

Statistical Characterisation of Porous Media at the Pore Scale

Covariance, Minkowski Tensors and Permeability Samples S1-S9,C1- C2, Berea

Lukas Mosser

Student:

Supervisors:

Professor Olivier Dubrule

Professor Martin Blunt

Presentation Outline

- Summary of Results
- Comparison Permeability vs. W102, W202
- Comparison Formation Factor vs. W102, W202
- Covariance, MT Plot and Permeability
 - Sandstone Samples S1-S9
 - Carbonate Samples C1 and C2
 - Berea
- Covariance and MT Plot
 - Beadpack, Ketton, Estaillades, Doddington, Bentheimer

Summary of Results

- 1. Attempt at correlating Minkowski Tensors to Permeability
 - For most samples:
 - Magnitude of permeability in Cartesian directions decreases with increase in Minkowski Tensor Components
 - Direction K highest > Direction W102 lowest
 - Direction K lowest -> Direction W102 highest
 - Individual Cases:
 - Weak Discrepancy:
 - Permeability values on 2 axis very similar
 - -> W102 very similar
 - Samples S6, S7, S8
 - Strong discrepancy: Completely different orientation of W102
 - Sample S3: Possibly wrong computation of K Tensor?
 - Sample C1
- 2. Most Sandstone Samples S1-S9 show anisotropy from Minkowski Tensor
 - Only Berea close to isotropic
- 3. REV estimation:
 - Sandstone Sample S6 and Carbonate C2:
 - Upwards trend of MT Plot -> REV not reached
- 4. Comparison Covariance and Minkowski Tensor Plot
 - Sample S4: X and Y Covariance very similar trend, Z direction different
 - Minkowski Tensor indicates Anisotropy in Z direction

Sandstone Samples S1-S5

	$\overline{\overline{k}}$ (md)	$ \widehat{W}_1^{0,2} $	$\widehat{W}_{2}^{0,2}$
S1	$\begin{bmatrix} 1969 & 0 & 0 \\ 0 & 1752 & 0 \\ 0 & 0 & 1312 \end{bmatrix}$	$\begin{bmatrix} 0.286 & 0 & 0 \\ 0 & 0.294 & 0 \\ 0 & 0 & 0.419 \end{bmatrix}$	$\begin{bmatrix} 0.312 & 0 & 0 \\ 0 & 0.350 & 0 \\ 0 & 0 & 0.337 \end{bmatrix}$
S2	$\begin{bmatrix} 4318 & 0 & 0 \\ 0 & 3983 & 0 \\ 0 & 0 & 3394 \end{bmatrix}$	$\begin{bmatrix} 0.298 & 0 & 0 \\ 0 & 0.302 & 0 \\ 0 & 0 & 0.398 \end{bmatrix}$	$\begin{bmatrix} 0.314 & 0 & 0 \\ 0 & 0.305 & 0 \\ 0 & 0 & 0.379 \end{bmatrix}$
S3	$\begin{bmatrix} 143 & 0 & 0 \\ 0 & 420 & 0 \\ 0 & 0 & 109 \end{bmatrix}$	$\begin{bmatrix} 0.2761 & 0 & 0 \\ 0 & 0.2762 & 0 \\ 0 & 0 & 0.447 \end{bmatrix}$	$\begin{bmatrix} 0.309 & 0 & 0 \\ 0 & 0.307 & 0 \\ 0 & 0 & 0.383 \end{bmatrix}$
S4	$\begin{bmatrix} 273 & 0 & 0 \\ 0 & 289 & 0 \\ 0 & 0 & 215 \end{bmatrix}$	$\begin{bmatrix} 0.283 & 0 & 0 \\ 0 & 0.269 & 0 \\ 0 & 0 & 0.446 \end{bmatrix}$	$\begin{bmatrix} 0.313 & 0 & 0 \\ 0 & 0.306 & 0 \\ 0 & 0 & 0.379 \end{bmatrix}$
S5	$\begin{bmatrix} 4638 & 0 & 0 \\ 0 & 4874 & 0 \\ 0 & 0 & 4440 \end{bmatrix}$	$\begin{bmatrix} 0.303 & 0 & 0 \\ 0 & 0.301 & 0 \\ 0 & 0 & 0.395 \end{bmatrix}$	$\begin{bmatrix} 0.310 & 0 & 0 \\ 0 & 0.315 & 0 \\ 0 & 0 & 0.374 \end{bmatrix}$

