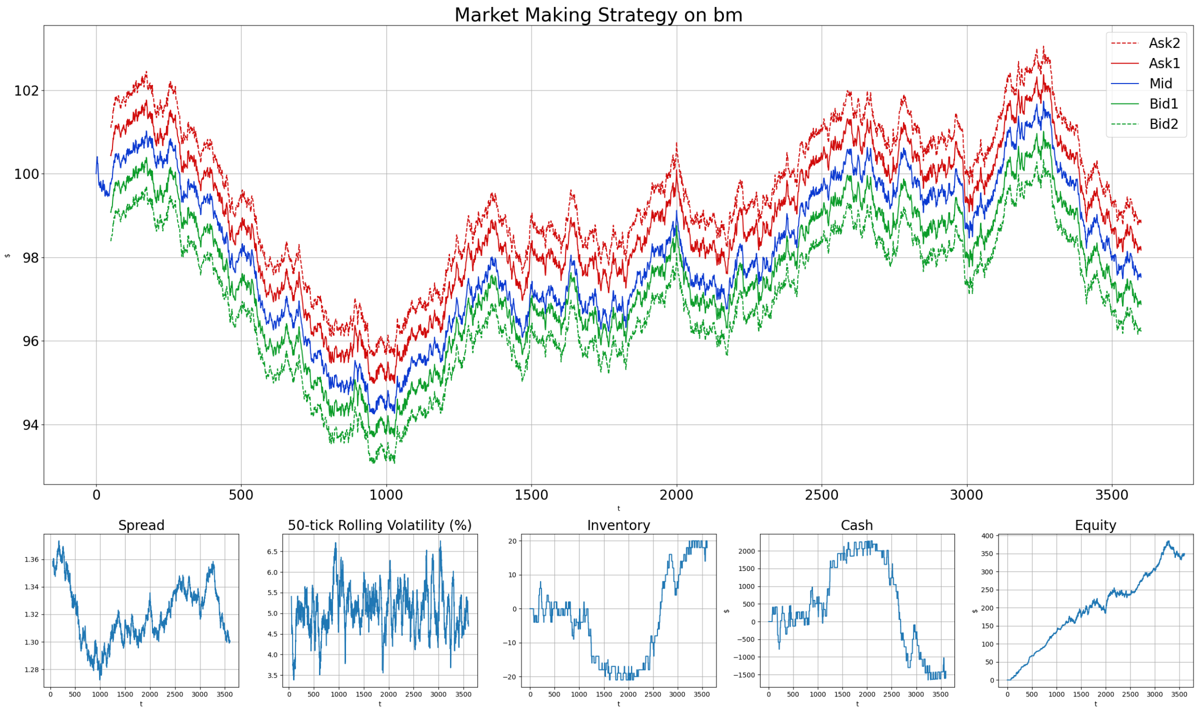

Figure 8: Backtest on Brownian Motion where $\sigma = 5$

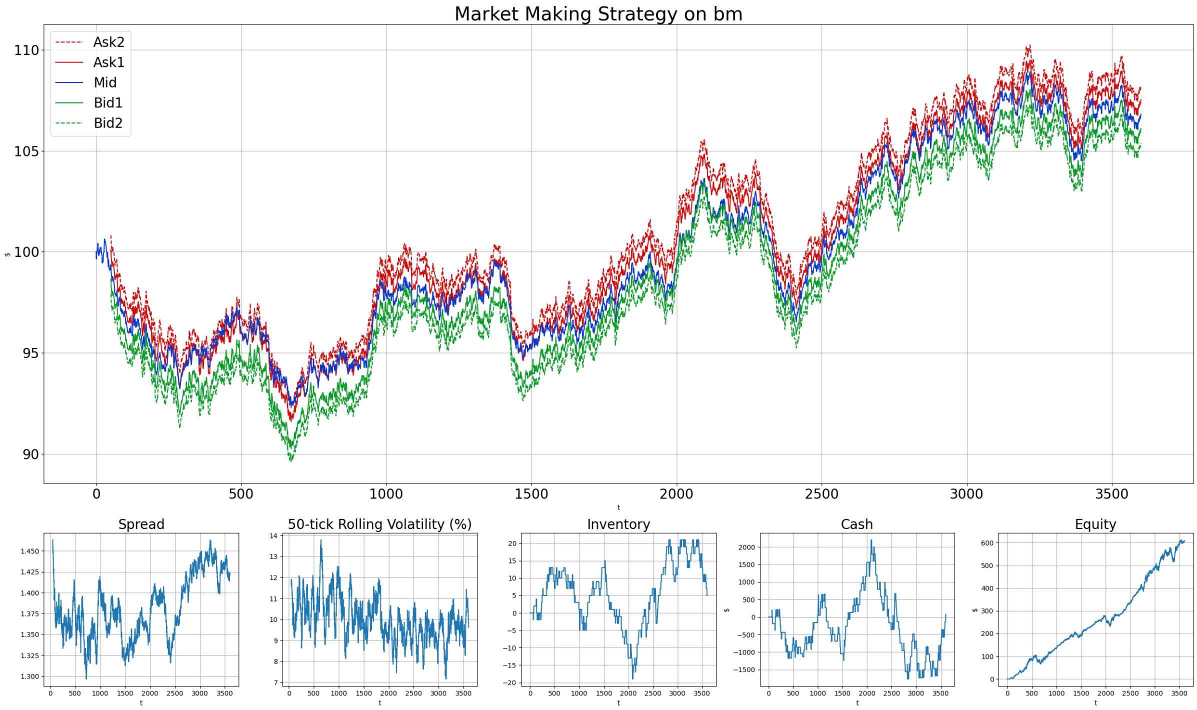

