Imperial College London

Statistical Characterisation of Porous Media at the Pore Scale

Progress Report 5

Student:
Lukas Mosser

Supervisors:
Professor Olivier Dubrule
Professor Martin Blunt

Imperial College London

Report Outline

- Current Status
  - Linking Minkowski Tensors to Permeability
  - Linking Covariance and Minkowski Tensors
  - Stereonet visualisation of MT principal directions

Macroscopic Momentum Balance -> Permeability

- **After Bear and Bachmat 1990:**
  - **Macroscopic Momentum Balance Equation:**

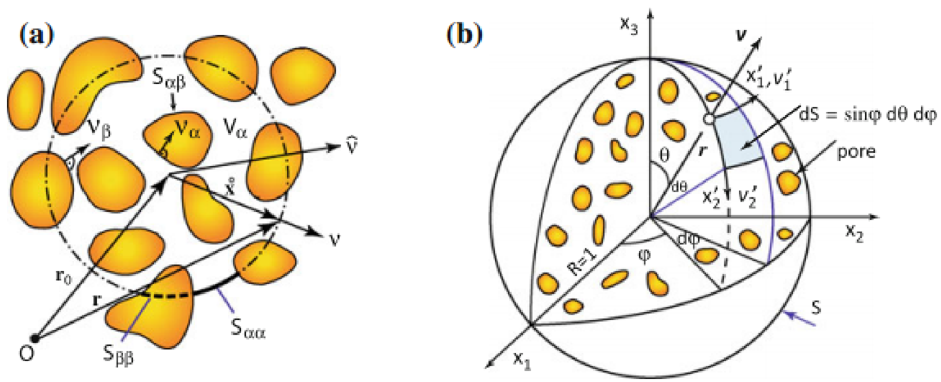


Fig. 1 **a** REV of a porous medium and **b** a unit sphere

$$\bar{\mu}^{\alpha}_{\alpha}\alpha_{ij}C_{\alpha}\phi_0\frac{\bar{V}^{\alpha}_{\alpha j}-\bar{V}^{\beta}_{\beta j}}{\Delta^2_{\alpha}}=-\phi_0 T^{*}_{\alpha ij}\left(\frac{\partial \bar{p}^{\alpha}}{\partial x_j}+\bar{\rho}^{\alpha}g\frac{\partial z}{\partial x_j}\right),$$

$$\alpha_{ij}=\delta_{ij}-\widetilde{\nu_i\nu_j}^{\alpha\beta}=\delta_{ij}-\frac{1}{S_{\alpha\beta}}\int_{S_{\alpha\beta}}\nu_{\alpha i}\nu_{\alpha j}\,\mathrm{d}S, \qquad T^{*}_{\alpha ij}=\frac{1}{V_{\alpha}}\int_{S_{\alpha\alpha}}\mathring{x}_i\,\nu_{\alpha j}\,\mathrm{d}S$$

Review definition MT

- Recall from Schröder-Türk, Mickel and Kapfer 2010
  - Definition of Minkowski Tensors:

$$W_\nu^{r,s}(K) := \frac{1}{3}\int_{\partial K} G_\nu \, \mathbf{x}^r \mathbf{n}^s \, \mathrm{d}A.$$

with $\nu = 1, 2, 3$ and $(r, s) = (2, 0), (1, 1)$ or $(0, 2)$.

$$G_0 = 1, G_1 = 1, G_2 = \frac{\kappa_1 + \kappa_2}{2}$$

- $\widehat{W}_1^{0,2}(K) := \frac{\int_{dK} n^2 dA}{\int_{dK} dA} = \quad \alpha_{ij} = \delta_{ij} - \widetilde{\nu_i \nu_j}^{\alpha\beta} = \delta_{ij} - \frac{1}{S_{\alpha\beta}} \int_{S_{\alpha\beta}} \nu_{\alpha i} \nu_{\alpha j} \mathrm{d}S,$

- $\frac{3W_1^{1,1}(K)}{V_\alpha} = \frac{1}{V_\alpha}\int_{dK} x \quad n \quad dA = \quad T^*_{\alpha ij} = \frac{1}{V_\alpha}\int_{S_{\alpha\alpha}} \mathring{x}_i \, \nu_{\alpha j} \mathrm{d}S$

