



Faculteit Bedrijf en Organisatie

How will quantum computing affect the mainframe environment and its applications?

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Preface

Why did I choose to explore the quantum realm without any specific pre-knowledge/education? Quantum is being transformed to a real buzzword much like data science used to be. The field has been opened up from highly specialised academics to an open source community willing to teach outsiders from the very start.

Also the mere fact that the field of quantum computing is developing to a profitable and sustainable business models so rapidly has astonished me from my very first contacts with the environment.

I would like to thank Frank Harkins from IBM for always being available to have a discussion about quantum computing and how it will influence our societies and even our very nature of problem-solvings.

But most of all I would like to congratulate the research environment as a around quantum computing on how accepting and supportive they are in all interested parties. It has become as comparable to learning a new sector inside computer science. In the next decade this will only further increase to where quantum computing becomes an essential part of solving anything in a fair time scheme.

So that is exactly why this paper should serve as the starting tool for computer scientist willing to expand their skill sets far outside the skill sets that is to be expected of a single computer scientist.

Samenvatting

Zoals het onderzoek aantoont is het onderzoeksgebied van kwantum computers nog in zijn beginjaren en moeten we kritisch blijven ten opzichte van elke nieuwe uitgave in verband met nieuw onderzoek. Echter is het ook belangrijk dat we buiten het puur theoretische deel ook effectief op zoek gaan naar de praktische toepassingen en/ of inzichten in onze huidige processen met evt. de toepassing van kwantum verwerking.

In het onderzoek proberen we een duidelijk beeld weer te geven aan de lezer, zodat hij/ zij zelfstandig kan nadenken over toepassingen en/ of zelf toevoegingen maken aan de vele open source gemeenschappen op Github. Dit hebben we proberen te bereiken door enkele praktische vergelijkingen te maken met de hulp van het Python-framework Qiskit tussen de uitvoering op een klassiek systeem en een kwantum systeem. De resultaten wijzen inderdaad op een mogelijke versnelling, maar zoals te zien aan de werkelijke uitvoeringen op de echte kwantum computers van IBM is het moeilijk om deze nieuwe technologieën al meteen te gebruiken in bestaande productiesystemen.

In de paper is er ook een focus gelegd op het complete aspect van de mainframe in het verhaal over kwantum computers. De verwachtingen zijn enorm over de samenwerking tussen onze huidige supercomputers en een kwantum systeem, maar dit betekent dus ook dat er een enorm potentieel bestaat tussen een mainframe machine en een kwantum systeem. De winst van kwantum wordt voorzien in enorme versnellingen van data verwerkingen en dat is nu exact waar een mainframe machine zo krachtig in is, het genereren van enorme hoeveelheden data.

Er ligt ook een sterke nadruk op het tonen van alle kanten van quantumcomputing om er zeker van te zijn dat de lezer een volledig beeld heeft van het hele onderwerp.

Abstract

As the research shows, the field of quantum computers is still in its infancy and we must remain critical of any new publication related to new research. However, it is also important that beyond the purely theoretical part, we also effectively look for practical applications and/or insights into our current processes with the possible application of quantum processing.

In the research we try to present a more clear picture to the reader, so that he or she can independently think about applications and/or make additions to the many open source communities on Github. We have tried to achieve this by making some practical comparisons with the help of the Python framework Qiskit between classical computing and quantum computing. The results indeed point to possible acceleration, but as can be seen from the actual executions on IBM's real quantum computers, it is difficult to use these new technologies in existing production systems at this point in time.

In the paper there is also a focus on the complete aspect of the mainframe in the story about quantum computers. The expectations are huge about the cooperation between our current supercomputers and a quantum system, but this also means that there is a huge potential between a mainframe machine and a quantum system. The benefits of quantum are provided in huge accelerations of data processing and that is exactly what a mainframe machine is so powerful at, generating huge amounts of data.

There is also a strong emphasis on showing all sides of quantum computing as to make sure the reader has a full picture of the whole subject.

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1. Introduction

Why does everyone suddenly jump on the subject of quantum computing (QC) and why would it concern anyone at this point in time? Well we are rapidly reaching the limits of how small we are able to create the transistors on a chip, Moore's Law may very well be about to end. Currently we are able to create transistors so small that they themselves start being influenced by the quantum world which would undermine the whole point of building smaller and smaller components that are faster than its predecessors. (Hartnett, 2019)

The quantum field itself is also rapidly expanding due to the practical executions of QC, much in the same way data science has been expanding for the last 2 decades. Only 20 years ago data was something nice to have in a business to gain a possible edge over opponents, now data is the lifeblood for many of those companies. Data is what drives research, competitive advantage and various innovations. QC could offer our way of data handling and processing a surprising speed boost and expansion into regions we just were not able to even understand due to its quantum nature e.g. in sectors like Chemistry, Astronomy, Physics... And this is exactly why the mainframe environment could tie in so nicely into the research towards a classical and quantum computational combined environment. Mainframes drive the big enterprises who in turn drive the smaller ones that our societies are built upon. If QC could aid these big enterprises they would in turn let this information flow through into the lower sectors of our global economy. All this is why the paper will try and expose why we should start noticing about QC in the very same everyone suddenly started keeping track of data research with classical computing. (Arute et al., 2019) (Amico et al., 2019)

1.1 Problem Statement

What can QC actually solve as of this very moment, is a question every interested enterprise is trying to figure out first. The field has shown even in this very early state with the limited amount of computational resources much promise. Inside the fields of data processing there is a clear trend that as we further develop quantum computational expertise the possible business impacts are generated exponentially. Many big enterprises have finally figured out the most lucrative and easy-to-apply ways of capturing important data that could have business value. Now the actual issue that most definitely is worth addressing, is that the processing of the sheer amount of data has become unbearable in realistic time schemes.

Business needs all these data results as soon as possible to gain the edge over competitors and industry leaders. Quantum could in theory exponentially aid classical computers with the processing of this large amount of data. Mainframes especially are able to generate so much I/O with all sorts of data e.g. credit card spending, production analysis, transport optimization, that the mainframe together with QC could very well become the power couple of the 21st century. With platforms as Qiskit and Cirq everyone is able to contribute towards quantum research even within a mainframe minded environment. (Abraham et al., 2019) and (McClean et al., 2017)

1.2 Research question

The question of "How will QC affect the mainframe environment and its applications?" can be a really useful question to solve because it would allow the highly expertised environment of the mainframe to be able to think of possible applications of the quantum research with their mainframe systems. In this moment of time practical QC has come such a long way that this question could possibly give added business value to the mainframe industry and all of their users. Exploring this domain can provide valuable insights in all different kinds of sectors that make use of the mainframe's high-data capabilities.