Figure 9: Backtest on Brownian Motion where $\sigma = 10$

7.2 Market Data

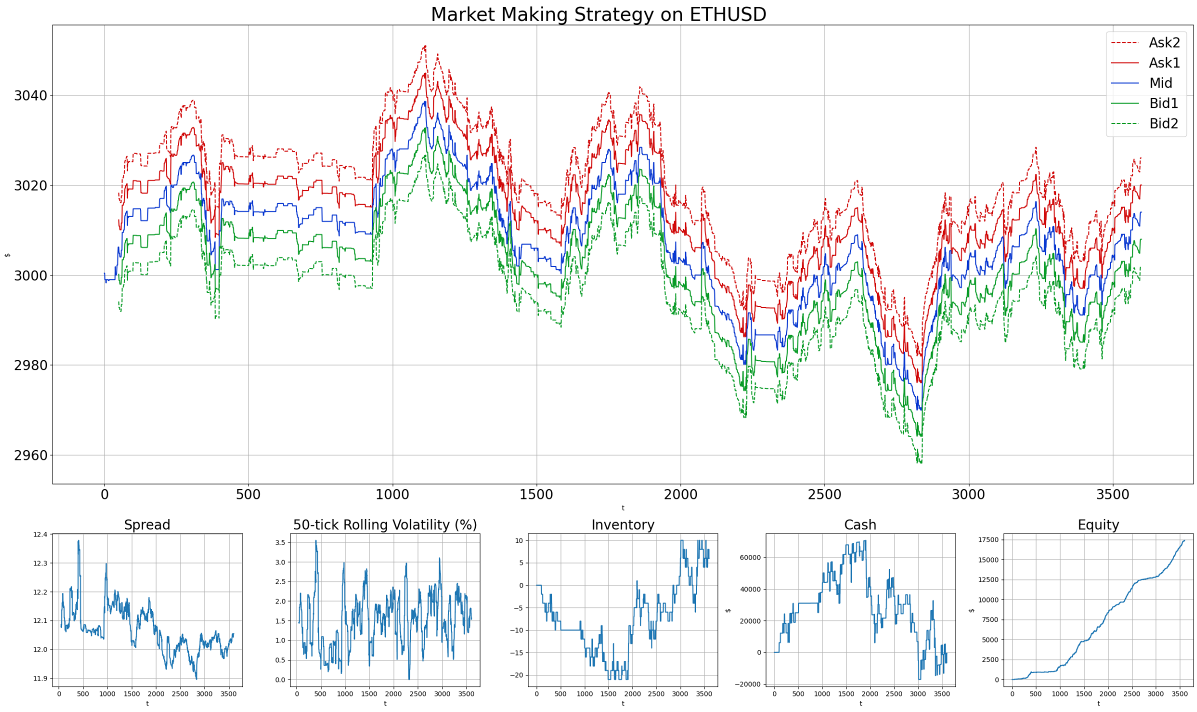