Permeability from MT

- Permeability after Bear and Bachmat 1990

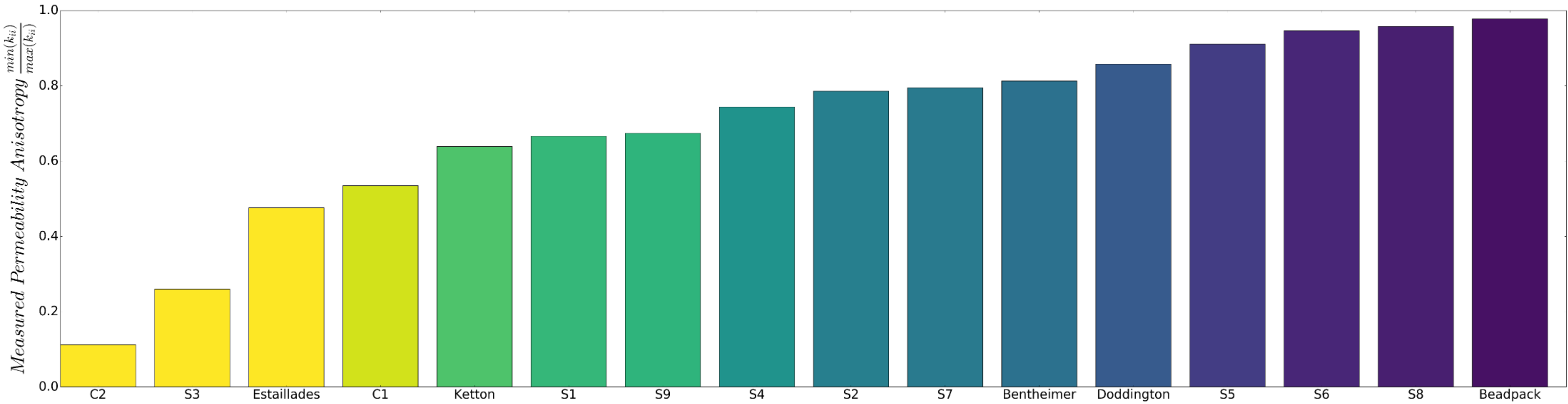
$$k_{ij} = \frac{\phi^3}{C_\alpha S_v} \alpha_{il}^{-1} T^*_{\alpha lj} \cong \frac{\phi^3}{C_\alpha S_v \tau^2}$$ (Kozeny Carman Isotropic Media)

Rewritten as Minkowski Tensors:

$$k_{ij} = \frac{\phi^3}{C_\alpha S_v} \{\delta_{ij} - \widehat{W}_1^{0,2}(S_{\alpha\beta})\}_{ij}^{-1} \{\frac{3W_1^{1,1}(S_{\alpha\alpha})}{V_\alpha}\}$$

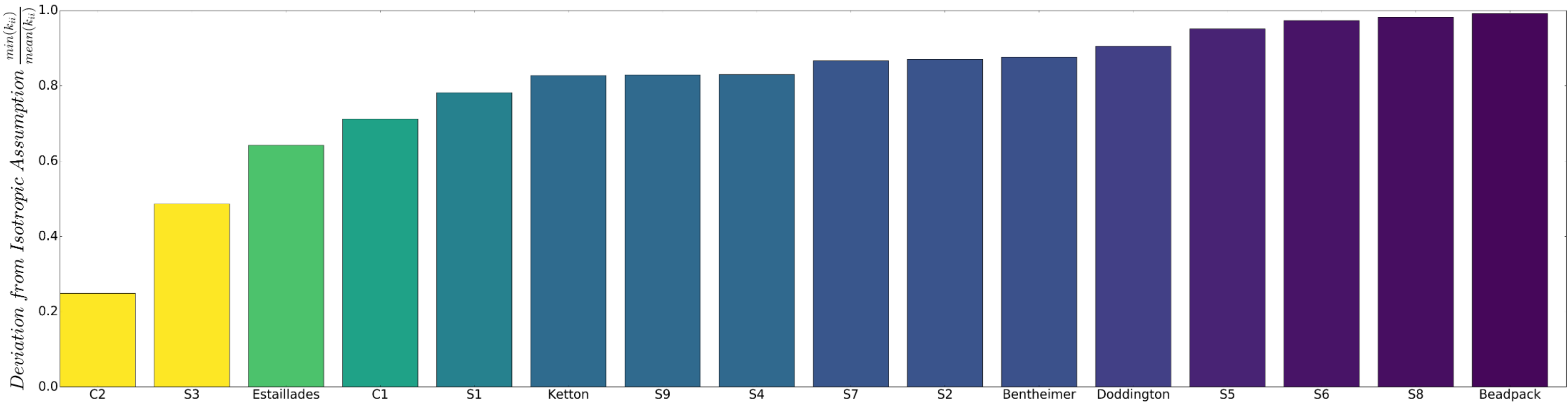
- Estimation of Permeability from MT:
  - Compute $\delta_{ij} - \widehat{W}_1^{0,2}(S_{\alpha\beta})$
  - Estimate Tortuosity by comparison with numerical simulation
  - Can we compute $\{\frac{3W_1^{1,1}(S_{\alpha\alpha})}{V_\alpha}\}$?
  - Follow approach found in literature i.e. estimate $C_\alpha \tau^2$

Numerical Results: min(kii)/max(kii)