Strong correlation

Weak correlation

Permeability - Sandstone Samples S6-S9

	$\overline{\overline{k}}$ (md)	$\widehat{W}_{\!\scriptscriptstyle 1}^{0,2}$	$\widehat{W}_{2}^{0,2}$
S6	$\begin{bmatrix} 11289 & 0 & 0 \\ 0 & 10683 & 0 \\ 0 & 0 & 10951 \end{bmatrix}$	$\begin{bmatrix} 0.296 & 0 & 0 \\ 0 & 0.306 & 0 \\ 0 & 0 & 0.397 \end{bmatrix}$	$\begin{bmatrix} 0.322 & 0 & 0 \\ 0 & 0.329 & 0 \\ 0 & 0 & 0.347 \end{bmatrix}$
S7	$\begin{bmatrix} 7268 & 0 & 0 \\ 0 & 7594 & 0 \\ 0 & 0 & 6037 \end{bmatrix}$	$\begin{bmatrix} 0.300 & 0 & 0 \\ 0 & 0.302 & 0 \\ 0 & 0 & 0.396 \end{bmatrix}$	$\begin{bmatrix} 0.303 & 0 & 0 \\ 0 & 0.322 & 0 \\ 0 & 0 & 0.374 \end{bmatrix}$
S8	$\begin{bmatrix} 13063 & 0 & 0 \\ 0 & 13507 & 0 \\ 0 & 0 & 12936 \end{bmatrix}$	$\begin{bmatrix} 0.287 & 0 & 0 \\ 0 & 0.288 & 0 \\ 0 & 0 & 0.423 \end{bmatrix}$	$\begin{bmatrix} 0.259 & 0 & 0 \\ 0 & 0.289 & 0 \\ 0 & 0 & 0.451 \end{bmatrix}$
S 9	$\begin{bmatrix} 2735 & 0 & 0 \\ 0 & 2093 & 0 \\ 0 & 0 & 1844 \end{bmatrix}$	$\begin{bmatrix} 0.300 & 0 & 0 \\ 0 & 0.359 & 0 \\ 0 & 0 & 0.339 \end{bmatrix}$	$\begin{bmatrix} 0.312 & 0 & 0 \\ 0 & 0.350 & 0 \\ 0 & 0 & 0.337 \end{bmatrix}$

Strong correlation

Weak correlation

Permeability - Samples C1, C2, Berea

$\overline{\overline{k}}$ (md)	$\widehat{W}_{1}^{0,2}$ $\widehat{W}_{2}^{0,2}$
$ \begin{array}{c cccc} \mathbf{C1} & \begin{bmatrix} 785 & 0 & 0 \\ 0 & 1469 & 0 \\ 0 & 0 & 1053 \end{bmatrix} \end{array} $	$\begin{bmatrix} 0.284 & 0 & 0 \\ 0 & 0.263 & 0 \\ 0 & 0 & 0.452 \end{bmatrix} \begin{bmatrix} 0.305 & 0 & 0 \\ 0 & 0.329 & 0 \\ 0 & 0 & 0.347 \end{bmatrix}$
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{bmatrix} 0.291 & 0 & 0 \\ 0 & 0.285 & 0 \\ 0 & 0 & 0.423 \end{bmatrix} \begin{bmatrix} 0.310 & 0 & 0 \\ 0 & 0.290 & 0 \\ 0 & 0 & 0.403 \end{bmatrix}$
Berea $\begin{bmatrix} 1360 & 0 & 0 \\ 0 & 1304 & 0 \\ 0 & 0 & 1193 \end{bmatrix}$	$\begin{bmatrix} 0.320 & 0 & 0 \\ 0 & 0.335 & 0 \\ 0 & 0 & 0.344 \end{bmatrix} \begin{bmatrix} 0.322 & 0 & 0 \\ 0 & 0.328 & 0 \\ 0 & 0 & 0.348 \end{bmatrix}$

Strong correlation

Weak correlation



Formation Factors - Sandstone Samples S1-S5

	FF	$\widehat{W}_1^{0,2}$	$\widehat{W}_{2}^{0,2}$
S1	$\begin{bmatrix} 36.3 & 0 & 0 \\ 0 & 37.9 & 0 \\ 0 & 0 & 53.5 \end{bmatrix}$	$\begin{bmatrix} 0.286 & 0 & 0 \\ 0 & 0.294 & 0 \\ 0 & 0 & 0.419 \end{bmatrix}$	$\begin{bmatrix} 0.312 & 0 & 0 \\ 0 & 0.350 & 0 \\ 0 & 0 & 0.337 \end{bmatrix}$
S2	$\begin{bmatrix} 10.9 & 0 & 0 \\ 0 & 11.3 & 0 \\ 0 & 0 & 12.6 \end{bmatrix}$	$\begin{bmatrix} 0.298 & 0 & 0 \\ 0 & 0.302 & 0 \\ 0 & 0 & 0.398 \end{bmatrix}$	$\begin{bmatrix} 0.314 & 0 & 0 \\ 0 & 0.305 & 0 \\ 0 & 0 & 0.379 \end{bmatrix}$
S3	$\begin{bmatrix} 52.2 & 0 & 0 \\ 0 & 41.9 & 0 \\ 0 & 0 & 70.3 \end{bmatrix}$	$\begin{bmatrix} 0.2761 & 0 & 0 \\ 0 & 0.2762 & 0 \\ 0 & 0 & 0.447 \end{bmatrix}$	$\begin{bmatrix} 0.309 & 0 & 0 \\ 0 & 0.307 & 0 \\ 0 & 0 & 0.383 \end{bmatrix}$
S4	$\begin{bmatrix} 74.5 & 0 & 0 \\ 0 & 71.0 & 0 \\ 0 & 0 & 114.4 \end{bmatrix}$	$\begin{bmatrix} 0.283 & 0 & 0 \\ 0 & 0.269 & 0 \\ 0 & 0 & 0.446 \end{bmatrix}$	$\begin{bmatrix} 0.313 & 0 & 0 \\ 0 & 0.306 & 0 \\ 0 & 0 & 0.379 \end{bmatrix}$
S5	$\begin{bmatrix} 14.1 & 0 & 0 \\ 0 & 14.6 & 0 \\ 0 & 0 & 15.9 \end{bmatrix}$	$\begin{bmatrix} 0.303 & 0 & 0 \\ 0 & 0.301 & 0 \\ 0 & 0 & 0.395 \end{bmatrix}$	$\begin{bmatrix} 0.310 & 0 & 0 \\ 0 & 0.315 & 0 \\ 0 & 0 & 0.374 \end{bmatrix}$

