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Statistical Characterisation of Porous Media at the Pore Scale

Covariance, Minkowski Tensors and Permeability

Student:
Lukas Mosser

Supervisors:
Professor Olivier Dubrule
Professor Martin Blunt

Presentation Outline

- Summary of Results
- Comparison Permeability vs. W102, W202
- Comparison Formation Factor vs. W102, W202
- Covariance, MT Plot and Permeability
  - Sandstone Samples S1-S9
  - Carbonate Samples C1 and C2
  - Berea
- Covariance and MT Plot
  - Beadpack, Ketton, Estaillades, Doddington, Bentheimer

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Summary of Results

1. Attempt at correlating Minkowski Tensors to Permeability
    - For most samples:
        - Magnitude of permeability in Cartesian directions decreases with increase in Minkowski Tensor Components
            - Direction K highest - > Direction W102 lowest
            - Direction K lowest -> Direction W102 highest
        - Individual Cases:
            - Weak Discrepancy:
                - Permeability values on 2 axis very similar
                    - -> W102 very similar
                - Samples S6, S7, S8
            - Strong discrepancy: Completely different orientation of W102
                - Sample S3: Possibly wrong computation of K Tensor?
                - Sample C1
2. Most Sandstone Samples S1-S9 show anisotropy from Minkowski Tensor
    - Only Berea close to isotropic
3. REV estimation:
    - Sandstone Sample S6 and Carbonate C2:
    - Upwards trend of MT Plot -> REV not reached
4. Comparison Covariance and Minkowski Tensor Plot
    - Sample S4: X and Y Covariance very similar trend, Z direction different
    - Minkowski Tensor indicates Anisotropy in Z direction

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Sandstone Samples S1-S5

	$\overline{\overline{k}}\,(md)$	$\widehat{W}_1^{0,2}$	$\widehat{W}_2^{0,2}$
S1	$\begin{bmatrix} 1969 & 0 & 0 \\ 0 & 1752 & 0 \\ 0 & 0 & 1312 \end{bmatrix}$	$\begin{bmatrix} 0.286 & 0 & 0 \\ 0 & 0.294 & 0 \\ 0 & 0 & 0.419 \end{bmatrix}$	$\begin{bmatrix} 0.312 & 0 & 0 \\ 0 & 0.350 & 0 \\ 0 & 0 & 0.337 \end{bmatrix}$
S2	$\begin{bmatrix} 4318 & 0 & 0 \\ 0 & 3983 & 0 \\ 0 & 0 & 3394 \end{bmatrix}$	$\begin{bmatrix} 0.298 & 0 & 0 \\ 0 & 0.302 & 0 \\ 0 & 0 & 0.398 \end{bmatrix}$	$\begin{bmatrix} 0.314 & 0 & 0 \\ 0 & 0.305 & 0 \\ 0 & 0 & 0.379 \end{bmatrix}$
S3	$\begin{bmatrix} 143 & 0 & 0 \\ 0 & 420 & 0 \\ 0 & 0 & 109 \end{bmatrix}$	$\begin{bmatrix} 0.2761 & 0 & 0 \\ 0 & 0.2762 & 0 \\ 0 & 0 & 0.447 \end{bmatrix}$	$\begin{bmatrix} 0.309 & 0 & 0 \\ 0 & 0.307 & 0 \\ 0 & 0 & 0.383 \end{bmatrix}$
S4	$\begin{bmatrix} 273 & 0 & 0 \\ 0 & 289 & 0 \\ 0 & 0 & 215 \end{bmatrix}$	$\begin{bmatrix} 0.283 & 0 & 0 \\ 0 & 0.269 & 0 \\ 0 & 0 & 0.446 \end{bmatrix}$	$\begin{bmatrix} 0.313 & 0 & 0 \\ 0 & 0.306 & 0 \\ 0 & 0 & 0.379 \end{bmatrix}$
S5	$\begin{bmatrix} 4638 & 0 & 0 \\ 0 & 4874 & 0 \\ 0 & 0 & 4440 \end{bmatrix}$	$\begin{bmatrix} 0.303 & 0 & 0 \\ 0 & 0.301 & 0 \\ 0 & 0 & 0.395 \end{bmatrix}$	$\begin{bmatrix} 0.310 & 0 & 0 \\ 0 & 0.315 & 0 \\ 0 & 0 & 0.374 \end{bmatrix}$

Strong correlation Weak correlation No correlation

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Permeability - Sandstone Samples S6-S9

	$\bar{\bar{k}}\,(md)$	$\widehat{W}_1^{0,2}$	$\widehat{W}_2^{0,2}$
S6	$\begin{bmatrix} 11289 & 0 & 0 \\ 0 & 10683 & 0 \\ 0 & 0 & 10951 \end{bmatrix}$	$\begin{bmatrix} 0.296 & 0 & 0 \\ 0 & 0.306 & 0 \\ 0 & 0 & 0.397 \end{bmatrix}$	$\begin{bmatrix} 0.322 & 0 & 0 \\ 0 & 0.329 & 0 \\ 0 & 0 & 0.347 \end{bmatrix}$
S7	$\begin{bmatrix} 7268 & 0 & 0 \\ 0 & 7594 & 0 \\ 0 & 0 & 6037 \end{bmatrix}$	$\begin{bmatrix} 0.300 & 0 & 0 \\ 0 & 0.302 & 0 \\ 0 & 0 & 0.396 \end{bmatrix}$	$\begin{bmatrix} 0.303 & 0 & 0 \\ 0 & 0.322 & 0 \\ 0 & 0 & 0.374 \end{bmatrix}$
S8	$\begin{bmatrix} 13063 & 0 & 0 \\ 0 & 13507 & 0 \\ 0 & 0 & 12936 \end{bmatrix}$	$\begin{bmatrix} 0.287 & 0 & 0 \\ 0 & 0.288 & 0 \\ 0 & 0 & 0.423 \end{bmatrix}$	$\begin{bmatrix} 0.259 & 0 & 0 \\ 0 & 0.289 & 0 \\ 0 & 0 & 0.451 \end{bmatrix}$
S9	$\begin{bmatrix} 2735 & 0 & 0 \\ 0 & 2093 & 0 \\ 0 & 0 & 1844 \end{bmatrix}$	$\begin{bmatrix} 0.300 & 0 & 0 \\ 0 & 0.359 & 0 \\ 0 & 0 & 0.339 \end{bmatrix}$	$\begin{bmatrix} 0.312 & 0 & 0 \\ 0 & 0.350 & 0 \\ 0 & 0 & 0.337 \end{bmatrix}$

Strong correlation Weak correlation No correlation

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Permeability - Samples C1, C2, Berea

	$\bar{\bar{k}}\,(md)$	$\widehat{W}_1^{0,2}$	$\widehat{W}_2^{0,2}$
C1	$\begin{bmatrix} 785 & 0 & 0 \\ 0 & 1469 & 0 \\ 0 & 0 & 1053 \end{bmatrix}$	$\begin{bmatrix} 0.284 & 0 & 0 \\ 0 & 0.263 & 0 \\ 0 & 0 & 0.452 \end{bmatrix}$	$\begin{bmatrix} 0.305 & 0 & 0 \\ 0 & 0.329 & 0 \\ 0 & 0 & 0.347 \end{bmatrix}$
C2	$\begin{bmatrix} 38 & 0 & 0 \\ 0 & 161 & 0 \\ 0 & 0 & 18 \end{bmatrix}$	$\begin{bmatrix} 0.291 & 0 & 0 \\ 0 & 0.285 & 0 \\ 0 & 0 & 0.423 \end{bmatrix}$	$\begin{bmatrix} 0.310 & 0 & 0 \\ 0 & 0.290 & 0 \\ 0 & 0 & 0.403 \end{bmatrix}$
Berea	$\begin{bmatrix} 1360 & 0 & 0 \\ 0 & 1304 & 0 \\ 0 & 0 & 1193 \end{bmatrix}$	$\begin{bmatrix} 0.320 & 0 & 0 \\ 0 & 0.335 & 0 \\ 0 & 0 & 0.344 \end{bmatrix}$	$\begin{bmatrix} 0.322 & 0 & 0 \\ 0 & 0.328 & 0 \\ 0 & 0 & 0.348 \end{bmatrix}$

Strong correlation Weak correlation No correlation

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Formation Factors - Sandstone Samples S1-S5

	FF	$\widehat{W}_1^{0,2}$	$\widehat{W}_2^{0,2}$
S1	$\begin{bmatrix} 36.3 & 0 & 0 \\ 0 & 37.9 & 0 \\ 0 & 0 & 53.5 \end{bmatrix}$	$\begin{bmatrix} 0.286 & 0 & 0 \\ 0 & 0.294 & 0 \\ 0 & 0 & 0.419 \end{bmatrix}$	$\begin{bmatrix} 0.312 & 0 & 0 \\ 0 & 0.350 & 0 \\ 0 & 0 & 0.337 \end{bmatrix}$
S2	$\begin{bmatrix} 10.9 & 0 & 0 \\ 0 & 11.3 & 0 \\ 0 & 0 & 12.6 \end{bmatrix}$	$\begin{bmatrix} 0.298 & 0 & 0 \\ 0 & 0.302 & 0 \\ 0 & 0 & 0.398 \end{bmatrix}$	$\begin{bmatrix} 0.314 & 0 & 0 \\ 0 & 0.305 & 0 \\ 0 & 0 & 0.379 \end{bmatrix}$
S3	$\begin{bmatrix} 52.2 & 0 & 0 \\ 0 & 41.9 & 0 \\ 0 & 0 & 70.3 \end{bmatrix}$	$\begin{bmatrix} 0.2761 & 0 & 0 \\ 0 & 0.2762 & 0 \\ 0 & 0 & 0.447 \end{bmatrix}$	$\begin{bmatrix} 0.309 & 0 & 0 \\ 0 & 0.307 & 0 \\ 0 & 0 & 0.383 \end{bmatrix}$
S4	$\begin{bmatrix} 74.5 & 0 & 0 \\ 0 & 71.0 & 0 \\ 0 & 0 & 114.4 \end{bmatrix}$	$\begin{bmatrix} 0.283 & 0 & 0 \\ 0 & 0.269 & 0 \\ 0 & 0 & 0.446 \end{bmatrix}$	$\begin{bmatrix} 0.313 & 0 & 0 \\ 0 & 0.306 & 0 \\ 0 & 0 & 0.379 \end{bmatrix}$
S5	$\begin{bmatrix} 14.1 & 0 & 0 \\ 0 & 14.6 & 0 \\ 0 & 0 & 15.9 \end{bmatrix}$	$\begin{bmatrix} 0.303 & 0 & 0 \\ 0 & 0.301 & 0 \\ 0 & 0 & 0.395 \end{bmatrix}$	$\begin{bmatrix} 0.310 & 0 & 0 \\ 0 & 0.315 & 0 \\ 0 & 0 & 0.374 \end{bmatrix}$

Strong correlation Weak correlation No correlation

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Formation Factors - Sandstone Samples S6-S9

	FF	$\widehat{W}_1^{0,2}$	$\widehat{W}_2^{0,2}$
S6	$\begin{bmatrix} 10.3 & 0 & 0 \\ 0 & 11.2 & 0 \\ 0 & 0 & 12.9 \end{bmatrix}$	$\begin{bmatrix} 0.296 & 0 & 0 \\ 0 & 0.306 & 0 \\ 0 & 0 & 0.397 \end{bmatrix}$	$\begin{bmatrix} 0.322 & 0 & 0 \\ 0 & 0.329 & 0 \\ 0 & 0 & 0.347 \end{bmatrix}$
S7	$\begin{bmatrix} 9.6 & 0 & 0 \\ 0 & 9.3 & 0 \\ 0 & 0 & 11.2 \end{bmatrix}$	$\begin{bmatrix} 0.300 & 0 & 0 \\ 0 & 0.302 & 0 \\ 0 & 0 & 0.396 \end{bmatrix}$	$\begin{bmatrix} 0.303 & 0 & 0 \\ 0 & 0.322 & 0 \\ 0 & 0 & 0.374 \end{bmatrix}$
S8	$\begin{bmatrix} 6.02 & 0 & 0 \\ 0 & 5.97 & 0 \\ 0 & 0 & 6.53 \end{bmatrix}$	$\begin{bmatrix} 0.287 & 0 & 0 \\ 0 & 0.288 & 0 \\ 0 & 0 & 0.423 \end{bmatrix}$	$\begin{bmatrix} 0.259 & 0 & 0 \\ 0 & 0.289 & 0 \\ 0 & 0 & 0.451 \end{bmatrix}$
S9	$\begin{bmatrix} 16.4 & 0 & 0 \\ 0 & 19.9 & 0 \\ 0 & 0 & 18.1 \end{bmatrix}$	$\begin{bmatrix} 0.300 & 0 & 0 \\ 0 & 0.359 & 0 \\ 0 & 0 & 0.339 \end{bmatrix}$	$\begin{bmatrix} 0.312 & 0 & 0 \\ 0 & 0.350 & 0 \\ 0 & 0 & 0.337 \end{bmatrix}$

