Directional Measures to Characterize Anisotropy in Pore-Scale Morphology and Permeability

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Introduction

The presence of anisotropy in sedimentary rocks has critical implications on the effective recovery of oil and gas from hydrocarbon accumulations. The permeability of a porous medium can show strong anisotropy in the presence of clays or lamination.

Using available Micro-CT images we characterise the anisotropy of sedimentary rocks using two directional measures, the covariance and tensorial Minkowski functionals.

We compare the anisotropy in pore morphology to the single-phase permeability obtained from numerical solution of the Stokes equation direct on Micro-CT images (Mostaghimi 2013).

Our results show that the covariance and tensorial Minkowski functionals allow qualitative estimates of the anisotropy in permeability and provide a rich framework to characterise morphology and material properties of porous media at the pore-scale.

Directional Covariance

The covariance or two-point probability function is the probability that two points at distance $\mathbf{r}$ lie within the same phase, the pore or grain phase, of the porous medium.

$$C(\mathbf{r}) = P(\mathbf{o} \in \Xi, \mathbf{r} \in \Xi)$$

Estimating the covariance in the three Cartesian directions allows us to quantify two material characteristics:

- Porosity
- Specific Surface Area $\mathbf{S_v}$
- Characteristic pore size $\mathbf{r_c}$
- Shape of the covariance

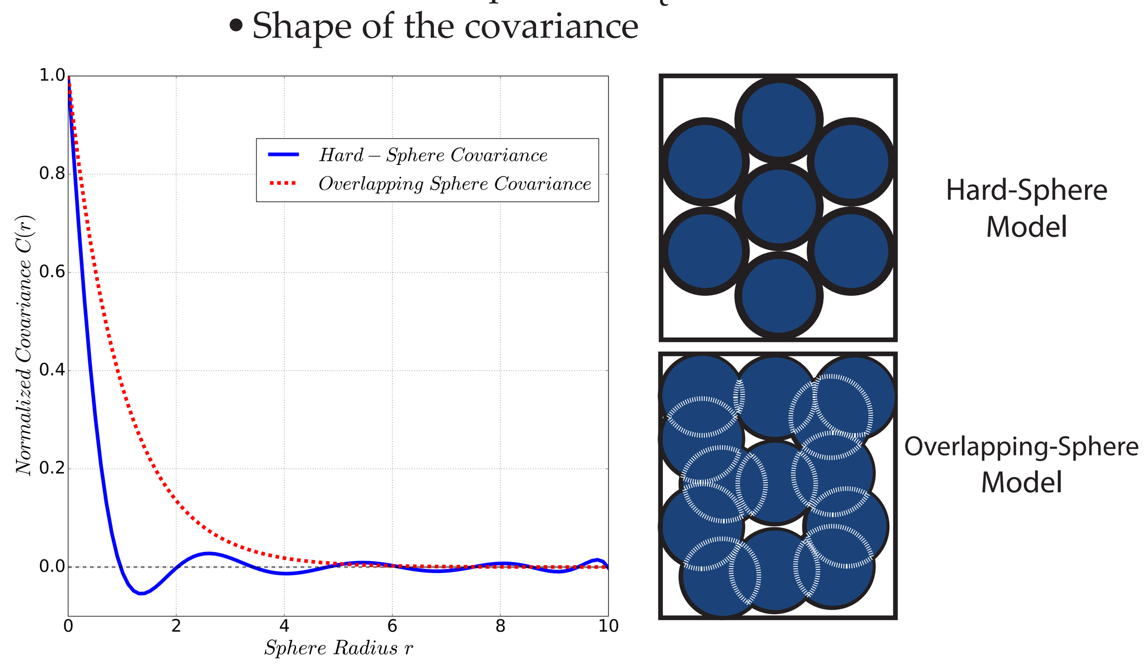


Figure 1: Comparison of hard-sphere and overlapping-sphere models of the covariance. Estimated directional covariance was compared with characteristic shapes of the covariance to characterise the pore-grain structure of sedimentary Micro-CT samples

The Kozeny-Carman equation relates the effective permeability of a porous medium can be correlated with the specific surface area and porosity of the porous medium.

$$k_{eff} = \frac{\phi^3}{C_\alpha \tau (1-\phi)^2}$$

The shape of the covariance gives an indication of the structure of the pore space.

