Appendix for Goods Trade, Factor Mobility and Welfare

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Stephen J. Redding*

Princeton University, NBER and CEPR, United States

2.1 Consumer Preferences

Preferences for worker ω residing in location n depend on goods consumption (C_n), residential land use (H_{Un}) and idiosyncratic amenity shocks to the utility from residing in each location n:

$$U_n(\omega) = b_n(\omega)\left(\frac{C_n(\omega)}{\alpha}\right)^{\alpha}\left(\frac{H_{Un}(\omega)}{1-\alpha}\right)^{1-\alpha} \tag{1}$$

The goods consumption index (C_n) is defined over consumption of a fixed continuum of goods $j \in [0;\ 1]$:

$$C_n = \left[\int_0^1 c_n(j)^{\rho}\, dj\right]^{\frac{1}{\rho}} \tag{2}$$

where the CES parameter (ρ) determines the elasticity of substitution between goods ($\sigma = 1/(1-\rho)$). The corresponding dual price index for goods consumption (P_n) is:

$$P_n = \left[\int_0^1 p_n(j)^{1-\sigma}\, dj\right]^{\frac{1}{1-\sigma}} \tag{3}$$

The idiosyncratic amenity shocks ($b_n(\omega)$) capture the idea that workers have heterogeneous preferences for living in each location (e.g., different preferences for climate, proximity to the coast etc). We assume that these amenity shocks are drawn independently across locations and workers from a Fréchet distribution:

$$G_n(b) = e^{-B_n b^{-\epsilon}} \tag{4}$$

where the scale parameter B_n determines average amenities for location n and the shape parameter controls the dispersion of amenities across workers for each location. Each worker is endowed with one unit of labor that is supplied inelasticity with zero disutility.

2.2 Production

Each location draws an idiosyncratic productivity $z(j)$ for each good j. Productivity is independently drawn across goods and locations from a Fréchet distribution:

$$F_i(z) = e^{-A_i z^{-\theta}} \tag{5}$$

where the scale parameter A_i determines average productivity for location i and the shape parameter θ controls the dispersion of productivity across goods.

Goods are homogeneous in the sense one unit of a given good is the same as any other unit of that good. Each good is produced with labor under conditions of perfect competition according to a linear technology.

The cost to a consumer in location n of purchasing one unit of good j from location i is therefore:

$$p_{ni}(j) = \frac{d_{ni} w_i}{z_i(j)} \tag{6}$$

2.3 Expenditure Shares and Price Indices

We begin by characterizing expenditure shares and price indices. The representative consumer in location n sources each good from the lowest-cost supplier to that location:

$$p_{ni}(j) = \min\{p_i(j); i \in N\} \tag{7}$$

Using equilibrium prices (6) and the properties of the Fréchet distribution following Eaton and Kortum (2002), the share of expenditure of location n on goods produced by location i is:

$$\pi_{ni} = \int_0^\infty \prod_{k \neq i} \Pr(P_{nk} \geq P_{ni} = p)\, dG_n(p)$$

$$\begin{aligned} G_{ni}(p) = \Pr[P_{ni} \leq p] = \Pr\left[\frac{d_{ni}w_i}{z_i(j)} \leq p\right] &= \Pr\left[\frac{d_{ni}w_i}{p} \leq z_i(j)\right] \\ &= 1 - \Pr\left[z_i(j) \leq \frac{d_{ni}w_i}{p}\right] \\ &= 1 - e^{-A_i(\frac{d_{ni}w_i}{p})^{-\theta}} \end{aligned}$$

$$\begin{aligned} &\prod_{k \neq i} \Pr(P_{nk} \geq P_{ni} = p) \\ &= \prod_{k \neq i} \Pr(P_{nk} \geq p) \\ &= \prod_{k \neq i} 1 - \Pr(P_{nk} < p) \\ &= \prod_{k \neq i} 1 - G_{nk}(p) \\ &= \prod_{k \neq i} 1 - [1 - e^{-A_k(\frac{d_{nk}w_k}{p})^{-\theta}}] \\ &= \prod_{k \neq i} e^{-A_k(\frac{d_{nk}w_k}{p})^{-\theta}} \\ &= e^{-p^\theta \sum_{k \neq i} A_k (d_{nk}w_k)^{-\theta}} \end{aligned}$$

$$\begin{aligned} \pi_{ni} &= \int_0^\infty \prod_{k \neq i} \Pr(P_{nk} \geq P_{ni} = p)\, dG_{ni}(p) \\ &= \int_0^\infty e^{-p^\theta \sum_{k \neq i} A_k (d_{nk}w_k)^{-\theta}} d[1 - e^{-p^\theta A_i (d_{ni}w_i)^{-\theta}}] \\ &= \int_0^\infty e^{-p^\theta \sum_{k \neq i} A_k (d_{nk}w_k)^{-\theta}} * \theta p^{\theta-1} A_i (d_{ni}w_i)^{-\theta} e^{-p^\theta A_i (d_{ni}w_i)^{-\theta}} dp \\ &= \int_0^\infty e^{-p^\theta \sum_{k \in N} A_k (d_{nk}w_k)^{-\theta}} * \theta p^{\theta-1} A_i (d_{ni}w_i)^{-\theta} dp \\ &= A_i (d_{ni}w_i)^{-\theta} \int_0^\infty e^{-p^\theta \sum_{k \in N} A_k (d_{nk}w_k)^{-\theta}} * \theta p^{\theta-1} dp \end{aligned}$$

$$= \frac{A_i(d_{ni}w_i)^{-\theta}}{\sum_{k\in N} A_k\,(d_{nk}w_k)^{-\theta}} \int_0^\infty \sum_{k\in N} A_k\,(d_{nk}w_k)^{-\theta} * e^{-p^\theta \sum_{k\in N} A_k (d_{nk}w_k)^{-\theta}} * \theta p^{\theta-1} dp$$

$$= \frac{A_i(d_{ni}w_i)^{-\theta}}{\sum_{k\in N} A_k\,(d_{nk}w_k)^{-\theta}} \int_0^\infty 1 d[e^{-p^\theta \sum_{k\in N} A_k (d_{nk}w_k)^{-\theta}}]$$

$$= \frac{A_i(d_{ni}w_i)^{-\theta}}{\sum_{k\in N} A_k\,(d_{nk}w_k)^{-\theta}} \int_0^\infty 1 d[1 - G_n(p)]$$

$$= \frac{A_i(d_{ni}w_i)^{-\theta}}{\sum_{k\in N} A_k\,(d_{nk}w_k)^{-\theta}} * [1 - G_n(p)]|_0^\infty$$

$$= \frac{A_i(d_{ni}w_i)^{-\theta}}{\sum_{k\in N} A_k\,(d_{nk}w_k)^{-\theta}} * \{[1 - G_n(\infty)] - [1 - G_n(0)]\}$$

$$= \frac{A_i(d_{ni}w_i)^{-\theta}}{\sum_{k\in N} A_k\,(d_{nk}w_k)^{-\theta}} * \{[1 - 0] - [1 - 1]\}$$

$$= \frac{A_i(d_{ni}w_i)^{-\theta}}{\sum_{k\in N} A_k (d_{nk}w_k)^{-\theta}} \tag{8}$$

where the elasticity of trade with respect to trade costs is determined by the Fréchet shape parameter for productivity θ. The price index dual to (2) can be expressed as:

$$P_n = \left[\int_0^1 p_n(j)^{1-\sigma}\, dj\right]^{\frac{1}{1-\sigma}}$$

$$P_n^{\;1-\sigma} = \int_0^1 p_n(j)^{1-\sigma}\, dj$$

$$= \int_0^\infty p^{1-\sigma}\, dG_n(p)$$

$$= \int_0^\infty p^{1-\sigma}\, d[1 - e^{-p^\theta \sum_k A_k (d_{nk}w_k)^{-\theta}}]$$

$$= \int_0^\infty p^{1-\sigma}\, \theta p^{\theta-1} * \sum_k A_k\,(d_{nk}w_k)^{-\theta} * e^{-p^\theta \sum_k A_k (d_{nk}w_k)^{-\theta}} dp$$

Define $x = p^\theta \sum_k A_k\,(d_{nk}w_k)^{-\theta}$, then $dx = \theta p^{\theta-1} * \sum_k A_k\,(d_{nk}w_k)^{-\theta} dp$. Hence,

$$P_n^{\;1-\sigma} = \int_0^\infty p^{1-\sigma}\, \theta p^{\theta-1} * \sum_k A_k\,(d_{nk}w_k)^{-\theta} * e^{-p^\theta \sum_k A_k (d_{nk}w_k)^{-\theta}} dp$$

$$= \int_0^\infty p^{1-\sigma} * e^{-x} dx$$

$$= \int_0^\infty \left(\frac{x}{\sum_k A_k\,(d_{nk}w_k)^{-\theta}}\right)^{\frac{1-\sigma}{\theta}} * e^{-x} dx$$

$$= \left[\sum_k A_k\,(d_{nk}w_k)^{-\theta}\right]^{\frac{\sigma-1}{\theta}} \int_0^\infty x^{\frac{1-\sigma}{\theta}} * e^{-x} dx$$

$$= \left[\sum_k A_k\,(d_{nk}w_k)^{-\theta}\right]^{\frac{\sigma-1}{\theta}} * \Gamma(1 + \frac{1-\sigma}{\theta})$$

$$P_n = \left\{ \left[\sum_k A_k \, (d_{nk} w_k)^{-\theta} \right]^{\frac{\sigma-1}{\theta}} * \Gamma(1 + \frac{1-\sigma}{\theta}) \right\}^{\frac{1}{1-\sigma}}$$

$$= \left\{ \Gamma(\frac{\theta+1-\sigma}{\theta}) \right\}^{\frac{1}{1-\sigma}} * \left[\sum_{k \in N} A_k \, (d_{nk} w_k)^{-\theta} \right]^{-\frac{1}{\theta}} \tag{9}$$

, where $\Gamma(.)$ is the Gamma function. To ensure a finite value for the price index, we require $\theta > \sigma - 1$. Using the trade share (8) and $d_{nn} = 1$, the goods price index can be equivalently written as:

$$P_n^{-\theta} = \sum_{k \in N} A_k \, (d_{nk} w_k)^{-\theta} * \left\{ \Gamma(\frac{\theta+1-\sigma}{\theta}) \right\}^{\frac{-\theta}{1-\sigma}}$$

$$= \frac{A_n (d_{nn} w_n)^{-\theta}}{\pi_{nn}} * \left\{ \Gamma(\frac{\theta+1-\sigma}{\theta}) \right\}^{\frac{-\theta}{1-\sigma}}$$

$$= \frac{A_n (1 * w_n)^{-\theta}}{\pi_{nn}} * \left\{ \Gamma(\frac{\theta+1-\sigma}{\theta}) \right\}^{\frac{-\theta}{1-\sigma}}$$

$$P_n^{-\theta} = \frac{A_n w_n^{-\theta}}{\pi_{nn}} * \left\{ \Gamma(\frac{\theta+1-\sigma}{\theta}) \right\}^{\frac{-\theta}{1-\sigma}} \tag{10}$$

2.4 Residential Choices and Income

The indirect utility function is:

$$U_n(\omega) = \frac{b_n(\omega) v_n(\omega)}{P_n^{\alpha} r_n^{1-\alpha}} \tag{11}$$

The indirect utility has a Fréchet distribution:

$$G_n(U) = e^{-\psi_n U^{-\epsilon}}, \qquad \psi_n = B_n (\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}})^{\epsilon}$$

Each worker chooses the location that offers her the highest utility after taking into account her idiosyncratic preferences. Using the above distribution of indirect utility, the probability that a worker chooses to live in location $n \in N$ is:

$$\prod_{k \neq n} \Pr(U_k \leq U) = \prod_{k \neq n} (e^{-\psi_k U^{-\epsilon}}) = e^{-U^{-\epsilon} \sum_k \psi_k} = e^{-U^{-\epsilon} \sum_k B_k (\frac{v_k}{P_k^{\alpha} r_k^{1-\alpha}})^{\epsilon}}$$

Integrating over all utilities U gives:

$$\pi_n = \frac{L_n}{\overline{L}} = \int_0^{\infty} e^{-U^{-\epsilon} \sum_k \psi_k} \, dG_n(U)$$

$$= \int_0^{\infty} e^{-U^{-\epsilon} \sum_k \psi_k} \, d[e^{-\psi_n U^{-\epsilon}}]$$

$$= \int_0^{\infty} e^{-U^{-\epsilon} \sum_k \psi_k} * e^{-\psi_n U^{-\epsilon}} * \psi_n \epsilon U^{-\epsilon-1} dU$$

$$= \int_0^{\infty} e^{-U^{-\epsilon} \sum_n \psi_n} * \psi_n \epsilon U^{-\epsilon-1} dU$$

$$= \frac{\psi_n}{\sum_n \psi_n} \int_0^{\infty} e^{-U^{-\epsilon} \sum_n \psi_n} * \sum_n \psi_n \, \epsilon U^{-\epsilon-1} dU$$

$$= \frac{\psi_n}{\sum_n \psi_n} \int_0^\infty \sum_n \psi_n * e^{-U^{-\epsilon} \sum_n \psi_n} * \epsilon U^{-\epsilon-1} dU$$

$$= \frac{\psi_n}{\sum_n \psi_n} \int_0^\infty 1\, d\left[e^{-U^{-\epsilon} \sum_k \psi_k}\right]$$

$$= \frac{\psi_n}{\sum_n \psi_n} * \prod_{k \neq n} \Pr(U_k \leq U) \mid_0^\infty$$

$$= \frac{\psi_n}{\sum_n \psi_n} * (1 - 0)$$

$$= \frac{\psi_n}{\sum_n \psi_n}$$

$$\frac{L_n}{L} = \frac{B_n \left(\frac{v_n}{P_n^\alpha r_n^{1-\alpha}}\right)^\epsilon}{\sum_{k \in N} B_k \left(\frac{v_k}{P_k^\alpha r_k^{1-\alpha}}\right)^\epsilon} \tag{12}$$

Each location faces a labor supply curve that is upward sloping in real income. Therefore, real income in general differs across locations, because higher real income has to be paid to attract workers with lower idiosyncratic tastes for a location. Expected utility for a worker across locations is:

$$\overline{U} = \int_0^\infty U\, d \prod_{k \in N} (G_n(U))$$

$$= \int_0^\infty U\, d\left(e^{-U^{-\epsilon} \sum_{k \in N} \psi_k}\right)$$

$$= \int_0^\infty U e^{-\sum_{k \in N} \psi_k U^{-\epsilon}} \sum_{k \in N} \psi_k\, \epsilon U^{-\epsilon-1}\, dU$$

Define $x = \sum_{k \in N} \psi_k\, U^{-\epsilon}$, then $dx = \sum_{k \in N} \psi_k\, \epsilon U^{-\epsilon-1} dU$. Hence,

$$\overline{U} = \int_0^\infty U e^{-\sum_k \psi_k U^{-\epsilon}} \sum_{k \in N} \psi_k\, \epsilon U^{-\epsilon-1}\, dU$$

$$= \int_0^\infty U e^{-x}\, dx$$

$$= \int_0^\infty \left(\frac{x}{\sum_{k \in N} \psi_k}\right)^{-\frac{1}{\epsilon}} e^{-x}\, dx$$

$$= \left(\sum_{k \in N} \psi_k\right)^{\frac{1}{\epsilon}} \int_0^\infty (x)^{-\frac{1}{\epsilon}} e^{-x}\, dx$$

$$= \left(\sum_{k \in N} \psi_k\right)^{\frac{1}{\epsilon}} * \Gamma\left(1 - \frac{1}{\epsilon}\right)$$

$$\overline{U} = \Gamma\left(\frac{\epsilon - 1}{\epsilon}\right) * \left(\sum_{k \in N} B_k \left(\frac{v_k}{P_k^\alpha r_k^{1-\alpha}}\right)^\epsilon\right)^{\frac{1}{\epsilon}} \tag{13}$$

, where $\Gamma(.)$ is the Gamma function. To ensure a finite value for expected utility, we require $\epsilon > 1$.