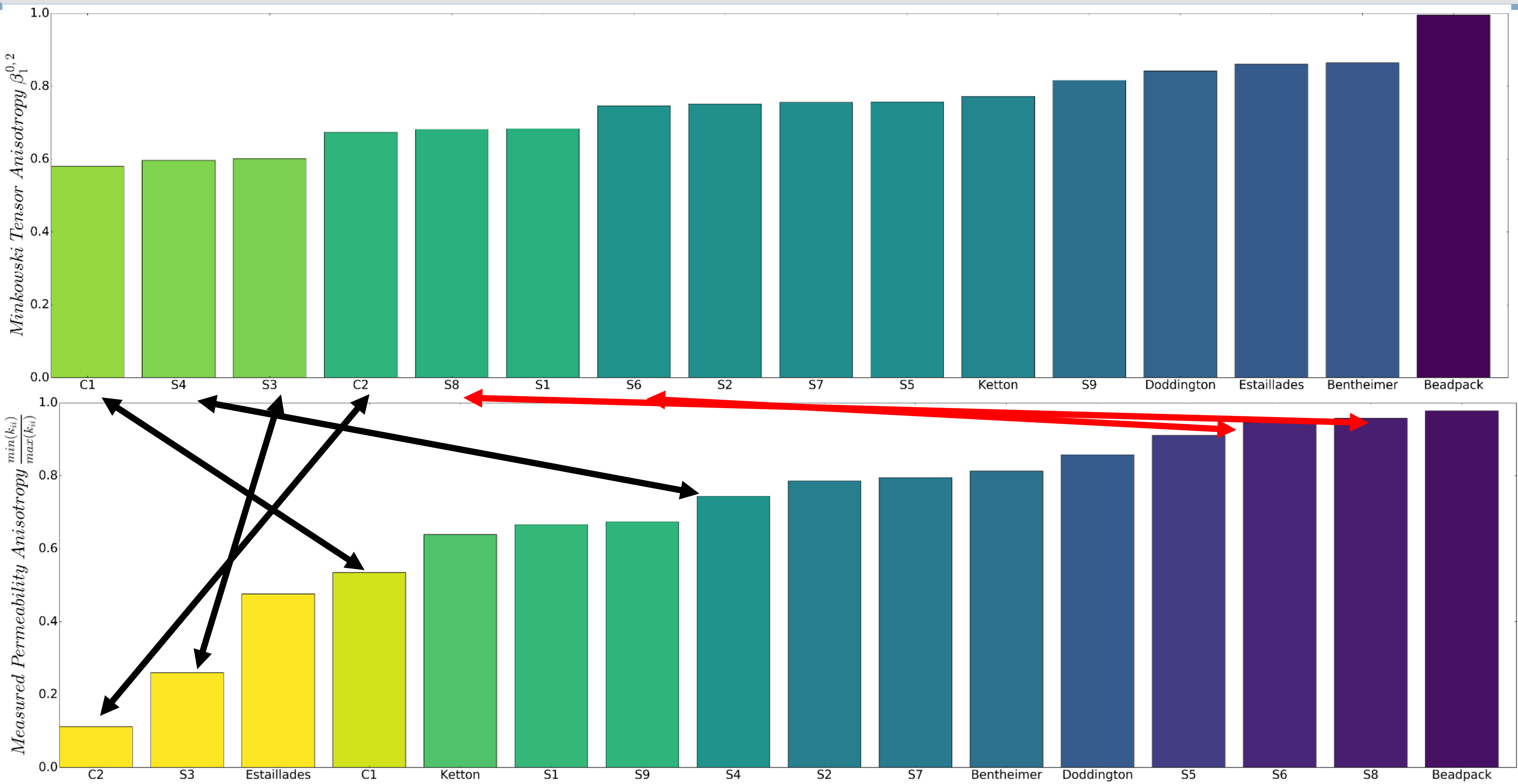
- Comparison of numerical estimated permeability anisotropy (min kij vs. max k)
- Almost all samples have anisotropic permeabilities (min/max < 0.9)
- Only Beadpack can be considered truly Isotropic

Numerical Results: min(kii)/mean(kii)



- Comparison of numerical estimated permeability anisotropy (min kij vs. Mean k)
- Defines a deviation from assumed isotropic behavior
- Assumption that lower perm. = worse than higher perm (=lower production)
  - Higher perms than expected could lead to early water breakthrough
- Main Takeaway: It‘s hard to say these samples are isotropic at this scale

