Feedback Systems:
Notes on Linear Systems Theory

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These notes are a supplement for the second edition of *Feedback Systems* by Åström and Murray (referred to as FBS2e), focused on providing some additional mathematical background and theory for the study of linear systems.

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