（二）理论模型相关推导与证明

Ⅰ. 青年购房者的终生效用最大化问题

首先，构建 Lagrange 函数可得

$$
\begin{aligned}
\mathcal{L} \quad = \quad & \ln C_{y,t} + \theta \ln H_{y,t} + \beta\left(\ln C_{m,t+1} + \theta \ln H_{m,t+2}\right) + \\
& \beta^2\left(\ln C_{o,t+2} + \theta \ln H_{o,t+2} + \xi \ln(1 + B_{t+2})\right) + \\
& \lambda_1\left[w_{y,t} + B_t - C_{y,t} - \psi(1+\phi)P_t^H H_{y,t} - R_t^m s_1 M_t - A_{y,t}\right] + \\
& \lambda_2\left[w_{m,t+1} + R^A A_{y,t} - C_{m,t+1} - R_{t+1}^m (1 - s_1) M_t - A_{m,t+1}\right] + \\
& \lambda_3\left(w_{o,t+2} + R^A A_{m,t+1} + P_{t+2}^H H_{y,t} - C_{o,t+2} - B_{t+2}\right) + \\
& \zeta_1\left[M_t - (1-\psi)P_t^H H_{y,t}\right] + \\
& \zeta_2\left[A_{y,t} + (1-v)\left(w_{y,t} + B_t\right)\right].
\end{aligned}
$$

$$\frac{\partial \mathcal{L}}{\partial C_{y,t}} = \frac{1}{C_{y,t}} - \lambda_1 = 0 \tag{1}$$

$$\frac{\partial \mathcal{L}}{\partial C_{m,t+1}} = \frac{\beta}{C_{m,t+1}} - \lambda_2 = 0 \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial C_{o,t+2}} = \frac{\beta^2}{C_{o,t+2}} - \lambda_3 = 0 \tag{3}$$

$$\frac{\partial \mathcal{L}}{\partial H_{y,t}} = \frac{\theta(1+\beta+\beta^2)}{H_{y,t}} - \lambda_1 \psi(1+\phi)P_t^H + \lambda_3 P_{t+2}^H - \zeta_1(1-\psi)P_t^H = 0 \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial A_{y,t}} = -\lambda_1 + \lambda_2 R^A + \zeta_2 = 0 \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial A_{m,t+1}} = -\lambda_2 + \lambda_3 R^A = 0 \tag{6}$$

$$\frac{\partial \mathcal{L}}{\partial M_t} = -\lambda_1 R_t^m s_1 - \lambda_2 R_{t+1}^m (1 - s_1) + \zeta_1 = 0 \tag{7}$$

$$\frac{\partial \mathcal{L}}{\partial B_{t+2}} = \frac{\beta^2 \xi}{1 + B_{t+2}} - \lambda_3 = 0 \tag{8}$$

由于$\zeta_1 = \lambda_1 R_t^m s_1 + \lambda_2 R_{t+1}^m (1 - s_1) > 0$,因此$M_t = (1-\psi)P_t^H H_{y,t}$。

（Ⅰ）当青年个体受到信贷约束时，即$\zeta_2 > 0$，此时有$A_{y,t} = -(1-v)\left(w_{y,t} + B_t\right)$，青年时期消费为

$$
\begin{aligned}
C_{y,t}(H_{y,t}) &= w_{y,t} + B_t - \psi(1+\phi)P_t^H H_{y,t} - R_t^m s_1 M_t - A_{y,t} \\
&= (2 - v)\left(w_{y,t} + B_t\right) - \psi(1+\phi)P_t^H H_{y,t} - R_t^m s_1 (1-\psi) P_t^H H_{y,t}.
\end{aligned} \tag{9}
$$

联立式(1)-(9)，可得

$$\frac{\theta(1+\beta+\beta^2)C_{y,t}}{1}-\psi(1+\phi)P_t^H H_{y,t}+\frac{\beta^2 C_{y,t}}{C_{o,t+2}}P_{t+2}^H H_{y,t}$$
$$=\left(R_t^m s_1+\frac{\beta^2 C_{y,t}}{C_{o,t+2}}R^A R_{t+1}^m(1-s_1)\right)(1-\psi)P_t^H H_{y,t},$$

$$\frac{\theta(1+\beta+\beta^2)C_{y,t}}{1}-\psi(1+\phi)P_t^H H_{y,t}-R_t^m s_1(1-\psi)P_t^H H_{y,t}$$
$$=\frac{\beta^2 C_{y,t}}{C_{o,t+2}}R^A R_{t+1}^m(1-s_1)(1-\psi)P_t^H H_{y,t}-\frac{\beta^2 C_{y,t}}{C_{o,t+2}}P_{t+2}^H H_{y,t},$$

$$C_{o,t+2}(H_{y,t})=\frac{\beta^2[P_{t+2}^H H_{y,t}-R^A R_{t+1}^m(1-s_1)(1-\psi)P_t^H H_{y,t}]C_{y,t}(H_{y,t})}{\psi(1+\phi)P_t^H H_{y,t}-\theta(1+\beta+\beta^2)C_{y,t}(H_{y,t})+R_t^m s_1(1-\psi)P_t^H H_{y,t}}. \quad (10)$$

将式(9)和(10)代入其终生预算约束中，可求得其最优住房面积

$$w_{m,t+1}-R^A(1-v)\left(w_{y,t}+B_t\right)-\frac{C_{o,t+2}(H_{y,t})}{\beta R^A}-R_{t+1}^m(1-s_1)(1-\psi)P_t^H H_{y,t}+$$

$$\frac{w_{o,t+2}}{R^A}+\frac{P_{t+2}^H H_{o,t+2}}{R^A}-\frac{C_{o,t+2}(H_{y,t})}{R^A}-\frac{\xi C_{o,t+2}(H_{y,t})-1}{R^A}=0,$$

$$\frac{(1+\beta+\beta\xi)C_{o,t+2}(H_{y,t})}{\beta R^A}$$
$$=w_{m,t+1}-R^A(1-v)\left(w_{y,t}+B_t\right)-R_{t+1}^m(1-s_1)(1-\psi)P_t^H H_{y,t}$$
$$+\frac{w_{o,t+2}+1}{R^A}+\frac{P_{t+2}^H H_{o,t+2}}{R^A},$$

$$H_{y,t}^*=\frac{w_{m,t+1}-R^A(1-v)\left(w_{y,t}+B_t\right)+\frac{w_{o,t+2}}{R^A}+\frac{1}{R^A}-\frac{(1+\beta+\beta\xi)C_{o,t+2}\left(H_{y,t}^*\right)}{\beta R^A}}{R_{t+1}^m(1-s_1)(1-\psi)P_t^H-P_{t+2}^H/R^A}.$$

