

# Statistical Characterisation of Porous Media at the Pore Scale

## Parametric Models

**Student:**

Lukas Mosser

**Supervisors:**

Professor Olivier Dubrule

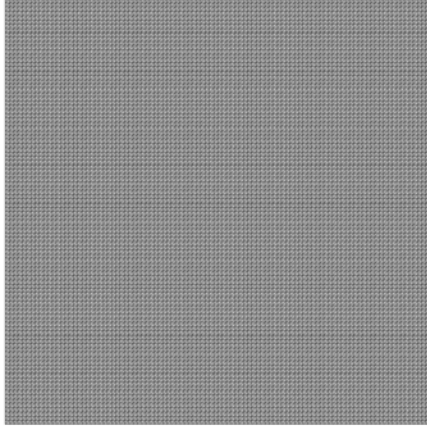
Professor Martin Blunt

## Presentation Outline

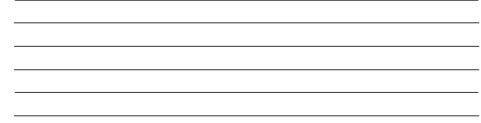
- **Plane Surface**
- **Regular Bundle of Tubes**
- **Regular Bundle of Tubes – High Mesh Res.**
- **Random Bundle of Tubes**
- **Array of Spheres**
- **Array of Ellipsoids**
- **Gaussian Porous Medium**
- **Beadpack: Anisotropy( $r$ )**
- **Ketton: Anisotropy( $r$ )**
- **Comparison Beadpack - Ketton**

# Plane Surfaces

X-Y View



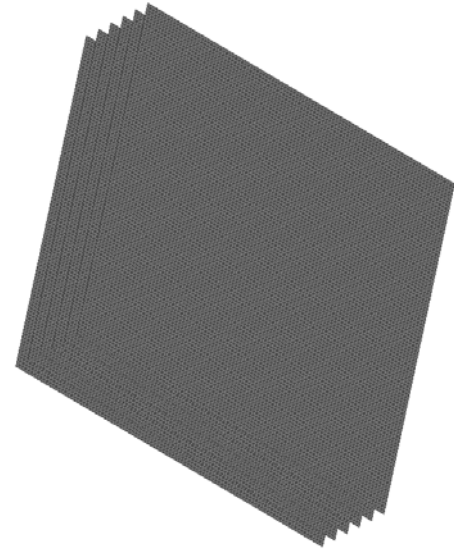
Y-Z View



X-Z View



3D View

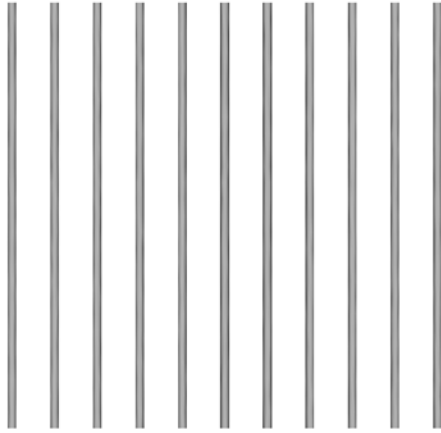


## Minkowski Tensors – Plane Surface

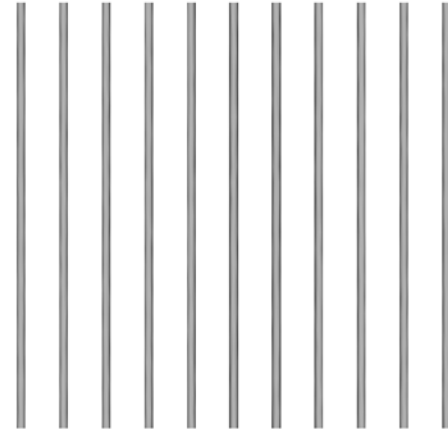
Minkowski Tensor	Eigenvectors	Eigenvalues	Anisotropy Index
$W_1^{0,2} = \begin{bmatrix} 0.0 & 0 & 0 \\ 0 & 0.0 & 0 \\ 0 & 0 & 2400. \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{0.0 \quad 0.0 \quad 2400.\}$	$\beta_1^{0,2} = not\ def.$
$W_2^{0,2} = \begin{bmatrix} 0.0 & 0 & 0 \\ 0 & 0.0 & 0 \\ 0 & 0 & 0.0 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{0.0 \quad 0.0 \quad 0.0\}$	$\beta_2^{0,2} = not\ def.$

## Regular Bundle of Tubes

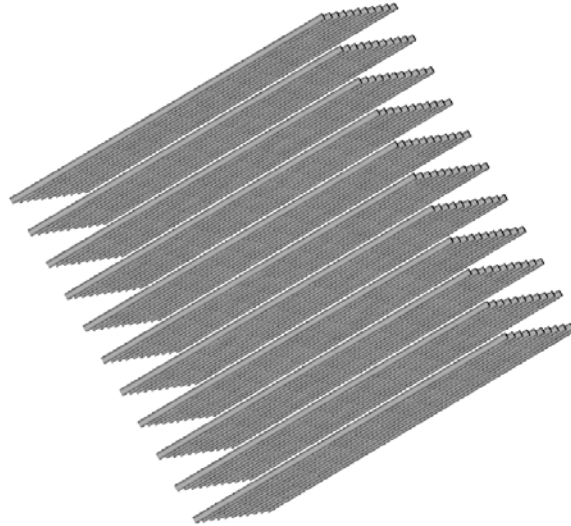
X-Z View



Y-Z View



3D View

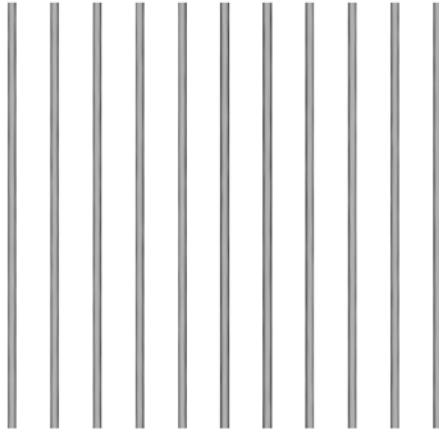


## Minkowski Tensors – Regular Bundle of Tubes

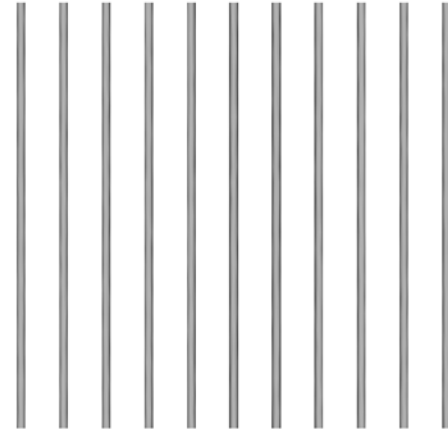
Minkowski Tensor	Eigenvectors	Eigenvalues	Anisotropy Index
$W_1^{0,2} = \begin{bmatrix} 12605 & 0 & 0 \\ 0 & 12605 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{12605 \quad 12605 \quad 0\}$	$\beta_1^{0,2} = \text{not def.}$
$W_2^{0,2} = \begin{bmatrix} 5960 & 1 & 0 \\ 1 & 6334 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{5960 \quad 6334 \quad 0\}$	$\beta_2^{0,2} = \text{not def.}$

## Regular Bundle of Tubes – High Mesh Resolution

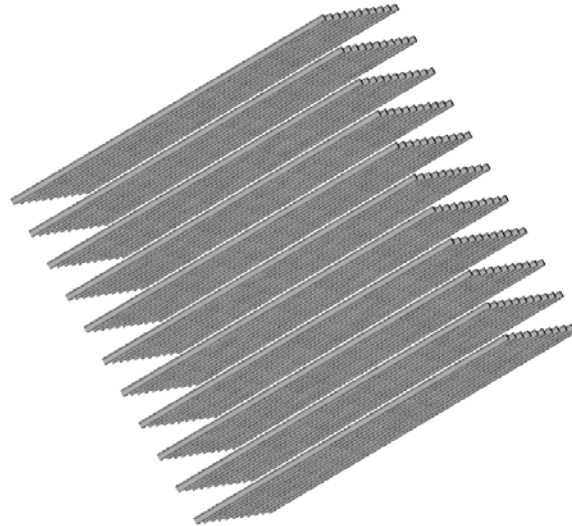
X-Z View



Y-Z View



3D View



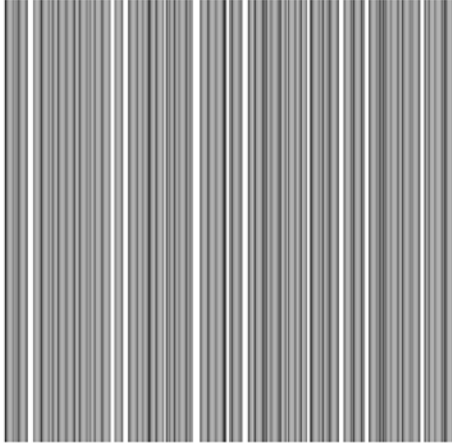
## Minkowski Tensors – Regular Bundle – High Mesh Res.

