# Prices, Plant Size, and Product Quality

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**4.1. Basic Set-up**

There are two symmetric countries and in each country two sectors, a monopolistically competitive final-good sector and a perfectly competitive, constant-returns-to-scale intermediate-input sector. Both final goods and inputs may have quality differences, in manners that will be made clear below. In each country, a representative consumer has the following standard asymmetric constant elasticity-of-substitution utility function over final goods:

$$U = \left[\int_{\omega\in\Omega}[q(\omega) * x(\omega)]^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}} \tag{2}$$

where $q(\omega)$ is output quality, can be interpreted as any product attribute that the representative consumer values and that is chosen by firms, $x(\omega)$ is the quantity consumed, $\sigma > 1$ is a parameter capturing the elasticity of substitution between varieties.

Consumer optimization yields the following demand for a particular variety, ω, in each country.

Write the Lagrangian:

$$\mathcal{L} = \left[\int_{\omega\in\Omega}(q(\omega)x(\omega))^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}} - \lambda\left[\int_{\omega\in\Omega}p(\omega)x(\omega)\, d\omega - R\right]$$

$$\frac{\partial\mathcal{L}}{\partial x(\omega)} = \frac{\sigma}{\sigma-1}\left[\int_{\omega\in\Omega}(q(\omega)x(\omega))^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{1}{\sigma-1}} * \frac{\sigma-1}{\sigma}\big(q(\omega)x(\omega)\big)^{\frac{-1}{\sigma}} * q(\omega) - \lambda p(\omega) = 0$$

$$\left[\int_{\omega\in\Omega}(q(\omega)x(\omega))^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{1}{\sigma-1}} * \big(q(\omega)x(\omega)\big)^{\frac{-1}{\sigma}} * q(\omega) = \lambda p(\omega)$$

$$U^{\frac{1}{\sigma}} * q(\omega)^{\frac{\sigma-1}{\sigma}} * x(\omega)^{\frac{-1}{\sigma}} = \lambda p(\omega)$$

$$U^{\frac{1}{\sigma}} * q(\omega)^{\frac{\sigma-1}{\sigma}} * \lambda^{-1}p(\omega)^{-1} = x(\omega)^{\frac{1}{\sigma}}$$

$$x(\omega) = U * q(\omega)^{\sigma-1} * \lambda^{-\sigma}p(\omega)^{-\sigma}$$

$$p(\omega)x(\omega) = U * q(\omega)^{\sigma-1} * \lambda^{-\sigma}p(\omega)^{1-\sigma}$$

$$= U * \lambda^{-\sigma} * \left(\frac{p(\omega)}{q(\omega)}\right)^{1-\sigma}$$

$$x(\omega)q(\omega)^{1-\sigma} = U * \lambda^{-\sigma} * p(\omega)^{-\sigma}$$

Integrate over $\Omega$:

$$\int_{\omega\in\Omega} p(\omega)x(\omega)\, d\omega = \int_{\omega\in\Omega} U * \lambda^{-\sigma} * \left(\frac{p(\omega)}{q(\omega)}\right)^{1-\sigma} d\omega$$

$$= U * \lambda^{-\sigma} * \int_{\omega\in\Omega} \left(\frac{p(\omega)}{q(\omega)}\right)^{1-\sigma} d\omega$$

Rewrite

$$U = \left[\int_{\omega\in\Omega} \left(q(\omega)x(\omega)\right)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}$$

$$= \left[\int_{\omega\in\Omega} \left(p(\omega)x(\omega) * q(\omega)/p(\omega)\right)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}$$

$$= \left[\int_{\omega\in\Omega} \left(U * \lambda^{-\sigma} * \left(\frac{p(\omega)}{q(\omega)}\right)^{1-\sigma} * q(\omega)/p(\omega)\right)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}$$

$$= \left[\int_{\omega\in\Omega} \left(U * \lambda^{-\sigma} * \left(\frac{p(\omega)}{q(\omega)}\right)^{-\sigma}\right)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}$$

$$= U * \lambda^{-\sigma} \left[\int_{\omega\in\Omega} \left(\frac{p(\omega)}{q(\omega)}\right)^{1-\sigma} d\omega\right]^{\frac{\sigma}{\sigma-1}}$$

$$\lambda^{\sigma} = \left[\int_{\omega\in\Omega} \left(\frac{p(\omega)}{q(\omega)}\right)^{1-\sigma} d\omega\right]^{\frac{\sigma}{\sigma-1}}$$

$$\lambda = \left[\int_{\omega\in\Omega} \left(\frac{p(\omega)}{q(\omega)}\right)^{1-\sigma} d\omega\right]^{\frac{1}{\sigma-1}}$$

