Credit Derivatives

Jacob Lundbeck Serup

November 11. 2024

What is credit risk?

Credit risk is very simply and very broadly the risk associated with changes in the ability of a party involved in a financial contract to meet his contractual obligations.

If the credit quality of financial asset changes, this can have huge impact on the value of the asset and managing credit risk is of main concern to any financial institution.

There will of course always be some credit risk involved in financial transactions, but many types of credit risk can be traded in financial markets allowing market participants to manage such risk.

Credit risk is typically traded in the form of credit derivatives and although the market did shrink in size after the financial crisis, it is now larger than ever before.

Credit derivatives

Often when credit risk is discussed in the broad public, the phrases *default* and *bankruptcy* and are used interchangeably but in practice, there is a clear distinction between the two.

The term default is a very wide concept that covers many often less severe breaches of agreement including delays in payments and other technical irregularities.

Bankruptcy, on the other hand is a legal framework under which the debtor can seek, at least temporary, protection from his creditors.

Credit derivatives

In the US, a company can file for 'Chapter 11 bankruptcy' under which the company can negotiate with its investors to renegotiate the terms of it's obligations to salvage the company. The closest Danish equivalent to chapter 11 bankruptcy is so called 'Betalingsstandsning'.

If either the company does not even seek chapter 11 protection or the negotiations fail to result in an agreement, the creditors will effectively take control of the company and the creditors can start to negotiate amongst themselves on how to split any asset that might remain in the company. The Danish equivalent of this state of bankruptcy is 'Konkurs'.

In the event of a bankruptcy shareholders are paid last and typically do not receive any payments and their investment is lost.

Credit derivatives

Two bonds with seemingly identical payment profile might trade at very different prices due to differences in credit quality and by trading in such bonds, it is not hard to get exposure to credit risk.

Even very similar bonds issued by the same company can traded at vastly different prices if one bond has seniority in the event of default compared to the other.

Credit derivatives very much have the flavor of insurance contracts and to find the fair value of such a derivative, one inevitably must account for the likelihood or probability that the some underlying asset fails to meet its contractual obligations.

Credit derivatives and their pricing in many cases will allow us to compute more or less explicitly, the probability of default as priced by the market.

Asset swap

An asset swap is a customized interest rate swap where the coupons on one leg are constructed to match the coupons of an existing asset. This leg is called the asset leg

The other leg is plain vanilla xIBOR(or some other reference rate) + some spread where the spread is called the Asset Swap (ASW) spread. This leg is called the funding leg.

Asset swaps are legally insulated from the underlying asset and the party paying the asset leg still has to make payments if the underlying asset defaults.

The party paying the asset leg thus retains all credit risk but can pass on other types of risks associated with the underlying asset such as interest rate risk or currency risk to the swap counterpart.

Asset swap

To find the 'price' of an asset swap, the parties have to agree on the spread on top of some reference rate that the swap counterpart must pay to the investor in exchange for the payments from the underlying asset.

Now, we have already developed a machinery for extracting the zero coupon term structure of interest rates from xIBOR fixings, FRAs and interest rate swaps, so we can use this curve as a reference rate.

The size of ASW spread for bonds with similar payment schedules but issued by different entities (corporations, governments, etc.) allows us to assess market implied differences in the credit quality of the issuing entities.

Asset swap

Many bond issuers issue bonds for a wide range of maturities and it is customary to plot their ASW spread over time to construct an *asset swap curve* much like the curve of zero coupon spot rates we have used before.

By looking at asset swap curves for a given corporation or government, we can get an idea of how the probability of a credit event occurring depends on time.

The existence of a term structure of ASW spreads allows the investor to single out exposure to credit risk involved with specific events.

From ASW spreads, investors who are not interested in trading asset swaps directly can get a gauge of market expectations with respect to the issuing entity and use this information in other strategies.

Par-par asset swap

It can be difficult to price asset swap packages with different upfront cash payments. Therefore, asset swap packages are very often constructed as a *par-par asset swap*.

