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Simulation of open channel network flows using finite element approach

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Abstract

This paper presents a finite element method (FEM) for simulating open channel network flows. Using a recursive formula derived from element equations, a relationship for channel junctions is established and a system equation is obtained. The system equation is much smaller in size in comparison with the simultaneous solution approach and its solution provides boundary conditions for single channels. The FEM is applied to solve individual channels, and a double sweep method is employed to save computation time. To demonstrate the potential of the approach, two channel networks were computed. It is shown that the results obtained with the proposed FEM are very close to those given by the widely used Preissmann scheme, indicating that it is an effective method to model flows in open channels.

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1. Introduction

Simulation of open channel flows has been an important topic in hydraulic engineering. Many numerical methods have been proposed for the solution of this problem, each with its advantages and disadvantages. Much of the research is focused on simulation of flows in single channels [1,4,7,11,13,17,20], though some of them deal with open channel networks (see [2,3,5,15,16,19]). Clearly, simulation of flows in channel networks is more complex due to the need to solve the

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equations at channel junctions, which constitute internal boundaries for each branch. Finite difference methods such as the Preissmann scheme [15] have been widely used in simulating flows in open channel networks.

Finite element method (FEM) is commonly employed to numerically solve partial differential equations, particularly in two and three dimensions. However it has traditionally been considered unsuitable for one-dimensional problems. Cunge et al. [6] who discussed solution methods of the equations of unsteady flow in open channels, did not consider FEM approaches to be appropriate for one-dimensional problems. Nevertheless, this paper applies a FEM to open channel networks and shows that for continuous problems (without the existence of shock or bore), this approach generates results nearly identical to that from the implicit Preissmann scheme. Moreover, a double sweep solution technique is used, saving a substantial amount of computation time, particularly for large-scale channel networks.

2. Governing equations

The governing equations for open channel network flows are St. Venant equations. When a FEM is used, the equations are usually written in a nonconservative form as [15]

$$B\frac{\partial Z}{\partial t}+\frac{\partial Q}{\partial x}=0 \tag{1}$$

$$\frac{\partial Q}{\partial t}+\frac{2\beta Q}{A}\frac{\partial Q}{\partial x}-\frac{\beta Q^2}{A^2}\frac{\partial A}{\partial x}+gA\frac{\partial Z}{\partial x}+\frac{gK}{AR^{4/3}}Q|Q|=0 \tag{2}$$

where A is the cross-sectional area of the channel; B is top width of a channel; Q is the discharge; Z is the water surface elevation; x is the longitudinal coordinate; t is the time; β is the coefficient which corrects for the nonuniform velocity distribution; K is the conveyance, defined as

$$K=\frac{C}{n}AR^{2/3} \tag{3}$$

Here n is Manning's roughness coefficient; R is the hydraulic radius; C is dimensional constant ($C=1$ for SI units and $C=1.49$ for the British units).

For channel networks, two additional equations for mass and energy conservation are needed at a channel junction. Assuming there is no change in storage volume ($\mathrm{d}s/\mathrm{d}t=0$, where s is storage) at a channel junction, the continuity equation may be written as [16,18]

$$\sum Q_{\mathrm{i}}+\sum Q_{\mathrm{o}}=0 \tag{4}$$

The energy equation for channel junctions may be given as [5]

$$\frac{V_{\mathrm{i}}^2}{2}+gZ_{\mathrm{i}}=\int_{x_{\mathrm{i}}}^{x_{\mathrm{o}}}\frac{\mathrm{d}V}{\mathrm{d}t}\,\mathrm{d}x+\frac{V_{\mathrm{o}}^2}{2}+gZ_{\mathrm{o}}+gh_{\mathrm{fi}} \tag{5}$$

where V is the velocity; h_{f} is the head loss due to friction and other local losses. In Eq. (5), the subscript 'i' and 'o' represent the inflow channel and the outflow channel section at the junction. The first term on the right-hand side of Eq. (5) represents energy losses due to acceleration of flow. If the change of velocity and head loss at a junction are ignored, then Eq. (5) becomes

$$Z_{\mathrm{i}} = Z_{\mathrm{o}} \tag{6}$$

St. Venant equations (1), (2) and junction equations (4), (5) or (6) with proper initial and boundary conditions constitute the mathematical model for unsteady flow in an open-channel network.

