DSP Coursera III   22.6.24

Ch. 10 A/D and D/A Conversion

- discretize amplitude values of signal

10.1. Quantization

- Sampling theorem: band-limited signals as discrete time sequences

- Transform real values of $x[n]$ into format suitable for the processor (DSP, CPU..)

- map real values to countable set of of values

- approximations errors → mapping is irreversible (loss of information)

- divide value interval $[A, B]$ into K non-overlapping subintervals I_k:

$$\bigcup_{k=0}^{K-1} I_k = [A, B]$$

- $K+1$ points i_k such that foreach subinterval : $I_k = [i_k, i_{k+1}]$, $k = 0, \ldots, K-1$
  ↳ no gaps between subintervalls

- Quantization Error
  - information loss: all (infinite) values from intervall I_k are mapped to same discrete value
  - higly nonlinear
  - quantization error: quantizized signal will oscillate randomly with 50% chance between two neighboring quantization levels

- Reconstruction
  - dependent on original / quantizer
  - error introduced by quantization / reconstruction:

$$e[n] = \varepsilon(x[n]) = (\hat{x}[n] - x[n])$$

DSP Coursera III

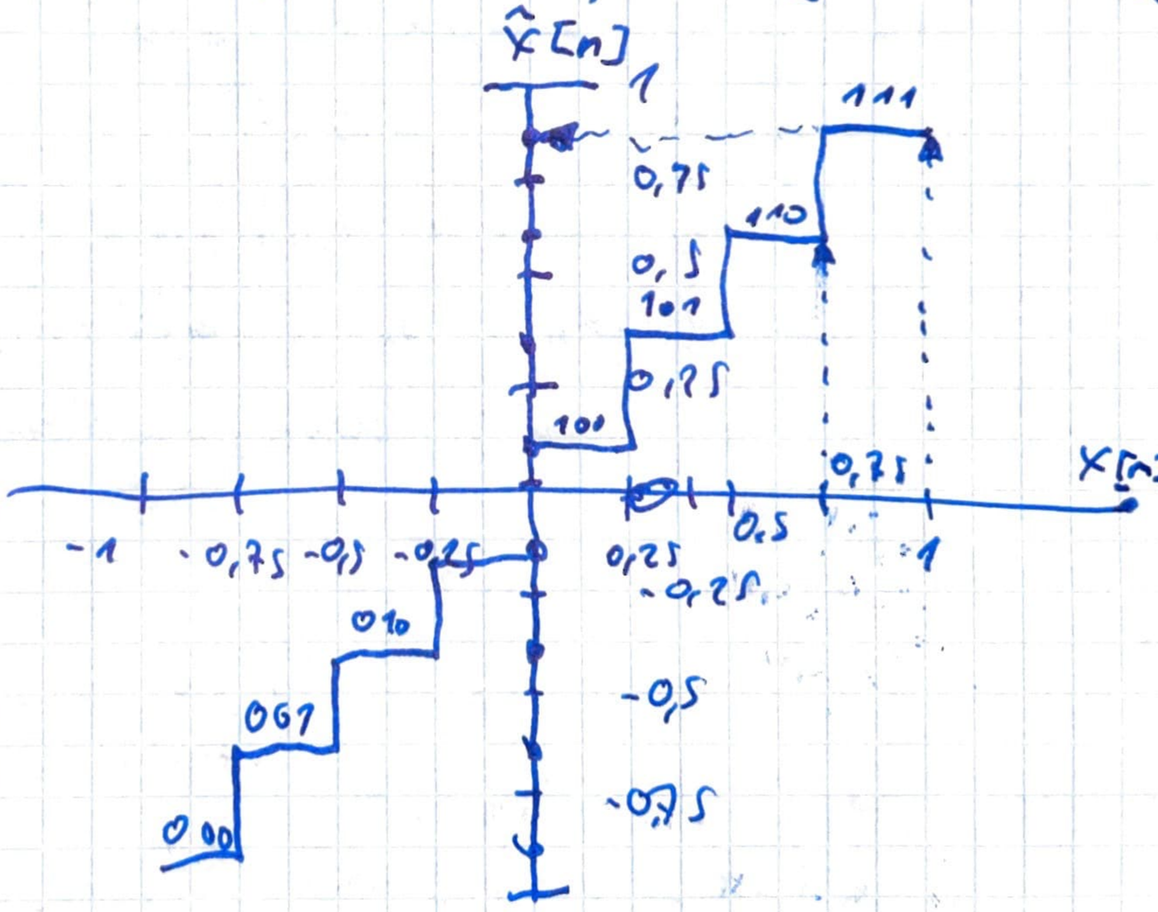
10.1.1. Uniform Scalar Quantization

- non-linear operation: quantization
- dificult to analyse
- statistical description of quantization error
- simplified models often used

- Scalar quantization:
  - each input sample $x[n]$ is quantized independently
  - more sophisticated techiques
    ↳ correlation between neighboring samples ⇒ "vector quantization"

- Uniform quantization
  - given budget of R bits per sample (rate)
    ↳ quantize input signal into $K = 2^R$ levels
  - split range of signal into K intervals
    - intervals should be disjoint
    - cover whole range of input

- Design for uniform quantization
  - range of input $x[n]$
    ↳ interval $[A, B]$, $A, B \in \mathbb{R}$
  - range $[A, B]$ is split into $K = 2^R$ continuous intervals I_k of equal width $\Delta = (B-A)/K$
  - each reconstruction point $\hat{x}_k$ is chosen to be the midpoint of the corresponding interval I_k



DSP Coursera III 28.6.24

- Uniform Quantization of a Uniformly Distributed Input
  - evaluate distortion introduced by quantization
  - assumption about statistics of input signal
    - $x[n]$: uniformly distributed over interval $[A, B]$
    - $x[n]$ is i.i.d process

- signal is uniformly distributed ~~A~~:
  $P[x[n] \in I_k] = 1/K$ for all k
- average Power of input signal
  $\sigma_x^2 = E[x^2[n]] = \frac{(B-A)^2}{12}$
  with $\Delta = (B-A)/K = (B-A)/2^R$
  $\hookrightarrow P_e = \sigma_x^2 2^{-2R}$

  $\Rightarrow$ exponential decay of error as function of the rate !

$SNR = 2^{2R} \Leftrightarrow SNR_{dB} = 10 \log_{10}(2^{2R}) \approx 6R\,dB$

- Rule of thumb:
  each additional bit per sample improves SNR by 6 dB