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Statistical Characterisation of Porous Media at the Pore Scale

Normalization and Physical Interpretation

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Presentation Outline

- Normalization
- Mean Curvature Determination (Constant)
- Physical Interpretation MT 1 and MT 2
- Normalized Results Parametric Models
- Expanding Window:
  - Beadpack
  - Ketton
  - Estaillades
  - Doddington
  - Bentheimer
  - Comparison

Normalization

Normalization of Surface Tensor by 1st scalar Minkowski functional (surface area)

$$\widehat{W}_1^{0,2} = \frac{W_1^{0,2}}{W_1^{0,0}} = \frac{\frac{1}{3}\int_{\partial K} n \odot n dA}{\frac{1}{3}\int_{\partial K} dA}$$

Normalization of Curvature Tensor by 2nd scalar Minkowski functional (Mean Curvature)

$$\widehat{W}_2^{0,2} = \frac{W_2^{0,2}}{W_2^{0,0}} = \frac{\frac{1}{3}\int_{\partial K} G_2 n \odot n dA}{\frac{1}{3}\int_{\partial K} G_2 dA}, G_2 = \frac{\kappa_1 + \kappa_2}{2}$$

Finding mean (constant) curvature

$$G_2 = \frac{W_2^{0,2}}{W_1^{0,0}} = \frac{\frac{1}{3}\int_{\partial K} G_2 n \odot n dA}{\frac{1}{3}\int_{\partial K} dA} = tr\left(G_2 \frac{\frac{1}{3}\int_{\partial K} n \odot n dA}{\frac{1}{3}\int_{\partial K} dA}\right)$$

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Mean Curvature (simple)

	$W_1^{0,0}$	$W_2^{0,2}$	$\widehat{W}_2^{0,2}$	$tr(\widehat{W}_2^{0,2}) = \kappa$
Sphere (R=1, κ=1)	4.185	$\begin{bmatrix} 1.394 & 0 & 0 \\ 0 & 1.394 & 0 \\ 0 & 0 & 1.394 \end{bmatrix}$	$\begin{bmatrix} 0.33 & 0 & 0 \\ 0 & 0.33 & 0 \\ 0 & 0 & 0.33 \end{bmatrix}$	1.0
Cylinder (R=1, h=1, κ =0.5)	2.049	$\begin{bmatrix} 0.523 & 0 & 0 \\ 0 & 0.523 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.25 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	0.5
Two Cylinders (R=1, h=1, κ = 0.5)	4.188	$\begin{bmatrix} 1.031 & 0 & 0 \\ 0 & 1.047 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.25 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	0.5
N Cylinders (R=1, h=1, κ =0.5)	25301	$\begin{bmatrix} 5961 & 0 & 0 \\ 0 & 6334 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.24 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	0.5

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Physical Interpretation

First Translation Invariant MT:
Second moment of the distribution of surface normals.

Area Weighted „Variance" of surface normals

$$W_1^{0,2} = \frac{1}{3}\int_{\partial K} n_i n_j dA = \int_{\mathbb{S}_2} \varrho_{1(n)} \; n \odot n d\Omega$$

$$\varrho_{1(n)} = \int_{\partial K} \delta(n - n') dA$$

Second Translation Invariant MT:
Second moment of the distribution of surface normals and curvatures.

Area weighted „Variance" of curvatures and surface orientation.

$$W_2^{0,2} = \frac{1}{3}\int_{\partial K} G_2 \; n_i n_j dA$$

$$W_2^{0,2} = \frac{1}{3}\int_{-\infty}^{\infty} G_2 \int_{\mathbb{S}_2} \varrho_{2(n,G_2)} n \odot n d\Omega \, dG_2$$

$$\varrho_{2(n)} = \int_{\partial K} \delta(n - n') \delta\left(G_2 - G_2'\right) dA$$

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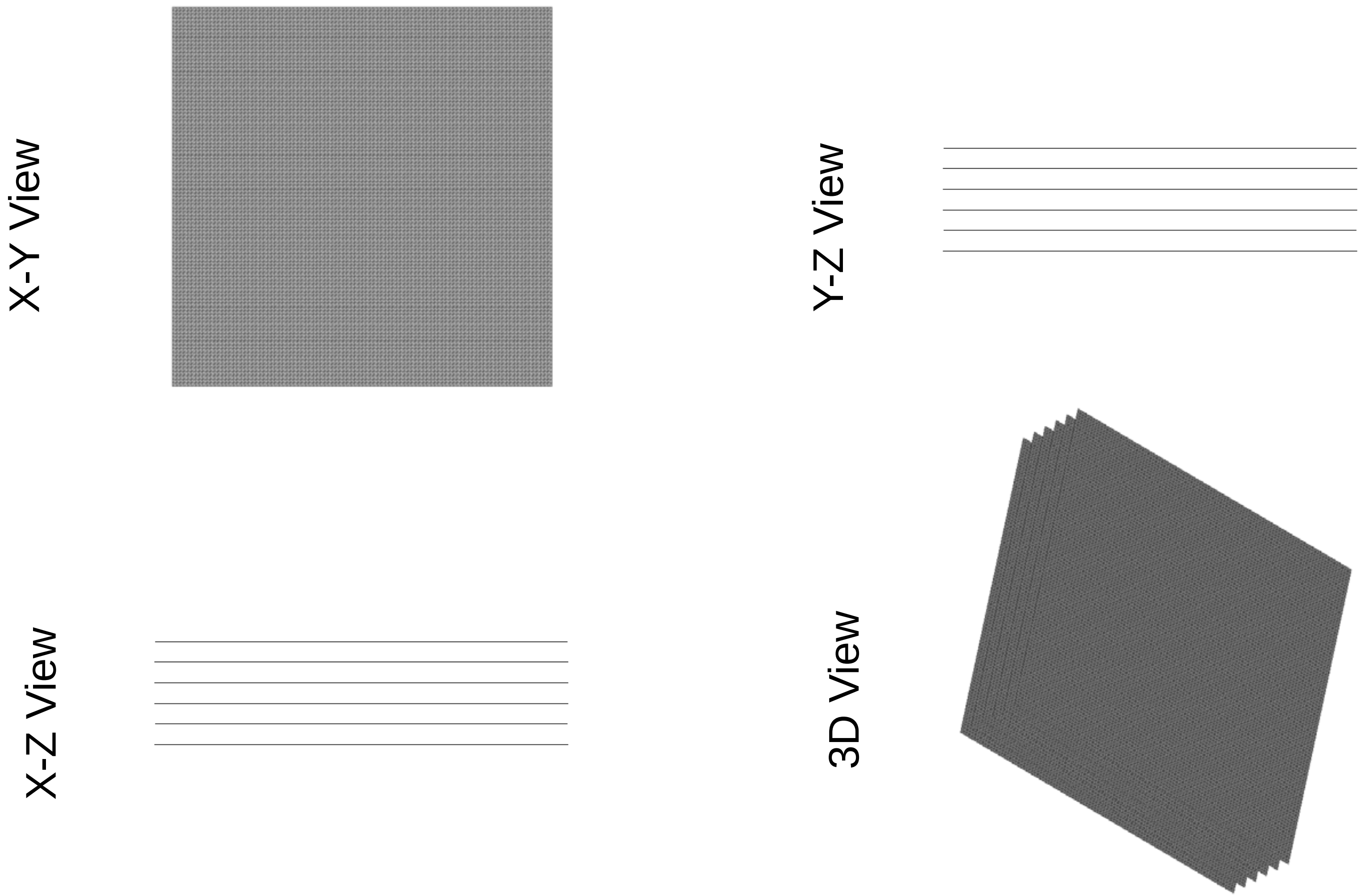
Parametric Models – Normalized Results

$$Parametric\ Models:\ Normalized\ Results$$

Plane Surfaces

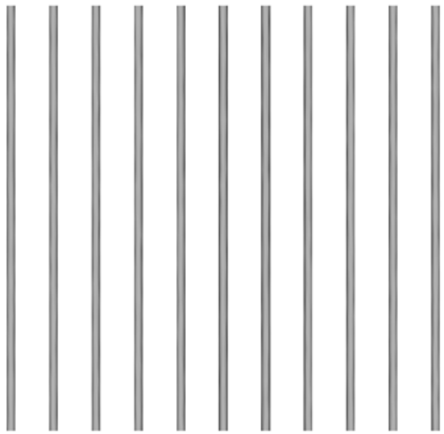


Minkowski Tensors – Plane Surface

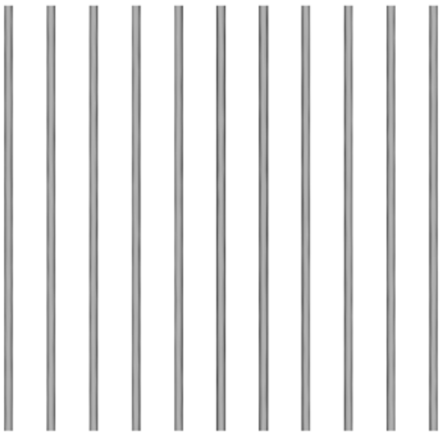
Minkowski Tensor	Eigenvectors	Eigenvalues	Anisotropy Index
$\widehat{W}_1^{0,2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{0.0 \quad 0.0 \quad 2400.\}$	$\beta_1^{0,2} = not\ def.$
$\widehat{W}_2^{0,2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{0.0 \quad 0.0 \quad 0.0\}$	$\beta_2^{0,2} = not\ def.$

Regular Bundle of Tubes

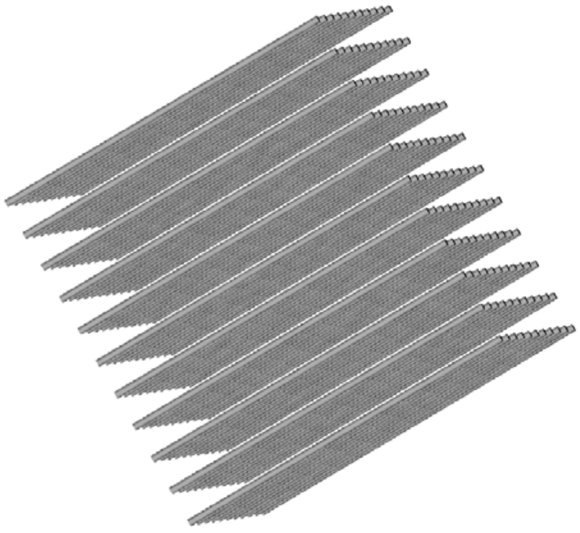
X-Z View



Y-Z View



3D View



Minkowski Tensors – Regular Bundle of Tubes

Minkowski Tensor	Eigenvectors	Eigenvalues	Anisotropy Index
$\widehat{W}_1^{0,2} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{12605 \quad 12605 \quad 0\}$	$\beta_1^{0,2} = not\ def.$
$\widehat{W}_2^{0,2} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{5960 \quad 6334 \quad 0\}$	$\beta_2^{0,2} = not\ def.$

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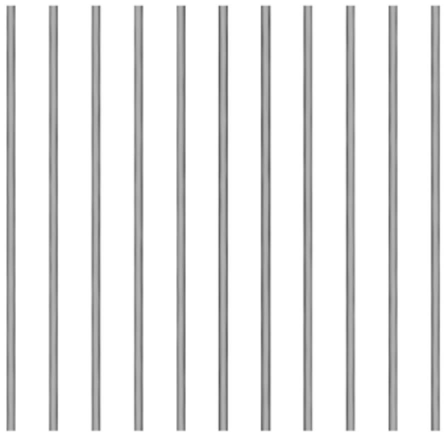
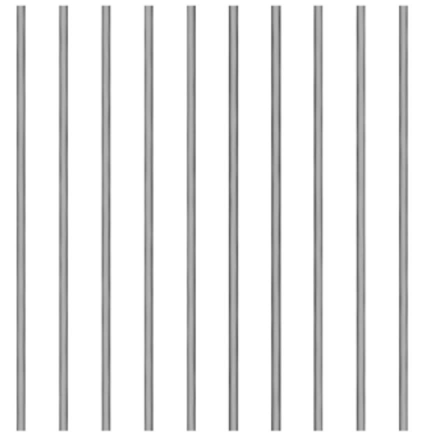
Regular Bundle of Tubes – High Mesh Resolution

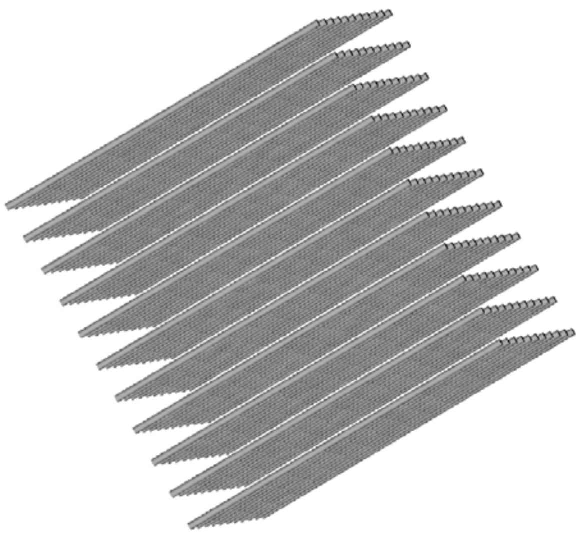
X-Z View



Y-Z View



3D View



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Minkowski Tensors – Regular Bundle – High Mesh Res.

Minkowski Tensor	Eigenvectors	Eigenvalues	Anisotropy Index
$\widehat{W}_1^{0,2} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{12605 \quad 12605 \quad 0\}$	$\beta_1^{0,2} = not\ def.$
$\widehat{W}_2^{0,2} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{5960 \quad 6334 \quad 0\}$	$\beta_2^{0,2} = not\ def.$

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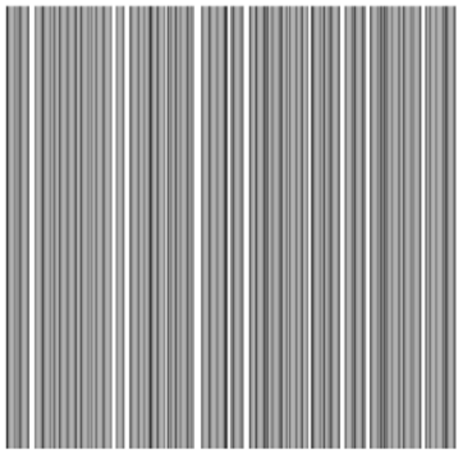
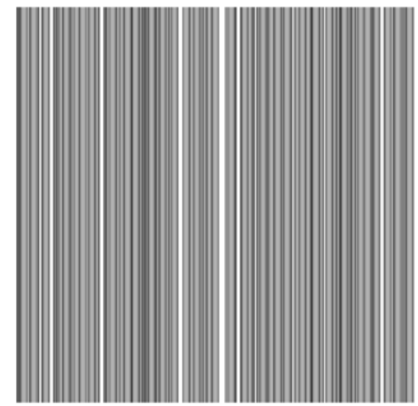
Random Bundle of Tubes

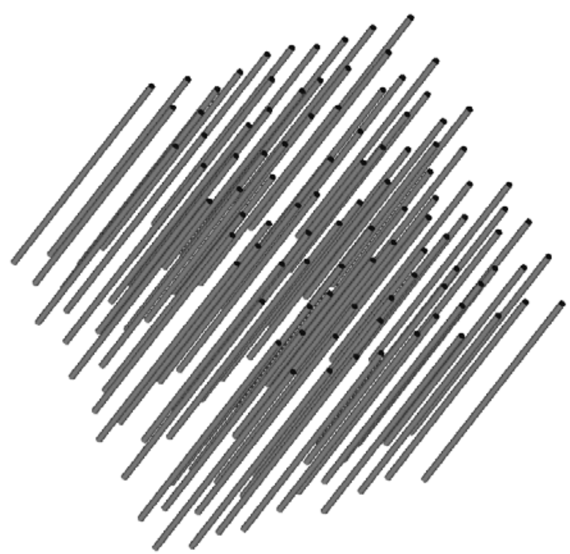
X-Z View



Y-Z View



3D View



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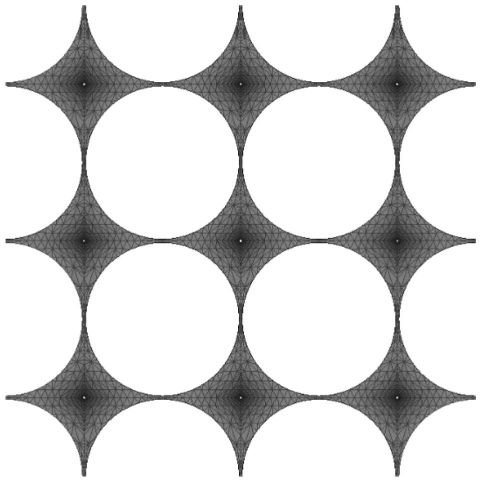
Minkowski Tensors – Random Bundle of Tubes