3. Algorithm

In simulation of flows in channel networks, a key issue is how to deal with junctions because they behave as internal boundary conditions which are parts of the solutions. Here, utilizing the decomposing approach which is similar to that of Schaffranek et al. [15], a channel network is considered as a large system consisting of a number of subsystems, i.e., branches. Segment equations of a branch are obtained by finite element approximation, while system equations for junction solution are established based on branch equations. Junctions are the connection between solutions of system equations and branch equation. By solving system equations, junction solutions are obtained, which are internal and external boundary conditions of branch equations. Then, branch equations are solved and the solution for each segment is obtained. Based on segment solutions, system equations are solved again. Solution is an iteration process between system equations and branch equations.

3.1. Finite element approximation for segment equations

Utilizing the Galerkin FEM and applying it to continuity equation (1), for any element (segment) i, one can obtain following element equation:

$$\begin{bmatrix} \frac{1}{3}\overline{B}\Delta x & \frac{1}{6}\overline{B}\Delta x \\ \frac{1}{6}\overline{B}\Delta x & \frac{1}{3}\overline{B}\Delta x \end{bmatrix} \begin{bmatrix} \frac{\mathrm{d}Z_i}{\mathrm{d}t} \\ \frac{\mathrm{d}Z_{i+1}}{\mathrm{d}t} \end{bmatrix} + \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} Q_i \\ Q_{i+1} \end{bmatrix} = 0 \tag{7}$$

where Δx is the interval length for element i.

Utilizing forward difference for $\mathrm{d}Z/\mathrm{d}t = (Z^{n+1} - Z^n)/\Delta t$ and letting variable:

$$Q = \psi Q^{n+1} + (1-\psi)Q^n \quad (0 \leqslant \psi \leqslant 1) \tag{8}$$

One can obtain following matrix equations:

$$[G^1]\begin{bmatrix} Z_{i+1}^{n+1} \\ Q_{i+1}^{n+1} \end{bmatrix} + [H^1]\begin{bmatrix} Z_i^{n+1} \\ Q_i^{n+1} \end{bmatrix} = \begin{bmatrix} bb_{11} \\ bb_{12} \end{bmatrix} \tag{9}$$

where Δt is time interval, n is time step, $[G^1]$ is 2×2 matrix, let $\omega = \Delta x B/\Delta t$, whose elements are $G^1_{11} = \omega/6$, $G^1_{12} = \psi/2$, $G^1_{21} = \omega/3$, $G^1_{22} = \psi/2$; $[H^1]$ is 2×2 matrix whose elements are $H^1_{11} = \omega/3$, $H^1_{12} = -\psi/2$, $H^1_{21} = \omega/6$, $H^1_{22} = -\psi/2$; $bb_{11} = \omega(Z_i^n + Z_{i+1}^n/2)/3 + (1-\psi)(Q_i^n - Q_{i+1}^n)/2$; $bb_{12} = \omega(Z_i^n/2 + Z_{i+1}^n)/3 + (1-\psi)(Q_i^n - Q_{i+1}^n)/2$.

Similarly, applying finite element approach to momentum equation (2) and letting:

$$\omega_1 = \frac{\Delta x}{3g\overline{A}t}; \quad \omega_2 = \frac{\beta\overline{Q}}{g\overline{A}^2}; \quad \omega_3 = \frac{\beta\overline{Q}^2}{2g\overline{A}^3}; \quad \omega_4 = \frac{\Delta x K|\overline{Q}|}{3\overline{A}^2\overline{R}^{4/3}} \tag{10}$$

where

$$\bar{f} = \chi \frac{f_{i+1}^{n+1} + f_i^{n+1}}{2} + (1-\chi)\frac{f_{i+1}^{n} + f_i^{n}}{2} \tag{11}$$

a matrix form for element equation is derived as

$$[G^2]\begin{bmatrix} Z_{i+1}^{n+1} \\ Q_{i+1}^{n+1} \end{bmatrix} + [H^2]\begin{bmatrix} Z_{i}^{n+1} \\ Q_{i}^{n+1} \end{bmatrix} = \begin{bmatrix} bb_{21} \\ bb_{22} \end{bmatrix} \tag{12}$$