Minkowski Tensor	Eigenvectors	Eigenvalues	Anisotropy Index
$W_1^{0,2} = \begin{bmatrix} 12605 & 0 & 0 \\ 0 & 12605 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{12605 \quad 12605 \quad 0\}$	$\beta_1^{0,2} = \text{not def.}$
$W_2^{0,2} = \begin{bmatrix} 5960 & 0 & 0 \\ 0 & 6334 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{5960 \quad 6334 \quad 0\}$	$\beta_2^{0,2} = \text{not def.}$

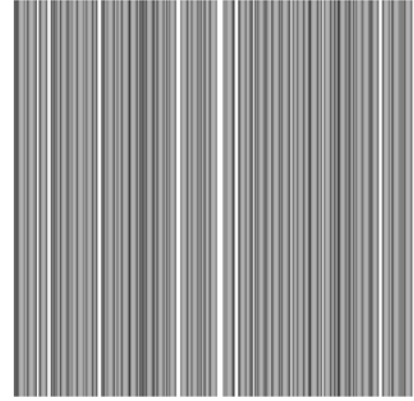


## Random Bundle of Tubes

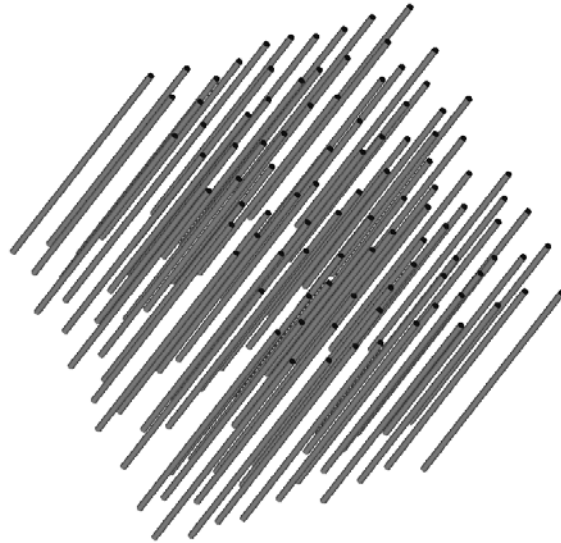
X-Z View



Y-Z View



3D View

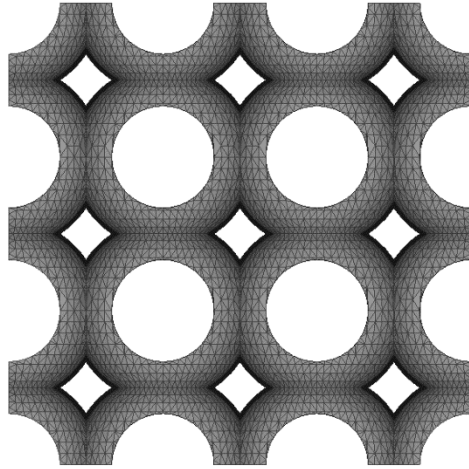


## Minkowski Tensors – Random Bundle of Tubes

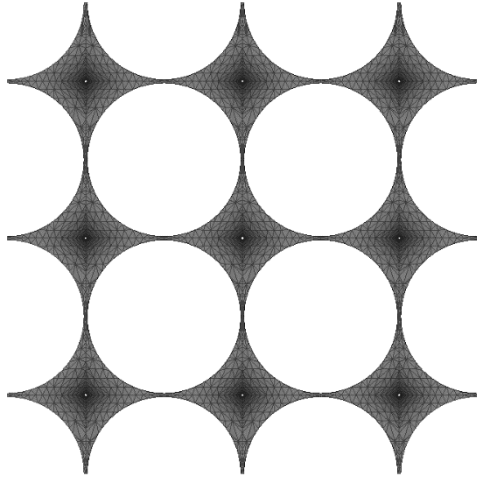
Minkowski Tensor	Eigenvectors	Eigenvalues	Anisotropy Index
$W_1^{0,2} = \begin{bmatrix} 12605 & 0 & 0 \\ 0 & 12605 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{12605 \quad 12605 \quad 0\}$	$\beta_1^{0,2} = \text{not def.}$
$W_2^{0,2} = \begin{bmatrix} 5960 & 0 & 0 \\ 0 & 6334 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{5960 \quad 6334 \quad 0\}$	$\beta_2^{0,2} = \text{not def.}$

## Symmetric Parametric Pore

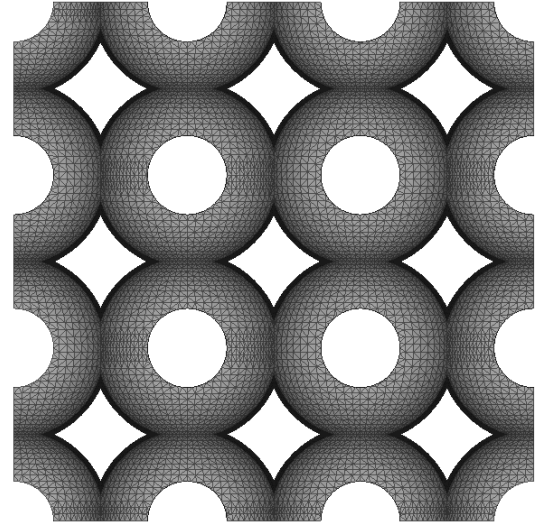
Radius 1.2



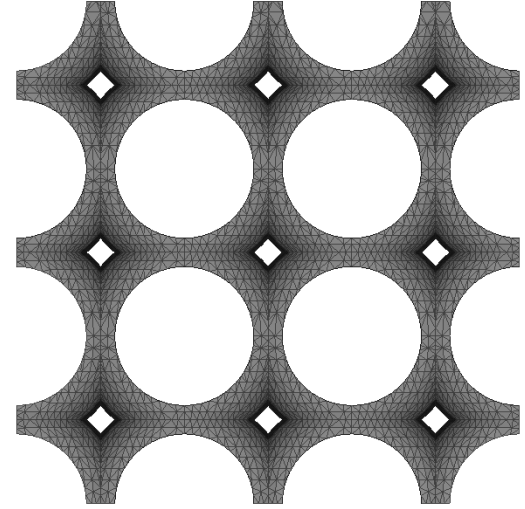
Radius 1.4



Radius 1.1



Radius 1.3



## Minkowski Tensors – Radius 1.4

Minkowski Tensor	Eigenvectors	Eigenvalues	Anisotropy Index
$W_1^{0,2} = \begin{bmatrix} 10.6 & 0 & 0 \\ 0 & 10.6 & 0 \\ 0 & 0 & 10.6 \end{bmatrix}$	$\begin{bmatrix} -0.76 \\ 0.66 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0.66 \\ -0.76 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{10.6 \quad 10.6 \quad 10.6\}$	$\beta_1^{0,2} = 1.0$
$W_2^{0,2} = \begin{bmatrix} -6.0 & 0 & 0 \\ 0 & -6.0 & 0 \\ 0 & 0 & -6.0 \end{bmatrix}$	$\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{-6.0 \quad -6.0 \quad -6.0\}$	$\beta_2^{0,2} = 1.0$

## Minkowski Tensors – Radius 1.3

Minkowski Tensor	Eigenvectors	Eigenvalues	Anisotropy Index
$W_1^{0,2} = \begin{bmatrix} 19.6 & 0 & 0 \\ 0 & 19.6 & 0 \\ 0 & 0 & 19.6 \end{bmatrix}$	$\begin{bmatrix} -0.7 \\ 0.7 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0.7 \\ -0.7 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{19.6 \quad 19.6 \quad 19.6\}$	$\beta_1^{0,2} = 1.0$
$W_2^{0,2} = \begin{bmatrix} -13 & 0 & 0 \\ 0 & -13 & 0 \\ 0 & 0 & -13 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{-13 \quad -13 \quad -13\}$	$\beta_2^{0,2} = 1.0$

## Minkowski Tensors – Radius 1.2

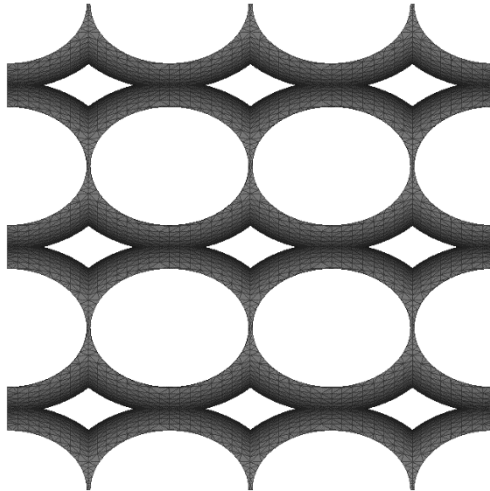
Minkowski Tensor	Eigenvectors	Eigenvalues	Anisotropy Index
$W_1^{0,2} = \begin{bmatrix} 27.1 & 0 & 0 \\ 0 & 27.1 & 0 \\ 0 & 0 & 27.1 \end{bmatrix}$	$\begin{bmatrix} -1.0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1.0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ -1.0 \\ 0 \end{bmatrix}$	$\{27.1 \quad 27.1 \quad 27.1\}$	$\beta_1^{0,2} = 1.0$
$W_2^{0,2} = \begin{bmatrix} -20.7 & 0 & 0 \\ 0 & -20.7 & 0 \\ 0 & 0 & -20.7 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \quad \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$	$\{-20.7 \quad -20.7 \quad -20.7\}$	$\beta_2^{0,2} = 1.0$

## Minkowski Tensors – Radius 1.1

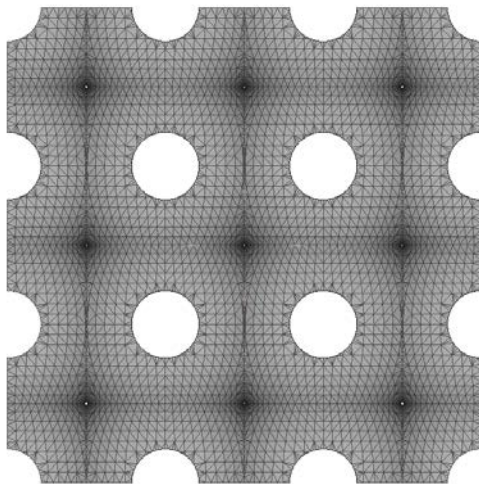
Minkowski Tensor	Eigenvectors	Eigenvalues	Anisotropy Index
$W_1^{0,2} = \begin{bmatrix} 33.1 & 0 & 0 \\ 0 & 33.1 & 0 \\ 0 & 0 & 33.1 \end{bmatrix}$	$\begin{bmatrix} -0.89 \\ 0.45 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0.45 \\ -0.89 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{33.1 \quad 33.1 \quad 33.1\}$	$\beta_1^{0,2} = 1.0$