Comparison Minkowski – Permeability Tensor

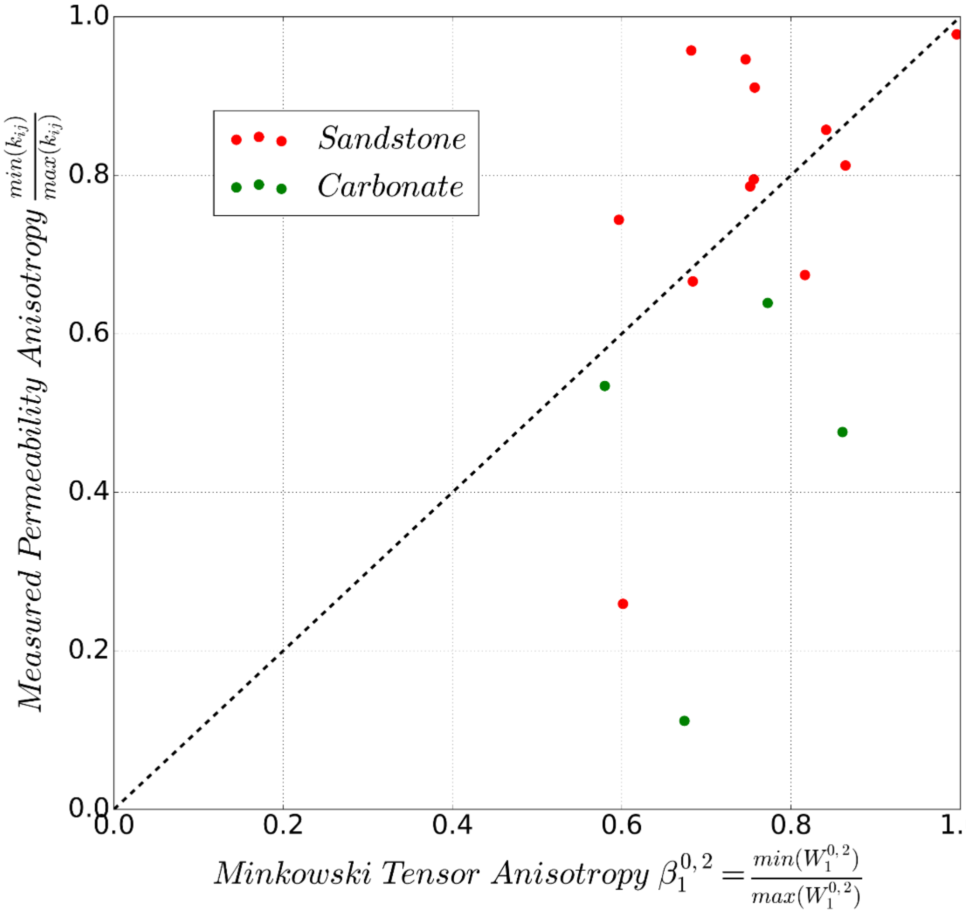


- Comparison Minkowski Tensor Anisotropy and Permeability Anisotropy
- Conclusion: Minkowski Tensor highly sensitive measure of Pore Grain Surface Anisotropy
- Not only contributing factor i.e. surface anisotropy can lead to permeability isotropy (S6, S8)

Imperial College London

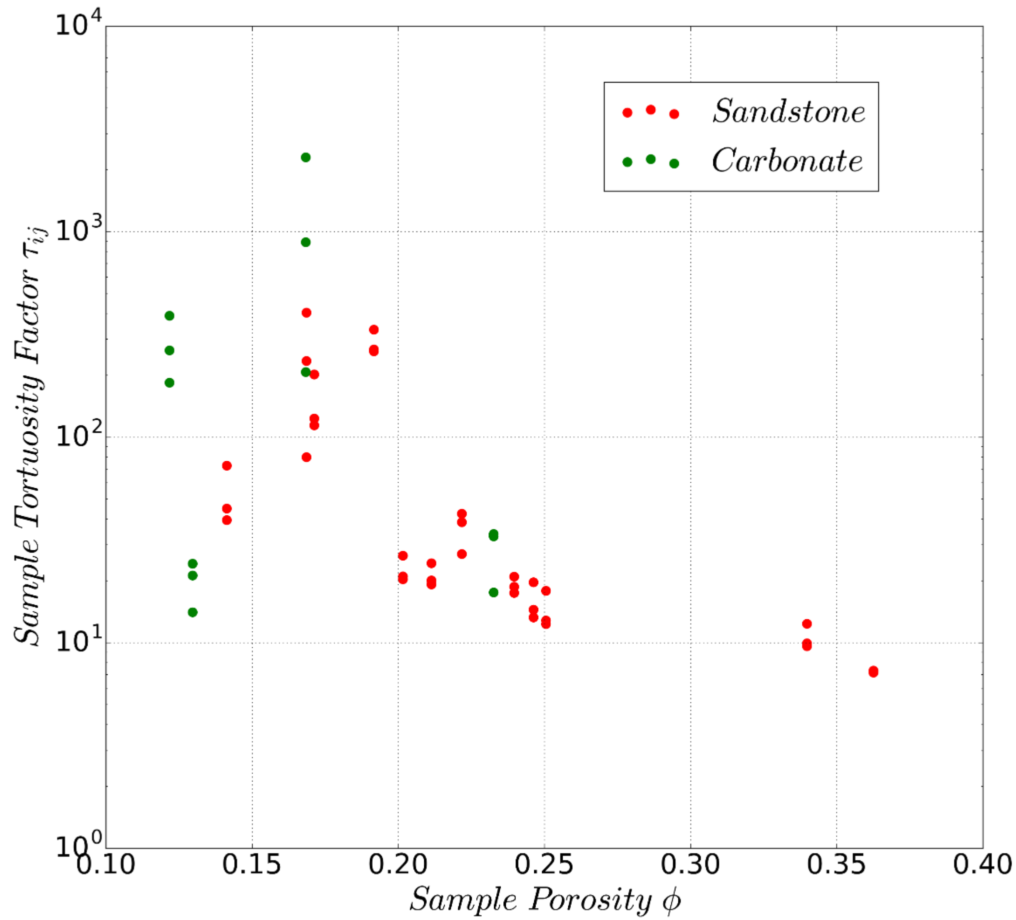
Computed Permeability vs. Minkowski Tensor