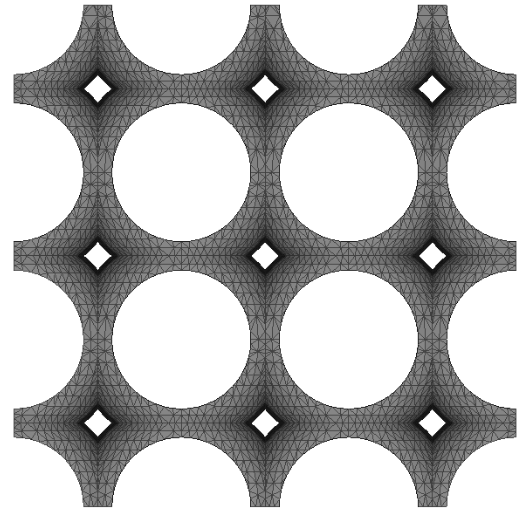
Minkowski Tensor	Eigenvectors	Eigenvalues	Anisotropy Index
$\widehat{W}_1^{0,2} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{12605 \quad 12605 \quad 0\}$	$\beta_1^{0,2} = not\ def.$
$\widehat{W}_2^{0,2} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{5960 \quad 6334 \quad 0\}$	$\beta_2^{0,2} = not\ def.$

Symmetric Parametric Pore

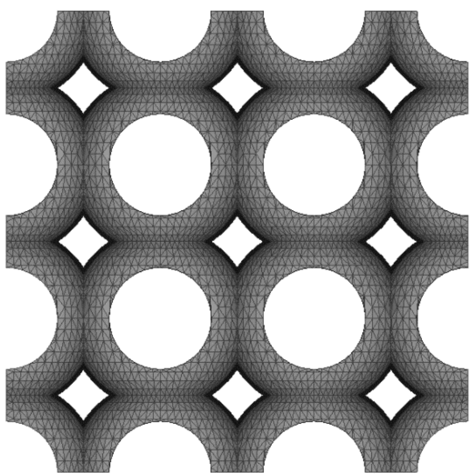
Radius 1.4



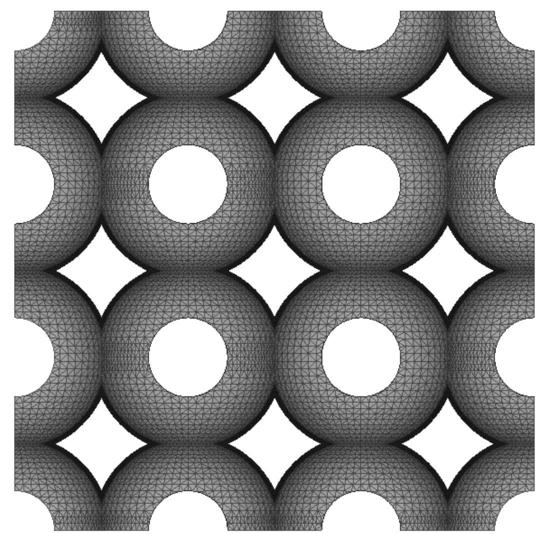
Radius 1.3



Radius 1.2



Radius 1.1



Minkowski Tensors – Radius 1.4

Minkowski Tensor	Eigenvectors	Eigenvalues	Anisotropy Index
$\widehat{W}_1^{0,2} = \begin{bmatrix} 0.333 & 0 & 0 \\ 0 & 0.333 & 0 \\ 0 & 0 & 0.333 \end{bmatrix}$	$\begin{bmatrix} -0.76 \\ 0.66 \\ 0 \end{bmatrix} \begin{bmatrix} 0.66 \\ -0.76 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{10.6 \quad 10.6 \quad 10.6\}$	$\beta_1^{0,2} = 1.0$
$\widehat{W}_2^{0,2} = \begin{bmatrix} 0.333 & 0 & 0 \\ 0 & 0.333 & 0 \\ 0 & 0 & 0.333 \end{bmatrix}$	$\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{-6.0 \quad -6.0 \quad -6.0\}$	$\beta_2^{0,2} = 1.0$

Minkowski Tensors – Radius 1.3

Minkowski Tensor	Eigenvectors	Eigenvalues	Anisotropy Index
$\widehat{W}_1^{0,2} = \begin{bmatrix} 0.333 & 0 & 0 \\ 0 & 0.333 & 0 \\ 0 & 0 & 0.333 \end{bmatrix}$	$\begin{bmatrix} -0.7 \\ 0.7 \\ 0 \end{bmatrix} \begin{bmatrix} 0.7 \\ -0.7 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{19.6 \quad 19.6 \quad 19.6\}$	$\beta_1^{0,2} = 1.0$
$\widehat{W}_2^{0,2} = \begin{bmatrix} 0.333 & 0 & 0 \\ 0 & 0.333 & 0 \\ 0 & 0 & 0.333 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{-13 \quad -13 \quad -13\}$	$\beta_2^{0,2} = 1.0$

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Minkowski Tensors – Radius 1.2

Minkowski Tensor	Eigenvectors	Eigenvalues	Anisotropy Index
$\widehat{W}_1^{0,2} = \begin{bmatrix} 0.333 & 0 & 0 \\ 0 & 0.333 & 0 \\ 0 & 0 & 0.333 \end{bmatrix}$	$\begin{bmatrix} -1.0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1.0 \end{bmatrix} \begin{bmatrix} 0 \\ -1.0 \\ 0 \end{bmatrix}$	$\{27.1 \quad 27.1 \quad 27.1\}$	$\beta_1^{0,2} = 1.0$
$\widehat{W}_2^{0,2} = \begin{bmatrix} 0.333 & 0 & 0 \\ 0 & 0.333 & 0 \\ 0 & 0 & 0.333 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$	$\{-20.7 \quad -20.7 \quad -20.7\}$	$\beta_2^{0,2} = 1.0$

Minkowski Tensors – Radius 1.1

Minkowski Tensor	Eigenvectors	Eigenvalues	Anisotropy Index
$\widehat{W}_1^{0,2} = \begin{bmatrix} 0.333 & 0 & 0 \\ 0 & 0.333 & 0 \\ 0 & 0 & 0.333 \end{bmatrix}$	$\begin{bmatrix} -0.89 \\ 0.45 \\ 0 \end{bmatrix} \begin{bmatrix} 0.45 \\ -0.89 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{33.1 \quad 33.1 \quad 33.1\}$	$\beta_1^{0,2} = 1.0$

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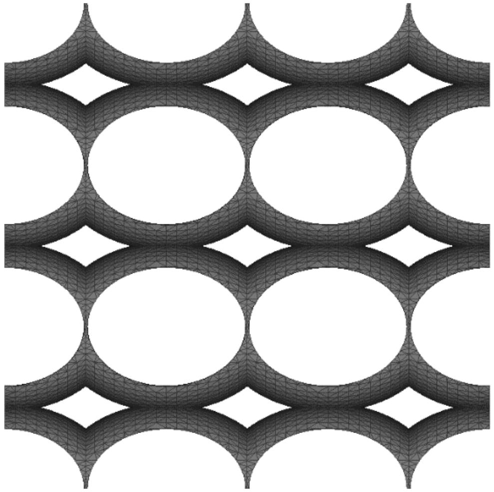
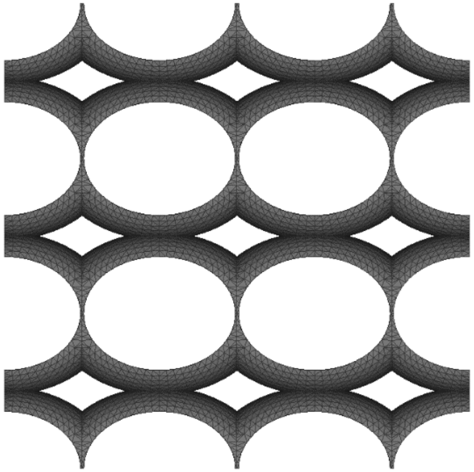
Asymmetric Parametric Pore – R1 = 1.4, R2=1.3

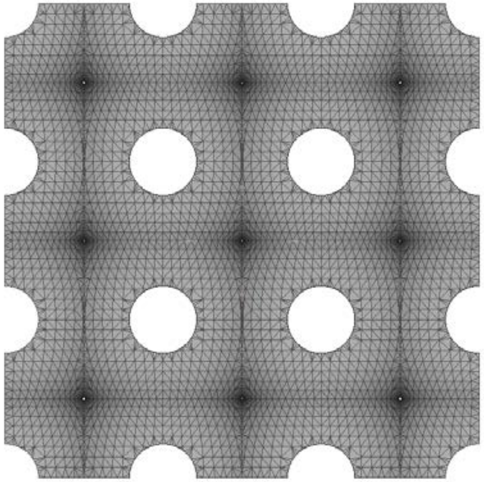
X-Y View



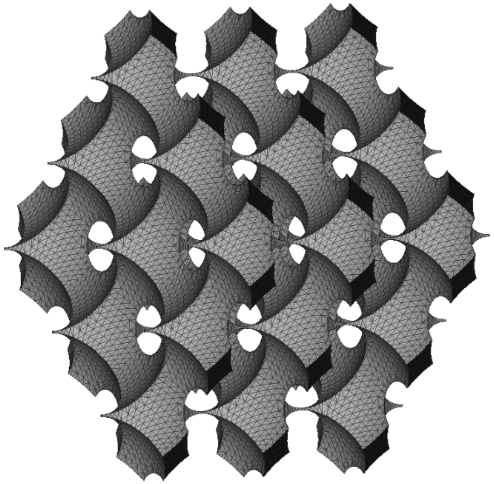
Y-Z View



X-Z View



3D View



Minkowski Tensors – R1 = 1.4, R2=1.3

Minkowski Tensor	Eigenvectors	Eigenvalues	Anisotropy Index
$\widehat{W}_1^{0,2} = \begin{bmatrix} 0.157 & 0 & 0 \\ 0 & 0.685 & 0 \\ 0 & 0 & 0.158 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{11.9 \quad 51.8 \quad 11.9\}$	$\beta_1^{0,2} = 0.23$
$\widehat{W}_2^{0,2} = \begin{bmatrix} 0.168 & 0 & 0 \\ 0 & 0.688 & 0 \\ 0 & 0 & 0.164 \end{bmatrix}$	$\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{-7.8 \quad -30.9 \quad -7.8\}$	$\beta_2^{0,2} = 0.25$

Minkowski Tensors – R1 = 1.4, R2=1.39

Minkowski Tensor	Eigenvectors	Eigenvalues	Anisotropy Index
$\widehat{W}_1^{0,2} = \begin{bmatrix} 0.328 & 0 & 0 \\ 0 & 0.342 & 0 \\ 0 & 0 & 0.329 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{11.9 \quad 51.8 \quad 11.9\}$	$\beta_1^{0,2} = 0.23$
$\widehat{W}_2^{0,2} = \begin{bmatrix} 0.328 & 0 & 0 \\ 0 & 0.344 & 0 \\ 0 & 0 & 0.327 \end{bmatrix}$	$\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\{-7.8 \quad -30.9 \quad -7.8\}$	$\beta_2^{0,2} = 0.25$

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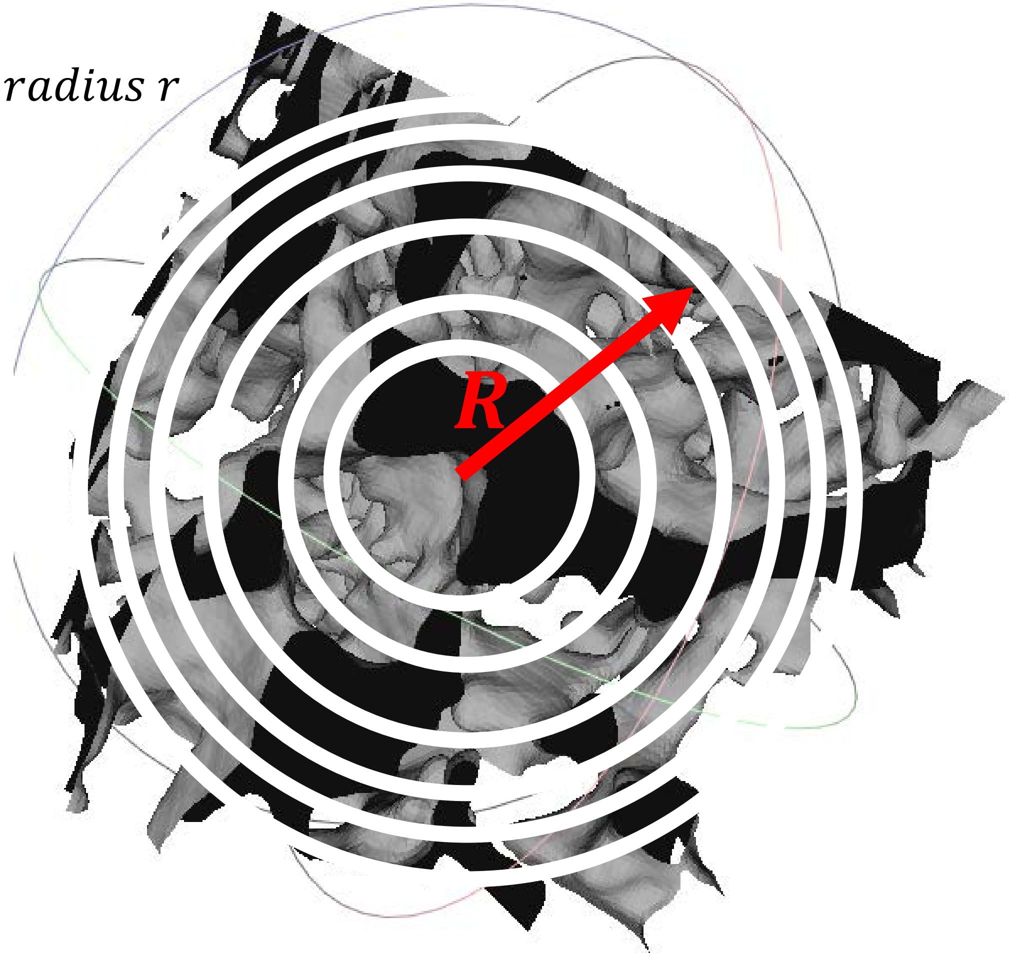
Beadpack/Ketton

Centered Expanding Window => REV – 500^3

Increasing Window of radius r



Cubic Volume used

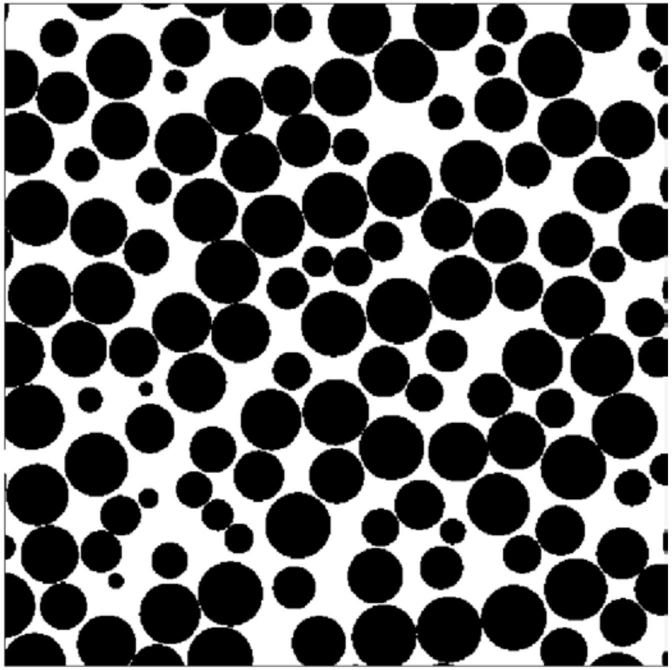
Determine Anisotropy as function of Window Size