## Asymmetric Parametric Pore – $R1 = 1.4$ , $R2=1.3$

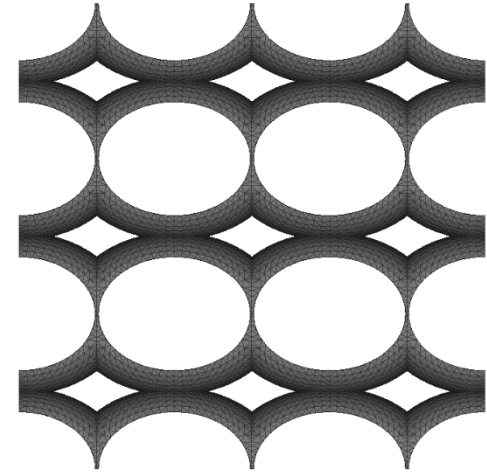
X-Y View



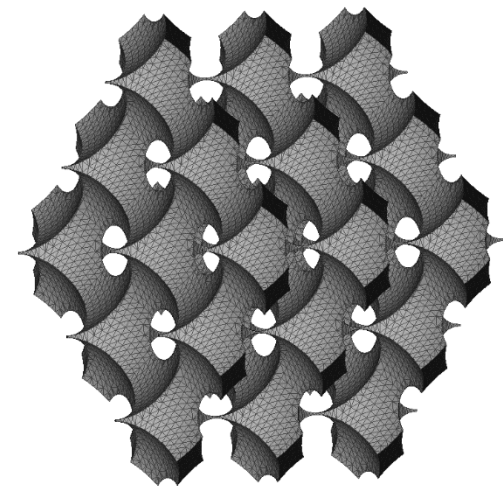
X-Z View



Y-Z View



3D View



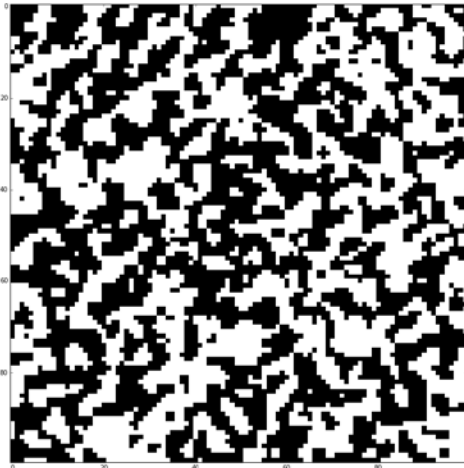


## Minkowski Tensors – R1 = 1.4, R2=1.3

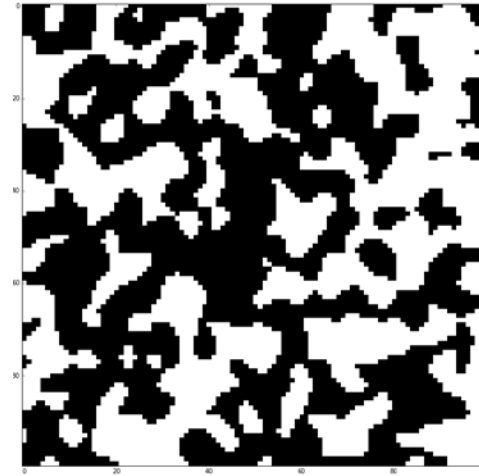
Minkowski Tensor	Eigenvectors	Eigenvalues	Anisotropy Index
$W_1^{0,2} = \begin{bmatrix} 11.9 & 0 & 0 \\ 0 & 51.8 & 0 \\ 0 & 0 & 11.9 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{11.9 \quad 51.8 \quad 11.9\}$	$\beta_1^{0,2} = 0.23$
$W_2^{0,2} = \begin{bmatrix} -7.8 & 0 & 0 \\ 0 & -30.9 & 0 \\ 0 & 0 & -7.6 \end{bmatrix}$	$\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{-7.8 \quad -30.9 \quad -7.8\}$	$\beta_2^{0,2} = 0.25$

