Fixed Income Derivatives - Problem Set Week 2

Problem 1

For this problem, assume that the year consists of 12 months each with exactly 30 days and that all payments occur at the end of day on the last day of the quarter. Also assume that there is no credit risk, that the principal of all bonds is 100 and that all interest rates are annualized. The date today is January 15. 2017 but you know the LIBOR fixings from December 30. 2016 and they were 3M LIBOR: 5.2%, 6M LIBOR: 4.9%, 12M LIBOR: 4.76%. The following three interest rates swaps are available in the market

i) A receiver swap with maturity December 30. 2017 receiving fixed semi-annual coupons of 5.1% and paying floating quarterly 3M LIBOR trading for 0.79492002.

ii) A payer swap with maturity December 30. 2018 paying fixed annual coupons of 4.4% and receiving floating semi-annual 6M LIBOR trading for -1.02540877.

iii) A receiver swap with maturity December 30. 2018 receiving fixed annual coupons of 4.9% and paying floating quarterly 3M LIBOR trading for 2.05066409.

In addition to the three interest rate swaps, there are also five fixed rate bullet bonds in the market.

iv) A fixed rate bullet bond maturing December 30. 2017 paying quarterly simple coupons of 7 % and a price of 103.02163487.

v) A fixed rate bullet bond maturing June 30. 2018 paying semi-annual simple coupons of 5 % and a price of 101.80152680.

vi) A fixed rate bullet bond maturing December 30. 2018 paying annual simple coupons of 6 % and a price of 104.48120266.

vii) A fixed rate bullet bond maturing June 30. 2018 paying quarterly simple coupons of 4.5 % and a price of 101.10990798.

viii) A fixed rate bullet bond maturing December 30. 2018 paying quarterly simple coupons of 5.5 % and a price of 103.67216735.

Now that you 8 traded assets in total, please answer the following

a) Set up equations for the value of both the fixed- and floating legs in terms of the LIBOR fixings and zero coupon bond prices for the three interest rate swaps.

b) Construct a cashflow matrix $\mathbf{C}$ with each row corresponding to a traded assets. What is the rank of this matrix? For which maturities can you compute zcb prices?

c) Compute ZCB prices for as many maturities as you can. Is the market complete? Is it arbitrage free?

d) Plot the term structures of ZCB prices, spot rates and 3M forward rates.

e) What would the prices of the 3 swaps and 5 bonds be if all spot rates were to suddenly drop by 10 basispoints (0.1 percentage points)?

f) Find the par swap rate for swap i) today on January 15. 2017. That is, find the size of the fixed coupon such that it would have had a value of 0 today. Can you determine if a trader, who has held this receiver swap since issuance, has made money simply by comparing the par swap rate at issuance to the par swap rate todas

g) Compute the accrual factor of swap i) and use the accrual factor along with the new par swap rate to find the PnL of this swap since the day of issuance.

h) Does your answer conform with other information given in this problem?

Now a trader calls you and offers to sell a fixed rate bullet bond with semi-annual coupons of 5.2% maturing on December 30. 2018 for 100.2.

i) Show that the inclusion of this bond into the market gives rise to an arbitrage?

j) Construct a portfolio that replicates the fixed rate bullet bond the trader is trying to sell you and use this portfolio to construct an arbitrage of type II.

k) Construct a portfolio containing one unit of the bond the trader is trying to sell you such that this portfolio has a price of 0 today, pays a positive amount only on March 30. 2017 and pays exactly 0 at all other future points in time. Check that the price of this portfolio is indeed 0 and hence that you have created an arbitrage of type I. Finally, find the payoff of this portfolio on March 30. 2017.

Problem 2

In this problem, we assume that the 6M Euribor rate has just been announced and that the data below for EUR FRA's and EUR denominated swaps has just been recorded. Recall that the coupon on the fixed leg is paid annually and the coupon on the floating leg semi-annually for EUR denominated swaps.

EURIBOR	Fixing	FRA	Midquote	IRS	Midquote
6M	0.02750	1X7	0.02980	2Y	0.03782
		2X8	0.03122	3Y	0.04152
		3X9	0.03257	4Y	0.04402
		4X10	0.03384	5Y	0.04577
		5X11	0.03504	7Y	0.04797
		6X12	0.03617	10Y	0.04971
		7X13	0.03724	15Y	0.05105
		8X14	0.03825	20Y	0.05170
		9X15	0.03920	30Y	0.05230

In addition to these securities, we will also consider a 10Y fixed rate bond paying a simple semi-annual coupon of 0.05 on a principal of 100 EUR. This fixed rate bond has also just been issued.

a) Calibrate a continuously compounded term structure of ZCB spot rates based on this data and plot the resulting spot rates and instantaneous forward rates. Does your calibration meet the criteria of a well-behaved yield curve?

In addition to the FRA's and the swaps above, we will also consider a 10Y fixed rate bond paying a simple semiannual coupon of 0.05 on a principal of 100 EUR. This fixed rate bond has also just been issued.

b) Compute the price, yield-to-maturity, Macauley duration, modified duration and convexity of the fixed rate bond.

c) Use an expansion to estimate the percentage change in the price of the fixed rate bond for a 10 bps increase in the yield-to-maturity first using only duration and second using both duration and convexity. Compare the two numbers and asses whether also using convexity has changed your estimate significantly.

d) Now compute the accrual factor and par swap rate of the 3Y payer swap knowing that the swap has just been issued and that it therefore has a value of 0.

e) Also compute the accrual factor and par swap rate of the 10Y receiver swap again using that the swap has just been issued and has a value of 0.

In the following, we will use the 3Y and 10Y swaps to hedge the risk to a long position in the fixed rate bond. First consider the case of a 10 bps increase in all ZCB spot rates.

f) Compute the PnL of the fixed rate bond, the 3Y payer swap and the 10Y receiver swap resulting from the increase in ZCB yields assuming the swaps have a principal of 100.

g) In order to hedge the long position in the fixed rate bond against a 10 bps increase in ZCB spot rates using only the 3Y payer swap, what should the principal of the swap be?

h) In order to hedge the long position in the fixed rate bond against a 10 bps increase in ZCB spot rates using only the 10Y receiver swap, what should the principal of the swap be?

i) Relate the size of the hedging positions to the duration of the fixed rate bond and the accrual factors of each of the swaps.

Next, consider a 10 bps increase in all market rates. Strictly speaking the 6M LIBOR rate, the FRA rates and the swap rates are of different types but nevertheless simple ad 10 bps to the data in the table above.

j) Compute the PnL of the fixed rate bond, the 3Y payer swap and the 10Y receiver swap resulting from the increase in market rates assuming the swaps have a principal of 100.

k) In order to hedge the long position in the fixed rate bond against a 10 bps increase in market rates using only the 3Y payer swap, what should the principal of the swap be?

l) In order to hedge the long position in the fixed rate bond against a 10 bps increase in market rates using only the 10Y receiver swap, what should the principal of the swap be?

m) Relate the size of the hedging positions to the duration of the fixed rate bond and the accrual factors of each of the swaps.

n) Discuss pros and cons of hedging the fixed rate bond using the 3Y versus the 10Y swap.