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Statistical Characterisation of Porous Media at the Pore Scale

Covariance Analysis

Student:

Lukas Mosser

Supervisors:

Professor Olivier Dubrule

Professor Martin Blunt

Presentation Outline

- Two-Point Probability Function
- Nomenclature
    - Covariance
    - Covariance Function
    - Normalized Covariance Function
    - Mean Grain Size
    - Characteristic Pore Size
- Covariance Analysis
    - Beadpack, Ketton, Estaillades, Doddington, Bentheimer

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Two – Point Probability Function Algorithm

Algorithm A.1 Algorithm for computing volume fraction for phase i

```
Read the image;                              ▷ Let's define the image as Ω
Read the x, y and z dimensions;              ▷ Let's say W, H, and D
N_pixels,i = 0;
for 0 ≤ a < W do
    for 0 ≤ b < H do
        for 0 ≤ c < D do
            if Ω(a, b, c) == i then
                N_pixels,i++;
            end if
        end for
    end for
end for
φ_i = N_pixels,i/(WHD);
```

After Pant 2016, PhD thesis

Procedure after Jin et. Al DOI: 10.1103/PhysRevE.93.013122

- Compute in each image direction for each pixel
  - directional two point probability function
- Average all directional two point probability functions
  - Averaged two point probability function

Nomenclature

Nomenclature after Ohser and Schladitz: 3D Images of Materials Structure

Remark 6.1

In various textbooks the covariance function is also called the two-point probability function or the two-point correlation function, see e. g. [361]. Sometimes it is called the auto-correlation function but this conflicts with the usual notation of stochastics where the autocorrelation function is the normalized covariance function $\mathrm{cov}_V/(V_V - V_V^2)$. In stochastic geometry the function $C(x) = \mathbb{P}(\{0, x\} \in \Xi)$ is known as the covariance of the random closed set Ξ. It follows that $C(x) = \mathrm{cov}_V(x) + V_V^2$.

- $Two - Point\ Probability\ Function = Covariance = C(x)$
- $Covariance\ Function = cov_V(r) = C(x) - \phi^2$
- $Normalized\ Covariance\ Function = \frac{cov_V(r)}{\phi - \phi^2}$
- $Mean\ Grain\ Size\ r_m = First\ Local\ Minimum\ of\ C(x)$
- $Characteristic\ Pore\ Size\ r_m = \phi(\phi - 1)/\frac{d}{dr}cov(r)|_{r=0}$

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Summary of Results

- Beadpack
  - Isotropic
  - Equal mean grain size in x, y, z-directions
- Ketton:
  - Anisotropic behavior – grain size between 159 and 204 μm
  - Well defined local minima to determine mean grain size
- Estaillades:
  - Difficult to determine mean grain size
  - Varies strong between directional and averaged covariance
- Doddington:
  - Anisotropic covariance and mean grain size
  - Distinct normalized covariance function, nearly exponential
- Bentheimer:
  - Isotropic covariance and therefore mean grain size
  - Position of local minima same
  - Different absolute values of covariance at first minimum

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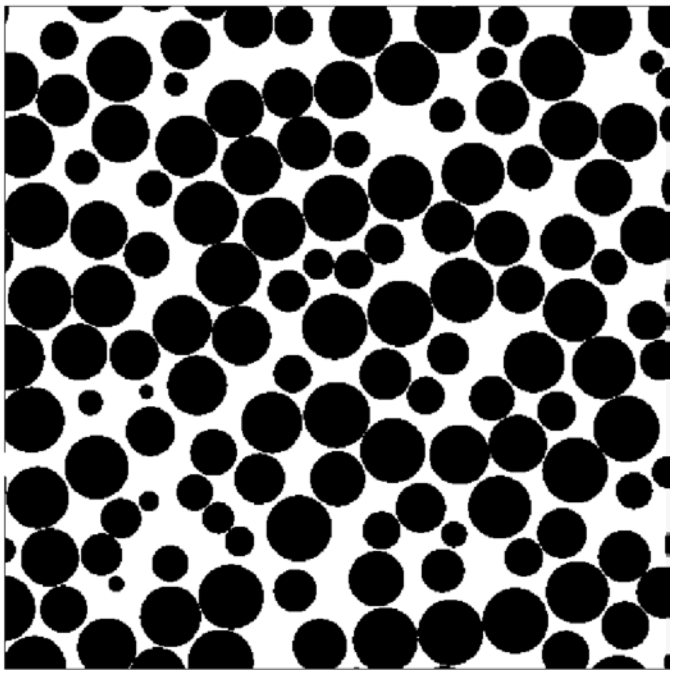
Beadpack – Covariance Analysis



$Voxel\ Size\ \mu m/\mathrm{voxel}$

$Image\ Size: 500\ voxel^3$

Mean Grain Size r_m		
	pixels	μm
$x - Direction$	34	
$y - Direction$	33	
$z - Direction$	33	
$Averaged$	33	
$Ratio$	0.99	

Char. Pore Size r_c		
	pixels	μm
$x - Direction$	15	
$y - Direction$	15	
$z - Direction$	15	
$Averaged$	15	
$Ratio$	1.0	

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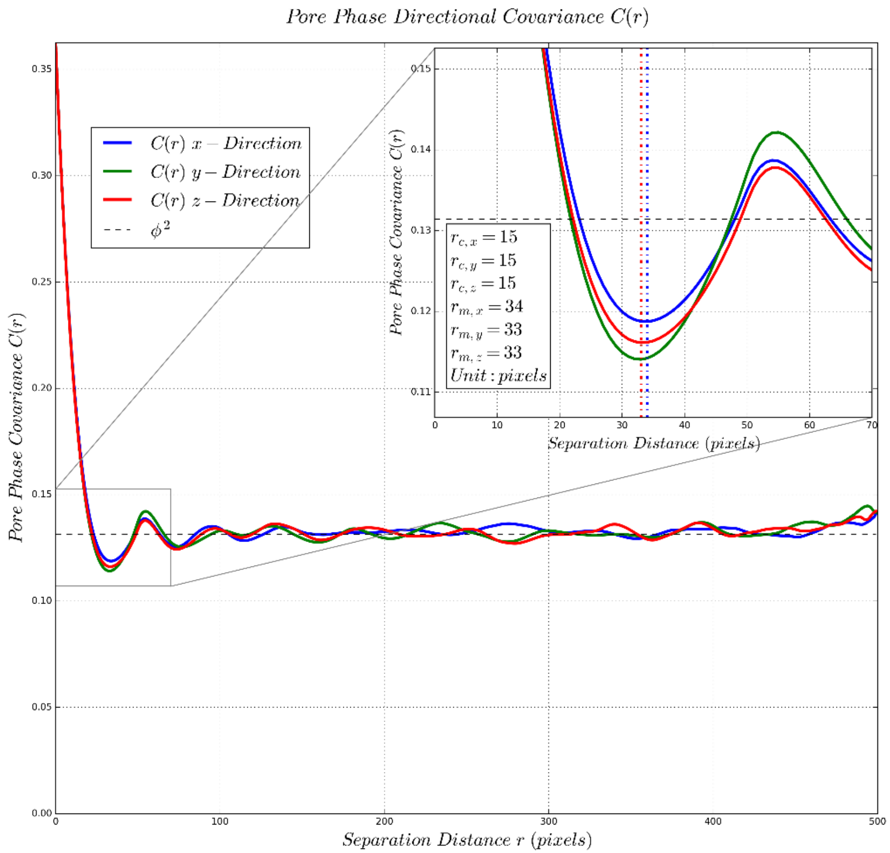
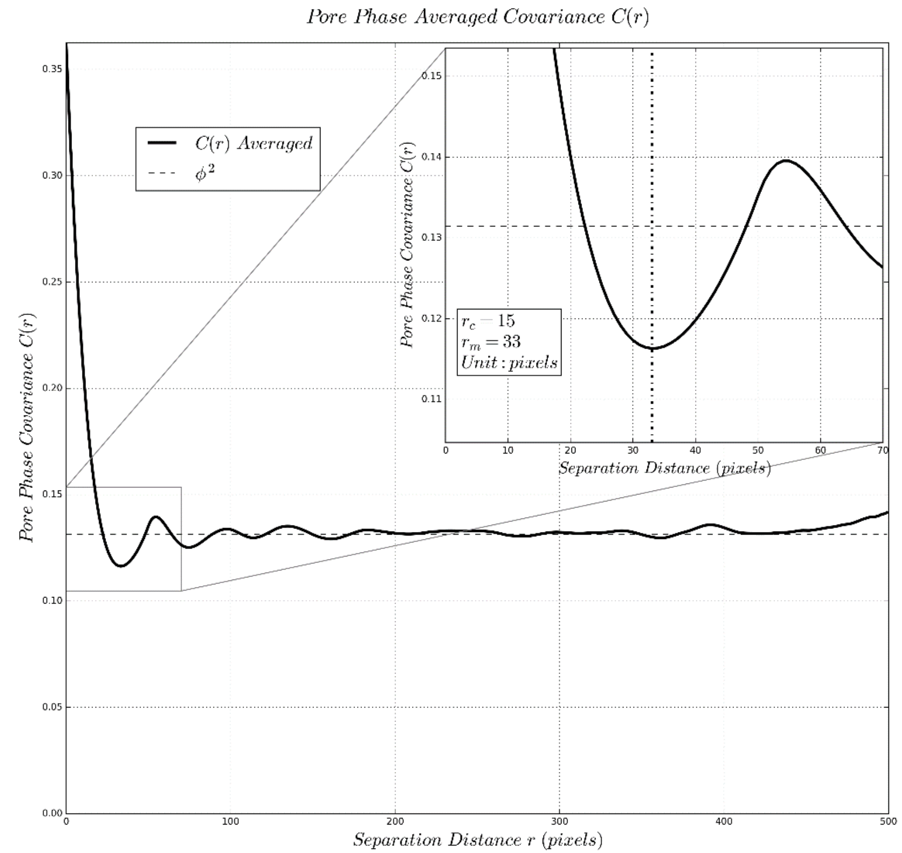
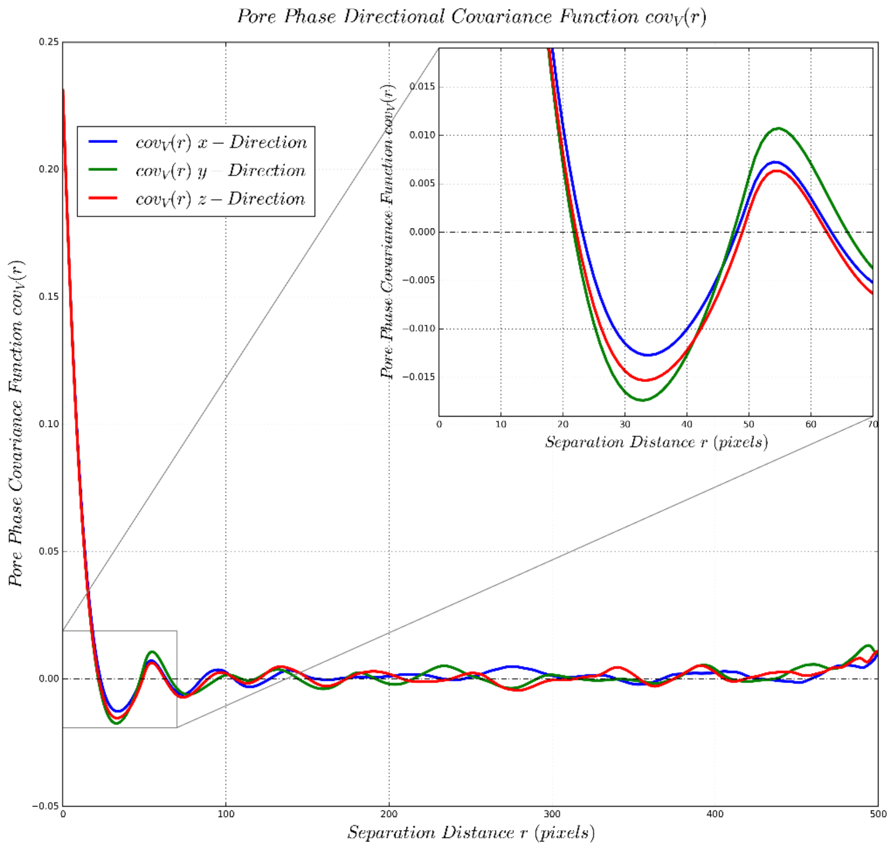
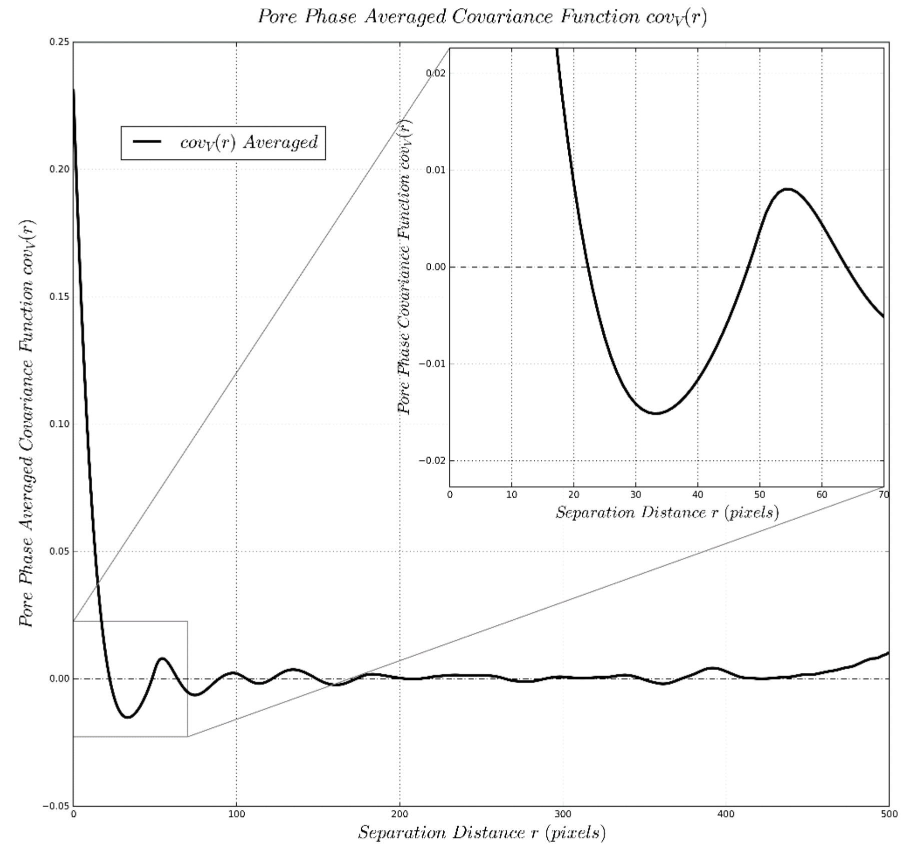
Beadpack – Covariance





