Foreign Exchange Markets

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Foreign Exchange markets

We will consider a situation where there are two bond markets, a domestic market and a foreign market and will, in line with our previous exposition, use zero coupon bonds as the building blocks of each of these two markets.

We will denote the price at present time t, in units of the domestic currency, of one unit of domestic currency delivered at time T by $P^d(t, T)$.

Similarly, we will denote the price denote the price at present time t, in units of the foreign currency, of one unit of foreign currency delivered at time T by $P^f(t, T)$.

The spot exchange rate

In addition to the domestic and foreign bonds markets, we will assume that there exists a foreign market allowing investors to exchange units of domestic currency for units of foreign currency and vice versa.

We will denote the spot exchange rate by $X(t)$ and define $X(t)$ as

$$\begin{aligned} X(t) &:= \frac{\text{units of domestic currency}}{\text{units of foreign currency}} \\ &= \text{Price of the foreign currency delivered at present time } t \qquad (1) \end{aligned}$$

Thus, to acquire one unit of *foreign* currency at present time t, you have to pay $X(t)$ units of *domestic* currency.

The exchange rate can equivalently be seen as the price in terms of *domestic* currency for one unit of *foreign* currency.

The forward exchange rate

In addition to a spot market for foreign exchange, we also assume that there exists a *forward* foreign exchange market.

That is, it is possible to arrange at present t the exchange of domestic currency for foreign currency delivered in the future at some time T.

The forward exchange rate contracted at present t for the exchange of domestic currency to foreign currency is denoted by $X(t, T)$ and defined as

$$\begin{aligned} X(t,T) &:= \frac{\text{units of domestic currency paid at time } T}{\text{units of foreign currency paid at time } T} \\ &= \text{Price at time } t \text{ of the foreign currency delivered at time } T \quad (2) \end{aligned}$$

So, you can arrange at present time t for one unit of foreign currency to be delivered at future time T and pay $X(t, T)$ units of domestic currency for that one unit of foreign currency at time T.

A spot foreign exchange trade

It is of course possible to exchange both domestic currency for foreign currency and to exchange foreign currency for domestic currency using foreign exchange markets and it is easy to get confused about which exchange rate means what.

As an example, the spot exchange rate to exchange Euro for dollar was on November 7. 0.9272 and in financial market lingo, this exchange rate is denoted *USDEUR* or *USD/EUR* and referred to as *dollareuro*.

The price in Euros for one USD was 0.9219 and hence, you have to forego 0.92720 Euros for each USD you wish to purchase. The *dollareuro* is thus NOT the number of USD you get per one Euro but rather the number of Euros you get for one USD.

If the USD/EUR increases, the USD is strengthened versus the Euro.

FX swaps

Forward exchange rates are quoted in units of $\frac{1}{10000}$ also referred to as *pips*, so if for example the spot USD/EUR is 0.92720 and the 1Y USD/EUR forward exchange rate is quoted as -281 pips, the one year forward USD/EUR exchange rate is 0.92439.

A foreign exchange swap or simply FX swap is the agreement to exchange a given number of units of domestic(foreign) currency for foreign(domestic) currency today and then exchange the foreign(domestic) currency for the given amount of domestic(foreign) currency in the future.

In the given example 1 USD would be exchanged for 0.92720 EUR at present time and in one year, 1 USD would be exchanged for 0.92439 EUR.

FX forwards and FX swaps also involve not only currency risk but also interest rate risk why these types of contracts typically have maturities less than 2 years.

The covered interest rate parity

As always, we are assuming the absence of arbitrage which has implications for the relationship between ZCB prices $p^d(t,T)$, $p^f(t,T)$ and $X(t,T)$ or equivalently between spot rates $R^d(t,T)$, $R^f(t,T)$ and $X(t,T)$.

This relation is well-known and commonly referred to as the *covered interest rate parity*.

This relation arises because of the simple fact that the investor should receive the same return form a risk-free investment in domestic currency as he would from a risk-free investment in the foreign currency arranged when eliminating exchange rate risk using the spot and forward foreign exchange markets.

