University of Innsbruck
Faculty of Mathematics, Computer Science and Physics
Institute for Theoretical Physics

Bachelor Thesis
submitted for the degree of
Bachelor of Science

An information-theoretical approach to internal models in a Partially Observable Markov Decision Process

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Abstract

Lorem ipsum

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1 Introduction

2 Methods

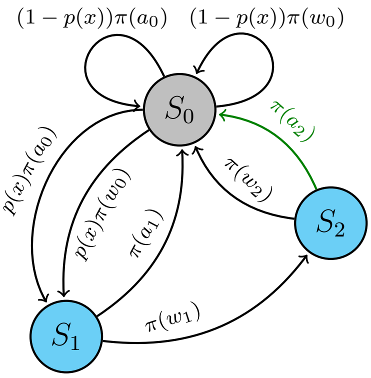
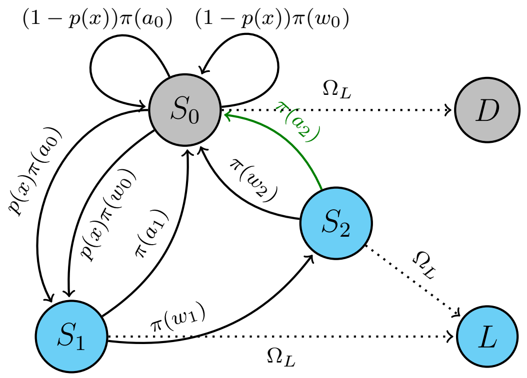


Figure 1: Graph showing the delayed action MDP



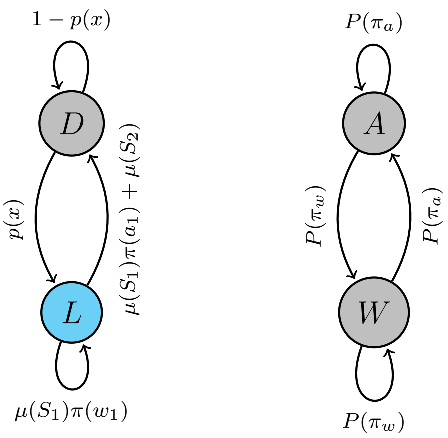


Figure 2: Reduced models for Observation of light (left) and action (right).

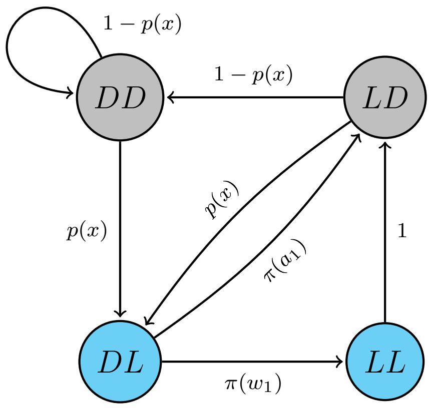


Figure 3: Internal model with sufficient complexity.

3 Results

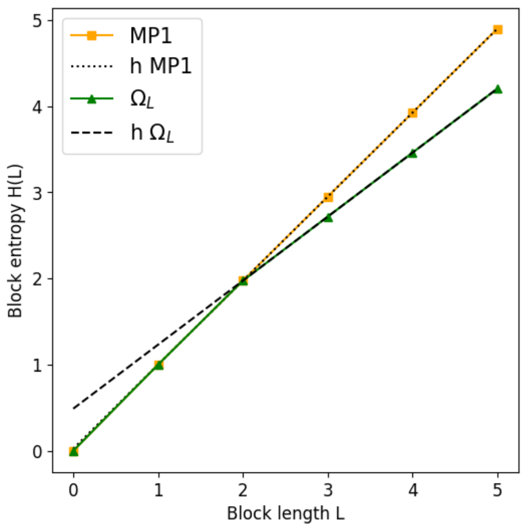


Figure 4: Plot showing the difference between MP1 and the MDP.

Talk about behaviour correlating to order 2 of process, in agreement with Crutchfield. Also do the same plot for MP2 and action as well, showing hidden Markov convergence.

