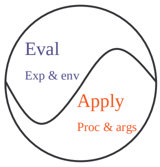
Evaluation and universal machines

- What is the role of evaluation in defining a language?
- How can we use evaluation to design a language?

The Eval/Apply Cycle



- Eval and Apply execute a cycle that unwinds our abstractions
  - Reduces to simple applications of built in procedure to primitive data structures
- Key:
  - Evaluator determines meaning of programs (and hence our language)
- Evaluator is just another program!!

Examining the role of Eval

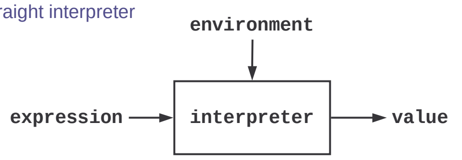
- From perspective of a language designer
- From perspective of a theoretician

Eval from perspective of language designer

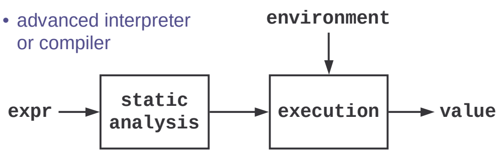
- Applicative order
- Dynamic vs. lexical scoping
- Lazy evaluation
  - Full normal order
  - By specifying arguments
  - Just for pairs
- Decoupling analysis from evaluation

static analysis: work done before execution

- straight interpreter



- advanced interpreter or compiler



Reasons to do static analysis

- Improve execution performance
  - avoid repeating work if expression contains loops
  - simplify execution engine
- Catch common mistakes early
  - garbled expression
  - operand of incorrect type
  - wrong number of operands to procedure
- Prove properties of program
  - will be fast enough, won't run out of memory, etc.
  - significant current research topic

Eval is expensive

```
(eval '(define (fact n)
        (if (= n 1) 1 (* n (fact (- n 1))))) GE)
 ==> undef
              ... (eval '(fact 4) GE) ...
              ... (eval '(= n 1) E1) ...
```

which executes the case statement in **eval** four times

```
              ... (eval '(fact 3) E1) ...
              ... (eval '(= n 1) E2) ...
```

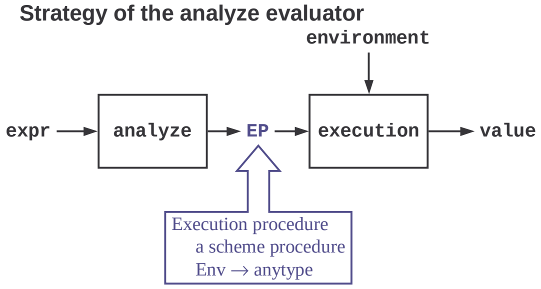
which executes the case statement in **eval** four times

- The analyze evaluator avoids this cost

Summary of part 1

- static analysis
  - work done before execution
  - performance
  - catch mistakes
  - prove program properties
- analyze evaluator
  - static analysis: eliminate execution cost of eval

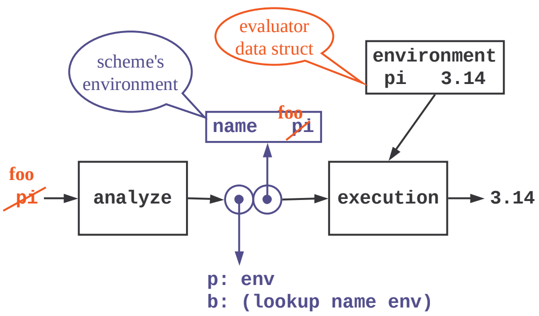
Strategy of the analyze evaluator



analyze: expression → (Env → anytype)

```
(define (a-eval exp env)
   ((analyze exp) env))
```


Example of analyze: variable name lookup



Implementing variable name lookup

```
(define (analyze exp)
  (cond
     ((number? exp)   (analyze-number exp))
     ((variable? exp) (analyze-variable exp))
     ...
     ))

(define (analyze-variable exp)
  (lambda (env) (lookup-variable exp env)))
```

(black: analysis phase)   (blue: execution phase)

Implementing number analysis

- Implementing **analyze-number** is also easy

```
(define (analyze-number exp)
    (lambda (env) exp))
```