- Individual samples show nearly isotropic permeability where Minkowski Tensor shows anisotropic behavior
- No linear correlation of permeability anisotropy and minkowski tensor anisotropy
- If defined as a false-positive assumption:
  - Minkowski tensor shows anisotropy on 15 out of 16 samples
  - Of 15 samples 3 could be considered isotropic
- If anisotropy will influence the process you are considering, computing the minkowski tensor and checking for anisotropy is a cheap first guess if it really is anisotropic
  => chances high that it truly is anisotropic
  But MT won't tell you straight how anisotropic
- Other factors such as tortuosity play a role here



Computed Kozeny Factors (after Bear and Bachmat 1990)

- Scattering of data and larger Kozeny Factors as porosity decreases
  i.e. Tortuosity has to increase
  to explain permeabilities at lower porosity than accounted for by specific surface area and orientation only
- High Tortuosity not restricted to carbonates
  But high porosity > 0.20 samples show
  low tortuosity factors



Imperial College London

Linking Covariance to Minkowski Tensors

- Classical Theory on Covariance shows that derivative of covariance at r=0 = Sv (specific surface area)

$$\lim_{r\to 0}[\langle P_{ij}(r)\rangle/r] = [S_V]_{ij}/4.$$

- For directional covariance this is not valid (Gokhale 2005):
- Directional Covariance can be defined as:

$$\lim_{r\to 0}[P_{ij}(r,\theta,\phi)/r] = [S_V]_{ij}\int_0^{2\pi}\int_0^{\pi/2}|\cos\beta|G_{ij}(\theta',\phi')\,\mathrm{d}\theta'\,\mathrm{d}\phi'/2$$
$$\text{for } i \neq j. \tag{20}$$

- Here Gij is defined as:

In Eq. (10), β is the angle between the directions (θ,ϕ) and (θ',ϕ'), and $G_{ij}(\theta',\phi')$ is the morphological orientation distribution of the i–j interfaces in the microstruc-

i–j interfacial area in the orientation range θ' to $(\theta' + \mathrm{d}\theta')$ and ϕ' to $(\phi' + \mathrm{d}\phi')$. For randomly oriented interfaces, $G_{ij}(\theta',\phi')$ is equal to $\sin\theta'/2\pi$. Combining

- For binary voxel images assume every grain-pore interface is square interface of area (resolution²) and normal vectors:

Imperial College London

Linking Covariance to Minkowski Tensors

- Directional Covariance can be defined as:

$$\lim_{r\to 0}[P_{ij}(r,\theta,\phi)/r] = [S_V]_{ij}\int_0^{2\pi}\int_0^{\pi/2}|\cos\beta|G_{ij}(\theta',\phi')\,d\theta'\,d\phi'/2$$

$$\text{for } i \neq j. \qquad (20)$$

- For binary voxel images assume every grain-pore interface is square interface of area (resolution²) and normal vectors:

$$e_1 = \pm\begin{bmatrix}1\\0\\0\end{bmatrix}, e_2 = \pm\begin{bmatrix}1\\0\\0\end{bmatrix}, e_3 = \pm\begin{bmatrix}1\\0\\0\end{bmatrix}$$

