Fiscal Transfers in the Spatial Economy

Marcel Henkel, Tobias Seidel, and Jens Suedekum

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2 A quantitative geography model with fiscal transfers

We consider an economy consisting of N regions. The economy is populated by a mass $\bar{L}$ of homogeneous workers, who are (imperfectly) mobile across regions. Governments in every region collect income taxes to provide local public goods, and a fiscal transfer scheme reallocates resources across jurisdictions.

2.1 Preferences

Similarly, as in Fajgelbaum et al. (2019), we assume that households in region i derive utility from consumption of a private good $C(i)$ and public services $G(i)$ according to the following Cobb-Douglas preferences, where $0 < \gamma < 1$:

$$U(i) = u(i) \cdot \left(\frac{G(i)}{L(i)^{\eta}}\right)^{\gamma} \cdot C(i)^{1-\gamma} \tag{1}$$

The parameter $\eta \in [0,1]$ governs the degree of rivalry in public services, with $\eta = 0$ capturing the case of a pure local public good and $\eta = 1$ the case of fully rival per capita transfers (Flatters, Henderson and Mieszkowski, 1974). The term $u(i)$ represents a local amenity including fixed features like scenery or climate, but also endogenous local characteristics such as congestion or housing prices. Our specification also allows us to account for idiosyncratic locational preferences, as we discuss in greater detail in Section 2.6 below.

2.2 Production technologies

Every region i produces a unique variety of a differentiated intermediate good under perfect competition using labor as the sole input. Locations differ in productivity, such that every worker produces $A(i)$ units of this good. A final good $Q(i)$ is assembled from the continuum of intermediates according to the following CES aggregator:

$$Q(i) = \left[\int_0^N q(n,i)^{\frac{\sigma-1}{\sigma}} dn\right]^{\frac{\sigma}{\sigma-1}} \tag{2}$$

Here, $q(n,i)$ denotes the quantity of the variety produced in location n and used for assembly in location i, and $\sigma > 1$ represents the elasticity of substitution between intermediates. We assume that $\tau(n,i) \geq 1$ units must be sent from n for one unit to arrive in i, and we abstract from intra-regional transport costs ($\tau(i,i) = 1$).

Final goods are not traded across regions, and assembly has no extra costs. The price of the final good in location i is therefore given by

$$P(i) = \left[\int_0^N p(n,i)^{1-\sigma} dn\right]^{\frac{1}{1-\sigma}} \tag{3}$$

The final good $Q(i)$ can either be used directly for private consumption $C(i)$, or by local governments to provide public services $G(i)$. Thus, we have $Q(i) = L(i)C(i) + G(i)$. The aggregate demand for the variety from n in location i is given by

$$q(n,i) = \frac{p(n,i)^{-\sigma}}{P(i)^{1-\sigma}} E(i), \tag{4}$$

where $E(i)$ denotes overall (private and public) expenditure in i.

Proof of (4):

$$\max_{q(n,i)} P(i)\, Q(i) - \int_0^N p(n,i) q(n,i) dn$$

$$\mathcal{L} = \left[\int_0^N p(n,i)^{1-\sigma} dn\right]^{\frac{1}{1-\sigma}} \cdot \left[\int_0^N q(n,i)^{\frac{\sigma-1}{\sigma}} dn\right]^{\frac{\sigma}{\sigma-1}} - \int_0^N p(n,i) q(n,i) dn$$

$$\frac{\partial \mathcal{L}}{\partial q(n,i)} = \left[\int_0^N p(n,i)^{1-\sigma} dn\right]^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma-1} \left[\int_0^N q(n,i)^{\frac{\sigma-1}{\sigma}} dn\right]^{\frac{\sigma}{\sigma-1}-1} \frac{\sigma-1}{\sigma} q(n,i)^{\frac{-1}{\sigma}} - p(n,i) = 0$$