Beadpack – Covariance Function





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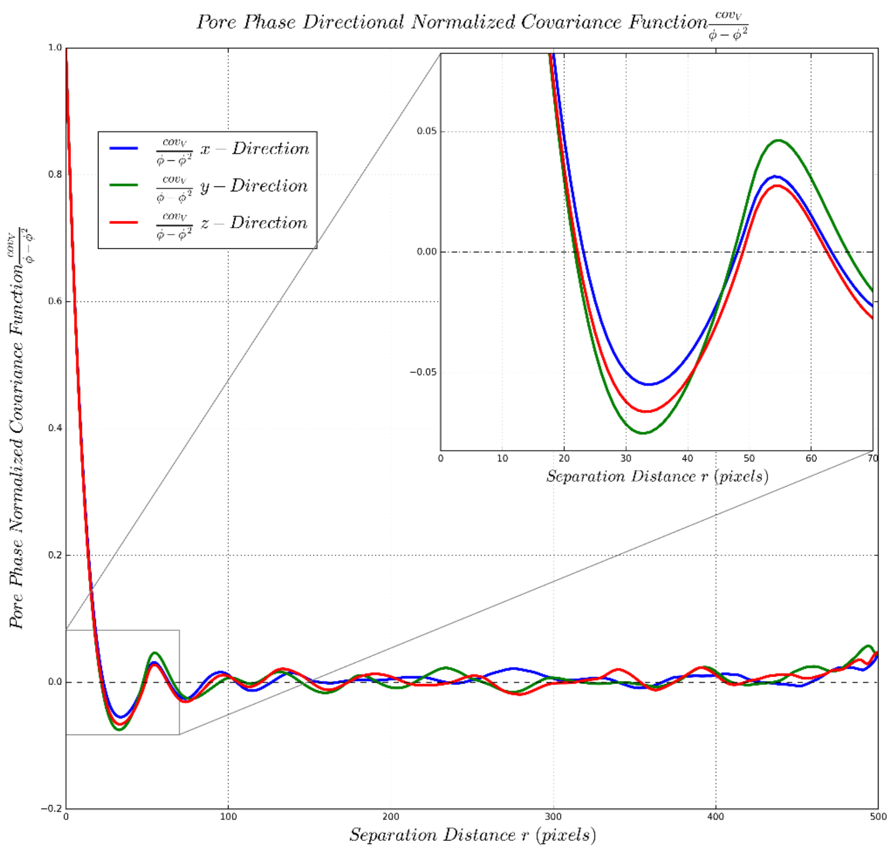
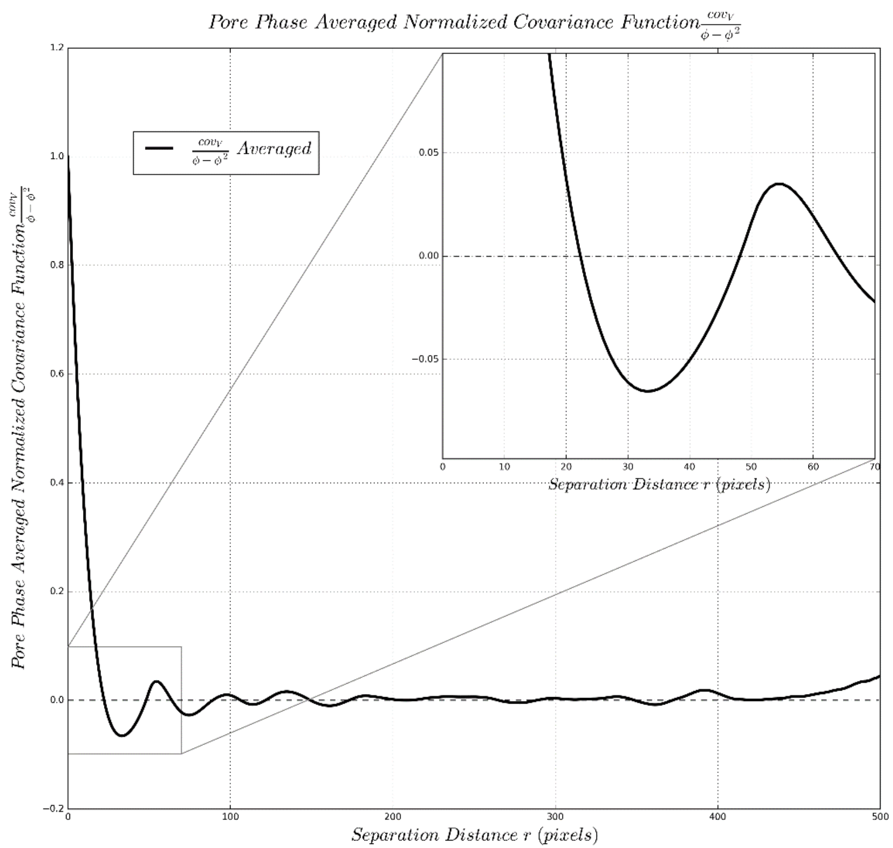
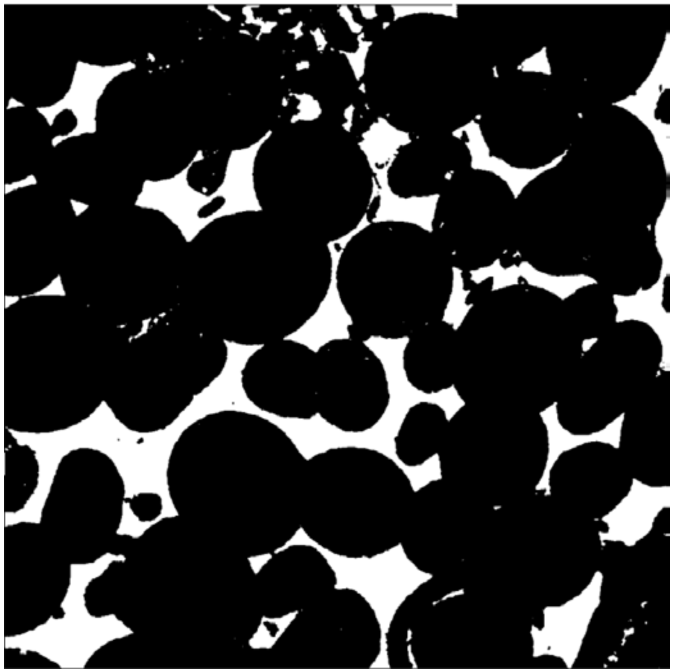
Beadpack – Normalized Covariance Function