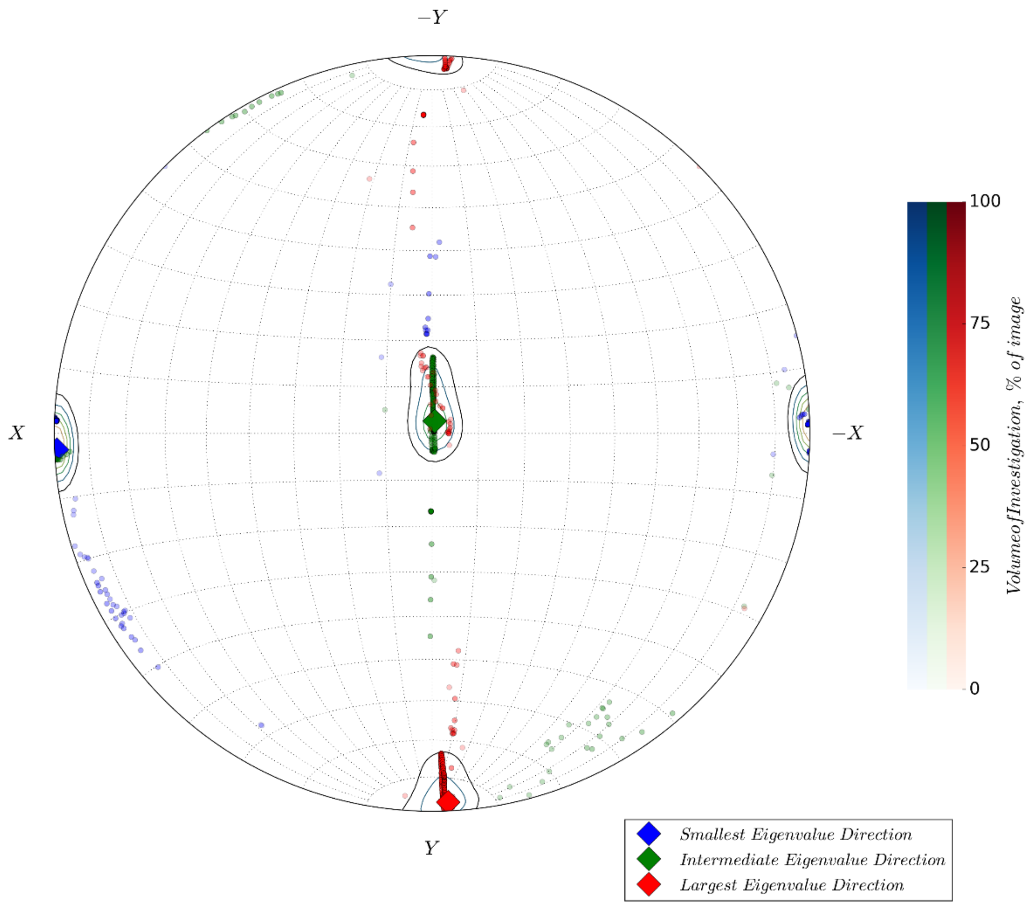
- Angle Beta defined as angle between (theta-phi) and (theta'-phi):
- => abs(cos(beta)) = 1 for e_{ii}
- Can write double integral above as sum over the surfaces oriented in each direction divided by the total surfaces i.e. the fraction of surfaces oriented in the direction theta: phi
- Hypothesis:

$$G_{ij}\ \boldsymbol{related\ to}\ \widehat{\boldsymbol{W}}_1^{0,2}(\boldsymbol{K}) := \frac{\int_{dK}\boldsymbol{n}^2\,d\boldsymbol{A}}{\int_{dK}d\boldsymbol{A}}$$

- Observation:
  Can not deduce directional permeability from covariance as everything relates to total specific surface area.
  Definition of effective specific surface area possible but questionable.
- Possibly justifieable using theory of Bear Bachmat 1990 if linked to α_{ij}^{-1}

Imperial College London

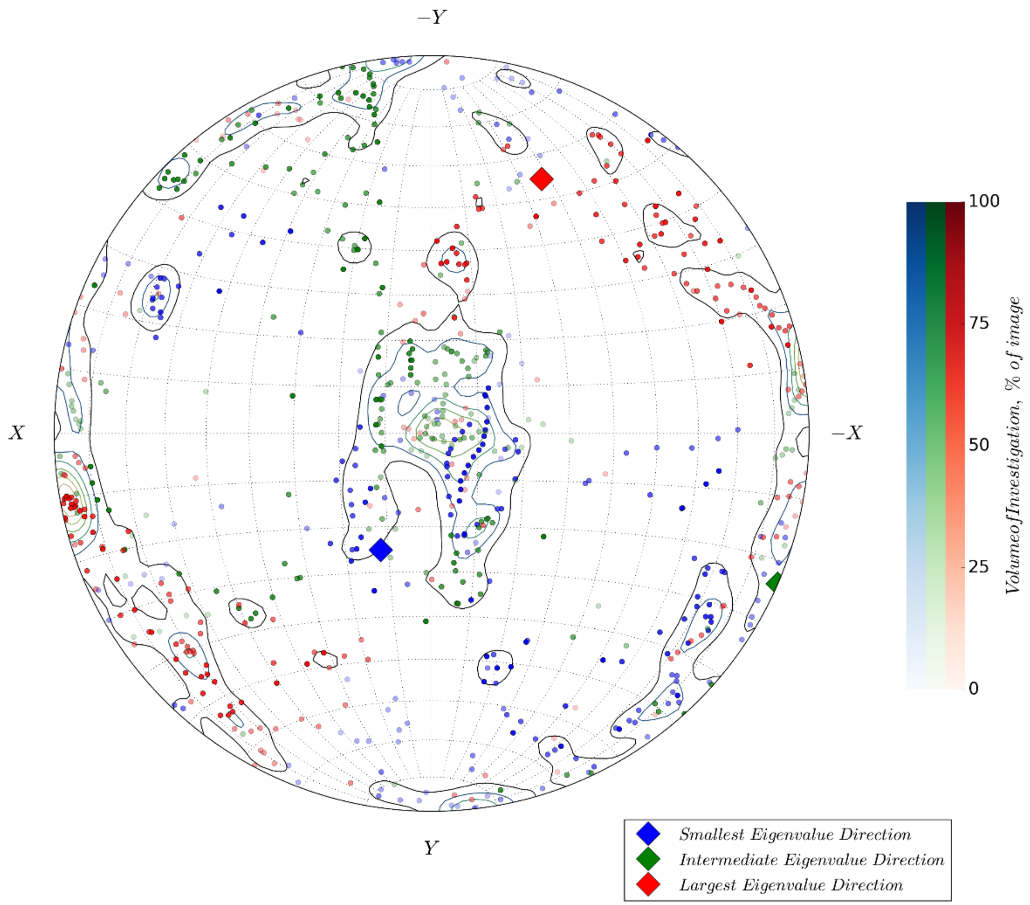
Visualisation of Minkowski Tensor Orientation