$$\left[\int_0^N p(n,i)^{1-\sigma} dn\right]^{\frac{1}{1-\sigma}} \left[\int_0^N q(n,i)^{\frac{\sigma-1}{\sigma}} dn\right]^{\frac{\sigma}{\sigma-1}} \frac{q(n,i)^{\frac{-1}{\sigma}}}{\left[\int_0^N q(n,i)^{\frac{\sigma-1}{\sigma}} dn\right]} = p(n,i)$$

$$P(i)Q(i) * \frac{q(n,i)^{\frac{-1}{\sigma}}}{\left[\int_0^N q(n,i)^{\frac{\sigma-1}{\sigma}} dn\right]} = p(n,i)$$

$$P(i)Q(i) * \frac{q(n,i)^{\frac{-1}{\sigma}}}{Q(i)^{\frac{\sigma-1}{\sigma}}} = p(n,i)$$

$$\frac{P(i)Q(i)^{\frac{1}{\sigma}}}{q(n,i)^{\frac{1}{\sigma}}} = p(n,i)$$

$$q(n,i)^{\frac{1}{\sigma}} = \frac{P(i)Q(i)^{\frac{1}{\sigma}}}{p(n,i)}$$

$$q(n,i) = \frac{P(i)^{\sigma} Q(i)}{p(n,i)^{\sigma}}$$

$$= \frac{P(i)^{\sigma} * P(i) * Q(i)}{p(n,i)^{\sigma} * P(i)}$$

$$= \frac{P(i)^{\sigma-1} * E(i)}{p(n,i)^{\sigma}}$$

$$= \frac{p(n,i)^{-\sigma} * E(i)}{P(i)^{1-\sigma}}$$

$$q(n,i) = \frac{p(n,i)^{-\sigma}}{P(i)^{1-\sigma}} E(i), \tag{4}$$

2.3 Profit maximization and inter-regional trade

As the differentiated varieties are produced under perfect competition, prices equal effective marginal costs including transport costs. That is, $p(n,i) = \tau(n,i)w(n)/A(n)$ for the intermediate produced in n and sold in i, where $w(n)$ is the wage in n. Using those prices in (3), $P(i) = \left[\int_0^N p(n,i)^{1-\sigma} dn\right]^{\frac{1}{1-\sigma}}$, and (4), $q(n,i) = \frac{p(n,i)^{-\sigma}}{P(i)^{1-\sigma}} E(i)$, we obtain total sales from n to i as follows,

$$X(n,i) = \left(\frac{\tau(n,i)w(n)}{A(n)P(i)}\right)^{1-\sigma} * E(i), \tag{5}$$

and the CES price index in location i becomes:

$$P(i) = \left[\int_0^N \left(\frac{\tau(n,i)w(n)}{A(n)}\right)^{1-\sigma} dn\right]^{\frac{1}{1-\sigma}} \tag{3'}$$

Proof:

$$P(i) = \left[\int_0^N p(n,i)^{1-\sigma} dn\right]^{\frac{1}{1-\sigma}}$$

$$= \left[\int_0^N \left(\frac{\tau(n,i)w(n)}{A(n)}\right)^{1-\sigma} dn\right]^{\frac{1}{1-\sigma}}$$

$$q(n,i) = \frac{\left[\frac{\tau(n,i)w(n)}{A(n)}\right]^{-\sigma}}{P(i)^{1-\sigma}} E(i)$$

$$= \frac{\left[\frac{\tau(n,i)w(n)}{A(n)}\right]^{-\sigma}}{P(i)^{1-\sigma}} E(i)$$

$$p(n,i) * q(n,i) = X(n,i) = \frac{\left[\frac{\tau(n,i)w(n)}{A(n)}\right]^{1-\sigma}}{P(i)^{1-\sigma}} E(i)$$

