University of Innsbruck
Faculty of Mathematics, Computer Science and Physics
Institute for Theoretical Physics

Bachelor Thesis
submitted for the degree of
Bachelor of Science

An information-theoretical approach to internal models in a Partially Observable Markov Decision Process

by
Lukas Prader
Matriculation Nr.: 12115058
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Supervisors: Alexander Vining, Hans J Briegel

Abstract

Lorem ipsum

Contents

1 Introduction

A large reason for success in many, if not all scientific disciplines has been the adoption of a reductionist perspective on phenomena. Widely adopted and successful, this hypothesis assumes that all processes in our universe are in the end governed by a set of fundamental laws, which can be used do describe any higher order of phenomena as well. Especially in physics, the idea of a final, unified theory of everything has been seen as the ultimate goal of the discipline for centuries.

Yet, in many disciplines it has been shown that there are so-called emergent phenomena which are not easily explained by lower level fundamental laws, requiring additional concepts to accurately explain them (Anderson 1972). Exactly how these complex phenomena can emerge from the set of currently known fundamental laws is an ongoing field of research, with interdisciplinary approaches taking from the fields of physics, chemistry, biology, psychology, philosophy computer science and others.

Especially the rise of artificial intelligence in recent years has produced new research trying to create complex models able to perform intelligent tasks. Research into so-called complex adaptive systems is also closely connected to research in behavioural biology, trying to understand the emergence of intelligent and adaptive behaviour in animals.

get to information theory and crutchfield

2 Methods

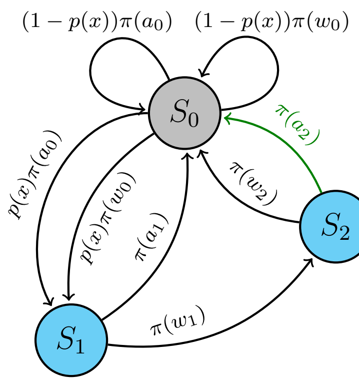


Figure 1: Graph showing the delayed action MDP

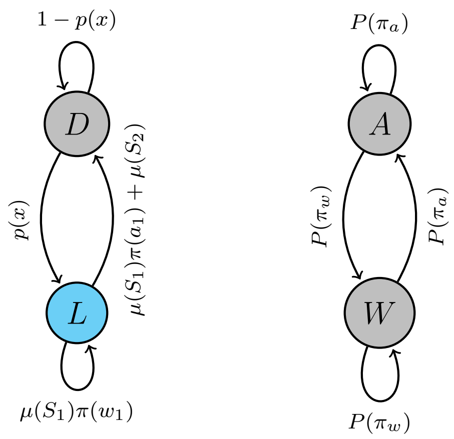


Figure 2: Reduced models for Observation of light (left) and action (right).

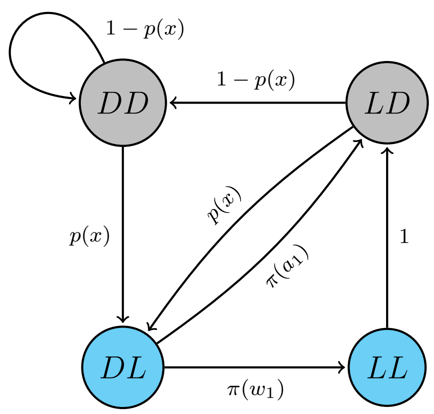


Figure 3: Internal model with sufficient complexity.

3 Results

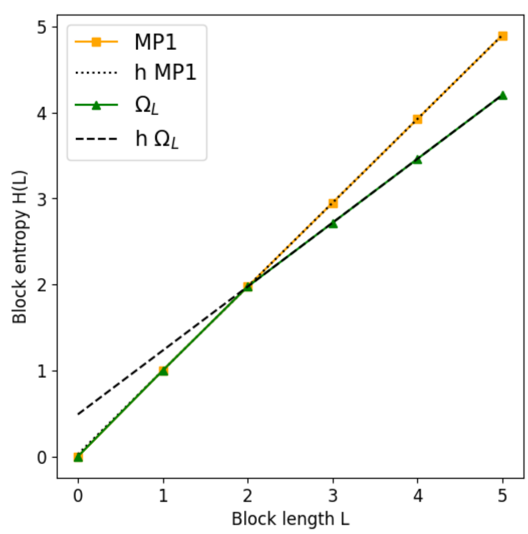


Figure 4: Plot showing the difference between MP1 and the MDP.

Talk about behaviour correlating to order 2 of process, in agreement with Crutchfield. Also do the same plot for MP2 and action as well, showing hidden Markov convergence.