Expenditure on land in each location is redistributed lump sum to the workers residing in that location. Therefore, total income in each location (v_n) equals labor income plus expenditure on residential land:

$$v_n L_n = w_n L_n + (1-\alpha) v_n L_n$$

$$v_n L_n = \frac{w_n L_n}{\alpha} \tag{14}$$

Labor income in each location equals expenditure on goods produced in that location:

$$w_i L_i = \sum_{n \in N} \pi_{ni} \, w_n L_n \tag{15}$$

Land market clearing implies that the equilibrium land rent can be determined from the equality of land income and expenditure:

$$r_n = \frac{(1-\alpha) v_n L_n}{H_n} = \frac{(1-\alpha)\frac{w_n L_n}{\alpha}}{H_n} = \frac{(1-\alpha)}{\alpha} \frac{w_n L_n}{H_n} \tag{16}$$

$$\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}} = \frac{\frac{w_n}{\alpha}}{\left[\frac{A_n w_n{}^{-\theta}}{\pi_{nn}} * \left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{-\theta}{1-\sigma}}\right]^{-\frac{\alpha}{\theta}} \left[\frac{(1-\alpha)}{\alpha} \frac{w_n L_n}{H_n}\right]^{1-\alpha}}$$

$$= \frac{\frac{w_n}{\alpha}}{\left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{\alpha}{1-\sigma}} \left[\frac{A_n}{\pi_{nn}}\right]^{-\frac{\alpha}{\theta}} w_n{}^{\alpha} \left[\frac{(1-\alpha)}{\alpha} \frac{w_n L_n}{H_n}\right]^{1-\alpha}}$$

$$= \frac{\left[\frac{A_n}{\pi_{nn}}\right]^{\frac{\alpha}{\theta}}}{\alpha \left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{\alpha}{1-\sigma}} \left[\frac{(1-\alpha)}{\alpha} \frac{L_n}{H_n}\right]^{1-\alpha}}$$

$$= \frac{\left[\frac{A_n}{\pi_{nn}}\right]^{\frac{\alpha}{\theta}} \left[\frac{L_n}{H_n}\right]^{-(1-\alpha)}}{\alpha \left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{\alpha}{1-\sigma}} \left[\frac{(1-\alpha)}{\alpha}\right]^{1-\alpha}}$$

2.5 General Equilibrium

The general equilibrium of the model can be represented by the measure of workers (L_n) in each location $n \in N$, the share of each location's expenditure on goods produced in other locations (π_{ni}) and the wage in each location (w_n). Using labor income (15), the trade share (8), the price index (10), residential choice probabilities (12) and land market clearing (16), this equilibrium triple $\{L_n, w_n, \pi_{ni}\}$ solves the following system of equations for all $i, n \in N$.

First, each location's income must equal expenditure on the goods produced in that location:

$$w_i L_i = \sum_{n \in N} \pi_{ni} \, w_n L_n \tag{17}$$

Second, location expenditure shares are:

$$\pi_{ni} = \frac{A_i (d_{ni} w_i)^{-\theta}}{\sum_{k \in N} A_k (d_{nk} w_k)^{-\theta}} \tag{18}$$

Third, residential choice probabilities imply:

$$\frac{L_n}{\bar{L}} = \frac{B_n(\frac{v_n}{P_n^\alpha r_n^{1-\alpha}})^\epsilon}{\sum_{k\in N} B_k(\frac{v_k}{P_k^\alpha r_k^{1-\alpha}})^\epsilon}$$

$$= \frac{B_n(\frac{\left[\frac{A_n}{\pi_{nn}}\right]^{\frac{\alpha}{\theta}}\left[\frac{L_n}{H_n}\right]^{-(1-\alpha)}}{\alpha\left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{\alpha}{1-\sigma}}\left[\frac{(1-\alpha)}{\alpha}\right]^{1-\alpha}})^\epsilon}{\sum_{k\in N} B_k(\frac{\left[\frac{A_k}{\pi_{kk}}\right]^{\frac{\alpha}{\theta}}\left[\frac{L_k}{H_k}\right]^{-(1-\alpha)}}{\alpha\left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{\alpha}{1-\sigma}}\left[\frac{(1-\alpha)}{\alpha}\right]^{1-\alpha}})^\epsilon}$$

$$= \frac{B_n(\left[\frac{A_n}{\pi_{nn}}\right]^{\frac{\alpha}{\theta}}\left[\frac{L_n}{H_n}\right]^{-(1-\alpha)})^\epsilon}{\sum_{k\in N} B_k(\left[\frac{A_k}{\pi_{kk}}\right]^{\frac{\alpha}{\theta}}\left[\frac{L_k}{H_k}\right]^{-(1-\alpha)})^\epsilon}$$

$$= \frac{B_n\left[\frac{A_n}{\pi_{nn}}\right]^{\frac{\alpha\epsilon}{\theta}}\left[\frac{L_n}{H_n}\right]^{-\epsilon(1-\alpha)}}{\sum_{k\in N} B_k\left[\frac{A_k}{\pi_{kk}}\right]^{\frac{\alpha\epsilon}{\theta}}\left[\frac{L_k}{H_k}\right]^{-\epsilon(1-\alpha)}} \tag{19}$$

2.6 Existence and Uniqueness

We now show that there exists a unique general equilibrium that solves the system of equations (17)-(19).

Using the trade share (8), $\pi_{ni} = \frac{A_i(d_{ni}w_i)^{-\theta}}{\sum_{k\in N} A_k(d_{nk}w_k)^{-\theta}}$, the requirement that income equals expenditure for each location $n \in N$ can be re-written as:

$$w_i L_i = \sum_{n\in N} \pi_{ni}\, w_n L_n$$

$$w_i L_i = \sum_{n\in N} \frac{A_i(d_{ni}w_i)^{-\theta}}{\sum_{k\in N} A_k\,(d_{nk}w_k)^{-\theta}} w_n L_n$$

$$w_i L_i \frac{(w_i)^\theta}{A_i} = \sum_{n\in N} \frac{(d_{ni})^{-\theta}}{\sum_{k\in N} A_k\,(d_{nk}w_k)^{-\theta}} w_n L_n$$

$$\frac{w_i^{1+\theta} L_i}{A_i} = \sum_{n\in N} d_{ni}^{-\theta} \frac{1}{\sum_{k\in N} A_k\,(d_{nk}w_k)^{-\theta}} w_n L_n$$

$$= \sum_{n\in N} d_{ni}^{-\theta} \frac{A_n(d_{nn}w_n)^{-\theta}}{\sum_{k\in N} A_k\,(d_{nk}w_k)^{-\theta}} * \frac{w_n L_n}{A_n(d_{nn}w_n)^{-\theta}}$$

$$= \sum_{n\in N} d_{ni}^{-\theta} \pi_{nn} \frac{w_n L_n}{A_n(1 * w_n)^{-\theta}}$$

$$= \sum_{n\in N} d_{ni}{}^{-\theta} \pi_{nn} \frac{w_n{}^{1+\theta} L_n}{A_n}$$

$$\frac{w_i^{1+\theta} L_i}{A_i} = \sum_{n\in N} d_{ni}^{-\theta} \pi_{nn} \frac{w_n^{1+\theta} L_n}{A_n} \tag{20}$$

Additionally, using labor income (15), $w_i L_i = \sum_{n\in N} \pi_{ni}\, w_n L_n$, the residential choice probabilities (12), $\frac{L_n}{\overline{L}} = \frac{B_n(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}})^{\epsilon}}{\sum_{k\in N} B_k(\frac{v_k}{P_k^{\alpha} r_k^{1-\alpha}})^{\epsilon}}$, expected utility (13), $\overline{U} = \Gamma\left(\frac{\epsilon-1}{\epsilon}\right) * \left(\sum_{k\in N} B_k(\frac{v_k}{P_k^{\alpha} r_k^{1-\alpha}})^{\epsilon}\right)^{\frac{1}{\epsilon}}$, and land market clearing (16), $r_n = \frac{(1-\alpha)}{\alpha} \frac{w_n L_n}{H_n} = \frac{(1-\alpha) v_n L_n}{H_n}$, $v_n L_n = \frac{w_n L_n}{\alpha}$, we obtain the following alternative expression for the goods price index (P_n):

$$\overline{U} = \Gamma\left(\frac{\epsilon-1}{\epsilon}\right) * \left(\sum_{k\in N} B_k(\frac{v_k}{P_k^{\alpha} r_k^{1-\alpha}})^{\epsilon}\right)^{\frac{1}{\epsilon}}$$

$$\left(\frac{\overline{U}}{\delta}\right)^{\epsilon} = \sum_{k\in N} B_k(\frac{v_k}{P_k^{\alpha} r_k^{1-\alpha}})^{\epsilon} = \frac{B_n(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}})^{\epsilon}}{\frac{L_n}{\overline{L}}}$$

$$(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}})^{\epsilon} = \left(\frac{\overline{U}}{\delta}\right)^{\epsilon} \frac{L_n}{\overline{L}} B_n^{-1}$$

$$\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}} = \frac{\overline{U}}{\delta} \left(\frac{L_n}{\overline{L}}\right)^{1/\epsilon} B_n^{-1/\epsilon}$$

$$P_n^{\alpha} r_n^{1-\alpha} = v_n \left(\frac{\overline{U}}{\delta}\right)^{-1} \left(\frac{L_n}{\overline{L}}\right)^{-1/\epsilon} B_n^{1/\epsilon} = \frac{w_n}{\alpha} \left(\frac{\overline{U}}{\delta}\right)^{-1} \left(\frac{L_n}{\overline{L}}\right)^{-1/\epsilon} B_n^{1/\epsilon}$$

$$P_n^{\alpha} \left[\frac{(1-\alpha)}{\alpha} \frac{w_n L_n}{H_n}\right]^{1-\alpha} = \frac{w_n}{\alpha} \left(\frac{\overline{U}}{\delta}\right)^{-1} \left(\frac{L_n}{\overline{L}}\right)^{-1/\epsilon} B_n^{1/\epsilon}$$

$$P_n^{\alpha} \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} \frac{w_n^{1-\alpha} L_n^{1-\alpha}}{H_n^{1-\alpha}} = \frac{w_n}{\alpha} \left(\frac{\overline{U}}{\delta}\right)^{-1} \left(\frac{L_n}{\overline{L}}\right)^{-1/\epsilon} B_n^{1/\epsilon}$$

$$P_n^{\alpha} = \alpha^{1-\alpha} (1-\alpha)^{\alpha-1} w_n^{\alpha-1} L_n^{\alpha-1} H_n^{1-\alpha} \frac{w_n}{\alpha} \left(\frac{\overline{U}}{\delta}\right)^{-1} \left(\frac{L_n}{\overline{L}}\right)^{-1/\epsilon} B_n^{1/\epsilon}$$

$$= \alpha^{-\alpha} (1-\alpha)^{\alpha-1} w_n^{\alpha} L_n^{\alpha-1-1/\epsilon} H_n^{1-\alpha} \left(\frac{\overline{U}}{\delta}\right)^{-1} (\overline{L})^{1/\epsilon} B_n^{1/\epsilon}$$

$$P_n^{\alpha\epsilon} = \alpha^{-\alpha\epsilon} (1-\alpha)^{(\alpha-1)\epsilon} w_n^{\alpha\epsilon} L_n^{\alpha\epsilon-\epsilon-1} H_n^{(1-\alpha)\epsilon} \left(\frac{\overline{U}}{\delta}\right)^{-\epsilon} \overline{L} B_n$$

$$P_n = \alpha^{-1} (1-\alpha)^{\frac{(\alpha-1)\epsilon}{\alpha\epsilon}} w_n L_n^{\frac{\alpha\epsilon-\epsilon-1}{\alpha\epsilon}} H_n^{\frac{(1-\alpha)\epsilon}{\alpha\epsilon}} \left(\frac{\overline{U}}{\delta}\right)^{-\frac{1}{\alpha}} \overline{L}^{\frac{1}{\alpha\epsilon}} B_n^{\frac{1}{\alpha\epsilon}}$$

$$= \alpha^{-1}(1-\alpha)^{\frac{(\alpha-1)\epsilon}{\alpha\epsilon}} \left(\frac{\overline{U}}{\delta}\right)^{-\frac{1}{\alpha}} \overline{L}^{\frac{1}{\alpha\epsilon}} * w_n L_n^{\frac{\alpha\epsilon-\epsilon-1}{\alpha\epsilon}} H_n^{\frac{(1-\alpha)\epsilon}{\alpha\epsilon}} B_n^{\frac{1}{\alpha\epsilon}}$$

$$= \alpha^{-1}(1-\alpha)^{\frac{(\alpha-1)\epsilon}{\alpha\epsilon}} \left(\frac{\overline{U}}{\delta}\right)^{-\frac{1}{\alpha}} \overline{L}^{\frac{1}{\alpha\epsilon}} * w_n L_n^{\frac{\alpha\epsilon-\epsilon-1}{\alpha\epsilon}} H_n^{\frac{(1-\alpha)\epsilon}{\alpha\epsilon}} B_n^{\frac{1}{\alpha\epsilon}}$$

$$P_n^{-\theta} = \left[\alpha^{-1}(1-\alpha)^{\frac{(\alpha-1)\epsilon}{\alpha\epsilon}} \left(\frac{\overline{U}}{\delta}\right)^{-\frac{1}{\alpha}} \overline{L}^{\frac{1}{\alpha\epsilon}}\right]^{-\theta} * w_n^{-\theta} L_n^{\theta(\frac{1+\epsilon-\alpha\epsilon}{\alpha\epsilon})} H_n^{-\theta\frac{(1-\alpha)\epsilon}{\alpha\epsilon}} B_n^{-\frac{\theta}{\alpha\epsilon}}$$

$$= \left[\alpha^{-1}(1-\alpha)^{\frac{(\alpha-1)\epsilon}{\alpha\epsilon}} \left(\frac{\overline{U}}{\delta}\right)^{-\frac{1}{\alpha}} \overline{L}^{\frac{1}{\alpha\epsilon}}\right]^{-\theta} * w_n^{-\theta} B_n^{-\frac{\theta}{\alpha\epsilon}} H_n^{-\theta\frac{(1-\alpha)\epsilon}{\alpha\epsilon}} L_n^{\theta(\frac{1+\epsilon-\alpha\epsilon}{\alpha\epsilon})}$$

$$= \left(\frac{w_n}{\alpha^{1}(1-\alpha)^{-\frac{(\alpha-1)\epsilon}{\alpha\epsilon}} \left(\frac{\overline{U}}{\delta}\right)^{\frac{1}{\alpha}} \overline{L}^{-\frac{1}{\alpha\epsilon}}}\right)^{-\theta} * B_n^{-\frac{\theta}{\alpha\epsilon}} H_n^{-\theta\frac{(1-\alpha)}{\alpha}} L_n^{\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}$$