## Gaussian Porous Medium – $100^3$

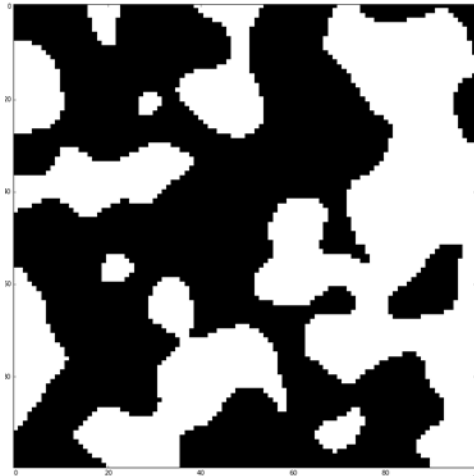
$$\sigma = 1.0$$



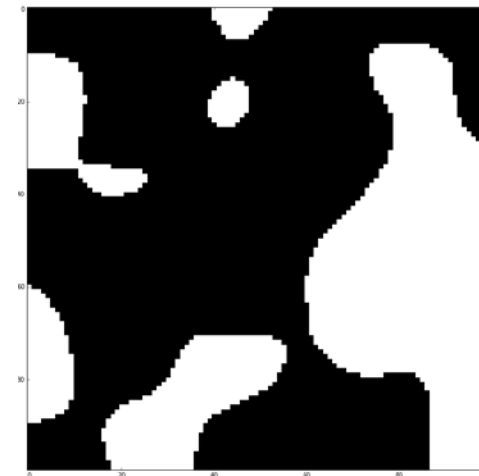
$$\sigma = 2.0$$



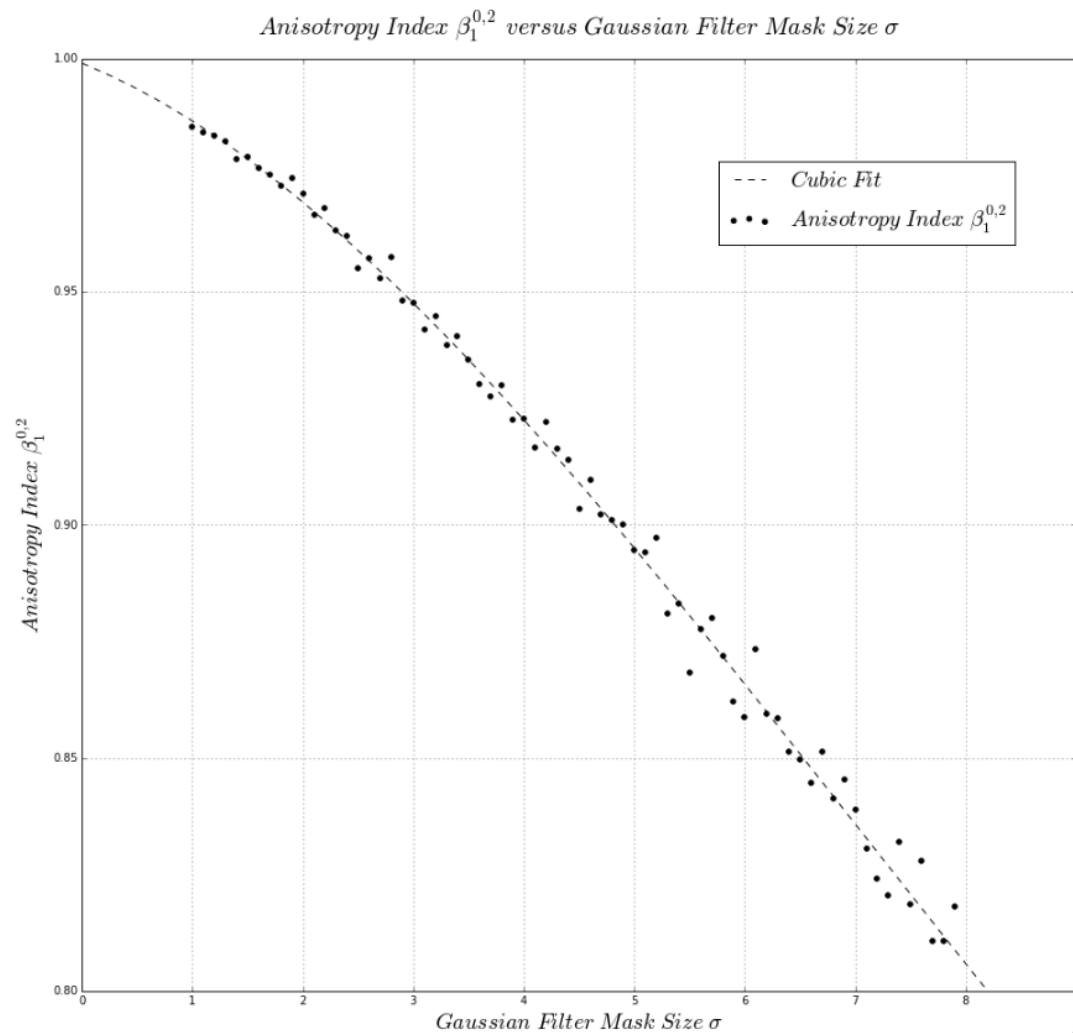
$$\sigma = 4.0$$



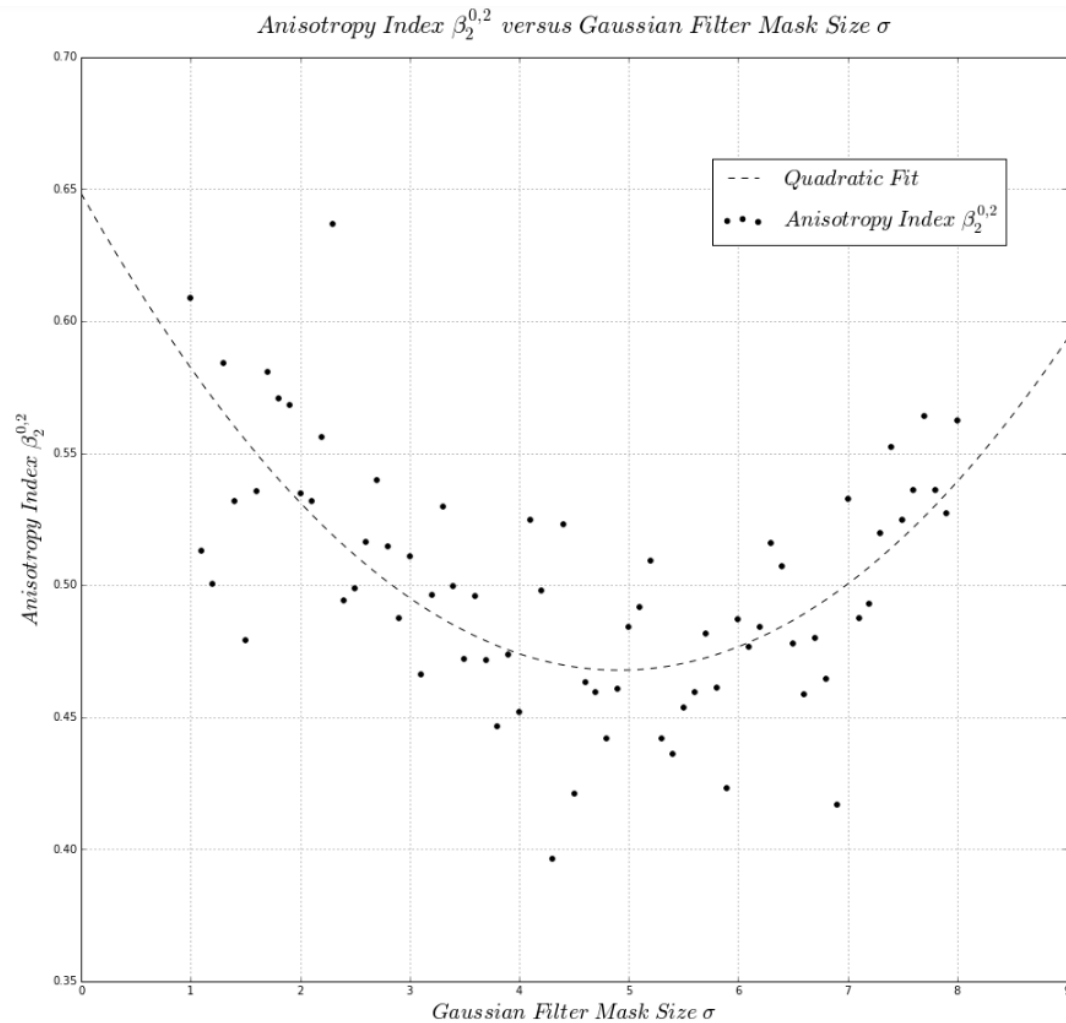
$$\sigma = 8.0$$



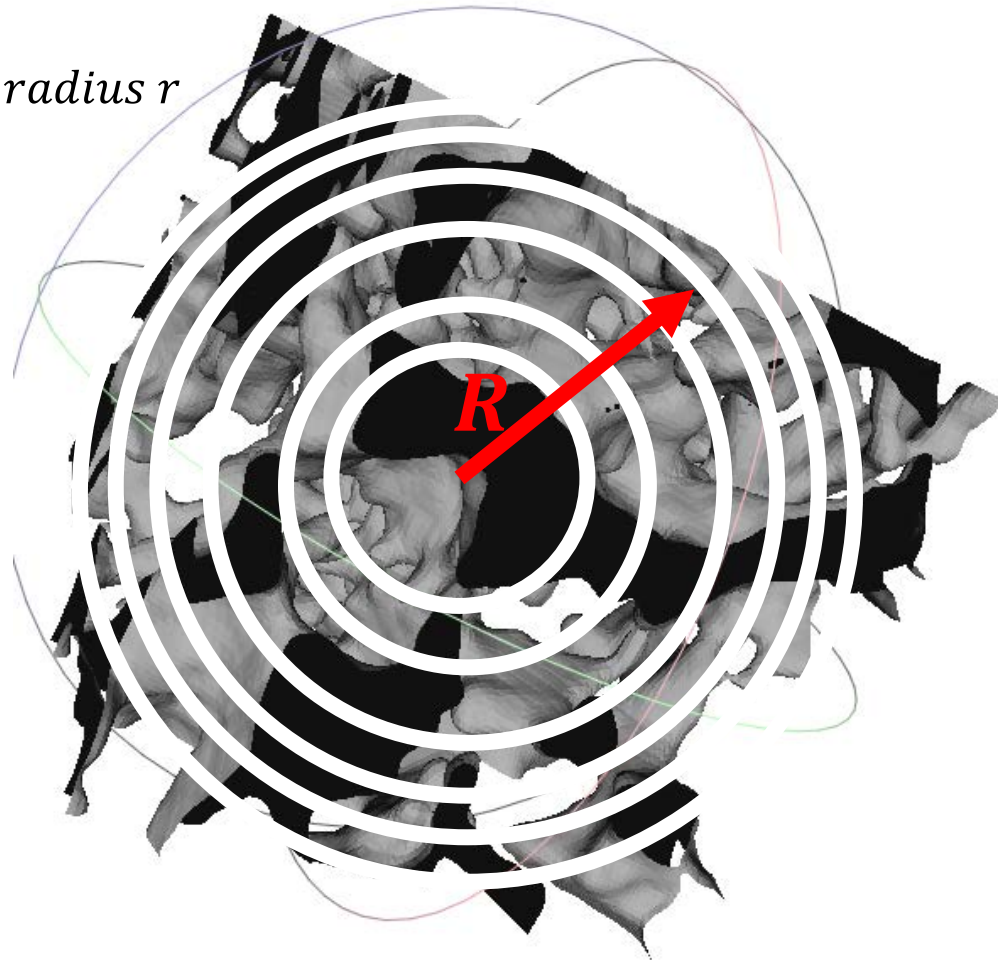
## Anisotropy Index as function of filter size (W102)