where $[G^2]$ is 2×2 matrix whose elements are $G_{11}^2 = \psi/2$, $G_{12}^2 = \omega_1/2 + \psi\omega_2 + \psi\omega_4/2$, $G_{21}^2 = \psi/2$, $G_{22}^2 = \omega_1 + \psi\omega_2 + \psi\omega_4$; $[H^2]$ is 2×2 matrix whose elements are $H_{11}^2 = -\psi/2$, $H_{12}^2 = \omega_1 - \psi\omega_2 + \psi\omega_4$, $H_{21}^2 = -\psi/2$, $H_{22}^2 = \omega_1/2 - \psi\omega_2 + \psi\omega_4/2$; $bb_{21} = [\omega_1 - (1-\psi)(\omega_4 - \omega_2)]Q_i^n + [\omega_1/2 - (1-\psi)(\omega_2 + \omega_4/2)]Q_{i+1}^n + (1-\psi)(Z_i^n - Z_{i+1}^n)/2 + \omega_3(A_{i+1}^{n+1} - A_i^{n+1})$; $bb_{22} = [\omega_1/2 - (1-\psi)(\omega_4/2 - \omega_2)]Q_i^n + [\omega_1 - (1-\psi)(\omega_2 + \omega_4)]Q_{i+1}^n + (1-\psi)(Z_i^n - Z_{i+1}^n)/2 + \omega_3(A_{i+1}^{n+1} - A_i^{n+1})$.

Above results are obtained when the coefficients of equations are considered as constants. More accurate results can be obtained by considering them to be variables.

Finally, combining Eqs. (9) and (12), one can obtain segment equations as

$$[G]\begin{bmatrix} Z_{i+1}^{n+1} \\ Q_{i+1}^{n+1} \end{bmatrix} + [H]\begin{bmatrix} Z_{i}^{n+1} \\ Q_{i}^{n+1} \end{bmatrix} = \begin{bmatrix} BB_1 \\ BB_2 \end{bmatrix} \tag{13}$$

where $[G]$ is 2×2 matrix whose elements are $G_{11} = G_{11}^1 + G_{21}^1$, $G_{12} = G_{12}^1 + G_{22}^1$, $G_{21} = G_{11}^2 + G_{21}^2$, $G_{22} = G_{12}^2 + G_{22}^2$; $[H]$ is 2×2 matrix whose elements are $H_{11} = H_{11}^1 + H_{21}^1$, $H_{12} = H_{12}^1 + H_{22}^1$, $H_{21} = H_{11}^2 + H_{21}^2$, $H_{22} = H_{12}^2 + H_{22}^2$; $BB_1 = bb_{11} + bb_{12}$; $BB_2 = bb_{21} + bb_{22}$.

3.2. Branch transformation equation

From Eq. (13) and its recursive relationship, one can obtain the branch equation. Letting:

$$U_i^{n+1} = \begin{bmatrix} Z_i^{n+1} \\ Q_i^{n+1} \end{bmatrix} \tag{14}$$

Eq. (13) becomes

$$U_{i+1}^{n+1} = \Pi_{(i)} U_i^{n+1} + \Omega_{(i)} \tag{15}$$

where

$$\Pi_{(i)} = -[G]^{-1}[H] \tag{16}$$

$$\Omega_{(i)} = [G]^{-1}\begin{bmatrix} BB_1 \\ BB_2 \end{bmatrix} \tag{17}$$

Utilizing this recursive formula from the first element (1) to the last element (m), one can obtain branch transformation equation as follows:

$$U_m^{n+1} = \Pi_N U_1^{n+1} + \Omega_N \tag{18}$$

where

$$\Pi_N = \Pi_{(1)}\Pi_{(2)}\cdots\Pi_{(m-2)}\Pi_{(m-1)} \tag{19}$$

$$\Omega_N = \Omega_{(m-1)} + \Pi_{(m-1)}(\Omega_{(m-2)} + \Pi_{(m-2)}(\Omega_{(m-3)} + \cdots + \Pi_{(3)}(\Omega_{(2)} + \Pi_{(2)}\Omega_{(1)})\cdots)) \tag{20}$$

Eq. (18) describes the relationship between the first node and the last node of a branch. Using this relationship, one can establish a system of equations for junction solutions including internal and external boundaries, which will be shown later.