通过求解上述关于$H_{y,t}^*$的函数，我们可以求得最优房产面积$H_{y,t}^*$，进而得到各个生命阶段的最优消费水平$C_{y,t}(H_{y,t}^*)$，$C_{o,t+2}\left(H_{y,t}^*\right)$，$C_{m,t+1}\left(H_{y,t}^*\right)=\frac{C_{o,t+2}\left(H_{y,t}^*\right)}{\beta R^A}$，最优遗产$B_{t+2}^*=\xi C_{o,t+2}\left(H_{y,t}^*\right)-1\geq 0$，以及中年时期金融资产配置情况

$$A_{m,t+1}\left(H_{y,t}^*\right)=w_{m,t+1}-R^A(1-v)\left(w_{y,t}+B_t\right)-$$
$$C_{m,t+1}\left(H_{y,t}^*\right)-R_{t+1}^m(1-s_1)(1-\psi)P_t^H H_{y,t}^*.$$

基于上述结果，我们可以推出受信贷约束下的青年购房者的最优福利水平为

$$U_{y,t}^{c*}=\ln C_{y,t}^*+\theta\ln H_{y,t}^*+\beta\left(\ln C_{m,t+1}^*+\theta\ln H_{y,t}^*\right)$$
$$+\beta^2\left(\ln C_{o,t+2}^*+\theta\ln H_{y,t}^*+\xi\ln(1+B_{t+2}^*)\right).$$

（II）当青年个体不受信贷约束时，即$\zeta_2=0$。联立式(1)-(3)和(5)-(6)有

$$C_{m,t+1} = \beta R^A C_{y,t}, \quad (11)$$
$$C_{o,t+2} = (\beta R^A)^2 C_{y,t}. \quad (12)$$

将上两式代入式(4)可得

$$C_{y,t} = \frac{\left\{\psi(1+\phi)+[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)](1-\psi)\right\}P_t^H-\frac{P_{t+2}^H}{(R^A)^2}}{\theta(1+\beta+\beta^2)}H_{y,t}. \quad (13)$$

将式(11)-(13)代入青年个体的终生预算约束，可得

$$H_{y,t}^{**} = \frac{w_{y,t}+\frac{w_{m,t+1}}{R^A}+\frac{w_{o,t+2}}{\left(R^A\right)^2}+B_t+\frac{1}{\left(R^A\right)^2}}{\left[1+\frac{1+\beta+(1+\xi)\beta^2}{\theta(1+\beta+\beta^2)}\right]\left\{\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}\right\}},$$

由此可知，当未来房价（预期）P_{t+2}^H上升时，家庭所配置的房产价值$P_t^H H_{y,t}^{**}$会增加，反之则会下降；当房贷利率（R_t^m和R_{t+1}^m）上升时，其所配置的房产价值会减少，反之则会上升。

（2）对于计划购房家庭，其持有的金融资产是否具有生命周期特征，取决于金融资产回报率$\mathrm{R^A}$与房贷利率$\mathrm{R^m}$、未来房价（预期）$\mathrm{P_{t+2}^H}$的相对大小：当房贷利率低，金融资产回报增长小于未来房价预期涨幅时，家庭将会增加其对于房产的配置，且在$\frac{P_{t+2}^H}{P_t^H}<\Upsilon_1<\Upsilon_2$时，房产上升导致的替代效应高于财富效应，家庭会降低其青年和中年时期金融资产的配置，此时家庭的金融资产配置不呈现明显的生命周期特征。反之，当金融资产回报增长大于未来房价预期涨幅时，家庭将会增加其对于金融资产的配置。对于无房家庭，除保留一定遗产外，为平滑各期消费，其金融资产配置将呈现出生命周期特征。

$$\begin{aligned}
A_{y,t}^{**} &= w_{y,t}+B_t-C_{y,t}^{**}-\psi(1+\phi)P_t^H H_{y,t}^{**}-R_t^m s_1(1-\psi)P_t^H H_{y,t}^{**} \\
&= w_{y,t}+B_t-\frac{\left\{\psi(1+\phi)+[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)](1-\psi)\right\}P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}}{\theta(1+\beta+\beta^2)}H_{y,t}^{**} \\
&\quad -\psi(1+\phi)P_t^H H_{y,t}^{**}-R_t^m s_1(1-\psi)P_t^H H_{y,t}^{**} \\
&= w_{y,t}+B_t+\underbrace{\frac{1}{\theta(1+\beta+\beta^2)}\frac{P_{t+2}^H H_{y,t}^{**}}{\left(R^A\right)^2}}_{\text{财富效应}}- \\
&\quad \underbrace{\left\{\frac{\left\{\psi(1+\phi)+[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)](1-\psi)\right\}}{\theta(1+\beta+\beta^2)}+\psi(1+\phi)+R_t^m s_1(1-\psi)\right\}P_t^H H_{y,t}^{**}}_{\text{替代效应}}.
\end{aligned}$$

$$
\begin{aligned}
A_{m,t+1}^{**} &= w_{m,t+1}+R^{A}A_{y,t}^{**}-\beta R^{A}C_{y,t}^{**}-R_{t+1}^{m}(1-s_1)(1-\psi)P_t^H H_{y,t}^{**}\\
&= w_{m,t+1}+R^{A}A_{y,t}^{**}-R_{t+1}^{m}(1-s_1)(1-\psi)P_t^H H_{y,t}^{**}\\
&\quad -\beta R^{A}\frac{\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}}{\theta\left(1+\beta+\beta^2\right)}H_{y,t}^{**}.\\
&= w_{m,t+1}+R^{A}\left(w_{y,t}+B_t\right)+\underbrace{\frac{1+\beta}{\theta\left(1+\beta+\beta^2\right)}\frac{P_{t+2}^H H_{y,t}^{**}}{R^A}}_{\text{财富效应}}-\\
&\quad \underbrace{R^A\left\{\psi(1+\phi)+R_t^m s_1(1-\psi)+\frac{\left\{\psi(1+\phi)+[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)](1-\psi)\right\}}{\theta(1+\beta+\beta^2)}\right\}P_t^H H_{y,t}^{**}}_{\text{替代效应}}\\
&\quad -\underbrace{\left\{R_{t+1}^m(1-s_1)(1-\psi)+\beta R^A\frac{\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}}{\theta\left(1+\beta+\beta^2\right)}\right\}P_t^H H_{y,t}^{**}}_{\text{替代效应}}
\end{aligned}
$$

