Problems accompanying the lectures about "Probabilistic and Bayesian Reasoning in Physics" by Jörg Enderlein

Problem 1

A three-man jury has two members each of whom independently has probability p of making the correct decision and a third member who flips a fair coin for each decision (majority rules). A one-man jury has probability p of making the correct decision. Which jury has the better probability of making the correct decision?

Problem 2

Coupons in cereal boxes are numbered 1 to 5, and a full set with one of each is required for a prize. Calculate the probability of having a full set of 5 different coupons (winning a prize) after having acquired n cereal boxes.

Problem 3

A contestant in a television game show must choose between three doors. An expensive automobile awaits the contestant behind one of the three doors, and gag prizes await him behind the other two. The contestant must try to pick the door leading to the automobile. He chooses a door randomly (without opening it), and then the host opens one of the other two doors concealing one of the gag prizes. With two doors remaining unopened, the host now asks the contestant whether he wants to remain with his choice of door, or whether he wishes to switch to the other remaining door. What is the better choice, or is there none?

Problem 4

Consider a continuous death process that starts at time zero and for which the probability that a death event happens at time t within time interval dt is given by a *time-dependent* function $r(t) = a\cdot\exp(b\cdot t)$ with some positive constants a and b (death becomes more and more likely with age t). Find the probability $P(t)$ that a decay did not yet happen until time t.

Problem 5

Consider n identical stochastic variables x_j with mean value μ and variance σ^2. Calculate the variance $\langle\sum_{j=1}^{n}\left(x_j-\bar{x}\right)^2\rangle$ where $\bar{x}$ is the mean value $\bar{x}=\frac{1}{n}\sum_{j=1}^{n}x_j$.