Strong correlation Weak correlation No correlation

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Formation Factors - Samples C1, C2, Berea

	FF	$\widehat{W}_1^{0,2}$	$\widehat{W}_2^{0,2}$
C1	$\begin{bmatrix} 33.7 & 0 & 0 \\ 0 & 21.9 & 0 \\ 0 & 0 & 27.2 \end{bmatrix}$	$\begin{bmatrix} 0.284 & 0 & 0 \\ 0 & 0.263 & 0 \\ 0 & 0 & 0.452 \end{bmatrix}$	$\begin{bmatrix} 0.305 & 0 & 0 \\ 0 & 0.329 & 0 \\ 0 & 0 & 0.347 \end{bmatrix}$
C2	$\begin{bmatrix} 18 & 0 & 0 \\ 0 & 121 & 0 \\ 0 & 0 & 249 \end{bmatrix}$	$\begin{bmatrix} 0.291 & 0 & 0 \\ 0 & 0.285 & 0 \\ 0 & 0 & 0.423 \end{bmatrix}$	$\begin{bmatrix} 0.310 & 0 & 0 \\ 0 & 0.290 & 0 \\ 0 & 0 & 0.403 \end{bmatrix}$
Berea	$\begin{bmatrix} 23.1 & 0 & 0 \\ 0 & 24.0 & 0 \\ 0 & 0 & 25.2 \end{bmatrix}$	$\begin{bmatrix} 0.320 & 0 & 0 \\ 0 & 0.335 & 0 \\ 0 & 0 & 0.344 \end{bmatrix}$	$\begin{bmatrix} 0.322 & 0 & 0 \\ 0 & 0.328 & 0 \\ 0 & 0 & 0.348 \end{bmatrix}$

Strong correlation Weak correlation No correlation

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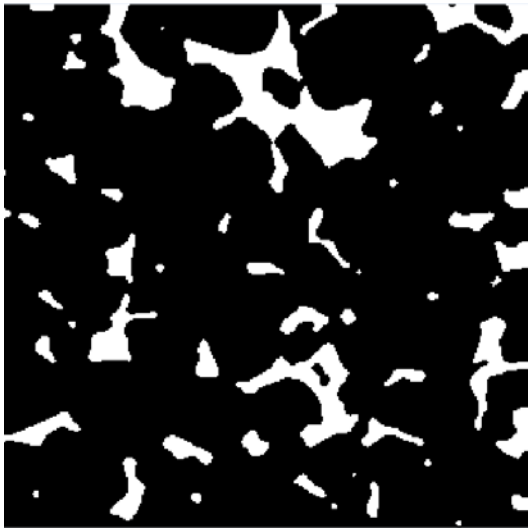
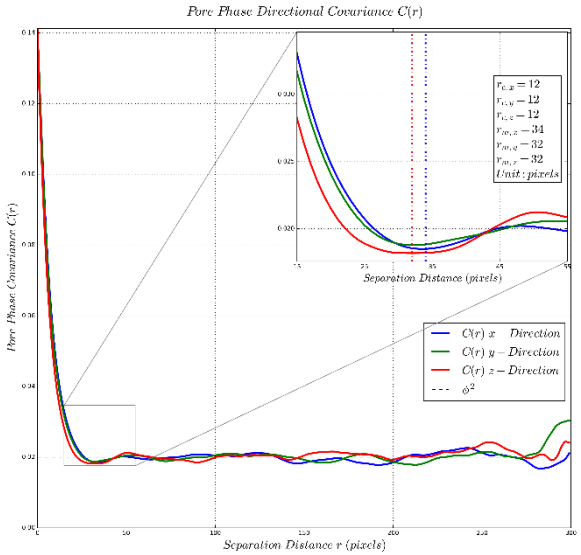
Sandstone Sample - S1

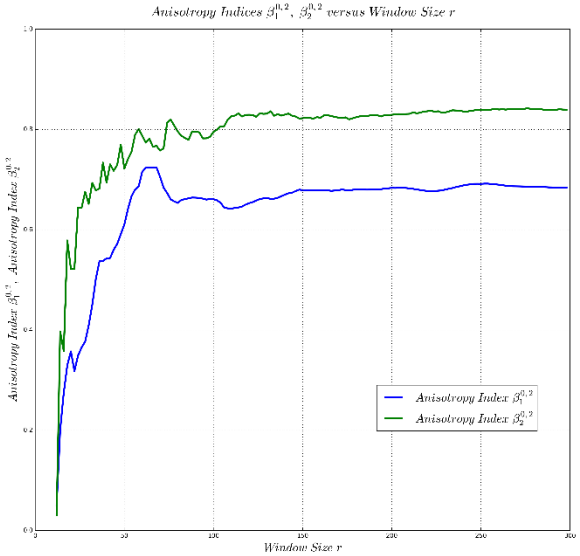
Image



Covariance



Minkowski Tensor



$$\phi_{cov} = 0.141 \qquad \overline{\overline{k}}\,(md) \begin{bmatrix} 1969 & 0 & 0 \\ 0 & 1752 & 0 \\ 0 & 0 & 1312 \end{bmatrix} \qquad \widehat{W}_1^{0,2} \begin{bmatrix} 0.286 & 0 & 0 \\ 0 & 0.294 & 0 \\ 0 & 0 & 0.419 \end{bmatrix} \qquad \widehat{W}_2^{0,2} \begin{bmatrix} 0.312 & 0 & 0 \\ 0 & 0.350 & 0 \\ 0 & 0 & 0.337 \end{bmatrix}$$

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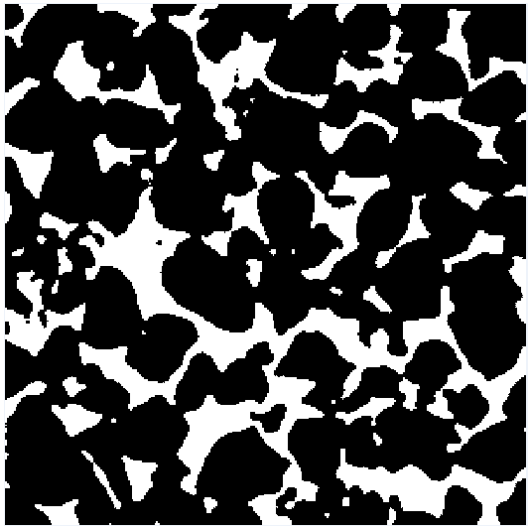
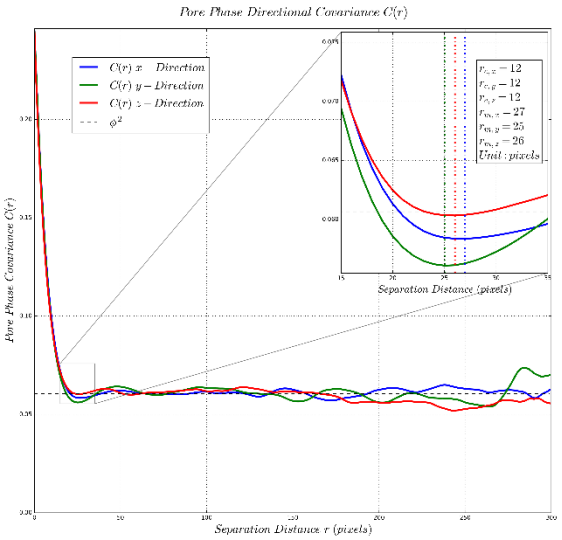
Sandstone Sample – S2

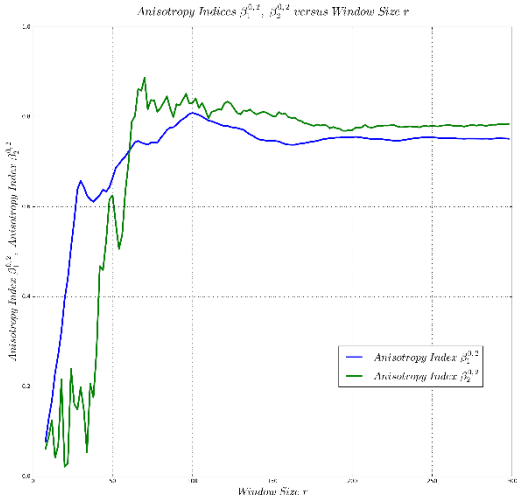
Image



Covariance



Minkowski Tensor



$$\phi_{cov} = 0.246 \qquad \overline{\overline{k}}\,(md) \begin{bmatrix} 4318 & 0 & 0 \\ 0 & 3983 & 0 \\ 0 & 0 & 3394 \end{bmatrix} \qquad \widehat{W}_1^{0,2} \begin{bmatrix} 0.298 & 0 & 0 \\ 0 & 0.302 & 0 \\ 0 & 0 & 0.398 \end{bmatrix} \qquad \widehat{W}_2^{0,2} \begin{bmatrix} 0.314 & 0 & 0 \\ 0 & 0.305 & 0 \\ 0 & 0 & 0.379 \end{bmatrix}$$

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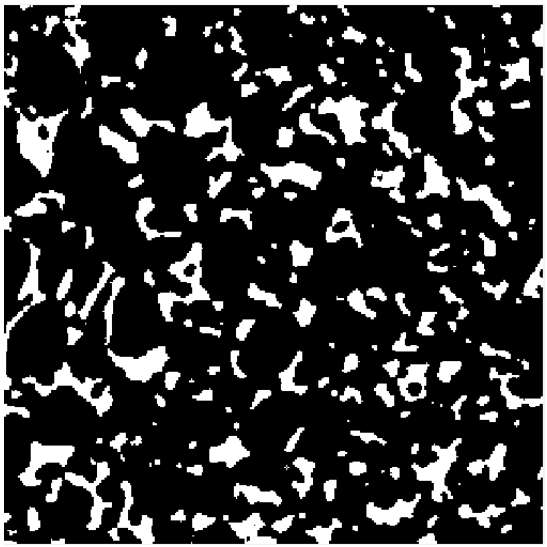
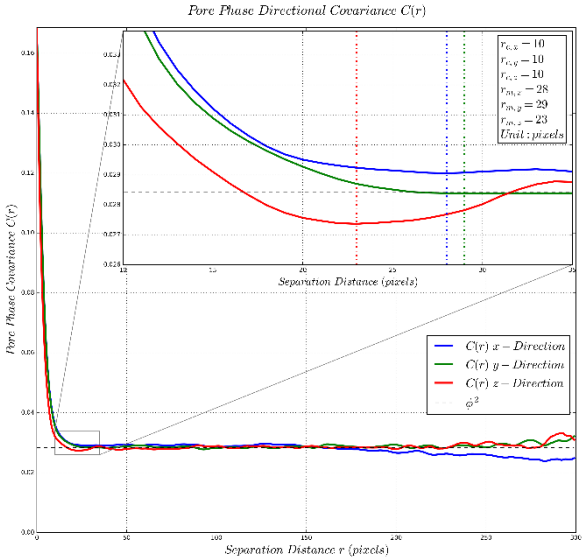
Sandstone Sample – S3