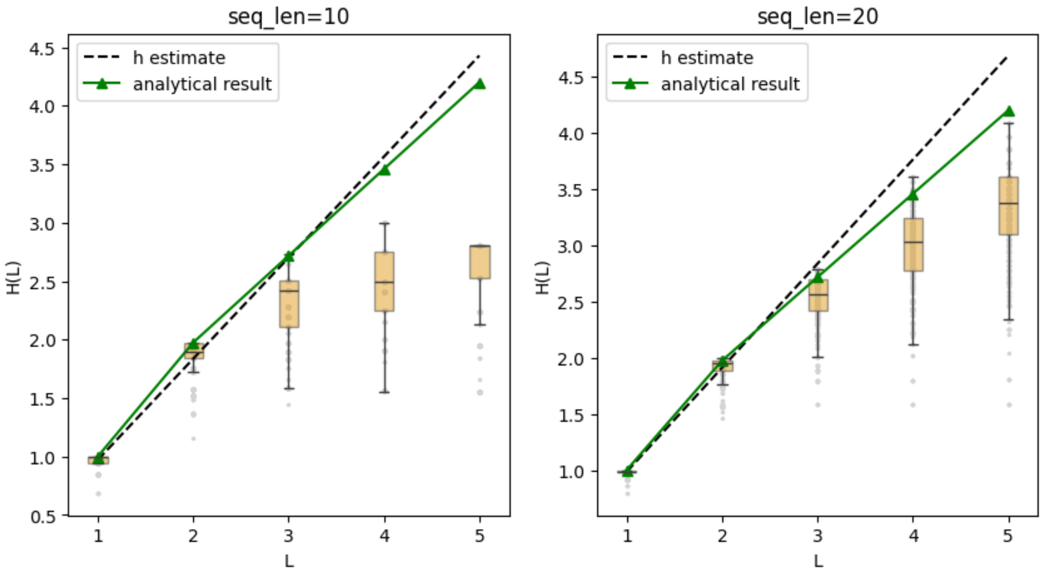


Figure 5: Plot showing the bias and variance of the plug in estimator for block Entropy.

$$\hat{H}(s^L) = -\sum_i \frac{n_i}{N} \log_2 \frac{n_i}{N}$$

(Lesne et al.) (10.1103/PhysRevE.79.046208) Entropy estimation of short (time correlated) sequences Upper bound on block length given a sequence of observations, in order to have good estimates:

$$n \leq \frac{N h_\mu}{\ln(k)}$$

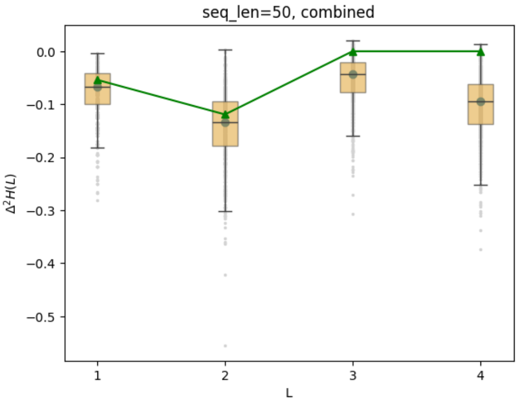
with word length n, length of observation sequence N and number of symbols k. They propose first estimating the entropy rate using Lempel-Ziv complexity(iterative algorithm moving through sequence), then setting the sequence length/word length accordingly.

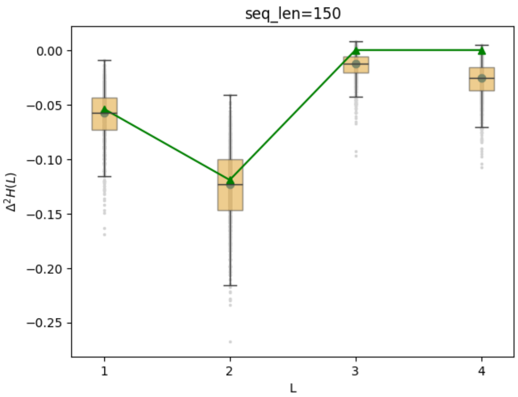
$$\hat{L}_0 = \frac{\mathcal{N}_w \ln(N)}{N}, \ \hat{L} = \frac{\mathcal{N}_w[1 + \log_k \mathcal{N}_w]}{N}, \ \lim_{n\to\infty} \hat{L} = \frac{h_\mu}{\ln(k)}$$

With N the length of the observation sequence and $\mathcal{N}_w$ parsed words from the Lempel-Ziv algorithm. For the plug in estimator, error bars are computable, meaning computable confidence intervals.
Larson et al. (10.1016/j.procs.2011.04.172) use

$$\mathcal{L} h_\mu \geq L|A|^L \ln |A|$$

solving for L given $\mathcal{L}$ returns $W(\mathcal{L} h_\mu)/\ln|A|$ (mathematica), with the Lambert W function. (gives better estimate)





deviation from analytical: [-0.00365034217876947 -0.00365907607011157 -0.0124851947847702 -0.0251811425394552]

4 Conclusion

5 Acknowledgements

Declaration of Authorship

I hereby solemnly declare, by my own signature, that I have independently authored the presented work and have not used any sources or aids other than those indicated. All passages taken verbatim or in content from the specified sources are identified as such.

I consent to the archiving of this Bachelor thesis.

Innsbruck, 8th June 2024 Lukas Prader

References

Anderson, Philip W (1972). 'More Is Different: Broken symmetry and the nature of the hierarchical structure of science.' In: *Science* 177.4047, pp. 393–396. URL: https://www.jstor.org/stable/pdf/1734697.pdf.