家庭房产配置上升对于家庭金融资产配置存在财富效应和替代效应两种影响。当未来房价（预期）$\frac{P_{t+2}^H}{P_t^H}<\Upsilon_1\equiv\left\{\psi(1+\phi)+[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)](1-\psi)\right\}(R^A)^2+\theta(1+\beta+\beta^2)\psi(1+\phi)(R^A)^2+\theta(1+\beta+\beta^2)(R^A)^2R_t^m s_1(1-\psi)$时，房产配置上升会使得替代效应大于财富效应，进而降低青年时期的金融资产配置；房产配置下降则会提高青年时期的金融资产配置。

类似地，当$\frac{P_{t+2}^H}{P_t^H}<\Upsilon_2\equiv\frac{\theta(1+\beta+\beta^2)R^A}{1+\beta}\left\{\psi(1+\phi)R^A+R^AR_t^m s_1(1-\psi)+\frac{R^A\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}}{\theta(1+\beta+\beta^2)}+R_{t+1}^m(1-s_1)(1-\psi)+\beta R^A\frac{\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}}{\theta(1+\beta+\beta^2)}\right\}=\left\{\left\{\psi(1+\phi)+[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)](1-\psi)\right\}(R^A)^2+\theta(1+\beta+\beta^2)\psi(1+\phi)\frac{(R^A)^2}{1+\beta}+\theta(1+\beta+\beta^2)\frac{(R^A)^2}{1+\beta}R_t^m s_1(1-\psi)+\theta(1+\beta+\beta^2)\frac{R^A}{1+\beta}R_{t+1}^m(1-s_1)(1-\psi)\right\}$时，家庭房产配置上升会使得替代效应大于财富效应，进而降低中年时期的金融资产配置；家庭房产配置下降则会提高中年时期的金融资产配置。

当有$\Upsilon_1<\Upsilon_2$且$\frac{P_{t+2}^H}{P_t^H}<\Upsilon_1$时，房产配置上升会使得对于金融资产替代效应大于财富效应，会降低青年时期和中年时期的金融资产配置，从而减弱金融资产的生命周期特征；房产配置下降会使得对于金融资产替代效应小于财富效应，会提高青年时期和中年时期的金融资产配置，从而加强金融资产的生命周期特征。

当有$\Upsilon_1 < \Upsilon_2$且$\Upsilon_1 < \frac{P_{t+2}^H}{P_t^H} < \Upsilon_2$时，房产配置上升（下降）会使得青年时期对于金融资产的替代效应小于（大于）财富效应，中年时期的财富效应小于（大于）替代效应，最终增加（降低）青年时期的金融资产配置，降低（增加）中年时期的金融资产配置，从而减弱（加强）金融资产的生命周期特征。

当有$\Upsilon_1 < \Upsilon_2$且$\frac{P_{t+2}^H}{P_t^H} > \Upsilon_2$时，房产配置上升会使得对于金融资产替代效应小于财富效应，会提高青年时期和中年时期的金融资产配置，从而加强金融资产的生命周期特征。

$$\frac{\partial H_{y,t}^{**}}{\partial R^A} = \frac{-\frac{w_{m,t+1}}{\left(R^A\right)^2}-2\frac{w_{o,t+2}}{\left(R^A\right)^3}-\frac{2}{\left(R^A\right)^3}}{\Omega\left\{\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}\right\}}-$$

$$\frac{\left[w_{y,t}+\frac{w_{m,t+1}}{R^A}+\frac{w_{o,t+2}}{\left(R^A\right)^2}+B_t+\frac{1}{\left(R^A\right)^2}\right]\Omega\cdot\left[-\frac{R_{t+1}^m}{\left(R^A\right)^2}(1-s_1)(1-\psi)P_t^H+\frac{2P_{t+2}^H}{\left(R^A\right)^3}\right]}{\left\{\Omega\left\{\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}\right\}\right\}^2}$$

$$\Leftrightarrow$$

$$-\left[\frac{w_{m,t+1}}{\left(R^A\right)^2}+2\frac{w_{o,t+2}}{\left(R^A\right)^3}+\frac{2}{\left(R^A\right)^3}\right]\Omega\left\{\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}\right\}+$$

$$\left[w_{y,t}+\frac{w_{m,t+1}}{R^A}+\frac{w_{o,t+2}}{\left(R^A\right)^2}+B_t+\frac{1}{\left(R^A\right)^2}\right]\Omega\cdot\left[\frac{R_{t+1}^m}{\left(R^A\right)^2}(1-s_1)(1-\psi)P_t^H-\frac{2P_{t+2}^H}{\left(R^A\right)^3}\right]$$

$$= -\left[\frac{w_{m,t+1}}{\left(R^A\right)^2}+2\frac{w_{o,t+2}}{\left(R^A\right)^3}+\frac{2}{\left(R^A\right)^3}\right]\Omega\left\{\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}\right\}+$$

$$\left[\frac{w_{y,t}}{R^A}+\frac{w_{m,t+1}}{\left(R^A\right)^2}+\frac{w_{o,t+2}}{\left(R^A\right)^3}+\frac{B_t}{R^A}+\frac{1}{\left(R^A\right)^3}\right]\Omega\cdot\left[\frac{R_{t+1}^m}{R^A}(1-s_1)(1-\psi)P_t^H-\frac{2P_{t+2}^H}{\left(R^A\right)^2}\right]$$

$$= -\left[\frac{w_{m,t+1}}{\left(R^A\right)^2}+2\frac{w_{o,t+2}}{\left(R^A\right)^3}+\frac{2}{\left(R^A\right)^3}\right]\Omega\left\{\{\psi(1+\phi)+[R_t^m s_1](1-\psi)\}P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}\right\}-$$