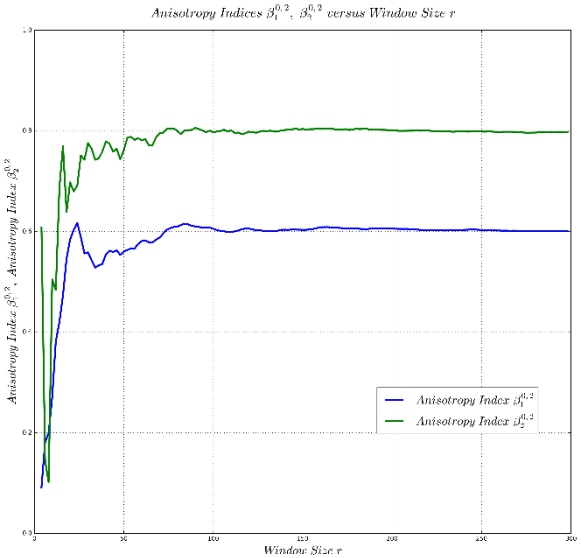
Image



Covariance



Minkowski Tensor



$\phi_{cov} = 0.169$

$\bar{\bar{k}}\ (md)$

$$\begin{bmatrix} 143 & 0 & 0 \\ 0 & 420 & 0 \\ 0 & 0 & 109 \end{bmatrix}$$

$\widehat{W}_1^{0,2}$

$$\begin{bmatrix} 0.2761 & 0 & 0 \\ 0 & 0.2762 & 0 \\ 0 & 0 & 0.447 \end{bmatrix}$$

$\widehat{W}_2^{0,2}$

$$\begin{bmatrix} 0.309 & 0 & 0 \\ 0 & 0.307 & 0 \\ 0 & 0 & 0.383 \end{bmatrix}$$

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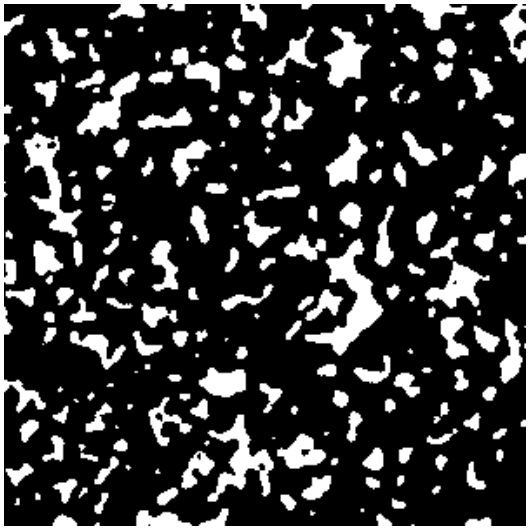
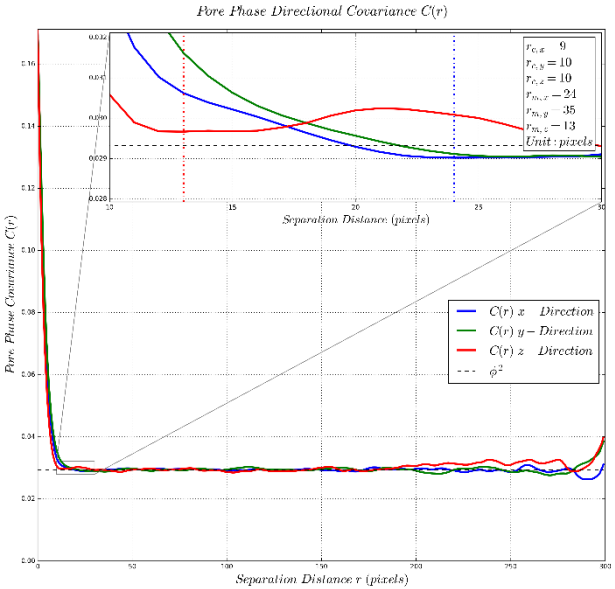
Sandstone Sample – S4

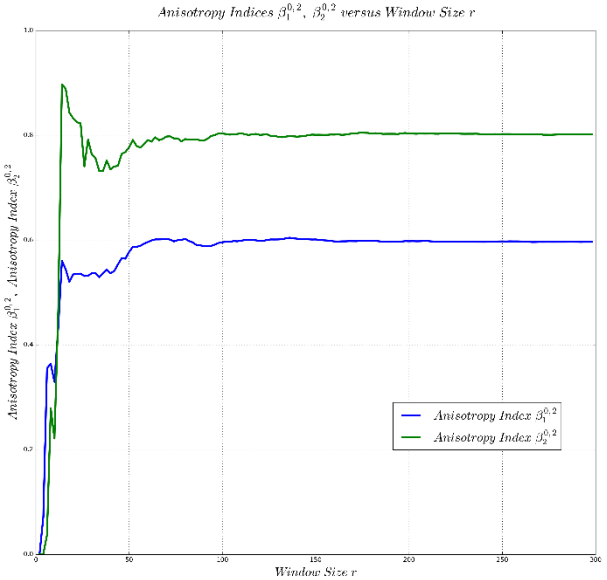
Image



Covariance



Minkowski Tensor



$$\phi_{cov} = 0.171 \qquad \overline{\overline{k}}\,(md) \begin{bmatrix} 273 & 0 & 0 \\ 0 & 289 & 0 \\ 0 & 0 & 215 \end{bmatrix} \qquad \widehat{W}_1^{0,2} \begin{bmatrix} 0.283 & 0 & 0 \\ 0 & 0.269 & 0 \\ 0 & 0 & 0.446 \end{bmatrix} \qquad \widehat{W}_2^{0,2} \begin{bmatrix} 0.313 & 0 & 0 \\ 0 & 0.306 & 0 \\ 0 & 0 & 0.379 \end{bmatrix}$$

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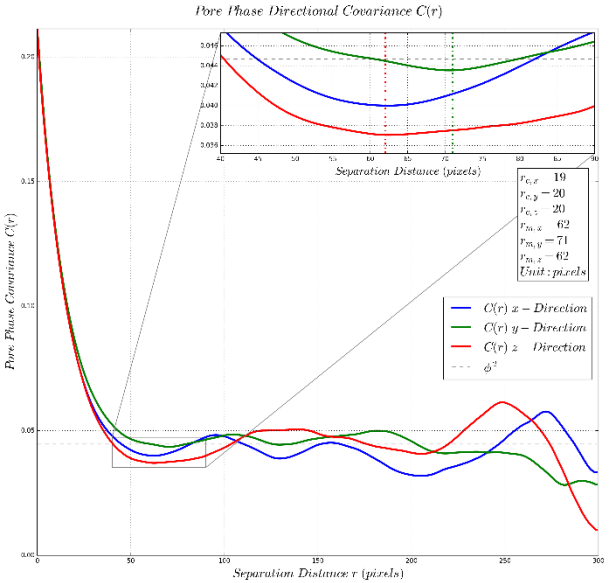
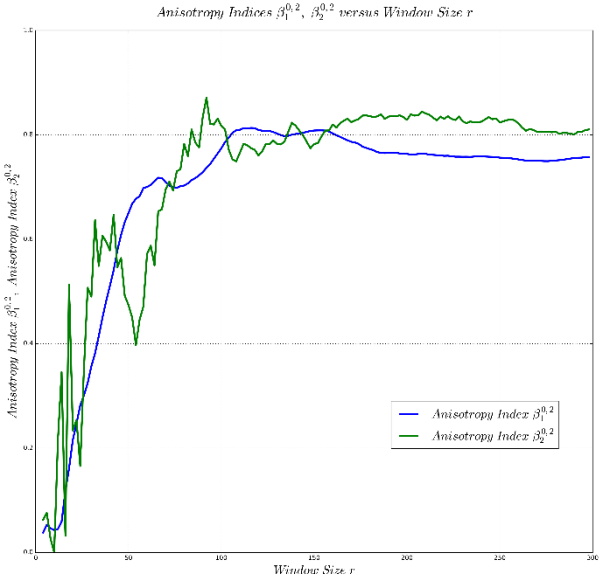
Sandstone Sample – S5

Image

Covariance



Minkowski Tensor



$$\phi_{cov} = 0.211 \qquad \overline{\overline{k}}\,(md) \begin{bmatrix} 4638 & 0 & 0 \\ 0 & 4874 & 0 \\ 0 & 0 & 4440 \end{bmatrix} \qquad \widehat{W}_1^{0,2} \begin{bmatrix} 0.303 & 0 & 0 \\ 0 & 0.301 & 0 \\ 0 & 0 & 0.395 \end{bmatrix} \qquad \widehat{W}_2^{0,2} \begin{bmatrix} 0.310 & 0 & 0 \\ 0 & 0.315 & 0 \\ 0 & 0 & 0.374 \end{bmatrix}$$

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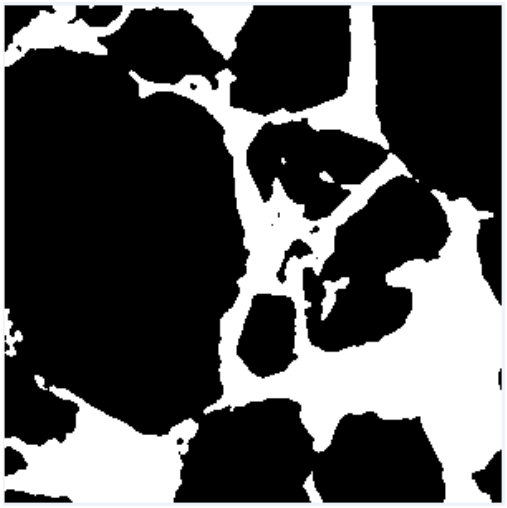
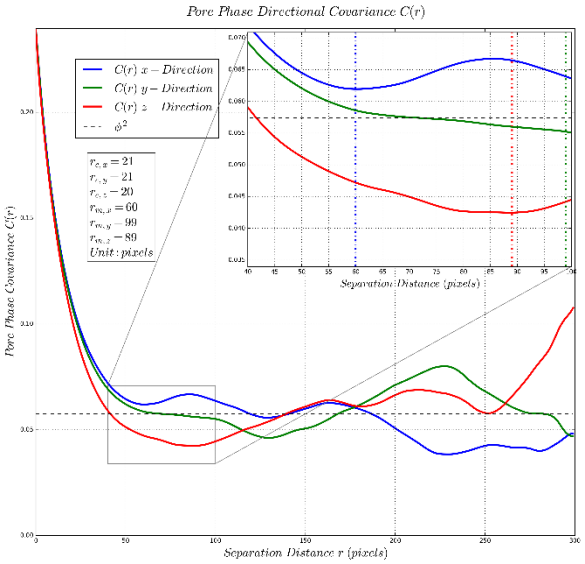
Sandstone Sample – S6

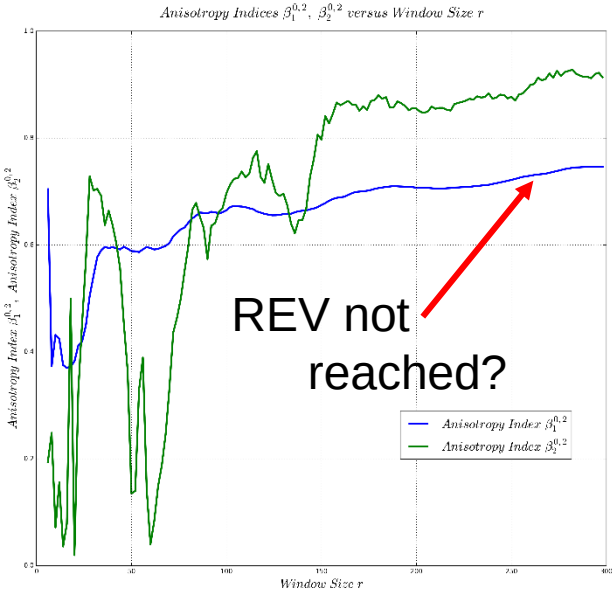
Image



Covariance



Minkowski Tensor



$$\phi_{cov} = 0.240$$

$$\overline{\overline{k}}\ (md) \begin{bmatrix} 11289 & 0 & 0 \\ 0 & 10683 & 0 \\ 0 & 0 & 10951 \end{bmatrix}$$

$$\widehat{W}_1^{0,2} \begin{bmatrix} 0.296 & 0 & 0 \\ 0 & 0.306 & 0 \\ 0 & 0 & 0.397 \end{bmatrix}$$

$$\widehat{W}_2^{0,2} \begin{bmatrix} 0.322 & 0 & 0 \\ 0 & 0.329 & 0 \\ 0 & 0 & 0.347 \end{bmatrix}$$