# Anisotropy Index as function of filter size (W202)



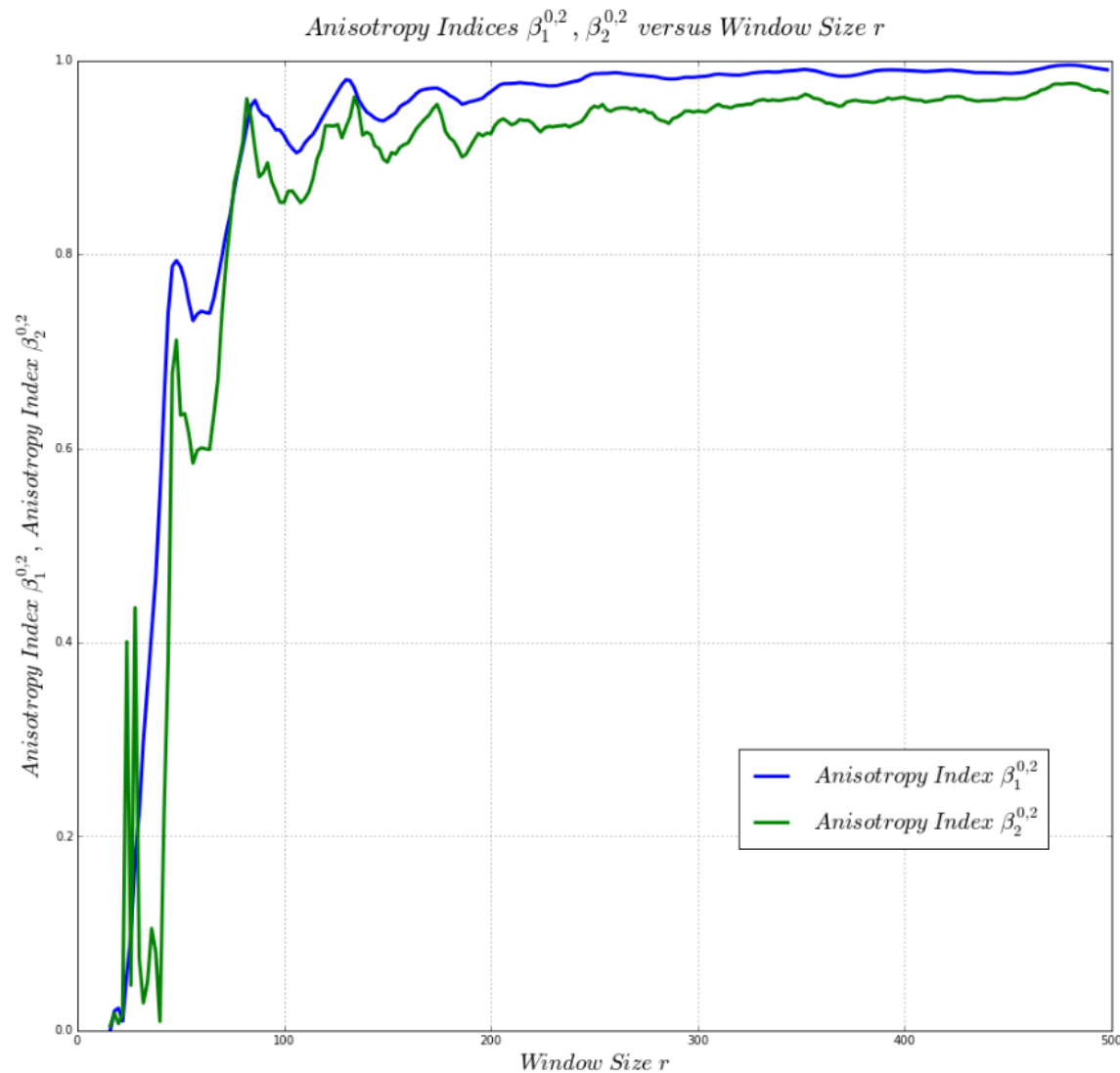
*Increasing Window of radius  $r$*



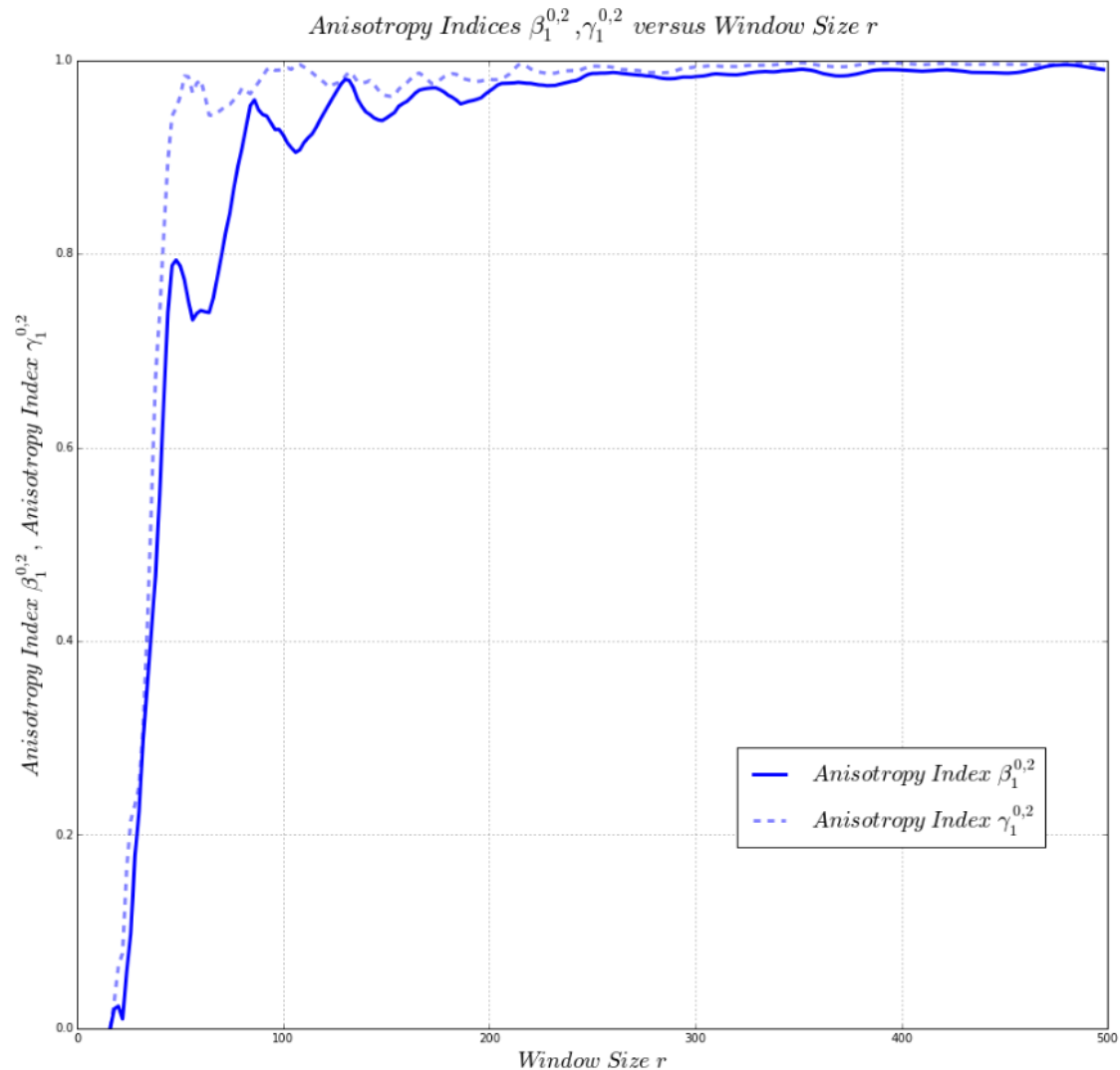
*Cubic Volume used*

*Determine Anisotropy as function of Window Size*

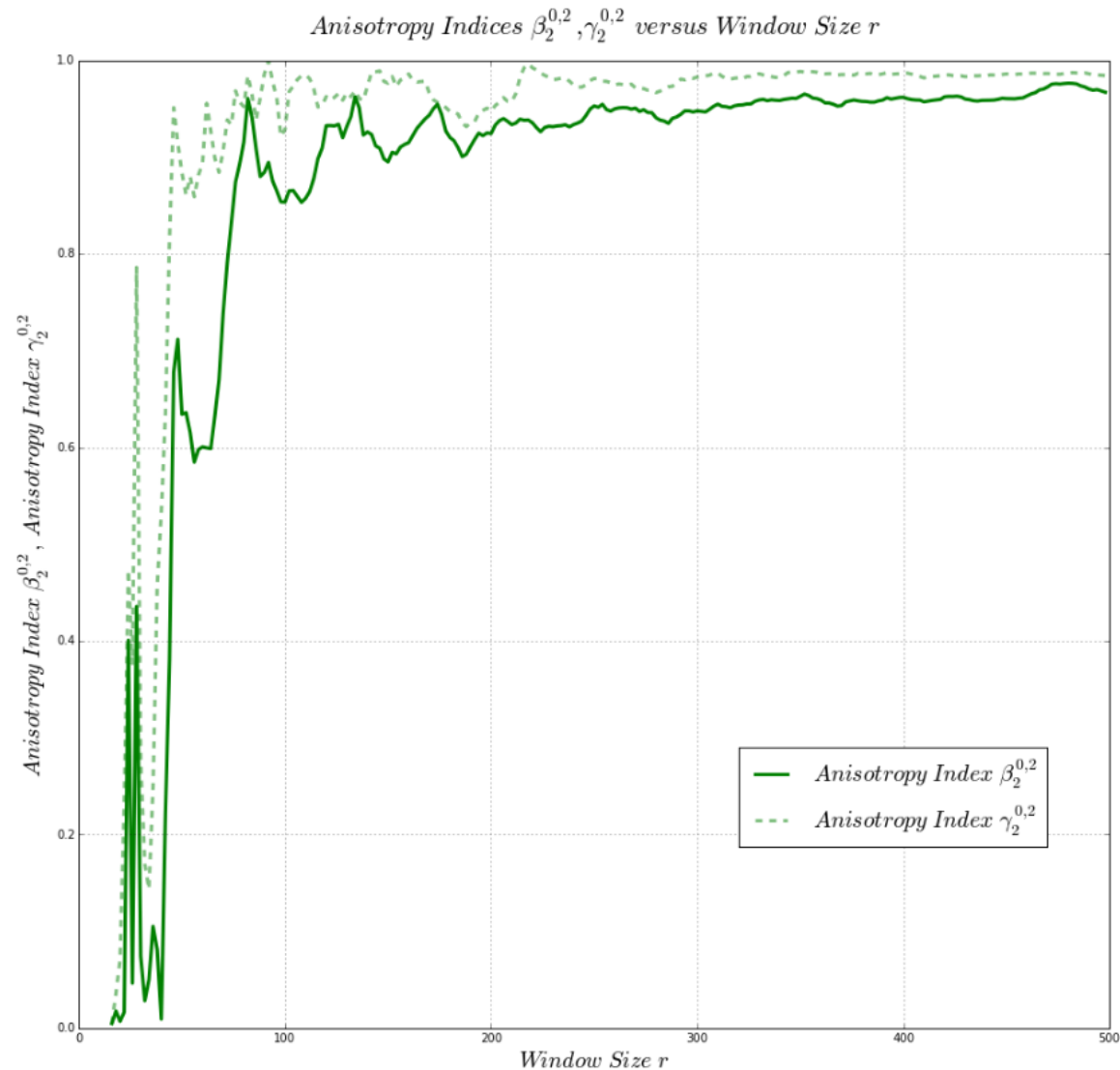
# Beadpack – Centered Expanding Window => REV – 500^3