$$\left[\frac{w_{m,t+1}}{\left(R^A\right)^2}+2\frac{w_{o,t+2}}{\left(R^A\right)^3}+\frac{2}{\left(R^A\right)^3}\right]\Omega\left\{\frac{R_{t+1}^m}{R^A}(1-s_1)(1-\psi)P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}\right\}+$$

$$\left[\frac{w_{y,t}}{R^A}+\frac{w_{m,t+1}}{\left(R^A\right)^2}+\frac{w_{o,t+2}}{\left(R^A\right)^3}+\frac{B_t}{R^A}+\frac{1}{\left(R^A\right)^3}\right]\Omega\cdot\left[\frac{R_{t+1}^m}{R^A}(1-s_1)(1-\psi)P_t^H-\frac{2P_{t+2}^H}{\left(R^A\right)^2}\right]$$

$$= -\left[\frac{w_{m,t+1}}{\left(R^A\right)^2}+2\frac{w_{o,t+2}}{\left(R^A\right)^3}+\frac{2}{\left(R^A\right)^3}\right]\Omega\left\{\{\psi(1+\phi)+R_t^m s_1(1-\psi)\}P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}\right\}-$$

$$\left[\frac{w_{o,t+2}}{\left(R^A\right)^3}-\frac{w_{y,t}}{R^A}-\frac{B_t}{R^A}+\frac{1}{\left(R^A\right)^3}\right]\Omega\left\{\frac{R_{t+1}^m}{R^A}(1-s_1)(1-\psi)P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}\right\},$$

其中$\Omega\equiv 1+\frac{1+\beta+(1+\xi)\beta^2}{\theta(1+\beta+\beta^2)}$。

给定最优房产面积$H_{y,t}^{*}$，同样地，我们可以得到其各个时期的最优消费水平$C_{y,t}(H_{y,t}^{*})$，$C_{m,t+1}\left(H_{y,t}^{**}\right)=\beta R^{A}C_{y,t}(H_{y,t}^{**})$，$C_{o,t+2}\left(H_{y,t}^{**}\right)=(\beta R^{A})^{2}C_{y,t}(H_{y,t}^{**})$，最优遗产$B_{t+2}^{**}=\xi C_{o,t+2}\left(H_{y,t}^{**}\right)-1>0$，以及青年、中年时期金融资产配置情况

$$\begin{aligned}A_{y,t}^{**} &= w_{y,t}+B_{t}-C_{y,t}\left(H_{y,t}^{**}\right)-\psi(1+\phi)P_{t}^{H}H_{y,t}^{**}-R_{t}^{m}s_{1}(1-\psi)P_{t}^{H}H_{y,t}^{**},\\ A_{m,t+1}^{**} &= w_{m,t+1}+R^{A}A_{y,t}^{**}-C_{m,t+1}\left(H_{y,t}^{**}\right)-R_{t+1}^{m}(1-s_{1})(1-\psi)P_{t}^{H}H_{y,t}^{**}.\end{aligned}$$

$$C_{y,t} = \frac{\left\{\psi(1+\phi)+[R_{t}^{m}s_{1}+\frac{R_{t+1}^{m}}{R^{A}}(1-s_{1})](1-\psi)\right\}P_{t}^{H}-\frac{P_{t+2}^{H}}{(R^{A})^{2}}}{\theta(1+\beta+\beta^{2})}H_{y,t}.$$

$$H_{y,t}^{**} = \frac{w_{y,t}+\frac{w_{m,t+1}}{R^{A}}+\frac{w_{o,t+2}}{(R^{A})^{2}}+B_{t}+\frac{1}{(R^{A})^{2}}}{\Omega\left\{\left\{\psi(1+\phi)+\left[R_{t}^{m}s_{1}+\frac{R_{t+1}^{m}}{R^{A}}(1-s_{1})\right](1-\psi)\right\}P_{t}^{H}-\frac{P_{t+2}^{H}}{(R^{A})^{2}}\right\}},$$

其中$\Omega\equiv 1+\frac{1+\beta+(1+\xi)\beta^{2}}{\theta(1+\beta+\beta^{2})}$。

$$\frac{\partial A^{**}_{\mathrm{m},t+1}}{\partial B_t} = R^A\frac{\partial A^{**}_{y,t}}{\partial B_t} - \frac{\beta R^A}{\theta(1+\beta+\beta^2)\Omega} - \frac{R^m_{t+1}(1-s_1)(1-\psi)P^H_t}{\Omega\left\{\left\{\psi(1+\phi)+\left[R^m_t s_1+\frac{R^m_{t+1}}{R^A}(1-s_1)\right](1-\psi)\right\}P^H_t-\frac{P^H_{t+2}}{(R^A)^2}\right\}}$$

$$= \frac{R^A\left[\theta\left(1+\beta+\beta^2\right)\right]\left\{\left\{\left[\frac{R^m_{t+1}}{R^A}(1-s_1)\right](1-\psi)\right\}P^H_t-\frac{P^H_{t+2}}{(R^A)^2}\right\}}{\theta\left(1+\beta+\beta^2\right)\Omega\left\{\left\{\psi(1+\phi)+\left[R^m_t s_1+\frac{R^m_{t+1}}{R^A}(1-s_1)\right](1-\psi)\right\}P^H_t-\frac{P^H_{t+2}}{(R^A)^2}\right\}} + \frac{R^A[\beta+(1+\xi)\beta^2]\left\{\left\{\psi(1+\phi)+\left[R^m_t s_1+\frac{R^m_{t+1}}{R^A}(1-s_1)\right](1-\psi)\right\}P^H_t-\frac{P^H_{t+2}}{(R^A)^2}\right\}}{\theta\left(1+\beta+\beta^2\right)\Omega\left\{\left\{\psi(1+\phi)+\left[R^m_t s_1+\frac{R^m_{t+1}}{R^A}(1-s_1)\right](1-\psi)\right\}P^H_t-\frac{P^H_{t+2}}{(R^A)^2}\right\}} - \frac{\beta R^A\left\{\left\{\psi(1+\phi)+\left[R^m_t s_1+\frac{R^m_{t+1}}{R^A}(1-s_1)\right](1-\psi)\right\}P^H_t-\frac{P^H_{t+2}}{(R^A)^2}\right\}}{\theta(1+\beta+\beta^2)\Omega\left\{\left\{\psi(1+\phi)+\left[R^m_t s_1+\frac{R^m_{t+1}}{R^A}(1-s_1)\right](1-\psi)\right\}P^H_t-\frac{P^H_{t+2}}{(R^A)^2}\right\}} - \frac{R^A\theta(1+\beta+\beta^2)\frac{R^m_{t+1}}{R^A}(1-s_1)(1-\psi)P^H_t}{\theta(1+\beta+\beta^2)\Omega\left\{\left\{\psi(1+\phi)+\left[R^m_t s_1+\frac{R^m_{t+1}}{R^A}(1-s_1)\right](1-\psi)\right\}P^H_t-\frac{P^H_{t+2}}{(R^A)^2}\right\}}$$