$$= \left(\frac{w_n}{\left[\alpha^{\epsilon}\left(\frac{1-\alpha}{\alpha}\right)^{\epsilon(1-\alpha)} \left(\frac{\overline{U}}{\delta}\right)^{\epsilon} (\overline{L})^{-1}\right]^{\frac{1}{\alpha\epsilon}}}\right)^{-\theta} * B_n^{-\frac{\theta}{\alpha\epsilon}} H_n^{-\theta\frac{(1-\alpha)}{\alpha}} L_n^{\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}$$

$$P_n^{-\theta} = \left(\frac{w_n}{\overline{W}}\right)^{-\theta} B_n^{-\frac{\theta}{\alpha\epsilon}} H_n^{-\theta(\frac{1-\alpha}{\alpha})} L_n^{\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})} \tag{21}$$

where $\overline{W} = \left[\alpha^{\epsilon}\left(\frac{1-\alpha}{\alpha}\right)^{\epsilon(1-\alpha)} \left(\frac{\overline{U}}{\delta}\right)^{\epsilon} (\overline{L})^{-1}\right]^{\frac{1}{\alpha\epsilon}}$ captures the expected utility ($\overline{U}$) across locations and parameters, $\delta = \Gamma\left(\frac{\epsilon-1}{\epsilon}\right)$. Combining this expression for the goods price index with (10), $P_n^{-\theta} = \frac{A_n w_n{}^{-\theta}}{\pi_{nn}} * \left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{-\theta}{1-\sigma}}$, the domestic trade share (π_{nn}) can be written as:

$$\pi_{nn} = A_n w_n{}^{-\theta} P_n^{\theta} \left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{-\theta}{1-\sigma}}$$

$$= A_n w_n{}^{-\theta} \left(\frac{w_n}{\overline{W}}\right)^{\theta} B_n^{\frac{\theta}{\alpha\epsilon}} H_n^{\theta(\frac{1-\alpha}{\alpha})} L_n^{-\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})} \left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{-\theta}{1-\sigma}}$$

$$= \overline{W}^{-\theta} \left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{-\theta}{1-\sigma}} A_n B_n^{\frac{\theta}{\alpha\epsilon}} H_n^{\theta(\frac{1-\alpha}{\alpha})} L_n^{-\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})} \tag{22}$$

Using this expression for the domestic trade share in the equality of income and expenditure (20), $\frac{w_i^{1+\theta} L_i}{A_i} = \sum_{n\in N} d_{ni}^{-\theta} \pi_{nn} \frac{w_n^{1+\theta} L_n}{A_n}$, we obtain a first system of equations linking the wages and populations of locations $n \in N$ as a function of parameters and expected utility:

$$\frac{w_i^{1+\theta}L_i}{A_i} = \sum_{n\in N} d_{ni}^{-\theta}\pi_{nn}\frac{w_n^{1+\theta}L_n}{A_n}$$

$$= \sum_{n\in N} d_{ni}^{-\theta}\{\overline{W}^{-\theta}\left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{-\theta}{1-\sigma}} A_n B_n^{\frac{\theta}{\alpha\epsilon}} H_n^{\theta(\frac{1-\alpha}{\alpha})} L_n^{-\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}\}\frac{w_n^{1+\theta}L_n}{A_n}$$

$$\overline{W}^{-\theta} = \frac{\frac{w_i^{1+\theta}L_i}{A_i}}{\left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{-\theta}{1-\sigma}} * \sum_{n\in N} d_{ni}^{-\theta}\{A_n B_n^{\frac{\theta}{\alpha\epsilon}} H_n^{\theta(\frac{1-\alpha}{\alpha})} L_n^{-\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}\}\frac{w_n^{1+\theta}L_n}{A_n}}$$

$$= \frac{\frac{w_n^{1+\theta}L_n}{A_n}}{\left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{-\theta}{1-\sigma}} * \sum_{k\in N} d_{nk}^{-\theta} B_k^{\frac{\theta}{\alpha\epsilon}} H_k^{\theta(\frac{1-\alpha}{\alpha})} w_k^{1+\theta} L_k^{1-\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}} \tag{23}$$

Returning to (21), $P_n^{-\theta} = \left(\frac{w_n}{\overline{W}}\right)^{-\theta} B_n^{-\frac{\theta}{\alpha\epsilon}} H_n^{-\theta(\frac{1-\alpha}{\alpha})} L_n^{\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}$, and using the expression for the price index (9), $P_n = \left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{1}{1-\sigma}} * \left[\sum_{k\in N} A_k\,(d_{nk}w_k)^{-\theta}\right]^{-\frac{1}{\theta}}$, we obtain a second system of equations linking the wages and populations of locations $n \in N$ as a function of parameters and expected utility:

$$P_n^{-\theta} = \left(\frac{w_n}{\overline{W}}\right)^{-\theta} B_n^{-\frac{\theta}{\alpha\epsilon}} H_n^{-\theta(\frac{1-\alpha}{\alpha})} L_n^{\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})} = \left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{-\theta}{1-\sigma}} * \sum_{k\in N} A_k\,(d_{nk}w_k)^{-\theta}$$

$$\overline{W}^{-\theta} = \frac{w_n{}^{-\theta} B_n^{-\frac{\theta}{\alpha\epsilon}} H_n^{-\theta(\frac{1-\alpha}{\alpha})} L_n^{\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}}{\left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{-\theta}{1-\sigma}} * \sum_{k\in N} A_k (d_{nk}w_k)^{-\theta}} \tag{24}$$

We characterize the properties of the general equilibrium of the model under the assumption that transport costs (d_{ni}) are "quasi-symmetric," which implies that they can be partitioned into an importer component (D_n), an exporter component (D_i) and a symmetric bilateral component ($D_{ni} = D_{in}$):

$$d_{ni} = \begin{cases} 1 & if\ n = i \\ D_n D_i D_{ni} & if\ n \neq i \end{cases} \tag{25}$$

where $D_n > 1$, $D_i > 1$ and $D_{ni} = D_{in} > 1$.
Under this assumption, the two wage systems (23) and (24) imply the following closed form solution linking the endogenous variables for each location $n \in N$:

$$\frac{\frac{w_n^{1+\theta}L_n}{A_n}}{\left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{-\theta}{1-\sigma}} * \sum_{k\in N} d_{nk}^{-\theta} B_k^{\frac{\theta}{\alpha\epsilon}} H_k^{\theta(\frac{1-\alpha}{\alpha})} w_k^{1+\theta} L_k^{1-\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}} = \frac{w_n{}^{-\theta} B_n^{-\frac{\theta}{\alpha\epsilon}} H_n^{-\theta(\frac{1-\alpha}{\alpha})} L_n^{\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}}{\left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{-\theta}{1-\sigma}} * \sum_{k\in N} A_k\,(d_{nk}w_k)^{-\theta}}$$

$$\frac{\frac{w_n^{1+2\theta}L_n}{A_n}}{\sum_{k\in N} d_{nk}^{-\theta} B_k^{\frac{\theta}{\alpha\epsilon}} H_k^{\theta(\frac{1-\alpha}{\alpha})} w_k^{1+\theta} L_k^{1-\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}} = \frac{B_n^{-\frac{\theta}{\alpha\epsilon}} H_n^{-\theta(\frac{1-\alpha}{\alpha})} L_n^{\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}}{\sum_{k\in N} A_k\,(d_{nk}w_k)^{-\theta}}$$

$$\frac{w_n^{1+2\theta} A_n^{-1} L_n^{1-\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})} B_n^{\frac{\theta}{\alpha\epsilon}} H_n^{\theta(\frac{1-\alpha}{\alpha})}}{\sum_{k\in N} d_{nk}^{-\theta} B_k^{\frac{\theta}{\alpha\epsilon}} H_k^{\theta(\frac{1-\alpha}{\alpha})} w_k^{1+\theta} L_k^{1-\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}} = \frac{1}{\sum_{k\in N} A_k\,(d_{nk}w_k)^{-\theta}}$$

$$w_n^{1+2\theta}A_n^{-1}B_n^{\frac{\theta}{\alpha\epsilon}}H_n^{\theta(\frac{1-\alpha}{\alpha})}L_n^{1-\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}=\frac{\sum_{k\in N}d_{nk}^{-\theta}B_k^{\frac{\theta}{\alpha\epsilon}}H_k^{\theta(\frac{1-\alpha}{\alpha})}w_k^{1+\theta}L_k^{1-\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}}{\sum_{k\in N}A_k(d_{nk}w_k)^{-\theta}}\equiv\kappa, \tag{26}$$

where κ is a scalar.

If equation (26) holds, then any functions w_n and L_n that satisfy the system of equations (23) will also satisfy the system of equations (24) (and vice versa). In the proposition below, we prove below that equation (26) is the unique relationship between w_n and L_n that satisfies both systems. Substituting this relationship (26) into (24), $\overline{W}^{-\theta}=\frac{w_n{}^{-\theta}B_n{}^{-\frac{\theta}{\alpha\epsilon}}H_n{}^{-\theta(\frac{1-\alpha}{\alpha})}L_n{}^{\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}}{\{\Gamma(\frac{\theta+1-\sigma}{\theta})\}^{\frac{-\theta}{1-\sigma}}*\sum_{k\in N}A_k(d_{nk}w_k)^{-\theta}}$, we obtain the following system of equations for equilibrium populations.

$$\overline{W}^{-\theta}=\frac{w_n{}^{-\theta}B_n{}^{-\frac{\theta}{\alpha\epsilon}}H_n{}^{-\theta(\frac{1-\alpha}{\alpha})}L_n{}^{\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}}{\{\Gamma(\frac{\theta+1-\sigma}{\theta})\}^{\frac{-\theta}{1-\sigma}}*\sum_{k\in N}A_k(d_{nk}w_k)^{-\theta}} \tag{24}$$

$$\overline{W}^{-\theta}\left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{-\theta}{1-\sigma}}*\sum_{k\in N}A_k\,(d_{nk}w_k)^{-\theta}=w_n{}^{-\theta}B_n{}^{-\frac{\theta}{\alpha\epsilon}}H_n{}^{-\theta(\frac{1-\alpha}{\alpha})}L_n{}^{\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}$$

$$w_n{}^{-\theta}B_n{}^{-\frac{\theta}{\alpha\epsilon}}H_n{}^{-\theta(\frac{1-\alpha}{\alpha})}L_n{}^{\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}=\overline{W}^{-\theta}\left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{-\theta}{1-\sigma}}*\sum_{k\in N}A_k\,(d_{nk}w_k)^{-\theta} \tag{24}$$

Given eq. (26), $w_n^{1+2\theta}A_n^{-1}B_n^{\frac{\theta}{\alpha\epsilon}}H_n^{\theta(\frac{1-\alpha}{\alpha})}L_n^{1-\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}=\frac{\sum_{k\in N}d_{nk}^{-\theta}B_k^{\frac{\theta}{\alpha\epsilon}}H_k^{\theta(\frac{1-\alpha}{\alpha})}w_k^{1+\theta}L_k^{1-\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}}{\sum_{k\in N}A_k(d_{nk}w_k)^{-\theta}}\equiv\kappa$, we have

$$w_n^{1+2\theta}=\kappa\left(A_n^{-1}B_n^{\frac{\theta}{\alpha\epsilon}}H_n^{\theta(\frac{1-\alpha}{\alpha})}L_n^{1-\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}\right)^{-1}$$

$$w_n^{1+\theta}=\kappa\left(A_n^{-1}B_n^{\frac{\theta}{\alpha\epsilon}}H_n^{\theta(\frac{1-\alpha}{\alpha})}L_n^{1-\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}\right)^{-\frac{1+\theta}{1+2\theta}}$$

$$w_k^{1+\theta}=\kappa\left(A_k^{-1}B_k^{\frac{\theta}{\alpha\epsilon}}H_k^{\theta(\frac{1-\alpha}{\alpha})}L_k^{1-\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}\right)^{-\frac{1+\theta}{1+2\theta}}$$

Plug (26) into (24):

$$w_n^{1+2\theta}A_n^{-1}B_n^{\frac{\theta}{\alpha\epsilon}}H_n^{\theta(\frac{1-\alpha}{\alpha})}L_n^{1-\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}*w_n{}^{-\theta}B_n{}^{-\frac{\theta}{\alpha\epsilon}}H_n{}^{-\theta(\frac{1-\alpha}{\alpha})}L_n{}^{\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}=\overline{W}^{-\theta}\left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{-\theta}{1-\sigma}}*\sum_{k\in N}d_{nk}^{-\theta}B_k^{\frac{\theta}{\alpha\epsilon}}H_k^{\theta(\frac{1-\alpha}{\alpha})}w_k^{1+\theta}L_k^{1-\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}$$

$$w_n^{1+\theta}A_n^{-1}L_n=\overline{W}^{-\theta}\left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{-\theta}{1-\sigma}}*\sum_{k\in N}d_{nk}^{-\theta}B_k^{\frac{\theta}{\alpha\epsilon}}H_k^{\theta(\frac{1-\alpha}{\alpha})}w_k^{1+\theta}L_k^{1-\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}$$

$$\kappa\left(A_n^{-1}B_n^{\frac{\theta}{\alpha\epsilon}}H_n^{\theta\left(\frac{1-\alpha}{\alpha}\right)}L_n^{1-\theta\left(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha}\right)}\right)^{-\frac{1+\theta}{1+2\theta}}A_n^{-1}L_n=$$

$$\overline{W}^{-\theta}\left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{-\theta}{1-\sigma}}*\sum_{k\in N}d_{nk}^{-\theta}B_k^{\frac{\theta}{\alpha\epsilon}}H_k^{\theta(\frac{1-\alpha}{\alpha})}\kappa\left(A_k^{-1}B_k^{\frac{\theta}{\alpha\epsilon}}H_k^{\theta(\frac{1-\alpha}{\alpha})}L_k^{1-\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}\right)^{-\frac{1+\theta}{1+2\theta}}L_k^{1-\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}$$

$$A_n^{\frac{1+\theta}{1+2\theta}-1}B_n^{-\frac{\theta}{\alpha\epsilon}*\frac{1+\theta}{1+2\theta}}H_n^{-\frac{1+\theta}{1+2\theta}\theta\left(\frac{1-\alpha}{\alpha}\right)}L_n^{1-\frac{1+\theta}{1+2\theta}[1-\theta\left(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha}\right)]}=$$

$$\overline{W}^{-\theta}\left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{-\theta}{1-\sigma}}*\sum_{k\in N}d_{nk}^{-\theta}B_k^{\frac{\theta}{\alpha\epsilon}}H_k^{\theta(\frac{1-\alpha}{\alpha})}*A_k^{\frac{1+\theta}{1+2\theta}}B_k^{-\frac{\theta}{\alpha\epsilon}\frac{1+\theta}{1+2\theta}}H_k^{-\frac{1+\theta}{1+2\theta}\theta(\frac{1-\alpha}{\alpha})}L_k^{-\frac{1+\theta}{1+2\theta}[1-\theta\left(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha}\right)]}*L_k^{1-\theta(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha})}$$

$$A_n^{-\frac{\theta}{1+2\theta}} B_n^{-\frac{\theta}{\alpha\epsilon}*\frac{1+\theta}{1+2\theta}} H_n^{-\frac{1+\theta}{1+2\theta}\theta\left(\frac{1-\alpha}{\alpha}\right)} L_n^{\frac{\theta}{1+2\theta}\left[1-\theta\left(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha}\right)\right]} =$$

$$\overline{W}^{-\theta}\left\{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)\right\}^{\frac{-\theta}{1-\sigma}} * \sum_{k\in N} d_{nk}^{-\theta} * A_k^{\frac{1+\theta}{1+2\theta}} B_k^{\frac{\theta}{\alpha\epsilon}\frac{\theta}{1+2\theta}} H_k^{\frac{\theta}{1+2\theta}\theta\left(\frac{1-\alpha}{\alpha}\right)} L_k^{\frac{\theta}{1+2\theta}\left[1-\theta\left(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha}\right)\right]}$$

$$L_n^{\frac{\theta}{1+2\theta}\left[1+(1+\theta)\left(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha}\right)\right]} A_n^{-\frac{\theta}{1+2\theta}} B_n^{-\frac{\theta}{1+2\theta}\frac{(1+\theta)}{\alpha\epsilon}} H_n^{-\frac{\theta}{1+2\theta}\frac{(1+\theta)(1-\alpha)}{\alpha}} =$$

$$\overline{W}^{-\theta}\left\{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)\right\}^{\frac{-\theta}{1-\sigma}} * \sum_{k\in N} d_{nk}^{-\theta} A_k^{\frac{\theta}{1+2\theta}\frac{1+\theta}{\theta}} B_k^{\frac{\theta}{1+2\theta}\frac{\theta}{\alpha\epsilon}} H_k^{\frac{\theta}{1+2\theta}\theta\left(\frac{1-\alpha}{\alpha}\right)} L_k^{\frac{\theta}{1+2\theta}\left[1-\theta\left(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha}\right)\right]} \tag{27}$$

This system of equations (27) uniquely determines the equilibrium population of each location $n \in N$ up to a normalization for expected utility ($\overline{W}$) (a choice of units in which to measure expected utility).