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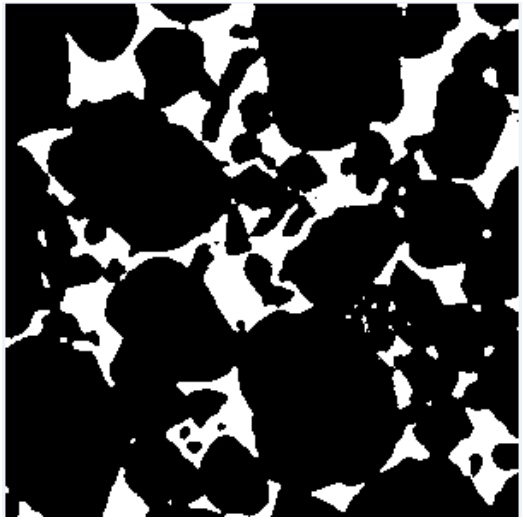
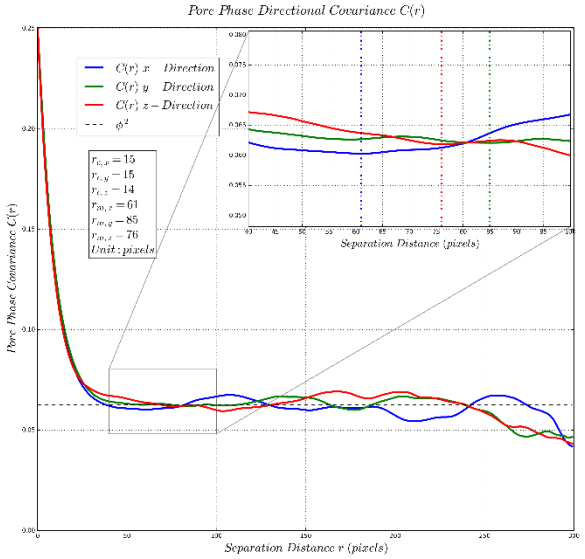
Sandstone Sample – S7

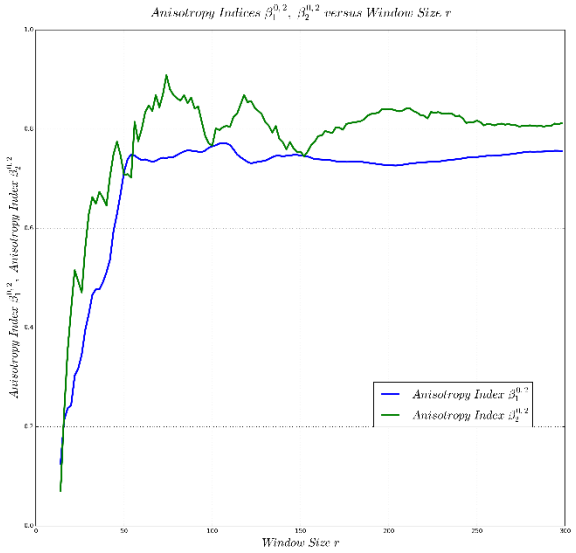
Image



Covariance



Minkowski Tensor



$$\phi_{cov} = 0.250 \quad \overline{\overline{k}}\,(md) \begin{bmatrix} 7268 & 0 & 0 \\ 0 & 7594 & 0 \\ 0 & 0 & 6037 \end{bmatrix} \qquad \widehat{W}_1^{0,2} \begin{bmatrix} 0.300 & 0 & 0 \\ 0 & 0.302 & 0 \\ 0 & 0 & 0.396 \end{bmatrix} \qquad \widehat{W}_2^{0,2} \begin{bmatrix} 0.303 & 0 & 0 \\ 0 & 0.322 & 0 \\ 0 & 0 & 0.374 \end{bmatrix}$$

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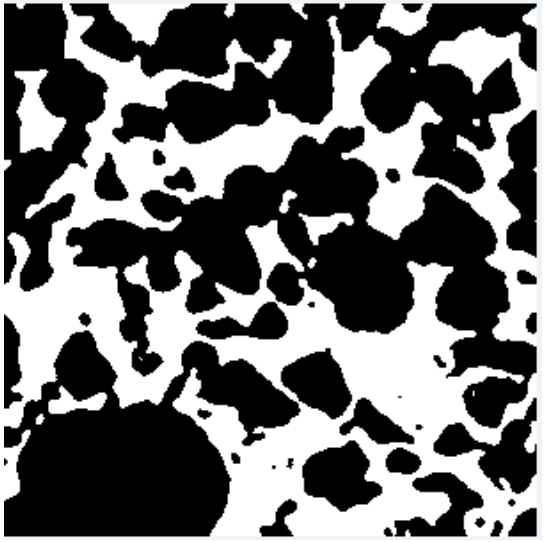
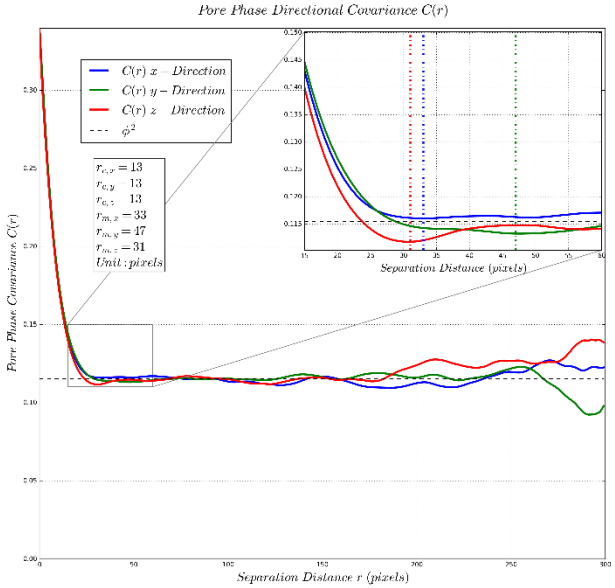
Sandstone Sample – S8

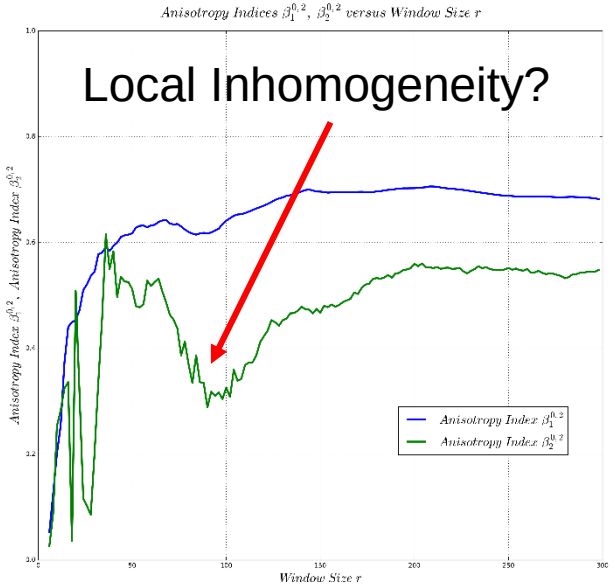
Image



Covariance



Minkowski Tensor



$$\phi_{cov} = 0.340 \qquad \overline{\overline{k}}\,(md) \begin{bmatrix} 13063 & 0 & 0 \\ 0 & 13507 & 0 \\ 0 & 0 & 12936 \end{bmatrix} \qquad \widehat{W}_1^{0,2} \begin{bmatrix} 0.287 & 0 & 0 \\ 0 & 0.288 & 0 \\ 0 & 0 & 0.423 \end{bmatrix} \qquad \widehat{W}_2^{0,2} \begin{bmatrix} 0.259 & 0 & 0 \\ 0 & 0.289 & 0 \\ 0 & 0 & 0.451 \end{bmatrix}$$

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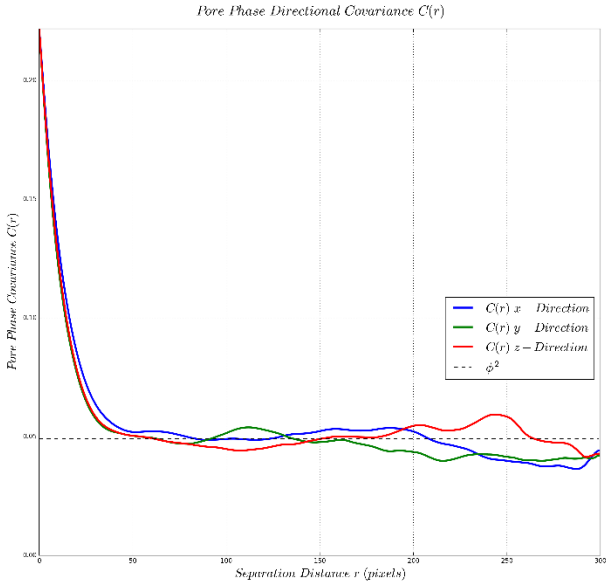
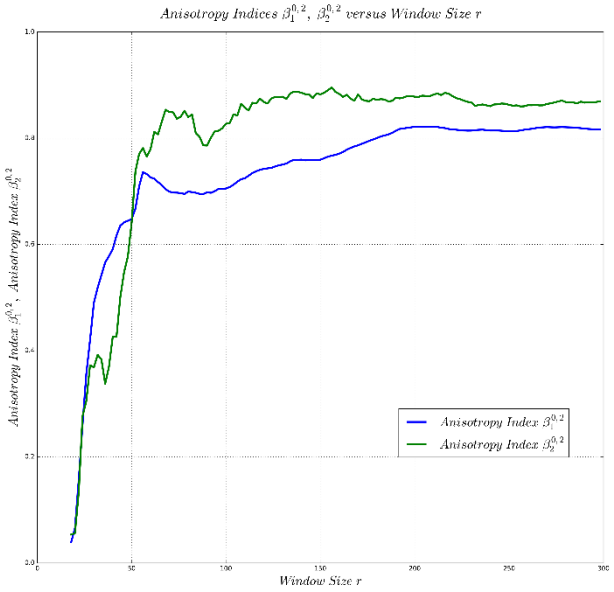
Sandstone Sample – S9

Image

Covariance



Minkowski Tensor



$$\phi_{cov} = 0.222 \qquad \overline{\overline{k}}\,(md) \begin{bmatrix} 2735 & 0 & 0 \\ 0 & 2093 & 0 \\ 0 & 0 & 1844 \end{bmatrix} \qquad \widehat{W}_1^{0,2} \begin{bmatrix} 0.300 & 0 & 0 \\ 0 & 0.359 & 0 \\ 0 & 0 & 0.339 \end{bmatrix} \qquad \widehat{W}_2^{0,2} \begin{bmatrix} 0.312 & 0 & 0 \\ 0 & 0.350 & 0 \\ 0 & 0 & 0.337 \end{bmatrix}$$

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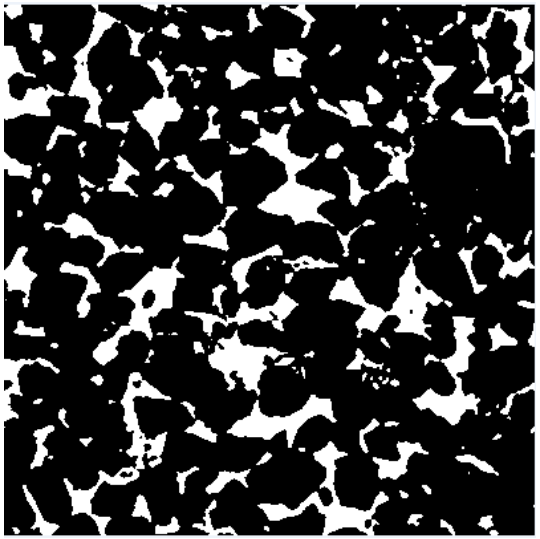
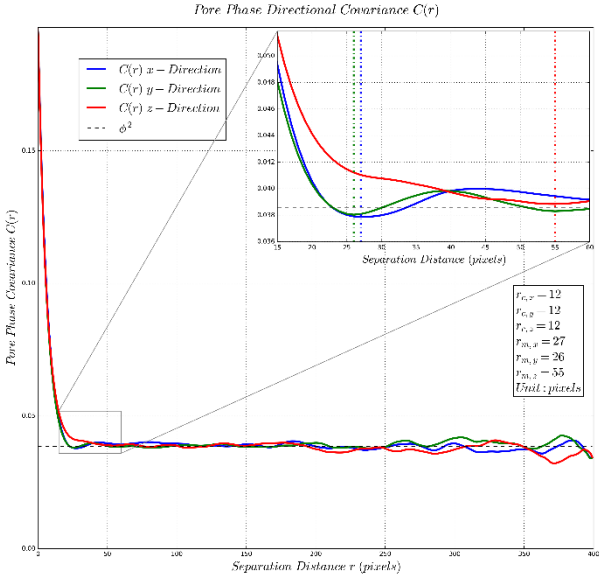
Sandstone Sample – Berea