Ketton – Covariance Analysis



$Voxel\ size\ 3\mu m/voxel$

$Image\ Size{:}\ 500\ voxel^3$

Mean Grain Size r_m		
	pixels	μm
$x - Direction$	53	159
$y - Direction$	68	204
$z - Direction$	59	177
$Averaged$	57	174
$Ratio$	0.78	

Char. Pore Size r_c		
	pixels	μm
$x - Direction$	16	48
$y - Direction$	18	54
$z - Direction$	18	54
$Averaged$	17	51
$Ratio$	0.89	

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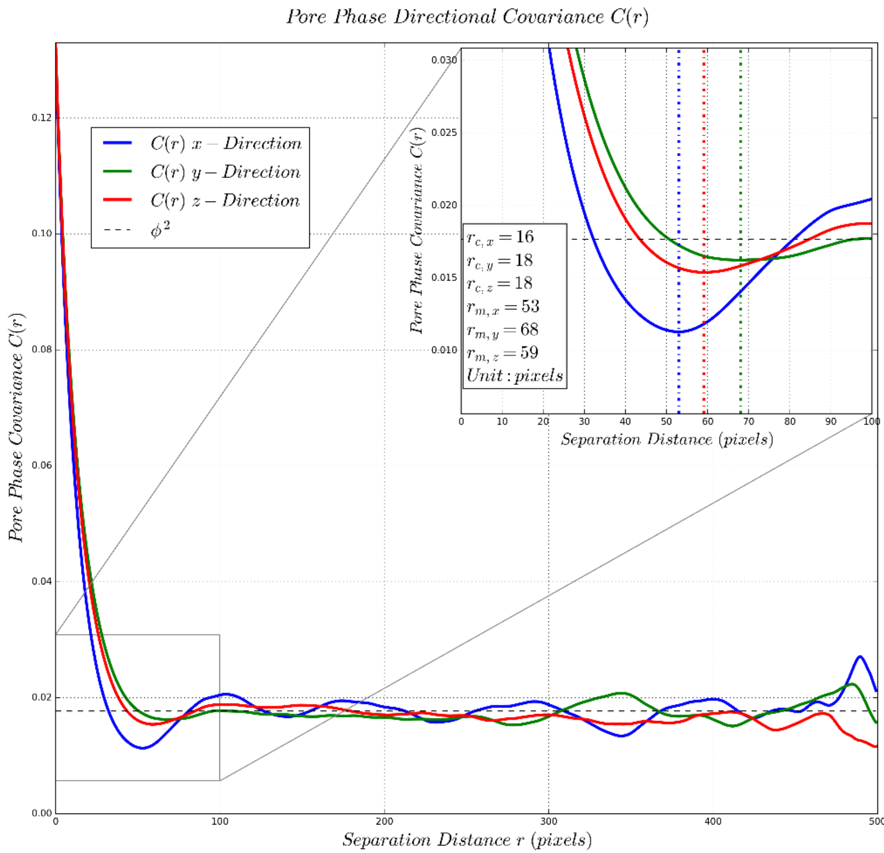
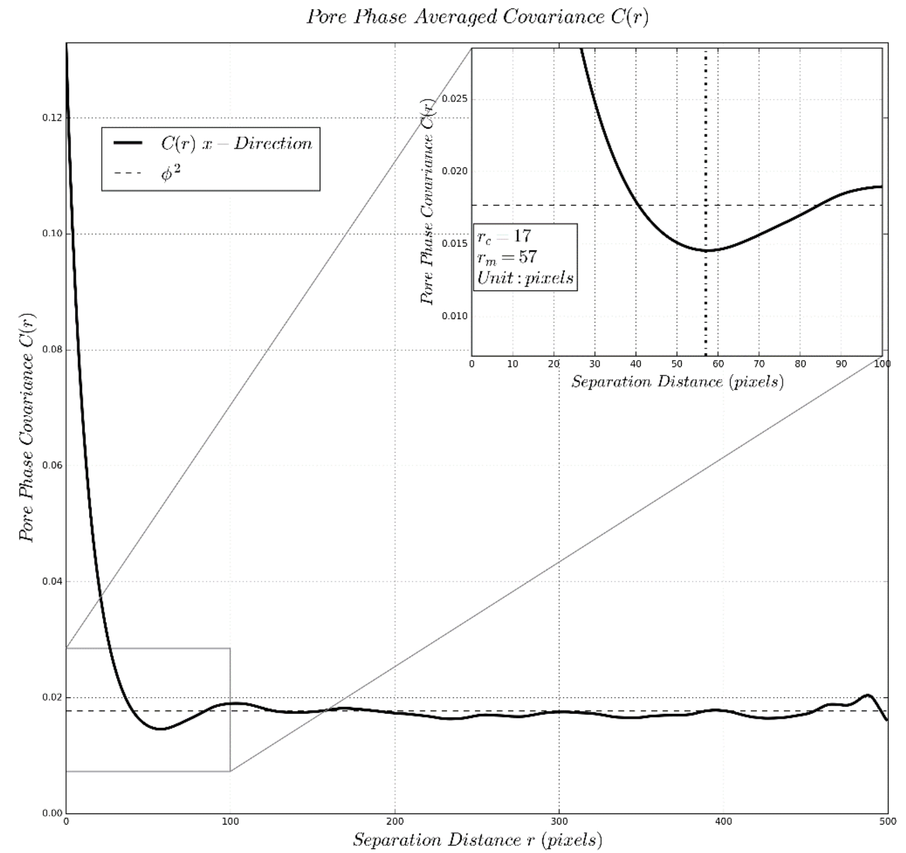
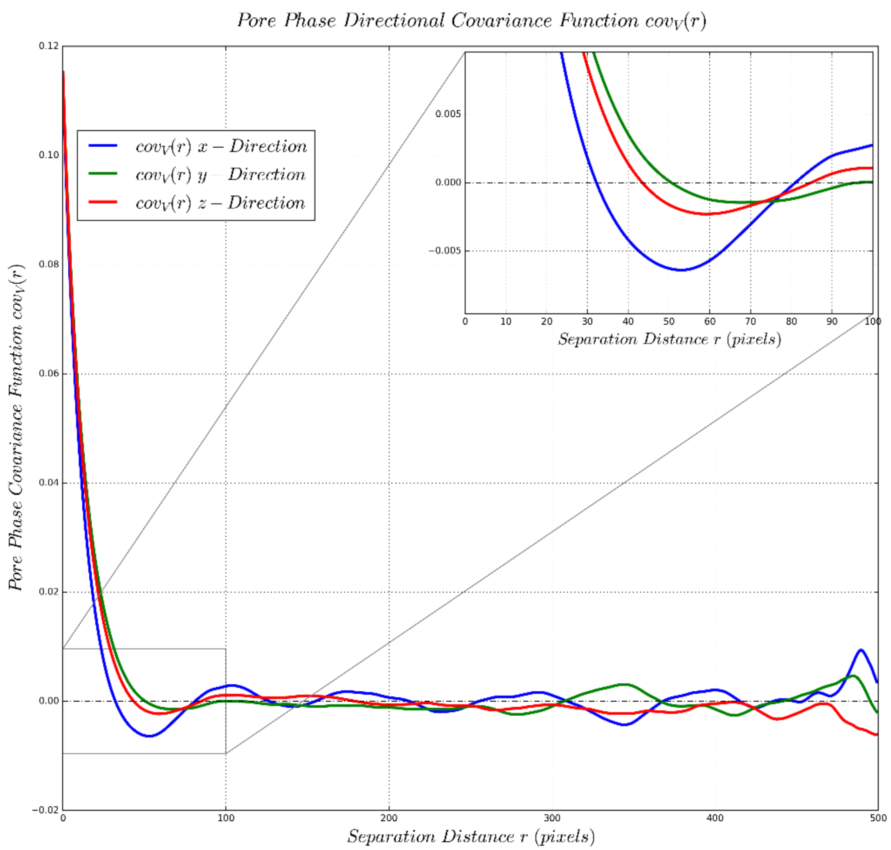
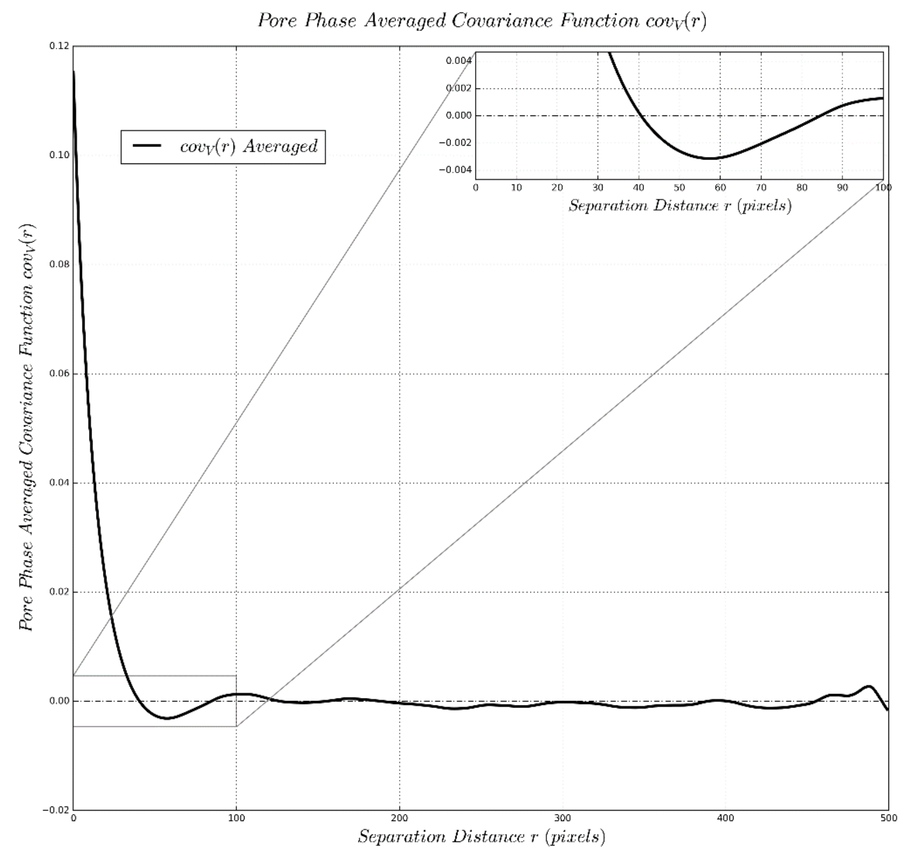
Ketton – Covariance