- Stereonet shows direction of Smallest, Intermediate and Largest Eigenvalues
- Increasing color hue indicates larger sample volumes:
  - Diamonds indicate orientation at largest (image) size
- Indicates changes in orientation as function of REV size

Imperial College London

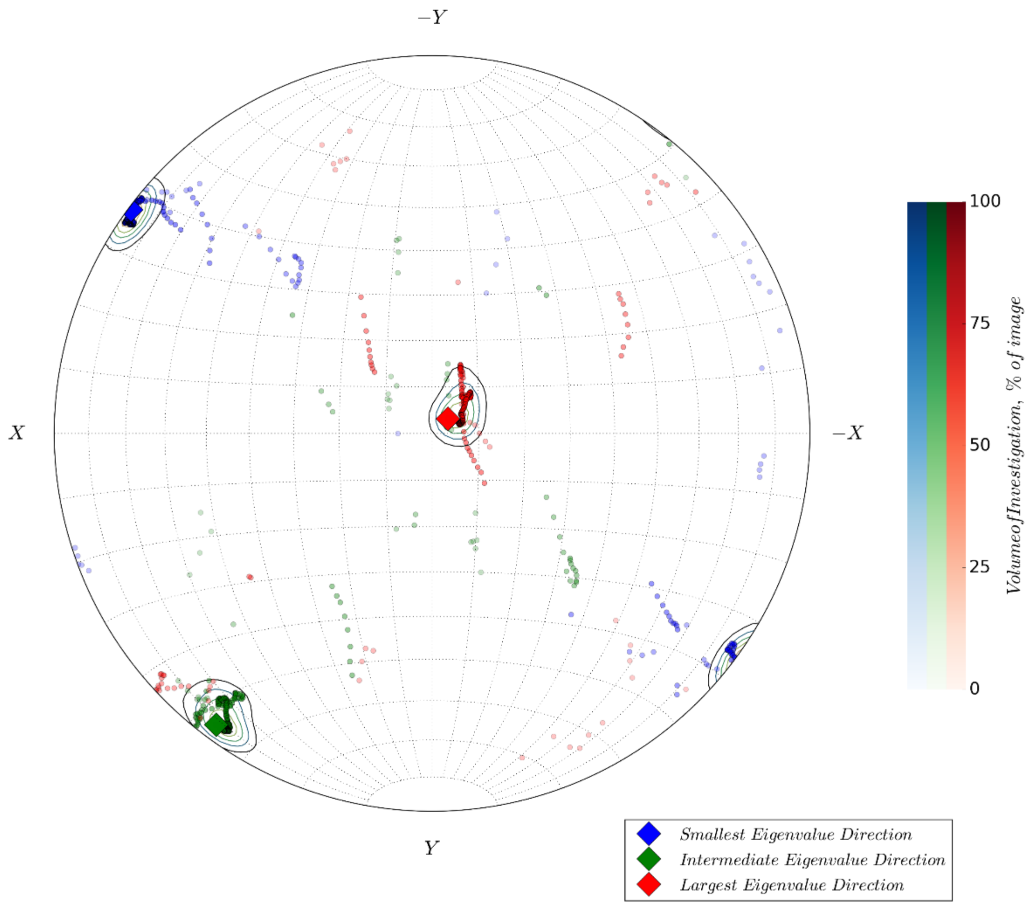
Beadpack MT Directional Visualisation



- Truly Isotropic behavior: Eigenvalues very similar => large spread
- No Preferential orientation across evaluated REV scales