$$= \frac{R^A\left[\theta\left(1+\beta+\beta^2\right)\right]\left\{-\frac{P^H_{t+2}}{(R^A)^2}\right\}}{\theta\left(1+\beta+\beta^2\right)\Omega\left\{\left\{\psi(1+\phi)+\left[R^m_t s_1+\frac{R^m_{t+1}}{R^A}(1-s_1)\right](1-\psi)\right\}P^H_t-\frac{P^H_{t+2}}{(R^A)^2}\right\}} + \frac{R^A[(1+\xi)\beta^2]\left\{\left\{\psi(1+\phi)+\left[R^m_t s_1+\frac{R^m_{t+1}}{R^A}(1-s_1)\right](1-\psi)\right\}P^H_t-\frac{P^H_{t+2}}{(R^A)^2}\right\}}{\theta\left(1+\beta+\beta^2\right)\Omega\left\{\left\{\psi(1+\phi)+\left[R^m_t s_1+\frac{R^m_{t+1}}{R^A}(1-s_1)\right](1-\psi)\right\}P^H_t-\frac{P^H_{t+2}}{(R^A)^2}\right\}}$$

$$R^A\left[\theta\left(1+\beta+\beta^2\right)\right]\left\{-\frac{P_{t+2}^H}{(R^A)^2}\right\}+$$

$$R^A[(1+\xi)\beta^2]\left\{\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H-\frac{P_{t+2}^H}{(R^A)^2}\right\}$$

$$\leq \quad 0$$

$$(R^A)^2[(1+\xi)\beta^2]\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}$$

$$\leq \quad \left\{[(1+\xi)\beta^2]+\left[\theta\left(1+\beta+\beta^2\right)\right]\right\}\frac{P_{t+2}^H}{P_t^H}$$

$$\frac{P_{t+2}^H}{P_t^H} \geq \Psi_2 \equiv \frac{(R^A)^2[(1+\xi)\beta^2]\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}}{[(1+\xi)\beta^2]+\left[\theta\left(1+\beta+\beta^2\right)\right]}$$

$$\frac{\partial A_{\mathrm{m},t+1}^{**}}{\partial B_t} \leq 0$$

$$\left\{[\theta(1+\beta+\beta^2)]+\left[\beta+(1+\xi)\beta^2\right]\right\}\frac{P_{t+2}^H}{\left(R^A\right)^2}$$

$$\geq [\theta(1+\beta+\beta^2)]\left\{\left[\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H$$

$$+\left[\beta+(1+\xi)\beta^2\right]\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H$$

$$\frac{P_{t+2}^H}{P_t^H} \geq \Psi_1 \equiv \frac{(R^A)^2\left[\theta\left(1+\beta+\beta^2\right)\right]\left\{\left[\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}}{\left[\theta\left(1+\beta+\beta^2\right)\right]+[\beta+(1+\xi)\beta^2]}+$$

$$\frac{(R^A)^2[\beta+(1+\xi)\beta^2]\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}}{\left[\theta\left(1+\beta+\beta^2\right)\right]+[\beta+(1+\xi)\beta^2]}$$

$$\frac{\partial A_{y,t}^{**}}{\partial B_t} \leq 0.$$

$$
\begin{aligned}
\frac{\partial A_{y,t}^{**}}{\partial B_t} &= 1-\frac{1}{\theta(1+\beta+\beta^2)\Omega}-\frac{[\psi(1+\phi)+R_t^m s_1(1-\psi)]P_t^H}{\Omega\left\{\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}\right\}} \\
&= \frac{\theta(1+\beta+\beta^2)\Omega\left\{\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}\right\}}{\theta(1+\beta+\beta^2)\Omega\left\{\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}\right\}}- \\
&\quad \frac{\left\{\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}\right\}}{\theta(1+\beta+\beta^2)\Omega\left\{\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}\right\}}- \\
&\quad \frac{\theta(1+\beta+\beta^2)[\psi(1+\phi)+R_t^m s_1(1-\psi)]P_t^H}{\theta(1+\beta+\beta^2)\Omega\left\{\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}\right\}} \\
&= \frac{\left[\theta(1+\beta+\beta^2)+\beta+(1+\xi)\beta^2\right]\left\{\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}\right\}}{\theta(1+\beta+\beta^2)\Omega\left\{\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}\right\}}- \\
&\quad \frac{\theta(1+\beta+\beta^2)[\psi(1+\phi)+R_t^m s_1(1-\psi)]P_t^H}{\theta(1+\beta+\beta^2)\Omega\left\{\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}\right\}} \\
&= \frac{[\theta(1+\beta+\beta^2)]\left\{\left\{\left[\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}\right\}}{\theta(1+\beta+\beta^2)\Omega\left\{\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}\right\}}+ \\
&\quad \frac{\left[\beta+(1+\xi)\beta^2\right]\left\{\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}\right\}}{\theta(1+\beta+\beta^2)\Omega\left\{\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}\right\}}
\end{aligned}
$$

$$[\theta(1+\beta+\beta^2)]\left\{\left\{\left[\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}\right\}$$

$$+\left[\beta+(1+\xi)\beta^2\right]\left\{\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H-\frac{P_{t+2}^H}{\left(R^A\right)^2}\right\}$$

$$\left\{[\theta(1+\beta+\beta^2)]+\left[\beta+(1+\xi)\beta^2\right]\right\}\frac{P_{t+2}^H}{\left(R^A\right)^2}$$