Strong correlation

Weak correlation

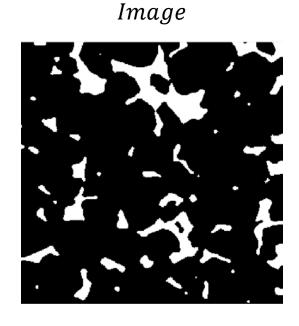
Formation Factors - Sandstone Samples S6-S9

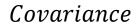
	FF	$\widehat{W}_{\!\scriptscriptstyle 1}^{0,2}$	$\widehat{W}_{2}^{0,2}$
S6	$\begin{bmatrix} 10.3 & 0 & 0 \\ 0 & 11.2 & 0 \\ 0 & 0 & 12.9 \end{bmatrix}$	$\begin{bmatrix} 0.296 & 0 & 0 \\ 0 & 0.306 & 0 \\ 0 & 0 & 0.397 \end{bmatrix}$	$\begin{bmatrix} 0.322 & 0 & 0 \\ 0 & 0.329 & 0 \\ 0 & 0 & 0.347 \end{bmatrix}$
S7	$\begin{bmatrix} 9.6 & 0 & 0 \\ 0 & 9.3 & 0 \\ 0 & 0 & 11.2 \end{bmatrix}$	$\begin{bmatrix} 0.300 & 0 & 0 \\ 0 & 0.302 & 0 \\ 0 & 0 & 0.396 \end{bmatrix}$	$\begin{bmatrix} 0.303 & 0 & 0 \\ 0 & 0.322 & 0 \\ 0 & 0 & 0.374 \end{bmatrix}$
S8	$\begin{bmatrix} 6.02 & 0 & 0 \\ 0 & 5.97 & 0 \\ 0 & 0 & 6.53 \end{bmatrix}$	$\begin{bmatrix} 0.287 & 0 & 0 \\ 0 & 0.288 & 0 \\ 0 & 0 & 0.423 \end{bmatrix}$	$\begin{bmatrix} 0.259 & 0 & 0 \\ 0 & 0.289 & 0 \\ 0 & 0 & 0.451 \end{bmatrix}$
S9	$\begin{bmatrix} 16.4 & 0 & 0 \\ 0 & 19.9 & 0 \\ 0 & 0 & 18.1 \end{bmatrix}$	$\begin{bmatrix} 0.300 & 0 & 0 \\ 0 & 0.359 & 0 \\ 0 & 0 & 0.339 \end{bmatrix}$	$\begin{bmatrix} 0.312 & 0 & 0 \\ 0 & 0.350 & 0 \\ 0 & 0 & 0.337 \end{bmatrix}$

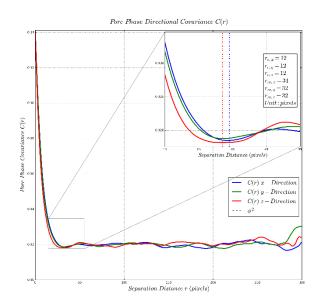
Formation Factors - Samples C1, C2, Berea

	FF	$\widehat{W}_{\!\scriptscriptstyle 1}^{0,2}$	$\widehat{W}_2^{0,2}$
C1	$\begin{bmatrix} 33.7 & 0 & 0 \\ 0 & 21.9 & 0 \\ 0 & 0 & 27.2 \end{bmatrix}$	$\begin{bmatrix} 0.284 & 0 & 0 \\ 0 & 0.263 & 0 \\ 0 & 0 & 0.452 \end{bmatrix}$	$\begin{bmatrix} 0.305 & 0 & 0 \\ 0 & 0.329 & 0 \\ 0 & 0 & 0.347 \end{bmatrix}$
C2	$\begin{bmatrix} 18 & 0 & 0 \\ 0 & 121 & 0 \\ 0 & 0 & 249 \end{bmatrix}$	$\begin{bmatrix} 0.291 & 0 & 0 \\ 0 & 0.285 & 0 \\ 0 & 0 & 0.423 \end{bmatrix}$	$\begin{bmatrix} 0.310 & 0 & 0 \\ 0 & 0.290 & 0 \\ 0 & 0 & 0.403 \end{bmatrix}$
Berea	$\begin{bmatrix} 23.1 & 0 & 0 \\ 0 & 24.0 & 0 \\ 0 & 0 & 25.2 \end{bmatrix}$	$\begin{bmatrix} 0.320 & 0 & 0 \\ 0 & 0.335 & 0 \\ 0 & 0 & 0.344 \end{bmatrix}$	$\begin{bmatrix} 0.322 & 0 & 0 \\ 0 & 0.328 & 0 \\ 0 & 0 & 0.348 \end{bmatrix}$