Figure 10: Backtest on ETH/USD

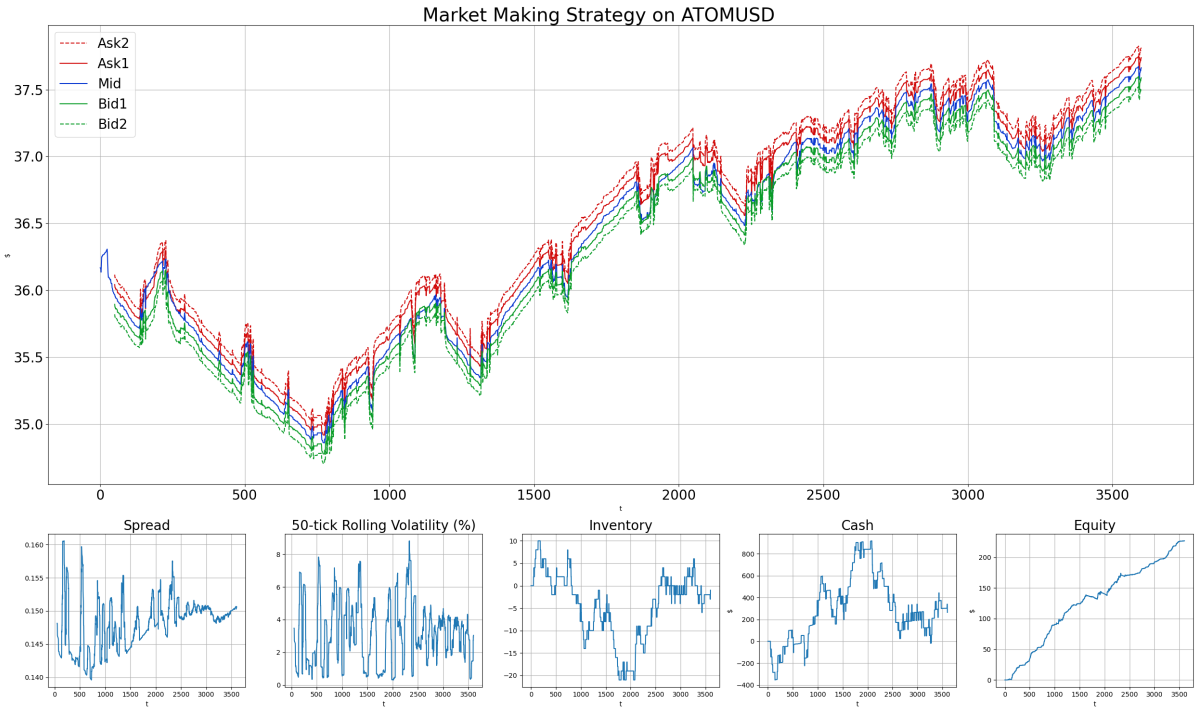

Figure 11: Backtest on ATOM/USD