Beadpack – Centered Expanding Window

$$Beadpack\ Centered\ Expanding\ Window$$

$$Grain\ Size\ 26\ \mu m$$

$$Image\ Size: 500\ voxel^3$$



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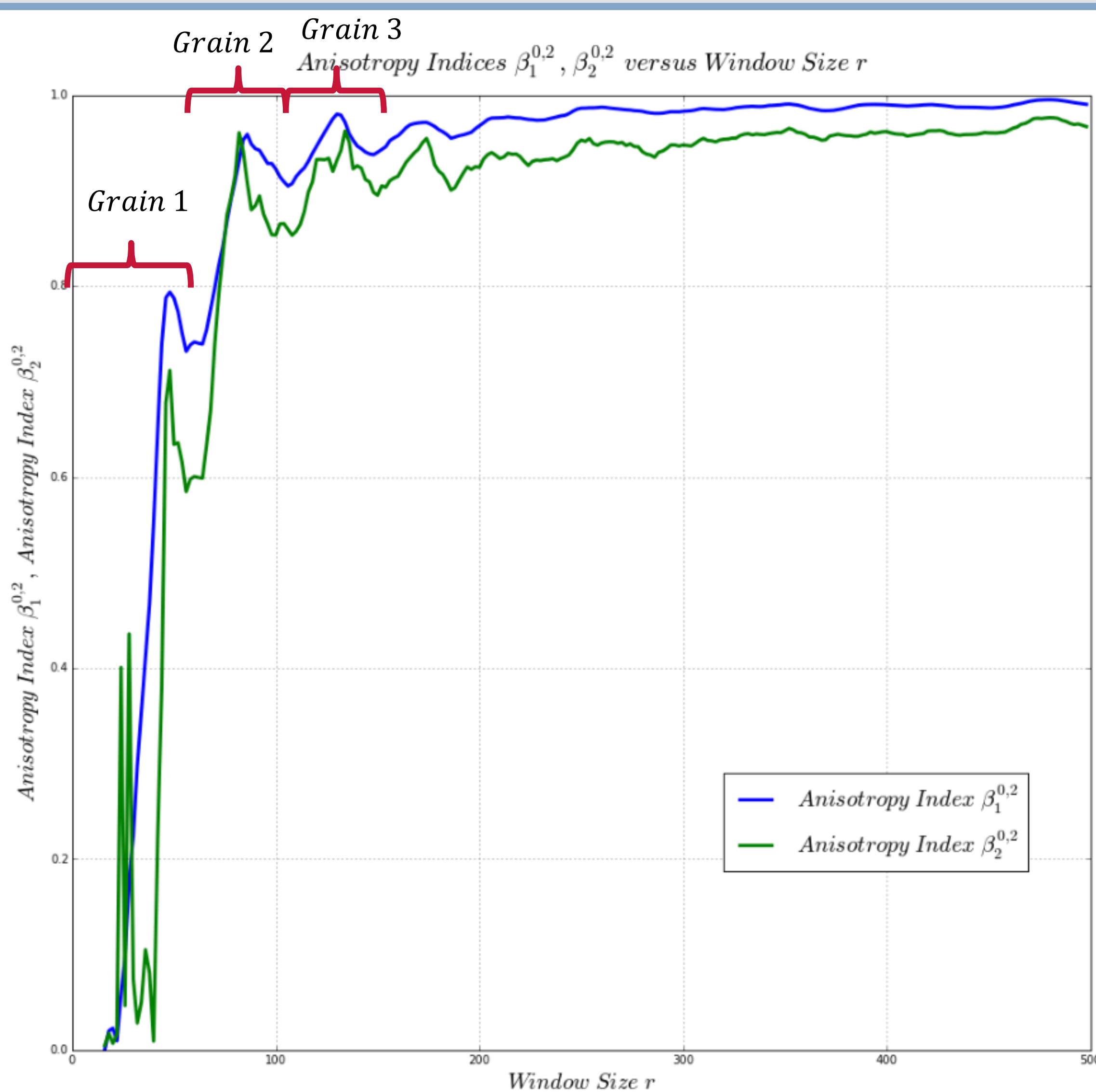
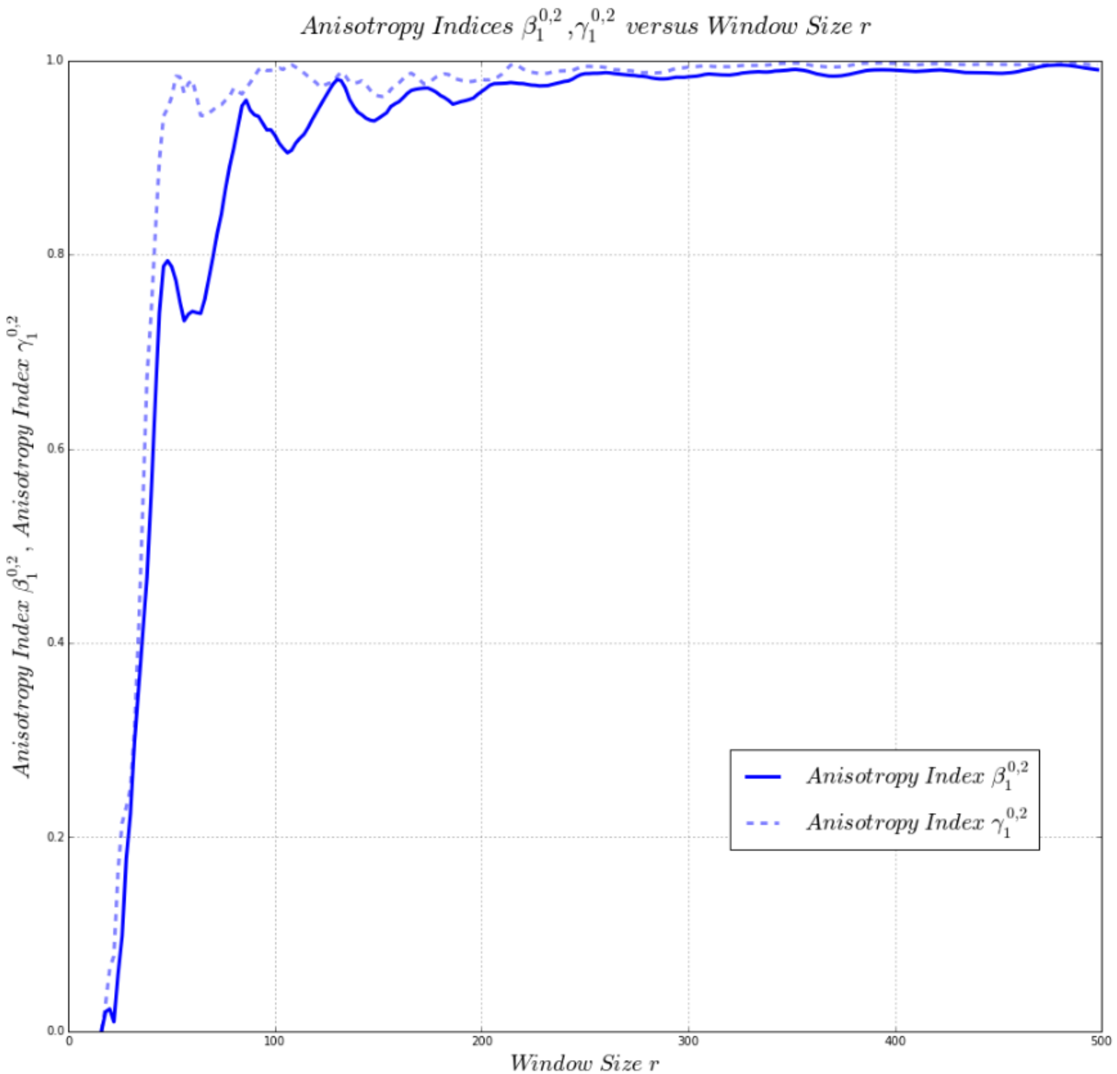
Beadpack – Centered Expanding Window => REV – 500^3

Grain Diameter 26 μm

$50\ voxels^3/grain$



Beadpack – Centered Expanding Window => REV – 500^3



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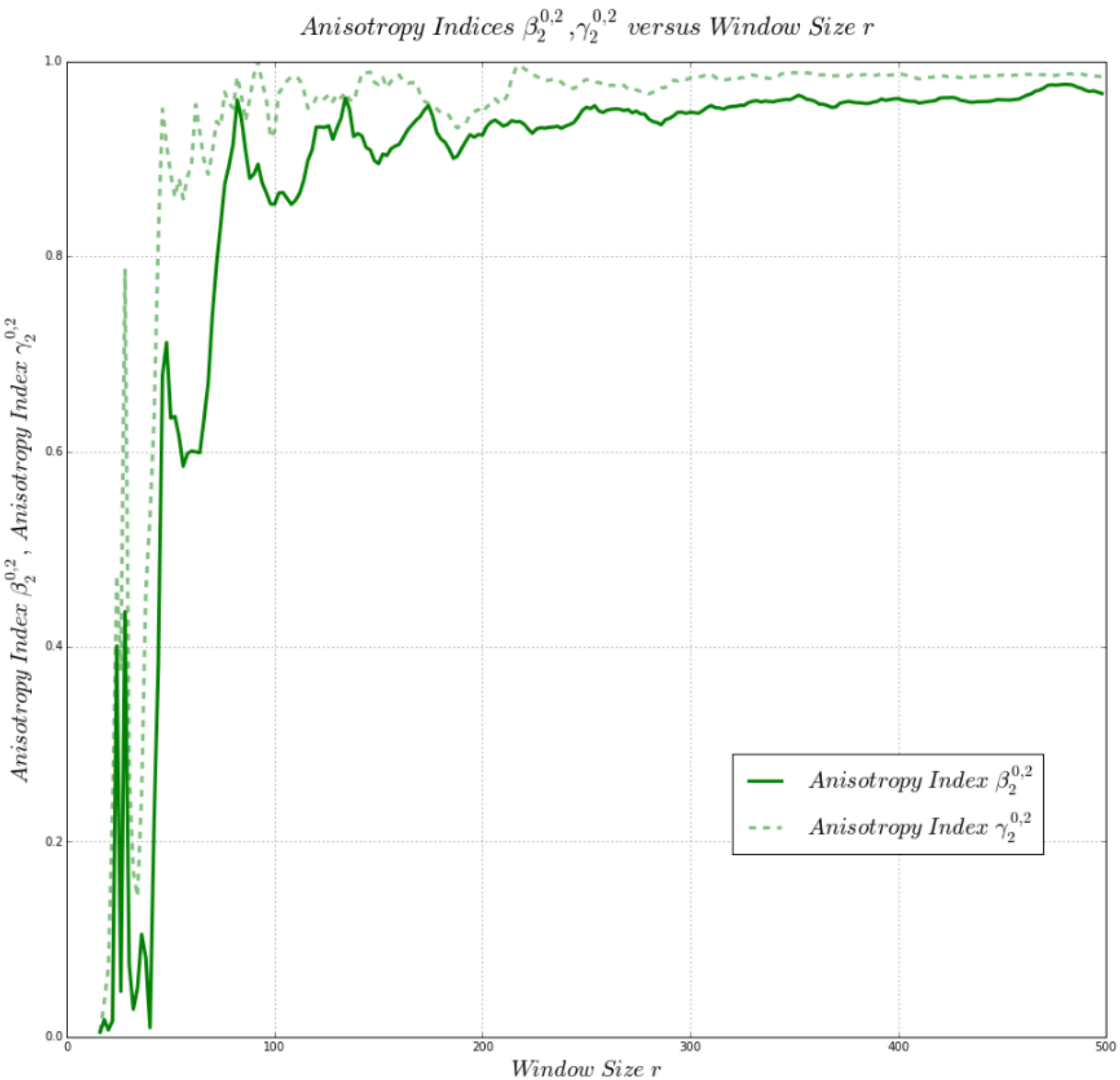
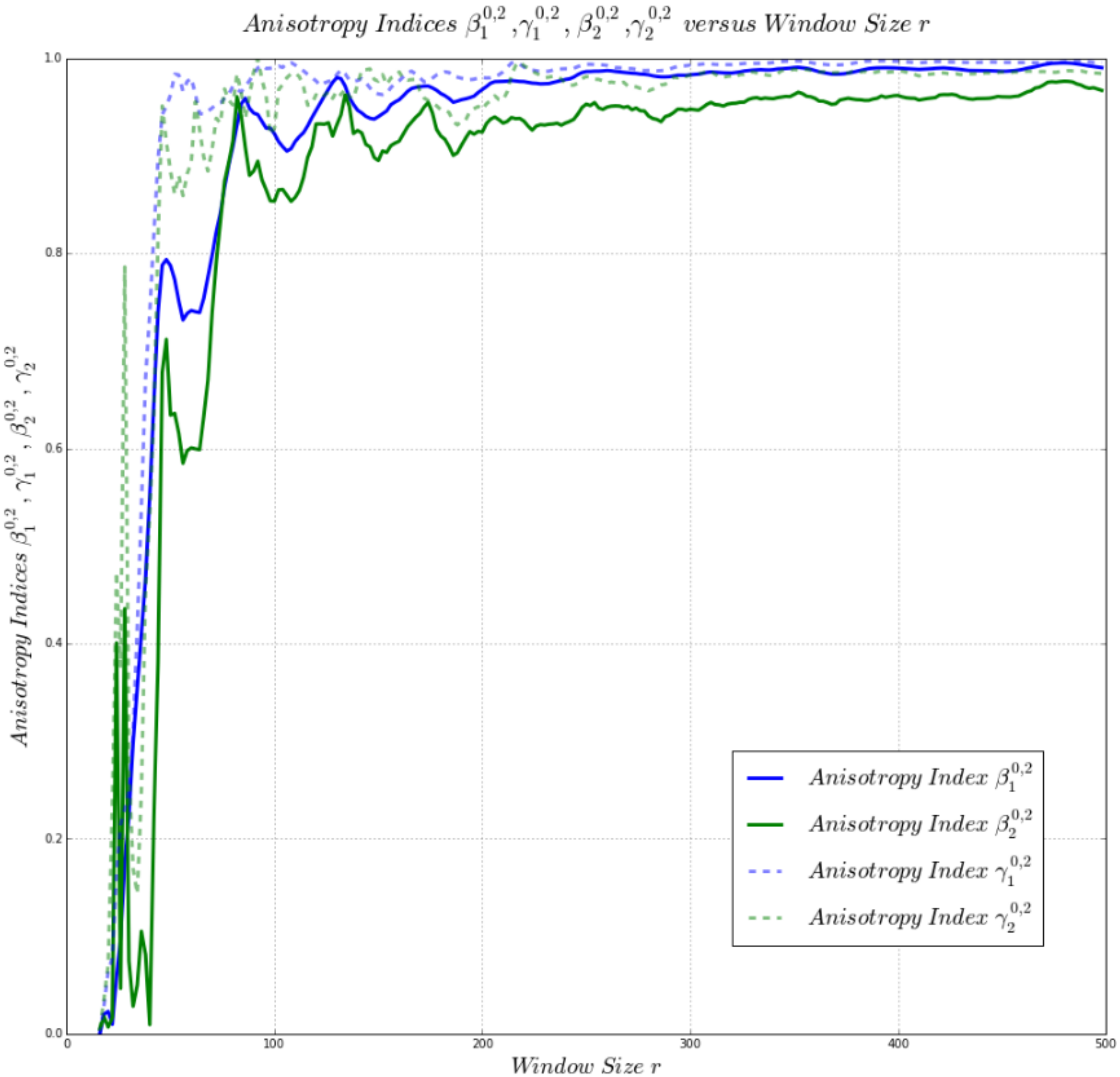
Beadpack – Centered Expanding Window => REV – 500^3