# Beadpack – Centered Expanding Window => REV – 500<sup>3</sup>

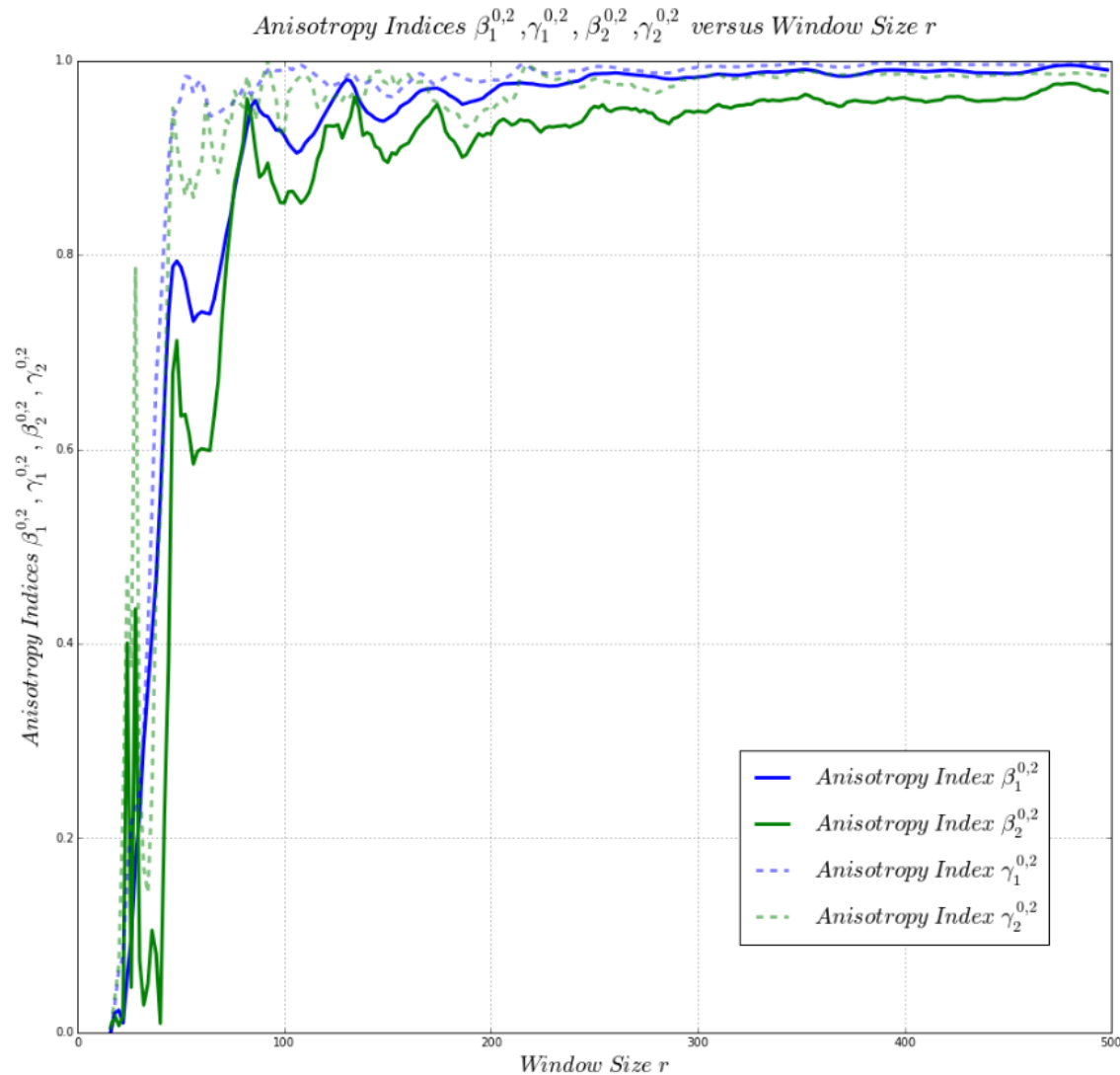


# Beadpack – Centered Expanding Window => REV – 500^3





# Beadpack – Centered Expanding Window => REV – 500<sup>3</sup>

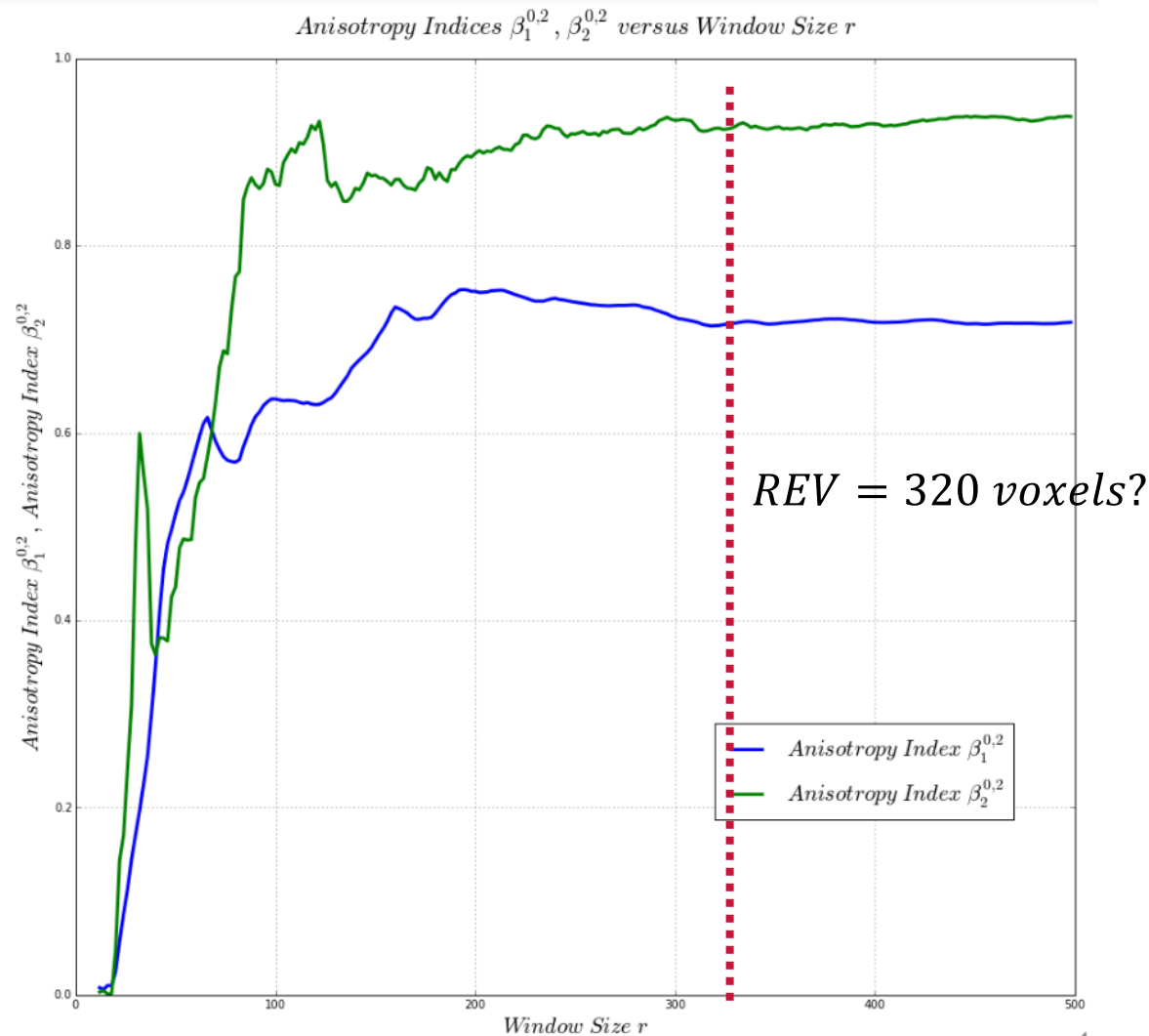