$$\geq [\theta(1+\beta+\beta^2)]\left\{\left[\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H$$

$$+\left[\beta+(1+\xi)\beta^2\right]\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}P_t^H$$

$$\begin{aligned}
\frac{P_{t+2}^H}{P_t^H} &\geq \frac{(R^A)^2\left[\theta\left(1+\beta+\beta^2\right)\right]\left\{\left[\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}}{\left[\theta\left(1+\beta+\beta^2\right)\right]+[\beta+(1+\xi)\beta^2]}+ \\
&\quad \frac{(R^A)^2[\beta+(1+\xi)\beta^2]\left\{\psi(1+\phi)+\left[R_t^m s_1+\frac{R_{t+1}^m}{R^A}(1-s_1)\right](1-\psi)\right\}}{\left[\theta\left(1+\beta+\beta^2\right)\right]+[\beta+(1+\xi)\beta^2]} \\
\frac{\partial A_{y,t}^{**}}{\partial B_t} &\leq 0.
\end{aligned}$$

基于上述结果，我们可以求出无信贷约束下的青年购房者的最优福利水平为

$$\begin{aligned}
U_{y,t}^{f*} &= \ln C_{y,t}^{**}+\theta\ln H_{y,t}^{**}+\beta\left(\ln C_{m,t+1}^{**}+\theta\ln H_{y,t}^{**}\right) \\
&\quad +\beta^2\left(\ln C_{o,t+2}^{**}+\theta\ln H_{y,t}^{**}+\xi\ln(1+B_{t+2}^{**})\right).
\end{aligned}$$

II. 中年购房者的终生效用最大化问题

同样，我们首先构建 Lagrange 函数可得

$$\begin{aligned}
\mathcal{L} = {} & \ln C_{y,t}+\theta\ln H_{y,t}+\beta\left(\ln C_{m,t+1}+\theta\ln H_{m,t+1}\right)+ \\
& \beta^2\left(\ln C_{o,t+2}+\theta\ln H_{m,t+1}+\xi\ln(1+B_{t+2})\right)+ \\
& \lambda_1\left[w_{y,t}+B_t-C_{y,t}-\alpha P_t^H H_{y,t}-A_{y,t}\right]+ \\
& \lambda_2\left[w_{m,t+1}+R^A A_{y,t}-C_{m,t+1}-\psi(1+\phi)P_{t+1}^H H_{m,t+1}-R_{t+1}^m M_{t+1}-A_{m,t+1}\right]+ \\
& \lambda_3\left(w_{o,t+2}+R^A A_{m,t+1}+P_{t+2}^H H_{m,t+1}-C_{o,t+2}-B_{t+2}\right)+ \\
& \zeta_1\left[M_{t+1}-(1-\psi)P_{t+1}^H H_{m,t+1}\right]+ \\
& \zeta_2\left[A_{y,t}+(1-\nu)\left(w_{y,t}+B_t\right)\right].
\end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial C_{y,t}} = \frac{1}{C_{y,t}} - \lambda_1 = 0 \tag{14}$$

$$\frac{\partial \mathcal{L}}{\partial C_{m,t+1}} = \frac{\beta}{C_{m,t+1}} - \lambda_2 = 0 \tag{15}$$

$$\frac{\partial \mathcal{L}}{\partial C_{o,t+2}} = \frac{\beta^2}{C_{o,t+2}} - \lambda_3 = 0 \tag{16}$$

$$\frac{\partial \mathcal{L}}{\partial H_{y,t}} = \frac{\theta}{H_{y,t}} - \lambda_1 \alpha P_t^H = 0 \tag{17}$$

$$\frac{\partial \mathcal{L}}{\partial H_{m,t+1}} = \frac{\theta(\beta+\beta^2)}{H_{m,t+1}} - \lambda_2 \psi(1+\phi)P_{t+1}^H + \lambda_3 P_{t+2}^H - \zeta_1(1-\psi)P_{t+1}^H = 0 \tag{18}$$

$$\frac{\partial \mathcal{L}}{\partial A_{y,t}} = -\lambda_1 + \lambda_2 R^A + \zeta_2 = 0 \tag{19}$$

$$\frac{\partial \mathcal{L}}{\partial A_{m,t+1}} = -\lambda_2 + \lambda_3 R^A = 0 \tag{20}$$

$$\frac{\partial \mathcal{L}}{\partial M_{t+1}} = -\lambda_2 R_{t+1}^m + \zeta_1 = 0 \tag{21}$$

$$\frac{\partial \mathcal{L}}{\partial B_{t+2}} = \frac{\beta^2 \xi}{1+B_{t+2}} - \lambda_3 = 0 \tag{22}$$

联立式(14)和(17)可得$\alpha P_t^H H_{y,t} = \theta C_{y,t}$；联立式(16)和(22)可得$B_{t+2} = \xi C_{o,t+2} - 1$。

（I）当中年购房者在青年时期受到信贷约束时，$\zeta_2 > 0$，有

$$A_{y,t} = -(1-v)\left(w_{y,t} + B_t\right),$$

$$C_{y,t} = \frac{(2-v)(w_{y,t}+B_t)}{1+\theta}.$$

联立式(14)-(16)和式(18)-(20)可得

$$C_{o,t+2} = \frac{\beta^2 R^A[\psi(1+\phi) + R_{t+1}^m(1-\psi)]P_{t+1}^H - \beta^2 P_{t+2}^H}{\theta\beta(1+\beta)} H_{m,t+1}.$$

将式(14)-(22)，$A_{y,t}$和上式代入其终生预算约束中可得，

$$H_{m,t+1}^* = \frac{w_{m,t+1} - R^A(1-v)\left(w_{y,t}+B_t\right) + \frac{w_{o,t+2}+1}{R^A}}{\left[1+\frac{1+\beta+\beta\xi}{\theta(1+\beta)}\right]\left\{\left[\psi(1+\phi)+R_{t+1}^m(1-\psi)\right]P_{t+1}^H - \frac{P_{t+2}^H}{R^A}\right\}}. \tag{23}$$