(black: analysis phase)   (blue: execution phase)

Summary of part 2

- output of analyze is an execution procedure
  - given an environment
  - produces value of expression
- within analyze
  - execution phase code appears inside `(lambda (env) ...)`
  - all other code runs during analysis phase

Subexpressions (hardest concept today)

```
(analyze '(if (= n 1) 1 (* n (...))))
```

- analysis phase:

```
(analyze '(= n 1))     ==> pproc
(analyze 1)            ==> cproc
(analyze '(* n (...)))==> aproc
```

- execution phase

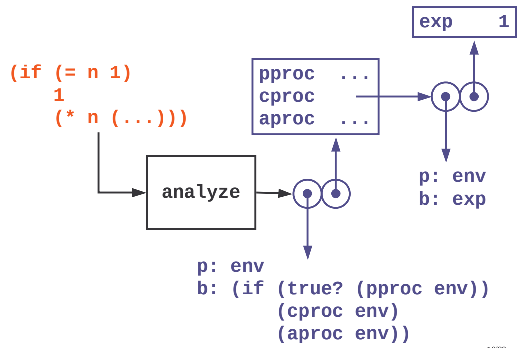
```
(pproc env) ==> #t or #f   (depending on n)
if #t, (cproc env)
if #f, (aproc env)
```


Implementation of `analyze-if`

```
(define (analyze-if exp)
  (let ((pproc (analyze (if-predicate exp)))
        (cproc (analyze (if-consequent exp)))
        (aproc (analyze (if-alternative exp))))
    (lambda (env)
      (if (true? (pproc env))
          (cproc env)
          (aproc env)))))
```

black: analysis phase   blue: execution phase

Visualization of analyze-if



Your turn

- Assume the following procedures for definitions like `(define x (+ y 1))`

`(definition-variable exp)`	`x`
`(definition-value exp)`	`(+ y 1)`
`(define-variable! name value env)`	add binding to env

- Implement **`analyze-definition`**
  - The only execution-phase work is **`define-variable!`**
  - The definition-value might be an arbitrary expression

Implementation of `analyze-definition`

```
(define (analyze-definition exp)
 (let ((var (definition-variable exp))
       (vproc (analyze (definition-value exp))))
    (lambda (env)
      (define-variable! var (vproc env) env))))
```

black: analysis phase   blue: execution phase

Summary of part 3

- Within `analyze`
  - recursively call analyze on subexpressions
  - create an execution procedure which stores the EPs for subexpressions as local state

Implementing lambda

- Body stored in double bubble is an execution procedure
- old **make-procedure**
  list<symbol>, expression, Env → Procedure
- new **make-procedure**
  list<symbol>, (Env->anytype), Env → Procedure

```
(define (analyze-lambda exp)
  (let ((vars (lambda-parameters exp))
        (bproc (analyze (lambda-body exp))))
    (lambda (env)
      (make-procedure vars bproc env))))
```


Implementing apply: execution phase

```
(define (execute-application proc args)
  (cond
   ((primitive-procedure? proc)
    ...)
   ((compound-procedure? proc)
    ((procedure-body proc)
     (extend-environment (parameters proc)
                         args
                         (environment proc))))
   (else ...)))
```


Implementing apply: analysis phase

```
(define (analyze-application exp)
  (let ((fproc  (analyze (operator exp)))
        (aprocs (map analyze (operands exp))))
    (lambda (env)
      (execute-application
          (fproc env)
          (map (lambda (aproc) (aproc env))
               aprocs)))))
```


Summary of part 4

- In the analyze evaluator,
  - double bubble stores execution procedure, not expression

What is Eval really?