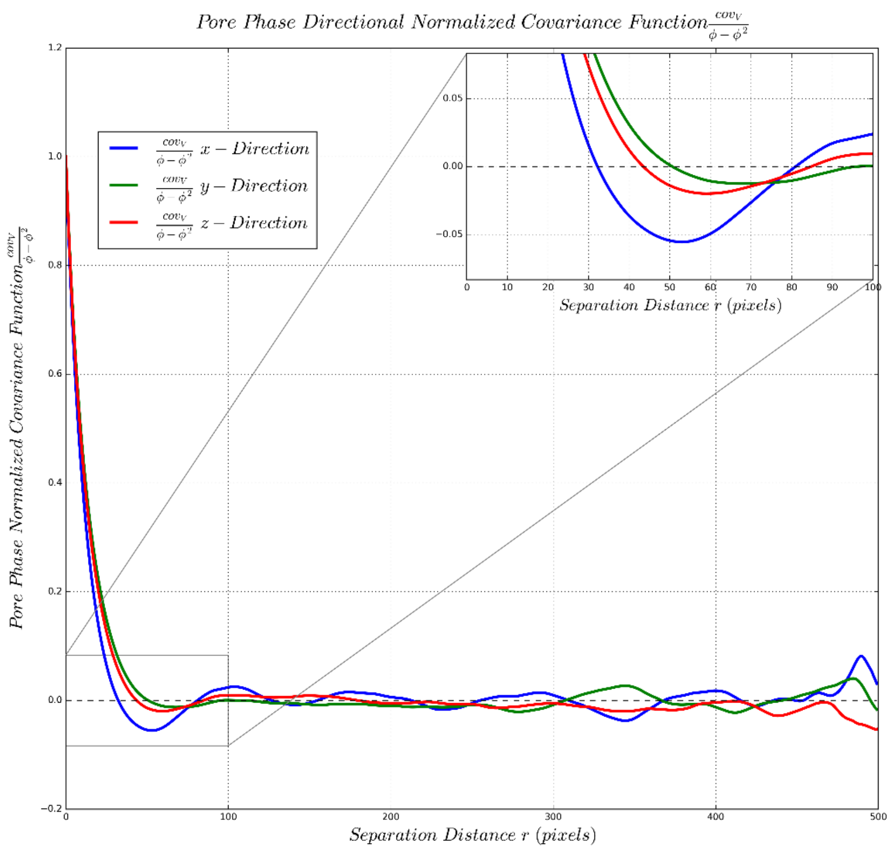


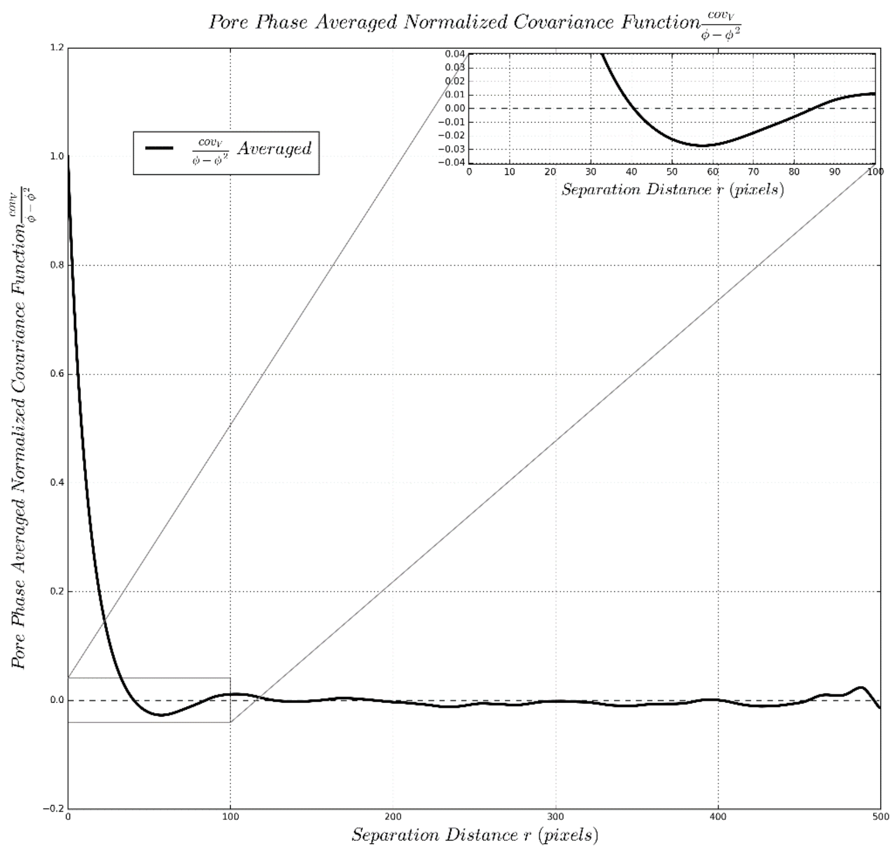
Ketton– Covariance Function



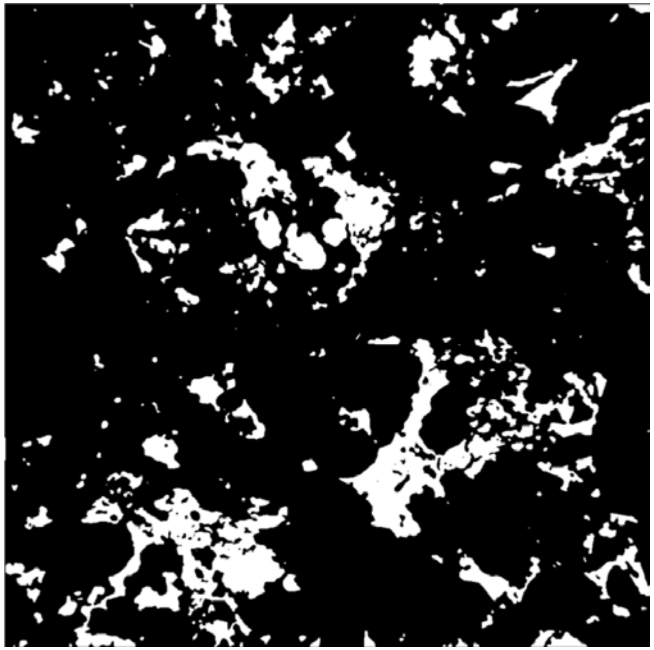


Ketton – Normalized Covariance Function





Estaillades – Covariance Analysis



$Voxel\ size\ 3.31\mu m/voxel$

$Image\ Size: 500\ voxel^3$

Mean Grain Size r_m		
	pixels	μm
$x-Direction$	143	473
$y-Direction$	135	446
$z-Direction$	102	337
$Averaged$	145	479
$Ratio$	0.71	

Char. Pore Size r_c		
	pixels	μm
$x-Direction$	14	46
$y-Direction$	14	46
$z-Direction$	14	46
$Averaged$	14	46
$Ratio$	1.0	