基于最优房产面积，我们可以求得各个时期的最优消费水平，$C_{y,t} = \frac{(2-v)(w_{y,t}+B_t)}{1+\theta}$，$C_{o,t+2}(H_{m,t+1}^*)$，$C_{m,t+1}(H_{m,t+1}^*) = C_{o,t+2}(H_{m,t+1}^*)/(\beta R^A)$，遗产 $B_{t+2}^* = \xi C_{o,t+2}(H_{m,t+1}^*) - 1$，以及中年时期的金融资产水平

$$A_{m,t+1}^* = w_{m,t+1} - R^A(1-v)\left(w_{y,t}+B_t\right) - C_{m,t+1}\left(H_{m,t+1}^*\right) - \psi(1+\phi)P_{t+1}^H H_{m,t+1}^* - R_{t+1}^m(1-\psi)P_{t+1}^H H_{m,t+1}^*.$$

最后，基于上述结果，我们可以求得中年购房者在受到信贷约束下的福利水平

$$\begin{aligned} U_{m,t+1}^{c*} \quad = \quad & \ln C_{y,t}^{*} + \theta \ln H_{y,t}^{*} + \beta\left(\ln C_{m,t+1}^{*} + \theta \ln H_{y,t}^{*}\right) \\ & +\beta^{2}\left(\ln C_{o,t+2}^{*} + \theta \ln H_{y,t}^{*} + \xi \ln(1 + B_{t+2}^{*})\right). \end{aligned}$$

（II）当中年购房者在青年时期无信贷约束时，即$\zeta_2 = 0$。联立式(14)-(16)和(19)-(20)有

$$C_{m,t+1} \quad = \quad \beta R^{A} C_{y,t}, \tag{24}$$

$$C_{o,t+2} \quad = \quad (\beta R^{A})^{2} C_{y,t}. \tag{25}$$

将式(19)-(21)和式(24)-(25)代入(18)可得

$$C_{y,t} \quad = \quad \frac{[\psi(1+\phi) + R_{t+1}^{m}(1-\psi)]P_{t+1}^{H}/R^{A} - P_{t+2}^{H}/(R^{A})^{2}}{\theta\beta(1+\beta)} H_{m,t+1} \tag{26}$$

最后，将式(24)-(26)和(22)代入中年购房者的终生预算约束方程可得

$$H_{m,t+1}^{*} = \frac{w_{y,t} + B_t + \frac{w_{m,t+1}}{R^{A}} + \frac{w_{o,t+2}}{(R^{A})^{2}} + \frac{1}{(R^{A})^{2}}}{\left[1 + \frac{1+\theta+\beta(1+\beta+\beta\xi)}{\theta\beta(1+\beta)}\right]\left\{\frac{[\psi(1+\phi) + R_{t+1}^{m}(1-\psi)]P_{t+1}^{H}}{R^{A}} - \frac{P_{t+2}^{H}}{(R^{A})^{2}}\right\}}.$$

由此，我们可以计算出中年购房者无信贷约束下的各期最优消费$C_{y,t}(H_{m,t+1}^{*})$，$C_{m,t+1}\left(H_{m,t+1}^{*}\right) = \beta R^{A} C_{y,t}(H_{m,t+1}^{*})$，$C_{o,t+2}\left(H_{m,t+1}^{*}\right) = (\beta R^{A})^{2} C_{y,t}(H_{m,t+1}^{*})$，青年时期的租房面积$H_{y,t}^{**} = \frac{\theta C_{y,t}(H_{m,t+1}^{*})}{\alpha P_t^{H}}$，遗产水平$B_{t+2}^{**} = \xi C_{o,t+2}(H_{m,t+1}^{*}) - 1$，以及青年和中年时期的金融资产水平

$$\begin{aligned} A_{y,t}^{**} \quad &= \quad w_{y,t} + B_t - (1+\theta) C_{y,t}\left(H_{m,t+1}^{*}\right), \\ A_{m,t+1}^{**} \quad &= \quad w_{m,t+1} + R^{A} A_{y,t}^{**} - C_{m,t+1}\left(H_{m,t+1}^{*}\right) - \\ & \quad\quad \psi(1+\phi) P_{t+1}^{H} H_{m,t+1}^{*} - R_{t+1}^{m}(1-\psi) P_{t+1}^{H} H_{m,t+1}^{*}. \end{aligned}$$

给定上述最优决策，我们可以计算其最优福利水平

$$\begin{aligned} U_{\mathrm{m},t+1}^{f*} \quad = \quad & \ln C_{y,t}^{**} + \theta \ln H_{y,t}^{**} + \beta\left(\ln C_{m,t+1}^{**} + \theta \ln H_{m,t+1}^{**}\right) \\ & +\beta^{2}\left(\ln C_{o,t+2}^{**} + \theta \ln H_{m,t+1}^{**} + \xi \ln(1 + B_{t+2}^{**})\right). \end{aligned}$$

III. 求解终生未购房者的效用最大化问题

首先，构建 Lagrange 函数可得

$$
\begin{aligned}
\mathcal{L} \quad = \quad & \ln C_{y,t} + \theta \ln H_{y,t} + \beta\left(\ln C_{m,t+1} + \theta \ln H_{m,t+1}\right) + \\
& \beta^2\left(\ln C_{o,t+2} + \theta \ln H_{m,t+1} + \xi \ln(1 + B_{t+2})\right) + \\
& \lambda_1\left[w_{y,t} + B_t - C_{y,t} - \alpha P_t^H H_{y,t} - A_{y,t}\right] + \\
& \lambda_2\left[w_{m,t+1} + R^A A_{y,t} - C_{m,t+1} - \alpha P_{t+1}^H H_{m,t+1} - A_{m,t+1}\right] + \\
& \lambda_3\left(w_{o,t+2} + R^A A_{m,t+1} - C_{o,t+2} - \alpha P_{t+2}^H H_{o,t+2} - B_{t+2}\right) + \\
& \zeta_1\left[A_{y,t} + (1 - v)\left(w_{y,t} + B_t\right)\right].
\end{aligned}
$$

$$\frac{\partial \mathcal{L}}{\partial C_{y,t}} = \frac{1}{C_{y,t}} - \lambda_1 = 0 \tag{27}$$

$$\frac{\partial \mathcal{L}}{\partial C_{m,t+1}} = \frac{\beta}{C_{m,t+1}} - \lambda_2 = 0 \tag{28}$$

$$\frac{\partial \mathcal{L}}{\partial C_{o,t+2}} = \frac{\beta^2}{C_{o,t+2}} - \lambda_3 = 0 \tag{29}$$