1.3 Research objective

The paper is designed to allow individuals that are interested in QC and general computer science to get a better grasp on the real business impacts of the quantum realm. We expect that we can evaluate the real value of QC inside a business, but also we want to find actual value and applications inside the sector as a whole. Following up with the real practical showcase of these new quantum technologies using a Python framework that is freely available for anyone to download and use. The framework is called Qiskit and is at the moment of writing the only framework that allows the connection to real quantum devices. This connection is free of charge, where you are able to create quantum circuits and test them out on simulators of the framework itself to then follow these circuits up by sending them off to a quantum device and get back the results from it. By being able to look at

real executions you instantaneously receive a better grasp of the whole aspect of quantum phenomena.

1.4 Structure of this bachelor thesis

The paper will consist of the following chapters:

In chapter 2, we will introduce the very basic usage of QC to make sure every reader is able to understand the basic necessary principles to understand this paper.

In chapter 3, we will expose how classical and QC could offer a valuable partnership in their effort to speed up all research.

In chapter 4, we will finally show the real usages of quantum algorithms of the future through simulations or even executions on real devices with Qiskit. The algorithm that we will use will be an adaptation of Grover's algorithm for a unstructured search, more specifically an algorithm that is able to solve the 3-SAT problem in an advantageous manner.

This chapter will be designed for computer scientist that want to really understand the technology and want to learn and maybe even contribute themselves towards the many open source options out there surrounding QC.

In chapter 5, there will be an extensive discussion where we take in the results of the practical compartment of this paper. Furthermore we would still like to take a critical look at how QC has its benefits but also its disadvantages.

2. Quantum Essentials

To make sure everyone starts off on the same baseline to understand the full potential of this paper, we will introduce a few of the basic quantum principles. This paper is not targeting these specific principles but does use them to explain different practical consequences of the use of them within quantum computation. If there is any further interest regarding these ground fundamentals, we would refer you to the following papers, Rieffel and Polak (1998) and Shor (2000).

2.0.1 The Qubit and its representations

The foundation of any quantum related paper is and will always be the **qubit**. A qubit is just like a classical computing **bit** the foundational unit of its computer. Whilst a bit can either be on or off, a qubit has a certain statistical measurement to it. To be able to program on a quantum computer you need to think of the issue of computing an equation in a completely different way.

A qubit is not an infinite on or off, meaning that any type of computation needs to happen during the time frame qubit remaining stable without being thrown off its state by decoherence or any other external factors. One of the biggest issues with qubits is that they can behave unstable when influenced by the slightest of external influences, which also means the influence of an external observer. Once a qubit reaches an unstable state it has lost its quantum advantages and becomes a determined particle, which is not available for calculations any more. Meaning that during the execution of your program you are simply not able to look at the intermediary results as this would affect the final result, which would make the whole computation worthless. This means that debugging and looking at variables whilst you are executing a piece of code simply is not possible, which in turn

makes writing actual code for a quantum computer a lot more difficult. This aspect will be explained more clearly further on.

First of all to comprehend the nature of a qubit we need to understand that representing a qubit is only possible in a complex field, which shows of a certain amplitude of the state of the qubit in a point in time. When we think about a classical bit, we like to imagine a switch being turned on or off. With a qubit you would have to think about a sphere that is being transformed and shifted around its axes to transform its state. *Felix Bloch* was the individual that came up with the Bloch sphere that we currently use to clearly represent what a qubit is at a certain point in time. Looking at figure 2.1 you are able to see a representation of the state of a qubit in its elevated state. You need to think about this representation in a complex field on which transformations and rotations can be applied. During runtime a qubit is in a probabilistic state where it can not be fully determined in which state the qubit currently resides, which makes the whole execution harder. The moment we measure the state of the qubit it will degrade back to a determined result that shows us exactly the state of the qubit.

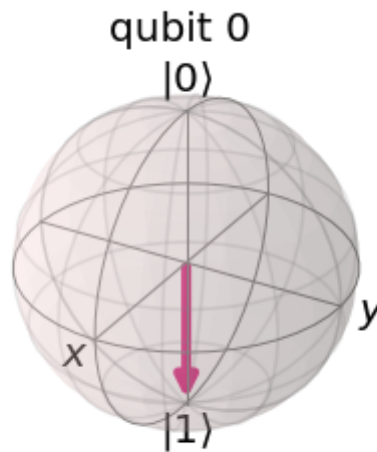


Figure 2.1: A Bloch sphere representation of a qubit in the $|1\rangle$ state. The Bloch sphere clearly indicates that the state of a qubit has a certain complex aspect to it.

Another way of demystifying what a qubit exactly means is by representing it through the use of matrices and the Bra-ket notation. By using these matrices and using matrix transformations we can more easily expose the way a qubit can be influenced during execution. If you take a look at the way of representing a qubit in its elevated $|1\rangle$ state or base $|0\rangle$ state, by representing the qubits as a matrix we are able to more clearly show how operations on a quantum device have an impact on the state of the qubit.

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

This image is mostly preferred by computer scientist because it gives them a clear image

of transformation in the same way an ordinary logic gate can influence an electrical signal. The combination of qubit basis states can be achieved by the utilisation of a tensor product. In the formulae below you are able to see how a 2-qubit system is represented through their matrix-representation. So look at the following transformations in much the same way one would look at an electrical signal would flow through a set of gates. All transformations are applied by performing a Kronecker product on the 2 states.

$$\begin{aligned} |00\rangle &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} & |01\rangle &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \\ |10\rangle &= \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} & |11\rangle &= \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \end{aligned}$$

To sum it up a bit can be only be in a state of on or off and this can be checked throughout execution, whilst a qubit is in a uncertain state during execution much like Schrödinger's cat. So once we observe the qubit it becomes just as determined as a normal bit would be. But determining the state during execution will affect the rest of the experiment and will remove the potential benefits of quantum in much the same way if Schrödinger went on with the experiment after observation that the cat died, he would be certain that cat would still be dead at a later point.

2.0.2 Superposition and entanglement

Now the next 2 principles are fully responsible of giving quantum computing its exponential speed-up compared to classical computing in certain tasks like factorisation and database searches. However these principles only offer that powerful advantage when they operate together in solving a certain quantum algorithm.

Superposition is a term a lot of people have heard about and how it could achieve major breakthroughs in the scientific world, but what it exactly represents is the real question.

We should think about qubits in superposition in a statistical way to receive a clearer picture. When a qubit is put into a state of superposition, the qubit operates between its elevated state and its ground state. By referring back to the representation of the Bloch sphere the qubit lives in a three dimensional complex field, where the only knowledge we can keep track of is its statistical chance of the state of the qubit. Until we have truly observed the qubit and have measured it extensively, the state of a qubit remains a statistical probability.