Proposition 1 Given the land area, productivity and amenity parameters $\{H_n, A_n, B_n\}$ and quasi-symmetric bilateral trade frictions $\{d_{ni}\}$ for each location $n \in N$, there exist unique equilibrium populations (${L_n}^*$), trade shares (${\pi_{ni}}^*$) and wages (${w_n}^*$).

Proof. The proof follows the same structure as in Allen and Arkolakis (2014). Given the land area, productivity and amenity parameters $\{H_n, A_n, B_n\}$ and bilateral trade frictions $\{d_{ni}\}$, there exists a unique fixed point in the system of equations (27) because $\gamma_2/\gamma_1 < 1$ (see for example Fujimoto and Krause 1985). This unique fixed point determines the unique equilibrium population (L_n^*) for each location $n \in N$ up to a normalization determined by the expected utility ($\bar{W}$). This normalization ($\bar{W}$) is determined by combining the system of equations (27) with the requirement that the labor market clear: $\sum_{n\in N} L_n = \bar{L}$. Having determined the unique equilibrium population (L_n^*) for each location, the closed-form relationship between the endogenous variables (26) immediately yields the unique equilibrium wage (w_n^*) for each location $n \in N$. Having solved for equilibrium wages and populations $\{L_n^*, w_n^*\}$, the expenditure shares (18) determine unique equilibrium trade shares (π_{ni}^*) for each pair of locations $n, i \in N$. ■

2.7 Comparative Statics

Although we allow for both trade costs and heterogeneity in worker preferences, and consider a large number of locations that can differ from one another in productivity, amenities, land supplies and bilateral trade costs, the model admits closed-form expressions for the comparative statics of the endogenous variables with respect to the relative value of these location characteristics. To characterize these comparative statics, we re-write the system of equations for equilibrium populations (27) as the following implicit function:

$$\begin{pmatrix}\Omega_1\\ \vdots\\ \Omega_n\\ \vdots\\ \Omega_N\end{pmatrix} = \begin{pmatrix}\Omega_1^I\\ \vdots\\ \Omega_n^I\\ \vdots\\ \Omega_N^I\end{pmatrix} - \begin{pmatrix}\Omega_1^{II}\\ \vdots\\ \Omega_n^{II}\\ \vdots\\ \Omega_N^{II}\end{pmatrix} = \begin{pmatrix}\Omega_1^I\\ \vdots\\ \Omega_n^I\\ \vdots\\ \Omega_N^I\end{pmatrix} - \begin{pmatrix}\Omega_1^{II}\\ \vdots\\ \sum_{k\in N}\Omega_{nk}^{II}\\ \vdots\\ \Omega_N^{II}\end{pmatrix} = \begin{pmatrix}0\\ \vdots\\ 0\\ \vdots\\ 0\end{pmatrix} \tag{28}$$

$$\Omega_n^I = L_n^{\frac{\theta}{1+2\theta}\left[1+(1+\theta)\left(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha}\right)\right]} A_n^{-\frac{\theta}{1+2\theta}} B_n^{-\frac{\theta}{1+2\theta}\frac{(1+\theta)}{\alpha\epsilon}} H_n^{-\frac{\theta}{1+2\theta}\frac{(1+\theta)(1-\alpha)}{\alpha}}$$

$$\Omega_n^{II} = \sum_{k\in N} \Omega_{nk}^{II}$$

$$\Omega_{nk}^{II} = \overline{W}^{-\theta} \left\{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)\right\}^{\frac{-\theta}{1-\sigma}} * d_{nk}^{-\theta} A_k^{\frac{\theta}{1+2\theta}\frac{1+\theta}{\theta}} B_k^{\frac{\theta}{1+2\theta}\frac{\theta}{\alpha\epsilon}} H_k^{\frac{\theta}{1+2\theta}\theta\left(\frac{1-\alpha}{\alpha}\right)} L_k^{\frac{\theta}{1+2\theta}\left[1-\theta\left(\frac{1}{\alpha\epsilon}+\frac{1-\alpha}{\alpha}\right)\right]}$$

where Ω_n^{II} has an interpretation as a market access term that captures the goods market access of each location (depending on trade costs d_{nk}) to the characteristics of other locations.

We first show that the implicit function (28) is monotonically decreasing in the productivity (A_n), amenities (B_n) and land supply (H_n) of each location. Consider the derivative of the implicit function (Ω_n) for location n with respect to these characteristics of location n:

$$\begin{aligned} \frac{\partial \Omega_n}{\partial A_n} &= -\frac{\theta}{1+2\theta}\frac{\Omega_n^I}{A_n} - \frac{\theta}{1+2\theta}\frac{1+\theta}{\theta}\frac{\Omega_{nn}^{II}}{A_n} = -\tilde{\theta}\frac{\Omega_n^I}{A_n} - \tilde{\theta}\frac{1+\theta}{\theta}\frac{\Omega_{nn}^{II}}{A_n} \\ &= -\tilde{\theta}\frac{\Omega_n^I}{A_n} - \tilde{\theta}\frac{1+\theta}{\theta}\frac{\Omega_n^I}{A_n}\frac{\Omega_{nk}^{II}}{\Omega_n^I} = -\left[\tilde{\theta} + \tilde{\theta}\frac{1+\theta}{\theta}\frac{\Omega_{nn}^{II}}{\Omega_n^I}\right]\frac{\Omega_n^I}{A_n} < 0. \qquad \left(\tilde{\theta} \equiv \frac{\theta}{1+2\theta}\right). \end{aligned} \tag{29}$$

$$\begin{aligned} \frac{\partial \Omega_n}{\partial B_n} &= -\frac{\theta}{1+2\theta}\frac{(1+\theta)}{\alpha\epsilon}\frac{\Omega_n^I}{B_n} - \frac{\theta}{1+2\theta}\frac{\theta}{\alpha\epsilon}\frac{\Omega_{nn}^{II}}{B_n} \\ &= -\tilde{\theta}\frac{(1+\theta)}{\alpha\epsilon}\frac{\Omega_n^I}{B_n} - \tilde{\theta}\frac{\theta}{\alpha\epsilon}\frac{\Omega_{nn}^{II}}{B_n} \\ &= -\tilde{\theta}\frac{(1+\theta)}{\alpha\epsilon}\frac{\Omega_n^I}{B_n} - \tilde{\theta}\frac{\theta}{\alpha\epsilon}\frac{\Omega_n^I}{B_n}\frac{\Omega_{nn}^{II}}{\Omega_n^I} \\ &= -\left[\tilde{\theta}\frac{(1+\theta)}{\alpha\epsilon} + \tilde{\theta}\frac{\theta}{\alpha\epsilon}\frac{\Omega_{nn}^{II}}{\Omega_n^I}\right]\frac{\Omega_n^I}{B_n} < 0 \end{aligned} \tag{30}$$

$$\begin{aligned} \frac{\partial \Omega_n}{\partial H_n} &= -\frac{\theta}{1+2\theta}\frac{(1+\theta)(1-\alpha)}{\alpha}\frac{\Omega_n^I}{H_n} - \frac{\theta}{1+2\theta}\theta\left(\frac{1-\alpha}{\alpha}\right)\frac{\Omega_{nn}^{II}}{H_n} \\ &= -\tilde{\theta}\frac{(1+\theta)(1-\alpha)}{\alpha}\frac{\Omega_n^I}{H_n} - \tilde{\theta}\theta\left(\frac{1-\alpha}{\alpha}\right)\frac{\Omega_{nn}^{II}}{\Omega_n^I}\frac{\Omega_n^I}{H_n} \\ &= -\left[\tilde{\theta}\frac{(1+\theta)(1-\alpha)}{\alpha} + \tilde{\theta}\theta\left(\frac{1-\alpha}{\alpha}\right)\frac{\Omega_{nn}^{II}}{\Omega_n^I}\right]\frac{\Omega_n^I}{H_n} < 0 \end{aligned} \tag{31}$$

Note that $\Omega_n^I = \Omega_n^{II} = \sum_{k\in N}\Omega_{nk}^{II}$, i.e., (27), then $\frac{\Omega_{nn}^{II}}{\Omega_n^I} = \frac{\Omega_{nn}^{II}}{\Omega_n^{II}}$ in the above three equations.

Consider the derivative of the implicit function (Ω_k) for location $k \neq n$ with respect to these characteristics of location n:

$$\frac{\partial \Omega_k}{\partial A_n} = -\frac{\theta}{1+2\theta}\frac{1+\theta}{\theta}\frac{\Omega_{nk}^{II}}{A_n} = -\tilde{\theta}\frac{1+\theta}{\theta}\frac{\Omega_{nk}^{II}}{\Omega_k^{II}}\frac{\Omega_k^{II}}{A_n} < 0, \qquad \forall\ k \neq n, \tag{32}$$

$$\frac{\partial \Omega_k}{\partial B_n} = -\frac{\theta}{1+2\theta}\frac{\theta}{\alpha\epsilon}\frac{\Omega_{nk}^{II}}{B_n} = -\tilde{\theta}\frac{\theta}{\alpha\epsilon}\frac{\Omega_{nk}^{II}}{\Omega_k^{II}}\frac{\Omega_k^{II}}{B_n} < 0, \qquad \forall\ k \neq n, \tag{33}$$

$$\frac{\partial \Omega_k}{\partial H_n} = -\frac{\theta}{1+2\theta}\theta\left(\frac{1-\alpha}{\alpha}\right)\frac{\Omega_{nk}^{II}}{H_n} = -\tilde{\theta}\theta\left(\frac{1-\alpha}{\alpha}\right)\frac{\Omega_{nk}^{II}}{\Omega_k^{II}}\frac{\Omega_k^{II}}{H_n} < 0, \qquad \forall\ k \neq n, \tag{34}$$

where we have used $\Omega_k^I = \Omega_k^{II}$.

We next show that the implicit function (28), $\begin{pmatrix}\Omega_1\\ \vdots\\ \Omega_N\end{pmatrix} = \begin{pmatrix}\Omega_1^I\\ \vdots\\ \Omega_N^I\end{pmatrix} - \begin{pmatrix}\Omega_1^{II}\\ \vdots\\ \Omega_N^{II}\end{pmatrix} = \begin{pmatrix}0\\ \vdots\\ 0\end{pmatrix}$, is monotonically increasing in trade costs to other locations. Under our assumption of quasi-symmetric trade costs, we have:

$$d_{ni} = \begin{cases}1 & if\ n = i\\ D_n D_i D_{ni} & if\ n \neq i\end{cases} \tag{25}$$

where $D_n > 1$, $D_i > 1$ and $D_{ni} = D_{in} > 1$.

Consider the derivative of the implicit function (Ω_n) for location n with respect to the common component of trade costs (D_n) for location n:

$$\frac{\partial \Omega_n}{\partial D_n} = -(-\theta)\sum_{k\neq n}\frac{\Omega_{nk}^{II}}{D_n} = \theta\sum_{k\neq n}\frac{\Omega_{nk}^{II}}{D_n} > 0, \tag{35}$$

Now consider the derivative of the implicit function (Ω_k) for location $k \neq n$ with respect to the common component of trade costs (D_n) for location n:

$$\frac{\partial \Omega_k}{\partial D_n} = -(-\theta)\frac{\Omega_{kn}^{II}}{D_n} = \theta\frac{\Omega_{kn}^{II}}{D_n} > 0, \qquad \forall\ k \neq n. \tag{36}$$

Finally, we show that the implicit function for each location is monotonically increasing in its own population and monotonically decreasing in the population of other locations:

$$\begin{aligned}\frac{\partial \Omega_n}{\partial L_n} &= \frac{\theta}{1+2\theta}\left[1 + (1+\theta)\left(\frac{1}{\alpha\epsilon} + \frac{1-\alpha}{\alpha}\right)\right]\frac{\Omega_n^I}{L_n} - \frac{\theta}{1+2\theta}[1 - \theta(\frac{1}{\alpha\epsilon} + \frac{1-\alpha}{\alpha})]\frac{\Omega_{nn}^{II}}{L_n}\\ &= \tilde{\theta}\left[1 + (1+\theta)\left(\frac{1}{\alpha\epsilon} + \frac{1-\alpha}{\alpha}\right)\right]\frac{\Omega_n^I}{L_n} - \tilde{\theta}[1 - \theta(\frac{1}{\alpha\epsilon} + \frac{1-\alpha}{\alpha})]\frac{\Omega_{nn}^{II}}{L_n}\\ &= \tilde{\theta}\gamma_1\frac{\Omega_n^I}{L_n} - \tilde{\theta}\gamma_2\frac{\Omega_{nn}^{II}}{L_n}\\ &= \tilde{\theta}\gamma_1\frac{\Omega_n^I}{L_n} - \tilde{\theta}\gamma_2\frac{\Omega_{nn}^{II}}{\Omega_n^I}\frac{\Omega_n^I}{L_n}\\ &= \tilde{\theta}\gamma_1\left[1 - \frac{\gamma_2}{\gamma_1}\frac{\Omega_{nn}^{II}}{\Omega_n^I}\right]\frac{\Omega_n^I}{L_n} > 0\end{aligned} \tag{37}$$

$$\begin{aligned}\frac{\partial \Omega_k}{\partial L_n} &= -\frac{\theta}{1+2\theta}[1 - \theta(\frac{1}{\alpha\epsilon} + \frac{1-\alpha}{\alpha})]\frac{\Omega_{nk}^{II}}{L_n}\\ &= -\frac{\theta}{1+2\theta}[1 - \theta(\frac{1}{\alpha\epsilon} + \frac{1-\alpha}{\alpha})]\frac{\Omega_{nk}^{II}}{\Omega_k^{II}}\frac{\Omega_k^I}{L_n}\\ &= -\tilde{\theta}\gamma_2\frac{\Omega_{nk}^{II}}{\Omega_k^{II}}\frac{\Omega_k^I}{L_n} = -\tilde{\theta}\,\gamma_1\frac{\gamma_2}{\gamma_1}\frac{\Omega_{nk}^{II}}{\Omega_k^{II}}\frac{\Omega_k^I}{L_n} < 0 \quad \forall\ k \neq n\end{aligned} \tag{38}$$

, given $\gamma_1 \equiv 1 + (1+\theta)\left(\frac{1}{\alpha\epsilon} + \frac{1-\alpha}{\alpha}\right)$, $\gamma_2 \equiv 1 - \theta\left(\frac{1}{\alpha\epsilon} + \frac{1-\alpha}{\alpha}\right)$, $\Omega_n^{II} = \Omega_n^{II}$.