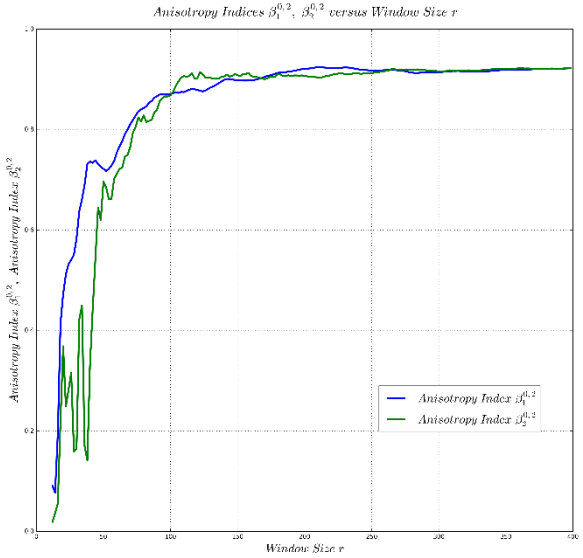
Image



Covariance



Minkowski Tensor



$$\phi_{cov} = 0.197 \qquad \overline{\overline{k}}\ (md) \begin{bmatrix} 1360 & 0 & 0 \\ 0 & 1304 & 0 \\ 0 & 0 & 1193 \end{bmatrix} \qquad \widehat{W}_1^{0,2} \begin{bmatrix} 0.320 & 0 & 0 \\ 0 & 0.335 & 0 \\ 0 & 0 & 0.344 \end{bmatrix} \qquad \widehat{W}_2^{0,2} \begin{bmatrix} 0.322 & 0 & 0 \\ 0 & 0.328 & 0 \\ 0 & 0 & 0.348 \end{bmatrix}$$

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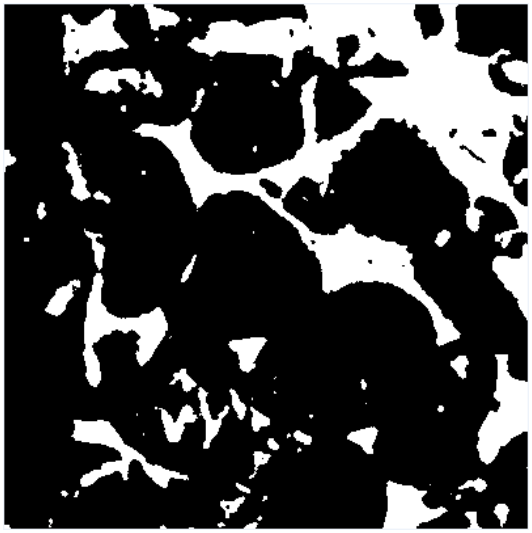
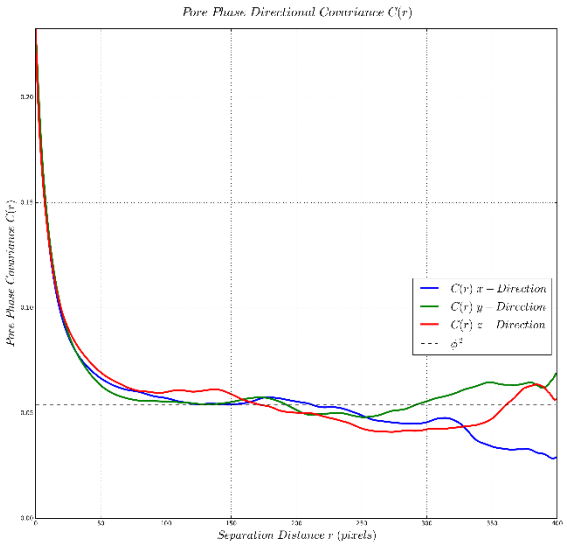
Carbonate Sample – C1

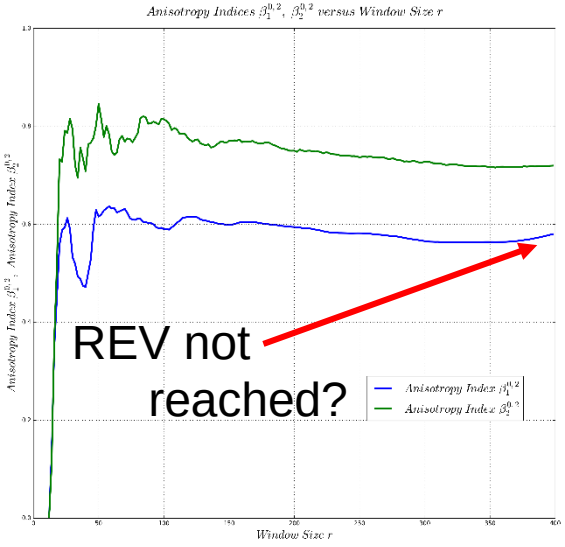
Image



Covariance



Minkowski Tensor



$\phi_{cov} = 0.233$

$\overline{\overline{k}}\,(md)$

$$\begin{bmatrix} 785 & 0 & 0 \\ 0 & 1469 & 0 \\ 0 & 0 & 1053 \end{bmatrix}$$

$\widehat{W}_1^{0,2}$

$$\begin{bmatrix} 0.284 & 0 & 0 \\ 0 & 0.263 & 0 \\ 0 & 0 & 0.452 \end{bmatrix}$$

$\widehat{W}_2^{0,2}$

$$\begin{bmatrix} 0.305 & 0 & 0 \\ 0 & 0.329 & 0 \\ 0 & 0 & 0.347 \end{bmatrix}$$

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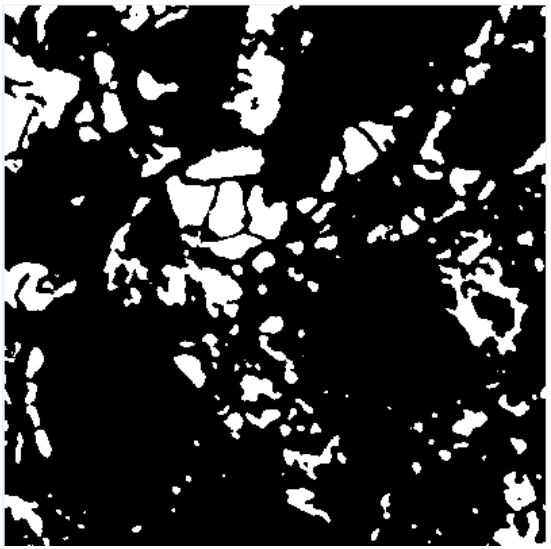
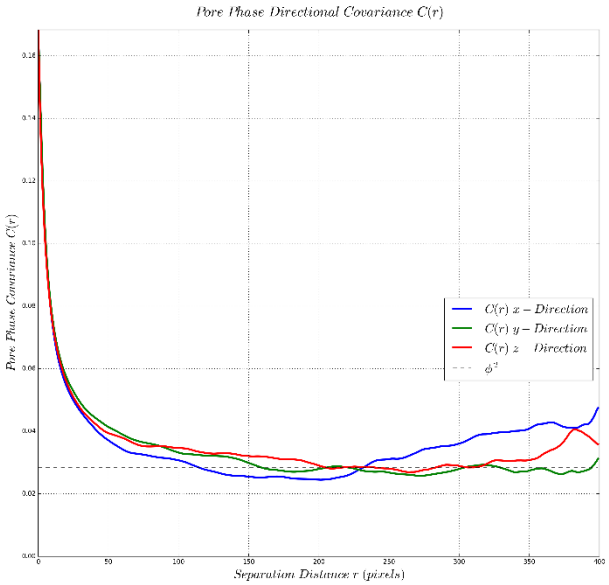
Carbonate Sample – C2

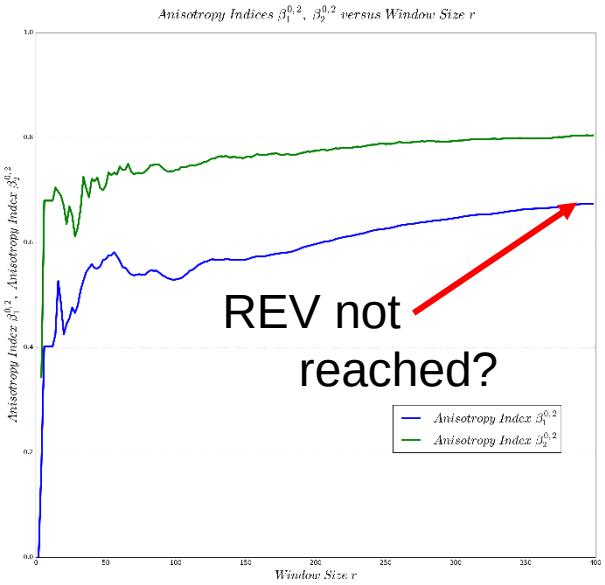
Image



Covariance



Minkowski Tensor



$\bar{\bar{k}}\ (md)$ $\qquad$ $\widehat{W}_1^{0,2}$ $\qquad$ $\widehat{W}_2^{0,2}$

$$\phi_{cov} = 0.168 \begin{bmatrix} 38 & 0 & 0 \\ 0 & 161 & 0 \\ 0 & 0 & 18 \end{bmatrix} \quad \begin{bmatrix} 0.291 & 0 & 0 \\ 0 & 0.285 & 0 \\ 0 & 0 & 0.423 \end{bmatrix} \quad \begin{bmatrix} 0.310 & 0 & 0 \\ 0 & 0.290 & 0 \\ 0 & 0 & 0.403 \end{bmatrix}$$

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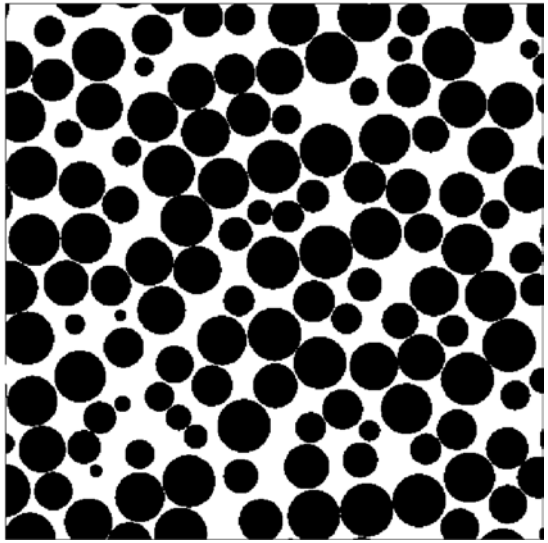
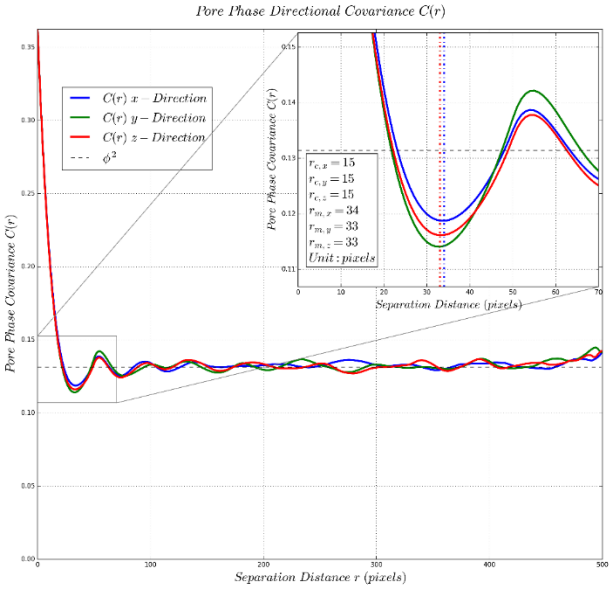
Beadpack

