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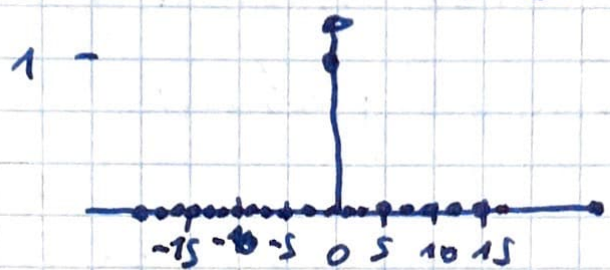
2.1.2 Basic Signals

- Impulse (delta function)

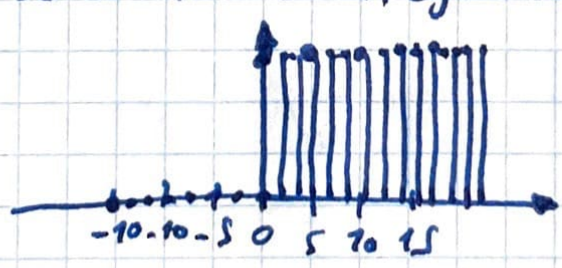
$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



- Unit Step

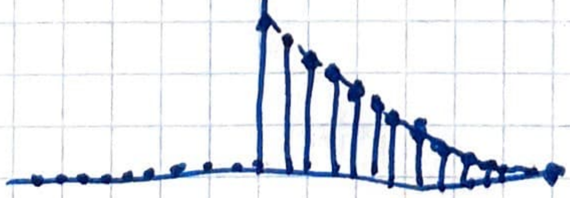
$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

↳ discrete-time integration of pulse



- Exponential Decay

$$x[n] = a^n \cdot u[n] \qquad a \in \mathbb{C}, \ |a| < 1$$



· Complex exponential

$$x[n] = e^{j(\omega_0 n + \phi)}$$

2.1.3 Digital Frequency

Index n represents dimensionless 'time'.
"digital" frequency: radians

Highest frequency manageable by discrete-time system:

$$\omega_{max} = 2\pi$$

↳ greater frequencies 'map back' to a frequency between 0 and 2π

⇒ aliasing

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2.1.4 Elementary Operators

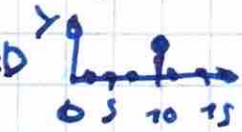
Operations on sequences

- Shift by k

$$y[n] = x[n-k]$$

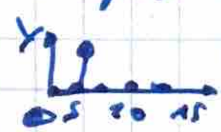
$k > 0$ delay (shift to the right)

$k=5$: 

$\Rightarrow$ 

$k < 0$ advance (shift to the left)

$k=-5$ 

$\Rightarrow$ 

Operator: $\mathcal{D}_k\{x[n]\} = x[n-k]$

- Scaling by a

$$y[n] = a \cdot x[n] \qquad a \in \mathbb{C}$$

$\left.\begin{array}{l} a > 0 : \text{amplification} \\ a < 0 : \text{attentuation} \end{array}\right\}$ if $a \in \mathbb{R}$

if $a \in \mathbb{C}$: attentuation/amplification with phase shift

- Sum of two signals/sequences

$$y[n] = x[n] + w[n]$$

- Product of two signals/sequences

$$y[n] = x[n] \cdot w[n]$$

- Integration

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

$\Rightarrow$ non-normalized running average

- Differentiation (first order difference)

$$y[n] = x[n] - x[n-1]$$

$$\begin{tikzcd}
\begin{array}{c}\text{unit step}\\ u[n]\end{array} \arrow[r, bend left, "\text{differentiation}"] & \begin{array}{c}\text{delta (impulse)}\\ \delta[n]\end{array} \arrow[l, bend left, "\text{integration}"]
\end{tikzcd}$$

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2.1.5 Signal Reproducing Formula

Any signal can be expressed by a linear combination of weighted and shifted impulses.

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$

The weights are the signal values themselves.

2.1.6 Energy and Power

Energy:

$$E_x = \|x\|_2^2 = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

The energy is only finite, if the sum converges (i.e. $x[n]$ is square-summable)
↳ finite-energy signal

! periodic signals are not square-summable

Power:

ratio of energy over time

$$P_x = \lim_{N \to \infty} \frac{1}{2N} \sum_{-N}^{N-1} |x[n]|^2$$

- Finite-energy signals have zero total power (energy dilutes over time)
- Many signals that have
  - infinite energy do have
  - finite power

  (↪ e.g. ~~[illegible]~~ periodic signals)

Power of periodic signals: average energy over a period:

$$P_x = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2$$

2.2. Classes of Discrete-Time Signals

2.2.1 Finite-Length Signals

$x = [\, x_0 \;\; x_1 \;\; \cdots x_{N-1} \,]^T \qquad x \in \mathbb{C}^N$

equivalent notation:

$x[n] \;,\; n = 0, \ldots, N-1$

2.2.2 Infinite-Length Signals

Aperiodic Signals

- Most general type of signal
- cannot be stored nor processed
- Useful for theory

Periodic Signals

- period N

$\tilde{x}[n] = \tilde{x}[n + kN] , \qquad k \in \mathbb{Z}$

$\tilde{x}$: explicit notation for periodic signals

Periodic Extensions

$$\tilde{x}[n] = x[n \bmod N], \quad n \in \mathbb{Z}$$

- 'natural' infinite extension of finite length signal
- repeat the finite signal periodically (period: N)
- shift is circular shift
- Energy of a periodic extension: ∞
- Power: energy of finite-length original signal scaled by $1/N$

Finite support signal

- values are zero for all indices outside of interval

$$\bar{x}[n] = 0, \text{ for } n < M \text{ and } n > M+N-1$$

$$\hookrightarrow \quad \bar{x}[n] = \begin{cases} x[n] & \text{if } 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases}$$

$\bar{x}$: explicit notation for finite support signals

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Signal type	Notation	Energy (E_x)	Power (P_x)
Finite-Length	$x[n],\ n=0,1,\dots,N-1$ $\mathbf{x}\ \mathbb{C}^N$	$\sum_{n=0}^{N-1} \lvert x[n]\rvert^2$	undefined
Infinite-Length	$x[n]\ ,\ n \in \mathbb{Z}$	$\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2 = \lVert x\rVert_2^2$	$\lim_{N\to\infty} \frac{1}{2N} \sum_{-N}^{N-1} \lvert x[n]\rvert^2$
N-Periodic	$\tilde{x}[n]\ ,\ n \in \mathbb{Z}$ $\tilde{x}[n] = \tilde{x}[n+kN]$	∞	$\frac{1}{N} \sum_{n=0}^{N-1} \lvert x[n]\rvert^2$
Finite-Support	$\bar{x}[n]\ ,\ n \in \mathbb{Z}$ $\bar{x}[n] \neq 0$ for $M \leq n \leq M+N-1$	$\sum_{n=M}^{M+N-1} \lvert x[n]\rvert^2$	0