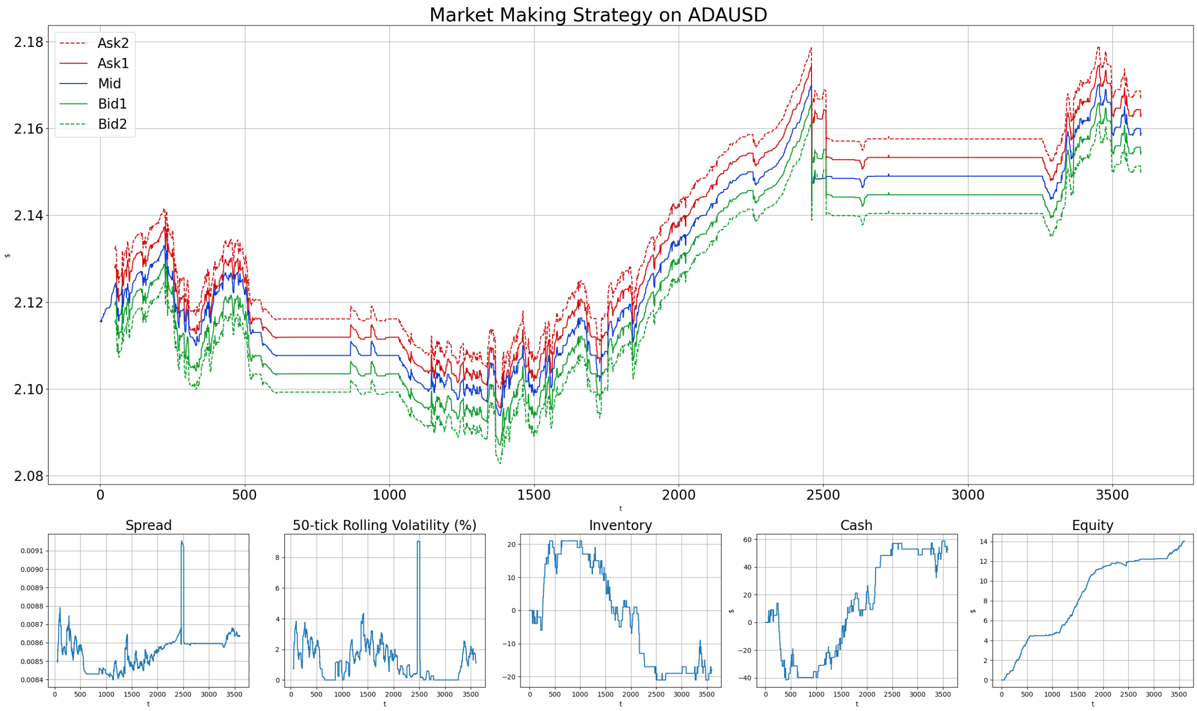

Figure 12: Backtest on ADA/USD

Figure 13: Backtest on AXS/USD

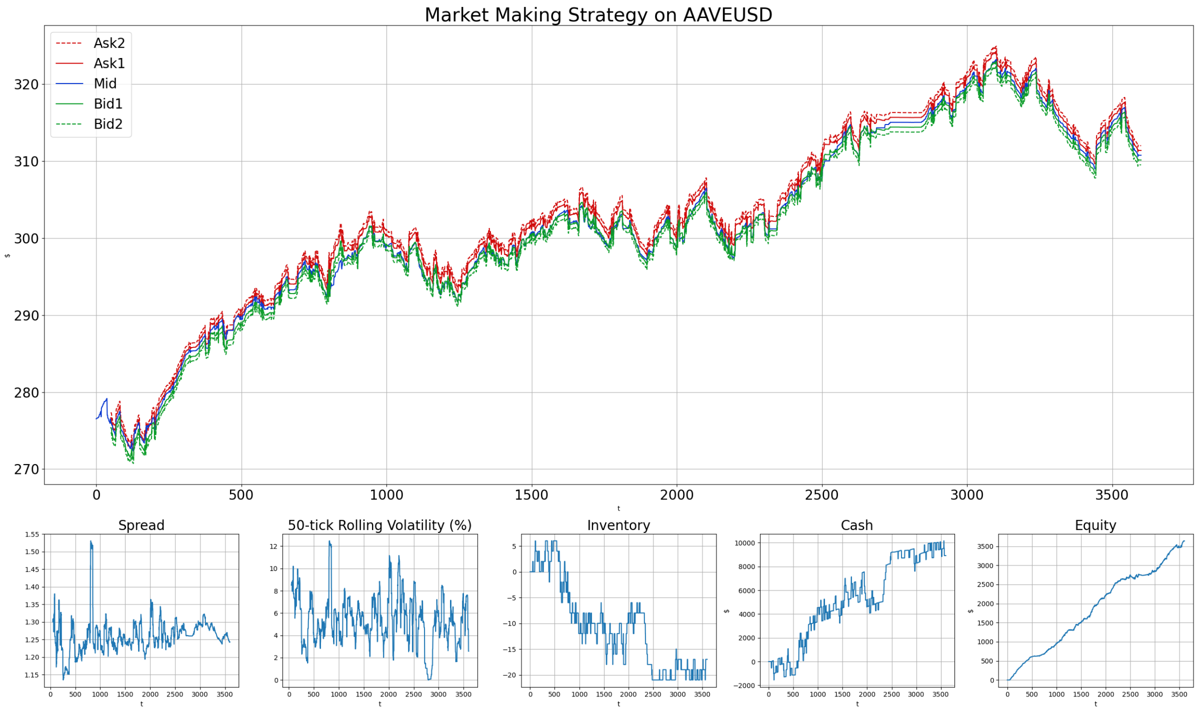