Beadpack – Centered Expanding Window => REV – 500^3

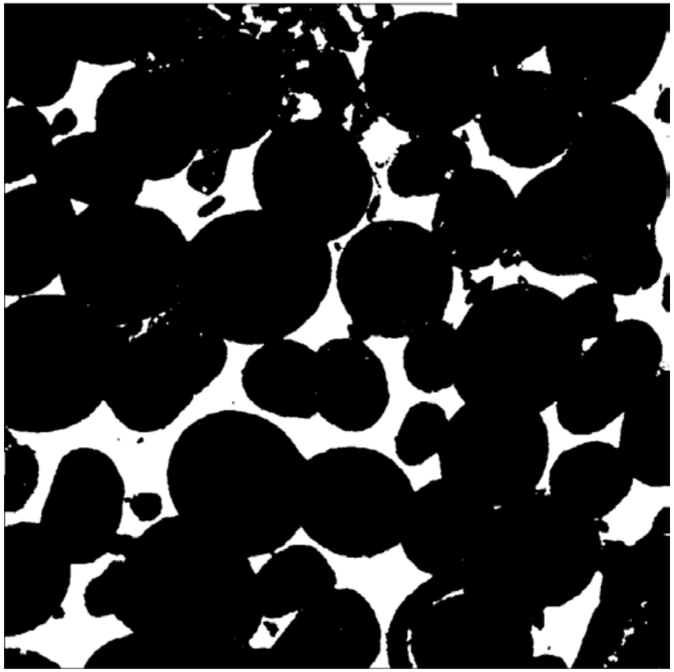


Ketton – Centered Expanding Window

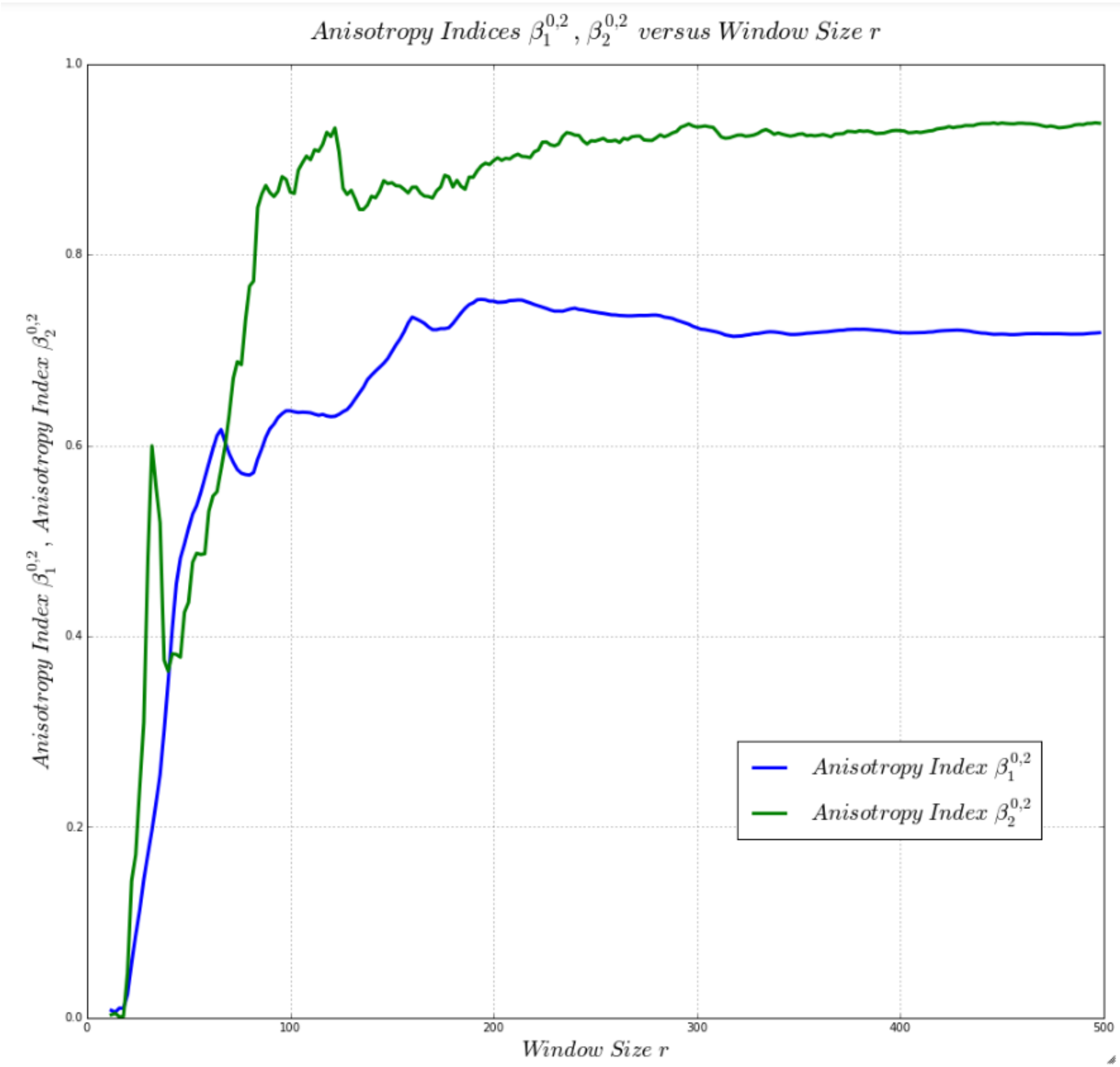
$$Ketton\ Centered\ Expanding\ Window$$

$$Voxel\ size\ 3\mu m/voxel$$

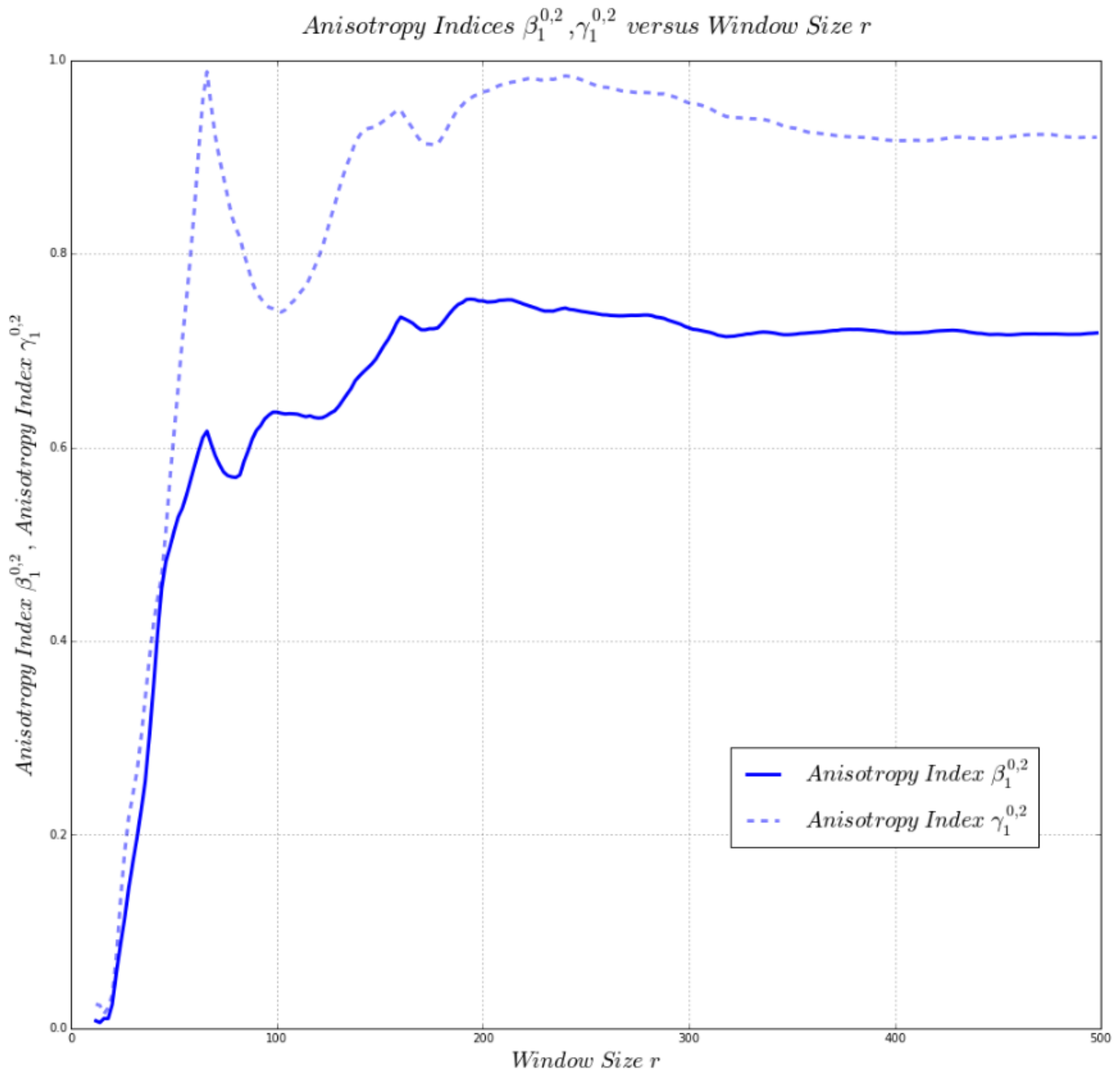
$$Image\ Size\text{: }500\ voxel^3$$



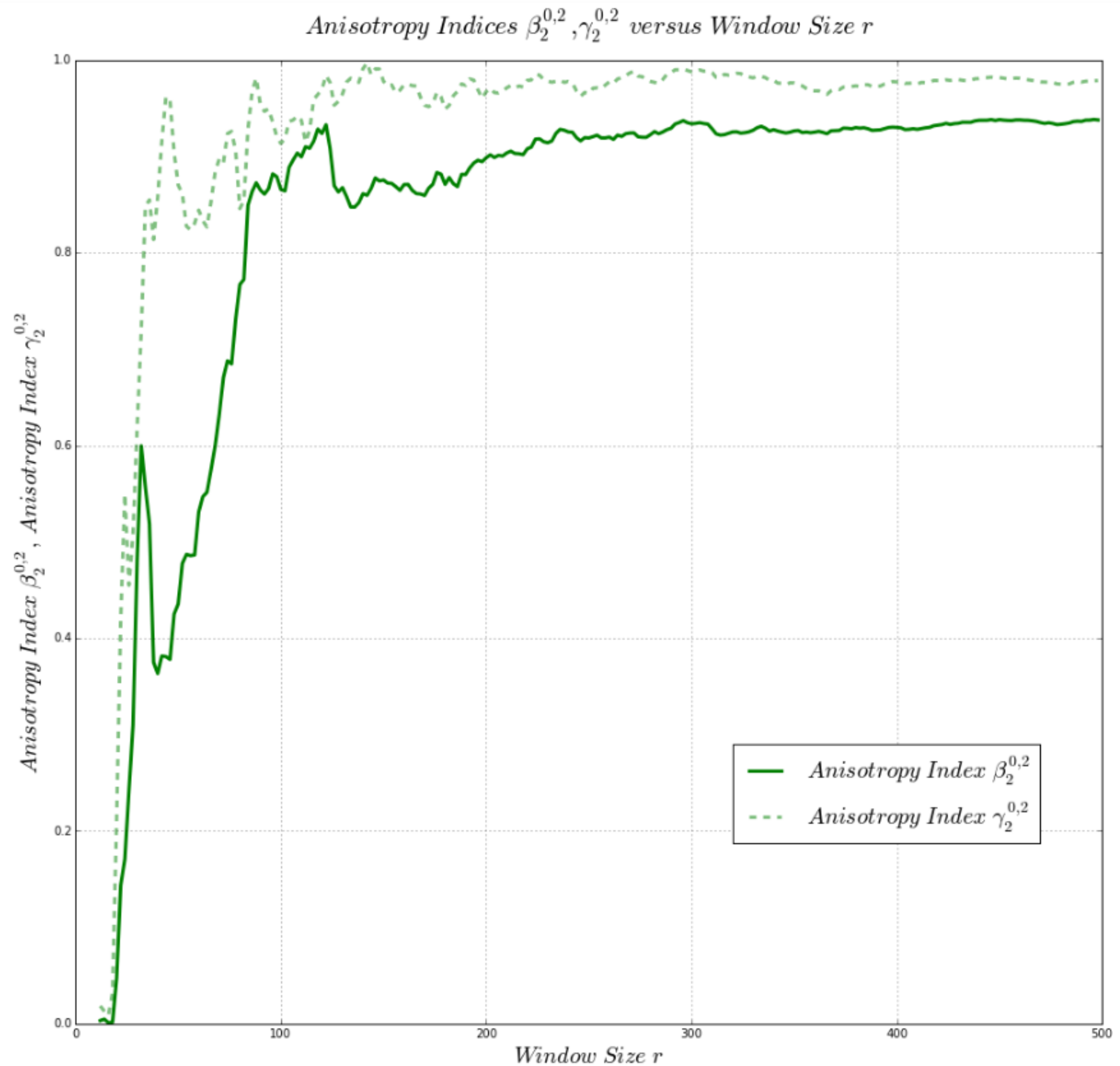
Ketton – Comparison Anisotropy Indices B102, B202



Ketton – Centered Expanding Window => REV – 500^3



Ketton – Centered Expanding Window => REV – 500^3



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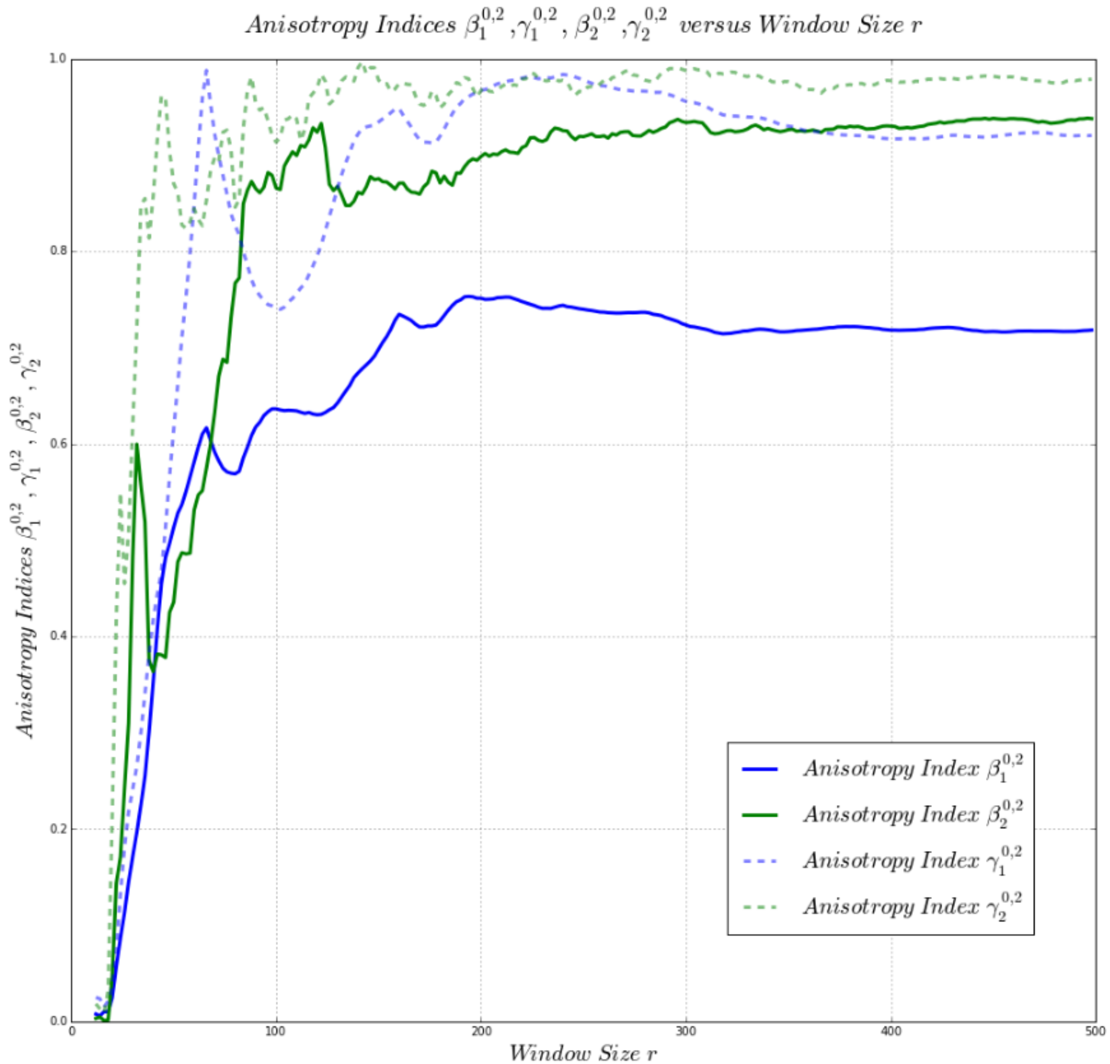
Ketton – Centered Expanding Window => REV – 500^3