- Suppose you were a circuit designer
  - Given a circuit diagram, you could transform it into an electric signal encoding the layout of the diagram
  - Now suppose you wanted to build a circuit that could take any such signal as input (any other circuit) and could then reconfigure itself to simulate that input circuit
  - What would this general circuit look like???
- Suppose instead you describe a circuit as a program
  - Can you build a program that takes any program as input and reconfigures itself to simulate that input program?
  - **Sure – that's just EVAL!! – it's a UNIVERSAL MACHINE**

It wasn't always this obvious

- "If it should turn out that the basic logics of a machine designed for the numerical solution of differential equations coincide with the logics of a machine intended to make bills for a department store, I would regard this as the most amazing coincidence that I have ever encountered"

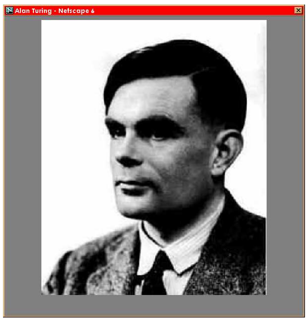
Howard Aiken, writing in 1956 (designer of the Mark I "Electronic Brain", developed jointly by IBM and Harvard starting in 1939)

Why a Universal Machine?

- If EVAL can simulate any machine, and if EVAL is itself a description of a machine, then EVAL can simulate itself
  - This was our example of *meval*
- In fact, EVAL can simulate an evaluator for any other language
  - Just need to specify syntax, rules of evaluation
- An evaluator for any language can simulate any other language
  - Hence there is a general notion of computability – idea that a process can be computed independent of what language we are using, and that anything computable in one language is computable in any other language

Turing's insight

- Alan Mathison Turing
- 1912-1954



Turing's insight

- Was fascinated by Godel's incompleteness results in decidability (1933)
  - In any axiomatic mathematical system there are propositions that cannot be proved or disproved within the axioms of the system
  - In particular the consistency of the axioms cannot be proved.
- Led Turing to investigate Hilbert's Entscheidungsproblem
  - Given a mathematical proposition could one find an algorithm which would decide if the proposition was true of false?
  - For many propositions it was easy to find such an algorithm.
  - The real difficulty arose in proving that for certain propositions no such algorithm existed.
  - In general – Is there some fixed definite process which, in principle, can answer any mathematical question?
  - E.g., Suppose want to prove some theorem in geometry
    - Consider all proofs from axioms in 1 step
    - … in 2 steps ….

Turing's insight

- Turing proposed a theoretical model of a simple kind of machine (now called a Turing machine) and argued that any "effective process" can be carried out by such a machine
  - Each machine can be characterized by its program
  - Programs can be coded and used as input to a machine
  - Showed how to code a universal machine
  - Wrote the first EVAL!

The halting problem

- If there is a problem that the universal machine can't solve, then no machine can solve, and hence no effective process
- Make list of all possible programs (all machines with 1 input)
- Encode all their possible inputs as integers
- List their outputs for all possible inputs (as integer, error or loops forever)
- Define f(n) = output of machine n on input n, plus 1 if output is a number
- Define f(n) = 0 if machine n on input n is error or loops
- But f can't be computed by any program in the list!!
- Yet we just described process for computing f??
- Bug is that can't tell if a machine will always halt and produce an answer

The Halting theorem

- Halting problem: Take as inputs the description of a machine M and a number n, and determine whether or not M will halt and produce an answer when given n as an input
- Halting theorem (Turing): There is no way to write a program (for any computer, in any language) that solves the halting problem.

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Turing's history

- Published this work as a student
  - Got exactly two requests for reprints
  - One from Alonzo Church (professor of logic at Princeton)
    - Had his own formalism for notion of an effective procedure, called the lambda calculus
- Completed Ph.D. with Church, proving Church-Turing Thesis:
  - Any procedure that could reasonably be considered to be an effective procedure can be carried out by a universal machine (and therefore by any universal machine)

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Turing's history

- Worked as code breaker during WWII
  - Key person in Ultra project, breaking German's Enigma coding machine
  - Designed and built the *Bombe,* machine for breaking messages from German Airforce
  - Designed statistical methods for breaking messages from German Navy
  - Spent considerable time determining counter measures for providing alternative sources of information so Germans wouldn't know Enigma broken
  - Designed general-purpose digital computer based on this work
- Turing test: argued that intelligence can be described by an effective procedure – foundation for AI
- World class marathoner – fifth in Olympic qualifying (2:46:03 – 10 minutes off Olympic pace)
- Working on computational biology – how nature "computes" biological forms.
- His death

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