$$\frac{\partial \mathcal{L}}{\partial H_{y,t}} = \frac{\theta}{H_{y,t}} - \lambda_1 \alpha P_t^H = 0 \tag{30}$$

$$\frac{\partial \mathcal{L}}{\partial H_{m,t+1}} = \frac{\theta\beta}{H_{m,t+1}} - \lambda_2 \alpha P_{t+1}^H = 0 \tag{31}$$

$$\frac{\partial \mathcal{L}}{\partial H_{\mathrm{o},t+2}} = \frac{\theta\beta^2}{H_{o,t+2}} - \lambda_3 \alpha P_{t+2}^H = 0 \tag{32}$$

$$\frac{\partial \mathcal{L}}{\partial A_{y,t}} = -\lambda_1 + \lambda_2 R^A + \zeta_1 = 0 \tag{33}$$

$$\frac{\partial \mathcal{L}}{\partial A_{m,t+1}} = -\lambda_2 + \lambda_3 R^A = 0 \tag{34}$$

$$\frac{\partial \mathcal{L}}{\partial B_{t+2}} = \frac{\beta^2 \xi}{1 + B_{t+2}} - \lambda_3 = 0 \tag{35}$$

（I）当终生未购房者受到信贷约束时，$\zeta_1 > 0$，此时有$A_{y,t} = -(1 - v)\left(w_{y,t} + B_t\right)$。

联立式(27)-(32)，可得

$$\alpha P_t^H H_{y,t} = \theta C_{y,t}, \tag{36}$$

$$\alpha P_{t+1}^H H_{m,t+1} = \theta C_{m,t+1}, \tag{37}$$

$$\alpha P_{t+2}^H H_{o,t+2} = \theta C_{o,t+2}, \tag{38}$$

因此，$C_{y,t}^* = \frac{(2-v)(w_{y,t}+B_t)}{1+\theta}$，$H_{y,t}^* = \frac{\theta(2-v)(w_{y,t}+B_t)}{(1+\theta)\alpha P_t^H}$。联立式(29)和(35)可得$B_{t+2} = \xi C_{o,t+2} - 1$。

将上述等式代入其终生预算约束方程可得

$$C_{o,t+2}^* = \frac{w_{m,t+1} - R^A(1 - v)\left(w_{y,t} + B_t\right) + \frac{w_{o,t+2}}{R^A} + \frac{1}{R^A}}{[(1 + \theta) + \beta(1 + \theta + \xi)]/(\beta R^A)},$$

$$
\begin{aligned}
H_{o,t+2}^{*} &= \frac{\theta C_{o,t+2}^{*}}{\alpha P_{t+2}^{H}},\\
C_{m,t+1}^{*} &= \frac{C_{o,t+2}^{*}}{\beta R^{A}},\\
H_{m,t+1}^{*} &= \frac{\theta C_{m,t+1}^{*}}{\alpha P_{t+1}^{H}},\\
B_{t+2}^{*} &= \xi C_{o,t+2}^{*} - 1.
\end{aligned}
$$

给定上述最优决策，我们可以推出终生未购房者在信贷约束下的最优福利水平

$$
\begin{aligned}
U_{o,t+2}^{c*} = {} & \ln C_{y,t}^{*} + \theta \ln H_{y,t}^{**} + \beta\left(\ln C_{m,t+1}^{*} + \theta \ln H_{m,t+1}^{*}\right)\\
& + \beta^{2}\left(\ln C_{o,t+2}^{*} + \theta \ln H_{o,t+2}^{*} + \xi \ln(1 + B_{t+2}^{*})\right).
\end{aligned}
$$

（II）当其不受到信贷约束时，即$\zeta_1 = 0$，联立式(27)-(30)和式(33)-(34)有

$$
\begin{aligned}
C_{m,t+1} &= \beta R^{A} C_{y,t},\\
C_{o,t+2} &= (\beta R^{A})^{2} C_{y,t}.
\end{aligned}
$$

将上面两式和式(36)-(38)代入其终生预算约束方程可得

$$
C_{y,\mathrm{t}}^{**} = \frac{w_{y,t} + B_t + \frac{w_{m,t+1}}{R^{A}} + \frac{w_{o,t+2}}{(R^{A})^{2}} + \frac{1}{(R^{A})^{2}}}{(1+\beta)(1+\theta) + \beta^{2}(1+\theta+\xi)},
$$

$$
\begin{aligned}
C_{m,\mathrm{t}+1}^{**} &= \beta R^{A} C_{y,\mathrm{t}}^{**},\\
C_{o,\mathrm{t}+2}^{**} &= (\beta R^{A})^{2} C_{y,\mathrm{t}}^{**},\\
H_{y,t}^{**} &= \frac{\theta C_{y,\mathrm{t}}^{**}}{\alpha P_{t}^{H}},\\
H_{m,t+1}^{**} &= \frac{\theta C_{m,\mathrm{t}+1}^{**}}{\alpha P_{t+1}^{H}},\\
H_{o,t+2}^{**} &= \frac{\theta C_{o,\mathrm{t}+2}^{**}}{\alpha P_{t+2}^{H}},\\
B_{t+2}^{**} &= \xi C_{o,\mathrm{t}+2}^{**} - 1.
\end{aligned}
$$

相应的，其最优金融资产配置水平

$$
A_{y,\mathrm{t}}^{**} = w_{y,t} + B_t - (1+\theta) C_{y,\mathrm{t}}^{**},
$$

$$
A_{m,\mathrm{t}+1}^{**} = w_{m,t+1} + R^{A} A_{y,\mathrm{t}}^{**} - (1+\theta) C_{m,\mathrm{t}+1}^{**}.
$$

基于上述最优决策，我们可以推导出终生不购房者的最优福利水平

$$
\begin{aligned}
U_{o,t+2}^{f*} = {} & \ln C_{y,t}^{**} + \theta \ln H_{y,t}^{**} + \beta\left(\ln C_{m,t+1}^{**} + \theta \ln H_{m,t+1}^{**}\right)\\
& + \beta^{2}\left(\ln C_{o,t+2}^{**} + \theta \ln H_{o,t+2}^{**} + \xi \ln(1 + B_{t+2}^{**})\right).
\end{aligned}
$$