Proposition 2 *Assuming that bilateral trade frictions (d_{ni}) are quasi-symmetric, an increase in the productivity (A_n), amenities (B_n) or land supply (H_n) of a location n relative to all other locations increases the equilibrium population of that location relative to all other locations $k \neq n$, other things equal. An increase in location n's trade costs to all other locations $k \neq n$ (D_n) decreases the equilibrium population of that location relative to all other locations $k \neq n$, other things equal.*

2.9 Counterfactuals

The system of equations for general equilibrium (17)-(19)

$$w_i L_i = \sum_{n\in N} \pi_{ni}\, w_n L_n \tag{17}$$

$$\pi_{ni} = \frac{A_i (d_{ni} w_i)^{-\theta}}{\sum_{k\in N} A_k (d_{nk} w_k)^{-\theta}} \tag{18}$$

$$\frac{L_n}{\bar{L}} = \frac{B_n \left[\frac{A_n}{\pi_{nn}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{L_n}{H_n}\right]^{-\epsilon(1-\alpha)}}{\sum_{k\in N} B_k \left[\frac{A_k}{\pi_{kk}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{L_k}{H_k}\right]^{-\epsilon(1-\alpha)}} \tag{19}$$

provides an approach for undertaking model-based counterfactuals that uses only parameters and the values of endogenous variables in the initial equilibrium (as in Dekle, Eaton, and Kortum 2007). In contrast to standard trade models, these model-based counterfactuals now yield predictions for the reallocation of the mobile factor labor across locations.

The system of equations for general equilibrium (17)-(19) must hold both before and after a change in trade frictions, productivity or amenities. We denote the value of variables in the counterfactual equilibrium with a prime (x') and the relative value of variables in the counterfactual and initial equilibria by a hat ($\hat{x} = x'/x$). Using this notation, the system of equations for the counterfactual equilibrium (17)-(19) can be re-written as follows:

$$given \left(\hat{x} = \frac{x'}{x}; Y_i = w_i L_i;\ \lambda_n = \frac{L_n}{\bar{L}}\right),$$

$$w_i' L_i' = \sum_{n\in N} \pi_{ni}' w_n' L_n'$$

$$\frac{w_i'}{w_i} w_i \frac{L_i'}{L_i} L_i = \sum_{n\in N} \pi_{ni}' \frac{w_n'}{w_n} w_n \frac{L_n'}{L_n} L_n$$

$$\hat{w}_i Y_i \frac{L_i'}{L_i} = \sum_{n\in N} \pi_{ni}' \hat{w}_n Y_n \frac{L_n'}{L_n}$$

$$\hat{w}_i Y_i \frac{L_i'/\bar{L}}{L_i/\bar{L}} = \sum_{n\in N} \pi_{ni}' \hat{w}_n Y_n \frac{L_n'/\bar{L}}{L_n/\bar{L}}$$

$$\hat{w}_i Y_i \frac{\lambda_i'}{\lambda_i} = \sum_{n\in N} \pi_{ni}' \hat{w}_n Y_n \frac{\lambda_n'}{\lambda_n}$$

$$\hat{w}_i Y_i \hat{\lambda}_i = \sum_{n\in N} \pi_{ni}' \hat{w}_n Y_n \hat{\lambda}_n$$

$$\hat{w}_i \hat{\lambda}_i Y_i = \sum_{n\in N} \pi_{ni}' \hat{w}_n \hat{\lambda}_n Y_n \tag{45}$$

$$\pi_{ni}' = \frac{A_i' (d_{ni}' w_i')^{-\theta}}{\sum_{k\in N} A_k' (d_{nk}' w_k')^{-\theta}}$$

$$\frac{\pi_{ni}'}{\pi_{ni}} \pi_{ni} = \frac{\frac{A_i'}{A_i} A_i \left(\frac{d_{ni}'}{d_{ni}} d_{ni} * \frac{w_i'}{w_i} w_i\right)^{-\theta}}{\sum_{k\in N} \frac{A_k'}{A_k} A_k \left(\frac{d_{nk}'}{d_{nk}} d_{nk} * \frac{w_k'}{w_k} w_k\right)^{-\theta}}$$

$$\hat{\pi}_{ni}\pi_{ni} = \frac{\hat{A}_i A_i (\hat{d}_{ni} d_{ni} * \hat{w}_i w_i)^{-\theta}}{\sum_{k\in N} \hat{A}_k A_k (\hat{d}_{nk} d_{nk} * \hat{w}_k w_k)^{-\theta}}$$

$$= \frac{A_i (d_{ni} * w_i)^{-\theta} * \hat{A}_i (\hat{d}_{ni} * \hat{w}_i)^{-\theta}}{\sum_{k\in N} A_k (d_{nk} * w_k)^{-\theta} * \hat{A}_k (\hat{d}_{nk} * \hat{w}_k)^{-\theta}}$$

$$= \frac{\frac{A_i (d_{ni} * w_i)^{-\theta}}{\sum_{k\in N} A_k (d_{nk} w_k)^{-\theta}} * \hat{A}_i (\hat{d}_{ni} * \hat{w}_i)^{-\theta}}{\sum_{k\in N} \frac{A_k (d_{nk} * w_k)^{-\theta}}{\sum_{k\in N} A_k (d_{nk} w_k)^{-\theta}} * \hat{A}_k (\hat{d}_{nk} * \hat{w}_k)^{-\theta}}$$

$$= \frac{\pi_{ni} * \hat{A}_i (\hat{d}_{ni} * \hat{w}_i)^{-\theta}}{\sum_{k\in N} \pi_{nk} * \hat{A}_k (\hat{d}_{nk} * \hat{w}_k)^{-\theta}} \tag{46}$$

$$given \left(\hat{x} = \frac{x'}{x}; Y_i = w_i L_i;\ \lambda_n = \frac{L_n}{\bar{L}}\right),$$

$$\frac{L_n{}'}{\bar{L}} = \frac{B_n{}' \left[\frac{A_n{}'}{\pi_{nn}{}'}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{L_n{}'}{H_n{}'}\right]^{-\epsilon(1-\alpha)}}{\sum_{k\in N} B_k{}' \left[\frac{A_k{}'}{\pi_{kk}{}'}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{L_k{}'}{H_k{}'}\right]^{-\epsilon(1-\alpha)}}$$

$$\frac{L_n{}'}{\bar{L}} = \frac{B_n{}' \left[\frac{A_n{}'}{\pi_{nn}{}'}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{L_n{}'}{H_n{}'}\right]^{-\epsilon(1-\alpha)}}{\sum_{k\in N} B_k{}' \left[\frac{A_k{}'}{\pi_{kk}{}'}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{L_k{}'}{H_k{}'}\right]^{-\epsilon(1-\alpha)}}$$

$$\frac{L_n{}'}{L_n}\frac{L_n}{\bar{L}} = \frac{\frac{B_n{}'}{B_n} B_n \left[\frac{\frac{A_n{}'}{A_n} A_n}{\frac{\pi_{nn}{}'}{\pi_{nn}} \pi_{nn}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{\frac{L_n{}'}{L_n} L_n}{\frac{H_n{}'}{H_n} H_n}\right]^{-\epsilon(1-\alpha)}}{\sum_{k\in N} \frac{B_k{}'}{B_k} B_k \left[\frac{\frac{A_k{}'}{A_k} A_k}{\frac{\pi_{kk}{}'}{\pi_{kk}} \pi_{kk}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{\frac{L_k{}'}{L_k} L_k}{\frac{H_k{}'}{H_k} H_k}\right]^{-\epsilon(1-\alpha)}}$$

$$\frac{L_n'/\bar{L}}{L_n/\bar{L}}\frac{L_n}{\bar{L}} = \frac{\frac{B_n{}'}{B_n} B_n \left[\frac{\frac{A_n{}'}{A_n} A_n}{\frac{\pi_{nn}{}'}{\pi_{nn}} \pi_{nn}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{\frac{L_n'/\bar{L}}{L_n/\bar{L}} L_n}{\frac{H_n{}'}{H_n} H_n}\right]^{-\epsilon(1-\alpha)}}{\sum_{k\in N} \frac{B_k{}'}{B_k} B_k \left[\frac{\frac{A_k{}'}{A_k} A_k}{\frac{\pi_{kk}{}'}{\pi_{kk}} \pi_{kk}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{\frac{L_k'/\bar{L}}{L_k/\bar{L}} L_k}{\frac{H_k{}'}{H_k} H_k}\right]^{-\epsilon(1-\alpha)}}$$

$$\frac{\lambda_n{}'}{\lambda_n}\lambda_n = \frac{\widehat{B}_n B_n \left[\frac{\widehat{A}_n A_n}{\widehat{\pi}_{nn}\pi_{nn}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{\widehat{\lambda}_n L_n}{\widehat{H}_n H_n}\right]^{-\epsilon(1-\alpha)}}{\sum_{k\in N} \widehat{B}_k B_k \left[\frac{\widehat{A}_k A_k}{\widehat{\pi}_{kk}\pi_{kk}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{\widehat{\lambda}_k L_k}{\widehat{H}_k{}' H_k}\right]^{-\epsilon(1-\alpha)}}$$

$$= \frac{\widehat{B}_n \left[\frac{\widehat{A}_n}{\widehat{\pi}_{nn}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{\widehat{\lambda}_n}{\widehat{H}_n}\right]^{-\epsilon(1-\alpha)} * B_n \left[\frac{A_n}{\pi_{nn}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{L_n}{H_n}\right]^{-\epsilon(1-\alpha)}}{\sum_{k\in N}\{\widehat{B}_k \left[\frac{\widehat{A}_k}{\widehat{\pi}_{kk}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{\widehat{\lambda}_k}{\widehat{H}_k{}'}\right]^{-\epsilon(1-\alpha)} * B_k \left[\frac{A_k}{\pi_{kk}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{L_k}{H_k}\right]^{-\epsilon(1-\alpha)}\}}$$

$$\frac{\lambda_n{}'}{\lambda_n}\lambda_n = \frac{\widehat{B}_n \left[\frac{\widehat{A}_n}{\widehat{\pi}_{nn}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{\widehat{\lambda}_n}{\widehat{H}_n}\right]^{-\epsilon(1-\alpha)} * B_n \left[\frac{A_n}{\pi_{nn}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{L_n}{H_n}\right]^{-\epsilon(1-\alpha)}}{\sum_{k\in N}\{\widehat{B}_k \left[\frac{\widehat{A}_k}{\widehat{\pi}_{kk}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{\widehat{\lambda}_k}{\widehat{H}_k{}'}\right]^{-\epsilon(1-\alpha)} * B_k \left[\frac{A_k}{\pi_{kk}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{L_k}{H_k}\right]^{-\epsilon(1-\alpha)}\}}$$

$$\widehat{\lambda}_n\lambda_n = \frac{\widehat{B}_n \left[\frac{\widehat{A}_n}{\widehat{\pi}_{nn}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{\widehat{\lambda}_n}{\widehat{H}_n}\right]^{-\epsilon(1-\alpha)} * B_n \left[\frac{A_n}{\pi_{nn}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{L_n}{H_n}\right]^{-\epsilon(1-\alpha)}}{\sum_{k\in N}\{\widehat{B}_k \left[\frac{\widehat{A}_k}{\widehat{\pi}_{kk}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{\widehat{\lambda}_k}{\widehat{H}_k{}'}\right]^{-\epsilon(1-\alpha)} * B_k \left[\frac{A_k}{\pi_{kk}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{L_k}{H_k}\right]^{-\epsilon(1-\alpha)}\}}$$

$$= \frac{\widehat{B}_n \left[\frac{\widehat{A}_n}{\widehat{\pi}_{nn}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{\widehat{\lambda}_n}{\widehat{H}_n}\right]^{-\epsilon(1-\alpha)} * \lambda_n * \sum_{k\in N} B_k \left[\frac{A_k}{\pi_{kk}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{L_k}{H_k}\right]^{-\epsilon(1-\alpha)}}{\sum_{k\in N}\{\widehat{B}_k \left[\frac{\widehat{A}_k}{\widehat{\pi}_{kk}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{\widehat{\lambda}_k}{\widehat{H}_k{}'}\right]^{-\epsilon(1-\alpha)} * B_k \left[\frac{A_k}{\pi_{kk}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{L_k}{H_k}\right]^{-\epsilon(1-\alpha)}\}}$$

$$= \frac{\widehat{B}_n \left[\frac{\widehat{A}_n}{\widehat{\pi}_{nn}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{\widehat{\lambda}_n}{\widehat{H}_n}\right]^{-\epsilon(1-\alpha)} * \lambda_n * 1}{\sum_{k\in N}\{\widehat{B}_k \left[\frac{\widehat{A}_k}{\widehat{\pi}_{kk}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{\widehat{\lambda}_k}{\widehat{H}_k{}'}\right]^{-\epsilon(1-\alpha)} * \frac{B_k \left[\frac{A_k}{\pi_{kk}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{L_k}{H_k}\right]^{-\epsilon(1-\alpha)}}{\sum_{k\in N} B_k \left[\frac{A_k}{\pi_{kk}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{L_k}{H_k}\right]^{-\epsilon(1-\alpha)}}\}}$$

$$= \frac{\widehat{B}_n \left[\frac{\widehat{A}_n}{\widehat{\pi}_{nn}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{\widehat{\lambda}_n}{\widehat{H}_n}\right]^{-\epsilon(1-\alpha)} * \lambda_n}{\sum_{k\in N}\{\widehat{B}_k \left[\frac{\widehat{A}_k}{\widehat{\pi}_{kk}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{\widehat{\lambda}_k}{\widehat{H}_k{}'}\right]^{-\epsilon(1-\alpha)} * \lambda_k\}}$$

$$\widehat{\lambda}_n\lambda_n = \frac{\widehat{B}_n \left[\frac{\widehat{A}_n}{\widehat{\pi}_{nn}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{\widehat{\lambda}_n}{1}\right]^{-\epsilon(1-\alpha)} * \lambda_n}{\sum_{k\in N}\{\widehat{B}_k \left[\frac{\widehat{A}_k}{\widehat{\pi}_{kk}}\right]^{\frac{\alpha\epsilon}{\theta}} \left[\frac{\widehat{\lambda}_k}{1}\right]^{-\epsilon(1-\alpha)} * \lambda_k\}} \tag{47}$$

where $Y_i = w_i L_i$ denotes labor income and $\lambda_n = \frac{L_n}{\bar{L}}$ denotes the population share in the initial equilibrium.

