Ch. 12 Grasping and Manipulation

29.9.23

Manipulation: grasping, pushing, rolling, throwing, catching, tapping, ...

Assumption: manipulator, objects and obstacles are rigid

3 relevant elements:

- contact kinematics
- forces applied through contacts
- dynamics of rigid bodies

Contact Kinematics

How rigid bodies can move relative to each other without penetration. Classify: rolling, sliding or separating

Contact force models

Normal and frictional forces that can be transmitted through rolling and sliding contact.

Dynamics

Actual motions of bodies satisfy kinematic constraints, contact force model and rigid-body dynamics

Linear Algebra basics

Linear span

Given j vectors $A = a_1, \dots, a_j \in \mathbb{R}^n$, set of linear combinations

$$\text{span}(A) = \left\{ \sum_{i=1}^{j} k_i a_i \,\middle|\, k_i \in \mathbb{R} \right\}$$

Nonnegative linear combinations (positive or conical span)

Give j vectors $A = a_1, \dots, a_j \in \mathbb{R}^n$, set of nonlinear combinations

$$\text{pos}(A) = \left\{ \sum_{i=1}^{j} k_i a_i \,\middle|\, K_i \geq 0 \right\}$$

Convex span

Given j vectors $A = a_1, \dots, a_j \in \mathbb{R}^n$

$$\text{conv}(A) = \left\{ \sum_{i=1}^{j} k_i a_i \,\middle|\, k_i \geq 0 \text{ and } \sum_i k_i = 1 \right\}$$

$\Rightarrow$ $\text{conv}(A) \subseteq \text{pos}(A) \subseteq \text{span}(A)$

- Space $\mathbb{R}^n$ can be <u>linearly</u> spanned by <u>n</u> vectors but not fewer
- Space $\mathbb{R}^n$ can be <u>positively</u> spanned by <u>n+1</u> vectors but not fewer.

Contact Kinematics

How two or more rigid bodies can move relative to each other. Respecting impenetrability constraint. Classify contacts as rolling or sliding.

First-Order Analysis of a Single Contact

2 rigid bodies, configurations (local coordinates): q_1 and q_2

Composite configuration : $q = (q_1, q_2)$

Distance function : $d(q) > 0$ if bodies seperated
$d(q) = 0$ touching
$d(q) < 0$ penetration

Constraints : $\cdot\, d(q) > 0$: no constraints on motion of bodies
$\cdot\, d(q) = 0$: looking at time derivatives $\dot{d}, \ddot{d}, \ldots$ to determine if bodies stay in contact or break apart

d	$\dot{d}$	$\ddot{d}$	...
>0			no contact
<0			infeasible (penetration)
$=0$	>0		in contact, but breaking free
$=0$	<0		infeasible (penetration)
$=0$	$=0$	>0	in contact, but breaking free
$=0$	$=0$	<0	infeasible (penetration)
$\vdots$	$\vdots$	$\vdots$	

Contact is maintained <u>only</u> if all time <u>derivatives</u> are <u>zero</u>!

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2 bodies that are initially in contact ($d=0$) at a single point:

$$\dot{d} = \frac{\partial d}{\partial q}\dot{q}$$

$$\ddot{d} = \dot{q}^T \frac{\partial^2 d}{\partial q^2}\dot{q} + \frac{\partial d}{\partial q}\ddot{q}$$

Terms $\frac{\partial d}{\partial q}$ and $\frac{\partial^2}{\partial q^2}$ carry information about local contact geometry

- Gradient vector $\frac{\partial d}{\partial q}$: separation direction in q space associated with contact normal
- Matrix $\frac{\partial^2 d}{\partial q^2}$: encodes information about relative curvature of the bodies at contact point.