Minkowski Functionals

Minkowski functionals are integral geometric parameters quantifying the morphology of the pore space. We can distinguish scalar, vectorial and tensorial Minkowski measures.

Scalar Minkowski measures such as the surface area or euler characteristic have no directional dependency and cannot be used to quantify anisotropy in porous media.

Tensorial Minkowski functionals allow the spatial dependency of these scalar properties to be determined. We define the Minkowski tensor $W_1^{0,2}$ that describes the orientational distribution of the pore-grain interface:

$$W_1^{0,2} = \frac{1}{3}\int_{\partial K} \mathbf{n}^2 dA$$

The anisotropy of the pore morphology can be quantified by defining the ratio of minimum and maximal eigenvalues of $W_1^{0,2}$:

$$\beta_1^{0,2} = \frac{min\{eig\{W_1^{0,2}\}\}}{max\{eig\{W_1^{0,2}\}\}}$$

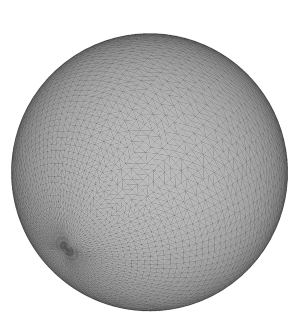
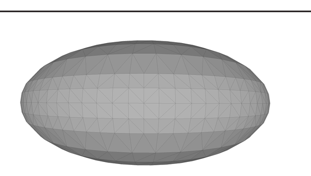
	$W_1^{0,2}$	$\beta_1^{0,2}$
	$\begin{bmatrix} 0.333 & 0 & 0 \\ 0 & 0.333 & 0 \\ 0 & 0 & 0.333 \end{bmatrix}$	1.0
	$\begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.6 \end{bmatrix}$	0.333

Figure 2: The Minkowski tensors and anisotropy index of two simple geometric objects. A sphere shows pure isotropic behavior. The diameter of the ellipsoid is the same as the sphere in the x-y plane and has been reduced by a factor of 0.5 in the vertical direction. The Minkowski tensor of the ellipsoid shows a strong anisotropy of surface.

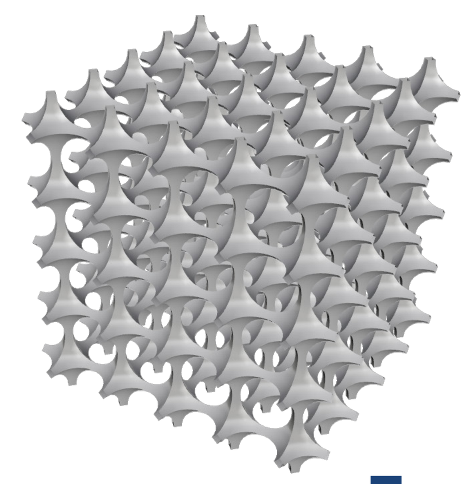
Integrated Workflow

An automated integrated workflow was developed to facilitate the computation of directional pore morphology measures and numerical estimates of the permeability.

Parametric models were created using a 3D modeling software and used for conceptual understanding of the Minkowski tensors and their behavior for anisotropic media.