3.3. Double sweep method for improvement of computational time

3.3.1. Double sweep method

By applying double sweep method [12], one can obtain more convenient expressions for Eq. (18) to save computational time. Eqs. (9) and (12) can be written in the following form:

$$H_i'\Delta Z_{i+1} + B_i'\Delta Q_{i+1} = C_i'\Delta Z_i + D_i'\Delta Q_i + G_i' \tag{21}$$

$$H_i''\Delta Z_{i+1} + B_i''\Delta Q_{i+1} = C_i''\Delta Z_i + D_i''\Delta Q_i + G_i'' \tag{22}$$

where Δ represents the difference of variable value between time step $n+1$ and n. The coefficients are as follows:

$$H_i' = \frac{\overline{B}\Delta x}{2\Delta t}; \quad B_i' = \psi \tag{23}$$

$$C_i' = -\frac{\overline{B}\Delta x}{2\Delta t}; \quad D_i' = \psi \tag{24}$$

$$G_i' = \psi(Q_i^n - Q_{i+1}^n) \tag{25}$$

$$H_i'' = \psi \tag{26}$$

$$B_i'' = \frac{3}{2}\omega_1 + 2\psi\omega_2 + \frac{3}{2}\psi\omega_4 \tag{27}$$

$$C_i'' = \psi \tag{28}$$

$$D_i'' = -\frac{3}{2}\omega_1 + 2\psi\omega_2 - \frac{3}{2}\psi\omega_4 \tag{29}$$

$$\begin{aligned} G_i'' = {} & \left[(3\omega_1 - 2\psi\omega_2 + \left(\frac{1}{2} + \psi\right)\omega_4\right]Q_i^n - \left(2\omega_2 + \frac{3}{2}\omega_4\right)Q_{i+1}^n \\ & + (1-2\psi)Z_i^n - Z_{i+1}^n + 2\omega_3(A_{i+1}^{n+1} - A_i^{n+1}) \end{aligned} \tag{30}$$

Now, double sweep method can be applied to Eqs. (21) and (22). Assume that there is a linear relationship between ΔZ_i and ΔQ_i as

$$\Delta Q_i = E_i'\Delta Z_i + F_i' \tag{31}$$

for a point i, it can be proven that an analogous linear relationship also exists for the next point $i+1$:

$$\Delta Q_{i+1} = E_{i+1}'\Delta Z_{i+1} + F_{i+1}' \tag{32}$$

Substituting Eq. (31) into Eq. (21), one can find the relation between ΔZ_i and ΔZ_{i+1}, ΔQ_{i+1}:

$$\Delta Z_i = \frac{H'_i}{C'_i + D'_i E'_i}\Delta Z_{i+1} + \frac{B'_i}{C'_i + D'_i E'_i}\Delta Q_{i+1} - \frac{G'_i + D'_i F'_i}{C'_i + D'_i E'_i} \tag{33}$$

That is

$$\Delta Z_i = LL_i \Delta Z_{i+1} + MM_i \Delta Q_{i+1} + NN_i \tag{34}$$

where

$$LL_i = \frac{H'_i}{C'_i + D'_i E'_i}; \quad MM_i = \frac{B'_i}{C'_i + D'_i E'_i}; \quad NN_i = -\frac{G'_i + D'_i F'_i}{C'_i + D'_i E'_i} \tag{35}$$

Substituting Eqs. (31) and (34) into Eq. (22) yields

$$\begin{aligned}\Delta Q_{i+1} = {} & \frac{H'_i(C''_i + D''_i E'_i) - H''_i(C'_i + D'_i E'_i)}{B''_i(C'_i + D'_i E'_i) - B'_i(C''_i + D''_i E'_i)}\Delta Z_{i+1} \\ & + \frac{(G''_i + D''_i F'_i)(C'_i + D'_i E'_i) - (G'_i + D'_i F'_i)(C''_i + D''_i E'_i)}{B''_i(C'_i + D'_i E'_i) - B'_i(C''_i + D''_i E'_i)}\end{aligned} \tag{36}$$

Eq. (36) can be written as

$$\Delta Q_{i+1} = E'_{i+1}\Delta Z_{i+1} + F'_{i+1} \tag{37}$$

where

$$\begin{aligned} E'_{i+1} &= \frac{H'_i(C''_i + D''_i E'_i) - H''_i(C'_i + D'_i E'_i)}{B''_i(C'_i + D'_i E'_i) - B'_i(C''_i + D''_i E'_i)} \\ F'_{i+1} &= \frac{(G''_i + D''_i F'_i)(C'_i + D'_i E'_i) - (G'_i + D'_i F'_i)(C''_i + D''_i E'_i)}{B''_i(C'_i + D'_i E'_i) - B'_i(C''_i + D''_i E'_i)}\end{aligned} \tag{38}$$