## Ketton – Centered Expanding Window

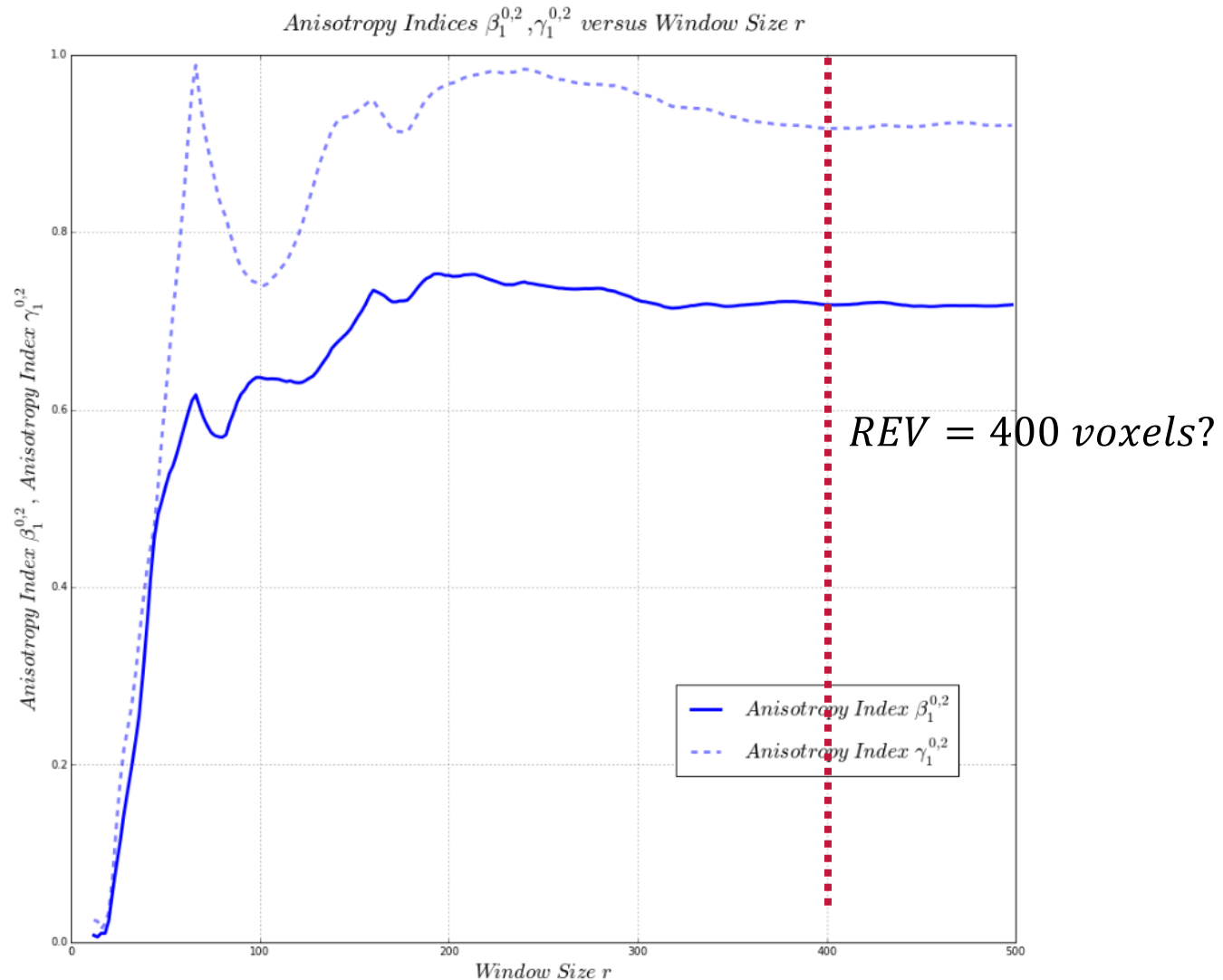
---

*Ketton Centered Expanding Window*

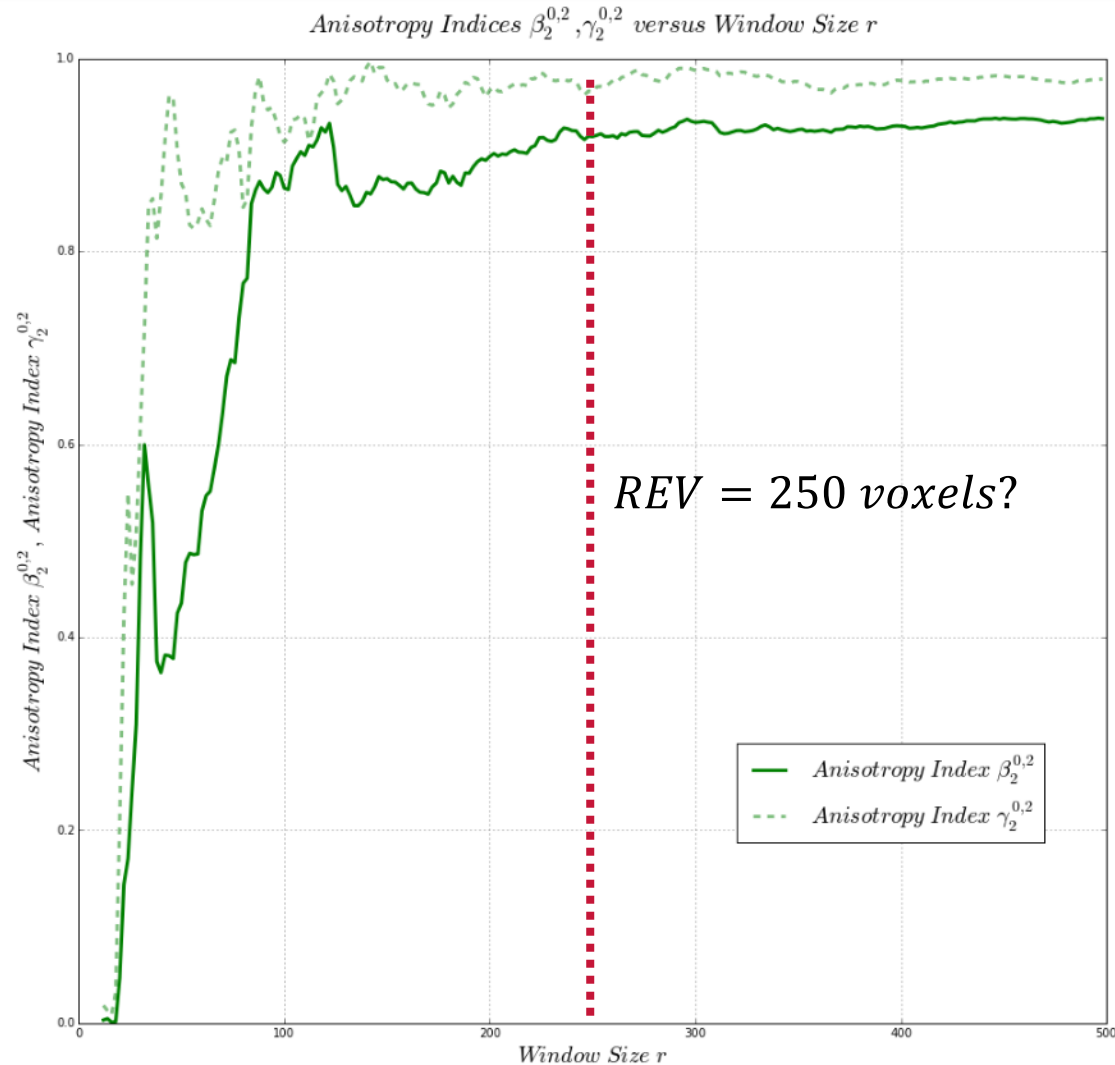
# Ketton – Comparison Anisotropy Indices B102, B202



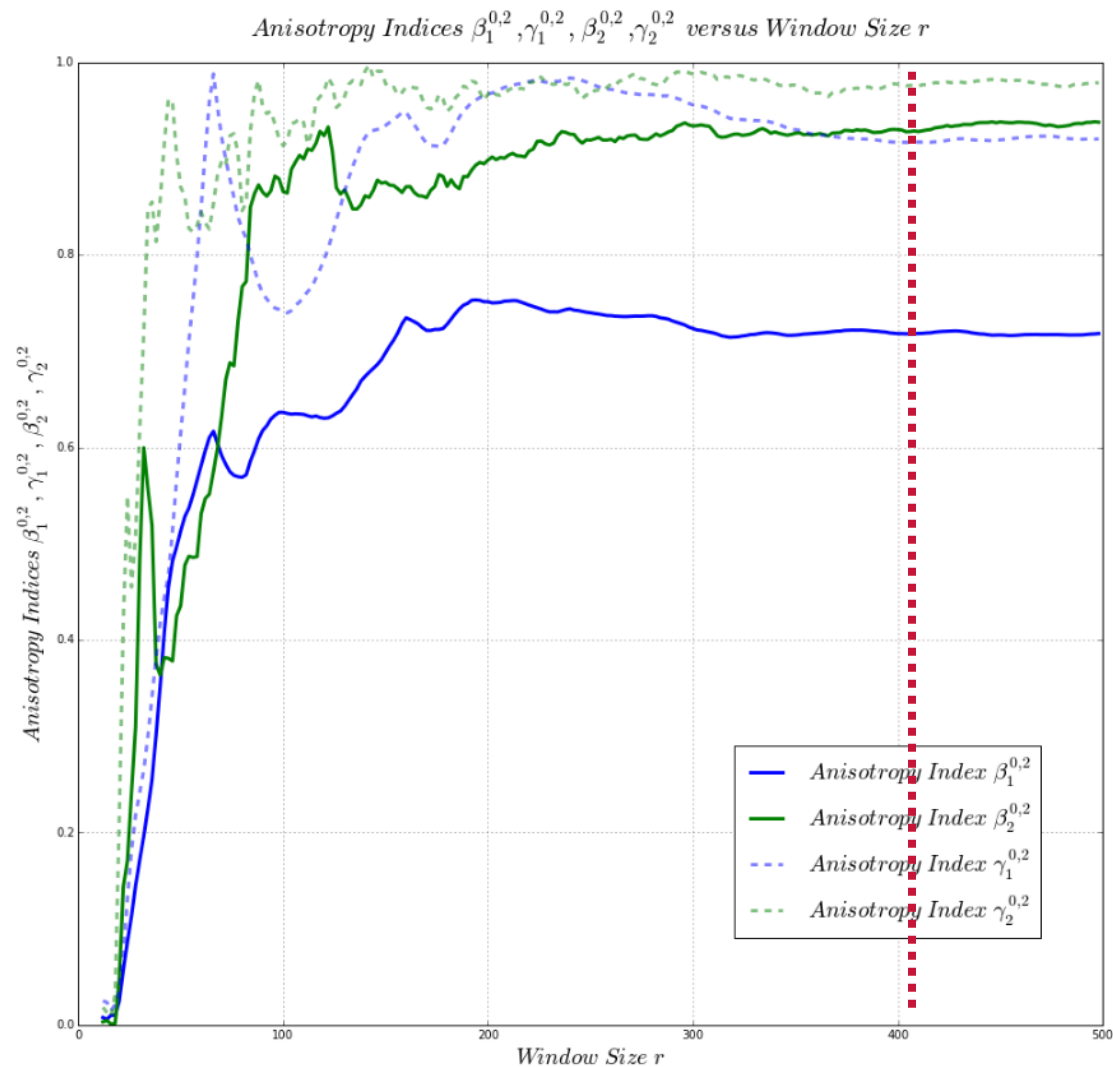
# Ketton – Centered Expanding Window => REV – 500<sup>3</sup>