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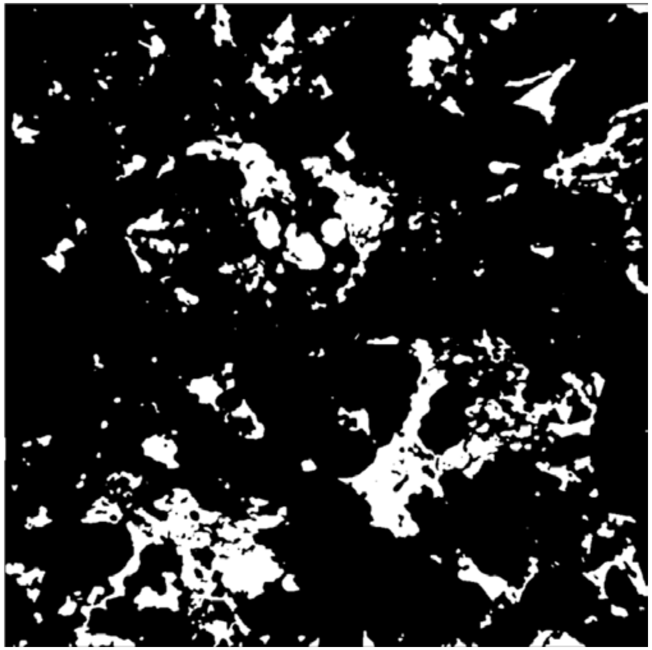
Estaillades – Centered Expanding Window

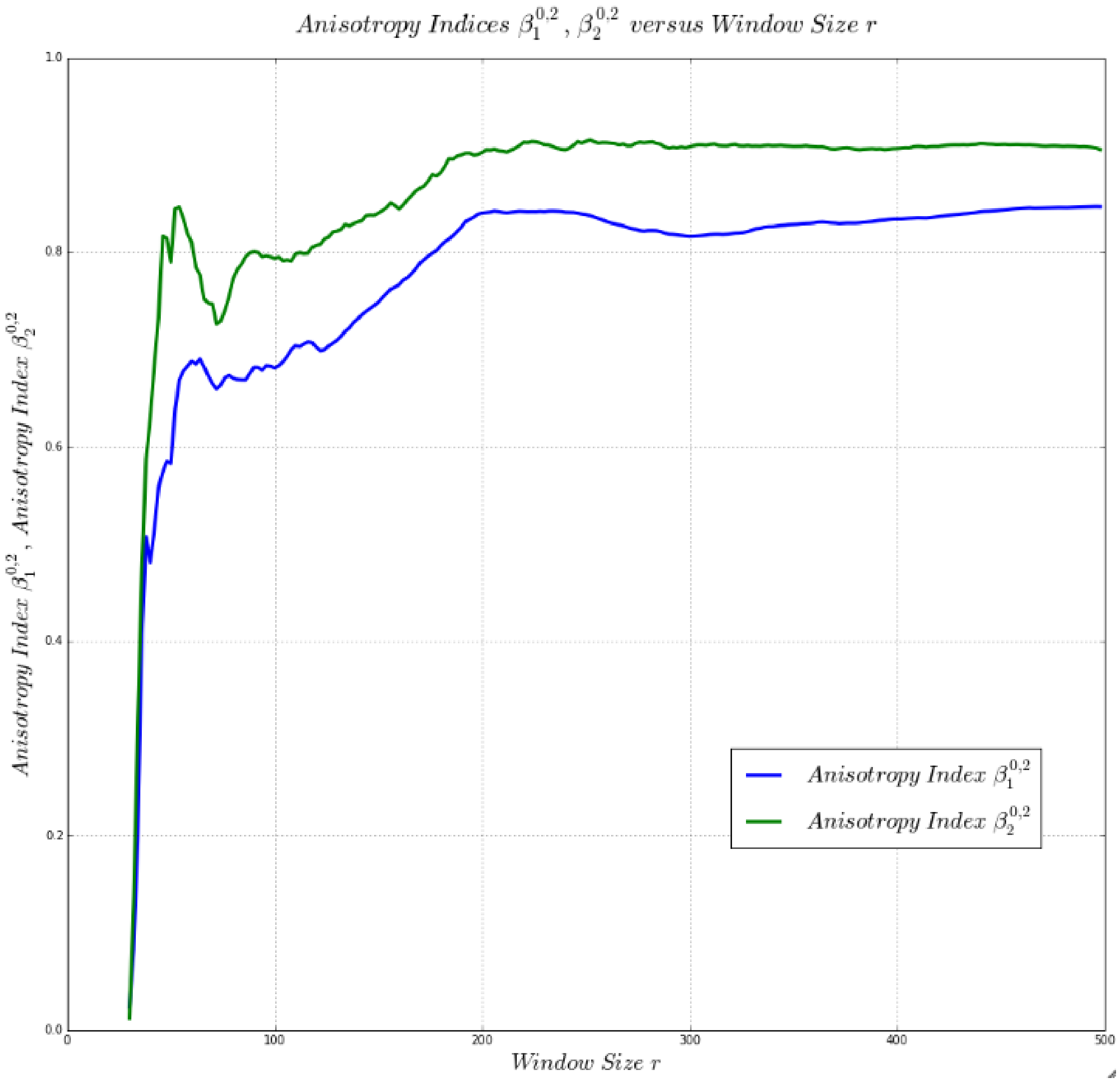
$$Estaillades\ Centered\ Expanding\ Window$$

$$Voxel\ size\ 3.31\mu m/voxel$$

$$Image\ Size: 500\ voxel^3$$



Estaillades – Comparison Anisotropy Indices B102, B202



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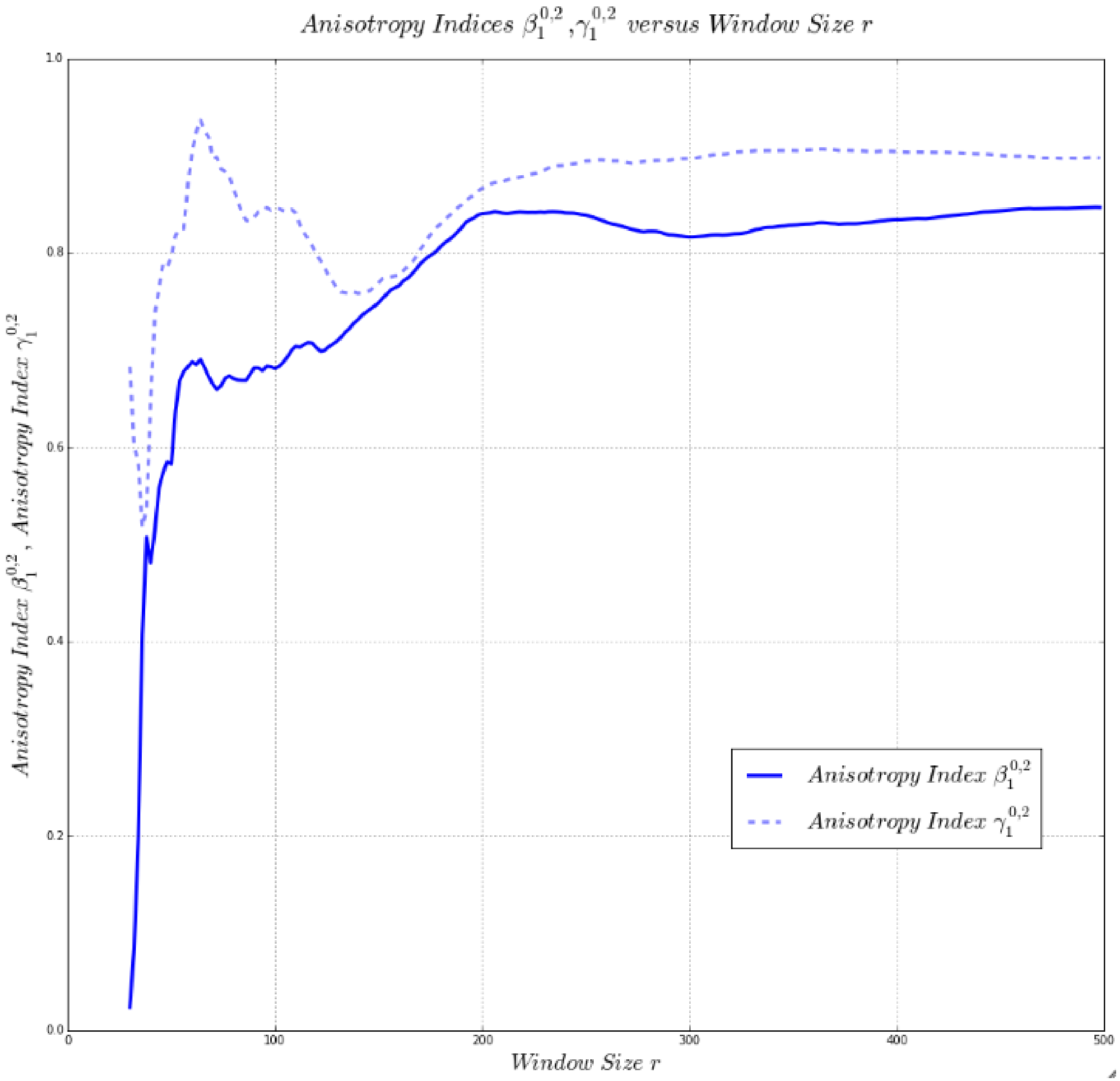
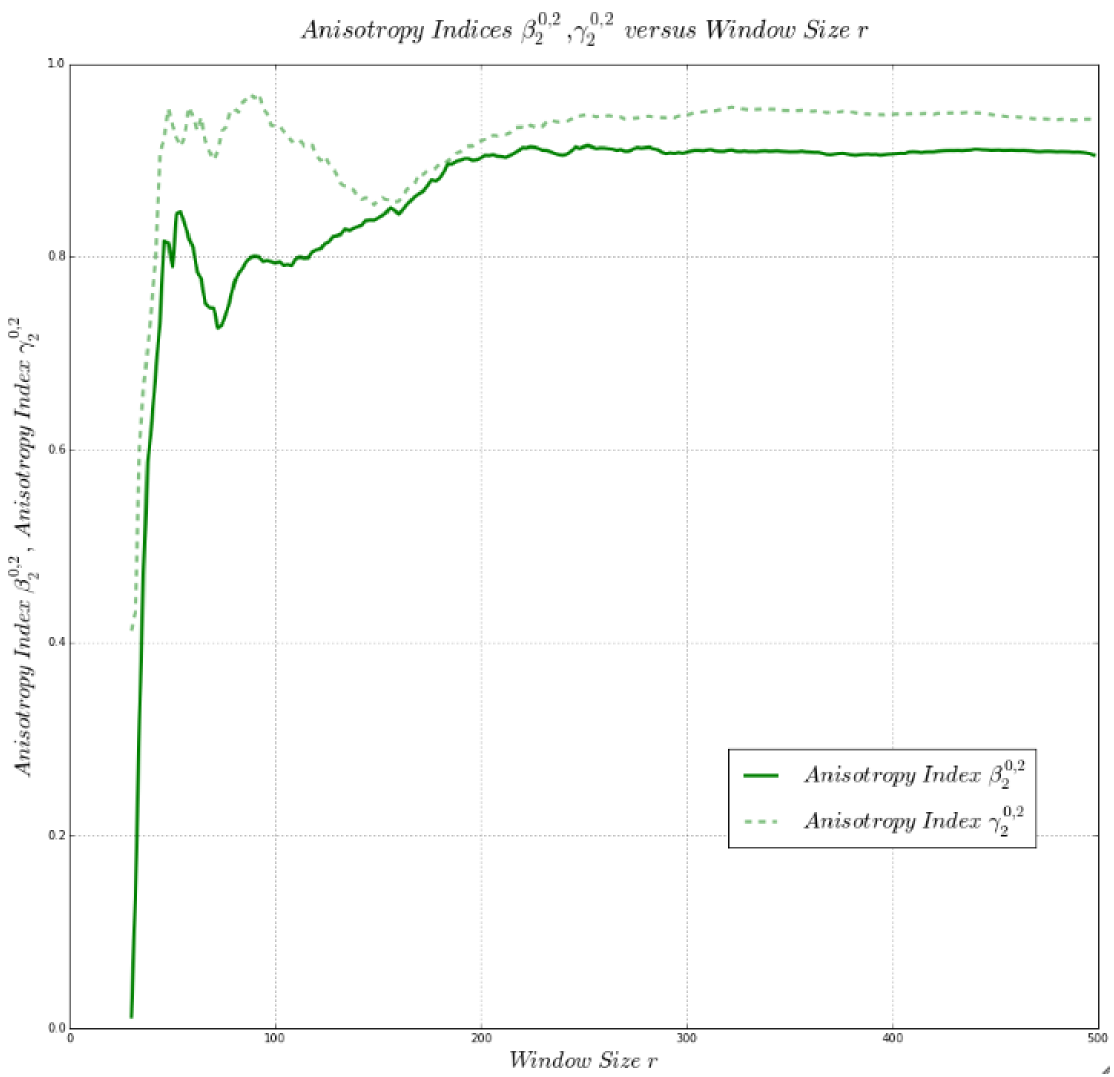
Estaillades– Centered Expanding Window



Estaillades– Centered Expanding Window



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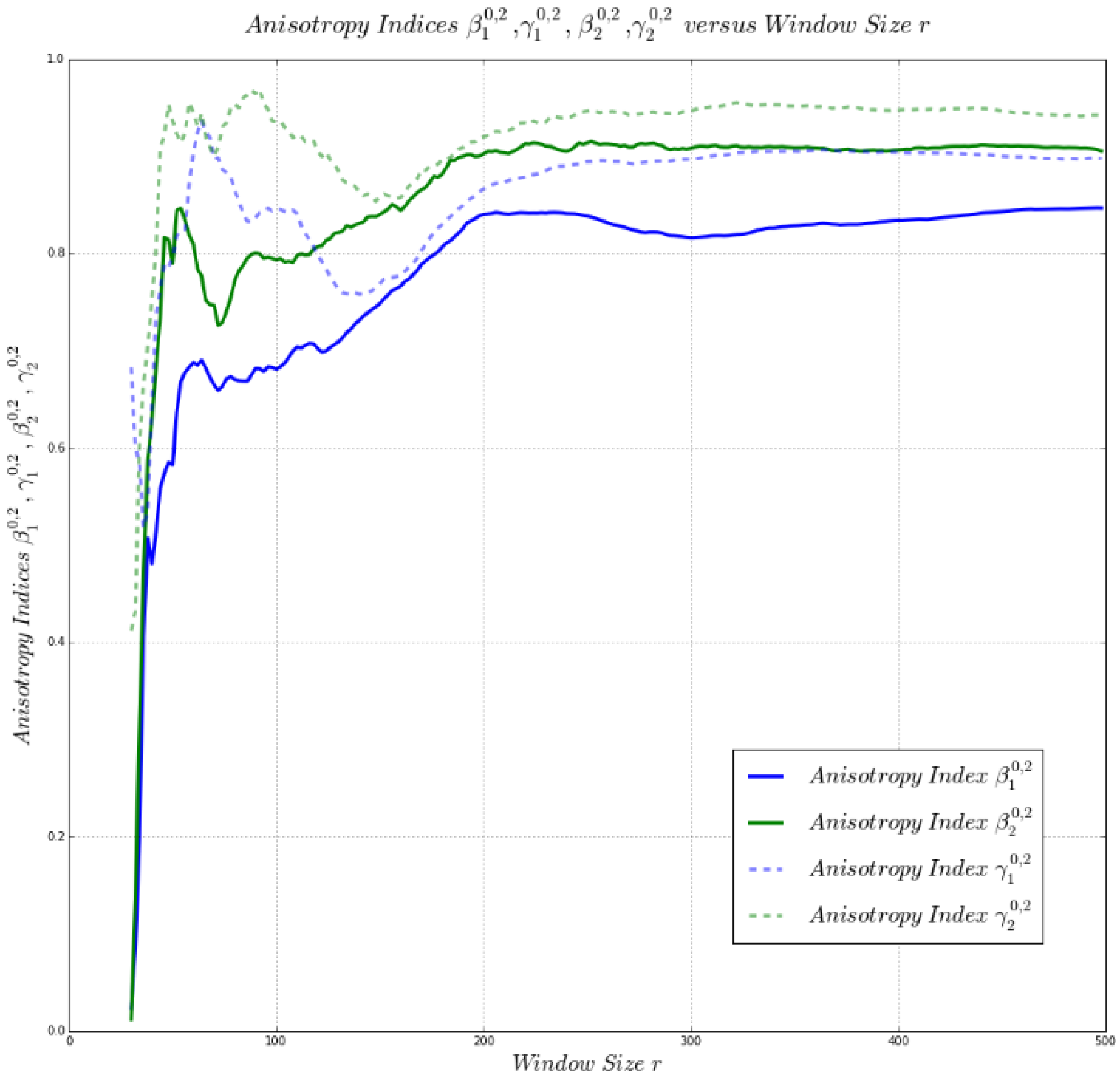
Estaillades – Centered Expanding Window