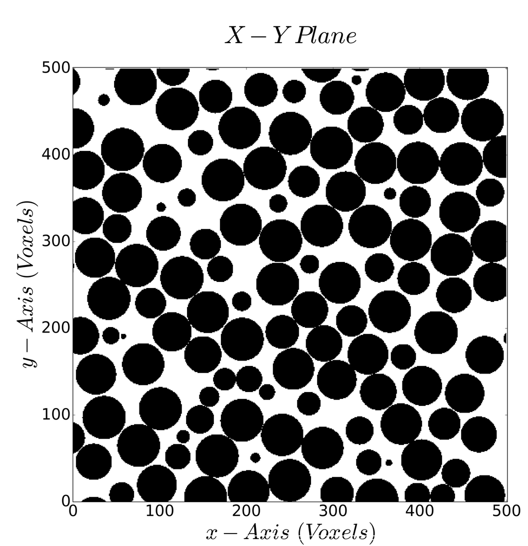
Micro-CT Images

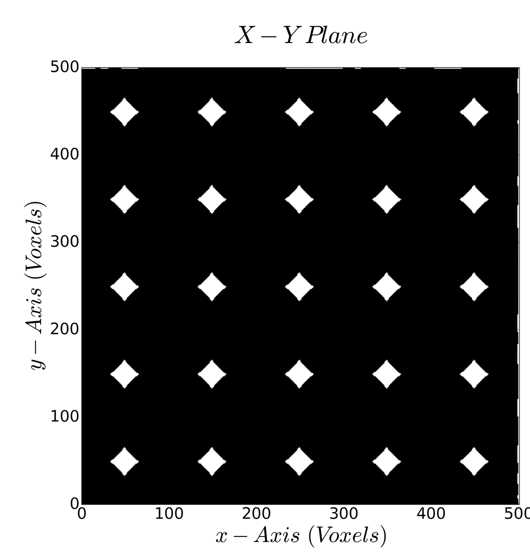
Parametric Models



Pre-Processing Segmentation

3D-Modeling Voxelization





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Covariance Estimation

Covariance Shape

Characteristic Pore Size

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Pore-Grain Interface Extraction

Minkowski Tensor

Anisotropy Index

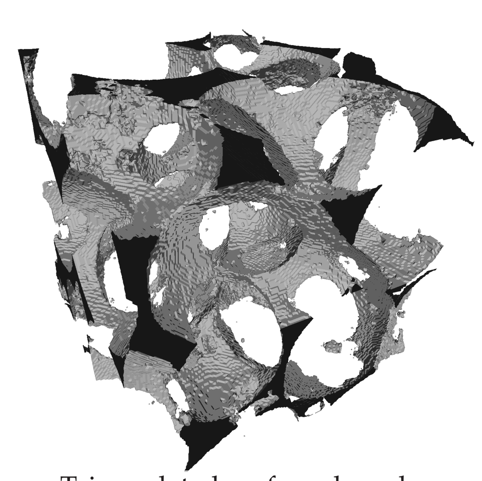
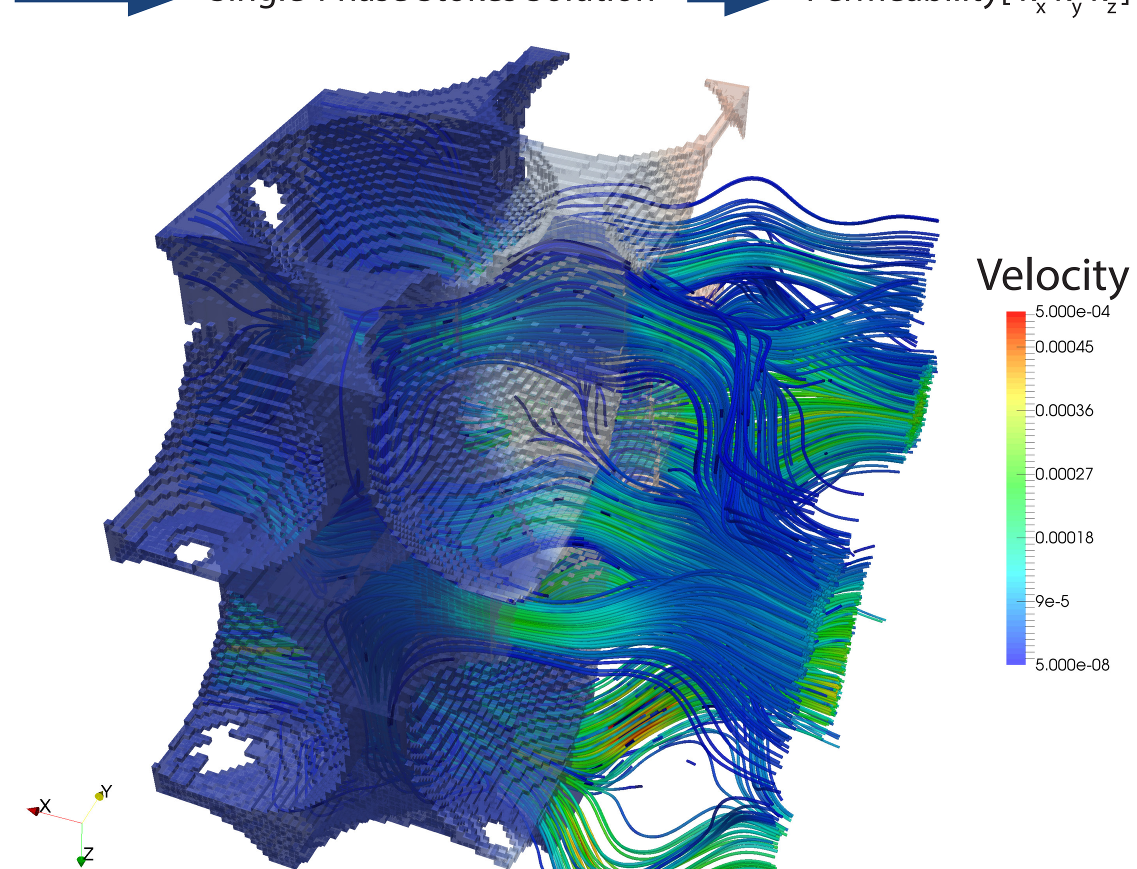