Remember that $R \equiv PC$, this gives:

$$R \equiv \int_{\omega\in\Omega} p(\omega)x(\omega)\, d\omega = \int_{\omega\in\Omega} U * \lambda^{-\sigma} * \left(\frac{p(\omega)}{q(\omega)}\right)^{1-\sigma} d\omega$$

$$= P * \int_{\omega\in\Omega} q(\omega)x(\omega)\, d\omega$$

(Given $x(\omega)q(\omega)^{1-\sigma} = U * \lambda^{-\sigma} * p(\omega)^{-\sigma}$ or $x(\omega)q(\omega) = U * q(\omega)^{\sigma} * \lambda^{-\sigma} * p(\omega)^{-\sigma}$)

$$= P * \int_{\omega\in\Omega} \left[U * q(\omega)^{\sigma} * \lambda^{-\sigma} p(\omega)^{-\sigma}\right] d\omega$$

$$= U\lambda^{-\sigma}P * \int_{\omega\in\Omega} [q(\omega)^{\sigma} * p(\omega)^{-\sigma}]\, d\omega$$

Since $p(\omega)x(\omega) = U * q(\omega)^{\sigma-1} * \lambda^{-\sigma}p(\omega)^{1-\sigma}$, then

$$x(\omega) = U * q(\omega)^{\sigma-1} * \lambda^{-\sigma} * p(\omega)^{-\sigma}$$

$$= U * q(\omega)^{\sigma-1} * \left[\int_{\omega\in\Omega} \left(\frac{p(\omega)}{q(\omega)}\right)^{1-\sigma} d\omega\right]^{-\frac{\sigma}{\sigma-1}} * p(\omega)^{-\sigma}$$

$$= \left[\int_{\omega\in\Omega} (q(\omega)x(\omega))^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}} * q(\omega)^{\sigma-1} * \left[\int_{\omega\in\Omega} \left(\frac{p(\omega)}{q(\omega)}\right)^{1-\sigma} d\omega\right]^{-\frac{\sigma}{\sigma-1}} * p(\omega)^{-\sigma}$$

$$= \left[\int_{\omega\in\Omega} (q(\omega)x(\omega))^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}} * q(\omega)^{\sigma-1} * \left(\frac{p(\omega)}{\left[\int_{\omega\in\Omega} \left(\frac{p(\omega)}{q(\omega)}\right)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}}\right)^{-\sigma}$$

$$x(\omega) = X * q(\omega)^{\sigma-1} * \left(\frac{p(\omega)}{P}\right)^{-\sigma} = X * q(\omega)^{\sigma-1} * \left(\frac{p_O(\omega)}{P}\right)^{-\sigma} \quad (3)$$

where $P \equiv \left[\int_{\omega\in\Omega} \left(\frac{p_O(\omega)}{q(\omega)}\right)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}} = \lambda^{-1}$ is an aggregate quality-adjusted price index, and $X \equiv \left[\int_{\omega\in\Omega} (q(\omega)x(\omega))^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}$ is a quality-adjusted consumption aggregate of the varieties available on the market.

$$R \equiv \int_{\omega\in\Omega} p(\omega)x(\omega)\, d\omega$$

$$= \int_{\omega\in\Omega} p(\omega) * X * q(\omega)^{\sigma-1} * \left(\frac{p(\omega)}{P}\right)^{-\sigma} d\omega$$

$$= P^{\sigma}X \int_{\omega\in\Omega} q(\omega)^{\sigma-1} * p(\omega)^{1-\sigma}\, d\omega$$

$$= P^{\sigma}XP^{1-\sigma} = PX$$

Like Melitz (2003), we begin with an inelastic labour supply $L$ (measured in labour-hours) with the hourly wage normalized to one. But we add the intermediate-input sector, which transforms homogeneous labour-hours into intermediate inputs of different qualities. In the intermediate-input sector, the production function is simply:

$$F_I(l, c) = \frac{l}{c}\,, \quad (5)$$

where $c$ is the quality of the intermediate input produced and $l$ is the number of labour-hours used.

In other words, producing one unit of an intermediate input of quality $c$ requires $c$ labour hours and, given the wage normalization, entails cost $c$. In equilibrium the price of each intermediate input equals the marginal cost of producing the input: $p_I(c) = c$.

As in Melitz (2003), to enter the final-good sector, entrants must pay an investment cost, $f_e$ (measured in labour-hours) in order to receive a capability draw, $\lambda$. We assume that capability is drawn from a Pareto distribution with c.d.f.

$$G(\lambda) = 1 - \left(\frac{\lambda_m}{\lambda}\right)^k, \text{ with } 0 < \lambda_m \leq \lambda.$$

There is an exogenous probability of exit, $\delta$, in each period. There is a fixed cost of production, $f$, and an additional fixed cost of exporting, $f_x > f$, in each period. In the interests of simplicity, we assume that there are no variable costs of trade. Since there is no cost of differentiation, each plant in final-good sector produces a distinct good and $\lambda$ can be used to index both plants and varieties.