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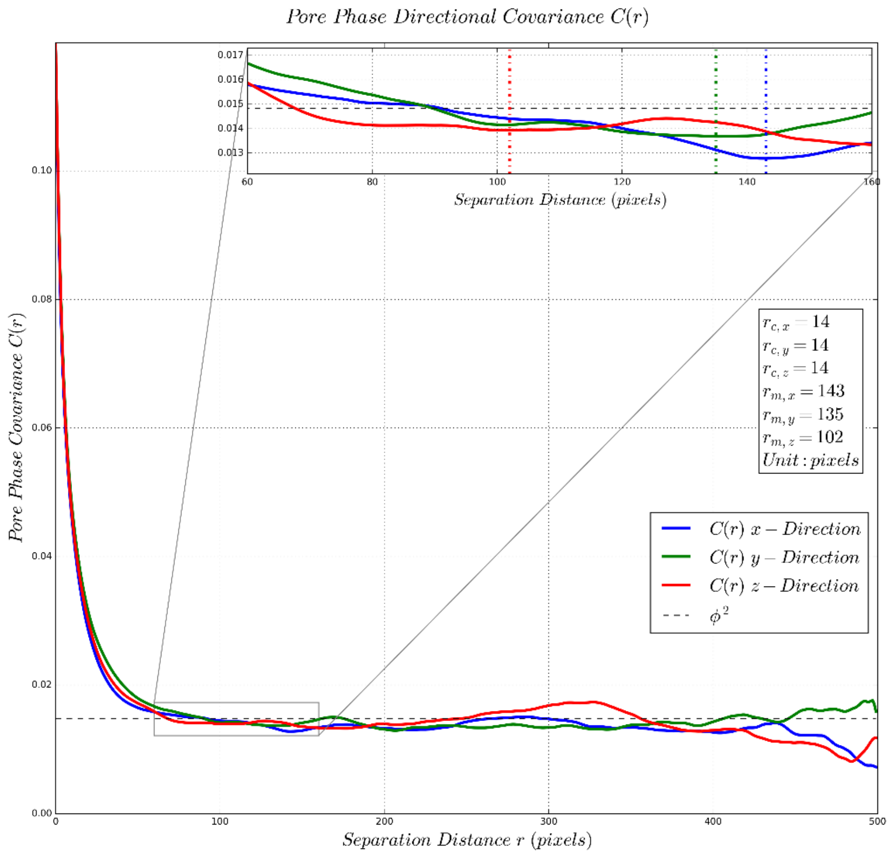
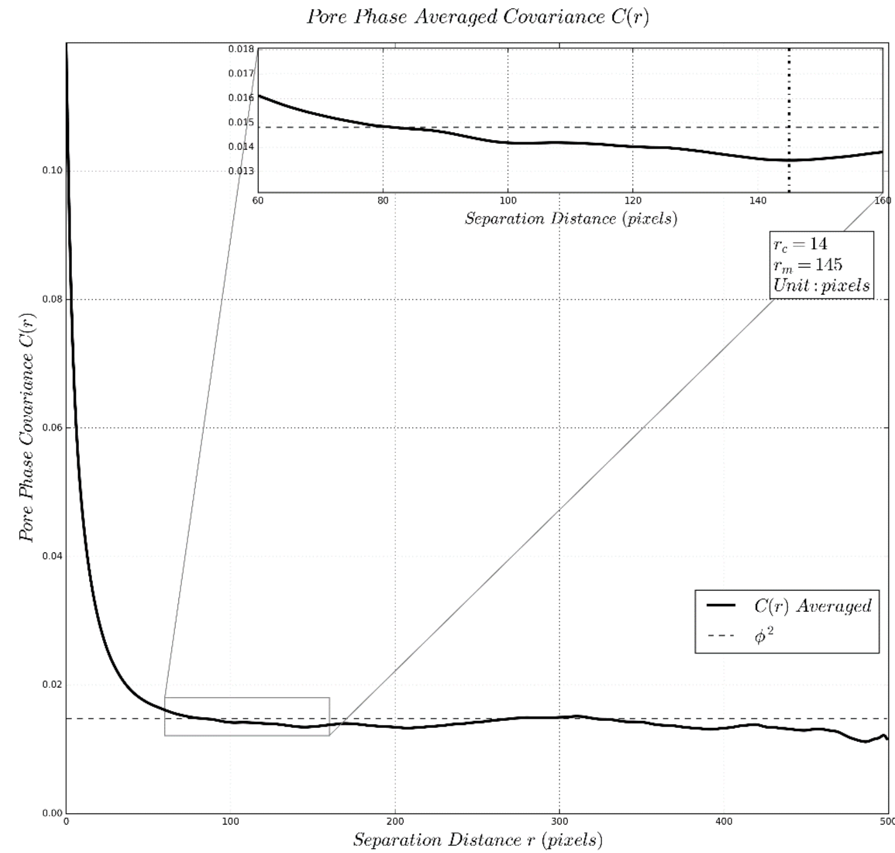
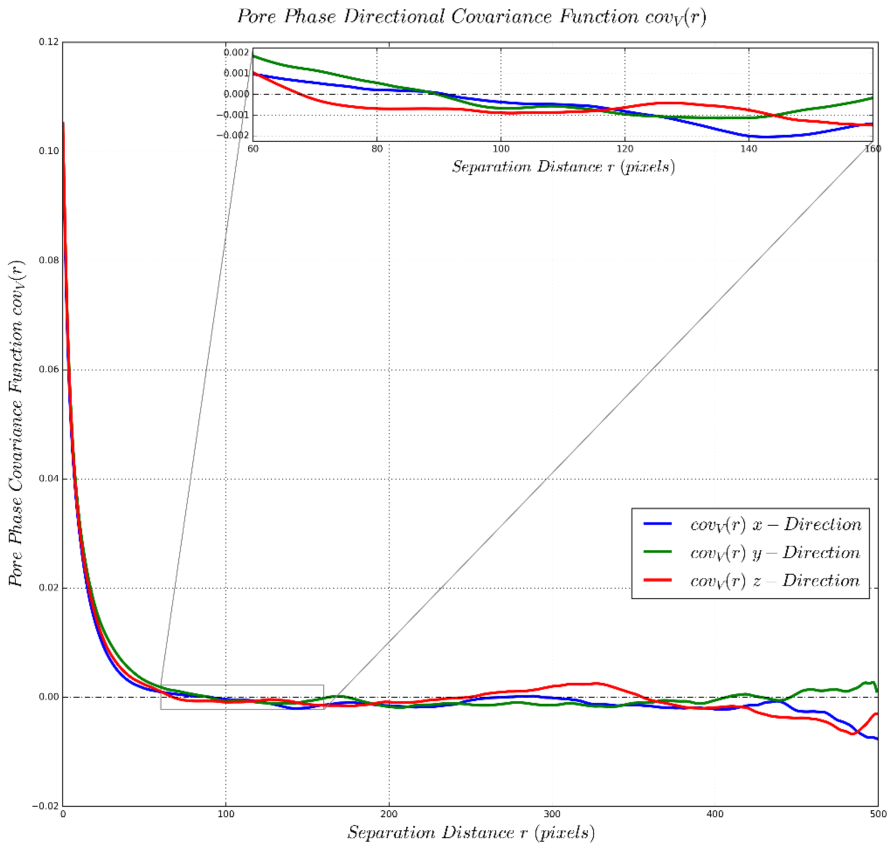
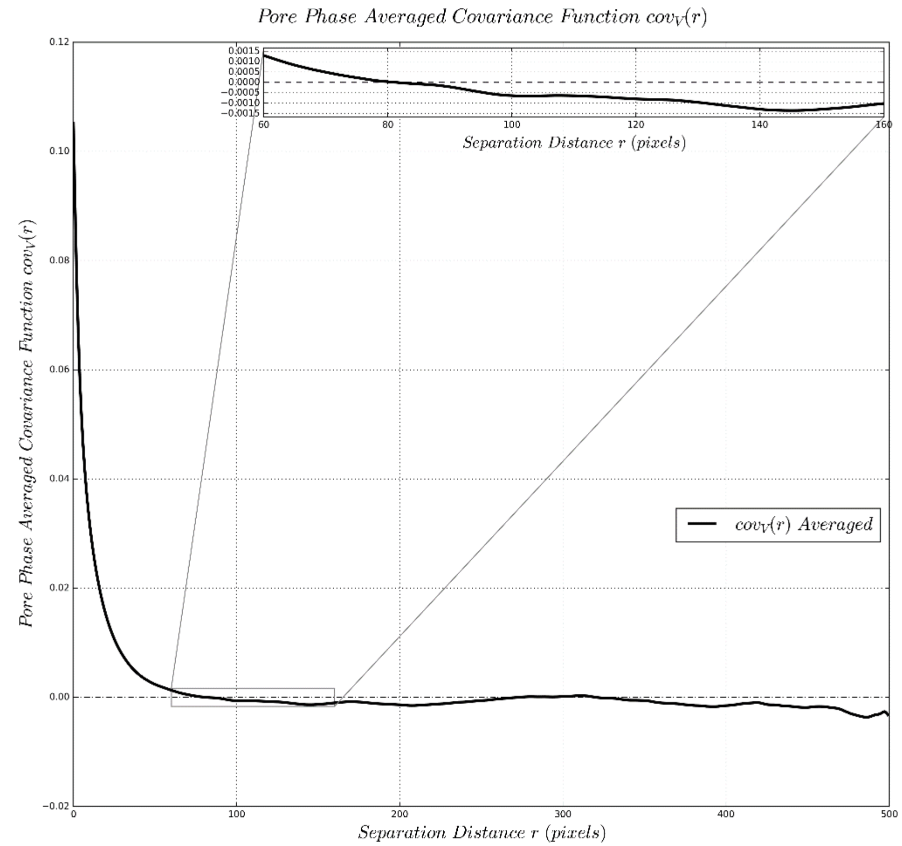
Estaillades – Covariance