Thus, above equations define following recursive relationship:

$$E'_{i+1} = f(E'_i, H'_i, \ldots) \tag{39}$$

$$F'_{i+1} = f(E'_i, F'_i, \ldots) \tag{40}$$

Substituting Eq. (34) into (31) yields

$$\Delta Q_i = E'_i LL_i \Delta Z_{i+1} + E'_i MM_i \Delta Q_{i+1} + E'_i NN_i + F'_i \tag{41}$$

Combining Eqs. (34) and (41), one derives the following matrix form for segment:

$$\begin{bmatrix}\Delta Z_i \\ \Delta Q_i\end{bmatrix} = \begin{bmatrix} LL_i & MM_i \\ E'_i LL_i & E'_i MM_i\end{bmatrix}\begin{bmatrix}\Delta Z_{i+1} \\ \Delta Q_{i+1}\end{bmatrix} + \begin{bmatrix} NN_i \\ E'_i NN_i + F'_i\end{bmatrix} \tag{42}$$

3.3.2. Branch transformation equation

Similarly as in Section 3.2, letting:

$$\Delta U_i = \begin{bmatrix}\Delta Z_i \\ \Delta Q_i\end{bmatrix} \tag{43}$$

Eq. (42) becomes

$$\Delta U_i = \Pi_{(i)}\Delta U_{i+1} + \Omega_{(i)} \tag{44}$$

where

$$\Pi_{(i)} = \begin{bmatrix} LL_i & MM_i \\ E_i'LL_i & E_i'MM_i \end{bmatrix} \tag{45}$$

$$\Omega_{(i)} = \begin{bmatrix} NN_i \\ E_i'NN_i + F_i' \end{bmatrix} \tag{46}$$

Utilizing this recursive formula, one can obtain branch Transformation equation as follow:

$$\Delta U_1 = \Pi_N \Delta U_m + \Omega_N \tag{47}$$

where

$$\Pi_N = \Pi_{(1)}\Pi_{(2)} \cdots \Pi_{(m-2)}\Pi_{(m-1)} \tag{48}$$

$$\Omega_N = \Omega_{(1)} + \Pi_{(1)}(\Omega_{(2)} + \Pi_{(2)}(\Omega_{(3)} + \cdots + \Pi_{(m-3)}(\Omega_{(m-2)} + \Pi_{(m-2)}\Omega_{(m-1)})\cdots)) \tag{49}$$

By using double sweep method, it is not necessary to compute an inverse matrix when computing $\Pi_{(i)}$ and $\Omega_{(i)}$. As a result, computational time is substantially reduced when computing Π_N and Ω_N in each branch equation, especially for large-scale network systems.

3.4. System equations for junction solution

For any branch, from Eq. (47), there are two branch equations associated with variables Z and Q at beginning node and end node, and there are two boundary conditions which may be one external and one internal boundary condition or two internal boundary conditions or two external boundary conditions. Therefore, there are total four equations for four unknown variables for any branch. Thus, for a network of N branches, the branch-transformation equations, internal boundary conditions and external boundary conditions form a linear system of $4N$ equations in $4N$ unknowns. Matrix expression for the system of equations is

$$[\mathrm{SA}]X = [\mathrm{SB}] \tag{50}$$

where [SA] is the $4N \times 4N$ coefficient matrix, X is vector of $4N$ unknowns and [SB] is the vector of $4N$ constrains. In this system equation, unknowns are the variables of junctions including internal and external boundaries. Solving this system equation, we can obtain the junction solutions. The scale of Eq. (50) is relatively small. For example, when the number of branches is 10 (i.e., $N = 10$), we only have 40×40 size coefficient matrix which are much smaller than that when solving all segment equations simultaneously.

After the junction solutions are obtained, the boundary conditions for each branch are determined. Then, one can use the segment equation (44) to obtain the solutions for every node. Then, repeat this procedure for next time steps.

4. Case studies

To test the effectiveness and generality of the proposed algorithm, two examples were solved to simulate flows in open-channel networks, a single-junction channel network and a multiple-junction channel system.