# Ketton – Centered Expanding Window => REV – 500^3



# Ketton – Centered Expanding Window => REV – 500<sup>3</sup>



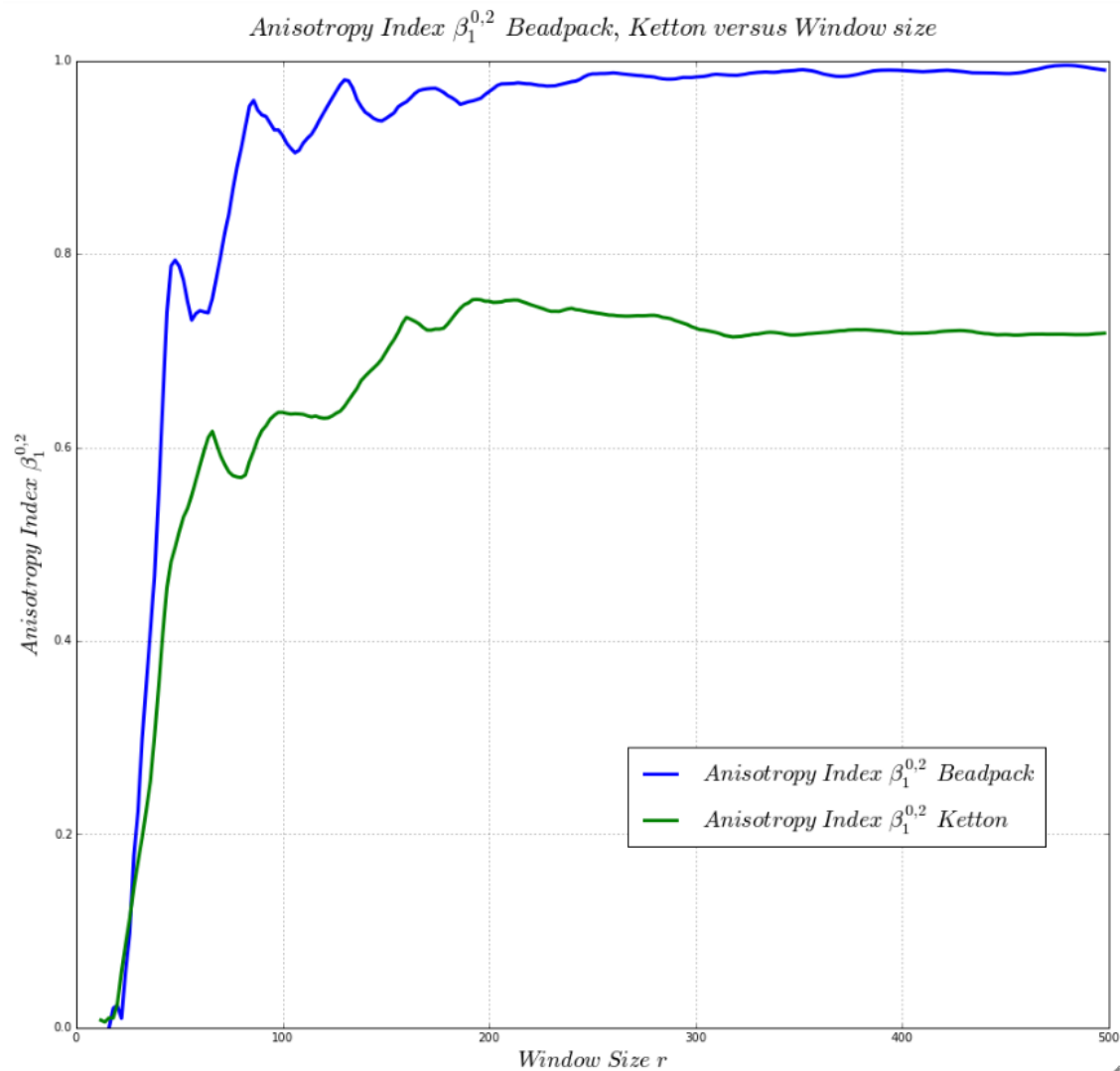
REV = 400 voxels

## Comparison Beadpack/Ketton

---

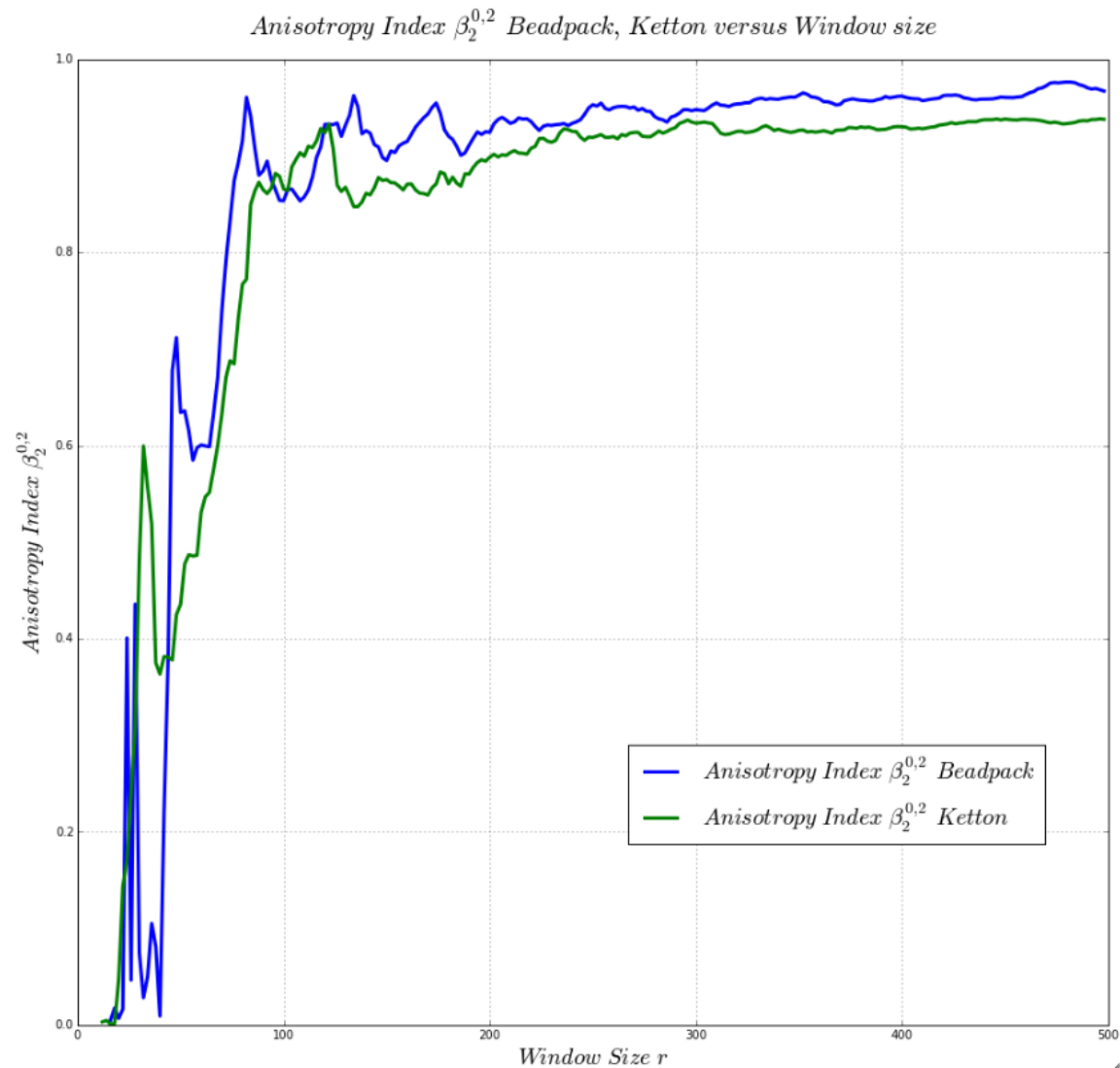
*Comparison Ketton and Beadpack Results*

## Comparison Beadpack/Ketton – Beta 102

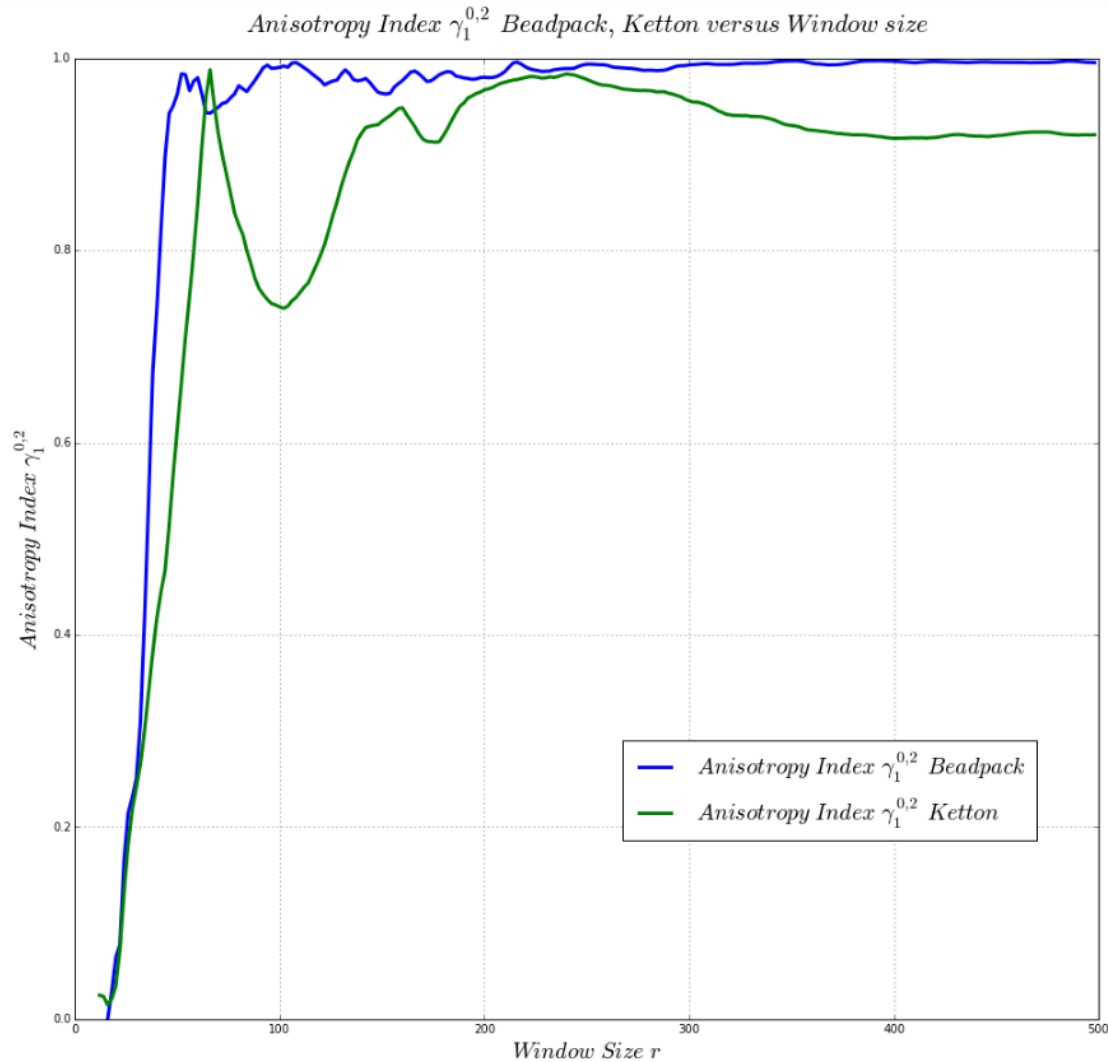




## Comparison Beadpack/Ketton – Beta 202



## Comparison Beadpack/Ketton – Gamma 102



## Comparison Beadpack Ketton – Gamma 202