Estaillades – Covariance Function





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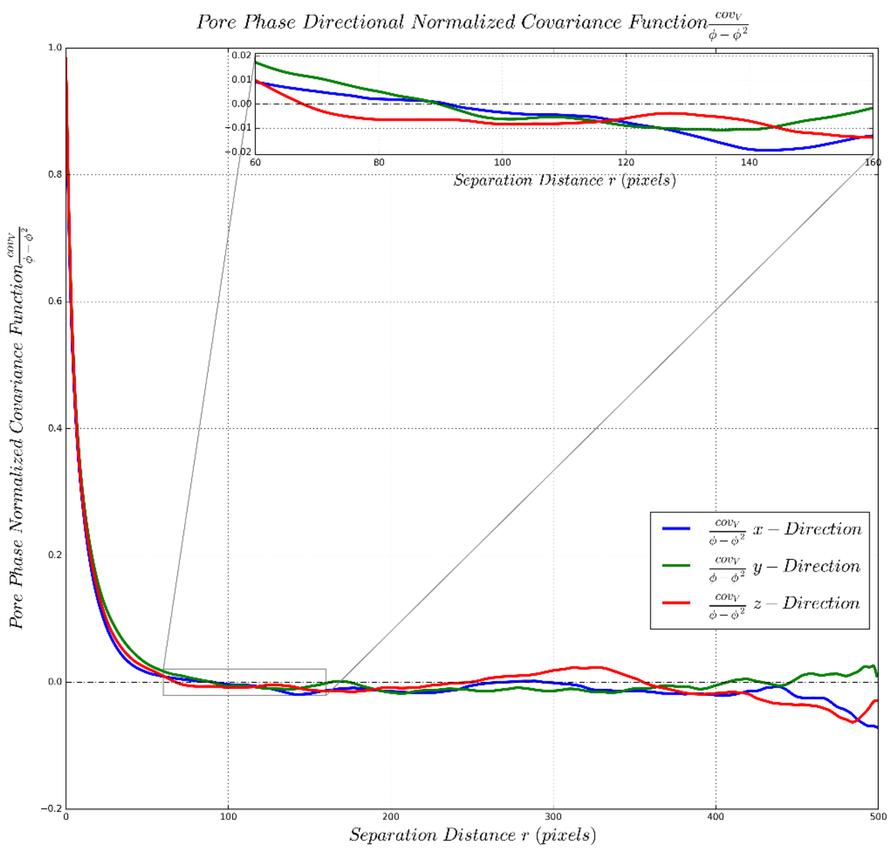
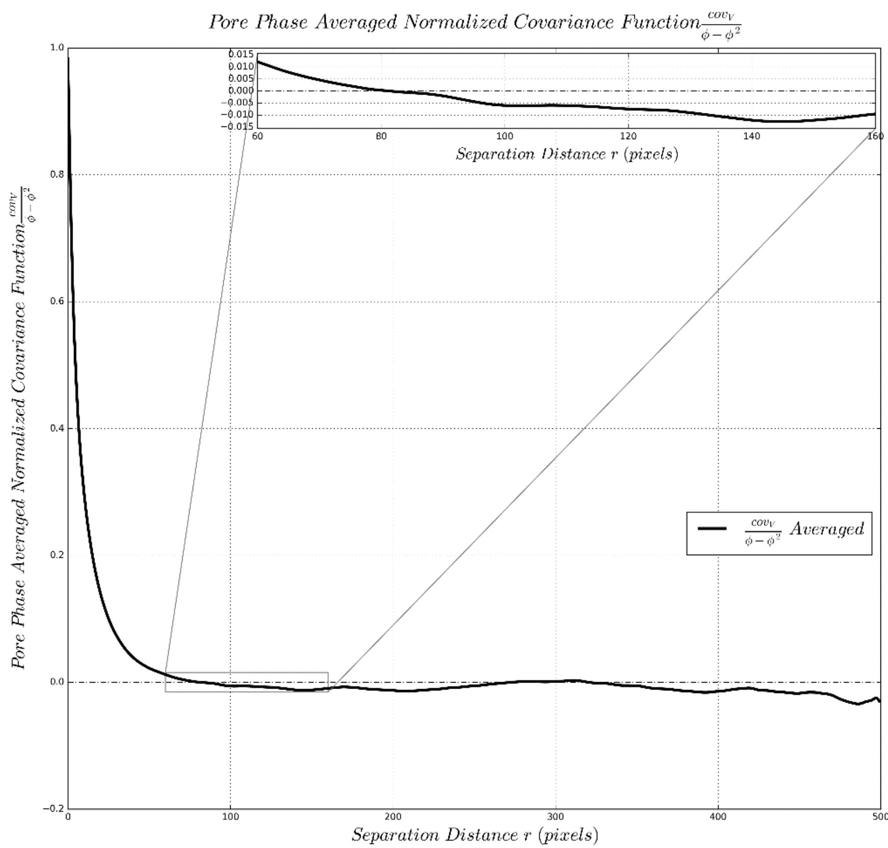
Estaillades – Normalized Covariance Function





Doddington – Covariance Analysis

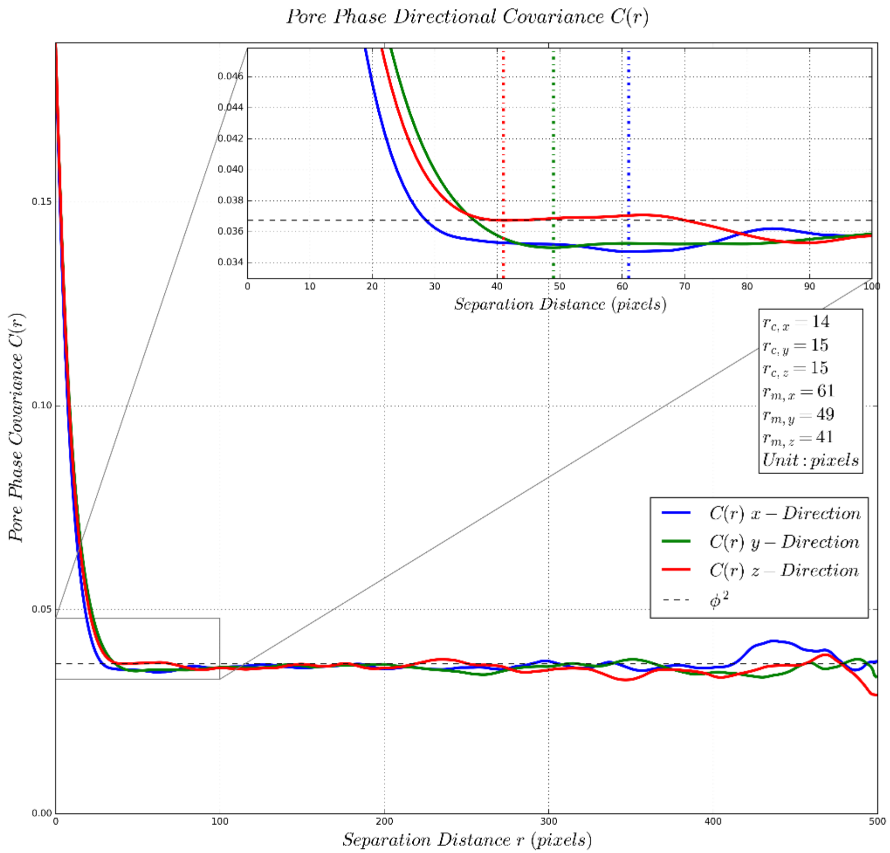
$Voxel\ size\ 3.31\mu m/voxel$

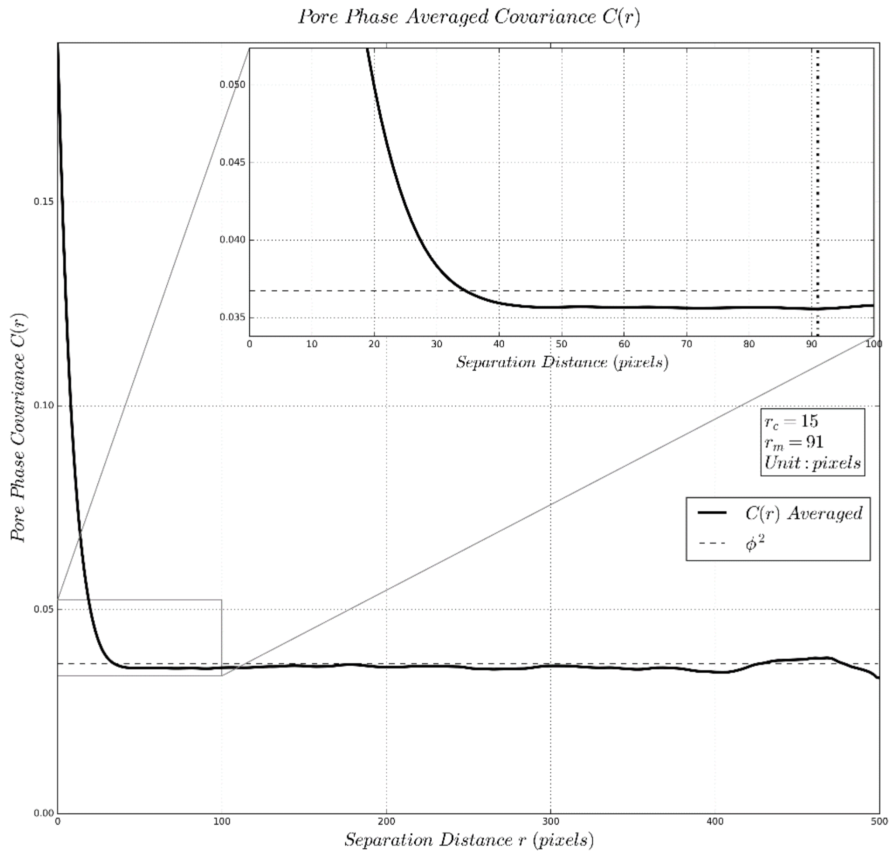
$Image\ Size: 500\ voxel^3$

Mean Grain Size r_m		
	pixels	μm
$x - Direction$	61	201
$y - Direction$	49	162
$z - Direction$	41	135
$Averaged$	91	301
$Ratio$	0.67	

Char. Pore Size r_c		
	pixels	μm
$x - Direction$	14	46
$y - Direction$	15	49
$z - Direction$	15	49
$Averaged$	15	49
$Ratio$	0.93	