This all meaning that quantum particle can remain in both states at once whilst it has not been observed. To explain this more clearly from a computer science perspective, a qubit in superposition is during execution behaving as 0 and 1 at the same time. A concept that seems impossible within a classical frame of mind but also very advantageous if you

know where to look. E.g. if you are processing a big array of data through your classical processor, your processor will take one item of the array, process, convert and output it before it will take another item of the array and perform the same thing. A quantum processor could go about this process in a similar yet much more ingenious way. It would put an amount of qubits in superposition to represent the full array as input, perform the needed amount of quantum gates and receive the output in a single go instead of needing to loop over the full array. (Draper, 2000)

Entanglement is another interesting principle within the realm of quantum physics. This principle truly offers the quantum advantage that is predicted over every single sector. It refers to the correlation between entangled qubits where the state of one qubit influences the state of the correlated qubit in a way that it can be exploited and theoretically infinitely speed up computation. This entanglement can be achieved inside a quantum computer by the use of quantum gates on qubits in a state of superposition. You are able to put every single qubit available in a entangled state with the qubits available in the system. By having this exponential factor inside your quantum system, it becomes more clear where the quantum advantage can truly be gained. Such a quantum system can represent an immense amount of classical bits by using this phenomenon. The deterministic result of the qubits at the end of an experiment will always show the correlation in the results, keeping in mind that enough experiments are performed to defend against quantum decoherence mistakes and other external influences. (Brandao et al., 2016)

These two principles are constantly being used by a quantum processor as the one that Google showed of in their latest showcase of their quantum supremacy, Arute et al. (2019). Together they are able to exponentially increase the computing power of a quantum computer, as you add more and more qubits you are exponentially increases the available data items such a processor could handle. For example to be able to simulate the biggest medicine of the 20th century, penicillin you would need 286 functional qubits, which in turn would be able to generate the 2^{286} bits of memory. It would straight up be impossible to get this amount of classical RAM, so it is impossible to simulate this medicine fully. Actually getting to such a stable amount of qubits in itself will still be a scientific miracle. The whole limitation of 'just adding qubits' to the system runs into a barrier by Quantum decoherence.

2.0.3 Quantum decoherence

QC is not solely composed of benefits, the biggest downside is that as of now technology has not yet progressed far enough to actually provide the necessary amount of stable qubits to perform trustworthy calculations with. For example the simulations of penicillin, a system would need 286 qubits that remain stable for a prolonged period, but as of now Google has only been able to keep 53 qubits stable for a prolonged period of time using quantum error correction throughout the calculations. The loss of these quantum aspects during execution is called *quantum decoherence*, it is the phenomenon that describes how a qubit falls in an unstable state after being influenced by external forces or even internal influences from the qubits around it inside the system.

Referring to figures 2.2 and 2.3 below you are able to see that quantum decoherence is even a measurable phenomenon. The circuit below is specially build to show that quantum decoherence even shows up in the smallest of computations, where a qubit gets thrown in an elevated state of $|1\rangle$ and then gets measured and pictured on a classical bit. If you analyse the results you are able to see that quantum decoherence has occurred and de-elevated the state of the qubit back to $|0\rangle$. In this specific case from the 1024 shots taking on the real quantum device, around 8 percent or 81 of the shots, were worthless due to the quantum decoherence.

Taking all of this into account the whole field has one giant, non-circumventable downside that goes with it, the larger our quantum systems become, the more internal decoherence we receive from the higher concentration of qubits near each other. This all could mean that there is a limit to how big we are able to make quantum machines, because at a certain point, without proper error correction, the internal decoherence would make every single calculation useless because of the high probability of faulty data throughout. However if we are able to find a solution to this internal decoherence, the amount of qubits inside a system could be limitless and our data processing with it could also be limitless. (Hartnett, 2019)

It is of utmost importance that we add qubits in a controlled manner where we are able to perform better and better error-correction before we just add more qubits in the system. If everyone started adding qubits to their systems without performing any research regarding quantum error correction, we would only receive an increase in computing power with more unreliable results. And in this case quality is a way more important factor compared to brute quantity.



Figure 2.2: This is the quantum circuit that puts a qubit in an elevated state followed up with a measurement, to show off how quantum decoherence can influence the results.

2.0.4 Quantum gates

Now one might wonder, how do we create calculations with particles that are not observable and not tangible at a point in time. Quantum gates offer the solution to this question, a quantum gate affects one or more qubits during execution so that a programmer is able to perform changes to the state of the qubit but does not create an unstable qubit. From a programmers perspective they function in a similar way that a normal logic gate functions on an electrical signal inside a regular processor. Furthermore we will provide some frequently used quantum gates. One major difference rule that a quantum computer has to

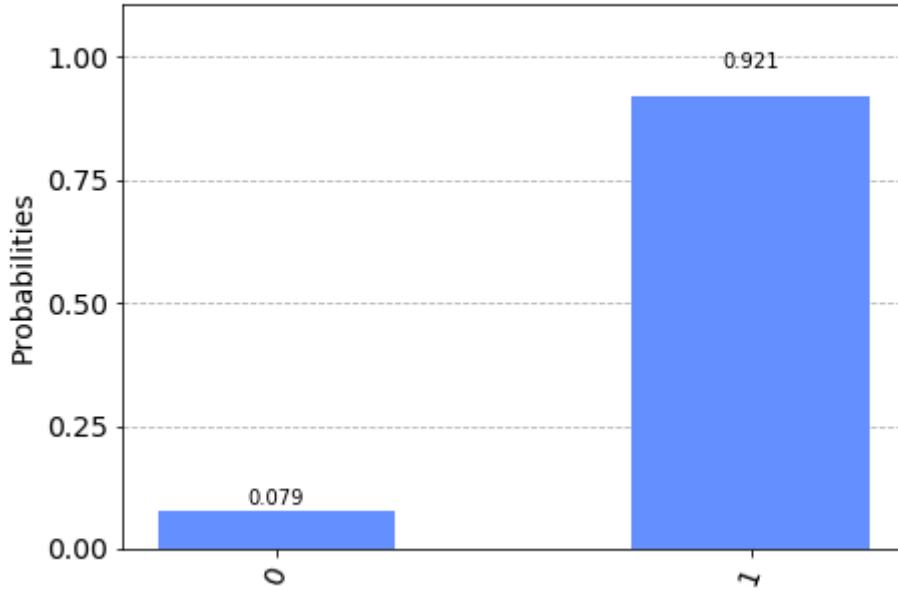


Figure 2.3: These are the results from which it is clearly visible that quantum decoherence has taken place on the initial $|1\rangle$ state to the $|0\rangle$ state.

listen to when it comes to its gates, is that quantum have to be 100 percent reversible while a classical computer does not have to deal with this limitation. This is clearly shown by the use of matrix representations when a qubit passes through a specific quantum gate and for a classical example, thing of an OR-gate you are able to see the outcome of the signal with an on or an off, but you can not know just from the outcome which initial signal influenced the OR-gate to be activated.

To understand the application of quantum gates inside a circuit on qubits, we would like to refer back to the representation of matrices where transformations are applied by performing a Kronecker product on the qubit in question.