This system of equations can be solved for $\{\hat{\lambda}_n, \hat{w}_n, \hat{\pi}_{ni}\}$ given the observed variables in the initial equilibrium $\{\lambda_n, Y_n, \pi_{ni}\}$ and an assumed comparative static. For example, a reduction in trade costs holding productivity and amenities constant corresponds to $\hat{d}_{ni} < 1$, $\hat{A}_n = 1$ and $\hat{B}_n = 1$, while an increase in productivity corresponds to $\hat{A}_n > 1$.

2.10 Welfare Gains from Trade

We now examine the implications of worker mobility with heterogeneous preferences for the welfare gains from trade. We first show that the common change in welfare between an actual and a counterfactual equilibrium across locations ($\hat{\bar{U}} = \bar{U}'/\bar{U}$) can be written as a weighted average of the change in real income in each location. From expected utility (13), $\bar{U} = \Gamma\left(\frac{\epsilon-1}{\epsilon}\right) * \left(\sum_{k\in N} B_k\left(\frac{v_k}{P_k^{\alpha} r_k^{1-\alpha}}\right)^{\epsilon}\right)^{\frac{1}{\epsilon}}$, and the residential choice probabilities (12), $\frac{L_n}{\bar{L}} = \frac{B_n\left(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}}\right)^{\epsilon}}{\sum_{k\in N} B_k\left(\frac{v_k}{P_k^{\alpha} r_k^{1-\alpha}}\right)^{\epsilon}} = \frac{B_n\left(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}}\right)^{\epsilon}}{\sum_{n\in N} B_n\left(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}}\right)^{\epsilon}}$, we have:

$$\hat{\bar{U}} = \frac{\bar{U}'}{\bar{U}} = \frac{\Gamma\left(\frac{\epsilon-1}{\epsilon}\right) * \left(\sum_{k\in N} B_k'\left(\frac{v_k'}{P_k'^{\alpha} r_k'^{1-\alpha}}\right)^{\epsilon}\right)^{\frac{1}{\epsilon}}}{\Gamma\left(\frac{\epsilon-1}{\epsilon}\right) * \left(\sum_{k\in N} B_k\left(\frac{v_k}{P_k^{\alpha} r_k^{1-\alpha}}\right)^{\epsilon}\right)^{\frac{1}{\epsilon}}}$$

$$= \frac{\left(\frac{B_n'\left(\frac{v_n'}{P_n'^{\alpha} r_n'^{1-\alpha}}\right)^{\epsilon}}{\frac{L_n'}{\bar{L}}}\right)^{\frac{1}{\epsilon}}}{\left(\frac{B_n\left(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}}\right)^{\epsilon}}{\frac{L_n}{\bar{L}}}\right)^{\frac{1}{\epsilon}}} = \left(\frac{B_n'\left(\frac{v_n'}{P_n'^{\alpha} r_n'^{1-\alpha}}\right)^{\epsilon}}{B_n\left(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}}\right)^{\epsilon}} \frac{\frac{L_n}{\bar{L}}}{\frac{L_n'}{\bar{L}}}\right)^{\frac{1}{\epsilon}}$$

$$= \left(\frac{\hat{B}_n\left(\frac{\hat{v}_n}{\hat{P}_n^{\alpha} \hat{r}_n^{1-\alpha}}\right)^{\epsilon}}{1} \frac{\frac{L_n}{\bar{L}}}{\frac{L_n'}{\bar{L}}}\right)^{\frac{1}{\epsilon}} = \left(L_n/\bar{L} \frac{\hat{B}_n\left(\frac{\hat{v}_n}{\hat{P}_n^{\alpha} \hat{r}_n^{1-\alpha}}\right)^{\epsilon}}{L_n'/\bar{L}}\right)^{\frac{1}{\epsilon}}$$

$$= \left(\frac{L_n}{\bar{L}} * \frac{\hat{B}_n \left(\frac{\hat{v}_n}{\hat{P}_n^{\alpha} \hat{r}_n^{1-\alpha}} \right)^{\epsilon}}{\frac{B_n{}' \left(\frac{v_n{}'}{P'^{\alpha}_n r'^{1-\alpha}_n} \right)^{\epsilon}}{\sum_{k \in N} B_k{}' \left(\frac{v_k{}'}{P'^{\alpha}_k r'^{1-\alpha}_k} \right)^{\epsilon}}} \right)^{\frac{1}{\epsilon}} = \left(\frac{L_n}{\bar{L}} * \frac{\hat{B}_n \left(\frac{\hat{v}_n}{\hat{P}_n^{\alpha} \hat{r}_n^{1-\alpha}} \right)^{\epsilon}}{\frac{B_n{}' \left(\frac{v_n{}'}{P'^{\alpha}_n r'^{1-\alpha}_n} \right)^{\epsilon}}{B_n \left(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}} \right)^{\epsilon}} \frac{B_n \left(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}} \right)^{\epsilon}}{\sum_{k \in N} B_k{}' \left(\frac{v_k{}'}{P'^{\alpha}_k r'^{1-\alpha}_k} \right)^{\epsilon}}} \right)^{\frac{1}{\epsilon}}$$

$$= \left(\frac{L_n}{\bar{L}} * \frac{1}{\frac{B_n \left(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}} \right)^{\epsilon}}{\sum_{k \in N} B_k{}' \left(\frac{v_k{}'}{P'^{\alpha}_k r'^{1-\alpha}_k} \right)^{\epsilon}}} \right)^{\frac{1}{\epsilon}} = \left(\frac{L_n}{\bar{L}} * \frac{\sum_{k \in N} B_k{}' \left(\frac{v_k{}'}{P'^{\alpha}_k r'^{1-\alpha}_k} \right)^{\epsilon}}{B_n \left(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}} \right)^{\epsilon}} \right)^{\frac{1}{\epsilon}}$$

$$= \left(\frac{L_n}{\bar{L}} * \sum_{k \in N} \frac{B_k{}' \left(\frac{v_k{}'}{P'^{\alpha}_k r'^{1-\alpha}_k} \right)^{\epsilon}}{B_n \left(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}} \right)^{\epsilon}} \right)^{\frac{1}{\epsilon}} = \left(\frac{L_n}{\bar{L}} * \sum_{k \in N} \frac{B_k{}' \left(\frac{v_k{}'}{P'^{\alpha}_k r'^{1-\alpha}_k} \right)^{\epsilon}}{B_n \left(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}} \right)^{\epsilon}} \right)^{\frac{1}{\epsilon}}$$

$$= \left(\frac{L_n}{\bar{L}} * \sum_{k \in N} \frac{B_k{}' \left(\frac{v_k{}'}{P'^{\alpha}_k r'^{1-\alpha}_k} \right)^{\epsilon}}{B_k \left(\frac{v_k}{P_k^{\alpha} r_k^{1-\alpha}} \right)^{\epsilon}} \frac{B_k \left(\frac{v_k}{P_k^{\alpha} r_k^{1-\alpha}} \right)^{\epsilon}}{B_n \left(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}} \right)^{\epsilon}} \right)^{\frac{1}{\epsilon}}$$

$$= \left(\frac{L_n}{L} * \sum_{k \in N} \hat{B}_k \left(\frac{\hat{v}_k}{\hat{P}_k^{\alpha} \hat{r}_k^{1-\alpha}} \right)^{\epsilon} \frac{B_k \left(\frac{v_k}{P_k^{\alpha} r_k^{1-\alpha}} \right)^{\epsilon}}{B_n \left(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}} \right)^{\epsilon}} \right)^{\frac{1}{\epsilon}}$$

$$= \left(\frac{B_n \left(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}} \right)^{\epsilon}}{\sum_{n \in N} B_n \left(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}} \right)^{\epsilon}} * \sum_{k \in N} \hat{B}_k \left(\frac{\hat{v}_k}{\hat{P}_k^{\alpha} \hat{r}_k^{1-\alpha}} \right)^{\epsilon} \frac{B_k \left(\frac{v_k}{P_k^{\alpha} r_k^{1-\alpha}} \right)^{\epsilon}}{B_n \left(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}} \right)^{\epsilon}} \right)^{\frac{1}{\epsilon}}$$

$$= \left(\sum_{k \in N} \frac{B_n \left(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}} \right)^{\epsilon}}{\sum_{n \in N} B_n \left(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}} \right)^{\epsilon}} \hat{B}_k \left(\frac{\hat{v}_k}{\hat{P}_k^{\alpha} \hat{r}_k^{1-\alpha}} \right)^{\epsilon} \frac{B_k \left(\frac{v_k}{P_k^{\alpha} r_k^{1-\alpha}} \right)^{\epsilon}}{B_n \left(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}} \right)^{\epsilon}} \right)^{\frac{1}{\epsilon}}$$

$$= \left(\sum_{k \in N} \hat{B}_k \left(\frac{\hat{v}_k}{\hat{P}_k^{\alpha} \hat{r}_k^{1-\alpha}} \right)^{\epsilon} \frac{B_k \left(\frac{v_k}{P_k^{\alpha} r_k^{1-\alpha}} \right)^{\epsilon}}{\sum_{n \in N} B_n \left(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}} \right)^{\epsilon}} \right)^{\frac{1}{\epsilon}} = \left(\sum_{k \in N} \frac{L_k}{L} * \hat{B}_k \left(\frac{\hat{v}_k}{\hat{P}_k^{\alpha} \hat{r}_k^{1-\alpha}} \right)^{\epsilon} \right)^{\frac{1}{\epsilon}}$$

$$= \left(\sum_{n \in N} \frac{L_n}{L} * \hat{B}_k \left(\frac{\hat{v}_k}{\hat{P}_k^{\alpha} \hat{r}_k^{1-\alpha}} \right)^{\epsilon} \right)^{\frac{1}{\epsilon}} \tag{48}$$

where the weights depend on location population shares. Using expenditure equals income (14), $v_n L_n = \frac{w_n L_n}{\alpha}$, the price index (10), $P_n^{-\theta} = \frac{A_n w_n^{-\theta}}{\pi_{nn}} * \left\{ \Gamma\left(\frac{\theta+1-\sigma}{\theta} \right) \right\}^{\frac{-\theta}{1-\sigma}}$, and land market clearing (16), $r_n = \frac{(1-\alpha)}{\alpha} \frac{w_n L_n}{H_n}$, real income for each location can be written in terms of its domestic trade share (π_{nn}), population (L_n) and parameters:

$$\frac{v_k}{P_k^{\alpha} r_k^{1-\alpha}} = \frac{\frac{w_k}{\alpha}}{\left\{ \frac{A_k w_k^{-\theta}}{\pi_{kk}} * \left\{ \Gamma\left(\frac{\theta+1-\sigma}{\theta} \right) \right\}^{\frac{-\theta}{1-\sigma}} \right\}^{-\frac{\alpha}{\theta}} * \left(\frac{(1-\alpha)}{\alpha} \frac{w_k L_k}{H_k} \right)^{1-\alpha}}$$

$$= \frac{\frac{w_k}{\alpha}}{\left(\frac{A_k}{\pi_{kk}} \right)^{-\frac{\alpha}{\theta}} w_k^{\alpha} * \left\{ \Gamma\left(\frac{\theta+1-\sigma}{\theta} \right) \right\}^{\frac{\alpha}{1-\sigma}} * \left(\frac{(1-\alpha)}{\alpha} \right)^{1-\alpha} \left(\frac{w_k L_k}{H_k} \right)^{1-\alpha}}$$

$$= \frac{\frac{1}{\alpha}}{\left(\frac{A_k}{\pi_{kk}}\right)^{-\frac{\alpha}{\theta}} * \left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{\alpha}{1-\sigma}} * \left(\frac{(1-\alpha)}{\alpha}\right)^{1-\alpha} \left(\frac{L_k}{H_k}\right)^{1-\alpha}}$$

$$= \frac{\left(\frac{A_k}{\pi_{kk}}\right)^{\frac{\alpha}{\theta}} \left(\frac{L_k}{H_k}\right)^{-1+\alpha}}{\left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{\alpha}{1-\sigma}} * \alpha \left(\frac{(1-\alpha)}{\alpha}\right)^{1-\alpha}}$$

$$\frac{v_k}{P_k^{\alpha} r_k^{1-\alpha}} = \frac{\left(\frac{A_k}{\pi_{kk}}\right)^{\frac{\alpha}{\theta}} \left(\frac{L_k}{H_k}\right)^{-1+\alpha}}{\alpha\left(\frac{(1-\alpha)}{\alpha}\right)^{1-\alpha} \left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{\alpha}{1-\sigma}}} \tag{49}$$

Combining (48) and (49), the common change in welfare between the two equilibria can be expressed as the weighted average of the changes in domestic trade shares and populations of each location:

$$\hat{\bar{U}} = \frac{\bar{U}'}{\bar{U}} = \left(\sum_{n \in N} \frac{L_n}{\bar{L}} * \hat{B}_k \left(\frac{\hat{v}_k}{\hat{P}_k^{\alpha} \hat{r}_k^{1-\alpha}}\right)^{\epsilon}\right)^{\frac{1}{\epsilon}}$$

$$\frac{\hat{v}_k}{\hat{P}_k^{\alpha} \hat{r}_k^{1-\alpha}} = \frac{\frac{v_k'}{P'^{\alpha}_k r'^{1-\alpha}_k}}{\frac{v_k}{P_k^{\alpha} r_k^{1-\alpha}}} = \frac{\dfrac{\left(\frac{A_k'}{\pi'_{kk}}\right)^{\frac{\alpha}{\theta}} \left(\frac{L_k'}{H_k'}\right)^{-1+\alpha}}{\alpha\left(\frac{(1-\alpha)}{\alpha}\right)^{1-\alpha} \left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{\alpha}{1-\sigma}}}}{\dfrac{\left(\frac{A_k}{\pi_{kk}}\right)^{\frac{\alpha}{\theta}} \left(\frac{L_k}{H_k}\right)^{-1+\alpha}}{\alpha\left(\frac{(1-\alpha)}{\alpha}\right)^{1-\alpha} \left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{\alpha}{1-\sigma}}}}$$

$$= \frac{\left(\frac{A_k'}{\pi'_{kk}}\right)^{\frac{\alpha}{\theta}} \left(\frac{L_k'}{H_k'}\right)^{-1+\alpha}}{\left(\frac{A_k}{\pi_{kk}}\right)^{\frac{\alpha}{\theta}} \left(\frac{L_k}{H_k}\right)^{-1+\alpha}} = \left(\frac{\hat{A}_k}{\hat{\pi}_{kk}}\right)^{\frac{\alpha}{\theta}} \left(\frac{\hat{L}_k}{\hat{H}_k}\right)^{-1+\alpha}$$

where the scale parameter A_i determines average productivity for location i.

$$\hat{\bar{U}} = \frac{\bar{U}'}{\bar{U}} = \left(\sum_{n \in N} \frac{L_n}{\bar{L}} * \hat{B}_k \left(\frac{\hat{v}_k}{\hat{P}_k^{\alpha} \hat{r}_k^{1-\alpha}}\right)^{\epsilon}\right)^{\frac{1}{\epsilon}}$$

$$= \left(\sum_{n \in N} \frac{L_n}{\bar{L}} * \hat{B}_k \left[\left(\frac{\hat{A}_k}{\hat{\pi}_{kk}}\right)^{\frac{\alpha}{\theta}} \left(\frac{\hat{L}_k}{\hat{H}_k}\right)^{-1+\alpha}\right]^{\epsilon}\right)^{\frac{1}{\epsilon}}$$

$$= \left(\sum_{n \in N} \frac{L_n}{\bar{L}} * \hat{B}_n \left[\left(\frac{\hat{A}_n}{\hat{\pi}_{nn}} \right)^{\frac{\alpha}{\theta}} \left(\frac{\hat{L}_n}{\hat{H}_n} \right)^{-1+\alpha} \right]^{\epsilon} \right)^{\frac{1}{\epsilon}} \tag{50}$$

