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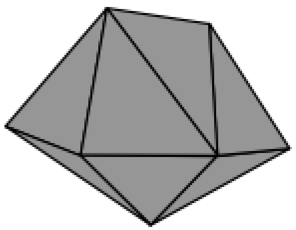
Minkowski Functionals - Integral Formulation

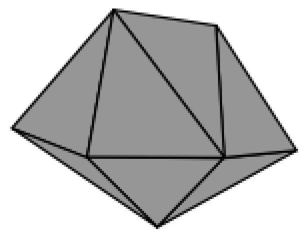
	scalar measures	
W_0	$\int_K \mathrm{d}V$	$\frac{1}{3}\sum_{T\in\mathcal{F}_2} \langle \mathbf{C}_T, \mathbf{n}_T\rangle \lvert T\rvert$
W_1	$\frac{1}{3}\int_{\partial K} \mathrm{d}A$	$\frac{1}{3}\sum_{T\in\mathcal{F}_2} \lvert T\rvert$
W_2	$\frac{1}{3}\int_{\partial K} G_2\,\mathrm{d}A$	$\frac{1}{12}\sum_{\mathbf{e}\in\mathcal{F}_1} \lvert\mathbf{e}\rvert\,\alpha_{\mathbf{e}}$
W_3	$\frac{1}{3}\int_{\partial K} G_3\,\mathrm{d}A$	$\frac{1}{3}\sum_{\mathbf{v}\in\mathcal{F}_0} (2\pi - \sum_{T\in\mathcal{F}_2(\mathbf{v})} \phi_T^{\mathbf{v}})$
	vectorial measures	
$(W_0^{1,0})_i$	$\int_K \mathbf{x}_i\,\mathrm{d}V$	$\sum_{T\in\mathcal{F}_2} (I_T)_{ik}(n_T)_k$, see sec. 2.4
$(W_1^{1,0})_i$	$\frac{1}{3}\int_{\partial K} \mathbf{x}_i\,\mathrm{d}A$	$\frac{1}{3}\sum_{T\in\mathcal{F}_2} \lvert T\rvert(\mathbf{C}_T)_i$
$(W_2^{1,0})_i$	$\frac{1}{3}\int_{\partial K} G_2\,\mathbf{x}_i\,\mathrm{d}A$	$\frac{1}{12}\sum_{\mathbf{e}\in\mathcal{F}_1} \lvert\mathbf{e}\rvert\alpha_{\mathbf{e}}(\mathbf{C}_{\mathbf{e}})_i$
$(W_3^{1,0})_i$	$\frac{1}{3}\int_{\partial K} G_3\,\mathbf{x}_i\,\mathrm{d}A$	$\frac{1}{3}\sum_{\mathbf{v}\in\mathcal{F}_0} (2\pi - \sum_{T\in\mathcal{F}_2(\mathbf{v})} \phi_T^{\mathbf{v}})\mathbf{v}_i$
	tensorial measures (rank two)	
$(W_0^{2,0})_{ij}$	$\int_K \mathbf{x}_i\mathbf{x}_j\,\mathrm{d}V$	$\sum_{T\in\mathcal{F}_2} (J_T)_{ijk}(n_T)_k$, see sec. 2.5
$(W_1^{2,0})_{ij}$	$\frac{1}{3}\int_{\partial K} \mathbf{x}_i\mathbf{x}_j\,\mathrm{d}A$	$\frac{1}{3}\sum_{T\in\mathcal{F}_2} (I_T)_{ij}$
$(W_2^{2,0})_{ij}$	$\frac{1}{3}\int_{\partial K} G_2\,\mathbf{x}_i\mathbf{x}_j\,\mathrm{d}A$	$\frac{1}{36}\sum_{\mathbf{e}\in\mathcal{F}_1} \alpha_{\mathbf{e}}\lvert\mathbf{e}\rvert\cdot((\mathbf{v}_1^2)_{ij} + (\mathbf{v}_1\mathbf{v}_2)_{ij} + (\mathbf{v}_2^2)_{ij})$
$(W_3^{2,0})_{ij}$	$\frac{1}{3}\int_{\partial K} G_3\,\mathbf{x}_i\mathbf{x}_j\,\mathrm{d}A$	$\frac{1}{3}\sum_{\mathbf{v}\in\mathcal{F}_0} \left(2\pi - \sum_{T\in\mathcal{F}_2(\mathbf{v})} \phi_T^{\mathbf{v}}\right)(\mathbf{v}^2)_{ij}$
$(W_1^{0,2})_{ij}$	$\frac{1}{3}\int_{\partial K} \mathbf{n}_i\mathbf{n}_j\,\mathrm{d}A$	$\frac{1}{3}\sum_{T\in\mathcal{F}_2} \lvert T\rvert\,(\mathbf{n}_T^2)_{ij}$
$(W_2^{0,2})_{ij}$	$\frac{1}{3}\int_{\partial K} G_2\,\mathbf{n}_i\mathbf{n}_j\,\mathrm{d}A$	$\frac{1}{24}\sum_{\mathbf{e}\in\mathcal{F}_1} \lvert\mathbf{e}\rvert\,((\alpha_{\mathbf{e}} + \sin\alpha_{\mathbf{e}})(\ddot{\mathbf{n}}_{\mathbf{e}}^2)_{ij} + (\alpha_{\mathbf{e}} - \sin\alpha_{\mathbf{e}})(\dot{\mathbf{n}}_{\mathbf{e}}^2)_{ij})$