Hadamard gate

The Hadamard gate is single most important gate for creating a quantum computation. This gate is responsible for putting a qubit inside a state of superposition and is also the one to get it out of this state. So in turn without this gate, quantum advantage would not exist. It maps $|0\rangle$ to $\frac{|0\rangle+|1\rangle}{\sqrt{2}}$ and $|1\rangle$ to $\frac{|0\rangle-|1\rangle}{\sqrt{2}}$, which are both superposition states.

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Pauli-X gate

Performs in a similar way a classical NOT-gate performs on an electrical signal or an absence of it. A qubit in $|0\rangle$ state going through a Paul-X gate would go in a $|1\rangle$ state.

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

CNOT gate

This gate works as a flip for a qubit. It is connected to 1 control qubit marked with an X and targets a target qubit marked with a small circle. If the control qubit is in an activated state it will flip the target qubit.

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Toffoli, CCNOT gate

The Toffoli gate works in the same way as a CNOT gate, but instead it has 2 control qubits. So both of them need to be activated to actually flip the target qubit, which logically requires at least 3 qubits in your system. This gate is the connector of the whole quantum system where we are able to entangle superposition states.

$$CCNOT = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

This is just a listing of the most prevalent quantum gates. These quantum gates form the bases for performing the simplest of quantum algorithms, which can be implemented through a programming interface or through many of the UI-editors where you can add gates along the circuit in a dynamic way. You are even able to send off these generated circuits through the UI tools off to real devices or simulators to analyse and compare your results.

3. Real-world solutions with Quantum

Once you start looking into quantum theory and everything it could possibly encompass for your scientific projects, you would find yourself in one of the deepest rabbit holes you could ever possibly find. The true value of quantum research is how we can actually use it for real-life solutions. One could easily imagine that being able to simulate an exact medicine within a couple of days instead of the many months it takes at the moment, would save a numerous amount of lives. So keeping this same train of thought throughout, it is of great importance that we actually focus our attention on what current developments could possibly mean for existing projects and research.

3.0.1 Quantum computing and traditional computing

QC is and will never be the sole solution to a problem. This new form of computing is made to be an addition to points where classical computing fails, e.g. searching through an extremely large dataset without having a clear index within a polynomial time frame such as in Terhal and Smolin (1998). QC also has its limits it takes a lot longer to actually set up your computation than it would take on a regular machine, but it could be able to solve a couple of non polynomial problems we are currently facing in computer science like factorization. Some problems have been left NP-complete even with quantum like this paper has tried, Wang et al. (2007). Quantum computing must not be looked at the single solution for every problem, it still needs the help classical computing to be able to perform its more advantageous tasks. Industries also won't be sitting idle waiting for quantum computers to evolve to a point where they would be able to break our RSA-encryption once and for all.

Classical computing is still great at organising stuff and performing parallel actions on

your device, but with the help of QC we would be able to shift the heavy long term calculations over to devices especially made for long term and hard calculations like a quantum processor. Calculating a machine learning model Schuld et al. (2014) or performing an accurate simulation of a new medicine could be exponentially reduced in time, which would in turn return the value of these calculations to the business side in a much faster way and with that would be even more valuable to them if the information is gathered in a proper time frame. (Schuld et al., 2015) (Troyer & Wiese, 2005)

3.0.2 Quantum computing and the mainframe

First of all we need to clarify what a mainframe is and what its main use is in our current business environments. A mainframe is a type of supercomputer that is different from other supercomputers because it is not specialised in solving 1 really hard problem, like simulations or factorisation, it is specialised to have the highest possible throughput for smaller calculations. The mainframe is widely used within the banking, production and logistical sector as it offers the most reliable way of managing your data that is generated by a certain business practice.

To clarify let us look at an example where a mainframe computer like the IBM Z15 shines. When millions of users throughout the world want to buy their flight tickets towards France around the end of April, a huge bottleneck is created at the end point of the booking system of the particular airport. A mainframe handles these types of atomic transactions to make sure every single booking will come through with the correct data and if the data is corrupted along the way, the mainframe is able to spot out these irregularities and discard this data so that the user received a proper notification as soon as possible. So look at a mainframe computer as a really good processor of input and output.

IBM has released the new mainframe in 2019, Z15, with a broad future perspective, because as one of the top researchers in quantum technology they have a clear image of how a quantum computer could influence themselves and others within their sector. They are emphasising on 2 very different aspects to make sure their devices are the most likely to take the biggest market share, modernisation and security. With modernisation IBM is trying to incorporate the mainframe in as much areas as possible to keep on attracting new developers so that their devices don't fall behind. And with this modernisation a lot of opportunities are opening up to connect different departments such as quantum research with data engineering etc.

Also emphasising on creating new security measures which focusses more on digital signing than the current RSA factorisation algorithm could secure the mainframe security status indefinitely. Quantum would in the future indeed be able to break these RSA based algorithms and that is why data-security has become such a high importance area at the moment for everyone in computer science.

So now that you are able to view what role the mainframe plays, we can more clearly look at how quantum computers could offer major benefits as a complementary service for solving the harder problems just like a super computer works with the mainframe in much

the same way. Nowadays all the data generated from the billions of transactions from the mainframe are preserved so that afterwards a supercomputer would be able to process all this information inside a reasonable time frame to get as most as possible business value out of it. If the quantum computer would be able to help process this data exponentially faster, the business value of this data would also exponentially increase. Mainframes offer this great amount of throughput of data that could offer the exponential force of QC the data-value it needs to become viable for business. Again this is something that could show how well QC can fit in our existing model of computation, to further enhance the processes that drive our economies. QC will become the most helpful powerhouse to make sure all our classical systems become even more valuable.

Mainframes are here to stay, with the dawn of fully data-driven worlds that operate on an autonomous platform and if QC can add to such an industry important device it will most definitely boost both devices in exposure and necessity. But for now there most certainly are questions of when QC would be able to reliably process and/ or handle the high throughput of a mainframe in a manner that would add to its business value. This will form another benchmark for QC to evolve to a point where full cooperation of these devices can transform into something practical and useful.

3.0.3 Quantum computing and Machine Learning

Another area where QC could have a major impact is the area of Machine Learning (ML). At this moment machine learning is running into a bottleneck where the amount of data has become so intense that ordinary classical computing is not able to process the data in time so that its value can be exploited to its maximum potential. QC could help with this issue in a couple of major aspects, like data model training and data capturing. This would greatly improve the impact of ML on the business side, because the relay of the captured information through the models could indeed be shortened in exponential ways in a purely theoretical way as of now. (Biamonte et al.,)

At this moment research is becoming quite prevalent in ML with a combination of QC-technology. Qiskit has also seen this opportunity opening up and they too try and attract businesses with these advantages. We are able to enhance supervised learning algorithms as well as unsupervised learning, with time series or without. Algorithms such as linear regression, k-means clustering and even neural networks can be enhanced during its training phases with QC. Because due to superposition and entanglement, these algorithms could train a model theoretically through one loop instead of having multiple epochs that contain a certain size batch, which obviously speeds up the models that require a large amount of data to become valuable.