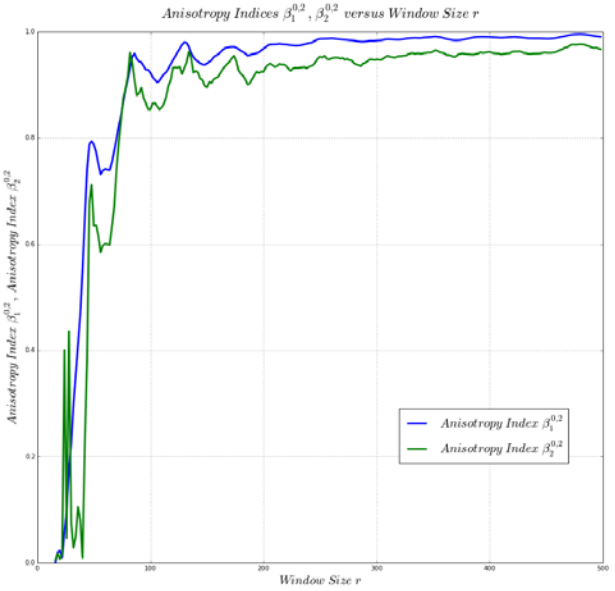
Image



Covariance



Minkowski Tensor



$\bar{\bar{k}}\,(md)$ $\qquad\qquad$ $\widehat{W}_1^{0,2}$ $\qquad\qquad$ $\widehat{W}_2^{0,2}$

$$\phi_{cov} = 0.363 \qquad \begin{bmatrix} 6106 & 0 & 0 \\ 0 & 6244 & 0 \\ 0 & 0 & 6117 \end{bmatrix} \qquad \begin{bmatrix} 0.335 & 0 & 0 \\ 0 & 0.334 & 0 \\ 0 & 0 & 0.331 \end{bmatrix} \qquad \begin{bmatrix} 0.334 & 0 & 0 \\ 0 & 0.337 & 0 \\ 0 & 0 & 0.327 \end{bmatrix}$$

$$\phi_{eff} = 0.362$$

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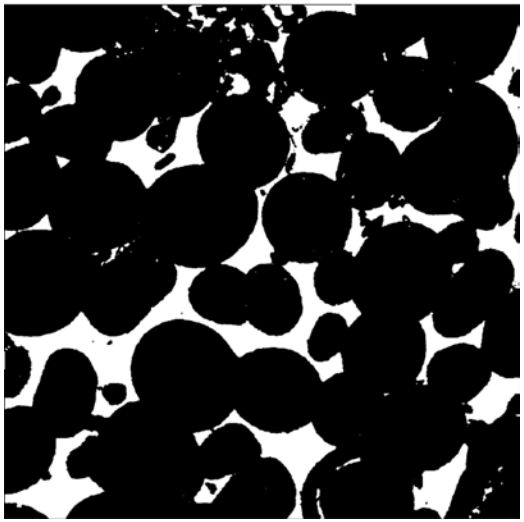
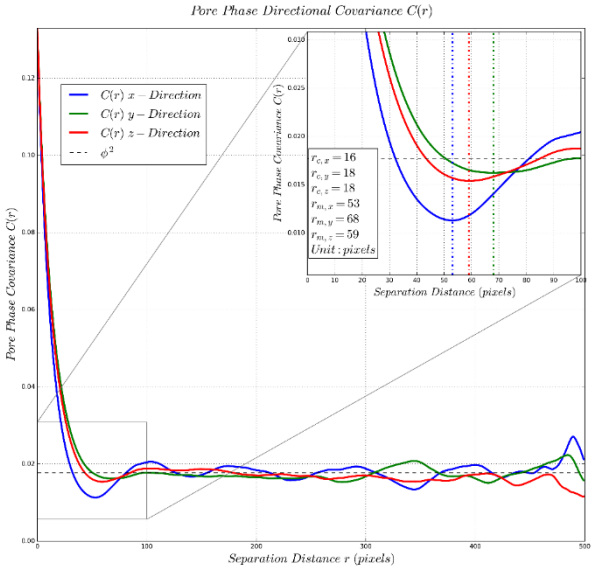
Ketton

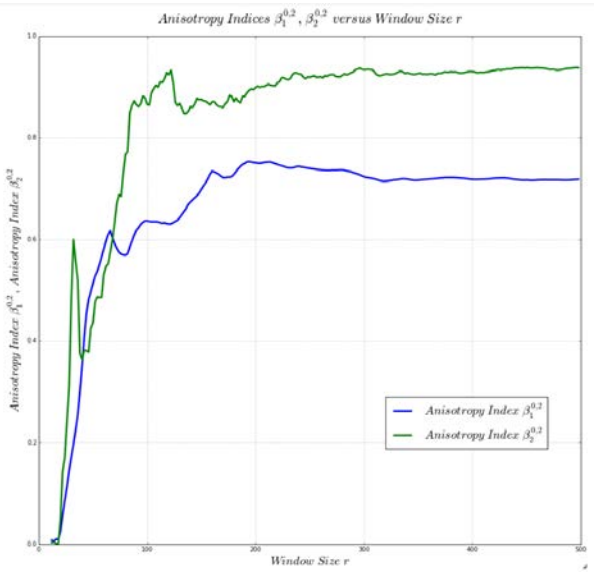
Image



Covariance



Minkowski Tensor



$\overline{\overline{k}}\,(md)$

$$\phi_{cov} = 0.133$$
$$\phi_{eff} = 0.128$$
$$\begin{bmatrix} 829 & 0 & 0 \\ 0 & 629 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

$\widehat{W}_1^{0,2}$

$$\begin{bmatrix} 0.300 & 0 & 0 \\ 0 & 0.321 & 0 \\ 0 & 0 & 0.378 \end{bmatrix}$$

$\widehat{W}_2^{0,2}$

$$\begin{bmatrix} 0.327 & 0 & 0 \\ 0 & 0.336 & 0 \\ 0 & 0 & 0.342 \end{bmatrix}$$

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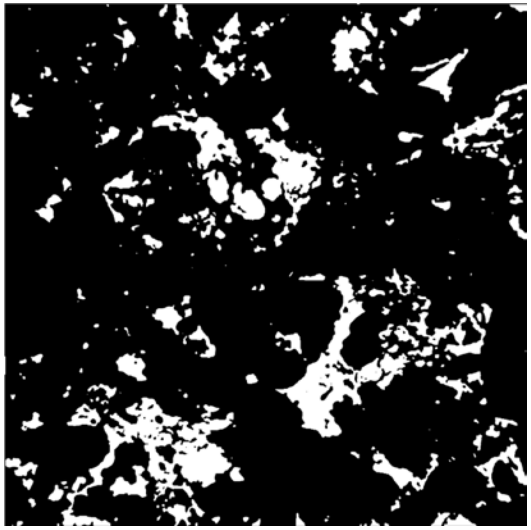
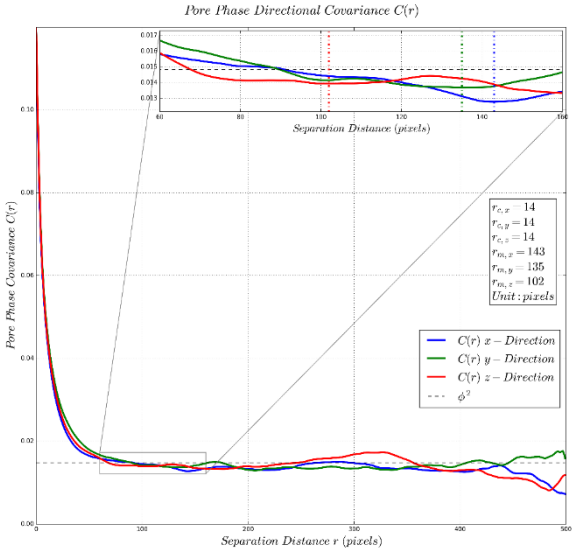
Estaillades

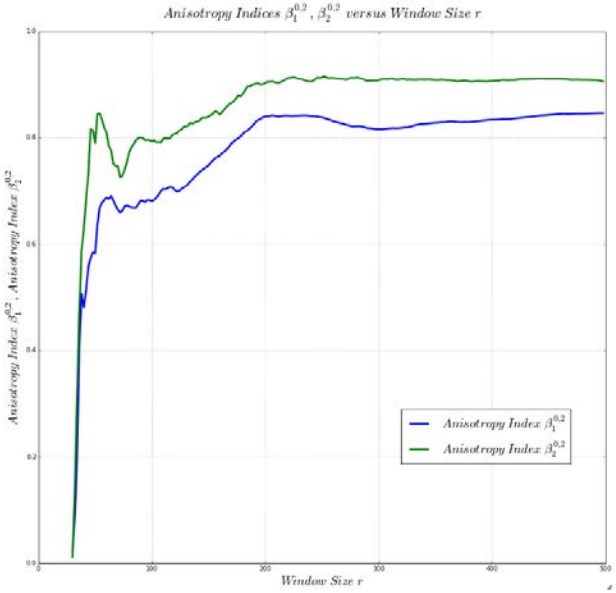
Image



Covariance



Minkowski Tensor



$\bar{\bar{k}}\ (md)$

$$\phi_{cov} = 0.122 \qquad \begin{bmatrix} 52 & 0 & 0 \\ 0 & 240 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\phi_{eff} = 0.085$$

$\widehat{W}_1^{0,2}$

$$\begin{bmatrix} 0.310 & 0 & 0 \\ 0 & 0.326 & 0 \\ 0 & 0 & 0.362 \end{bmatrix}$$

$\widehat{W}_2^{0,2}$

$$\begin{bmatrix} 0.320 & 0 & 0 \\ 0 & 0.329 & 0 \\ 0 & 0 & 0.350 \end{bmatrix}$$

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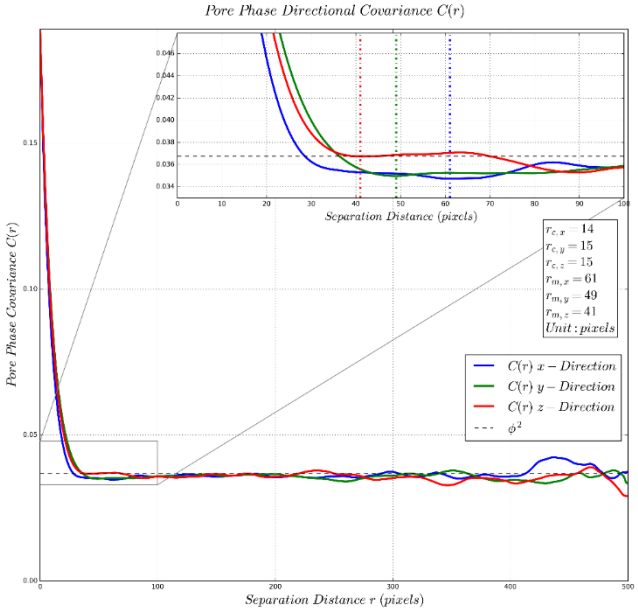
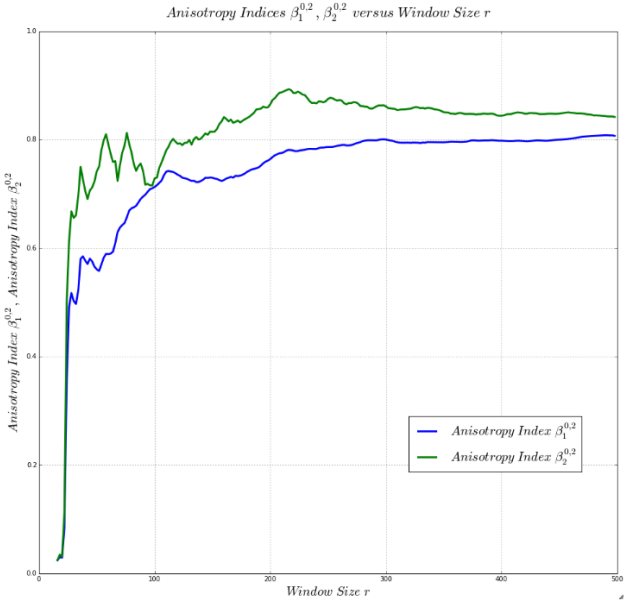
Doddington

Image

Covariance



Minkowski Tensor



$\bar{\bar{k}}\,(md)$

$$\phi_{cov} = 0.192 \quad \begin{bmatrix} 309 & 0 & 0 \\ 0 & 304 & 0 \\ 0 & 0 & 265 \end{bmatrix}$$

$$\phi_{eff} = 0.189$$

$\widehat{W}_1^{0,2}$

$$\begin{bmatrix} 0.306 & 0 & 0 \\ 0 & 0.315 & 0 \\ 0 & 0 & 0.378 \end{bmatrix}$$