While this expression features the changes in the domestic trade shares and populations of all locations, we now show that welfare also can be expressed in terms of the characteristics of any one individual location. From expected utility (13), $\bar{U} = \Gamma\left(\frac{\epsilon-1}{\epsilon}\right) * \left(\sum_{k \in N} B_k \left(\frac{v_k}{P_k^{\alpha} r_k^{1-\alpha}}\right)^{\epsilon}\right)^{\frac{1}{\epsilon}}$, the residential choice probabilities (12), $\frac{L_n}{\bar{L}} = \frac{B_n \left(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}}\right)^{\epsilon}}{\sum_{k \in N} B_k \left(\frac{v_k}{P_k^{\alpha} r_k^{1-\alpha}}\right)^{\epsilon}}$, and real income (49), $\frac{v_k}{P_k^{\alpha} r_k^{1-\alpha}} = \frac{\left(\frac{A_k}{\pi_{kk}}\right)^{\frac{\alpha}{\theta}} \left(\frac{L_k}{H_k}\right)^{-1+\alpha}}{\alpha\left(\frac{(1-\alpha)}{\alpha}\right)^{1-\alpha} \left\{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)\right\}^{\frac{\alpha}{1-\sigma}}}$, the common level of utility across locations can be expressed as:

$$\bar{U}_n = \bar{U} = \Gamma\left(\frac{\epsilon-1}{\epsilon}\right) * \left(\sum_{k \in N} B_k \left(\frac{v_k}{P_k^{\alpha} r_k^{1-\alpha}}\right)^{\epsilon}\right)^{\frac{1}{\epsilon}}$$

$$= \Gamma\left(\frac{\epsilon-1}{\epsilon}\right) * \left(\frac{B_n \left(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}}\right)^{\epsilon}}{\frac{L_n}{\bar{L}}}\right)^{\frac{1}{\epsilon}}$$

$$= \Gamma\left(\frac{\epsilon-1}{\epsilon}\right) * \left(\frac{\bar{L}}{L_n} B_n \left(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}}\right)^{\epsilon}\right)^{\frac{1}{\epsilon}}$$

$$= \Gamma\left(\frac{\epsilon-1}{\epsilon}\right) * \left(\frac{\bar{L}}{L_n} B_n \left(\frac{\left(\frac{A_n}{\pi_{nn}}\right)^{\frac{\alpha}{\theta}} \left(\frac{L_n}{H_n}\right)^{-1+\alpha}}{\alpha\left(\frac{(1-\alpha)}{\alpha}\right)^{1-\alpha} \left\{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)\right\}^{\frac{\alpha}{1-\sigma}}}\right)^{\epsilon}\right)^{\frac{1}{\epsilon}}$$

$$= \Gamma\left(\frac{\epsilon-1}{\epsilon}\right) * \left(\frac{\bar{L}}{L_n} B_n \left(\frac{\left(\frac{A_n}{\pi_{nn}}\right)^{\frac{\alpha}{\theta}} \left(\frac{L_n}{H_n}\right)^{-1+\alpha}}{\alpha\left(\frac{(1-\alpha)}{\alpha}\right)^{1-\alpha} \left\{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)\right\}^{\frac{\alpha}{1-\sigma}}}\right)^{\epsilon}\right)^{\frac{1}{\epsilon}}$$

$$= \delta * \left(\frac{\bar{L}}{L_n} B_n \left(\frac{\left(\frac{A_n}{\pi_{nn}}\right)^{\frac{\alpha}{\theta}} \left(\frac{L_n}{H_n}\right)^{-1+\alpha}}{\alpha\left(\frac{(1-\alpha)}{\alpha}\right)^{1-\alpha} \gamma^{\alpha}}\right)^{\epsilon}\right)^{\frac{1}{\epsilon}}$$

$$= \delta * \frac{\bar{L}^{\frac{1}{\epsilon}}}{L_n^{\frac{1}{\epsilon}}} B_n^{\frac{1}{\epsilon}} \frac{\left(\frac{A_n}{\pi_{nn}}\right)^{\frac{\alpha}{\theta}} \left(\frac{L_n}{H_n}\right)^{-1+\alpha}}{\alpha\left(\frac{(1-\alpha)}{\alpha}\right)^{1-\alpha} \gamma^{\alpha}}$$

$$= \frac{\delta B_n^{\frac{1}{\epsilon}} \left(\frac{A_n}{\pi_{nn}}\right)^{\frac{\alpha}{\theta}} (H_n)^{1-\alpha} (L_n)^{-1-\frac{1}{\epsilon}+\alpha}}{\alpha\left(\frac{(1-\alpha)}{\alpha}\right)^{1-\alpha} \gamma^{\alpha} \bar{L}^{-\frac{1}{\epsilon}}}, \qquad \forall\, n \tag{51}$$

where $\delta \equiv \Gamma\left(\frac{\epsilon-1}{\epsilon}\right)$, $\gamma \equiv \left\{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)\right\}^{\frac{1}{1-\sigma}}$.

Population mobility implies that this relationship must hold for each location. Locations with higher productivity (A_n), better amenities (B_n), better goods market access to other locations (lower π_{nn}) and higher supplies of land (H_n) have higher populations, which bids up the price of land until expected utility conditional on living in each location is the same for all locations.

An implication of this result is that the domestic trade share in the open economy equilibrium (π_{nn}^T), populations in the closed and open economies (L_n^A and L_n^T), the trade elasticity (θ), the elasticity of labor supply with respect to real income (ϵ) and the consumption goods share (α) are sufficient statistics for the welfare gains from trade:

$$\frac{\bar{U}_n^T}{\bar{U}_n^A} = \frac{\bar{U}^T}{\bar{U}^A} = \frac{\dfrac{\delta (B_n^T)^{\frac{1}{\epsilon}} \left(\frac{A_n^T}{\pi_{nn}^T}\right)^{\frac{\alpha}{\theta}} \left(H_n^T\right)^{1-\alpha} \left(L_n^T\right)^{-1-\frac{1}{\epsilon}+\alpha}}{\alpha\left(\frac{(1-\alpha)}{\alpha}\right)^{1-\alpha} \gamma^{\alpha} \bar{L}^{-\frac{1}{\epsilon}}}}{\dfrac{\delta (B_n^A)^{\frac{1}{\epsilon}} \left(\frac{A_n^A}{\pi_{nn}^A}\right)^{\frac{\alpha}{\theta}} \left(H_n^A\right)^{1-\alpha} \left(L_n^A\right)^{-1-\frac{1}{\epsilon}+\alpha}}{\alpha\left(\frac{(1-\alpha)}{\alpha}\right)^{1-\alpha} \gamma^{\alpha} \bar{L}^{-\frac{1}{\epsilon}}}}$$

$$= \frac{\delta (B_n^T)^{\frac{1}{\epsilon}} \left(\frac{A_n^T}{\pi_{nn}^T}\right)^{\frac{\alpha}{\theta}} \left(H_n^T\right)^{1-\alpha} \left(L_n^T\right)^{-1-\frac{1}{\epsilon}+\alpha}}{\delta (B_n^A)^{\frac{1}{\epsilon}} \left(\frac{A_n^A}{\pi_{nn}^A}\right)^{\frac{\alpha}{\theta}} \left(H_n^A\right)^{1-\alpha} \left(L_n^A\right)^{-1-\frac{1}{\epsilon}+\alpha}}$$

$$= \left(\frac{B_n^T}{B_n^A}\right)^{\frac{1}{\epsilon}} \left(\frac{A_n^T}{A_n^A}\right)^{\frac{\alpha}{\theta}} \left(\frac{\pi_{nn}^A}{\pi_{nn}^T}\right)^{\frac{\alpha}{\theta}} \left(\frac{H_n^T}{H_n^A}\right)^{1-\alpha} \left(\frac{L_n^A}{L_n^T}\right)^{\frac{1}{\epsilon}+1-\alpha}$$

$$= \left(\frac{1}{\pi_{nn}^T}\right)^{\frac{\alpha}{\theta}} \left(\frac{L_n^A}{L_n^T}\right)^{\frac{1}{\epsilon}+1-\alpha} \qquad \forall\, n \tag{52}$$

where we use the superscript T to denote the trade equilibrium and the superscript A to denote the autarky equilibrium; we have used $\pi_{nn}^A = 1$; and in general, $L_n^A \neq L_n^T$.

Intuitively, if some locations have better market access than others in the open economy (as reflected in a lower open economy domestic trade share π_{nn}^{T}), the opening of goods trade will lead to a larger reduction in the consumption price index in these locations. This larger reduction in the consumption price index in turn creates an incentive for migration from locations with worse market access to those with better market access. This labor mobility provides the mechanism that restores equilibrium, as the price of land is bid up in locations with better market access and bid down in those with worse market access, until expected utility is equalized across all locations. Therefore, computing the common value for the welfare gains from trade across all locations involves taking into account not only domestic trade shares (which affect consumption price indices) but also population redistributions (which affect the price of the immobile factor land).

Although labor mobility ensures the equalization of expected utility across all locations, real income is not equalized, because of the heterogeneity in workers' preferences for locations. Each location faces an upward sloping supply curve for workers, as higher real income has to be paid to attract workers with lower realizations for idiosyncratic tastes for that location. Only in the special case of no idiosyncratic heterogeneity in worker tastes ($\epsilon \to \infty$) is real income equalized across locations. In this special case, expected utility is given by:

$$\overline{U}_n = \overline{U} = \lim_{\epsilon \to \infty} \frac{\delta B_n^{\frac{1}{\epsilon}} \left(\frac{A_n}{\pi_{nn}}\right)^{\frac{\alpha}{\theta}} (H_n)^{1-\alpha} (L_n)^{-1-\frac{1}{\epsilon}+\alpha}}{\alpha \left(\frac{(1-\alpha)}{\alpha}\right)^{1-\alpha} \gamma^{\alpha} \overline{L}^{-\frac{1}{\epsilon}}}$$

$$= \frac{\delta * 1 * \left(\frac{A_n}{\pi_{nn}}\right)^{\frac{\alpha}{\theta}} (H_n)^{1-\alpha} (L_n)^{-1-0+\alpha}}{\alpha \left(\frac{(1-\alpha)}{\alpha}\right)^{1-\alpha} \gamma^{\alpha} \overline{L}^{-0}}$$

$$= \frac{\delta * \left(\frac{A_n}{\pi_{nn}}\right)^{\frac{\alpha}{\theta}} (H_n)^{1-\alpha} (L_n)^{-1+\alpha}}{\alpha \left(\frac{(1-\alpha)}{\alpha}\right)^{1-\alpha} \gamma^{\alpha}} \qquad \forall\, n \tag{53}$$

and the welfare gains from trade are:

$$\frac{\overline{U}_n^{T}}{\overline{U}_n^{A}} = \frac{\overline{U}^{T}}{\overline{U}^{A}} = \lim_{\epsilon \to \infty} \left(\frac{1}{\pi_{nn}^{T}}\right)^{\frac{\alpha}{\theta}} \left(\frac{L_n^{A}}{L_n^{T}}\right)^{\frac{1}{\epsilon}+1-\alpha}$$

$$= \left(\frac{1}{\pi_{nn}^{T}}\right)^{\frac{\alpha}{\theta}} \left(\frac{L_n^{A}}{L_n^{T}}\right)^{1-\alpha} \qquad \forall\, n \tag{54}$$

In another special case of perfect labor immobility, expected utility takes the same form as in (53), except that expected utility in general differs across locations:

$$\overline{U}_n = \frac{\delta * \left(\frac{A_n}{\pi_{nn}}\right)^{\frac{\alpha}{\theta}} (H_n)^{1-\alpha} (L_n)^{-1+\alpha}}{\alpha \left(\frac{(1-\alpha)}{\alpha}\right)^{1-\alpha} \gamma^{\alpha}} \neq \overline{U}_k, \qquad n \neq k \tag{55}$$

Similarly, the welfare gains from trade in general differ across locations under perfect labor immobility:

$$\frac{\overline{U}_n^T}{\overline{U}_n^A} = \left(\frac{1}{\pi_{nn}^T}\right)^{\frac{\alpha}{\theta}} \neq \frac{\overline{U}_k^T}{\overline{U}_k^A}, \qquad n \neq k \tag{56}$$

which corresponds to the limiting case of (52), in which $L_n^T = L_n^A$ because of labor immobility. Intuitively, in this limiting case, locations with better access to markets in the open economy experience larger welfare gains from trade, because labor mobility no longer provides a mechanism for utility equalization through changes in the price of the immobile factor land.

3 Agglomeration Forces

In this section, we examine the implications of introducing agglomeration forces in our setting with both trade costs and labor mobility with heterogeneous worker preferences. These agglomeration forces take the form of pecuniary externalities as a result of transport costs, increasing returns to scale and love of variety, as in the new economic geography literature following Krugman (1991), Krugman and Venables (1995) and Helpman (1998), and synthesized in Fujita, Krugman, and Venables (1999). This literature typically restricts attention to stylized settings with a small number of symmetric locations and assumes either perfect labor mobility, perfect labor immobility or a mechanical relationship between migration flows and relative wages. In contrast, we consider a rich geography with a large number of asymmetric locations, and allow for a positive finite elasticity of labor supply to each location.

3.1 Consumer Preferences

Preferences are again defined over goods consumption (C_n) and residential land use (H_{Un}) and take the same form as in (1), $U_n(\omega) = b_n(\omega)\left(\frac{C_n(\omega)}{\alpha}\right)^{\alpha}\left(\frac{H_{Un}(\omega)}{1-\alpha}\right)^{1-\alpha}$. The goods consumption index (C_n), however, is now defined over the endogenous measures of horizontally differentiated varieties supplied by each location (M_i):

$$C_n = \left[\sum_{i\in N}\int_0^{M_i} c_{ni}(j)^{\rho}\, dj\right]^{\frac{1}{\rho}} \tag{57}$$

where trade between locations i and n is again subject to iceberg variable trade costs of $d_{ni} \geq 1$.