$$X(n,i) = \left[\frac{\tau(n,i)w(n)}{A(n)P(i)}\right]^{1-\sigma} E(i)$$

2.4 Taxes, public spending, and fiscal transfers

We now describe the public sector in this economy. Income is taxed at rate $t(i)$ in region i, which generates an overall tax revenue equal to $t(i)w(i)L(i)$. Assuming that the public budget is balanced, the level of local public goods is thus given by $G(i) = t(i)w(i)L(i)/P(i)$ when there are no inter-regional transfers. When a fiscal transfer scheme is in place, every region is either a net recipient of public funds from other jurisdictions or respectively, a net donor. Net transfers are denoted as $\theta(i)w(i)L(i)$, where the transfer rate $\theta(i)$ (total transfers relative to local aggregate income) is positive for recipient and negative for donor regions. Given those transfers, the effective budget that is available for local public goods provision in region i is given by

$$G(i) = [t(i) + \theta(i)]w(i)L(i)/P(i), \tag{6}$$

and aggregate spending becomes

$$\begin{aligned} E(i) &= P(i) * Q(i) \\ &= P(i) * [L(i)C(i) + G(i)] \\ &= P(i) * \left[\big(1 - t(i)\big) * w(i)L(i)/P(i) + [t(i) + \theta(i)]w(i)L(i)/P(i)\right] \\ &= \big(1 + \theta(i)\big) * w(i)L(i). \end{aligned}$$

This specification of the public sector is kept as simple as possible, but it is flexible enough for our purpose of taking the model to the data. Three comments are in order about our setup.

First, the model abstracts from any optimizing behavior of governments in the setting of tax rates $t(i)$ or transfer rates $\theta(i)$, including strategic considerations such as horizontal tax competition. Instead, we consider $t(i)$ and $\theta(i)$ as being exogenously given and recover them from the data. Starting from those observed actual choices, we then study the economic effects of fiscal transfers in a counterfactual analysis.

Second, we abstract from a federal government (or any other vertical structure) and national public goods. In the empirical application below, however, we consider all layers of the public sector in Germany and break down Federal and State tax revenue to the local level. This approach allows us to include tax revenue from other sources than income taxes (such as value-added or corporate profits) as featured in our model.

Third, we abstract from progressive tax schedules and dead-weight losses of income taxation. However, although individuals supply labor inelastically, we will see later that they respond to regional differences in tax and transfer rates through migration. Local governments, therefore, do face a mobile tax base, as individuals choose their locations endogenously.

2.5 Indirect utility

Using (2)-(6) in (1), we can write indirect utility in region i as follows:

$$W(i) = u(i) \cdot \frac{w(i)}{P(i)} \cdot L(i)^{\gamma(1-\eta)} \cdot \{[t(i) + \theta(i)]^{\gamma} [1 - t(i)]^{1-\gamma}\} \tag{7}$$

The first two terms in (7) show that regions with better amenities $u(i)$ and higher real wages $w(i)/P(i)$ tend to be more attractive locations for households. The third term indicates that larger regions are more desirable when there is some non-rivalry in public services (if $\eta < 1$) because more inhabitants can share public facilities. Notice that local public goods provision establishes one agglomeration force in our model that is increasing in γ and decreasing in η. Finally, the fourth term shows that an inflow of fiscal transfers (an increase of $\theta(i)$) increases indirect utility, ceteris paribus, because it allows governments to expand local public goods.

Proof:

$$U(i) = u(i) \cdot (\frac{G(i)}{L(i)^{\eta}})^{\gamma} \cdot C(i)^{1-\gamma}$$

$$= u(i) \cdot (\frac{[t(i) + \theta(i)]w(i)L(i)/P(i)}{L(i)^{\eta}})^{\gamma} \cdot [(1 - t(i)) * w(i)/P(i)]^{1-\gamma}$$

$$= u(i) \cdot (\frac{[t(i) + \theta(i)]w(i)}{P(i)})^{\gamma} L(i)^{\gamma(1-\eta)} \cdot [\frac{(1 - t(i)) * w(i)}{P(i)}]^{1-\gamma}$$

$$= u(i) \cdot (\frac{w(i)}{P(i)})^{\gamma} L(i)^{\gamma(1-\eta)} [t(i) + \theta(i)]^{\gamma} \cdot (1 - t(i))^{1-\gamma} [\frac{w(i)}{P(i)}]^{1-\gamma}$$

$$= u(i) \cdot \frac{w(i)}{P(i)} \cdot L(i)^{\gamma(1-\eta)} \cdot \{[t(i) + \theta(i)]^{\gamma} [1 - t(i)]^{1-\gamma}\}$$