Example 1. A hypothetical single-junction network system consists of three rectangular channels is shown in Fig. 1. The length of each branch is chosen as 5000 m, channel bottom width is 50 m for two upstream channels and 100 m for the downstream channel. The bed slope is 0.0002. Manning's roughness coefficient is 0.025. Upstream boundary conditions are continuous linear functions for discharge and are shown in Fig. 1. The downstream boundary condition is constant water depth, $y = 1.43$ m. The initial condition is uniform flow with depth $y = 1.43$ m and discharge $Q = 50$ m^3/s in two upstream channels and 100 m^3/s in the downstream channel. Two

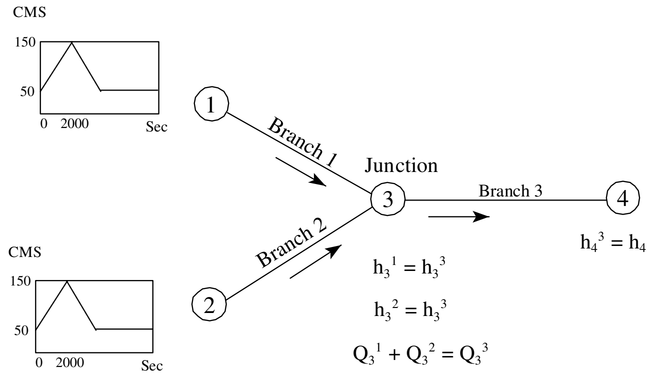

Fig. 1. One junction channel network for Example 1.

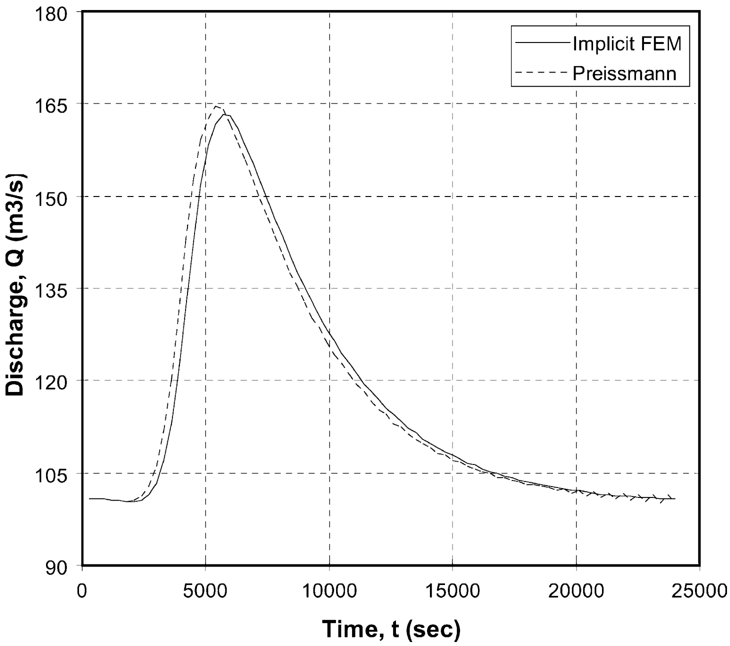

Fig. 2. Discharge hydrograph at $x = 4000$ m in Branch 3 of a channel network for Example 1.

methods were used to solve this problem. One is the implicit finite element approximation (implicit FEM) and another is the widely used Preissmann four-point implicit finite difference scheme. The results are shown in Figs. 2 and 3. From the results, we can see that the solutions are close both for discharge and depth hydrographs. For the discharge hydrographs, the Preissmann scheme generated a little higher peak discharge compared to finite element method, and the peak appears a little earlier in the Preissmann scheme. For the depth hydrographs, the implicit FEM yielded a higher peak depth. In this case, the time interval $\Delta t = 300$ s and weighting factor $\psi = 0.5$ and $\chi = 0.6$. The results can be adjusted by changing the weighting factor ψ. For example, by increasing ψ, peak discharge by the Preissmann scheme will decrease, therefore, leading to more close result between two methods. It was noted that there were some oscillations in the last parts of curves for Preissmann scheme. But if the weighting factor (currently 0.5) in the Preissmann increased, the oscillations would disappear. Also, in this case study, it was noted that the implicit FEM and implicit Preissmann scheme used here were not able to solve this problem when the bed slope is too large or there is a shock wave or bore occurs in flow regime. Both methods generate unstable results. For simulating flow in steep channels or with a shock wave involved, other shock-capture methods such as flux-limiter method should be applied (see [2,9,10,13,14]).