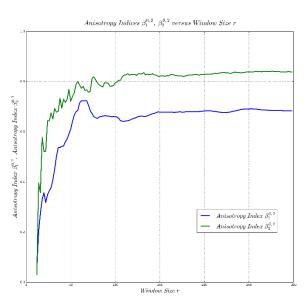








Minkowski Tensor

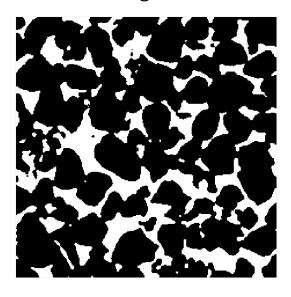


$$\begin{bmatrix} 1969 & 0 & 0 \\ 0 & 1752 & 0 \\ 0 & 0 & 1312 \end{bmatrix}$$

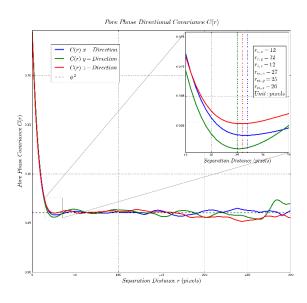
$$\begin{array}{cccc}
\widehat{W}_{1}^{0,2} \\
0.286 & 0 & 0 \\
0 & 0.294 & 0 \\
0 & 0 & 0.419
\end{array}$$

$\widehat{W}_{\!2}^{0,2}$				
0.312	0	0]		
0	0.350	0		
L 0	0	0.337		

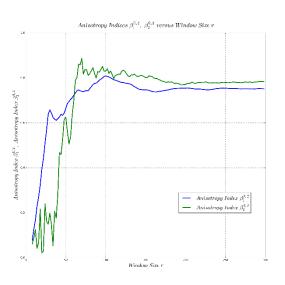
Image



Covariance



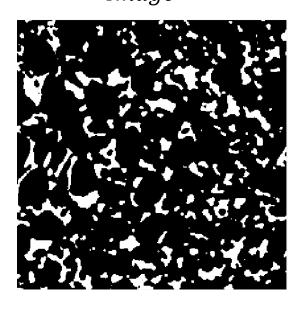
Minkowski Tensor



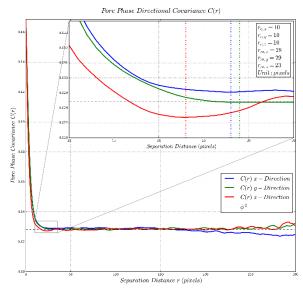
$$\begin{bmatrix} 4318 & 0 & 0 \\ 0 & 3983 & 0 \\ 0 & 0 & 3394 \end{bmatrix}$$

$$\begin{bmatrix} 0.298 & 0 & 0 \\ 0 & 0.302 & 0 \\ 0 & 0 & 0.398 \end{bmatrix}$$





Covariance



Minkowski Tensor

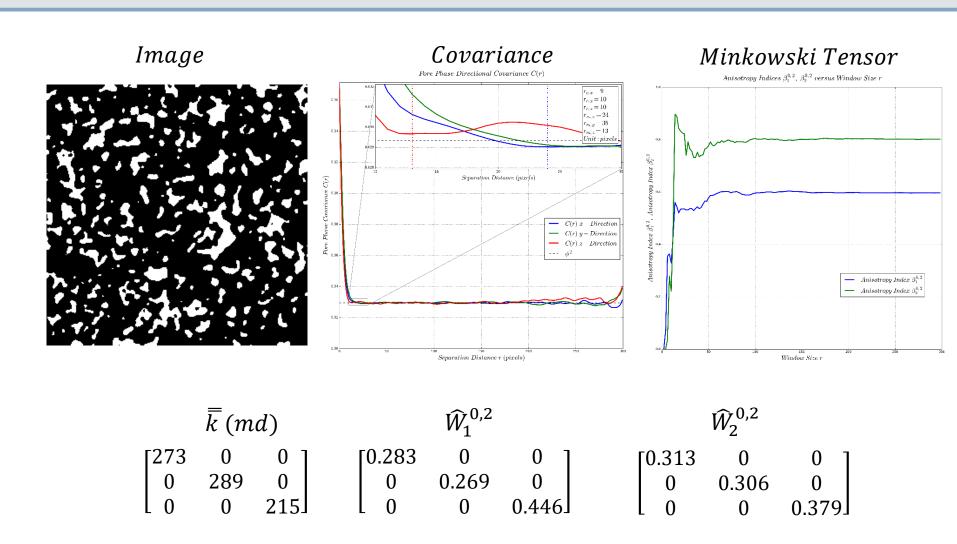
Anisotropy Indices $\beta_1^{0,2}$, $\beta_2^{0,2}$ versus Window Size r $- Anisotropy Index \beta_2^{0,1}$ $- Anisotropy Index \beta_2^{0,1}$ $- Anisotropy Index \beta_2^{0,1}$ $- Anisotropy Index \beta_2^{0,1}$