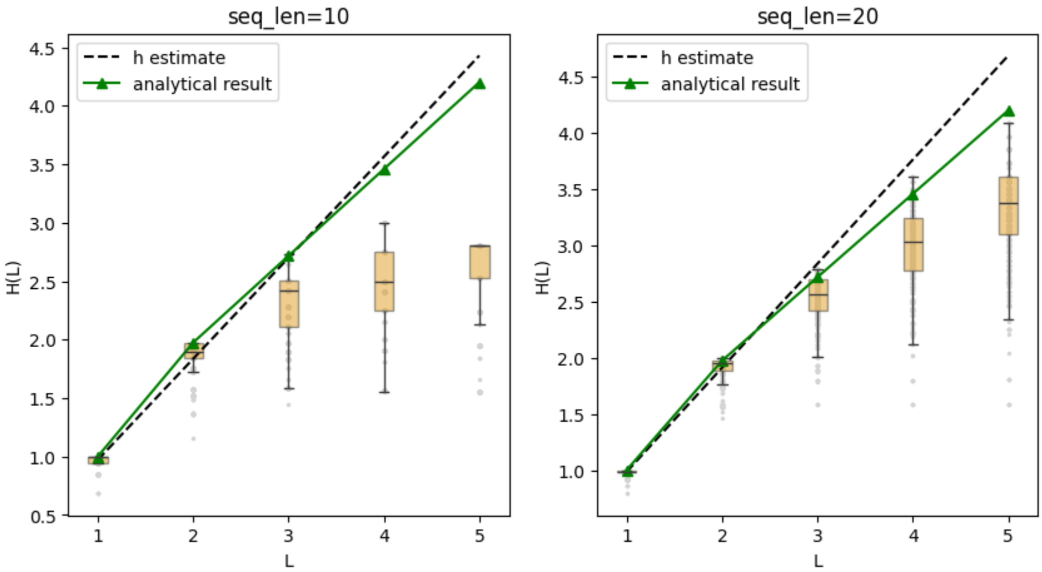


Figure 5: Plot showing the bias and variance of the plug in estimator for block Entropy.

$$\hat{H}(s^L) = -\sum_i \frac{n_i}{N} \log_2 \frac{n_i}{N}$$

(Lesne et al.) (10.1103/PhysRevE.79.046208) Entropy estimation of short (time correlated) sequences Upper bound on block length given a sequence of observations, in order to have good estimates:

$$n \leq \frac{N h_\mu}{\ln(k)}$$

with word length n, length of observation sequence N and number of symbols k. They propose first estimating the entropy rate using Lempel-Ziv complexity(iterative algorithm moving through sequence), then setting the sequence length/word length accordingly.

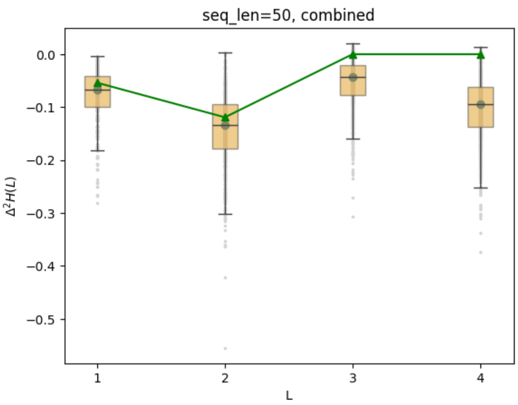
$$\hat{L}_0 = \frac{\mathcal{N}_w \ln(N)}{N},\ \hat{L} = \frac{\mathcal{N}_w[1 + \log_k \mathcal{N}_w]}{N},\ \lim_{n\to\infty} \hat{L} = \frac{h_\mu}{\ln(k)}$$

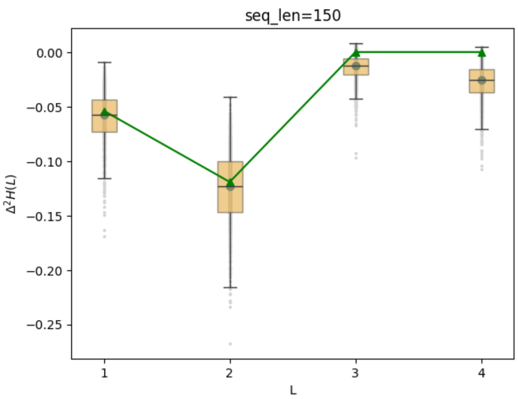
With N the length of the observation sequence and $\mathcal{N}_w$ parsed words from the Lempel-Ziv algorithm. For the plug in estimator, error bars are computable, meaning computable confidence intervals.

Larson et al. (10.1016/j.procs.2011.04.172) use

$$\mathcal{L} h_\mu \geq L|A|^L \ln|A|$$

solving for L given $\mathcal{L}$ returns $W(\mathcal{L}h_\mu)/\ln|A|$ (mathematica), with the Lambert W function. (gives better estimate)





deviation from analytical: [-0.00365034217876947 -0.00365907607011157 -0.0124851947847702 -0.0251811425394552]

4 Conclusion

5 Acknowledgements

Declaration of Authorship

I hereby solemnly declare, by my own signature, that I have independently authored the presented work and have not used any sources or aids other than those indicated. All passages taken verbatim or in content from the specified sources are identified as such.
I consent to the archiving of this Bachelor thesis.

Innsbruck, 4th June 2024 Lukas Prader