Assumption/Simplification:

- Only contact-normal information $\frac{\partial d}{\partial q}$ available
- other information about local contact geometry unknown
    - contact curvature $\frac{\partial^2 d}{\partial q^2}$ unknown
    - higher order derivatives unknown

↳ assuming that bodies remain in contact if $\dot{d} = 0$

<u>Contact types: Rolling, Sliding and Breaking free</u>

roll-slide motion: 2 bodies with single-point contact maintaining contact

Constraint that contact is maintained: holonomic constraint $d(q) = 0$

Necessary condition for maintaining contact: $\dot{d} = 0$

in a world frame

- $\hat{n} \in \mathbb{R}^3$: unit vector aligned with contact normal
- $p_A \in \mathbb{R}^3$: contact point on body A
- $p_B \in \mathbb{R}^3$: contact point on body B

p_A and p_B are initially identical

$\dot{p}_A$ and $\dot{p}_B$ may be different

Condition: $\dot{d} = 0$

$\hookrightarrow \hat{n}^T(\dot{p}_A - \dot{p}_B) = 0$

with twists $\mathcal{V}_A = (\omega_A, v_A)$ and $\mathcal{V}_B = (\omega_B, v_B)$

$$\hookrightarrow \dot{p}_A = v_A + \omega_A \times p_A = v_A + [\omega_A] p_A$$

$$\dot{p}_B = v_B + \omega_B \times p_B = v_B + [\omega_B] p_B$$

Wrench $\mathcal{F} = (m, f)$: unit force applied along contact normal

$$\mathcal{F} = (p_A \times \hat{n}, \hat{n}) = ([p_A]\hat{n}, \hat{n})$$

Impenetrability constraint $\dot{d} \geq 0$ (an inequality constraint) can be written:

$$\mathcal{F}^T(\mathcal{V}_A - \mathcal{V}_B) \geq 0$$

If active constraint $\mathcal{F}^T(\mathcal{V}_A - \mathcal{V}_B) = 0$ then (to first order) constraint is active and bodies remain in contact.

Twists $\mathcal{V}_A$ and $\mathcal{V}_B$ satisfying $\mathcal{F}^T(\mathcal{V}_A - \mathcal{V}_B) = 0$:

first-order roll-slide motions, contact may be either rolling or sliding

Rolling if bodies have no motion relative to each other at contact:

Rolling constraint $\dot{p}_A = v_A + [\omega_A] p_A = v_B + [\omega_B] p_B = \dot{p}_B$

Sliding if active constraint $\mathcal{F}^T(\mathcal{V}_A - \mathcal{V}_B) = 0$ but not rolling constraint.

Contact labels:

R: rolling

S: sliding

B: breaking free

Contact Types (summary)

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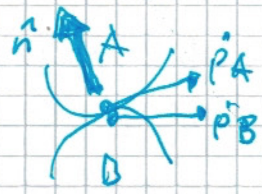
2 Bodies:



Twists of bodies:

$$\mathcal{V}_A = \begin{bmatrix} \omega_A \\ v_A \end{bmatrix} \quad \mathcal{V}_B = \begin{bmatrix} \omega_B \\ v_B \end{bmatrix}$$

expressed in space frame $\{s\}$

Velocities of bodies at current contact points:

$$\dot{p}_A = v_A + \omega_A \times p_A$$
$$\dot{p}_B = v_B + \omega_B \times p_B$$

Contact normal $\hat{n}$ pointing into body A

Impenetrability:

velocity of point A relative to point B in direction of $\hat{n}$ greater or equal to zero

$$\hat{n}^T (\dot{p}_A - \dot{p}_B) \geq 0$$

if greater than zero: two bodies break contact
single inequality constraint.

First-order roll-slide

Contact maintained by first-order analysis

$$\hat{n}^T (\dot{p}_A - \dot{p}_B) = 0$$

single equality constraint

First-order rolling

Stronger condition than first-order-roll-slide.
Bodies don't move relative to each other.

$$\dot{p}_A - \dot{p}_B = 0$$

Sticking, no sliding.
2 equality constraints on planar twists.
3 equality constraints on spacial twists.