A par-par asset swap is traded at par and the difference between the 'dirty' price of the underlying asset and 100 is transferred to the swap counterparty who will compensate the investor through the size of the spread paid on top of xIBOR.

The fact that a par-par asset swap initially trades at par implies that the ASW spread is a direct measure of the excess return on top of LIBOR that the investor receives in exchange for being exposed to the credit risk implied by the underlying asset.

Par-par asset swap

An investor entering into a par-par asset swap is exposed to the ASW spread falling.

The sensitivity of the investors investment to changes in the ASW spread is given directly by the accrual factor of the funding leg.

If the credit quality of the underlying asset is or becomes negative, this reflects that the credit quality of the underlying asset is higher than that of the payers of the institutions who's credit worthiness is implicit in the xIBOR rate.

If the underlying asset trades well below par, the difference from par becomes a 'deposit' with the swap counterpart that is repaid over time. If the underlying trades far below par, this deposit can be substantial and thus represents some credit risk to the investor.

Yield-to-maturity

To avoid the upfront transfer between the investor and the swap counterpart, many investors prefer a so called Yield-to-maturity (YTM) asset swap to the par-par asset swap.

The YTM asset swap matches the underlying asset with a plain vanilla interest rate swap of same maturity.

The YTM asset swap spread is then defined as the difference between the yield to maturity of the underlying bond and the par swap rate on the plain vanilla IRS.

Because of the mismatch between the timing of cashflows, the YTM asset swap spread is a less accurate measure of excess return but nevertheless, differences between ASW and YTM asset swap spreads are often small.

Modeling Credit Risk

Fundamentally, there are two approaches to modeling credit risk, the *structural* and the *reduced form* approach.

The structural form dates back to Mertons work from the early and mid 1970s in which the value of the company's assets evolve over time and are compared to the debt structure of the company.

Within this line of research, the credit risk of a company's debt is priced as a put option on the firms assets and because of it's use of pricing, the structural approach to modeling credit risk is often referred to as *options based modeling*.

The structural approach involves constructing an economic model for the occurrence of a default and gives good intuition to why defaults might happen. However, this approach relies on many unobservable factors and is therefore often not suitable for use in practice.

Modeling Credit Risk

The reduced form approach was pioneered by researchers such as Jarrow, Turnbull, Lando, etc. and relies on a probabilistic approach to credit risk.

This derection of research does not concern itself with what exactly causes default but rather tries to model the risk that such an event occurs.

Under some assumptions, default probabilities can be extracted from market data and these types of models can be fitted to market data.

This line of research makes heavy use of *point processes* in which the probability of default is governed by an intensity. Reduced form modeling is therefore also often referred to as *intensity based modeling*.

Intensity models

As the name 'Intensity Model' suggests, we will be interested in modeling the intensity with which defaults occur and compute probabilities of a default over given time horizons.

These probabilities will often be computed under the risk neutral measure.

We will denote the time of default by τ and not that τ is a random variable with some distribution.

Also, we will define a very simple point process N_t as

$$N_t := \mathbb{1}_{\tau \leq t} \tag{1}$$

N_t is thus a very simple stochastic process that takes the value 0 before default and the value 1 on and after default.

Intensity models

Depending on how exactly we specify the dynamics of N_t, the distribution of τ will change and with it prices of credit derivatives and other assets exposed to credit risk.

The intensity λ_t measures the probability of a default occurring infinitely short after time t under the risk neutral measure $\mathbb{Q}$ and is given by

$$\lambda_t = \lim_{\epsilon \nearrow 0} \mathbb{Q}(\tau \leq t + \epsilon \big| t < \tau) \tag{2}$$

In this expression $\mathbb{Q}(\tau \leq t + \epsilon \big| t < \tau)$ is the probability that default takes place between time t and $t + \epsilon$.