Production in the final-good sector is described by two functions, one describing the production of physical units of output and the other describing the production of quality. The production of physical units is assumed to be:

$$F(n) = n\lambda^a \tag{6}$$

where $n$ is the number of units of inputs used and $a$ is a parameter reflecting the extent to which capability lowers unit costs, with $a > 0$. This function implies that $1/\lambda^a$ units of inputs are used for each physical unit of output, and hence the marginal cost of each unit of output is $\frac{p_I(c)}{\lambda^a}$.

Plants in the final-good sector optimize over the choices of input quality, $c$, fixed quality investment, $f_q$, output price, $p_o$, and which markets to enter. Let $Z = 1$ if the plant enters the export market, and 0 otherwise. The profit function for each final-good producer is then:

$$\pi(p_o, c, f_q, Z; \lambda) = \left(p_o - \frac{p_I(c)}{\lambda^a}\right)x - f_q - f + Z\left[\left(p_o - \frac{p_I(c)}{\lambda^a}\right)x - f_x\right], \tag{7}$$

where demand, $x$, is given by equation (3) and depends on quality, $q$, and output price, $p_o$.

Each plant in the continuum of plants is small relative to the size of the market and ignores the effects of its decisions on the aggregates $X$ and $P$. Note that the symmetry of countries implies that, conditional on a choice of fixed quality costs, $f_q$, the optimal choices of $c$ and $p_o$ for exporters will be the same for both markets.

**4.2. Variant 1: Complementarity between input quality and plant capability**

In this variant, we assume that plant capability, $\lambda$, and input quality, $c$, are complements in generating output quality and that upgrading does not require fixed costs. In particular, we assume:

$$q = \left[\frac{1}{2}(\lambda^b)^\theta + \frac{1}{2}(c^2)^\theta\right]^{1/\theta} \tag{8}$$

The parameter $b$ in equation (8) reflects the availability of technology for translating higher plant capability into improved product quality. We impose the assumption that $\theta < 0$; this ensures that

$q(\cdot,\cdot)$ is log-supermodular in $\lambda$ and $c$. Intuitively, we are assuming that the marginal increase in output quality for a given increase in input quality is greater for more capable entrepreneurs.

As mentioned above, profit maximization and free entry in the intermediate-input sector imply that $p_I(c) = c$ for all levels of input quality produced in equilibrium.

$$\pi_I(l,c) = p_I(c)F_I(l,c) - l = p_I(c)\frac{l}{c} - l = 0$$

$$p_I(c) = c$$

The number of units of each quality produced is determined by demand from the final-good sector. In the final-good sector, the first-order conditions for the plant's maximization problem in equation (7) imply the following:

$$\max_{c,f_q,p_o,Z} \pi(p_o, c, f_q, Z; \lambda) = \left(p_o - \frac{p_I(c)}{\lambda^a}\right)x - f_q - f + Z\left[\left(p_o - \frac{p_I(c)}{\lambda^a}\right)x - f_x\right]$$

$$= \left(p_o - \frac{c}{\lambda^a}\right)x - f_q - f + Z\left[\left(p_o - \frac{c}{\lambda^a}\right)x - f_x\right]$$

$$= (1+Z)\left(p_o - \frac{c}{\lambda^a}\right)X * q(\omega)^{\sigma-1} * \left(\frac{p_o(\omega)}{P}\right)^{-\sigma} - f_q - f - Zf_x$$

$$= (1+Z)\left(p_o - \frac{c}{\lambda^a}\right) * X * \left[\frac{1}{2}(\lambda^b)^\theta + \frac{1}{2}(c^2)^\theta\right]^{(\sigma-1)/\theta} * \left(\frac{p_o}{P}\right)^{-\sigma} - f_q - f - Zf_x$$

where demand $x(\omega) = X * q(\omega)^{\sigma-1} * \left(\frac{p_o(\omega)}{P}\right)^{-\sigma}$, $q(\omega) = \left[\frac{1}{2}(\lambda^b)^\theta + \frac{1}{2}(c^2)^\theta\right]^{1/\theta}$ denotes output quality, $\lambda$ is plant's capability draw, $f_q$ is a fixed investment in quality, $f$ is a fixed cost of production, $f_x > f$ is an additional fixed cost of exporting, $c$ is input quality, $p_I(c)$ is the price of an intermediate input of quality $c$. Note that each plant in the continuum of plants is small relative to the size of the market and ignores the effects of its decisions on the aggregates $X$ and $P$.