Doddington– Centered Expanding Window

$$Doddington\ Centered\ Expanding\ Window$$

$$Voxel\ size\ 3.31\mu m/voxel$$

$$Image\ Size: 500\ voxel^3$$

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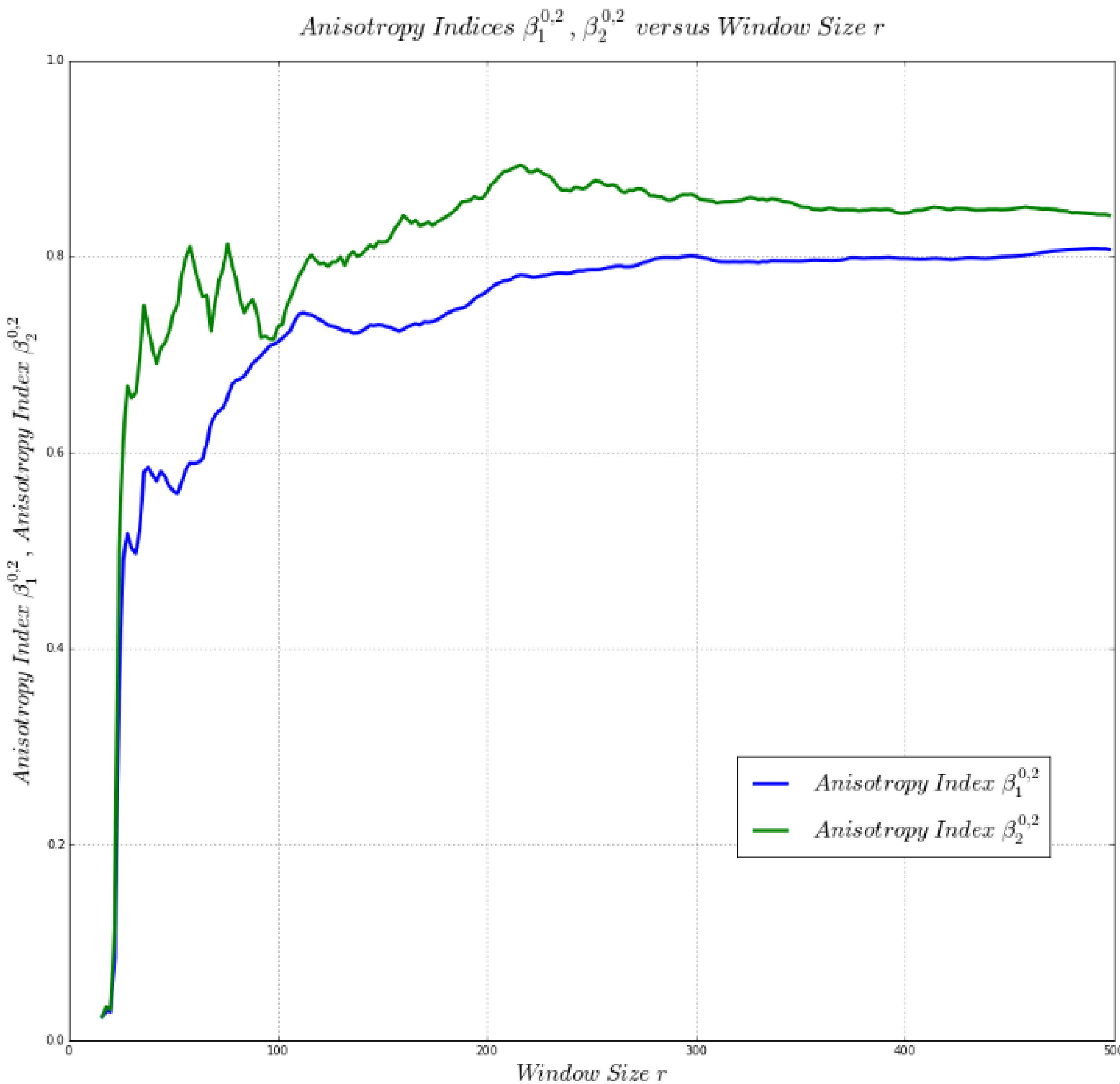
Doddington – Comparison Anisotropy Indices B102, B202



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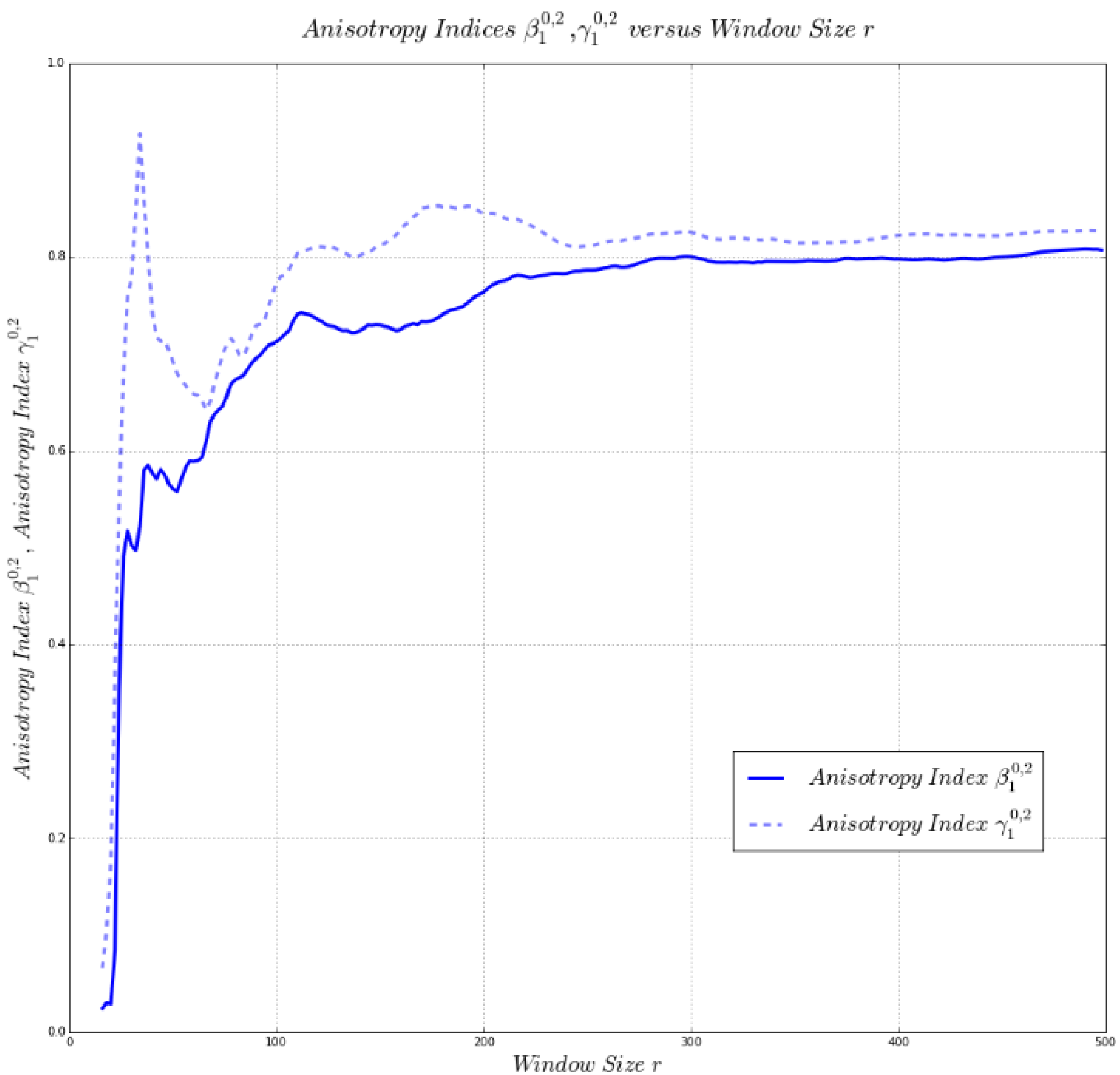
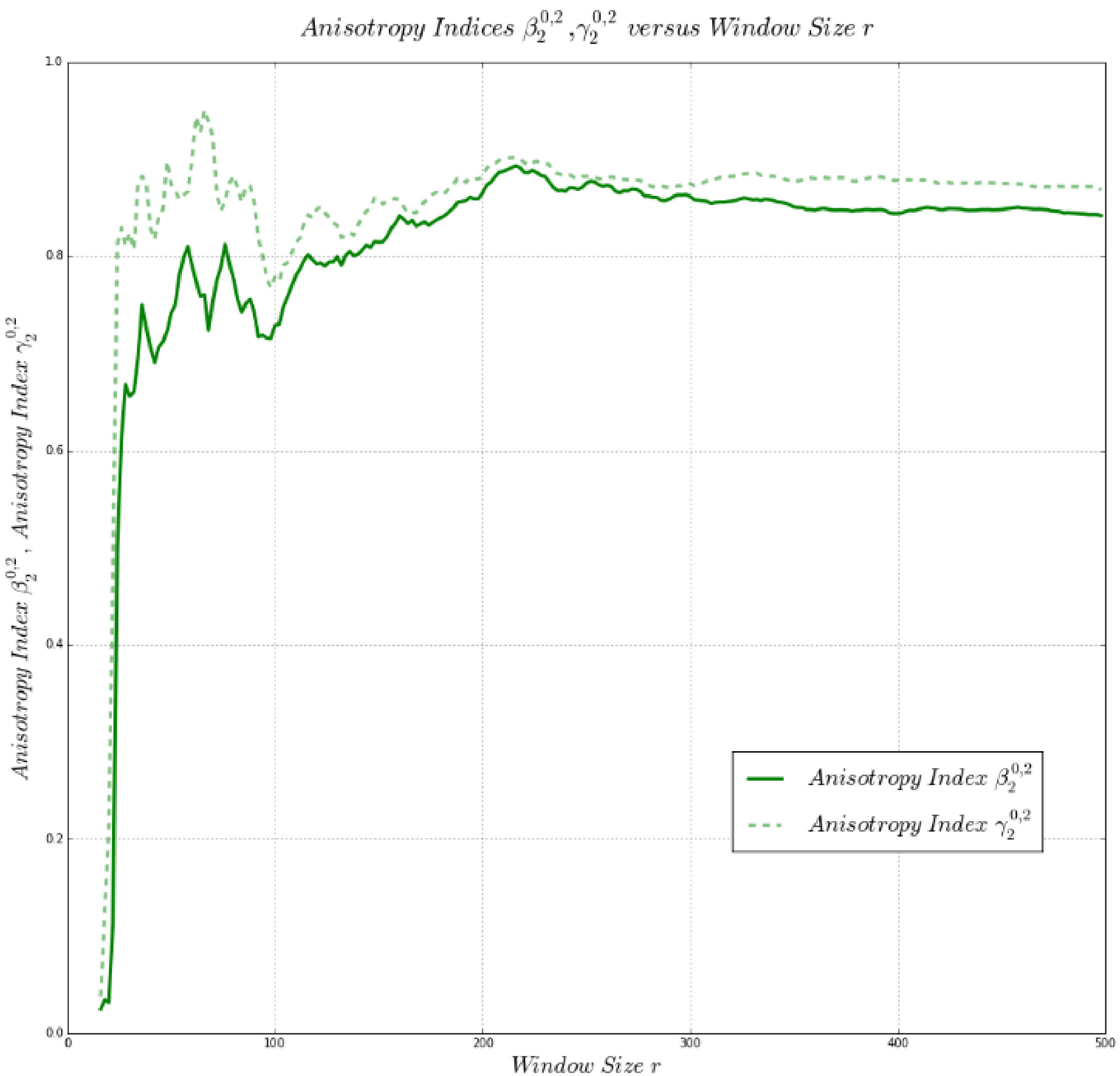
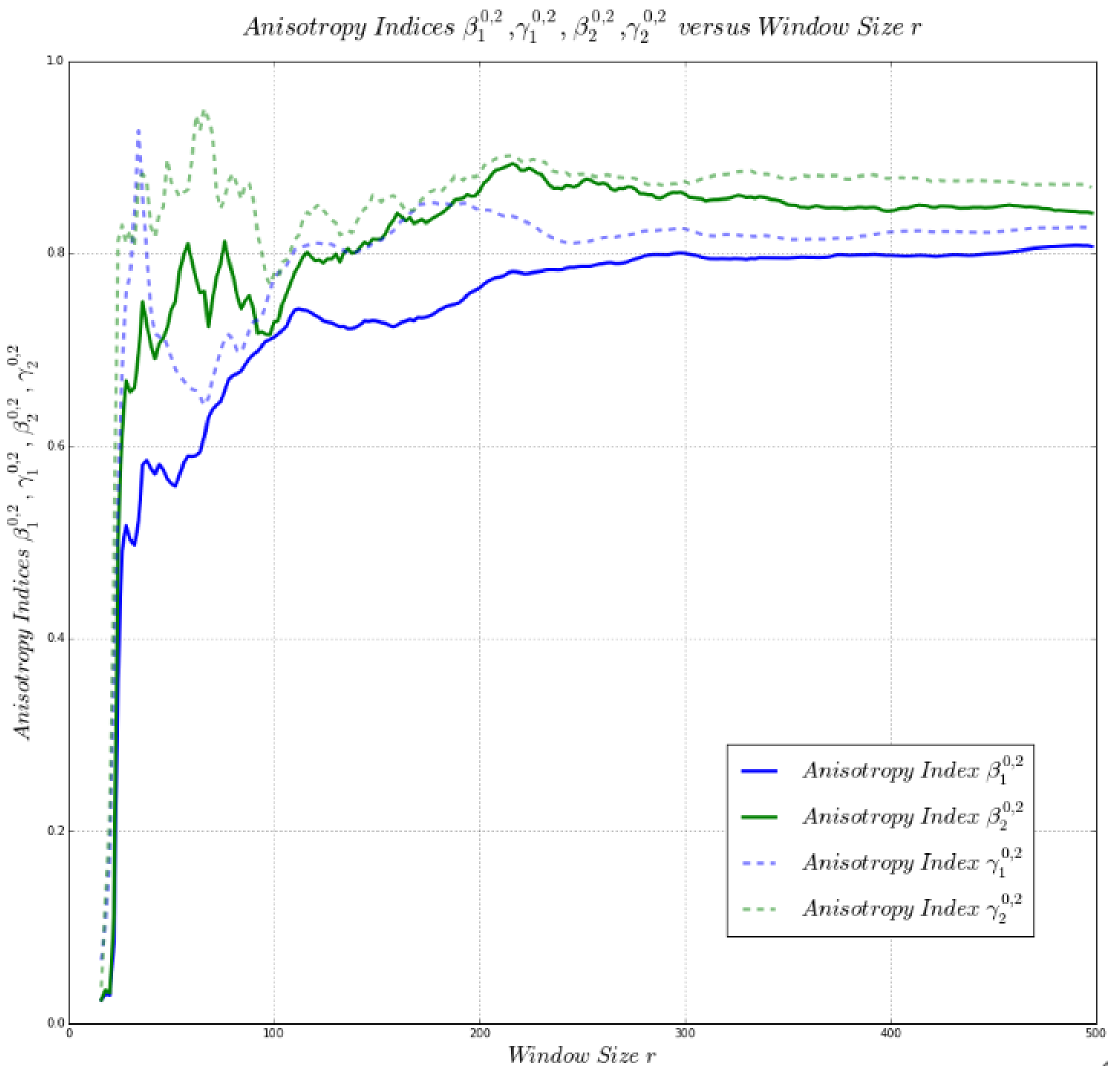
Doddington– Centered Expanding Window