2.6 Micro-foundations of agglomeration and dispersion forces

Apart from the sharing of local public goods, there is an additional agglomeration force in our model that operates via the local productivity level and thus local wages. More specifically, we follow Allen and Arkolakis (2014) and assume that $A(i)$ is given by:

$$A(i) = \overline{A}(i)L(i)^{\alpha}, \quad with\ \alpha \geq 0 \tag{8}$$

Here, $\overline{A}(i)$ is a location-specific exogenous productivity term, and the positive impact of $L(i)$ on $A(i)$ captures additional agglomeration economies such as knowledge spillovers. The strength of this force is measured by the elasticity α, which relates to a large empirical literature (e.g., Ciccone and Hall, 1996; Combes and Gobillon, 2015) that has estimated the impact of population size (or density) on productivity in various contexts.

The dispersion force in our model works through the common local amenity term $u(i)$. It, too, contains an exogenous component $\overline{u}(i)$, capturing scenery or climate, and an endogenous part that is negatively linked to local population size as follows:

$$u(i) = \overline{u}(i)L(i)^{-\beta}, \quad with\, \beta \geq 0 \tag{9}$$

where β governs the strength of that endogenous dispersion force. This simple notation is a short-cut for several possible micro-foundations of local congestion effects.

Housing costs.

For example, it may capture higher housing prices in larger cities. Allen and Arkolakis (2014, p. 1091) have formally established this isomorphism. First, defining $1 - \delta$ as the income share spent on a fixed local factor, and setting $\beta = (1 - \delta)/\delta$ renders this model isomorphic to Helpman (1998) or Redding (2016). The price of the immobile factor increases when workers migrate into a region, and this congestion externality (higher housing prices) as captured by β runs through (9) in our setup (Combes, Duranton, and Gobillon, 2019). Besides housing prices, β may also capture related urban costs such as worse traffic jams, longer commuting times, and more noise in larger cities.

Location tastes.

Moreover, we can add individual location tastes as another congestion force. In their theory appendix, Allen and Arkolakis (2014) show how to incorporate this feature, which is often used in the literature, into their framework and the isomorphism they establish carries over to our model. In short, adding location tastes effectively scales up the amenity spillover β, i.e., it leads to stronger congestion forces.

More specifically, suppose indirect utility (7) not only includes the amenity term $u(i)$ that is common to all individuals living and working in i, but an additional multiplicative term $v(i, \omega)$. That term represents idiosyncratic location tastes of individual ω for region i. It is the realization of a random variable, which is drawn from a Fréchet distribution with shape parameter k, similarly as in Redding (2016). Using (9), $u(i) = \overline{u}(i)L(i)^{-\beta}$, in (7), $W(i) = u(i) \cdot \frac{w(i)}{P(i)} \cdot L(i)^{\gamma(1-\eta)} \cdot \{[t(i) + \theta(i)]^{\gamma}\, [1 - t(i)]^{1-\gamma}\}$, we can rewrite indirect utility as follows:

$$W(i) = \overline{u}(i) \cdot w(i)/P(i) \cdot L(i)^{-\beta+\gamma(1-\eta)} \cdot \{[t(i) + \theta(i)]^{\gamma}\, [1 - t(i)]^{1-\gamma}\}.$$