$$\frac{\partial\pi}{\partial c} = 0 = (1+Z) * \left(-\frac{1}{\lambda^a}\right) * X * \left[\frac{1}{2}(\lambda^b)^\theta + \frac{1}{2}(c^2)^\theta\right]^{(\sigma-1)/\theta} * \left(\frac{p_o}{P}\right)^{-\sigma} +$$
$$(1+Z) * \left(p_o - \frac{c}{\lambda^a}\right) * X * (\sigma-1)/\theta * \left[\frac{1}{2}(\lambda^b)^\theta + \frac{1}{2}(c^2)^\theta\right]^{(\sigma-1)/\theta-1} * \frac{\theta}{2}(c^2)^{\theta-1} * 2c * \left(\frac{p_o}{P}\right)^{-\sigma}$$
$$= (1+Z) * \left(-\frac{1}{\lambda^a}\right) * X * \left[\frac{1}{2}(\lambda^b)^\theta + \frac{1}{2}(c^2)^\theta\right]^{(\sigma-1)/\theta} * \left(\frac{p_o}{P}\right)^{-\sigma} +$$
$$(1+Z) * (\sigma-1) * \left(p_o - \frac{c}{\lambda^a}\right) * X * \left[\frac{1}{2}(\lambda^b)^\theta + \frac{1}{2}(c^2)^\theta\right]^{(\sigma-1)/\theta-1} * c^{2\theta-1} * \left(\frac{p_o}{P}\right)^{-\sigma}$$

Then we get

$$(1+Z) * (\sigma-1) * \left(p_o - \frac{c}{\lambda^a}\right) * X * \left[\frac{1}{2}(\lambda^b)^\theta + \frac{1}{2}(c^2)^\theta\right]^{(\sigma-1)/\theta-1} * c^{2\theta-1} * \left(\frac{p_o}{P}\right)^{-\sigma}$$
$$= (1+Z) * \frac{1}{\lambda^a} * X * \left[\frac{1}{2}(\lambda^b)^\theta + \frac{1}{2}(c^2)^\theta\right]^{(\sigma-1)/\theta} * \left(\frac{p_o}{P}\right)^{-\sigma}$$

$$(\sigma-1) * \left(p_o - \frac{c}{\lambda^a}\right) * c^{2\theta-1} = \frac{1}{\lambda^a} * \left[\frac{1}{2}(\lambda^b)^\theta + \frac{1}{2}(c^2)^\theta\right]$$
$$(\sigma-1) * \left(2p_o\lambda^a c^{2\theta-1} - 2c^{2\theta}\right) = (\lambda^b)^\theta + c^{2\theta}$$

Given $\pi = (1+Z)\left(p_o - \frac{c}{\lambda^a}\right) * X * \left[\frac{1}{2}(\lambda^b)^\theta + \frac{1}{2}(c^2)^\theta\right]^{(\sigma-1)/\theta} * \left(\frac{p_o}{P}\right)^{-\sigma} - f_q - f - Zf_x$,

$$\frac{\partial \pi}{\partial p_o} = 0$$

$$(1+Z) * X * \left[\frac{1}{2}(\lambda^b)^\theta + \frac{1}{2}(c^2)^\theta\right]^{(\sigma-1)/\theta} * \left(\frac{p_o}{P}\right)^{-\sigma}$$

$$-\sigma(1+Z)\left(p_o - \frac{c}{\lambda^a}\right) * X * \left[\frac{1}{2}(\lambda^b)^\theta + \frac{1}{2}(c^2)^\theta\right]^{(\sigma-1)/\theta} * \left(\frac{p_o}{P}\right)^{-\sigma-1} * \frac{1}{P} = 0$$

$$(1+Z) * X * \left[\frac{1}{2}(\lambda^b)^\theta + \frac{1}{2}(c^2)^\theta\right]^{(\sigma-1)/\theta} * \left(\frac{p_o}{P}\right)^{-\sigma}$$

$$= \sigma(1+Z)\left(p_o - \frac{c}{\lambda^a}\right) * X * \left[\frac{1}{2}(\lambda^b)^\theta + \frac{1}{2}(c^2)^\theta\right]^{(\sigma-1)/\theta} * \left(\frac{p_o}{P}\right)^{-\sigma-1} * \frac{1}{P}$$

$$X * \left(\frac{p_o}{P}\right)^{-\sigma} = \sigma\left(p_o - \frac{c}{\lambda^a}\right) * X * \left(\frac{p_o}{P}\right)^{-\sigma-1} * \frac{1}{P}$$

$$1 = \sigma\left(p_o - \frac{c}{\lambda^a}\right) * \left(\frac{p_o}{P}\right)^{-1} * \frac{1}{P}$$