Doddington– Centered Expanding Window



Doddington – Centered Expanding Window

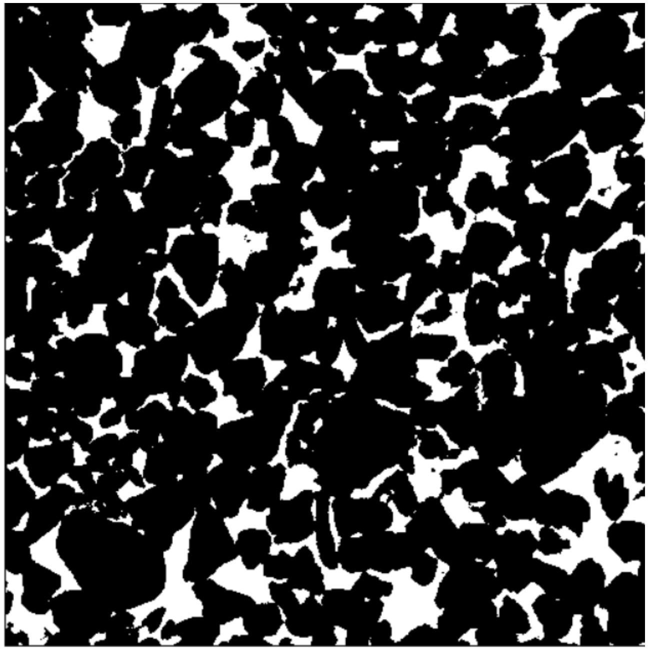


Bentheimer – Centered Expanding Window

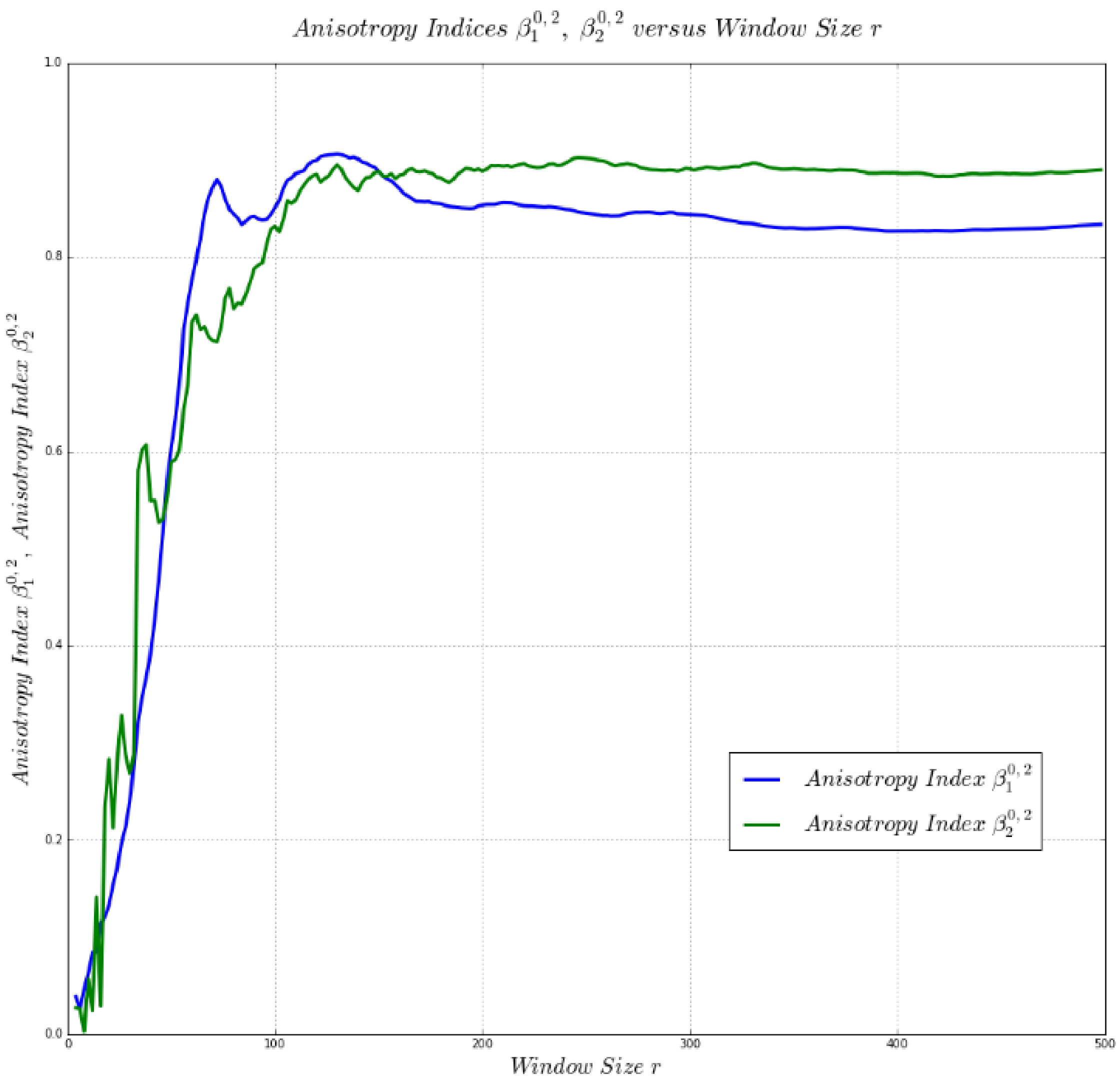
$$Bentheimer\ Centered\ Expanding\ Window$$

$$Voxel\ size\ 3.5\ \mu m/voxel$$

$$Image\ Size: 500\ voxel^3$$



Bentheimer – Comparison Anisotropy Indices B102, B202



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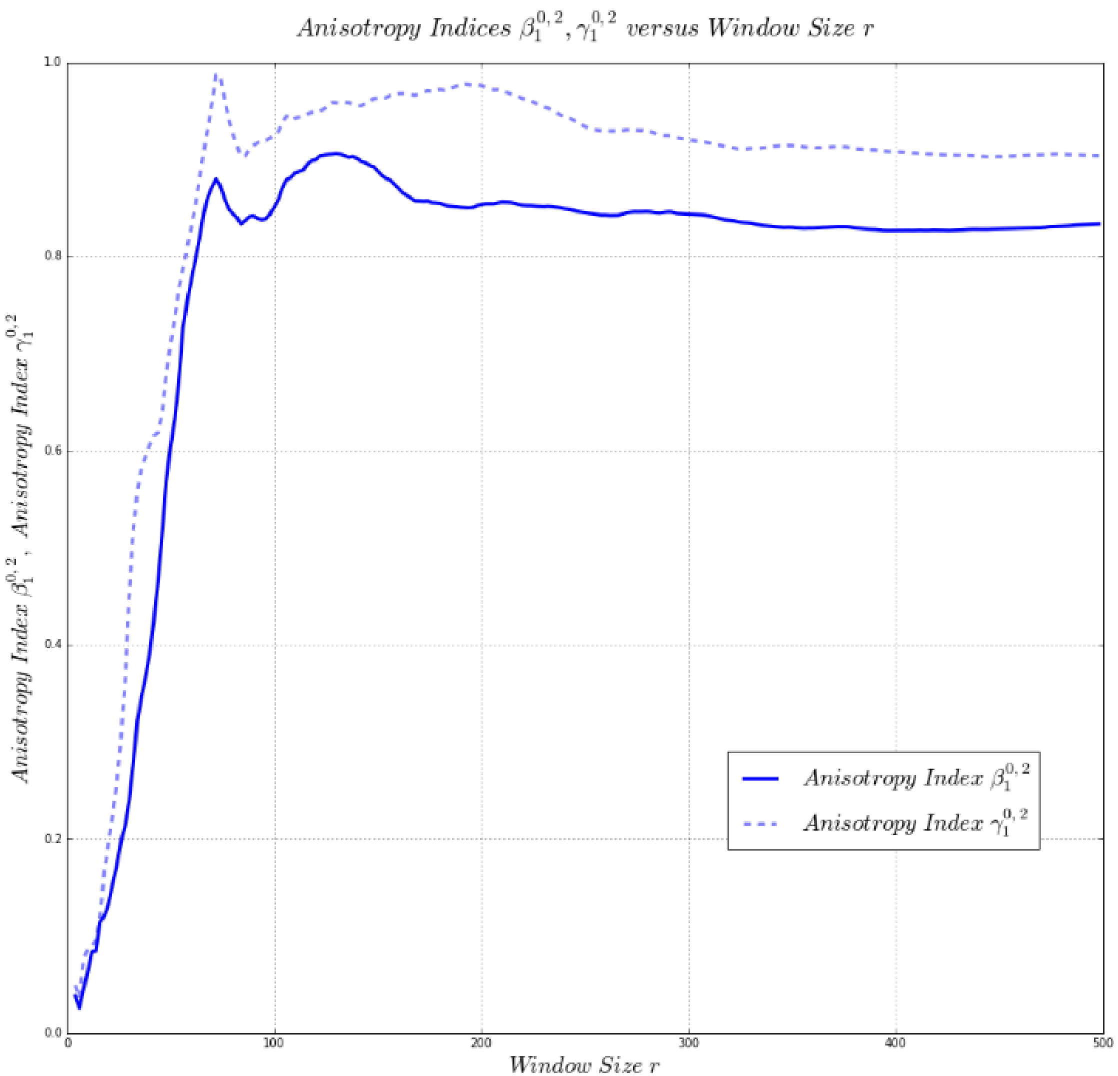
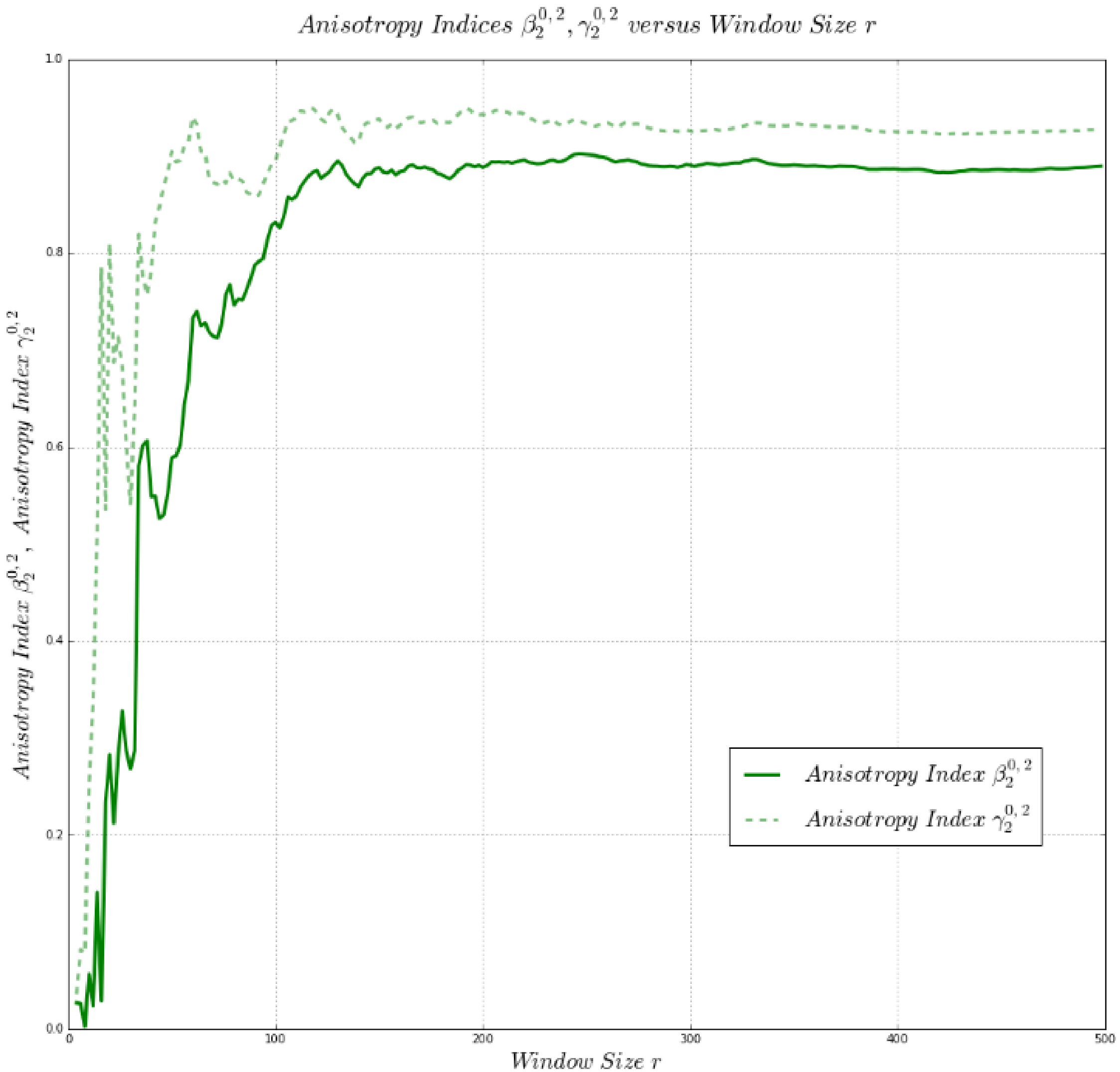
Bentheimer – Centered Expanding Window