There are several computation parameters in this model which are important since they affect the accuracy, convergence, and stability of the model. These parameters are time interval Δt, element (segment) length Δx and weighting factor ψ and χ. Usually, for an explicit finite difference method, to meet stability condition, the selections of time interval and element length are subject to Courant condition, i.e., a scheme is stable if Courant number is less than one. For an open channel flow, Courant number is define as

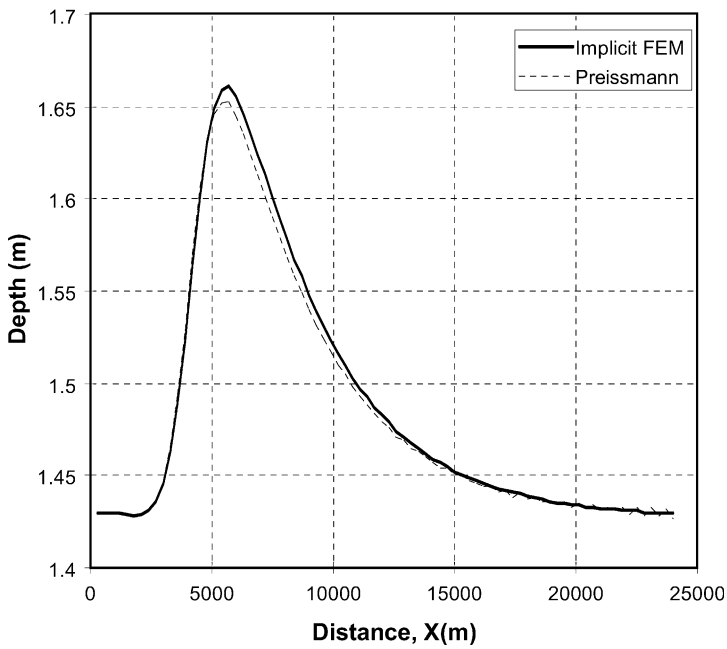

Fig. 3. Depth hydrograph at $x = 4000$ m in Branch 3 of a channel network for Example 1.

$$Cr = |V \pm \sqrt{gA/B}|\frac{\Delta t}{\Delta x} \leqslant 1 \tag{51}$$

Therefore, time interval Δt has to meet following condition:

$$\Delta t \leqslant \frac{\Delta x}{|V \pm \sqrt{gA/B}|} \tag{52}$$

Thus, for a fixed element length, the time interval is restricted for an explicit scheme in order to meet stability condition. However, similar to the Preissmann scheme, for the implicit FEM proposed here, Courant condition does not have to be met to keep numerical stability. Actually, the time interval for implicit FEM can be two to five times larger than that imposed by Courant condition depending on the weighting factor. However, when the time interval increases, the accuracy of the simulation will usually decrease. Therefore, the appropriate time interval needs to be determined based on various considerations.

Example 2. A more complicated dendritic network system is composed of five tributary channels and three main channels and is linked at three channel junctions (Fig. 4, [5]). All channel lengths are 6.0 miles. The channel widths are 100 ft for channels 1, 2, 4, 5 and 7, 200 ft for channel 3, 400 ft for channel 6, and 500 ft for the channel 8. The slope in each channel segment is 0.002 and Manning's roughness coefficient is 0.04. A linear discharge variation with base discharge of 500 ft^3/s and a maximum discharge of 2300 ft^3/s was provided as input data (upstream discharge boundary condition) for channels 1, 2, 4, 5 and 7. The initial discharge and depth are constant throughout the network. The depth of the outlet of channel network keeps unchanged.

The spatial increment of 316.8 ft and the temporal increment of 16 s were used in the simulation. The weighting factor $\psi = 0.6$ and $\chi = 0.6$. Similar as Example 1, above two methods were used to