Proof:

$$\begin{aligned} W(i) &= u(i) \cdot \frac{w(i)}{P(i)} \cdot L(i)^{\gamma(1-\eta)} \cdot \{[t(i) + \theta(i)]^{\gamma}\, [1 - t(i)]^{1-\gamma}\} \\ &= \overline{u}(i)L(i)^{-\beta} \cdot \frac{w(i)}{P(i)} \cdot L(i)^{\gamma(1-\eta)} \cdot \{[t(i) + \theta(i)]^{\gamma}\, [1 - t(i)]^{1-\gamma}\} \\ &= \overline{u}(i) \cdot \frac{w(i)}{P(i)} \cdot L(i)^{\gamma(1-\eta)-\beta} \cdot \{[t(i) + \theta(i)]^{\gamma}\, [1 - t(i)]^{1-\gamma}\} \end{aligned}$$

Applying the argument by Allen and Arkolakis (2014), we slightly change notation to obtain $\beta = \beta_0 + 1/k$, where $1/k$ captures the strength of individual location tastes and β_0 now measures all other congestion externalities such as housing or pollution. Intuitively, stronger location preferences (smaller k) make workers less mobile across regions. This is because their location choices are now also driven by idiosyncratic tastes, and not only by wages, prices and amenities that are valued equally by all individuals. Formally, this force materializes in the model *as if* the common congestion forces were stronger ($\beta > \beta_0$).

To keep our model parsimonious, we do not take a stance on specific micro-foundations. Instead, we use the congestion force (9) as a flexible reduced form, which may capture housing plus other urban costs, as well as location tastes. Similarly, the specification (8) represents various agglomeration externalities that have been studied in the literature, besides the sharing of local public goods that is inherent in our framework. Those assumptions ($\alpha \geq 0$ and $-\beta \leq 0$) are quite standard in the literature, but not undisputed. For instance, models with a fixed factor or heterogeneous individual productivity differences across locations would feature decreasing returns to labor as more workers enter a region, though not via technological externalities. Furthermore, some models of consumer cities feature net positive endogenous amenity spillovers. Below we briefly discuss how such assumptions, which are not represented in our framework, might impact on our main conclusions.

2.7 Equilibrium

A competitive equilibrium in this economy is defined by the following four conditions:

1. Labor market clearing.

$$\int_0^N L(i)di = \overline{L} \tag{10}$$

2. Goods market clearing.

Total labor income in region i, $w(i)L(i)$, must equal region i's total sales to all locations $n \in N$:

$$w(i)L(i) = \int_0^N X(i,n)dn \tag{11}$$

where $X(i,n)$ is given by (5), $X(n,i) = \left(\frac{\tau(n,i)w(n)}{A(n)P(i)}\right)^{1-\sigma} * E(i)$, and includes fiscal transfers across regions.

3. Balanced public budget.

The total amount of transfers paid must equal the total amount received:

$$\int_0^N \theta(i)w(i)L(i)di = 0 \tag{12}$$

Moreover, every local government spends its available budget entirely on local public goods, $[t(i) + \theta(i)]w(i)L(i) = P(i)G(i)$, as imposed above in (6), $G(i) = [t(i) + \theta(i)]w(i)L(i)/P(i)$.

4. Utility equalization.

Finally, free mobility of labor ensures that utility is equalized across all locations. That is,

$$W(i) = W(j) = W \qquad \forall i,j \in N.$$

Substituting utility (7), $W(i) = \overline{u}(i) \cdot w(i)/P(i) \cdot L(i)^{-\beta+\gamma(1-\eta)} \cdot \{[t(i) + \theta(i)]^{\gamma} [1 - t(i)]^{1-\gamma}\}$ and bilateral exports (5), $X(n,i) = \left(\frac{\tau(n,i)w(n)}{A(n)P(i)}\right)^{1-\sigma} * E(i)$, into the goods-market clearing condition (11), we obtain