In principle, any stochastic process taking values in the interval $[0, 1]$ can be used to govern λ_t, but in practice, one of three types of processes is often used:

Intensity models

i) **Constant intensity**, $\lambda_t = \lambda$. N_t follows a homogenous Poisson process with intensity λ, τ is exponentially distributed and the default time τ satisfies:

$$\mathbb{Q}(\tau \leq T) = 1 - \exp\left(-\lambda T\right) \tag{3}$$

ii) **Time-varying deterministic intensity**, $\lambda_t = \lambda(t)$. N_t follows an in-homogenous Poisson process with intensity $\lambda(t)$ and the default time τ satisfies:

$$\mathbb{Q}(\tau \leq T) = 1 - \exp\left(-\int_0^T \lambda(s)ds\right) \tag{4}$$

iii) **Stochastic intensity**, λ_t. N_t follows a Cox or doubly stochastic process and the default time τ satisfies:

$$\mathbb{Q}(\tau \leq T) = 1 - \mathrm{E}^{\mathbb{Q}}\left[\exp\left(-\int_0^T \lambda_s ds\right)\Big|\mathcal{F}_0\right] \tag{5}$$

Intensity models

When matching an intensity model to market prices it typically suffices to use a model where the intensity is deterministic but time-varying but for more complicated and realistic models, λ_t should be modelled as a stochastic process.

Often, it is more convenient to work with the probability of survival rather than the probability of default.

The survival probability is of course simply

$$\mathbb{Q}(\tau > T) = 1 - \mathbb{Q}(\tau \leq T) \tag{6}$$

Intensity models

We are now ready to introduce an important tool when modeling credit risk, namely the *Credit Risky Zero Coupon Bond without Recovery* that we will denote by $B(t, T)$.

The time t price of this bond can be computed as an expectation under $\mathbb{Q}$.

$$\begin{aligned} B(t, T) &= \mathrm{E}^{\mathbb{Q}}\Big[\exp\Big(-\int_0^T r_s ds\Big)\mathbb{1}_{T<\tau}\Big] = \mathrm{E}^{\mathbb{Q}}\Big[\exp\Big(-\int_0^T r_s ds\Big)\exp\Big(-\int_0^T \lambda_s ds\Big)\Big] \\ &= \mathrm{E}^{\mathbb{Q}}\Big[\exp\Big(-\int_0^T (r_s + \lambda_s) ds\Big)\Big] \qquad (7) \end{aligned}$$

The intensity with which default occurs can thus be seen as a spread applied directly on top of the short rate.

Intensity models

If the stochastic processes driving the short rate and the default intensity can further be assumed to be independent, computing $B(t, T)$ becomes even simpler

$$\begin{aligned} B(t,T) &= \mathrm{E}^{\mathbb{Q}}\Big[\exp\Big(-\int_0^T (r_s+\lambda_s)ds\Big)\Big] \\ &= \mathrm{E}^{\mathbb{Q}}\Big[\exp\Big(-\int_0^T r_s ds\Big)\Big]\cdot\mathrm{E}^{\mathbb{Q}}\Big[\exp\Big(-\int_0^T \lambda_s ds\Big)\Big] \\ &= P(t,T)\mathbb{Q}(\tau > T) \end{aligned} \tag{8}$$

Now $B(t, T)$ is the price of a derivative that pays 1 unit of currency if there has not been a default by time T.

The price $\tilde{B}(t, T)$ of a derivative that pays 1 unit of currency if there has been a default before time T must then be.

$$\tilde{B}(t,T) = P(t,T)\mathbb{Q}(\tau \leq T) \tag{9}$$

Credit Default Swaps

The credit Default Swap or simply CDS is an insurance type contract in which the credit risk on a given underlying asset is transferred in exchange for fixed periodic premium payments.

The CDS consists of a *fee leg* on which the protection buyer pays a fixed premium that we will denote by C to the protection seller so long as there has not been a credit event.

The other leg in a CDS is called the *protection leg* on which the protection seller pays the *Loss Given Default* (LGD) to the protection buyer in the event that a credit event takes place.

The *Recovery Rate* (RR) on some security is simply defined as $RR = 1 - LGD$.

Credit Default Swaps

Which types of events constitute credit events is of course vital to the valuation of a CDS but unfortunately it is not obvious how to classify credit events.

Typically, the market distinguishes between the more serious *hard* credit events and less serious *soft* credit events.