So the utilisation of QC with ML would not aid the accuracy of ML in the short run, but the time frame of processing the complete dataset could be exponentially decreased. Meaning if the relay of critical changes in the flow of data becomes quicker, business will become more valuable in the same manner.

4. Practical demonstration with Qiskit

For 2 decades now people have been receiving fully blown quantum mechanics courses where they are able to experiment with the thought of quantum experiments in a theoretical type of way, but never were truly interested parties able to perform their experiments in a free and fluid manner. QC is at a point where we are able to effectively experiment with the technology as a broader community. Platforms like Qiskit are excellent in their reach towards interested parties and are more than welcoming towards new developments that could aid the whole community in its research. The service is open source which truly pushes the whole movement of research out of this shroud of high costs and large enterprises. This will obviously influence other branches to follow in the same footsteps as to allow every party that is interested or has a passion to be able to participate in a costless and open manner. To remain objective and fair towards other companies outside of IBM, Google is also participating in the open source community with platforms like Cirq, McClean et al. (2017).

In the following part, we will lay out how any interested parties are able to perform their own executions on real devices and start applying what some of them have been learning theoretically for over 20 years. Whilst trying to resolve the main question of this specific paper ‘How will quantum computing affect the mainframe environment and its applications?’, a major roadblock has been discovered, which is interesting none the less because it shows where research and engineering has not yet ventured far enough to overcome them.

So the question remains is there a practical example where we would be able to show how a quantum algorithm could be applied on the large quantity of mainframe data generated each day? And the answer at the time of writing would be that as of now it is not yet feasible to process large quantities of batch data. So for now we have chosen for an

algorithm that could prove useful once implemented on a database structure inside the mainframe.

4.0.1 Grover's search algorithm in a practical fashion

Grover's search unstructured database search

We have chosen for an adaptation of an algorithm that could prove extremely useful for any implementation together with a mainframe, which is the Grover Search algorithm applied for the boolean satisfiability problem. The whole premise of the original algorithm is that we are able to speed up the search time in an unstructured database quadratically. This all meaning when a computer needs to find an item with a unique attribute that differentiates itself from the other items in the list, QC could become the main solution. The whole algorithm uses something called "amplitude amplification" where the algorithm influences the probabilities in such a manner that the specific item has the highest probability after the Quantum computation. (Grover, 1996)

To clarify amplitude amplification further, this is a principle that affects qubits in their superposition and entangled states where they would interfere with each other to eliminate the least likely outcomes and push up the statistical chance of the desired item. By applying this to an algorithm after the correct transformations, you would be able to decompose all the different results back to a highly probable result that does not collapse all the qubits in superposition like a normal measurement would.

Grover's search algorithm in an applied form

For the experiment itself, we have chosen for the "Boolean satisfiability problem" which uses Grover's way of amplitude amplification to find the correct result. This computer science question goes as follows, given a boolean comparison of multiple parts are we able to determine the outcome to get that specific TRUE. Being able to solve this comparison in a single execution could abuse the fact of superposition and could prove useful when we scale out the problem towards thousands or even millions of comparisons for other algorithms. For now the 3-SAT problem has been chosen to be performed using Qiskit to show off the potential of QC for now.

You are able to view the function stated below as the problem that we will try and solve using Quantum technology. The algorithm now needs to find which solutions are possible by interchanging x,y,z with TRUE/FALSE.

$$f(x, y, z) = (\neg x \vee y \vee \neg z) \wedge (x \vee \neg y \vee \neg z) \wedge (x \vee \neg y \vee z) \wedge (\neg x \vee \neg y \vee z) \wedge (x \vee y \vee z)$$

Executing the quantum algorithm

Using a simulator of an ideal quantum computer we are able to show the results in figure 4.1 below. The probabilities have been amplified to where they are the correct results of this boolean expression. Figure 4.2 shows the gathered results when we sent off the

identical circuit towards one of IBM's real quantum devices (IBMQMelbourne16 in this case).

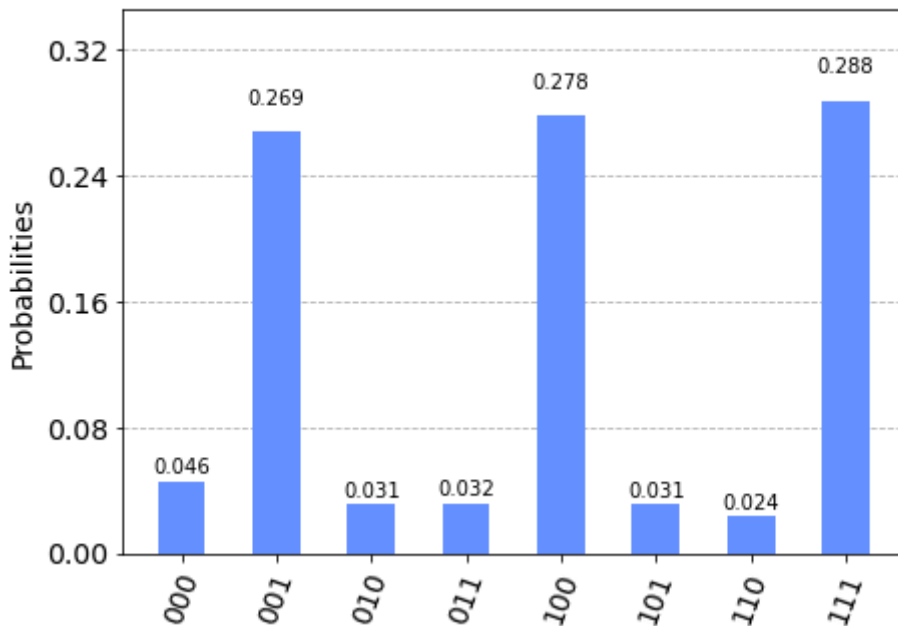


Figure 4.1: These are the results of executing the algorithm for the 3-SAT problem on a **quantum simulator** that comes with Qiskit. The encoding refers to the TRUE/FALSE value of the x,y,z respectively

The reason for choosing this specific experiment is to show that even problems that just require us to encode boolean statements we needed 694 quantum gates. IBMQ transpiles the sent-off circuit to the necessary amount of gates needed for this specific calculation. It does not keep in account that having this much gates on a single line of computation invites a multitude of quantum decoherence issues during runtime.

If you have looked closely at the image above (figure 4.2) you are able to see that decoherence for now breaks the probabilities of a computation too much to reliably trust any computation of this size out of a quantum computer. The values become distorted over time by all types of interference even if all the interference from inside the machine is not accounted for the machine can become influenced by a single external interaction like temperature, pressure etc.