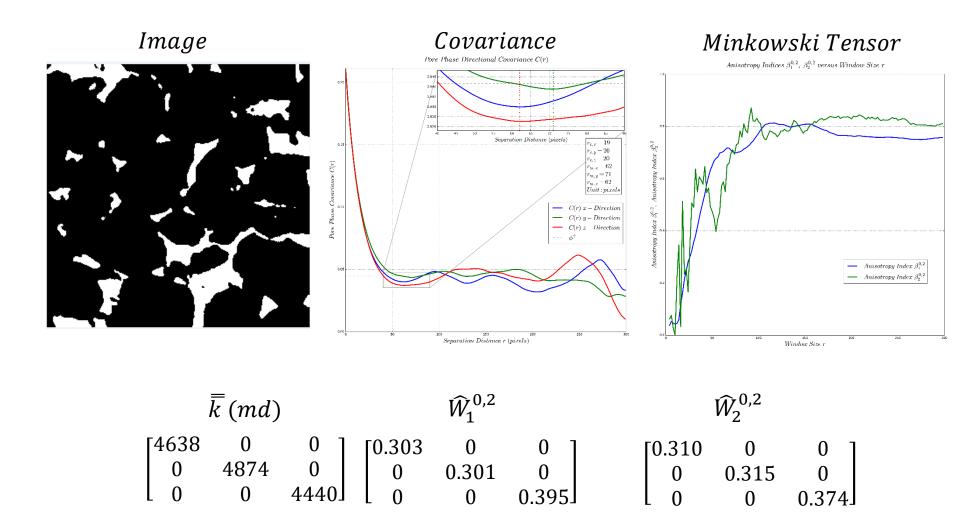
$$ar{k} \ (md)$$

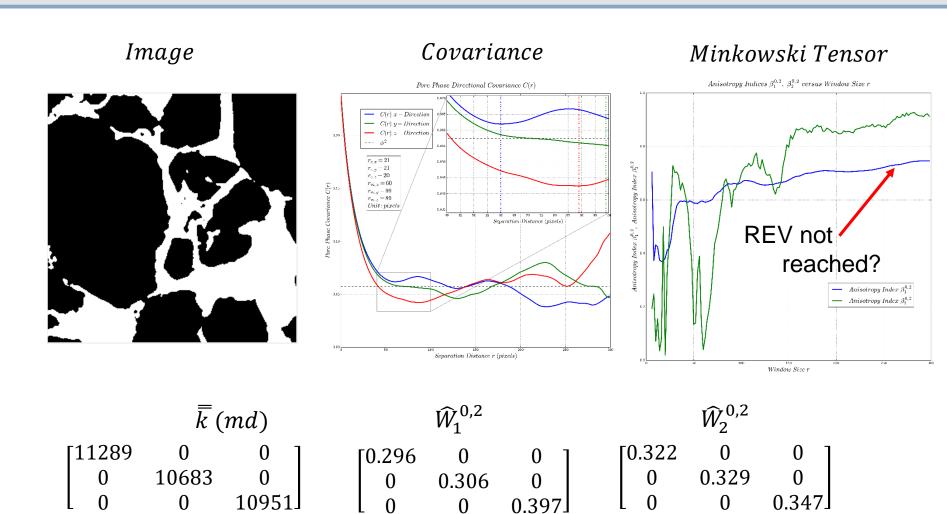
$$\begin{bmatrix} 143 & 0 & 0 \ 0 & 420 & 0 \ 0 & 0 & 109 \end{bmatrix}$$

$$\begin{bmatrix} 0.2761 & 0 & 0 \\ 0 & 0.2762 & 0 \\ 0 & 0 & 0.447 \end{bmatrix}$$

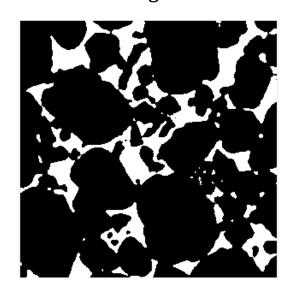
$$\begin{bmatrix} 0.309 & 0 & 0 \\ 0 & 0.307 & 0 \\ 0 & 0 & 0.383 \end{bmatrix}$$



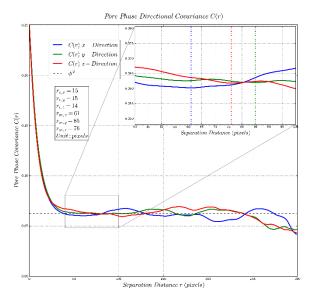








Covariance



Minkowski Tensor

Anisotropy Indices $\beta_1^{0.2}, \beta_2^{0.2}$ versus Window Size r $- Anisotropy Index \beta_1^{0.2}$ $- Anisotropy Index \beta_2^{0.2}$ $- Anisotropy Index \beta_2^{0.2}$

$$\overline{\overline{k}}$$
 (md)