Doddington – Covariance





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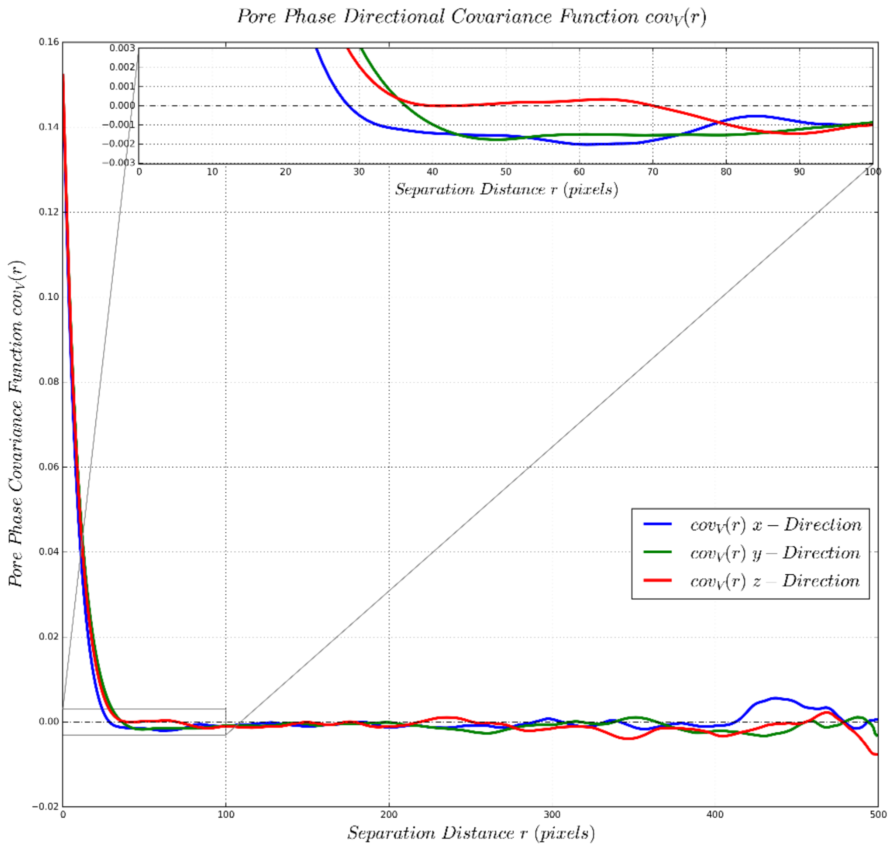
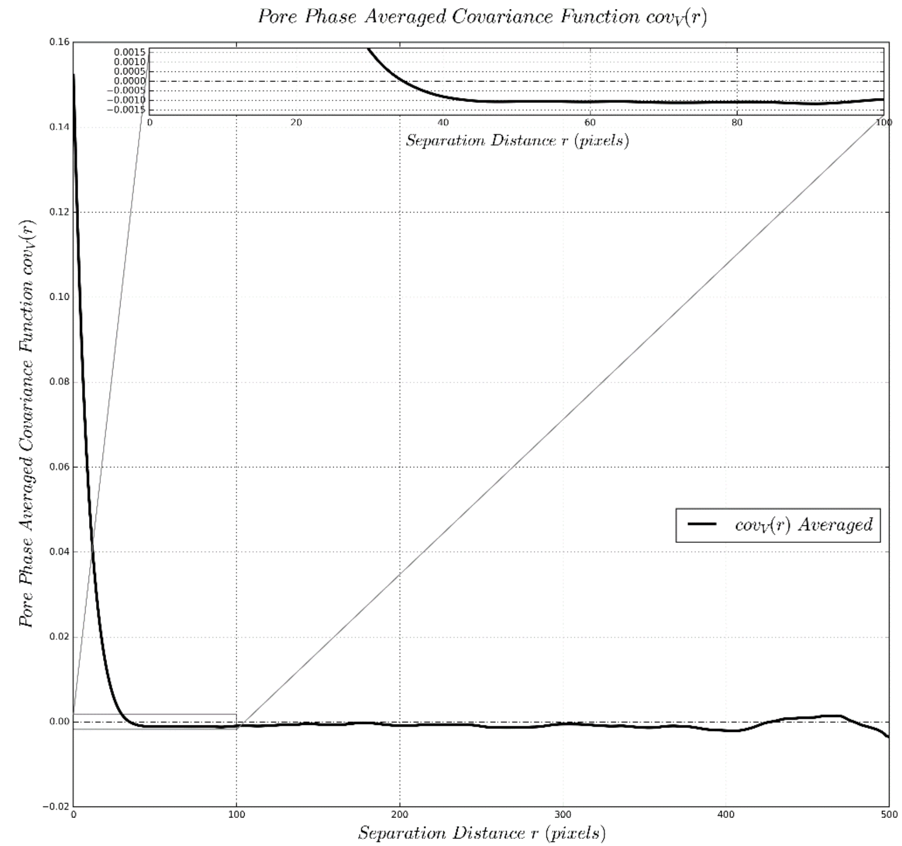
Doddington – Covariance Function





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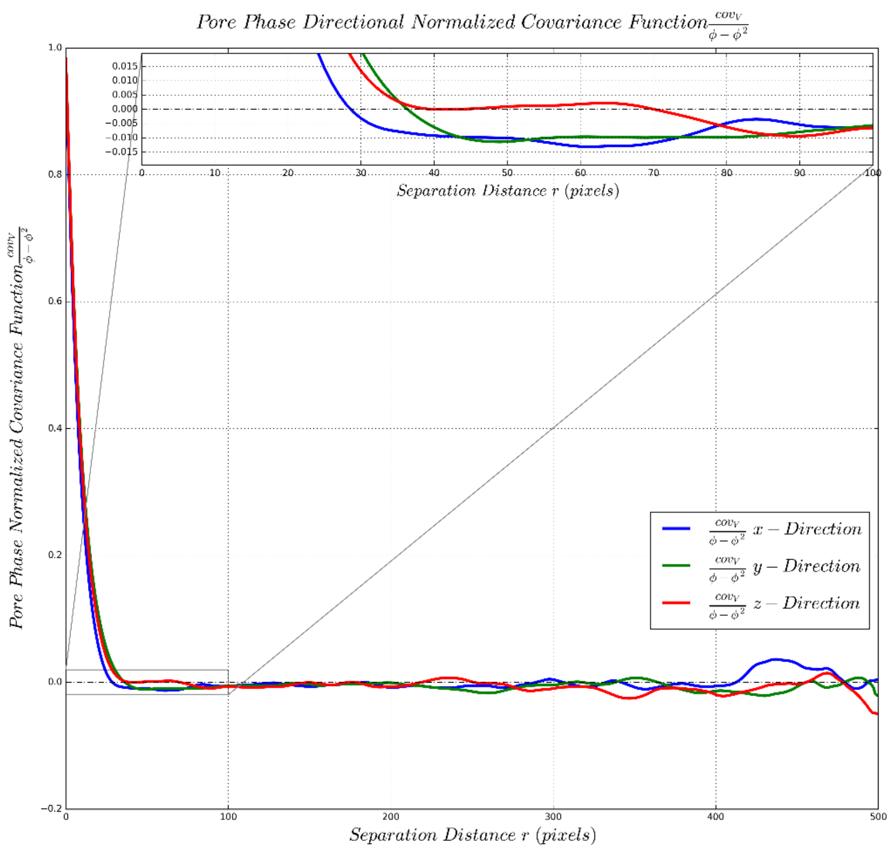
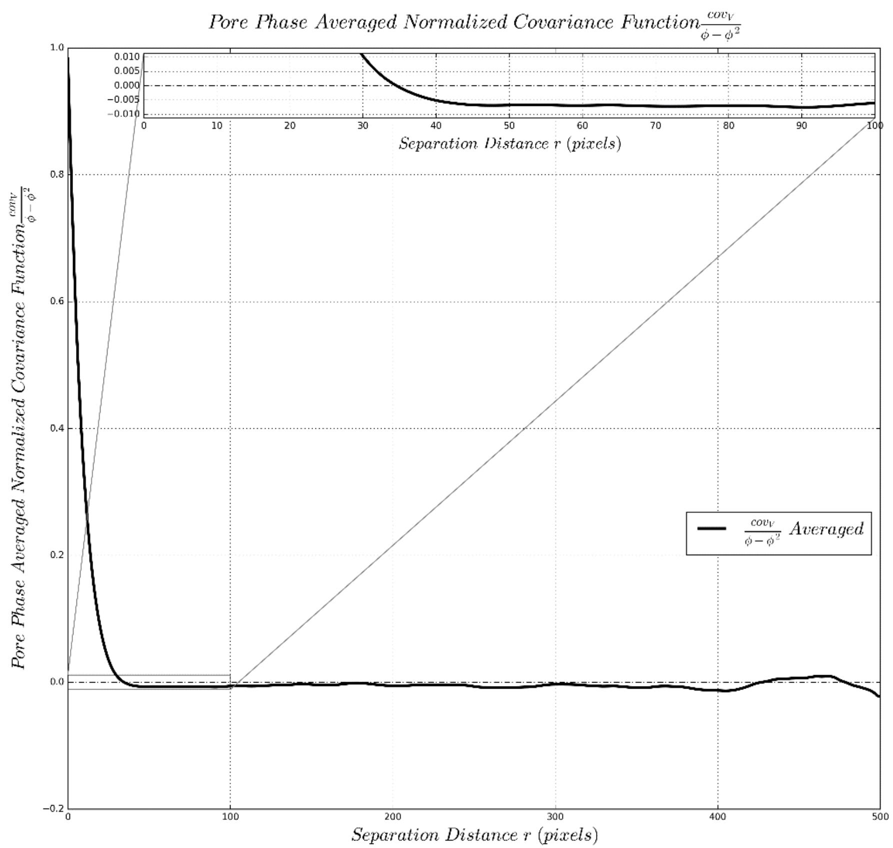
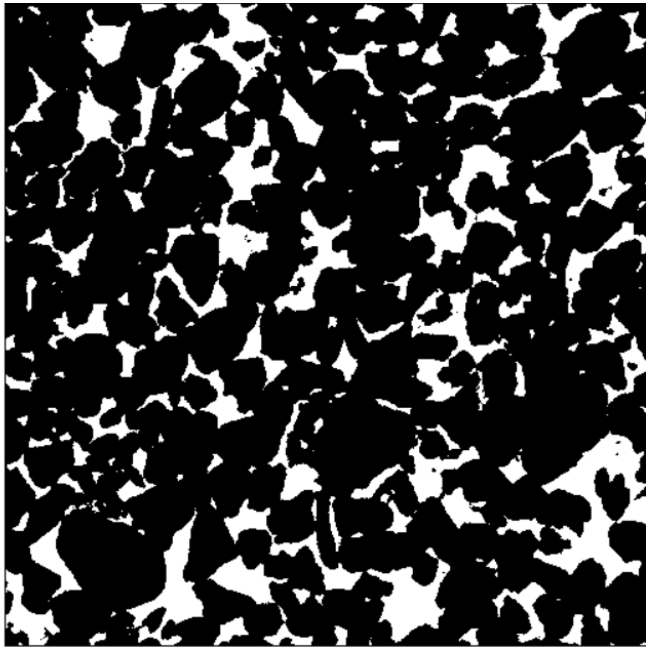
Doddington – Normalized Covariance Function