$$w(i)L(i) = \int_0^N X(i,n)dn = \int_0^N \left(\frac{\tau(n,i)w(n)}{A(n)P(i)}\right)^{1-\sigma} * E(i)dn$$

$$= \int_0^N \left(\frac{\tau(n,i)}{A(n)} * \frac{w(n)}{P(i)}\right)^{1-\sigma} * E(i)dn$$

$$= \int_0^N \left(\frac{\tau(n,i)}{A(n)} * \frac{W(i)}{\overline{u}(i)L(i)^{-\beta+\gamma(1-\eta)} \cdot \{[t(i) + \theta(i)]^\gamma [1 - t(i)]^{1-\gamma}\}}\right)^{1-\sigma} * E(i)dn$$

$$= \int_0^N \left(\frac{\tau(n,i)}{\overline{A}(i)L(i)^\alpha} * \frac{W}{\overline{u}(i)L(i)^{-\beta+\gamma(1-\eta)} \cdot \{[t(i) + \theta(i)]^\gamma [1 - t(i)]^{1-\gamma}\}}\right)^{1-\sigma} * \big(1 + \theta(i)\big) * w(i)L(i)dn$$

$$= \int_0^N W^{1-\sigma}\tau(n,i)^{1-\sigma}\overline{A}(i)^{\sigma-1}\left(\overline{u}(i)L(i)^{-\beta+\gamma(1-\eta)} \cdot \{[t(i) + \theta(i)]^\gamma [1 - t(i)]^{1-\gamma}\}\right)^{\sigma-1} L(i)^{-\alpha(1-\sigma)} * \big(1 + \theta(i)\big) * w(i)L(i)dn$$

$$w(i)L(i) = W^{1-\sigma}\overline{A}(i)^{\sigma-1} \int_0^N \tau(n,i)^{1-\sigma}\left(\overline{u}(i)L(i)^{-\beta+\gamma(1-\eta)} \cdot \{[t(i) + \theta(i)]^\gamma [1 - t(i)]^{1-\gamma}\}\right)^{\sigma-1} L(i)^{-\alpha(1-\sigma)} * \big(1 + \theta(i)\big) * w(i)L(i)dn$$

$$= W^{1-\sigma}\overline{A}(i)^{\sigma-1} \int_0^N \tau(n,i)^{1-\sigma}\overline{u}(i)^{\sigma-1}L(i)^{(\sigma-1)[-\beta+\gamma(1-\eta)]}\{[t(i) + \theta(i)]^\gamma [1 - t(i)]^{1-\gamma}\}^{\sigma-1} L(i)^{-\alpha(1-\sigma)} * \big(1 + \theta(i)\big) * w(i)L(i)dn$$

$$= W^{1-\sigma}\overline{A}(i)^{\sigma-1} \int_0^N \tau(n,i)^{1-\sigma}\overline{u}(i)^{\sigma-1}L(i)^{(\sigma-1)[-\beta+\gamma(1-\eta)]}\Theta(i)^{\sigma-1} L(i)^{-\alpha(1-\sigma)} * \big(1 + \theta(i)\big) * w(i)L(i)dn$$

$$w(i)L(i)L(i)^{\alpha(1-\sigma)} = W^{1-\sigma}\overline{A}(i)^{\sigma-1} \int_0^N \tau(n,i)^{1-\sigma}\overline{u}(i)^{\sigma-1}L(i)^{(\sigma-1)[-\beta+\gamma(1-\eta)]}\Theta(i)^{\sigma-1} * \big(1 + \theta(i)\big) * w(i)L(i)dn$$

$$w(i)L(i)^{1+\alpha(1-\sigma)} = W^{1-\sigma}\overline{A}(i)^{\sigma-1} \int_0^N \tau(n,i)^{1-\sigma}\overline{u}(i)^{\sigma-1}L(i)^{1+(\sigma-1)[-\beta+\gamma(1-\eta)]}\Theta(i)^{\sigma-1} * \big(1 + \theta(i)\big) * w(i)dn$$