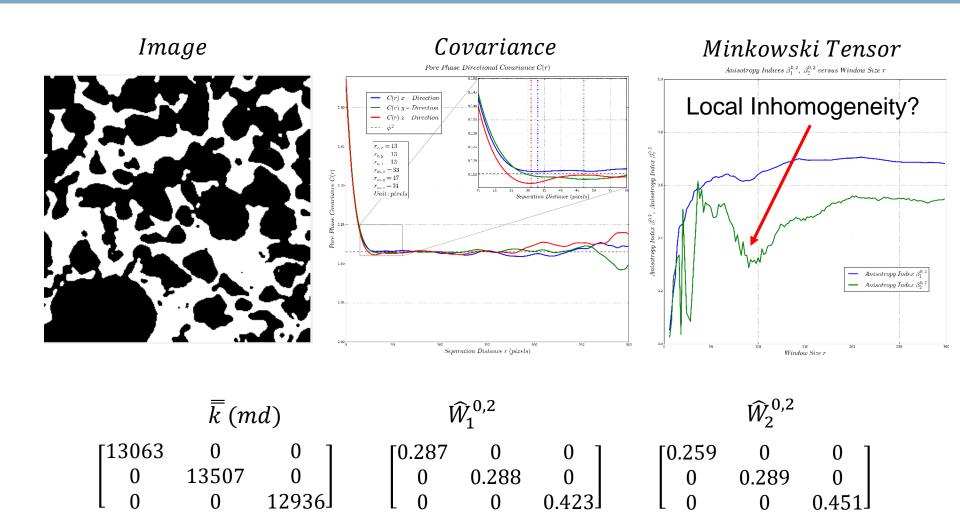
[7 <i>2</i>	268	0	0]
	0	7594	0
L	0	0	6037