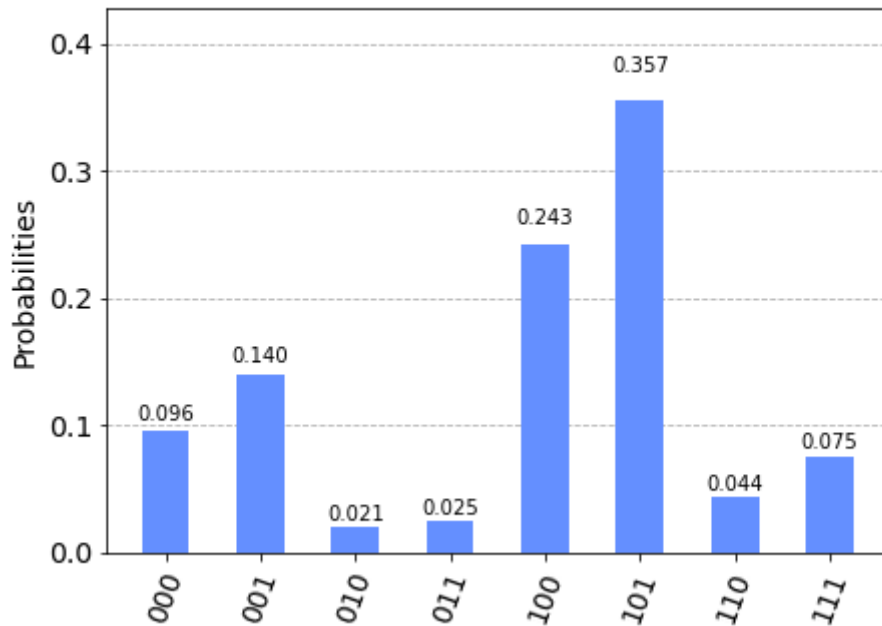


Figure 4.2: These are the results of executing the algorithm for the 3-SAT problem using 15 qubits on a **real quantum device** that comes with Qiskit. The encoding refers to the TRUE/FALSE value of the x,y,z respectively

$f(x,y,z)$	TRUE/FALSE
0 0 0	FALSE
0 0 1	TRUE
0 1 0	FALSE
0 1 1	FALSE
1 0 0	TRUE
1 0 1	FALSE
1 1 0	FALSE
1 1 1	TRUE

If we compare the manually gathered results in the table above across the simulated version and the real version, we are clearly able to see that decoherence has played too big of a role to be certain of any reliable output for these types of large computations. By having these big differences in results we once again prove that simply adding qubits will not be helpful for the reliability in QC. The results differ too much to ever use these results in business cases where speed is as critical as accuracy in reporting towards externals. Let us work out a clear example to make sure our probabilities generated by the real quantum device are incorrect. If we take the highest probability of the real execution which is the configuration of 101. Meaning that the quantum computer determined that when X and Z are true the whole boolean expression will result in a returned value of TRUE. This is simply not a valid option for this boolean expression. If we look at the first part of this boolean expression we can see that this configuration would return a FALSE resulting

in the whole expression being FALSE because all the parts are connected with a logical AND.

$$f(1,0,1) = (0 \vee 0 \vee 0 \vee) \wedge (1 \vee 1 \vee 0) \wedge (1 \vee 1 \vee 1) \wedge (0 \vee 1 \vee 1) \wedge (1 \vee 0 \vee 1)$$

4.0.2 Data-encoding in QC

As the experiment clearly shows there is an actual advantage reachable with quantum computing. Of course quantum decoherence is a main aspect of QC and solving it would be greatly beneficial for the whole sector, but there is also a different problem that arises with defining our classical way of problems in a quantum way. The way we represent data in a quantum circuit quickly becomes overly complicated for any large database structure.

For now quantum computers are great in predicting what quantum effects will occur and where quantum physics influences specific sectors. And by using them for use-cases like simulating quantum effects we can already see the advantages to the businesses itself. The issue lies in the fact that we want to input classical database structures into a quantum device and hopefully receive the results in a readable classical solution.

This shows an issue we are facing with the encoding of our classical data to quantum data and back. For the proof-of-concept experiments it does not matter as the encoding time really does not influence the experiment as a whole. But once we start scaling out the issue where we would want to find a specific item through the use of Grover's algorithm, we would run into the issue that the encoding and decoding of the input and output could take up a great amount of computational time. If however QC develops in such a way that we are able to gain the full benefits of qubits in superposition this encoding time could be overcome, but for now it remains a crucial factor in solving the whole feasibility of QC.

4.0.3 Mainframe computing with QC

As the paper has previously stated having QC together with the power of a mainframe could become extremely advantageous for the whole industry to provide the power of data crunching this immense layer of internal data that companies have collected over the years. So we needed to find a circuit that could show of where quantum computing indeed could benefit in the crunching of data in a better/ faster way than classical computing can at the moment. Soon it became clear that simulating anything of a mainframe is impossible for now, we can simulate how a new form of database search could work with Grover. But we are not able to simulate the main advantage of a mainframe device, which is performing quick, stable and secure input and output transformations. And as shown by the experiment it is obvious that having a stable output of a specific input is not one of the main strengths of QC. Then when we take into account the encoding and decoding of classical computations and problems, which would greatly slow down the performance of a mainframe.

So for now there is no clear advantage when we use the current developments of QC with the existing mainframe technology. It does not take away its immense potential when QC

is able to process the complete I/O of a mainframe in an exponentially smaller time frame than classical computing processing is able to do now.

Then obviously there remains the issue that quantum processors don't have the capability to actually perform algorithms that require a greater amount of qubits due to decoherence and previously stated problems.

Future prospects

As for now we are able to play around with the greater problems of quantum computing but to be able to reliably solve real-world solutions in a beneficial way remains an uncertainty.

So with the current state of engineering, computer scientists will have to wait to fully utilise the system in a reliable fashion. But as engineering develops the power of quantum computing will increase exponentially with each added qubit to the system, which would make algorithms like this extremely valuable for data-crunching. When we find a way to circumvent the interference of quantum decoherence or when we reliably fix the errors it produces, a quantum system could become an essential tool for every sector willing to innovate.

5. Discussion

Quantum computing will most definitely become one of the great buzzwords of the next decade. With the release of Google's paper around their interpretation of 'Quantum supremacy', the whole field was catapulted to the forefront of research. All the big players in Quantum research have been given a tremendous spotlight for the future of profitable quantum computing. And this is precisely why we need to make the whole concept more approachable for anybody interested.

5.0.1 Quantum computing for now

From the research around the subject, there are a couple of interesting conclusions to draw. Beginning with one of the most important ones, Quantum computing is here to stay. The technology has proven too much promise in too many sectors that have a strong financial backbone. Think about fields like medicine, where they analyse their drugs in such small dimensions that even those observations have their own quantum effects that could interfere with the effect of drugs as whole on the human body. At this point it is important to understand that the world around us that is visible with the naked eye is completely homogeneous with the 'quantum world', meaning that all research to exploring our world around is can only be profitable towards the future.