Bentheimer – Covariance Analysis



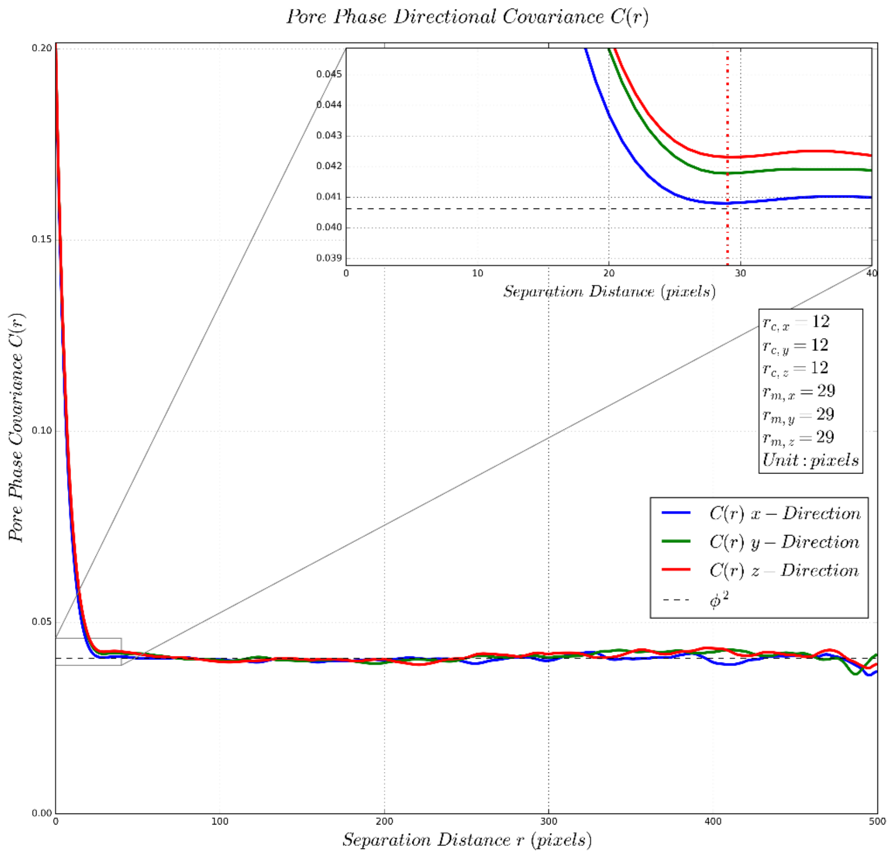
$Voxel\ size\ 3.5\ \mu m/voxel$

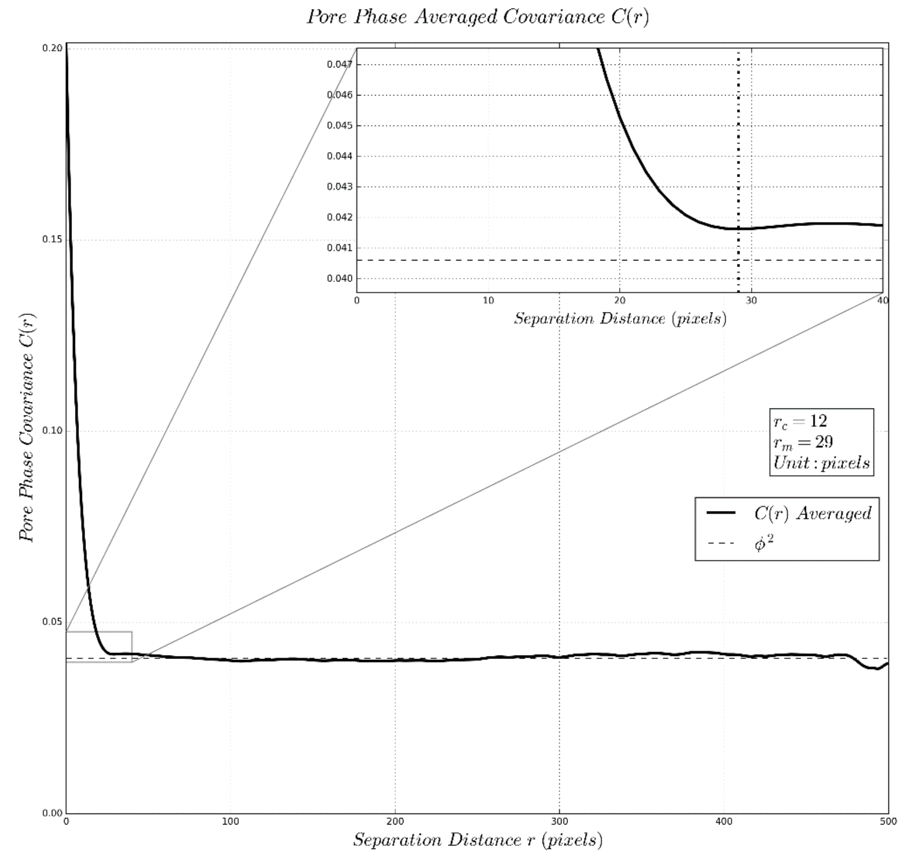
$Image\ Size: 500\ voxel^3$

Mean Grain Size r_m		
	pixels	μm
$x - Direction$	29	97
$y - Direction$	29	97
$z - Direction$	29	97
$Averaged$	29	97
$Ratio$	1.0	

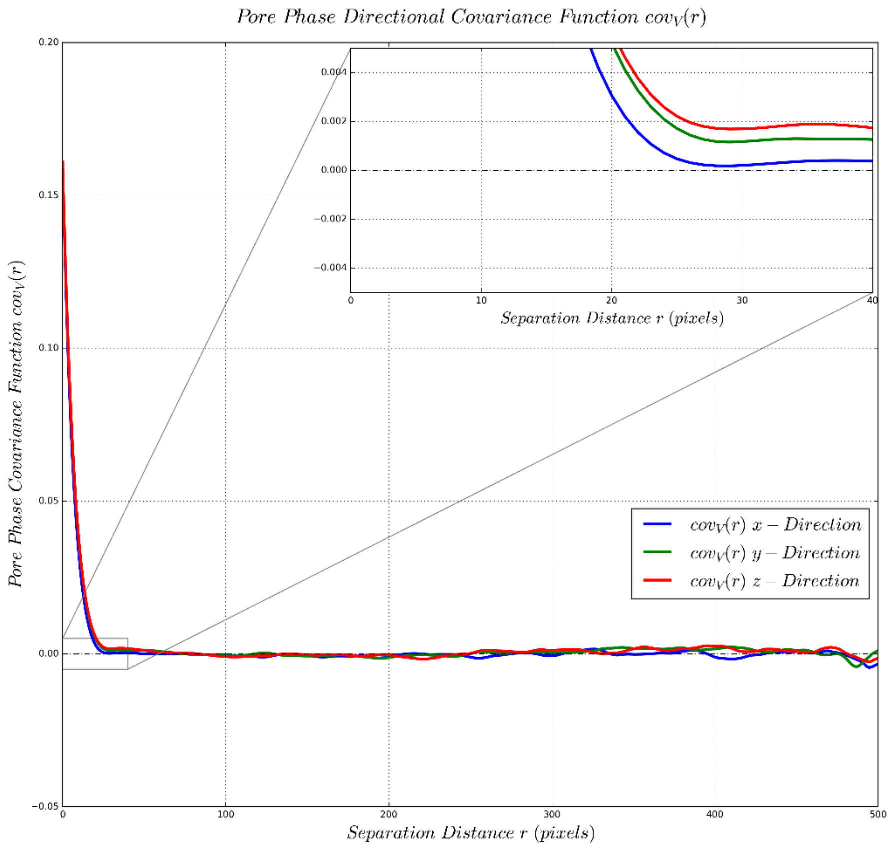
Char. Pore Size r_c		
	pixels	μm
$x - Direction$	12	40
$y - Direction$	12	40
$z - Direction$	12	40
$Averaged$	12	40
$Ratio$	1.0	

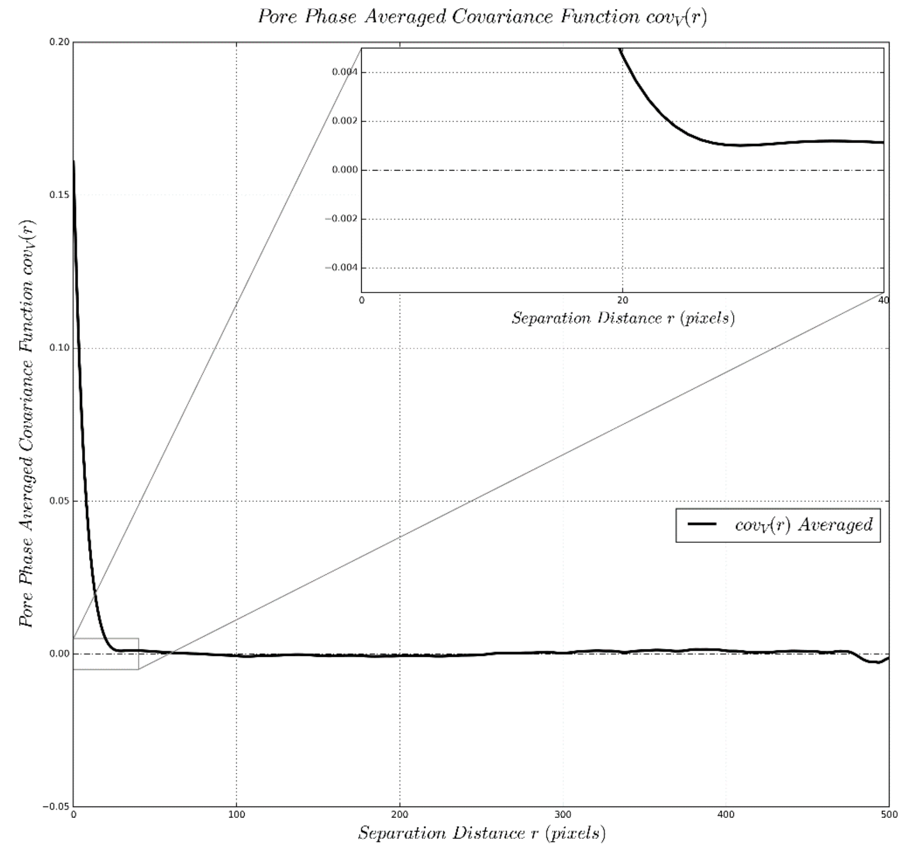
Bentheimer – Covariance





Bentheimer – Covariance Function





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Bentheimer – Normalized Covariance Function