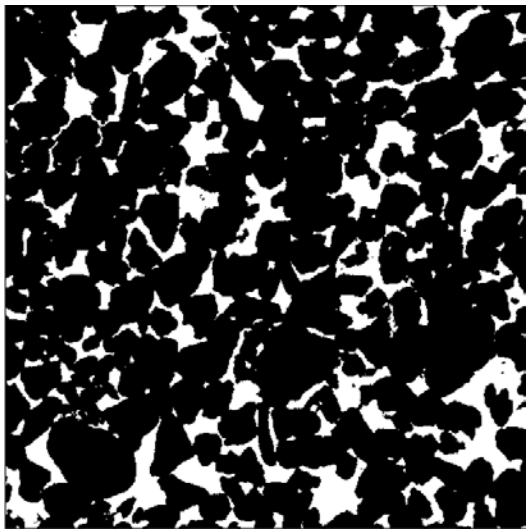
$\widehat{W}_2^{0,2}$

$$\begin{bmatrix} 0.319 & 0 & 0 \\ 0 & 0.312 & 0 \\ 0 & 0 & 0.368 \end{bmatrix}$$

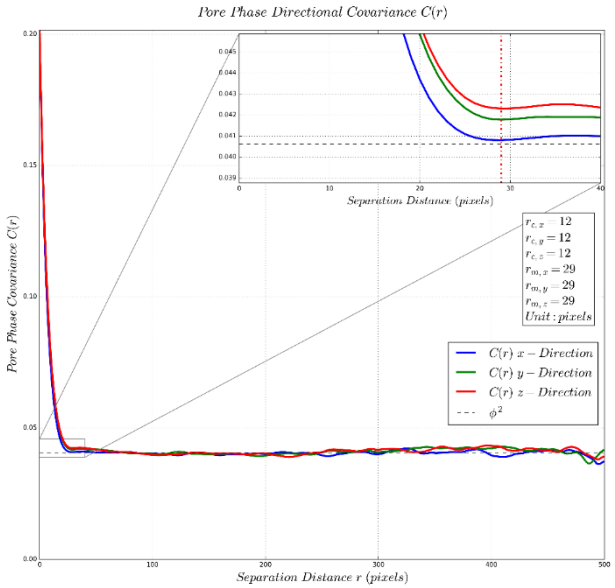
Imperial College London

Bentheimer

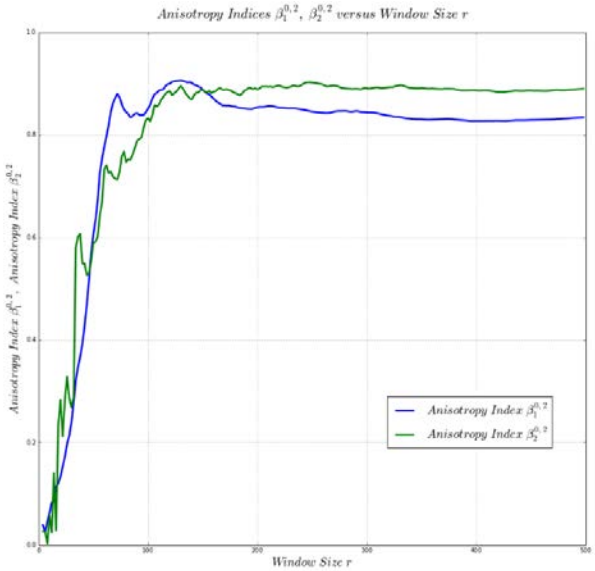
Image



Covariance



Minkowski Tensor



$$\bar{\bar{k}}\,(md) \qquad \widehat{W}_1^{0,2} \qquad \widehat{W}_2^{0,2}$$

$\phi_{cov} = 0.202$

$\phi_{eff} = 0.201$

$$\begin{bmatrix} 2743 & 0 & 0 \\ 0 & 2829 & 0 \\ 0 & 0 & 2299 \end{bmatrix} \quad \begin{bmatrix} 0.320 & 0 & 0 \\ 0 & 0.317 & 0 \\ 0 & 0 & 0.361 \end{bmatrix} \quad \begin{bmatrix} 0.325 & 0 & 0 \\ 0 & 0.322 & 0 \\ 0 & 0 & 0.351 \end{bmatrix}$$

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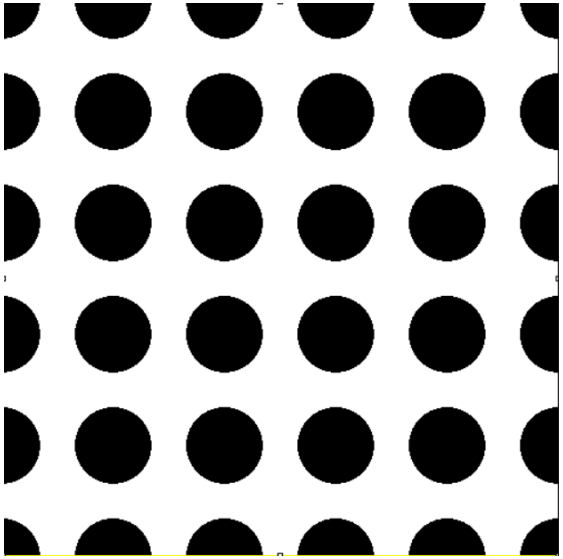
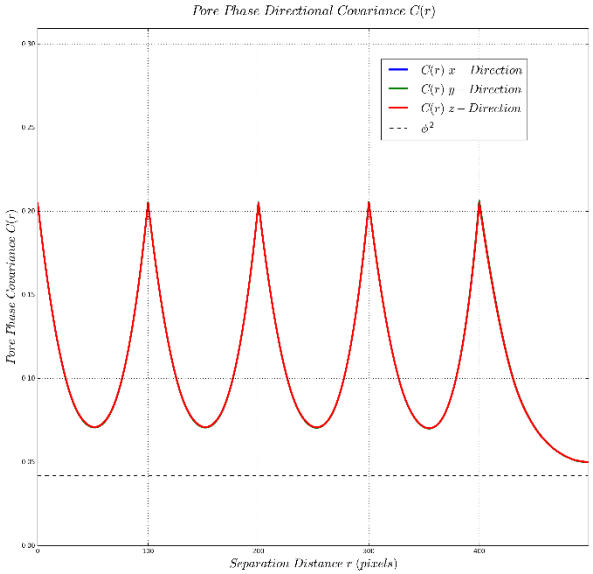
Isotropic R=1.2

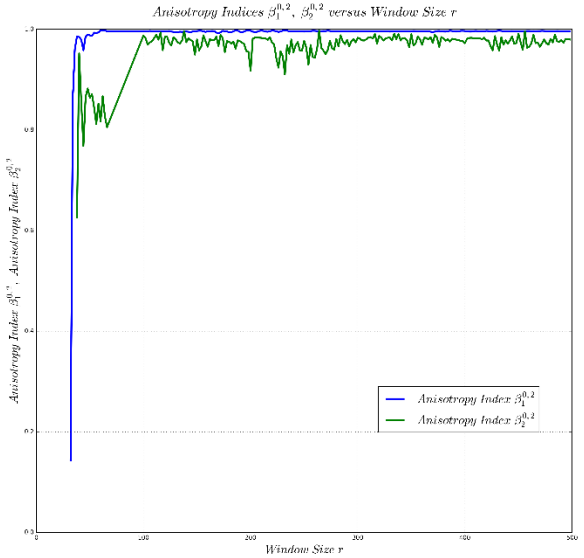
Image



Covariance



Minkowski Tensor



$$\bar{\bar{k}}\,(md) \qquad \widehat{W}_1^{0,2} \qquad \widehat{W}_2^{0,2}$$

$$\phi_{cov} = 0.205 \quad \begin{bmatrix} 8509 & 0 & 0 \\ 0 & 8506 & 0 \\ 0 & 0 & 8511 \end{bmatrix} \quad \begin{bmatrix} 0.333 & 0 & 0 \\ 0 & 0.333 & 0 \\ 0 & 0 & 0.333 \end{bmatrix} \quad \begin{bmatrix} 0.333 & 0 & 0 \\ 0 & 0.333 & 0 \\ 0 & 0 & 0.333 \end{bmatrix}$$

$$\phi_{eff} = 0.205$$

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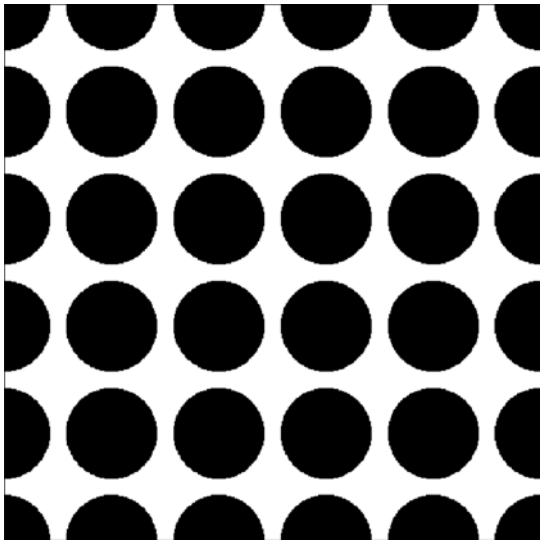
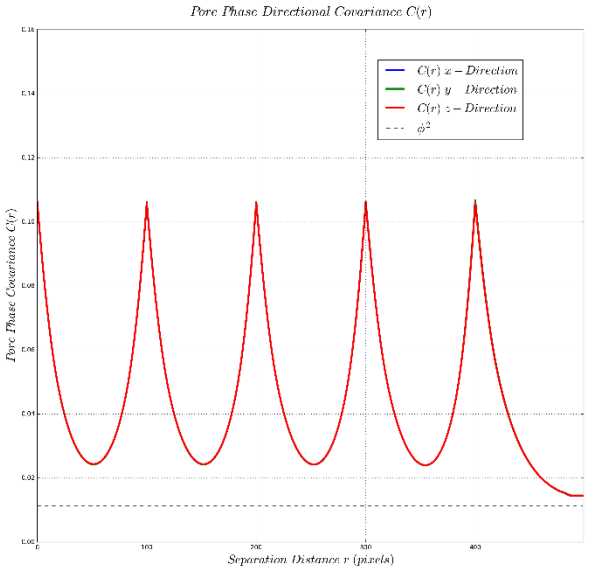
Isotropic R=1.3

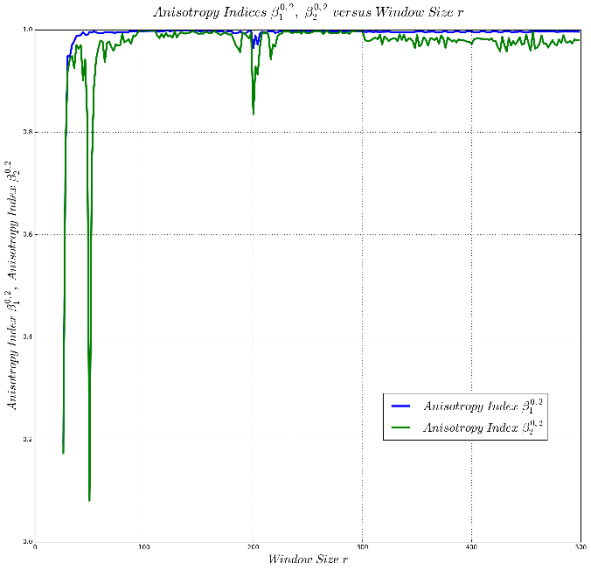
Image



Covariance



Minkowski Tensor



$\bar{\bar{k}}\,(md)$

$$\phi_{cov} = 0.106 \quad \begin{bmatrix} 919 & 0 & 0 \\ 0 & 920 & 0 \\ 0 & 0 & 932 \end{bmatrix}$$

$$\phi_{eff} = 0.106$$

$\widehat{W}_1^{0,2}$

$$\begin{bmatrix} 0.333 & 0 & 0 \\ 0 & 0.333 & 0 \\ 0 & 0 & 0.333 \end{bmatrix}$$

$\widehat{W}_2^{0,2}$

$$\begin{bmatrix} 0.333 & 0 & 0 \\ 0 & 0.333 & 0 \\ 0 & 0 & 0.333 \end{bmatrix}$$