To return back to the more concrete conclusions of this paper. While executing this specific version of Grover's algorithm, there was a clear trend visible between the theory and the practical example that does show there are some growing pains that come with the expansion of our quantum devices. Yes, the theory can be fully implemented at this point to visualise its results when we would work in the most perfect of environments like a simulator. But this perfect environment simply does not exist in the real world or at

least for now it doesn't. Meaning that we will have to be creative to reach this edge of perfect conditions to get to a point that these simulated algorithms can become reliable and profitable towards the future. There are 2 ways of trying to fix the issue of quantum decoherence. The first would be to actually create devices that are not influenced by any internal/ external interferences that could provide a stable platform for algorithms like Shor's encryption breaking algorithm. The other option would be to account for these interferences to happen anyway and try and compensate against them in a software way, much in the same way a computer does error correction for downloaded files. The latter seems to be the more reasonable option where we are already trying to implement these quantum error correction in to provide better results. For now there is no clear technique behind the whole principle except to play around with the length that a qubit needs to stay in its elevated $|1\rangle$ state and the length of the circuit as a whole

But this all can be tried out, because at the moment of writing we are able to extensively experiment with real or simulated quantum devices. There is a multitude of platforms available, where most of them are open-source and you are free to contribute to them as you want to expand their feature-set. Frameworks like Qiskit allow you to design quantum circuits and test them using their built in simulators, which are easy to pick up and hard to master. As of now IBMQ is the only service that allows you to push up your circuits to really test them on a real device owned and managed by IBM. By granting people the privilege to play around with the real devices and notice the shortcomings through raw data results, really shows off how dedicated the whole community of computer science is on pushing this technology to the forefront.

5.0.2 Quantum computing and its myths

As shown in the experiment, QC will not break our entire world in the next decade in any drastic manner. So theories that quantum computing could break our entire encryption standard in a matter of years seem absurd once you look at the real executions of these needed algorithms on real quantum devices. Nevertheless we do want to work proactively to have solutions ready-to-go once these machines do become powerful enough to brute force the RSA-encryption by finding the factors of the prime numbers that represent the private keys of encrypted files. There is also the notion that quantum decoherence will prove to have an insurmountable mountain the more qubits we start adding to the systems. This indeed is a major hurdle that needs to be overcome to make quantum computing the new standard for solving really hard to solve problems in the computer science community.

5.0.3 Quantum computing as an addition

The one thing we should take away from the dawn of quantum computing the following decade, is that quantum computing is not the one solution for every single issue in computing. It has its advantages and disadvantages just like classical computing. We should strive to make these 2 technologies as complementary as possible so that they can cancel out each others disadvantages and amplify their advantages respectively.

With this work, we have tried to inspire people to learn more about the subject of quantum computing and to hopefully entice them into writing their own 'Hello-world-Applications' on any of the freely available frameworks.

5.0.4 Future works

As to conclude back with the main subject of this paper, the potential powerhouse of a mainframe device and a quantum computer. If IBM keeps up its doubling pace of releasing quantum computational resources into the world each year and the quantum error correction evolves to an acceptable percentage, we could start thinking about implementing QC machine learning in our mainframes or even speeding up the database structures in the devices. But for now QC and mainframe are not able to cooperate in a useful manner to give back business value.

It is good to explore these options upfront so that we know when the time comes to expand QC towards the mainframe we have a clear scope to all the potential applications of these 2 devices.



Figure 5.1: The platform we used for executing the 3-SAT algorithm on a real quantum device. © Copyright 2020, Qiskit Development Team Last updated on 2020/05/14.

A. Research Proposition

Under this section you are able to view the original proposition for this paper to introduce the subject with schooling officials and technical promoters. This section can also serve as an introduction to this paper for any further interested readers.

To address the whole reason why this paper was created, the subject has become more and more influential in the Computer Science world. We have officially come at a point where we are able to think of real world utilisations of quantum computers to further our research in various subjects. Quantum has become a buzz word at this point, but not everyone that throws it around has a real grasp on what it exactly means. That is why this paper has been created to aid interested people in the subject to gain a real understanding of what quantum actually is and what it can do.

A.1 Introduction

A.1.1 Situating the subject

There has been a strong believe over the last 30 years that quantum computing can and will influence our sophisticated environment more than we think. In case of the mainframe environment it will maybe be the most influenced sector in *computer science*, because of its immense creation of data. Data will become or has already been the driving factor inside our societies, think of how much our daily lives are already controlled by data (e.g. online shopping, social media etc.). With the usage of mainframes we are able to create a sense of logic in this almost infinite pile of data. Now with *theoretical* utilisation of quantum computing, data can be searched more thoroughly and faster. (Grover, 1996)

If we are able to find and explore quantum applications for our current high-transactional business applications, a new wave of investment in research will open itself up. Which would obviously boost both fields at once. In this paper we will try and find these general applications that can prevail through the use of quantum technology.

A.2 State-of-the-art

A.2.1 Prior knowledge

Inside the paper a couple of physics specific terms will be utilised. If you are not familiar with basic quantum physics notations and or terms, it would be highly recommended to read one or both of the following papers, (Rieffel & Polak, 1998) or (Shor, 2000). For the general quantum notation that are used throughout the field, we refer towards Dirac (1939). It is also possible to read this paper as an informational piece without the implications of the mathematics and physics surrounding the subject. As previously stated the paper will not be going in depth technologically, because the scope is more focused on exposing the practical usages of quantum computing compared to classical computing or the combination of them both.

A.2.2 Recent developments

As of now Google has claimed to have won the *Quantum Supremacy race* (Arute et al., 2019) against IBM. They have realised this through the creation of their 54-Qubit quantum computer (53 functional qubits), that is able to perform a calculation exponentially faster than a classical system could ever hope to perform. In this case the *Sycamore* (Quantum processor) was able to perform a calculation within 200 seconds that could only be performed by a classical computer over 10.000 years (theoretically). Although it most definitely was an experimental calculation that has no real value in the business world, it does however prove the potential of quantum computing. It has been rumoured that IBM will release its counterpart of research in 2020. The fact that these 2 conglomerates are competing so fiercely will only further the technological developments in the realm of quantum mechanics. IBM has not been sitting idly either, they have released a paper regarding quantum algorithms applications. (Amico et al., 2019)

A.3 Methodology

While the field of practical quantum computing is still in its infancy, there are a lot of different possible angles to approach the subject with. First of all we will be introducing the guiding principles of quantum computing, as to all start on the same footing. Then we will explore the realistic potential that quantum computing can offer for economic gain, especially for mainframe development. This will mainly be comprised of an extensive literature study that will set its focus on economic applications of quantum computation and

thereby on the mainframe environment. Concretely the paper will use real-life economical batch data and will process this data through the use of quantum algorithms and classical algorithms. If there are any advantages in processing the nightly batch load by using quantum algorithms, it will become provable that quantum computing can also be extremely profitable. There will also be demonstrations of quantum computation software such as Qiskit by IBM (Abraham et al., 2019), Cirq by Google (McClean et al., 2017) and Q Sharp by Microsoft (Svore et al., 2018). Qiskit stands out because it is an IBM Python framework that solely offers the opportunity to actually execute your quantum circuits on real quantum devices as of today. (with limited qubits however)