Figure 14: Backtest on AAVE/USD

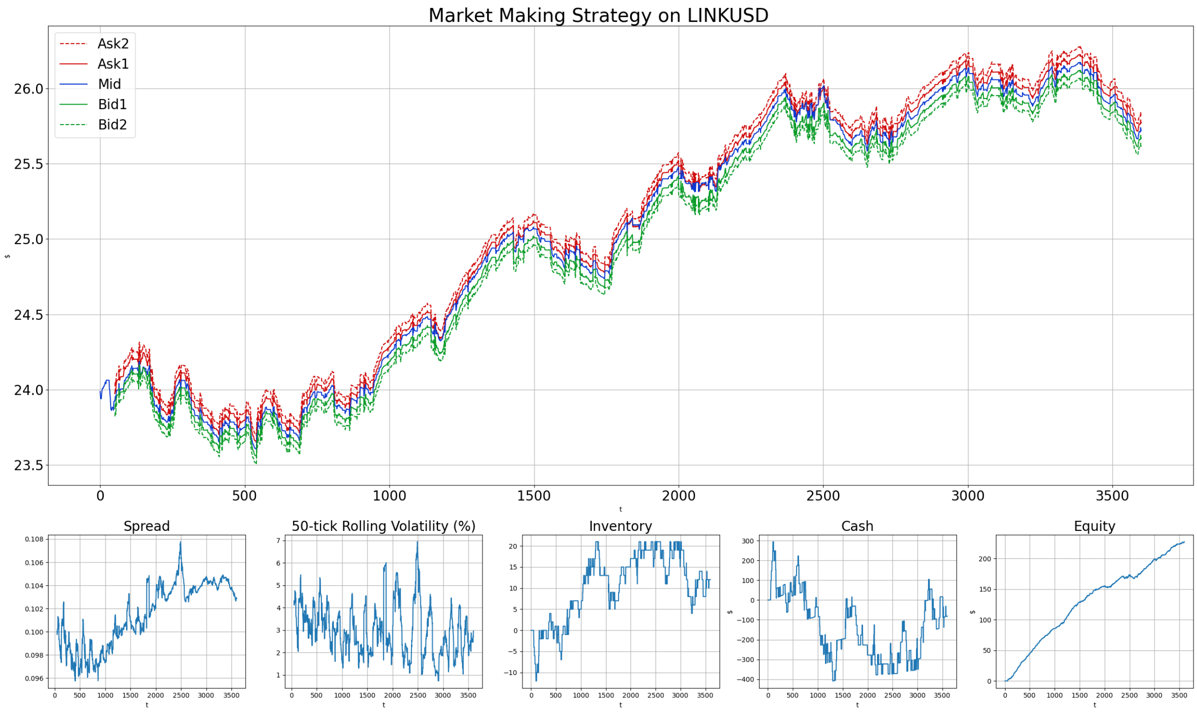

Figure 15: Backtest on LINK/USD