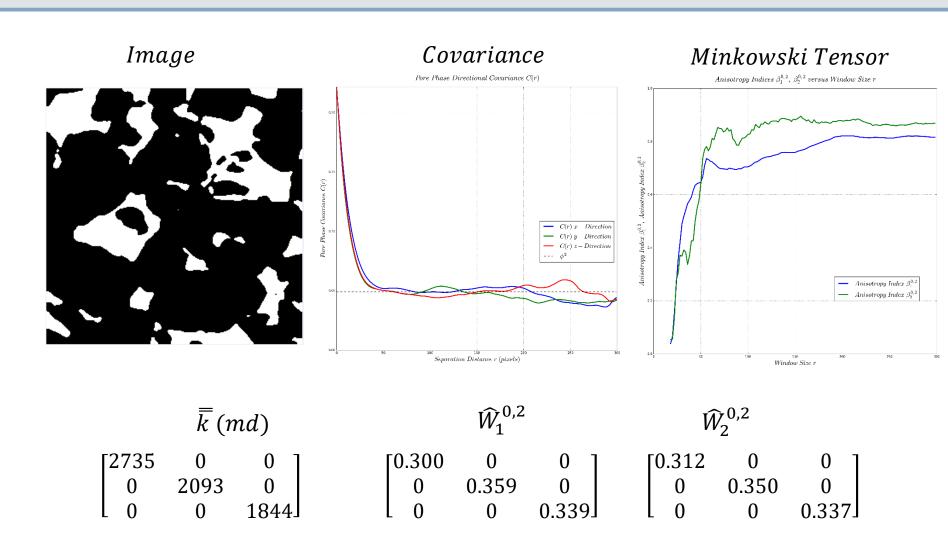
$$\widehat{W}_{1}^{0,2}$$

[0	.300	0	0]
	0	0.302	0
	0	0	0.396

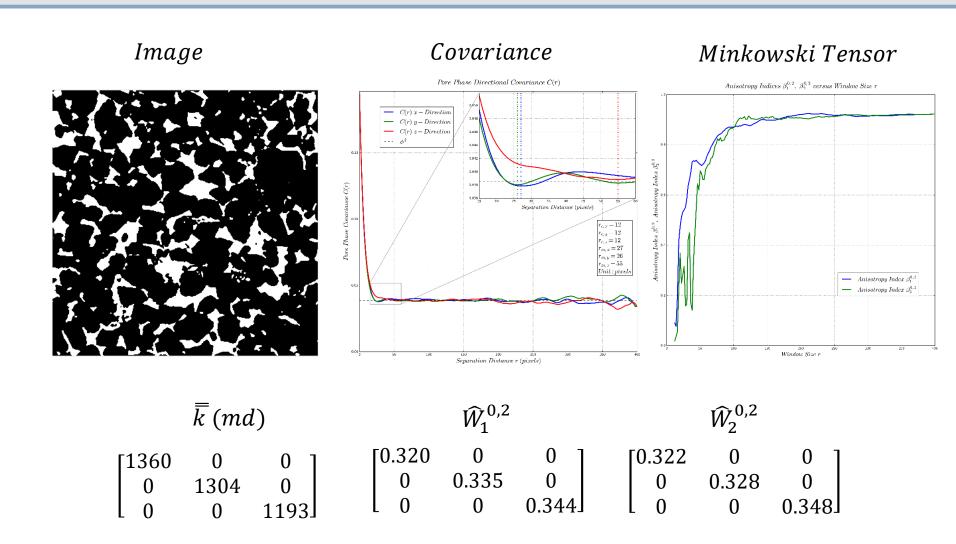
$$\widehat{W}_{2}^{0,2}$$

	77 2	
[0.303	0	0
0	0.322	0
Lo	0	0.374





Sandstone Sample – Berea

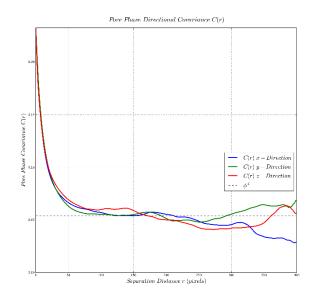


Carbonate Sample - C1

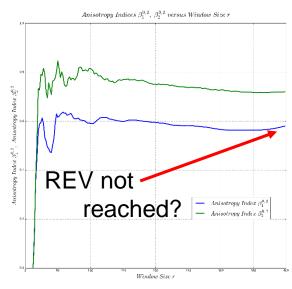
Image



Covariance



Minkowski Tensor



$$\overline{\overline{k}}$$
 (md)