In Europe, hard credit events include bankruptcy and failure to make a payments whereas various types of debt restructuring such as reductions or postponement of interest or principal payments, increased subordination and change of currency are considered soft credit events.

Credit Default Swaps

If a credit event takes place, CDS contracts can be settled in two different ways. Either by *physical* or *cash* settlement.

If physical settlement is to take place, the protection buyer is obligated to deliver the reference security.

In exchange, the protection seller must pay par value for the delivered bonds.

The protection buyer in other words received the difference between the value of the reference security after the credit event and par. That is, on a net basis, the protection buyer receives the LGD.

Credit Default Swaps

Physical settlement used to be the norm but this is problematic if and when the outstanding CDS contracts outnumber the underlying reference asset.

On several occasions during the financial crisis, assets that had undergone a credit event would rise sharply in price as protection buyers who did not already own it attempted to buy the reference asset.

In the case of cash settlement, an auction process is set up to determine the value of the reference security after the credit event.

THe CDS big-bang of 2009

As a result of the many problems with CDS contracts during the financial crisis, rules changed both in the US and in Europe in early 2009 and the following became standard conventions.

- Determination committees were formed to decide when a credit event has taken place.
- Cash settlement became the standard and all existing CDS contracts were converted to cash settlement.
- Fixed coupons replaced the practice of trading CDSs with different CDS spreads but the market still communicate CDS prices in terms of spreads.

Pricing and hedging of CDSs

To price CDS contacts, we will make the simplifying assumption that default can only take place on coupon dates.

That way, we do not have to deal with fractional coupons if a credit event takes place in-between coupon dates.

Also, we will assume that the intensity process is stepwise constant which given the above assumption is a harmless assumption that nonetheless makes it much easier to integrate $\lambda(t)$.

We could have chosen a more advanced interpolation method between a set of knot-points but a complicated interpolation method would likely be difficult to integrate.

Pricing and hedging of CDSs

We can now compute the value of the fee leg of a CDS starting at time T_n and ending at time T_N as

$$C \sum_{i=n+1}^{N} \alpha_i p(t, T_i) \mathbb{Q}(\tau > T_i) \tag{10}$$

The value of the protection leg is more tricky since it involves both the stochastic LGD payment and the stochastic default time.

However, it is a market standard to assume that $RR = 0.4$ and adjust intensities to match market prices. This practice is somewhat justified by arguing that intensities represent risk neutral probabilities. The value of the protection leg is essentially the expected loss and we get that

$$(1 - RR) \sum_{i=n+1}^{N} p(t, T_i) \mathbb{Q}(T_{i-1} \leq \tau < T_i) \tag{11}$$

Pricing and hedging of CDSs

Finally, we can define the par CDS spread as the value C^* of C that equates the fee leg and the protection leg.

$$C^* = \frac{LGD \cdot \sum_{i=n+1}^{N} p(t, T_i)\Big[\exp\Big(-\int_0^{T_{i-1}} \lambda(s)ds\Big) - \exp\Big(-\int_0^{T_i} \lambda(s)ds\Big)\Big]}{\sum_{i=n+1}^{N} \alpha_i p(t, T_i) \exp\Big(-\int_0^{T_i} \lambda(s)ds\Big)} \quad (12)$$

To compute the par CDS spread, we must integrate the intensity function.

However, if several the par CDS spread can be observed for several maturities, it should be possibly to construct and intensity curve and match this curve to market prices.

Just as we did for the swap curve we could then interpolate the intensity curve to price par CDS contracts for any maturity.

Pricing and hedging of CDSs

When marking CDS contracts to market and computing their value, we can simply compute the difference between the coupon of the CDS and the par CDS spread and multiply this difference with the accrual factor.

Just as was the case with interest rate swaps, it is not market practice to close a CDS.

If one wishes to get out of a CDS contract then, rather than closing the position, an offsetting trade is made.

The offsetting trade will cancel the protection leg but a residual on the fee leg will remain. Since the annuity on the fee leg terminates if a credit event takes place, some intensity risk remains.