Bentheimer – Centered Expanding Window



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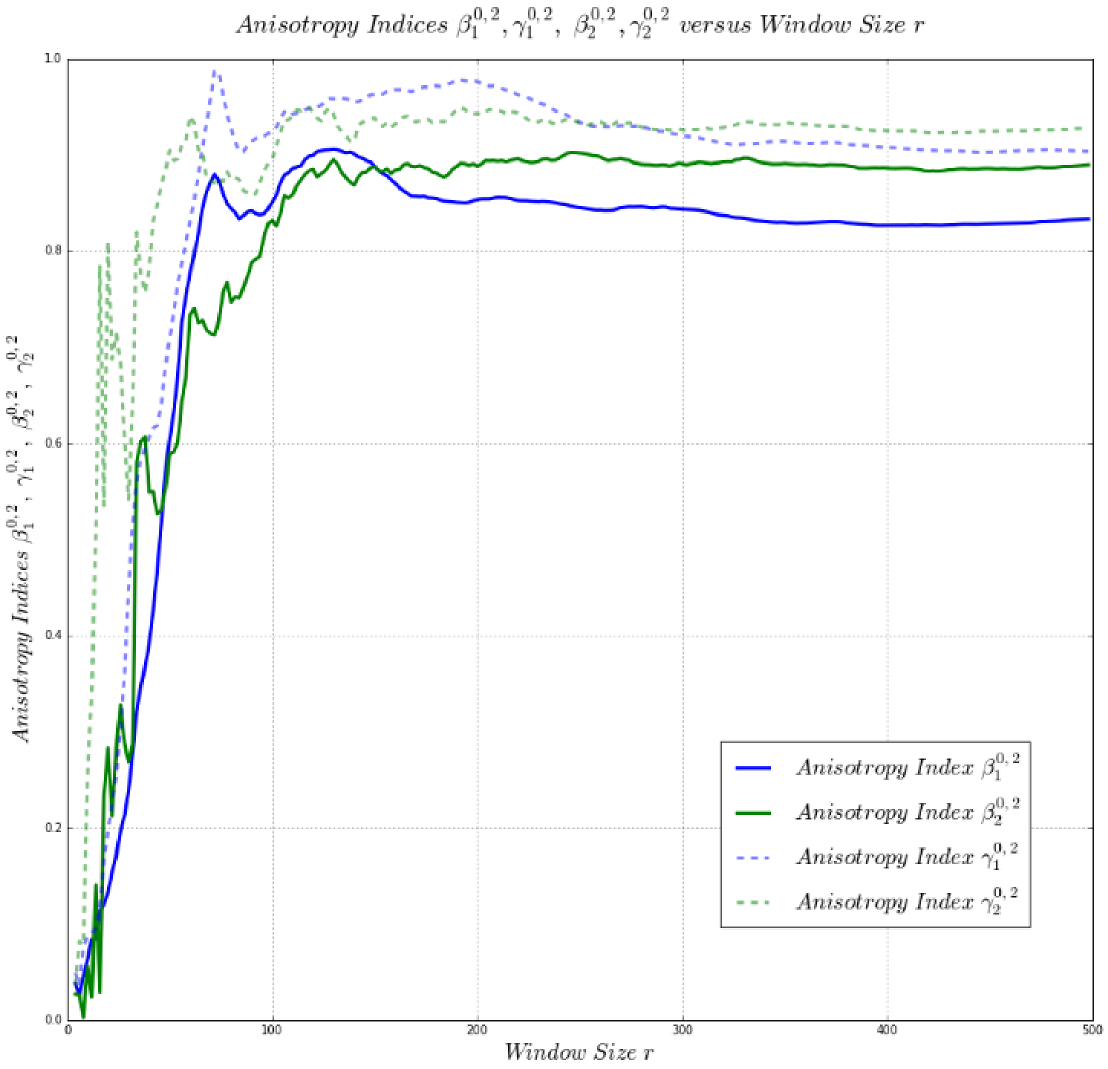
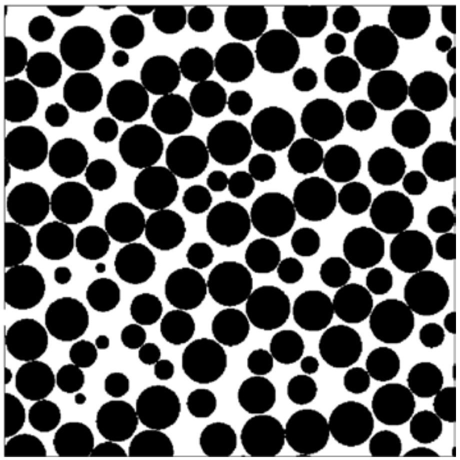
Bentheimer – Centered Expanding Window

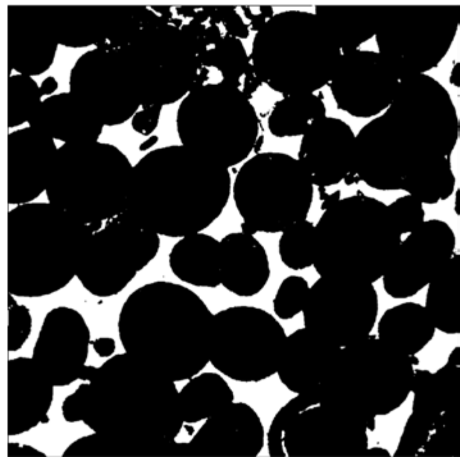


Comparison
Beadpack – Ketton – Estaillades – Doddington – Bentheimer

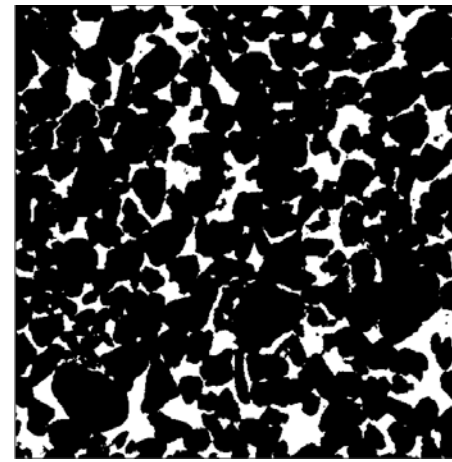
Beadpack



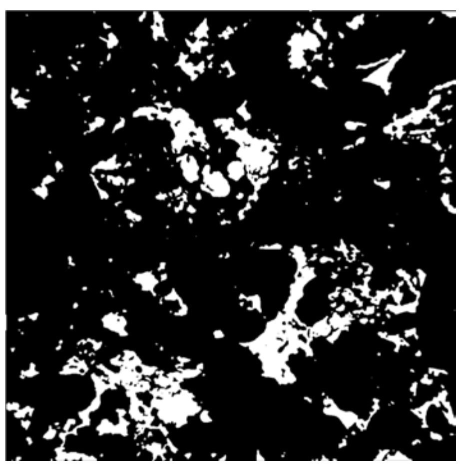
Ketton



Bentheimer



Estaillades



Doddington

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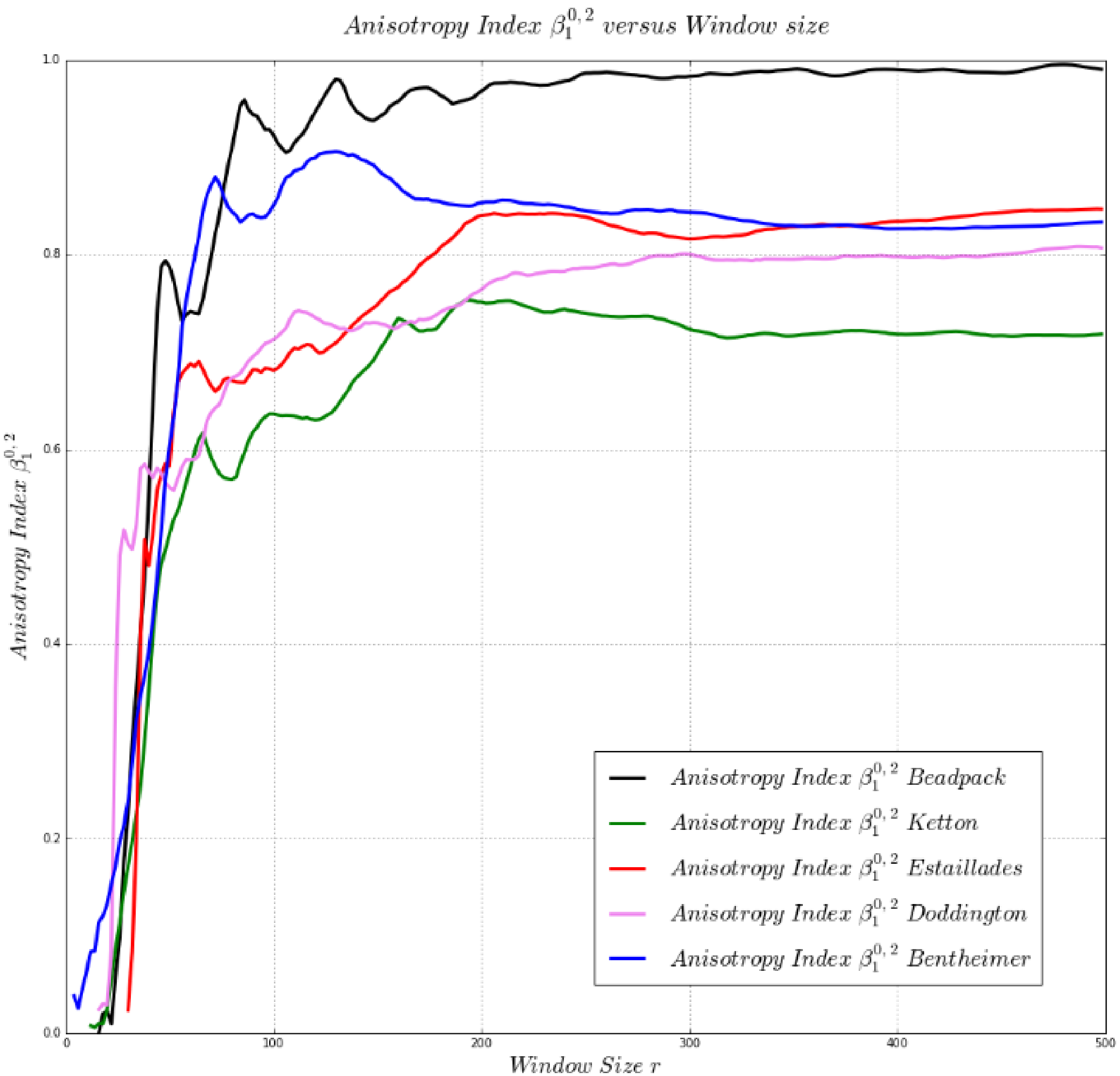
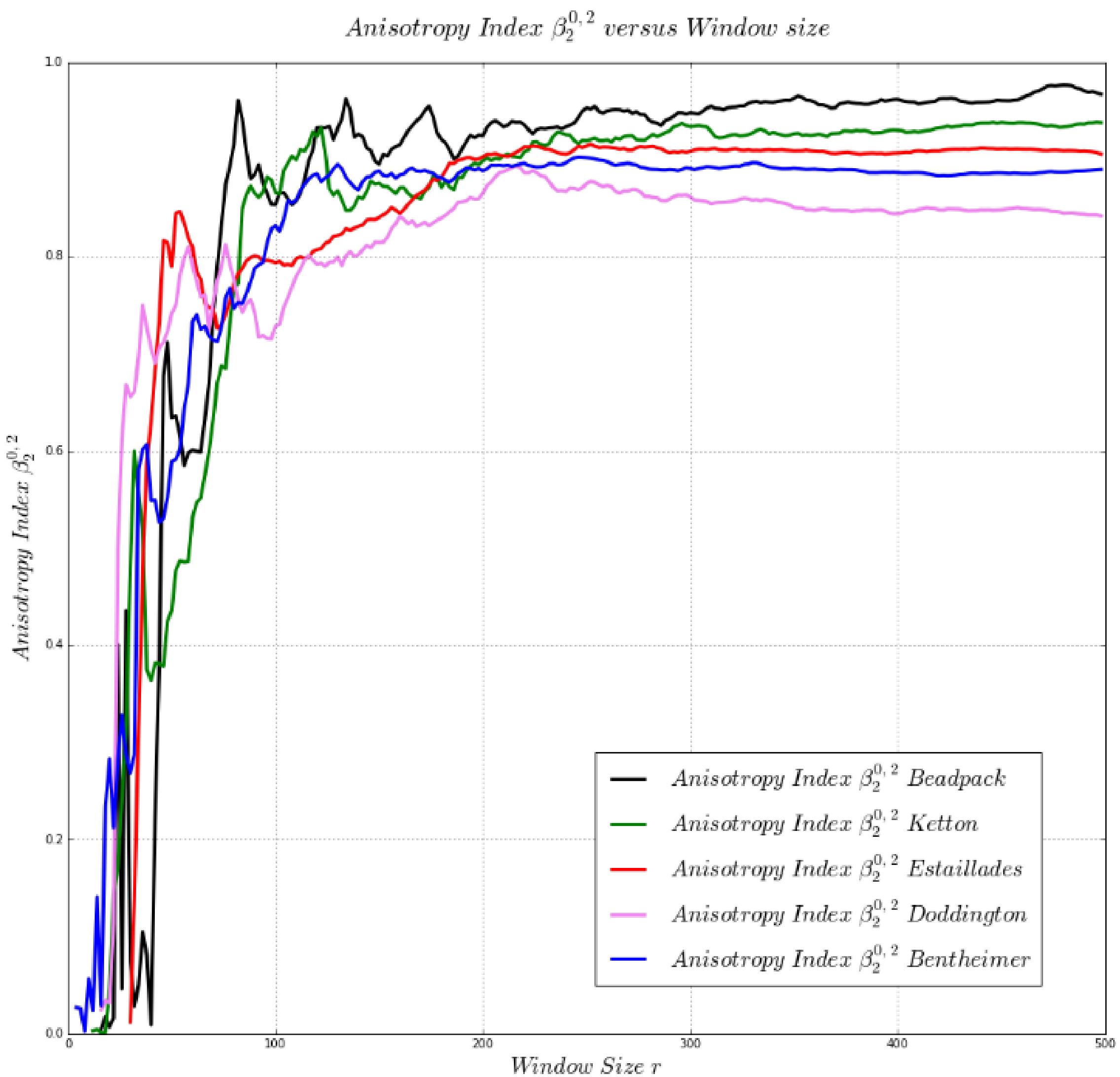
Comparison – Beta 102

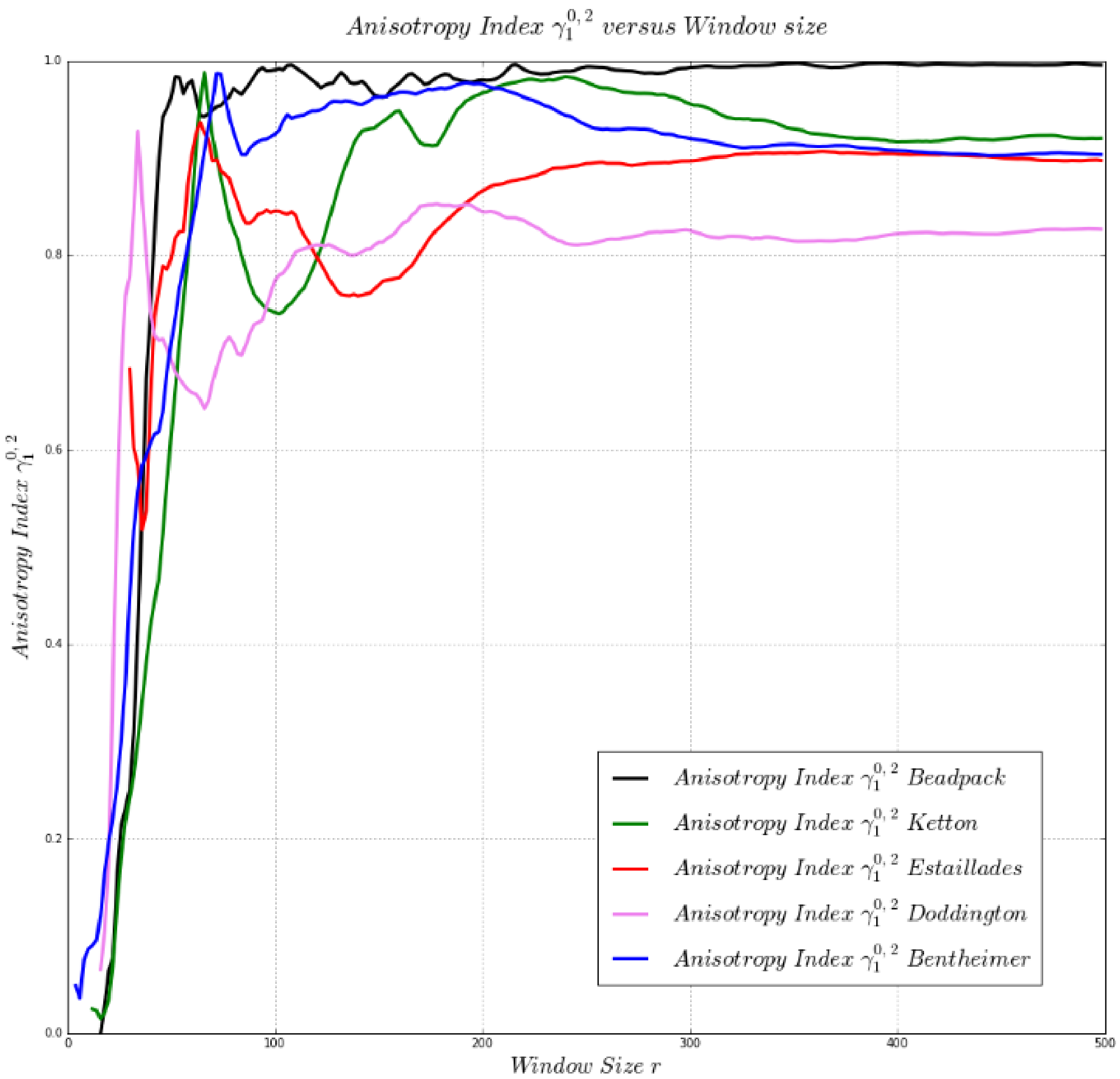


Imperial College London

Comparison – Beta 202



Comparison – Gamma 102



Comparison – Gamma 202