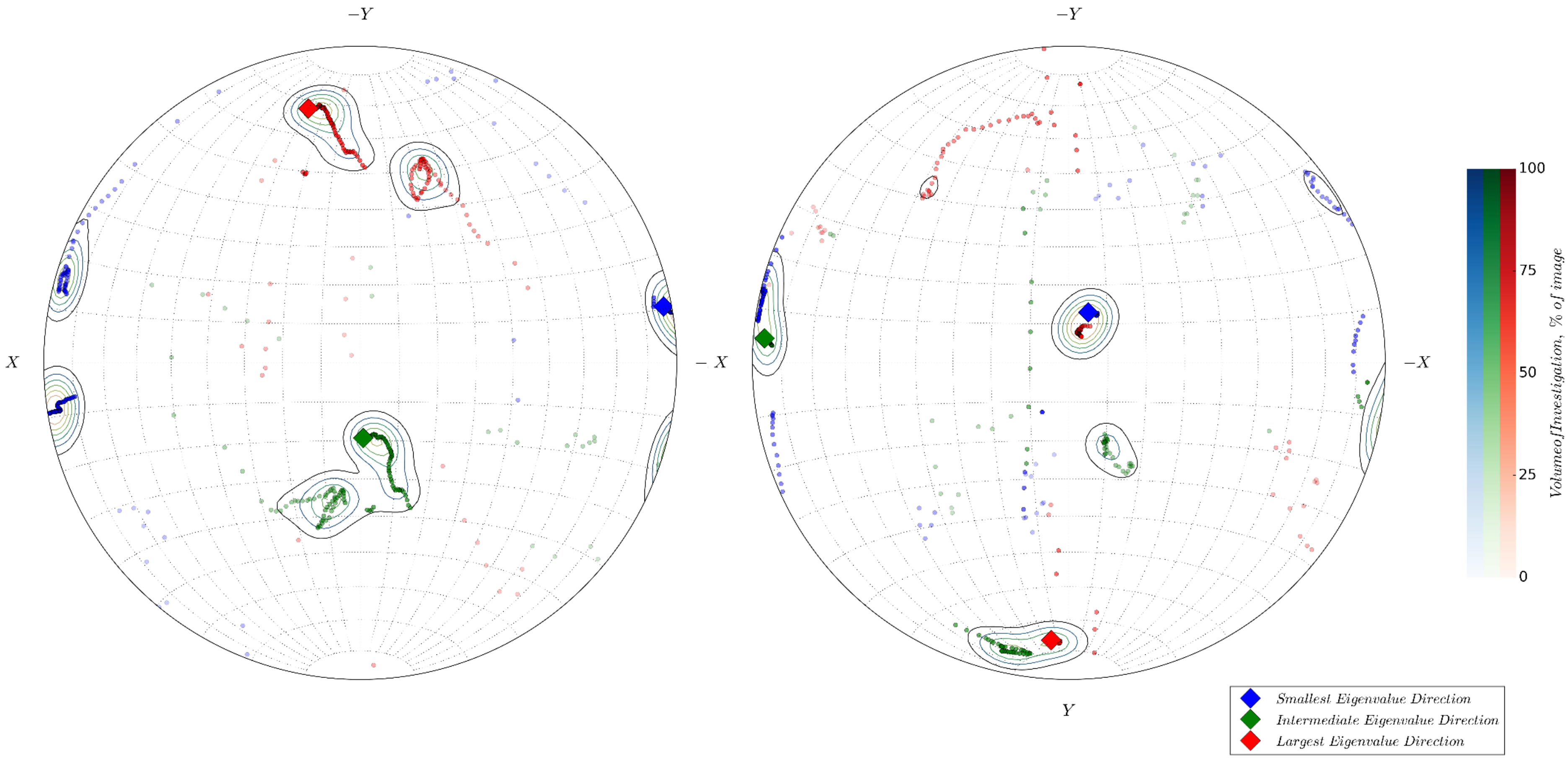
Imperial College London

Ketton MT Directional Visualisation



- Stabilisation of direction occurs quickly
  i.e. most tensors assume their final orientation quickly as volume of investigation grows

S9 (left) and S7(right) Directional Visualisation



- Orientation in these samples does not stabilise
- Could indicate the need for a directional criterion to establish an REV
- If Orientation of tensor changes within across a variety of length scales => no REV reached
  Or possibly multiple scales of interest in image (subscales or heterogeneities)

Imperial College London

Current Status

- Computation of directional permeability on 500 voxel³ images done
- Linked theory of Minkowski Tensors to Macroscopic Momentum Balance Theory Bear and Bachmat 1990
  - Simplifies to Kozeny-Carman for isotropic media
  - Tortuosity theoretically linked to Minkowski tensors but out of scope
    - » Follow approach of using tortuosity as fitting parameter
- Computed directional Kozeny-Carman factors for available samples
  - Results show that Minkowski Tensor is good indicator of anisotropy
  - Few cases show nearly isotropic behavior where MT shows anisotropy
  - Cheaper to compute than full tensor by simulation to rule out anisotropy behavior
- Covariance can not be used directly to infer directional permeability as linked to total specific surface area
  - Possible connection to Minkowski Tensor evaluated
  - Needs rigorous proof
- Covariance best used to infer material properties by comparison of different directions
  - Hole effect vs. No-hole effect
  - Periodicity in covariance
  - Qualitative indication of directional permeability
  - Steeper slope at origin => Smaller Characteristic Pore Size => Smaller Permeability
- Visualisation of Minkowski Tensor principal directions using Stereonets as a function of volume size
  - Change in prefferential orientation at various scales
  - Should average over ensemble of samples at one length scale to differentiate between true and local scale effects