Express constraints in terms of twists/wrenches

$$\mathcal{F} = \begin{bmatrix} m \\ f \end{bmatrix} = \begin{bmatrix} p_A \times \hat{n} \\ \hat{n} \end{bmatrix} = \begin{bmatrix} [p_A] \hat{n} \\ \hat{n} \end{bmatrix}$$

use wrench (instead of twist) as it's useful for analysing contact forces

$\rightarrow$ impenetrability: $\mathcal{F}^T (\mathcal{V}_A - \mathcal{V}_B) \geq 0$

first-order roll-slide: $\mathcal{F}^T (\mathcal{V}_A - \mathcal{V}_B) = 0$

first-order rolling: $\dot{p}_A - \dot{p}_B = 0$

Contact Label

Breaking (B) : $\mathcal{F}^T(V_A - V_B) > 0$
Sliding (S) : $\mathcal{F}^T(V_A - V_B) = 0$
Rolling (R) : $\dot{p}_A - \dot{p}_B = 0$

If only one body (A) is moving other body (B) has zero twist and zero velocity:

$$V_B = 0$$
$$\dot{p}_B = 0$$

Constraints in SE(3) and SE(2)

SE(3): $X_A, X_B \in SE(3)$ A and B are spacial bodies

relative twist:

$$V_A - V_B = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \\ v_x \\ v_y \\ v_z \end{bmatrix}$$

Sliding (S) : 1 constraint
relative twist must lie on a 5-d hyperplane of the 6d relative twist space.

Rolling (R) : 3 constraints
relative twist must lie on a 3-d hyperplane of the 6-d relative twist space

SE(2): $X_A, X_B \in SE(2)$ A and B are planar bodies

relative twist

$$V_A - V_B = \begin{bmatrix} \omega_z \\ v_x \\ v_y \end{bmatrix}$$

Sliding (S) :
plane of twists divides the 3-d : · half space of twists break (B) contact
· half space of twists that cause penetration

Rolling (R) :
line of twists for R lies on the S plane

Multiple Contacts

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- Body A
- n contacts with : $i = 1, \dots, n$
- m other bodies : $j = 1, \dots, m$

$\rightarrow n \geq m$

$j(i) \in \{1, \dots, m\}$ number of other body participating in contact i

Each contact i constrains V_A to a half-space of 6-d twist space : bounded by 5-d hyperplane of form $\mathcal{F}^T V_A = \mathcal{F}^T V_{j(i)}$

Union set of constraints from all contacts:

$$V = \{ V_A \mid \mathcal{F}_i^T (V_A - V_{j(i)}) \geq 0 \text{ for all } i \}$$

V: Set of constraints from all contacts : polyhedral convex set (polytope)
$\rightarrow$ feasible twists in V_A

$\mathcal{F}_i$: ith contact normal (pointing into body A)

$V_{j(i)}$: twist of other body at contact i

Constraint at contact i is redundant if half-space constraint contributed by contact i doesn't chance the feasible twist polytope V.

In general: the feasible twist polytope V for a body ca consist of:

- a six-dimensional interior (no contact constraint)
- five-dimensional faces (1 constraint)
- four-dimensional faces (2 constraints)

$\vdots$

- one-dimensional edges
- zero-dimensional points

Twist V_A on a k-dimensional facet of polytope
$\rightarrow$ 6-k independent (non-redundant) contact constraints.

Homogenous Constraint: 6.10.23

If all bodies providing constraints are **stationary**: $V_j = 0$ for all j

then each constraint hyperplane (for impenetrability) passes through origin of V_A space.

↳ feasible twist set:

cone rooted at origin

called (homogenous) polyhedral convex cone

Feasible twist cone V:

$$V = \{ V_A \mid F_i^T V_A \geq 0 \text{ for all } i \}$$

with F_i: constraint wrench of stationary contact i

- if F_i positively spans 6-d wrench space
- or equivalently the convex span of F_i contains origin in its interior

↳ then feasible twist polytope reduces to a point at origin

⇒ the stationary contacts completely constrain the motion of the body

⇒ form closure

Each point contact i can be given a label (B, R, S).
The contact mode for entire system can be written as concatenation of contact labels at contacts.

↳ n contacts ⇒ 3^n contact labels (at max)