Minkowski Tensor Visualization

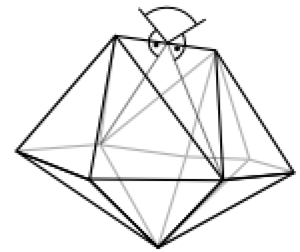
$W_0^{2,0}$ – moment tensor solid



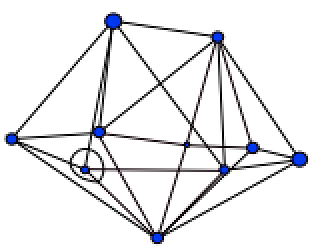
$W_1^{2,0}$ – moment tensor hollow



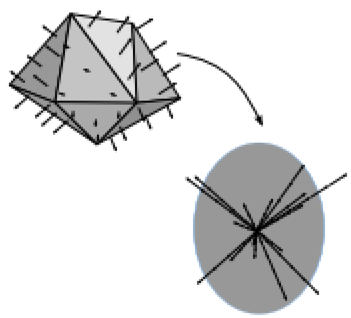
$W_0^{2,0}$ – moment tensor wire-frame



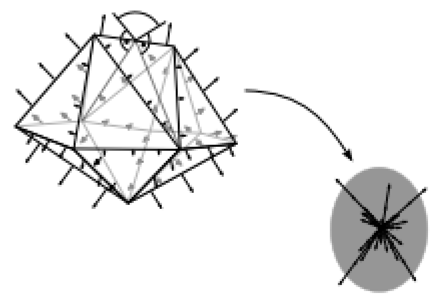
$W_3^{2,0}$ – moment tensor vertices



$W_1^{0,2}$ – normal distribution



$W_2^{0,2}$ – curvature distribution



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Minkowski Functionals – Translation Behavior

Homogeneity [unit]	rank 0	rank 1	rank 2	translation behaviour
λ^5 $[m^5]$	–	–	$W_0^{2,0}$	genuinely translation covariant
λ^4 $[m^4]$	–	$W_0^{1,0}$	$W_1^{2,0}$	genuinely translation covariant
λ^3 $[m^3]$	–	$W_1^{1,0}$	$W_2^{2,0}$	genuinely translation covariant
	W_0	–	$W_0\,Q$	translation invariant
λ^2 $[m^2]$	–	$W_2^{1,0}$	$W_3^{2,0}$	genuinely translation covariant
	–	–	$W_1^{0,2}$	translation invariant
	W_1	–	$W_1\,Q$	translation invariant
λ^1 $[m^1]$	–	$W_3^{1,0}$	–	genuinely translation covariant
	–	–	$W_2^{0,2}$	translation invariant
	W_2	–	$W_2\,Q$	translation invariant
λ^0 $[1]$	W_3	–	$W_3\,Q$	translation invariant