A.4 Expected results

The paper will try and create a more concrete point of view on the possible features quantum computation can offer. Through the analysis of multiple papers, we are hoping to find certain points of contest. These points indicate the highly debated subjects within quantum computing and are therefore extremely valuable. We will be trying to locate and display the business potential within these points of conflict. Currently IBM has created an extremely stable and performant business environment with its mainframe, Z15 and its older versions. Anything that can/ will affect this stable business platform can form a great threat or opportunity to the way we currently create and process our data. To protect this stable platform, we will be trying to index all the threats and opportunities that come with the introduction of quantum computation in our current computational environment. The second part of the paper will be more software-orientated, where we will be creating an application that processes the typical nightly batch data. This application will be performed on the different quantum platforms an on a classical device. The paper will visualise these probabilistic and timing differences between results of the different software platforms and will try to show attention points with simulating quantum computers compared to effectively executing on one. Through the demonstration of quantum computation we hope that readers are going to be personally inspired to be creative with the new technology and start developing their first 'Hello World' with their quantum circuits.

A.5 Expected conclusions

We are expecting to *debunk* the more absurd ideas of quantum computing. (e.g. destroying all our encryptions and our society) Concretely, we are going to put the whole subject inside a more realistic 'future' vision. This will hopefully offer readers ideas of possible applications of quantum computation inside their departments (e.g. Chemistry, Economics, Astronomy etc.) Also With software being so readily available for the general public, we expect that quantum computing applications will be created exponentially faster than with the dawn of classical computing 70 years ago. With this train of thought, we are hoping that real economical value can be available within the next decade. By processing our example night batch load we hope to find this necessary business value. Frameworks like Qiskit will be developed further and more powerful quantum computers will be made

available towards the public to boost the research in the subject. And with these thoughts we can be certain that interest in quantum computers will only increase in the future.

Bibliography

- Abraham, H., Akhalwaya, I. Y., Aleksandrowicz, G., Alexander, T., Alexandrowics, G., Arbel, E., Asfaw, A., Azaustre, C., Barkoutsos, P., Barron, G., Bello, L., Ben-Haim, Y., Bevenius, D., Bishop, L. S., Bosch, S., Bucher, D., CZ, Cabrera, F., Calpin, P., ... yotamvakninibm. (2019). Qiskit: An Open-source Framework for Quantum Computing. <https://doi.org/10.5281/zenodo.2562110>
- Amico, M., Saleem, Z. H., & Kumph, M. (2019). Experimental study of Shor's factoring algorithm using the IBM Q Experience. *Phys. Rev. A*, *100*, 012305. <https://doi.org/10.1103/PhysRevA.100.012305>
- Arute, F., Arya, K., Babbush, R., Bacon, D., Bardin, J. C., Barends, R., Biswas, R., Boixo, S., Brandao, F. G. S. L., Buell, D. A., Burkett, B., Chen, Y., Chen, Z., Chiaro, B., Collins, R., Courtney, W., Dunsworth, A., Farhi, E., Foxen, B., ... Martinis, J. M. (2019). Quantum supremacy using a programmable superconducting processor. *Nature*, *574*(7779), 505–510. <https://doi.org/10.1038/s41586-019-1666-5>
- Biamonte, P., Jacob and Wittek, Pancotti, N., Rebentrost, P., Wiebe, N., & Lloyd, S. (). Quantum machine learning. *Nature*.
- Brandao, F. G. S. L., Christandl, M., Harrow, A. W., & Walter, M. (2016). The Mathematics of Entanglement.
- Dirac, P. A. M. (1939). A new notation for quantum mechanics. *Mathematical Proceedings of the Cambridge Philosophical Society*, *35*(3), 416–418. <https://doi.org/10.1017/S0305004100021162>
- Draper, T. G. (2000). Addition on a Quantum Computer.
- Grover, L. K. (1996). A Fast Quantum Mechanical Algorithm for Database Search, In *Proceedings of the Twenty-eighth Annual ACM Symposium on Theory of Computing*, Philadelphia, Pennsylvania, USA, ACM. <https://doi.org/10.1145/237814.237866>
- Hartnett, K. (2019). A New “Law” Suggests Quantum Supremacy Could Happen This Year. *Scientific American*.

- McClean, J. R., Sung, K. J., Kivlichan, I. D., Cao, Y., Dai, C., Fried, E. S., Gidney, C., Gimby, B., Gokhale, P., Häner, T., Hardikar, T., Havlíček, V., Higgott, O., Huang, C., Izaac, J., Jiang, Z., Liu, X., McArdle, S., Neeley, M., . . . Babbush, R. (2017). OpenFermion: The Electronic Structure Package for Quantum Computers.
- Rieffel, E., & Polak. (1998). An Introduction to Quantum Computing for Non-Physicists. *ACM Computing Surveys*, 32. <https://doi.org/10.1145/367701.367709>
- Schuld, M., Sinayskiy, I., & Petruccione, F. (2014). Quantum Computing for Pattern Classification (D.-N. Pham & S.-B. Park, Eds.). In D.-N. Pham & S.-B. Park (Eds.), *PRICAI 2014: Trends in Artificial Intelligence*, Cham, Springer International Publishing.
- Schuld, M., Sinayskiy, I., & Petruccione, F. (2015). An introduction to quantum machine learning. *Contemporary Physics*, 56(2), <https://doi.org/10.1080/00107514.2014.964942>, 172–185. <https://doi.org/10.1080/00107514.2014.964942>
- Shor, P. W. (2000). Introduction to Quantum Algorithms.
- Svore, K., Roetteler, M., Geller, A., Troyer, M., Azariah, J., Granade, C., Heim, B., Kliuchnikov, V., Mykhailova, M., & Paz, A. (2018). QSharp. *Proceedings of the Real World Domain Specific Languages Workshop 2018 on - RWDSL2018*. <https://doi.org/10.1145/3183895.3183901>
- Terhal, B. M., & Smolin, J. A. (1998). Single quantum querying of a database. *Phys. Rev. A*, 58, 1822–1826. <https://doi.org/10.1103/PhysRevA.58.1822>
- Troyer, M., & Wiese, U.-J. (2005). Computational Complexity and Fundamental Limitations to Fermionic Quantum Monte Carlo Simulations. *Phys. Rev. Lett.*, 94, 170201. <https://doi.org/10.1103/PhysRevLett.94.170201>
- Wang, X.-Y., Feng, Y.-X., Huang, D.-B., Pu, W.-G., Zhou, Y.-C., Liang, C.-G., & null Zhou. (2007). Quantum swarm evolutionary algorithm, Quantum-inspired evolutionary algorithm, Particle swarm optimization, Knapsack problem, Traveling salesman problem. <https://doi.org/10.1016/j.neucom.2006.10.001>