The Lagrangian of a consumer is

$$\mathcal{L} = b_n(\omega)\left(\frac{C_n(\omega)}{\alpha}\right)^{\alpha}\left(\frac{H_{Un}(\omega)}{1-\alpha}\right)^{1-\alpha} - \lambda\left(\sum_{i\in N}\int_0^{M_i} p_{ni}(j)c_{ni}(j)\, dj + r_n H_{Un} - I\right)$$

$$= b_n(\omega)\left(\frac{\left[\sum_{i\in N}\int_0^{M_i} c_{ni}(j)^{\rho}\, dj\right]^{\frac{1}{\rho}}}{\alpha}\right)^{\alpha}\left(\frac{H_{Un}(\omega)}{1-\alpha}\right)^{1-\alpha} - \lambda\left(\sum_{i\in N}\int_0^{M_i} p_{ni}(j)c_{ni}(j)\, dj + r_n H_{Un} - I\right)$$

$$\frac{\partial\mathcal{L}}{\partial c_{ni}(j)} = b_n(\omega)\alpha\left(\frac{\left[\sum_{i\in N}\int_0^{M_i} c_{ni}(j)^{\rho}\, dj\right]^{\frac{1}{\rho}}}{\alpha}\right)^{\alpha-1}\frac{\frac{1}{\rho}\left[\sum_{i\in N}\int_0^{M_i} c_{ni}(j)^{\rho}\, dj\right]^{\frac{1}{\rho}-1}}{\alpha}\rho c_{ni}(j)^{\rho-1}\left(\frac{H_{Un}(\omega)}{1-\alpha}\right)^{1-\alpha} - \lambda p_{ni}(j)$$

$$= b_n(\omega)\left(\frac{\left[\sum_{i\in N}\int_0^{M_i} c_{ni}(j)^{\rho}\, dj\right]^{\frac{1}{\rho}}}{\alpha}\right)^{\alpha-1}\left[\sum_{i\in N}\int_0^{M_i} c_{ni}(j)^{\rho}\, dj\right]^{\frac{1}{\rho}-1} c_{ni}(j)^{\rho-1}\left(\frac{H_{Un}(\omega)}{1-\alpha}\right)^{1-\alpha} - \lambda p_{ni}(j)$$

$$= b_n(\omega)\left(\frac{\left[\sum_{i\in N}\int_0^{M_i} c_{ni}(j)^{\rho}\, dj\right]^{\frac{1}{\rho}}}{\alpha}\right)^{\alpha}\left(\frac{H_{Un}(\omega)}{1-\alpha}\right)^{1-\alpha}\frac{\alpha}{\left[\sum_{i\in N}\int_0^{M_i} c_{ni}(j)^{\rho}\, dj\right]^{\frac{1}{\rho}}}\frac{\left[\sum_{i\in N}\int_0^{M_i} c_{ni}(j)^{\rho}\, dj\right]^{\frac{1}{\rho}}}{\sum_{i\in N}\int_0^{M_i} c_{ni}(j)^{\rho}\, dj} c_{ni}(j)^{\rho-1} - \lambda p_{ni}(j)$$

$$= b_n(\omega)\left(\frac{\left[\sum_{i\in N}\int_0^{M_i} c_{ni}(j)^{\rho}\, dj\right]^{\frac{1}{\rho}}}{\alpha}\right)^{\alpha}\left(\frac{H_{Un}(\omega)}{1-\alpha}\right)^{1-\alpha}\frac{\alpha c_{ni}(j)^{\rho-1}}{\sum_{i\in N}\int_0^{M_i} c_{ni}(j)^{\rho}\, dj} - \lambda p_{ni}(j) = 0$$

$$\frac{b_n(\omega)\left(\frac{\left[\sum_{i\in N}\int_0^{M_i} c_{ni}(j)^\rho\,dj\right]^{\frac{1}{\rho}}}{\alpha}\right)^{\alpha}\left(\frac{H_{Un}(\omega)}{1-\alpha}\right)^{1-\alpha}\frac{\alpha c_{ni}(j_1)^{\rho-1}}{\sum_{i\in N}\int_0^{M_i} c_{ni}(j)^\rho\,dj}}{b_n(\omega)\left(\frac{\left[\sum_{i\in N}\int_0^{M_i} c_{ni}(j)^\rho\,dj\right]^{\frac{1}{\rho}}}{\alpha}\right)^{\alpha}\left(\frac{H_{Un}(\omega)}{1-\alpha}\right)^{1-\alpha}\frac{\alpha c_{ni}(j_2)^{\rho-1}}{\sum_{i\in N}\int_0^{M_i} c_{ni}(j)^\rho\,dj}}=\frac{\lambda p_{ni}(j_1)}{\lambda p_{ni}(j_2)}$$

$$\frac{c_{ni}(j_1)^{\rho-1}}{c_{ni}(j_2)^{\rho-1}}=\frac{p_{ni}(j_1)}{p_{ni}(j_2)}$$

$$\frac{c_{ni}(j_1)}{c_{ni}(j_2)}=\left(\frac{p_{ni}(j_1)}{p_{ni}(j_2)}\right)^{\frac{1}{\rho-1}}$$

From the above equation, the elasticity of substitution is the constant

$$\sigma=\frac{-d\ln\frac{c_{ni}(j_1)}{c_{ni}(j_2)}}{d\ln\frac{p_{ni}(j_1)}{p_{ni}(j_2)}}=\frac{-\frac{1}{\rho-1}d\ln\frac{p_{ni}(j_1)}{p_{ni}(j_2)}}{d\ln\frac{p_{ni}(j_1)}{p_{ni}(j_2)}}=\frac{1}{1-\rho}$$

Using σ and multiplying both sides by $c_{ni}(j_2)$ yields:

$$c_{ni}(j_1)=c_{ni}(j_2)\left(\frac{p_{ni}(j_1)}{p_{ni}(j_2)}\right)^{-\sigma}$$

Now multiply both sides by $p_{ni}(j_1)$ and take the integral with respect to j_1,

$$\int_0^{M_i} p_{ni}(j_1)c_{ni}(j_1)\,dj_1=\int_0^{M_i} p_{ni}(j_1)c_{ni}(j_2)\left(\frac{p_{ni}(j_1)}{p_{ni}(j_2)}\right)^{-\sigma}dj_1=c_{ni}(j_2)p_{ni}(j_2)^{\sigma}\int_0^{M_i} p_{ni}(j_1)^{1-\sigma}\,dj_1=\alpha I$$

$$c_{ni}(j_2)=\frac{\alpha I * p_{ni}(j_2)^{-\sigma}}{\int_0^{M_i} p_{ni}(j_1)^{1-\sigma}\,dj_1}$$

Define an index of all varieties' prices to be $P_n=\left(\int_0^{M_i} p_{ni}(j)^{1-\sigma}\,dj\right)^{\frac{1}{1-\sigma}}$, then

$$c_{ni}(j)=p_{ni}(j)^{-\sigma}{P_n}^{\sigma-1}\alpha I=\left(\frac{p_{ni}(j)}{P_n}\right)^{-\sigma}\frac{\alpha I}{P_n}$$

$$\frac{\partial c_{ni}(j)}{\partial p_{ni}(j)}=-\sigma\left(p_{ni}(j)\right)^{-\sigma-1}{P_n}^{\sigma-1}\alpha I$$

3.2 Production

Varieties are produced under conditions of monopolistic competition and increasing returns to scale. To produce a variety, a firm must incur a fixed cost of F units of labor and a constant variable cost in terms of labor that depends on a location's productivity A_i. Therefore, the total amount of labor ($l_i(j)$) required to produce $x_i(j)$ units of a variety j in country i is:

$$l_i(j)=F+\frac{x_i(j)}{A_i} \tag{58}$$

Profit maximization and zero profits imply that equilibrium prices are a constant mark-up over marginal cost:

$$\pi_i(j)=p_{ni}(j)x_i(j)-d_{ni}w_i l_i(j)$$
$$=p_{ni}(j)x_i(j)-d_{ni}w_i[F+\frac{x_i(j)}{A_i}]$$

$$= p_{ni}(j)x_i(j) - d_{ni}w_iF - d_{ni}w_i\frac{x_i(j)}{A_i}$$
$$\frac{\partial\pi_i(j)}{\partial p_{ni}(j)} = x_i(j) + [p_{ni}(j) - \frac{d_{ni}w_i}{A_i}]\frac{\partial x_i(j)}{\partial p_{ni}(j)} = 0$$
$$p_{ni}(j) = \frac{d_{ni}w_i}{A_i} + \frac{-x_i(j)}{\frac{\partial x_i(j)}{\partial p_{ni}(j)}}$$
$$= \frac{d_{ni}w_i}{A_i} + \frac{-p_{ni}(j)^{-\sigma}{P_n}^{\sigma-1}\alpha I}{-\sigma\left(p_{ni}(j)\right)^{-\sigma-1}{P_n}^{\sigma-1}\alpha I}$$
$$= \frac{d_{ni}w_i}{A_i} + \frac{p_{ni}(j)}{\sigma}$$
$$\frac{\sigma-1}{\sigma}p_{ni}(j) = \frac{d_{ni}w_i}{A_i}$$
$$p_{ni}(j) = \left(\frac{\sigma}{\sigma-1}\right)\frac{d_{ni}w_i}{A_i} \tag{59}$$

and equilibrium employment for each variety is equal to a constant:

$$\left(\frac{\sigma}{\sigma-1}\right)\frac{d_{ni}w_i}{A_i}x_i(j) - d_{ni}w_i\left[F + \frac{x_i(j)}{A_i}\right] = 0$$
$$\left(\frac{\sigma}{\sigma-1}\right)\frac{d_{ni}w_i}{A_i}x_i(j) - d_{ni}w_iF - \frac{d_{ni}w_i}{A_i}x_i(j) = 0$$
$$\left(\frac{1}{\sigma-1}\right)\frac{d_{ni}w_i}{A_i}x_i(j) = d_{ni}w_iF$$
$$\left(\frac{1}{\sigma-1}\right)\frac{x_i(j)}{A_i} = F$$
$$\left(\frac{1}{\sigma-1}\right)[l_i(j) - F] = F$$
$$l_i(j) - F = (\sigma-1)F$$
$$l_i(j) = \bar{l} = \sigma F \tag{60}$$

Given this constant equilibrium employment for each variety, labor market clearing implies that the total measure of varieties supplied by each location is proportional to the endogenous supply of workers choosing to locate there:

$$\int_0^{M_i} l_i(j)\,dj = L_i$$
$$\int_0^{M_i} \sigma F\,dj = L_i$$
$$M_i * \sigma F = L_i$$
$$M_i = \frac{L_i}{\sigma F} \tag{61}$$

3.3 Expenditure Shares and Price Indices

7 Commercial Land and Intermediate Inputs

In this section, we consider an extension of the model to allow land to be used commercially and to incorporate intermediate inputs in production. We develop this extension in the context of our baseline constant returns model from Section 2, but it is straightforward to instead consider our increasing returns model from Section 3 or our model with a distinction between regions and countries from Section 5.

7.1 Preferences, Endowments and Technology

Preferences are again defined over goods consumption (C_n) and residential land use (H_n) and take the same form as in (1). The goods consumption index (C_n) is defined over consumption of a fixed continuum of goods $j \in [0,1]$ as in (2). Each region draws an idiosyncratic productivity z_j for each good j as in (5) and goods are again homogeneous. Goods are produced with labor, land and intermediate inputs under conditions of perfect competition according to a Cobb-Douglas production technology. Each good uses all other goods as intermediate inputs with the same CES aggregator as for consumer preferences, as in Krugman and Venables (1995) and Eaton and Kortum (2002). The cost to a consumer in region n of purchasing one unit of good j from region i is:

$$p_{ni}(j) = \frac{d_{ni} {w_i}^{\beta} {r_i}^{\eta} {P_i}^{1-\beta-\eta}}{z_i(j)}, \quad 0 < \beta, \eta < 1, 0 < \beta + \eta < 1, \tag{99}$$

where P_i is the dual price index.

7.2 Expenditure Shares and Price Indices

The representative consumer in region n sources each good from the lowest cost supplier to that region. Using equilibrium prices (99) and the properties of the Fréchet distribution, the share of expenditure of region n on goods produced by region i is:

$$\pi_{ni} = \frac{A_i (d_{ni} {w_i}^{\beta} {r_i}^{\eta} {P_i}^{1-\beta-\eta})^{-\theta}}{\sum_{k \in N} A_k (d_{nk} {w_k}^{\beta} {r_k}^{\eta} {P_k}^{1-\beta-\eta})^{-\theta}} \tag{100}$$

while the price index for tradeable goods is:

$$P_n = \left\{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)\right\}^{\frac{1}{1-\sigma}} * \left[\sum_{k \in N} A_k \left(d_{nk} {w_k}^{\beta} {r_k}^{\eta} {P_k}^{1-\beta-\eta}\right)^{-\theta}\right]^{-\frac{1}{\theta}}, \quad \theta + 1 - \sigma > 0 \tag{101}$$

7.3 Residential Choices and Income

Residential choices take a similar form as in section 2. Using the Fréchet distribution of idiosyncratic shocks to amenities, the probability that a worker chooses to live in location $n \in N$ is:

$$\frac{L_n}{\bar{L}} = \frac{B_n \left(\frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}}\right)^{\epsilon}}{\sum_{k \in N} B_k \left(\frac{v_k}{P_k^{\alpha} r_k^{1-\alpha}}\right)^{\epsilon}} \tag{102}$$

Expected worker utility is:

$$\bar{U} = \Gamma\left(\frac{\epsilon-1}{\epsilon}\right) * \left(\sum_{k \in N} B_k \left(\frac{v_k}{P_k^{\alpha} r_k^{1-\alpha}}\right)^{\epsilon}\right)^{\frac{1}{\epsilon}} \tag{103}$$

Expenditure on land in each location is redistributed lump sum to the workers residing in that location, which implies that total income (v_n) equals labor income plus expenditure on commercial and residential land:

$$v_n L_n = \frac{\beta+\eta}{\alpha\beta} w_n L_n \tag{104}$$

Land market clearing implies that the equilibrium land rent again can be determined from the equality of land income and expenditure:

$$r_n = \frac{(1-\alpha)\beta+\eta}{\alpha\beta} \frac{w_n L_n}{H_n} \tag{105}$$

7.4 General Equilibrium

The general equilibrium of the model can be represented by the measure of workers (L_n), the trade share (π_{ni}), the wage (w_n) and the price index (P_n) for each location $n, i \in N$. Using labor income (104), the trade share (8), the price index (101), residential choice probabilities (102) and land market clearing (105), this equilibrium quadruple $\{L_n, \pi_{ni}, w_n, P_n\}$ solves the following system of equations for all $i, n \in N$.

First, each location's income must equal expenditure on the goods produced in that location:

$$w_i L_i = \sum_{n \in N} \pi_{ni} \, w_n L_n \tag{106}$$

Second, location expenditure shares are:

$$\pi_{ni} = \frac{A_i (d_{ni} w_i{}^{\beta+\eta} P_i{}^{1-\beta-\eta} (\frac{L_i}{H_i})^{\eta})^{-\theta}}{\sum_{k \in N} A_k (d_{nk} w_k{}^{\beta+\eta} P_k{}^{1-\beta-\eta} (\frac{L_k}{H_k})^{\eta})^{-\theta}} \tag{107}$$

Third, price indices are:

$$P_n = \left\{\Gamma(\frac{\theta+1-\sigma}{\theta})\right\}^{\frac{1}{1-\sigma}} * \left[\sum_{k \in N} A_k \left(d_{nk} w_i{}^{\beta+\eta} P_i{}^{1-\beta-\eta} \left(\frac{(1-\alpha)\beta+\eta}{\alpha\beta} \frac{L_i}{H_i}\right)^{\eta}\right)^{-\theta}\right]^{-\frac{1}{\theta}} \tag{108}$$

Fourth, residential choice probabilities imply:

$$\frac{L_n}{\bar{L}} = \frac{B_n \left(P_n^{\alpha} w_n^{\alpha} \left(\frac{L_n}{H_n}\right)^{1-\alpha}\right)^{-\epsilon}}{\sum_{k \in N} B_k \left(P_k^{\alpha} w_k^{\alpha} \left(\frac{L_k}{H_k}\right)^{1-\alpha}\right)^{-\epsilon}} \tag{109}$$

Therefore, the general equilibrium of the model with commercial land and intermediate inputs can be analyzed using an analogous approach as for our baseline model.