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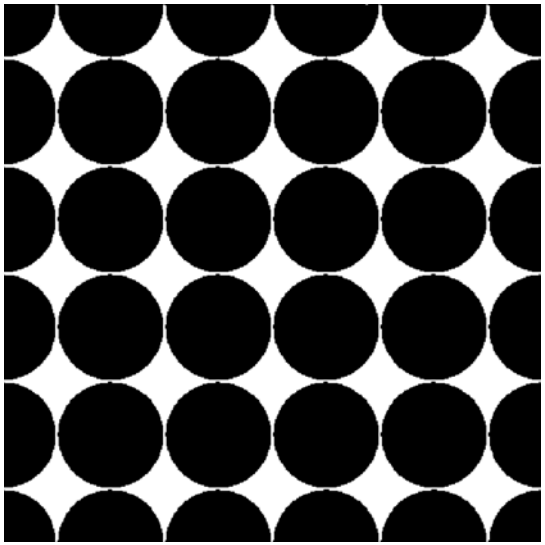
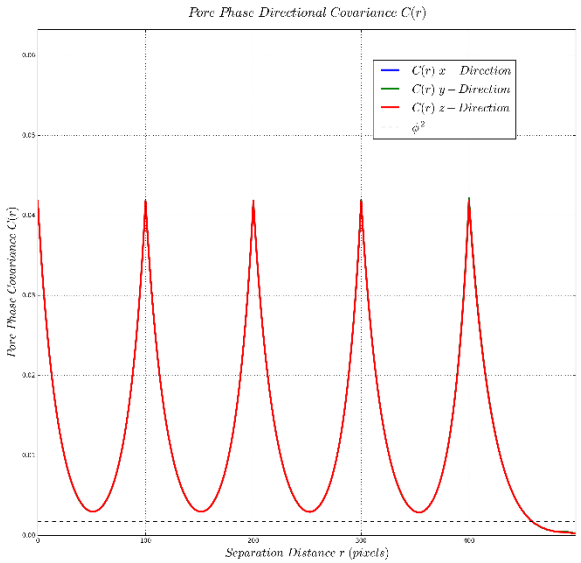
Isotropic R=1.4

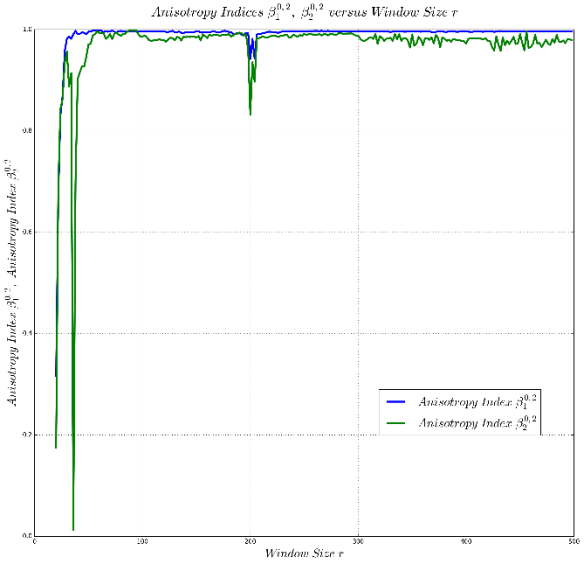
Image



Covariance



Minkowski Tensor



$\bar{\bar{k}}\,(md)$

$$\phi_{cov} = 0.042 \quad \begin{bmatrix} 0.3 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0 & 0 & 0.3 \end{bmatrix}$$

$$\phi_{eff} = 0.042$$

$\widehat{W}_1^{0,2}$

$$\begin{bmatrix} 0.333 & 0 & 0 \\ 0 & 0.333 & 0 \\ 0 & 0 & 0.333 \end{bmatrix}$$

$\widehat{W}_2^{0,2}$

$$\begin{bmatrix} 0.333 & 0 & 0 \\ 0 & 0.333 & 0 \\ 0 & 0 & 0.333 \end{bmatrix}$$

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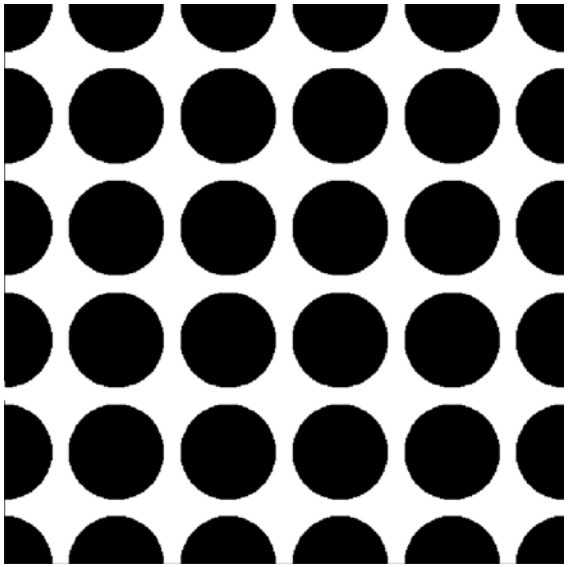
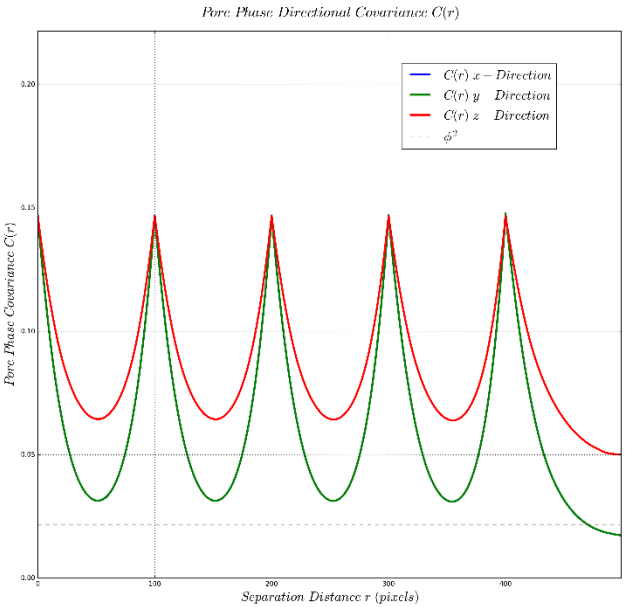
Isotropic R1=R2=1.2 R3=1.4

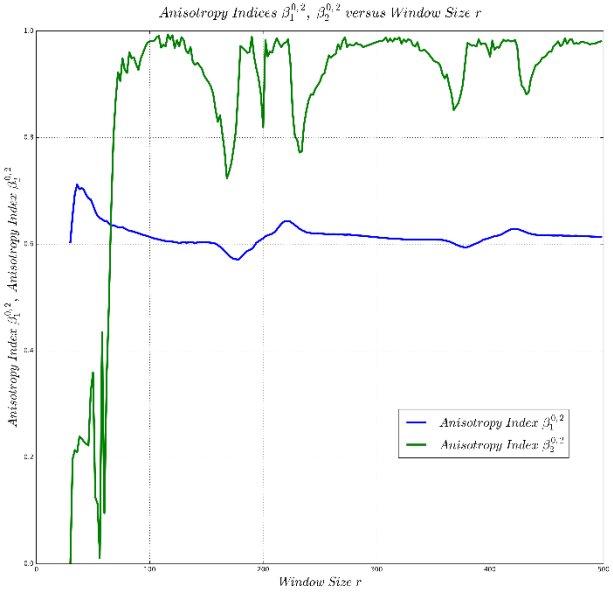
Image



Covariance



Minkowski Tensor



$$\phi_{cov} = 0.147$$

$$\phi_{eff} = 0.147$$

$$\bar{\bar{k}}\,(md) = \begin{bmatrix} 1346 & 0 & 0 \\ 0 & 1351 & 0 \\ 0 & 0 & 7459 \end{bmatrix}$$

$$\widehat{W}_1^{0,2} = \begin{bmatrix} 0.383 & 0 & 0 \\ 0 & 0.383 & 0 \\ 0 & 0 & 0.235 \end{bmatrix}$$

$$\widehat{W}_2^{0,2} = \begin{bmatrix} 0.334 & 0 & 0 \\ 0 & 0.334 & 0 \\ 0 & 0 & 0.331 \end{bmatrix}$$

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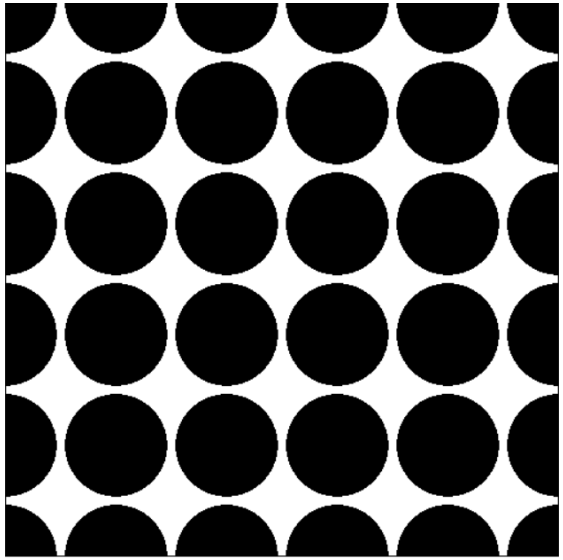
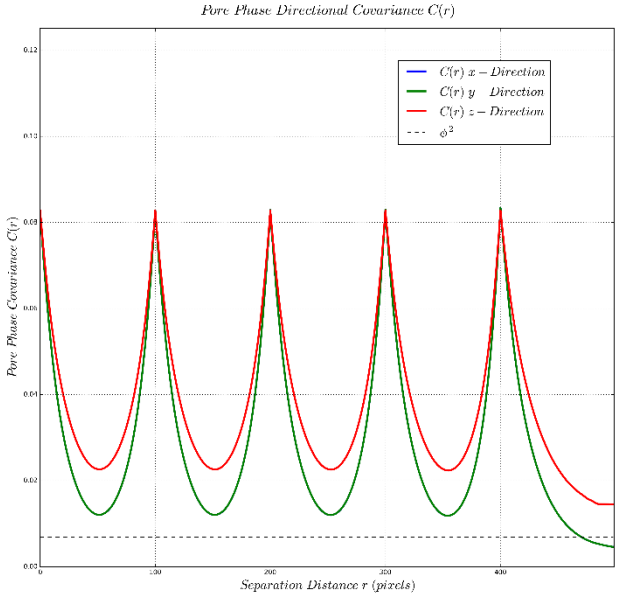
Isotropic R1=R2=1.3 R3=1.4

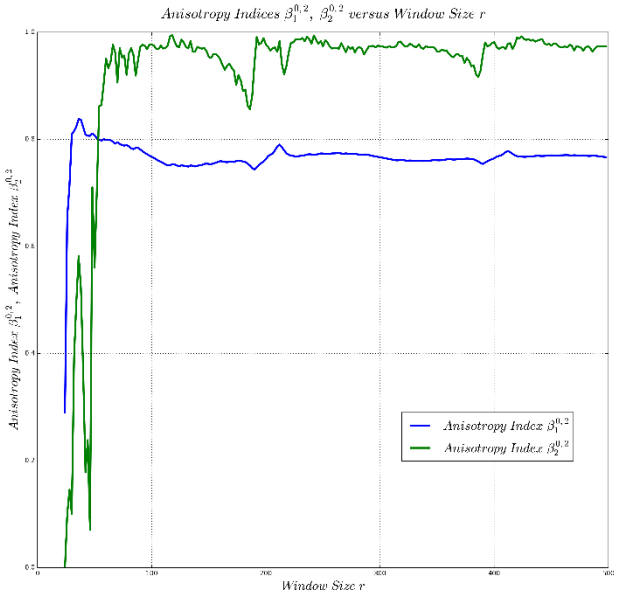
Image



Covariance



Minkowski Tensor



$$\bar{\bar{k}}\,(md) \qquad \widehat{W}_1^{0,2} \qquad \widehat{W}_2^{0,2}$$

$\phi_{cov} = 0.083$

$\phi_{eff} = 0.083$

$$\begin{bmatrix} 129 & 0 & 0 \\ 0 & 129 & 0 \\ 0 & 0 & 865 \end{bmatrix} \qquad \begin{bmatrix} 0.361 & 0 & 0 \\ 0 & 0.361 & 0 \\ 0 & 0 & 0.277 \end{bmatrix} \qquad \begin{bmatrix} 0.336 & 0 & 0 \\ 0 & 0.336 & 0 \\ 0 & 0 & 0.328 \end{bmatrix}$$