$$p_o = \sigma\left(p_o - \frac{c}{\lambda^a}\right)$$

$$(1-\sigma)p_o = -\frac{\sigma c}{\lambda^a}$$

$$p_o = \frac{\sigma c}{(\sigma-1)\lambda^a}$$

Then plug the above formula into the F.O.C. w.r.t. c gives

$$(\sigma-1) * \left(2p_o\lambda^a c^{2\theta-1} - 2c^{2\theta}\right) = \left(\lambda^b\right)^\theta + c^{2\theta}$$

$$(\sigma-1) * \left(2\frac{\sigma c}{(\sigma-1)\lambda^a}\lambda^a c^{2\theta-1} - 2c^{2\theta}\right) = \lambda^{b\theta} + c^{2\theta}$$

$$(\sigma-1) * \left(\frac{2\sigma}{(\sigma-1)\lambda^a}\lambda^a c^{2\theta} - 2c^{2\theta}\right) = \lambda^{b\theta} + c^{2\theta}$$

$$(2\sigma - (\sigma-1)2)c^{2\theta} = \lambda^{b\theta} + c^{2\theta}$$

$$(2\sigma - 1 - (\sigma-1)2)c^{2\theta} = \lambda^{b\theta}$$

$$c^*(\lambda) = \lambda^{b/2} \tag{9a}$$

*Then*

$$p_o(\lambda) = \frac{\sigma c}{(\sigma-1)\lambda^a} = \frac{\sigma\lambda^{b/2}}{(\sigma-1)\lambda^a} = \frac{\sigma}{(\sigma-1)} * \lambda^{\frac{b}{2}-a} \tag{9c}$$

$$c^*(\lambda) = p_I^*(\lambda) = \lambda^{b/2}$$

$$q^*(\lambda) = \left[\frac{1}{2}(\lambda^b)^\theta + \frac{1}{2}(c^2)^\theta\right]^{1/\theta} = \left[\frac{1}{2}(\lambda^b)^\theta + \frac{1}{2}(\lambda^b)^\theta\right]^{1/\theta} = \lambda^b \tag{9b}$$

$$r^*(\lambda) = (1+Z) * p_o * x = (1+Z) * p_o * X * \left[\frac{1}{2}(\lambda^b)^\theta + \frac{1}{2}(c^2)^\theta\right]^{(\sigma-1)/\theta} * \left(\frac{p_o}{P}\right)^{-\sigma}$$

$$= (1+Z) * XP^\sigma * \left[\frac{1}{2}(\lambda^b)^\theta + \frac{1}{2}(c^2)^\theta\right]^{(\sigma-1)/\theta} * p_o{}^{1-\sigma}$$

$$= (1+Z) * XP^\sigma * \left[\frac{1}{2}(\lambda^b)^\theta + \frac{1}{2}\left(\lambda^b\right)^\theta\right]^{(\sigma-1)/\theta} * \left(\frac{\sigma}{(\sigma-1)} * \lambda^{\frac{b}{2}-a}\right)^{1-\sigma}$$

$$= (1+Z) * XP^{\sigma} * \lambda^{(\sigma-1)b} * (\frac{\sigma}{(\sigma-1)} * \lambda^{\frac{b}{2}-a})^{1-\sigma}$$

$$= (1+Z) * (\frac{\sigma-1}{\sigma})^{\sigma-1} * XP^{\sigma} * \lambda^{(\sigma-1)b} * \lambda^{(\frac{b}{2}-a)(1-\sigma)}$$

$$= (1+Z) * (\frac{\sigma-1}{\sigma})^{\sigma-1} * XP^{\sigma} * \lambda^{(\sigma-1)(b-\frac{b}{2}+a)}$$

$$= (1+Z) * (\frac{\sigma-1}{\sigma})^{\sigma-1} * XP^{\sigma} * \lambda^{(\sigma-1)(\frac{b}{2}+a)}$$

$$= (1+Z) * (\frac{\sigma-1}{\sigma})^{\sigma-1} * XP^{\sigma} * \lambda^{\eta} \quad (9d)$$

where $\eta \equiv (\sigma-1)\left(\frac{b}{2}+a\right) > 0$.

To ensure that both the distribution of capability draws and the distribution of plant revenues in the final-good sector have finite means the assumption we need on the shape parameter of the Pareto distribution is $k > max(\eta, 1)$.