The covered interest rate parity

At present time, the domestic investor can either invest one unit of domestic currency (EUR) yielding him $e^{R^d(t,T)(T-t)}$ at time T, or

- Exchange one unit of domestic currency (EUR) to receive $\frac{1}{X(t)}$ of foreign currency (USD),
- invest $\frac{1}{X(t)}$ units of foreign currency in the foreign ZCB to receive $\frac{1}{X(t)}e^{R^f(t,T)(T-t)}$ units of foreign currency (USD) at time T,
- sell the $\frac{1}{X(t)}e^{R^f(t,T)(T-t)}$ units of foreign currency (USD) in the *forward* foreign exchange market for a price contracted at time t of $X(t,T)$ to receive $\frac{X(t,T)}{X(t)}e^{R^f(t,T)(T-t)}$ units of domestic currency at time T.

We must in other words have that

$$X(t,T) = X(t)\exp\left\{\left(R^d - R^f\right)(T-t)\right\} = X(t)\frac{p^f(t,T)}{p^d(t,T)} \qquad (3)$$

The uncovered interest rate parity

If there is no forward foreign exchange market, it can be argued under rational expectations that the *uncovered interest rate parity* should hold.

$$\mathrm{E}\big[X(T)\big|\mathcal{F}_t\big] = X(t)\exp\left\{\left(R^d - R^f\right)(T-t)\right\} = X(t)\frac{p^f(t,T)}{p^d(t,T)} \tag{4}$$

The uncovered interest rate parity thus relies on assumptions about the preferences and expectations of investors and is therefore not as firmly anchored as the covered interest rate parity.

Cross currency swaps

To manage FX exposure for longer maturities while avoiding interest rate risk, banks and financial institutions often engage in so called Cross Currency Swaps(CCS).

A cross currency swap is an OTC agreement to exchange a series of floating rate payments in one currency for a series of floating rate payments in another currency.

It used to be standard in CCS markets to exchange 3M xIBOR rates but with the end of xIBOR rates for currencies, cross currency swaps have largely shifted away from xIBOR rates and are now using other reference rates but the principles presented here still apply.

In our treatment, we will denote the reference rate paid at time T_i corresponding to the time period from T_{i-1} to T_i by $L(T_{i-1}, T_i)$ noting that the reference rate is a 3M rate often linked to some overnight funding rate.

Cross currency swaps

Unlike for interest rate swaps, cross currency swaps typically involve the exchange of notional both at the initial and the final date of the contract.

However, CCS is constructed such that it is a zero NPV contract at the initial exchange of notional.

The 'price' of a CCS is quoted in terms of the *par basis swap spread* which is a fixed spread applied to the non-USD leg set in such a way that the contract trades at par and hence is a 0 NPV agreement.

Cross currency swaps

Let us consider a 1Y EUR/USD CCS.

In this case, the domestic currency will be USD and the foreign currency will be EUR. The exchange rate $X(t)$ will be the price of EUR when paying with USD.

We will denote the EUR notional by N and note that the notional in USD becomes $N \cdot X(0)$.

The contract corresponds to

- Receiving a loan of N EUR at time 0 and paying interest payments of EUR LIBOR on the notional of N EUR in the future.
- Issuing a loan of $N \cdot X(0)$ USD at time 0 and receiving an interest of USD LIBOR on the notional of $N \cdot X(0)$ USD in the future.

Cross currency swaps

The cashflows from this EUR/USD cross currency swap then become.

T	Cashflow(EUR)	Cashflow(USD)
0	N	-$NX(0)$
0.25	$-\delta N\left(L^{EUR}(0,0.25)+C\right)$	$\delta X(0)NL^{USD}(0,0.25)$
0.5	$-\delta N\left(L^{EUR}(0.25,0.50)+C\right)$	$\delta X(0)NL^{USD}(0.25,0.50)$
0.75	$-\delta N\left(L^{EUR}(0.50,0.75)+C\right)$	$\delta X(0)NL^{USD}(0.50,0.75)$
1	$-\delta N\left(L^{EUR}(0.75,1)+C\right)-N$	$\delta X(0)NL^{USD}(0.75,1)+X(0)N$

Cross currency swaps

Now that we have seen an example of the cashflows to a CCS, we can compute its value and ultimately choose c such that it has a zero NPV.

Let us assume the notional is exchanged at time T_0, that LIBOR or some other 3M reference rate is exchanged at times $T_1, T_2, ..., T_N$ and the notional is exchanged but in reverse at time T_N.