$$w(i)L(i)^{1+\alpha(1-\sigma)} = W^{1-\sigma}\overline{A}(i)^{\sigma-1} \int_0^N \tau(n,i)^{1-\sigma}\overline{u}(i)^{\sigma-1}\Theta(i)^{\sigma-1} * \big(1 + \theta(i)\big) * w(i)L(i)^{1+(\sigma-1)[-\beta+\gamma(1-\eta)]}dn$$

$$w(i)^\sigma L(i)^{1-\alpha(\sigma-1)} = W^{1-\sigma}\overline{A}(i)^{\sigma-1} \int_0^N \tau(n,i)^{1-\sigma}\overline{u}(i)^{\sigma-1}\Theta(i)^{\sigma-1} * \big(1 + \theta(i)\big) * w(i)^\sigma L(i)^{1+(\sigma-1)[-\beta+\gamma(1-\eta)]}dn$$

$$L(i)^{1-\alpha(\sigma-1)}w(i)^\sigma = W^{1-\sigma}\overline{A}(i)^{\sigma-1} \int_0^N \tau(i,n)^{1-\sigma}\overline{u}(n)^{\sigma-1}\Theta(n)^{\sigma-1} * \big(1 + \theta(n)\big) * w(n)^\sigma L(n)^{1+(\sigma-1)[-\beta+\gamma(1-\eta)]}dn \qquad (13)$$

Second, combining (7), $W(i) = u(i) \cdot \frac{w(i)}{P(i)} \cdot L(i)^{\gamma(1-\eta)} \cdot \Theta(i)$, and (3'), $P(i) = \left[\int_0^N \left(\frac{\tau(n,i)w(n)}{A(n)}\right)^{1-\sigma} dn\right]^{\frac{1}{1-\sigma}}$, allows us to rewrite the price index equation as follows

$$W(i) = u(i) \cdot \frac{w(i)}{P(i)} \cdot L(i)^{\gamma(1-\eta)} \cdot \Theta(i)$$

$$= u(i) \cdot \frac{w(i)}{\left[\int_0^N \left(\frac{\tau(n,i)w(n)}{A(n)}\right)^{1-\sigma} dn\right]^{\frac{1}{1-\sigma}}} \cdot L(i)^{\gamma(1-\eta)} \cdot \Theta(i)$$

$$\left[\int_0^N \left(\frac{\tau(n,i)w(n)}{A(n)}\right)^{1-\sigma} dn\right]^{\frac{1}{1-\sigma}} = u(i) \cdot \frac{w(i)}{W(i)} \cdot L(i)^{\gamma(1-\eta)} \cdot \Theta(i)$$

$$\int_0^N \left(\frac{\tau(n,i)w(n)}{\overline{A}(i)L(i)^\alpha}\right)^{1-\sigma} dn = \left[u(i) \cdot \frac{w(i)}{W(i)} \cdot L(i)^{\gamma(1-\eta)} \cdot \Theta(i)\right]^{1-\sigma}$$

$$= \left[\overline{u}(i) L(i)^{-\beta} \cdot \frac{w(i)}{W(i)} \cdot L(i)^{\gamma(1-\eta)} \cdot \Theta(i) \right]^{1-\sigma}$$

$$= \overline{u}(i)^{1-\sigma} w(i)^{1-\sigma} W^{\sigma-1} L(i)^{(1-\sigma)[-\beta+\gamma(1-\eta)]} \Theta(i)^{1-\sigma}$$

$$\int_0^N \tau(n,i)^{1-\sigma} w(n)^{1-\sigma} \overline{A}(i)^{\sigma-1} L(i)^{\alpha(\sigma-1)}\, dn = \overline{u}(i)^{1-\sigma} w(i)^{1-\sigma} W^{\sigma-1} L(i)^{(1-\sigma)[-\beta+\gamma(1-\eta)]} \Theta(i)^{1-\sigma}$$