The values of the cut-offs (which, because of symmetry, are the same in each country) are pinned down by three conditions. First, the profit of the plant on the margin between remaining in the domestic market and stopping production is zero:

$$\pi_d(\lambda^*) = \left[p_o(\lambda) - \frac{c}{\lambda^a}\right] x - f_q - f$$

$$= \left[1 - \frac{c}{\lambda^a p_o(\lambda)}\right] r_d^*(\lambda^*) - f_q - f$$

$$= \left[1 - \frac{c}{\lambda^a \frac{\sigma c}{(\sigma-1)\lambda^a}}\right] r_d^*(\lambda^*) - f_q - f$$

$$= \left[1 - \frac{\sigma-1}{\sigma}\right] r_d^*(\lambda^*) - f_q - f$$

$$= \frac{1}{\sigma} r_d^*(\lambda^*) - 0 - f$$

$$= \frac{r_d^*(\lambda^*)}{\sigma} - f = 0 \quad (C1)$$

where $r_d^*(\lambda^*) = (\frac{\sigma-1}{\sigma})^{\sigma-1} * XP^{\sigma} * \lambda^{*\eta}$ represents revenues in the domestic market (given by (9d) when Z = 0).

Second, the additional profit of entering the export market for the plant on the margin between entering the export market and producing only for the domestic market is also zero:

$$\pi_x(\lambda_x^*) = \frac{r_x^*(\lambda_x^*)}{\sigma} - f_x = 0 \quad (C2)$$

Third, there is a free-entry condition: the ex-ante expected present discounted value of receiving a capability draw must be equal to the investment cost required to receive the draw, such that ex ante expected profits are zero. Formally:

$$[1 - G(\lambda^*)] \sum_{t=0}^{\infty} (1-\delta)^t \left\{\frac{E\left(r_d^*(\lambda)\right)}{\sigma} - f\right\} + [1 - G(\lambda_x^*)] \sum_{t=0}^{\infty} (1-\delta)^t \left\{\frac{E\left(r_d^*(\lambda)\right)}{\sigma} - f_x\right\} - f_e = 0 \quad (C3)$$

Using C1, C2, Pareto distribution $G(\lambda) = 1-\left(\frac{\lambda_m}{\lambda}\right)^k$ with $0 < \lambda_m \leq \lambda$, and the facts that $\frac{r_d^*(\lambda)}{r_d^*(\lambda^*)} = \frac{(\frac{\sigma-1}{\sigma})^{\sigma-1}*XP^{\sigma}*\lambda^{\eta}}{(\frac{\sigma-1}{\sigma})^{\sigma-1}*XP^{\sigma}*\lambda^{*\eta}} = \left(\frac{\lambda}{\lambda^*}\right)^{\eta}$ , $\frac{r_x^*(\lambda)}{r_x^*(\lambda_x^*)} = \left(\frac{\lambda}{\lambda_x^*}\right)^{\eta}$, we have that, conditional on entering each market,

$$E\left(r_d^*(\lambda)\right) = \int_{\lambda^*} k\left(\frac{\lambda^*}{\lambda}\right)^{k-1} * \lambda_m * \lambda^{-2} * r_d^*(\lambda)d\lambda$$

$$= \int_{\lambda^*} k\left(\frac{\lambda^*}{\lambda}\right)^{k} * \lambda^{-1} * \left(\frac{\lambda}{\lambda^*}\right)^{\eta} * r_d^*(\lambda^*)d\lambda$$

$$= \int_{\lambda^*} k\left(\frac{\lambda^*}{\lambda}\right)^{k} * \lambda^{-1} * \left(\frac{\lambda}{\lambda^*}\right)^{\eta} * \sigma f d\lambda$$

$$= \int_{\lambda^*} k{\lambda^*}^{k} * \lambda^{\eta-k-1} * \left(\lambda^*\right)^{-\eta} * \sigma f d\lambda$$

$$= \frac{k}{\eta-k}\lambda^{\eta-k} * {\lambda^*}^{k}\left(\lambda^*\right)^{-\eta} * \sigma f|_{\lambda^*}^{\infty}$$

$$= 0 - \frac{k}{\eta-k}{\lambda^*}^{\eta-k} * {\lambda^*}^{k} * \left(\lambda^*\right)^{-\eta} * \sigma f$$

$$= \frac{k}{k-\eta}\sigma f$$

$$E\left(r_x^*(\lambda)\right) = \frac{k}{k-\eta}\sigma f_x$$

Then using (C3) we can solve for the entry cut-offs:

$$[1-G(\lambda^*)]\sum_{t=0}^{\infty}(1-\delta)^t\left\{\frac{E\left(r_d^*(\lambda)\right)}{\sigma}-f\right\} + [1-G(\lambda_x^*)]\sum_{t=0}^{\infty}(1-\delta)^t\left\{\frac{E\left(r_d^*(\lambda)\right)}{\sigma}-f_x\right\} - f_e = 0$$

$$\left(\frac{\lambda_m}{\lambda^*}\right)^k \frac{\frac{\frac{k}{k-\eta}\sigma f}{\sigma}-f}{\delta} + \left(\frac{\lambda_m}{\lambda_x^*}\right)^k \frac{\frac{\frac{k}{k-\eta}\sigma f_x}{\sigma}-f_x}{\delta} = f_e$$