Now, at this point, let us recall that by the LIBOR replication argument, the time t value of the time T_i future LIBOR payment should be

$$p(t, T_{i-1}) - p(t, T_i) \tag{5}$$

Cross currency swaps

If this replication argument holds, the time t value $V^t(t)$ of the foreign leg in foreign currency should be

$$
\begin{aligned}
V^f(t) &= Np^f(t, T_0) - Np^f(t, T_N) - N\sum_{i=1}^{N}\left(p^f(t, T_{i-1}) - p^f(t, T_i)\right) - \delta NC\sum_{i=1}^{N} p^f(t, T_i) \\
&= -\,\delta NC\sum_{i=1}^{N} p^f(t, T_i) \qquad (6)
\end{aligned}
$$

and the time t value $V^d(t)$ of the domestic leg in domestic currency be

$$
\begin{aligned}
V^d(t) &= X(0)Np^d(t, T_0) - X(0)Np^d(t, T_N) - X(0)N\sum_{i=1}^{N}\left(p^d(t, T_{i-1}) - p^d(t, T_i)\right) \\
&= 0 \qquad (7)
\end{aligned}
$$

Now if that was the case, thew only sensible choice for the par basis swap spread should be 0.

Cross currency swaps

If that is indeed the case and par basis swap spreads were indeed always equal to 0, there should be no need to trade CCS contracts and the market should not exist at all.

The reason why there is a market for CCS contracts and that these do *not* trade at a par basis swap spread of 0 is that in practice, the covered interest rate parity does *not* quite hold.

If we go back to the covered interest from (3), we can use it to deduce an implied foreign(non-USD) LIBOR rate denoted $\hat{L}^f(t, T_{i-1}, T_i)$

$$X(t, T_i) = X(t, T_{i-1}) \frac{p^f(t, T_{i-1}, T_i)}{p^d(t, T_{i-1}, T_i)} = X(t, T_{i-1}) \frac{1 + \delta L^d(t; T_{i-1}, T_i)}{1 + \delta \hat{L}^f(t, T_{i-1}, T_i)} \quad \Rightarrow$$
$$\hat{L}^f(t, T_{i-1}, T_i) = \frac{1}{\delta} \left[\frac{X(t, T_{i-1})}{X(t, T_i)} \left(1 + L^d(t; T_{i-1}, T_i)\right) - 1 \right] \tag{8}$$

Cross currency swaps

The difference between the implied foreign forward LIBOR rate and the forward foreign LIBOR rate implied by foreign bond markets is referred to as the CCS break and will be denoted by $C(t, T_i)$.

$$C(t, T_i) = \hat{L}^f(t; T_{i-1}, T_i) - L^f(t; T_{i-1}, T_i) \tag{9}$$

This break is typically negative reflecting that foreign(non-USD) LIBOR rates implied by foreign exchange markets.

The main explanation why the CCS break deviates from 0 and is often negative is that European financial institutions are typically perceived to be of slightly lower credit quality causing non-USD LIBOR fixings to include a credit premium. The lower credit equality of non-USD financial institutions does not affect the value of a CCS and its break because here, the non-USD loan is collateralized by the equivalent of a USD deposit.

So, how do market participants handle this situation?

It is customary to apply the replication argument to the domestic (USD) leg implying that the value of this leg will be zero by construction.

The foreign leg is then priced by adding the CCS break to forward LIBOR fixings and then applying the replication argument. Since the value of the USD leg is 0, we now have the following equation for the non-USD leg

$$
\begin{aligned}
0 = {} & Np^f(t, T_0) - Np^f(t, T_N) - N\delta \sum_{i=1}^{N} \left[L^f(t; T_{i-1}, T_i) + p^f(t, T_i) C(t, T_i) \right] \\
& - \delta NC \sum_{i=1}^{N} p^f(t, T_i)
\end{aligned}
\tag{10}
$$

Applying the standard replication argument, we get the following relation

$$
0 = N\delta \sum_{i=1}^{N} p^f(t, T_i) C(t, T_i) - \delta NC \sum_{i=1}^{N} p^f(t, T_i) \tag{11}
$$

Cross currency swaps

The CCS break can then easily be found to be

$$C = C(t, T_0, T_N) = \sum_{i=1}^{N} \frac{p^f(t, T_i)}{\sum_{n=1}^{N} p^f(t, T_n)} C(t, T_i) = \sum_{i=1}^{N} w_i C(t, T_i) \qquad (12)$$

The par basis swap break then becomes a weighted average of the CCS breaks actually in a similar manner as the par swap rate was a weighted average of forward LIBOR rates.