Figure x: Triangulated surfaces have been extracted using the toolkit VTK. The software libraray Karambola was used to compute Minkowski tensor valuations.

Single-Phase Stokes Solution

Permeability[k_x k_y k_z]



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Discussion

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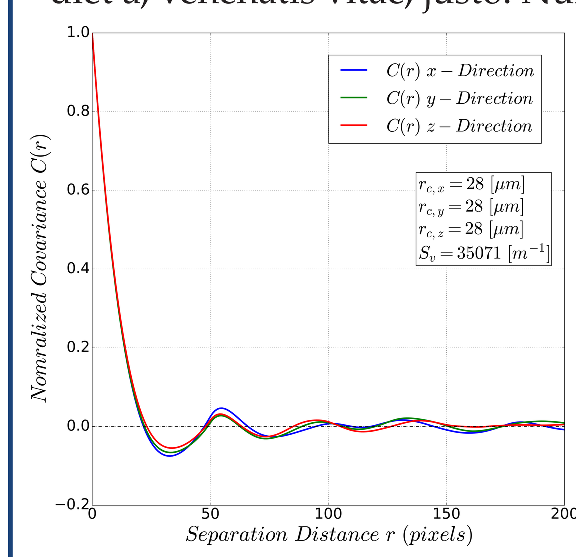