$$\left(\frac{\lambda_m}{\lambda^*}\right)^k \frac{\frac{k}{k-\eta}f-f}{\delta} + \left(\frac{\lambda_m}{\lambda_x^*}\right)^k \frac{\frac{k}{k-\eta}f_x-f_x}{\delta} = f_e$$

$$\left(\frac{\lambda_m}{\lambda^*}\right)^k \frac{\frac{\eta}{k-\eta}f}{\delta} + \left(\frac{\lambda_m}{\lambda_x^*}\right)^k \frac{\frac{\eta}{k-\eta}f_x}{\delta} = f_e$$

$$\left(\frac{\lambda_m}{\lambda^*}\right)^k \frac{\eta}{k-\eta}f + \left(\frac{\lambda_m}{\lambda_x^*}\right)^k \frac{\eta}{k-\eta}f_x = \delta f_e$$

$$\frac{\eta f}{k-\eta}\{\left(\frac{\lambda_m}{\lambda^*}\right)^k + \left(\frac{\lambda_m}{\lambda_x^*}\right)^k\frac{f_x}{f}\} = \delta f_e$$

$$\left(\frac{\lambda_m}{\lambda^*}\right)^k \frac{\eta f}{k-\eta}\{1+\left(\frac{\lambda^*}{\lambda_x^*}\right)^k\frac{f_x}{f}\} = \delta f_e$$

$$\frac{\eta f}{\delta f_e(k-\eta)}\{1+\left(\frac{\lambda^*}{\lambda_x^*}\right)^k\frac{f_x}{f}\} = \left(\frac{\lambda^*}{\lambda_m}\right)^k$$

$$\frac{\lambda^*}{\lambda_m} = \left\{\frac{\eta f}{\delta f_e(k-\eta)}[1+\left(\frac{\lambda^*}{\lambda_x^*}\right)^k\frac{f_x}{f}]\right\}^{1/k}$$

$$\lambda^* = \lambda_m\left\{\frac{\eta f}{\delta f_e(k-\eta)}[1+\left(\frac{\lambda^*}{\lambda_x^*}\right)^k\frac{f_x}{f}]\right\}^{1/k}$$

Since

$$(\frac{\sigma-1}{\sigma})^{\sigma-1} * XP^{\sigma} * \lambda^{*\eta} = \sigma f$$

$$(\frac{\sigma-1}{\sigma})^{\sigma-1} * XP^{\sigma} * \lambda_x^{*\eta} = \sigma f_x$$

then

$$\frac{\lambda^{*\eta}}{\lambda_x^{*\eta}} = \frac{f}{f_x}$$

$$\frac{\lambda^*}{\lambda_x^*} = \left(\frac{f}{f_x}\right)^{1/\eta}$$

Thus

$$\lambda^* = \lambda_m\left\{\frac{\eta f}{\delta f_e(k-\eta)}[1+\left(\frac{f}{f_x}\right)^{k/\eta}\frac{f_x}{f}]\right\}^{1/k}$$

$$= \lambda_m\left\{\frac{\eta f}{\delta f_e(k-\eta)}[1+\left(\frac{f}{f_x}\right)^{\frac{k}{\eta}-1}]\right\}^{1/k} \quad \text{(C4)}$$

$$\lambda_x^* = \lambda^* * \left(\frac{f_x}{f}\right)^{1/\eta} \quad \text{(C5)}$$

Let $M_e$ be the mass of entrepreneurs who pay the investment cost, $M$ be the mass of firms in business in the domestic market, and $M_x$ be the mass of exporters. Total payments by final-good producers for material inputs are equal to total payments by intermediate-input producers for labor-hours. The per-period fixed costs, $f$ and $f_x$, are also paid to workers.

Given the wage normalization, payments to workers are equal to the number of labor-hours utilized. Thus, the total effective utilization of labor-hours by existing final-good producers is the difference between total revenues and total profits of final-good producers, denoted Π.

The labor market clearing condition is that total effective labor-hours utilization for final-good production plus labor-hours utilization for investment equals total labor supply:

$$L = [ME\left(r_d^*(\lambda)\right) + M_xE\left(r_x^*(\lambda)\right) - \Pi] + M_ef_e \quad \text{(C6)}$$