$$\overline{u}(i)^{1-\sigma} w(i)^{1-\sigma} W^{\sigma-1} L(i)^{(1-\sigma)[-\beta+\gamma(1-\eta)]} \Theta(i)^{1-\sigma} = \int_0^N \tau(n,i)^{1-\sigma} w(n)^{1-\sigma} \overline{A}(i)^{\sigma-1} L(i)^{\alpha(\sigma-1)}\, dn$$

$$w(i)^{1-\sigma} L(i)^{(1-\sigma)[-\beta+\gamma(1-\eta)]} = W^{1-\sigma} \Theta(i)^{\sigma-1} \overline{u}(i)^{\sigma-1} \int_0^N \tau(n,i)^{1-\sigma} \overline{A}(i)^{\sigma-1} w(n)^{1-\sigma} L(i)^{\alpha(\sigma-1)}\, dn$$

$$w(i)^{1-\sigma} L(i)^{(1-\sigma)[-\beta+\gamma(1-\eta)]} = W^{1-\sigma} \Theta(i)^{\sigma-1} \overline{u}(i)^{\sigma-1} \int_0^N \tau(n,i)^{1-\sigma} \overline{A}(n)^{\sigma-1} w(n)^{1-\sigma} L(n)^{\alpha(\sigma-1)}\, dn \quad (14)$$

For given parameters, the system (13) and (14) can be solved for the equilibrium wages and populations (see Appendix A.1 for details).

Appendix A.1 Solving the system of non-linear equations

We employ a method of successive approximations to solve for the equilibrium of the system of non-linear equations (see Zabreiko et al., 1975). In the following, we briefly describe the steps of the iterative procedure. First, we choose an arbitrary vector of non-negative starting values for the endogenous variables in equations (13)-(14). Next, we simultaneously solve the system of equations (13)-(14) for a given parameterization of trade costs and parameter values to obtain new vectors of solutions for the endogenous variables. To ensure convergence, we normalize each new vector to sum up to one. We then update the starting values according to a weighted average of previous starting values and solutions of the previous iteration. Finally, we iteratively solve the system of equations until the metric distance between the starting values and solutions of the endogenous variables becomes sufficiently small.

Existence and uniqueness theorems for non-linear equations are described by Polyanin and Manzhirov (2008). Under the condition that the sequence of convergence is an element of a complete metric space, it will also converge to a limit point. Hence, the system of non-linear equations has at least one continuous solution.

Using data on tax rates $t(i)$, transfer rates $\theta(i)$, bilateral trade costs $\tau(i,n)$, as well as wages $w(i)$ and population sizes $L(i)$ in (13) and (14), we use the structure of the model to recover (up to a positive constant) the values $\overline{A}(i)$ and $\overline{u}(i)$ consistent with equilibrium.

In their model, Allen and Arkolakis (2014) have shown that $\beta > \alpha$ is a sufficient condition to ensure the existence and uniqueness of a stable equilibrium, although equilibria may also exist if that condition is not satisfied. In our framework with local public goods and fiscal transfers, the respective sufficient condition reads as:

Condition 1: $\boldsymbol{\beta \geq \alpha + \gamma(1-\eta)}$.

In words, the congestion force parameterized by $\beta \geq 0$ is at least as strong as the sum of the standard agglomeration force ($\alpha \geq 0$) and the sharing of public facilities ($\gamma(1-\eta) \geq 0$). Notice that the net agglomeration externality is then negative, $\alpha + \gamma(1-\eta) - \beta \leq 0$, so that an inflow of population into region i reduces indirect utility $W(i)$, ceteris paribus. In our baseline quantification, we choose parameter values for α, β, γ, and η such that this condition is satisfied. Those parameter choices are informed by available empirical estimates from the literature on agglomeration and dispersion forces. But we will also consider constellations where Condition 1 does not hold, in which case our model may still exhibit equilibria. We come back to those issues in the quantitative part below.