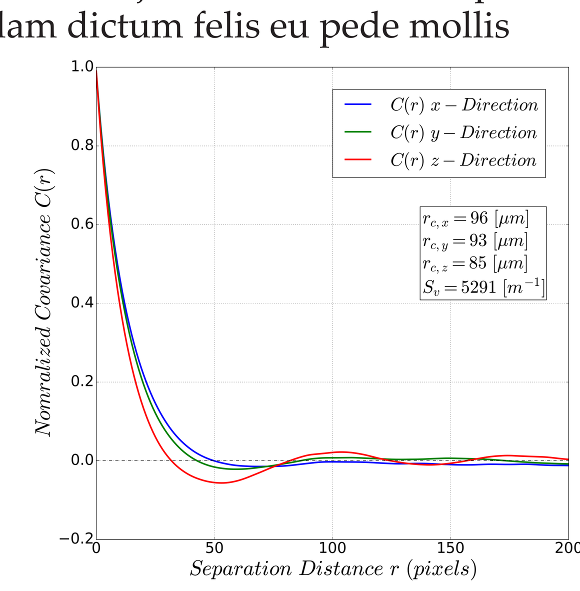

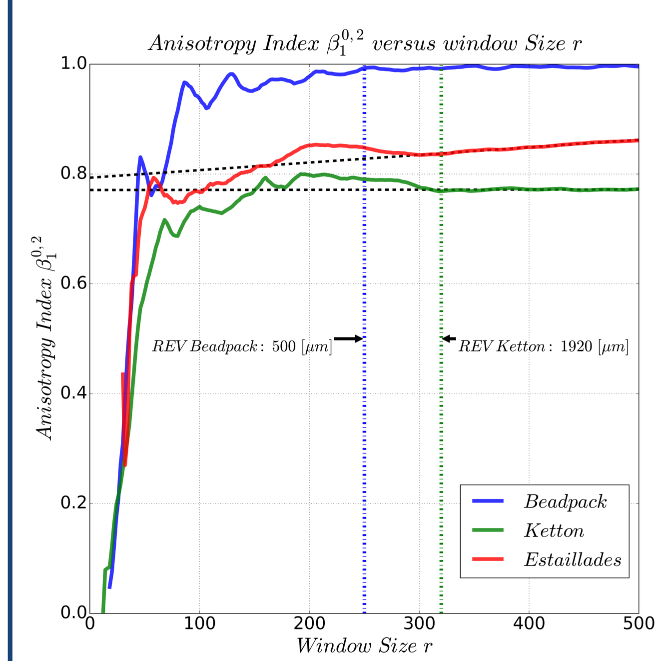

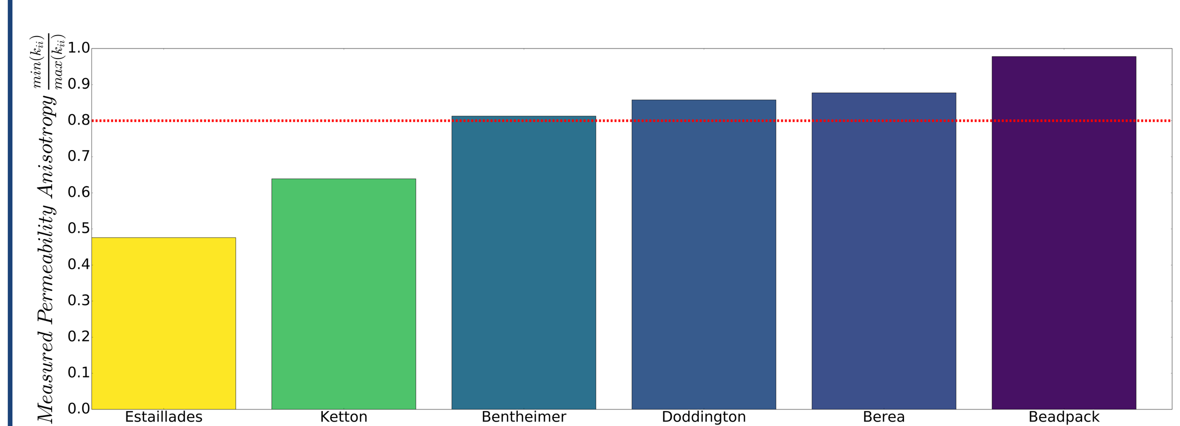

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Conclusions

The directional covariance and tensorial Minkowski functionals allow a qualitative

References

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