$$\begin{bmatrix} 785 & 0 & 0 \\ 0 & 1469 & 0 \\ 0 & 0 & 1053 \end{bmatrix}$$

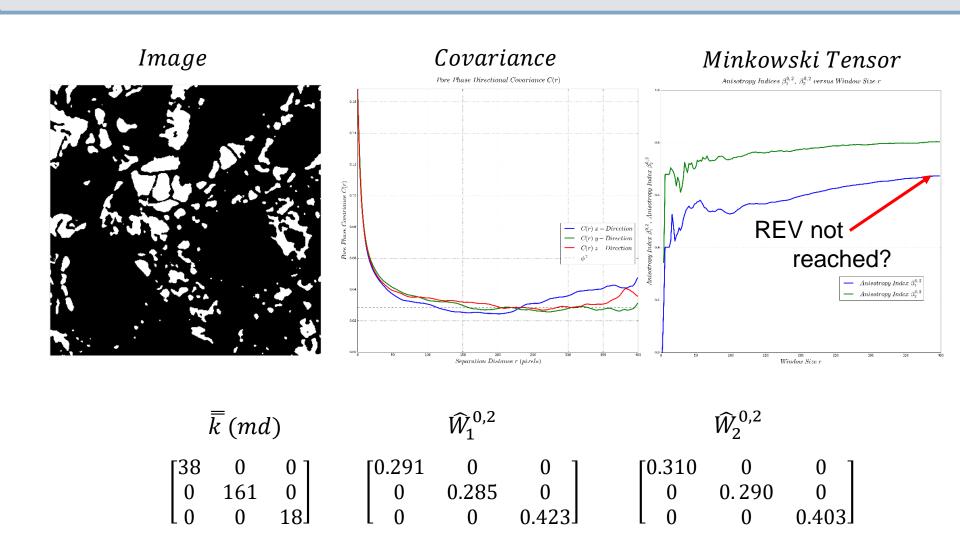
$$\widehat{W}_{1}^{0,}$$

[0	.284	0	0]
	0	0.263	0
L	0	0	0.452

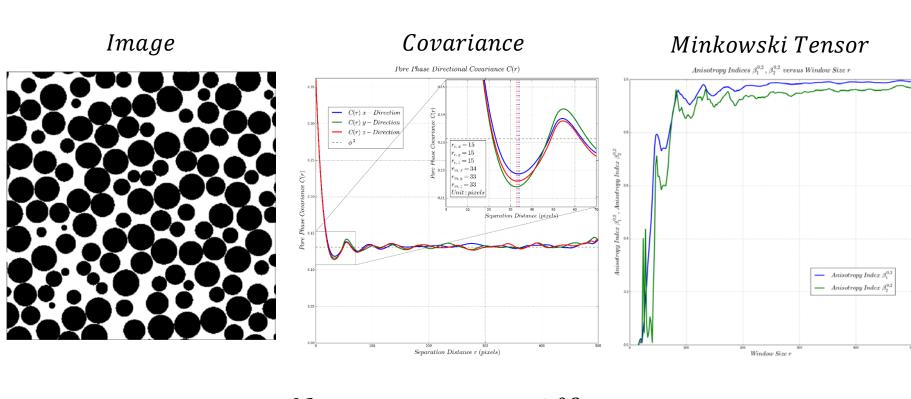
$$\widehat{W}_{2}^{0,2}$$

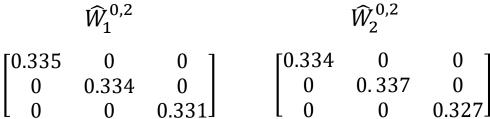
[0.305]	0	0]	
0	0.329	0	
0	0	0.347	

Carbonate Sample – C2



Beadpack

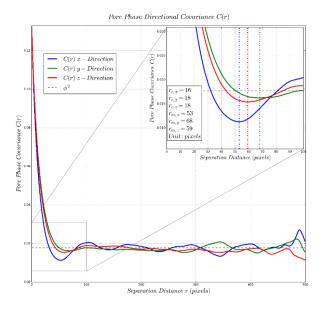




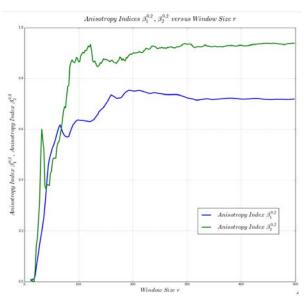
Ketton

Image

Covariance



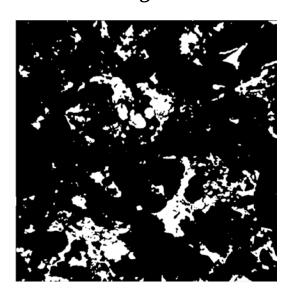
Minkowski Tensor



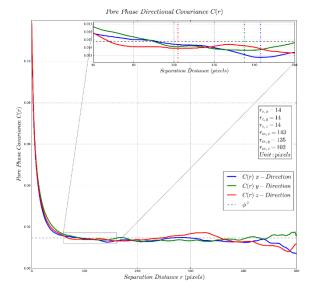
$$\begin{bmatrix} 0.327 & 0 & 0 \\ 0 & 0.336 & 0 \\ 0 & 0 & 0.342 \end{bmatrix}$$

Estaillades

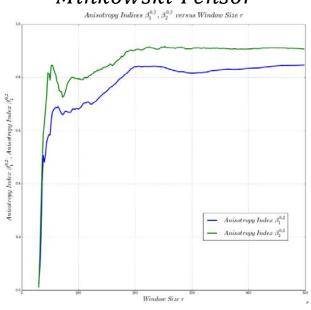
Image



Covariance



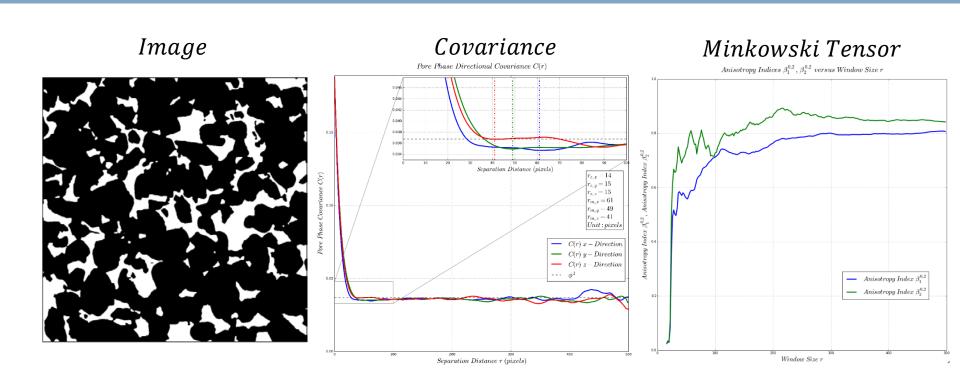
Minkowski Tensor

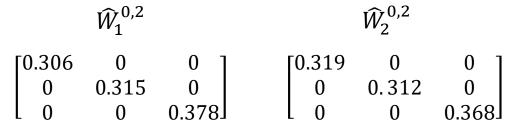


	$\widehat{W}_{1}^{0,2}$	
[0.310	0	0]
0	0.326	0
Lο	0	0.362

$$\begin{bmatrix} 0.320 & 0 & 0 \\ 0 & 0.329 & 0 \\ 0 & 0 & 0.350 \end{bmatrix}$$

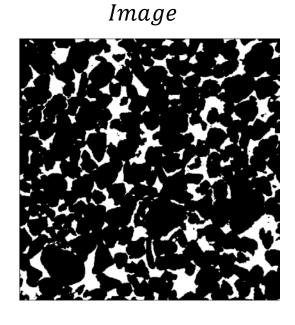
Doddington

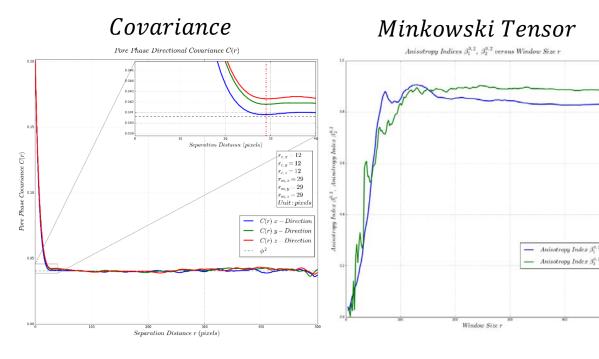




Bentheimer







0.351

Anisotropy Index $\beta_2^{0,1}$

$$\widehat{W}_{1}^{0,2} \qquad \widehat{W}_{2}^{0,2} \\
\begin{bmatrix}
0.320 & 0 & 0 \\
0 & 0.317 & 0 \\
0 & 0 & 0.361
\end{bmatrix} \qquad \begin{bmatrix}
0.325 & 0 \\
0 & 0.322 \\
0 & 0
\end{bmatrix}$$