In steady state, the mass of new entrants in each country is equal to the mass of plants that die:

$$M_e * \left(1-G(\lambda^*)\right) = \delta M \quad \text{(C7)}$$

Combining this with the free-entry condition (C3), we have:

$$[1-G(\lambda^*)]\sum_{t=0}^{\infty}(1-\delta)^t\left\{\frac{E\left(r_d^*(\lambda)\right)}{\sigma}-f\right\} + [1-G(\lambda_x^*)]\sum_{t=0}^{\infty}(1-\delta)^t\left\{\frac{E\left(r_d^*(\lambda)\right)}{\sigma}-f_x\right\} = f_e$$

$$\frac{\delta M}{1-G(\lambda^*)}[1-G(\lambda^*)]\sum_{t=0}^{\infty}(1-\delta)^t\left\{\frac{E\left(r_d^*(\lambda)\right)}{\sigma}-f\right\}+\frac{\delta M}{1-G(\lambda^*)}[1-G(\lambda_x^*)]\sum_{t=0}^{\infty}(1-\delta)^t\left\{\frac{E\left(r_d^*(\lambda)\right)}{\sigma}-f_x\right\}=M_e f_e$$

$$M\left\{\frac{E\left(r_d^*(\lambda)\right)}{\sigma}-f\right\}+M\frac{[1-G(\lambda_x^*)]}{1-G(\lambda^*)}\left\{\frac{E\left(r_d^*(\lambda)\right)}{\sigma}-f_x\right\}=M_e f_e$$

$$\Pi=M*\left\{\frac{E\left(r_d^*(\lambda)\right)}{\sigma}-f\right\}+M_x\left\{\frac{E\left(r_d^*(\lambda)\right)}{\sigma}-f_x\right\}=M_e f_e$$

$$M*\left\{\frac{E\left(r_d^*(\lambda)\right)}{\sigma}-f\right\}+\frac{M}{1-G(\lambda^*)}*[1-G(\lambda_x^*)]\left\{\frac{E\left(r_d^*(\lambda)\right)}{\sigma}-f_x\right\}=M_e f_e$$

$$M*\left\{\left\{\frac{E\left(r_d^*(\lambda)\right)}{\sigma}-f\right\}+\frac{1-G(\lambda_x^*)}{1-G(\lambda^*)}\left\{\frac{E\left(r_d^*(\lambda)\right)}{\sigma}-f_x\right\}\right\}=M_e f_e \tag{C8}$$

Together (C6) and (C8) imply:

$$L=\left[ME\left(r_d^*(\lambda)\right)+M_xE\left(r_x^*(\lambda)\right)-\Pi\right]+M_e f_e$$
$$=\left[ME\left(r_d^*(\lambda)\right)+M_xE\left(r_x^*(\lambda)\right)-\Pi\right]+\Pi$$
$$L=ME\left(r_d^*(\lambda)\right)+M_xE\left(r_x^*(\lambda)\right) \tag{C9}$$

Given the symmetry between countries, $M_xE\left(r_x^*(\lambda)\right)$ is equal to domestic expenditures on foreign varieties as well as export revenue of domestic firms. Thus (C9) is also the clearing condition for the final-good market: total income (and hence total expenditures, $1*L$) of workers is equal to total revenues of final-good producers.

Using the fact that $\frac{M_x}{M}=\frac{1-G(\lambda_x^*)}{1-G(\lambda^*)}=\frac{\left(\frac{\lambda_m}{\lambda_x^*}\right)^k}{\left(\frac{\lambda_m}{\lambda^*}\right)^k}=\left(\frac{\lambda^*}{\lambda_x^*}\right)^k=\left(\frac{f_x}{f}\right)^{-k/\eta}=\left(\frac{f}{f_x}\right)^{k/\eta}$, we can solve for the mass of final-good producers in steady state:

$$L=ME\left(r_d^*(\lambda)\right)+M_xE\left(r_x^*(\lambda)\right)$$
$$=ME\left(r_d^*(\lambda)\right)+M\left(\frac{f}{f_x}\right)^{k/\eta}E\left(r_x^*(\lambda)\right)$$
$$=M*\frac{k}{k-\eta}\sigma f+M\left(\frac{f}{f_x}\right)^{\frac{k}{\eta}}*\frac{k}{k-\eta}\sigma f_x$$
$$=M*\frac{k}{k-\eta}\sigma f\left[1+\left(\frac{f}{f_x}\right)^{\frac{k-\eta}{\eta}}\right]$$

$$M=\frac{L(k-\eta)}{k\sigma f\left[1+\left(\frac{f}{f_x}\right)^{\frac{k-\eta}{\eta}}\right]}=\frac{L(k-\eta)}{k\sigma f}\frac{1}{1+\left(\frac{f}{f_x}\right)^{\frac{k-\eta}{\eta}}} \tag{C10}$$

$$M_x=M\left(\frac{f}{f_x}\right)^{k/\eta}=\frac{L(k-\eta)}{k\sigma f}\frac{\left(\frac{f}{f_x}\right)^{k/\eta}}{1+\left(\frac{f}{f_x}\right)^{\frac{k-\eta}{\eta}}}$$