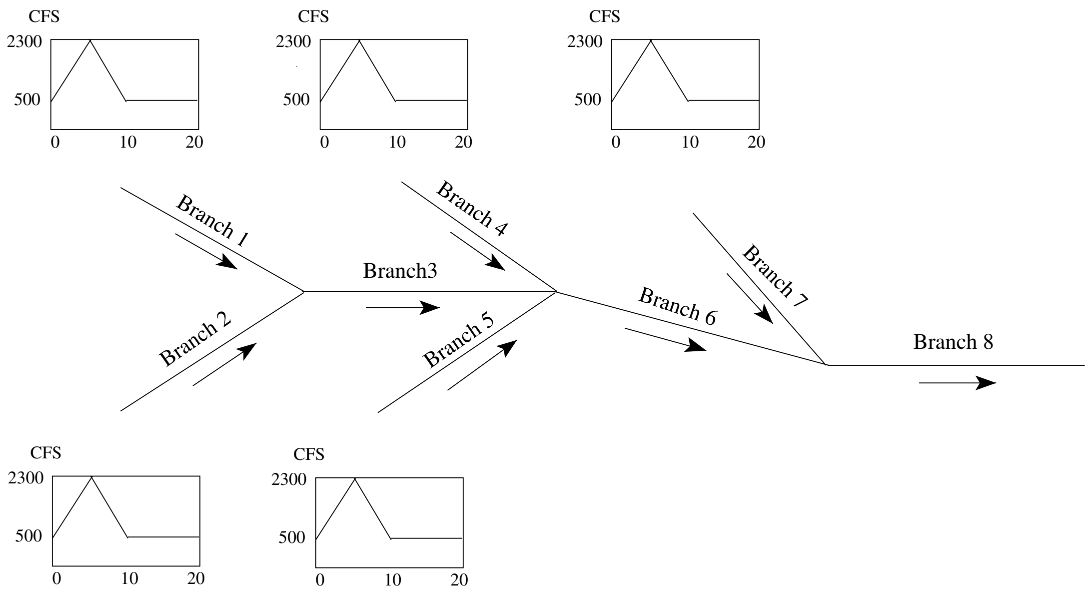

Fig. 4. Dendritic channel network for Example 2.

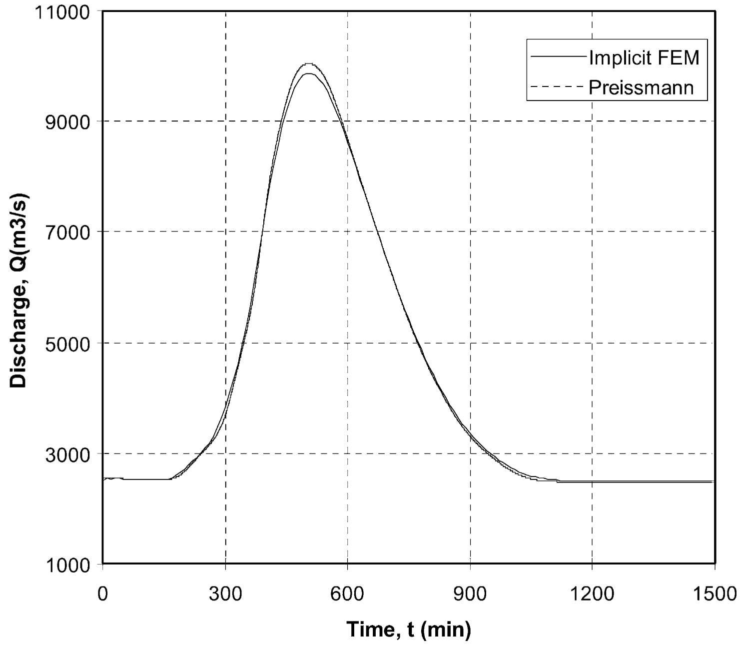

Fig. 5. Hydrograph at middle of channel 8 for Example 2.

solve this problem and a comparison was shown in Fig. 5 which is a discharge hydrograph at the middle of channel 8 ($x = 15840$ m in channel 8). This comparison also shows close agreement between these two methods, while implicit FEM generated a little lower peak discharge.

5. Summary

In this paper, a FEM is presented to compute flows in open channel networks. By using the recursive relation in the segment equation, a junction function is formulated and then a system equation is obtained. The size of this system equation is much smaller than that of the simultaneous solution approach and it is easy to solve. Furthermore, the double sweep method is adopted to save a large amount of computation time. Two test problems of open channel networks are calculated. It is shown that the results obtained with the finite element approximation are very close to those provided by the Preissmann scheme. This indicates that the FEM, together with the other techniques, is an effective method for modeling flows in open channel networks, especially in cases of mild slope channels. But, the approach is probably not appropriate for water jump problems in view that it is associated with the nonconservation form of St. Venant equations.

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