Time Series Analysis of Microsoft's Daily Stock Prices

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Abstract

Using time series analytic tools, this research thoroughly examines Microsoft's daily stock returns over a four-year period. The aim of this task is to forecast Microsoft's daily stock returns and to find underlying patterns and ways of modelling the stock data. The study has a strong emphasis on multivariate time series analysis, capturing the autocorrelation and volatility clustering common to financial time series with ARMA and GARCH models. Microsoft's sales volume and stock prices from April 1, 2015, to May 31, 2021 are included in the dataset. When market events such as the COVID-19 pandemic occur, analysis that focuses on the closing price reveals noteworthy volatility and significant price spikes. After data transformation, volatility analysis is performed. To stabilize variance and normalize returns, log-transformation is used.

The methodology comprises auto-correlation and partial auto-correlation function analysis, and stationarity analysis utilizing Dickey-Fuller and KPSS tests. After discussing the AR, MA, ARMA, and ARIMA processes, GARCH models are applied to identify volatility clustering in stock returns. The use of these techniques is described in full in the results section, which also confirms the stationarity of the log returns and shows the existence of volatility clustering. GARCH(1,1) x ARMA(1,2) is found to be the best fit model by grid search, as determined by AIC, BIC, Shibata, and HQ criteria.

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Contents

1 Introduction

This project aims to conduct a thorough analysis of Microsoft's daily stock returns over a four-year period through the lens of time series analysis. Using methods rooted in exploratory data analysis and time series modelling, the study seeks to identify the underlying patterns, especially forecasting Microsoft's daily stock returns.

One of the key aspects of this project is the exploration of time series analysis through the computation of the logarithmic returns of the Close sales price and the respective squared values to, consequently, focus on ARMA and GARCH models for time series modelling. These models are essential for capturing both the autocorrelation structure and volatility clustering present in financial time series, thereby providing a robust framework for forecasting and risk assessment.

1.1 Related work

The use of GARCH models and their variants is widespread in the field of financial time series analysis due to their ability to effectively capture and model volatility clustering, a common characteristic in financial data. These models have been extensively applied to estimate and forecast the volatility of stock returns, improve risk management practices, and enhance the accuracy of financial predictions. The following subsections review notable studies that have employed GARCH models to analyze stock market data, highlighting their methodologies and key findings.

1.1.1 S&P500 returns with GARCH models

Alfaro and Inzunza (2022) from the Central Bank of Chile investigate the use of GARCH models to estimate daily returns for the S&P500 index, aiming to enhance long-term volatility estimation by incorporating CBOE Volatility Index (VIX) data. Using data from January 2007 to December 2022, the study explores both standard GARCH and Threshold GARCH (TGARCH) models, with and without VIX data, and estimates parameters using maximum likelihood estimation (MLE). The inclusion of VIX data significantly improves the estimation of long-term volatility, with the TGARCH model using the logarithm of VIX showing the best fit. The findings indicate that even a small inclusion of VIX data enhances model performance and provides a reliable estimation of tail-risk measures, validated using the Federal Reserve of Minneapolis's Large Decrease Probability (LDP) index. The study concludes that incorporating VIX data in GARCH models offers more accurate long-term volatility and tail-risk estimations for the S&P500 index, and this approach can be extended to other financial markets.

1.1.2 Forecasting Volatility of Daily Stock Returns Using GARCH Models: Evidence from Dhaka Stock Exchange

Ahmed and Naher (2021) analyze the performance of various GARCH family models in modeling and forecasting the volatility of daily stock returns for the Dhaka Stock Exchange (DSE) over the period from January 2013 to November 2017. The study employs models such as GARCH, APARCH, EGARCH, TGARCH, and IGARCH under both normal and student's t-error distributions. The results indicate that the ARMA (1,1) - TGARCH (1,1) model under student's t distribution provides the best in-sample estimation accuracy, while the ARMA (1,1) - IGARCH (1,1) model excels in out-of-sample forecasting. The study highlights that negative shocks create more volatility than positive shocks, and using the student's t distribution improves the forecasting accuracy of these models.

1.2 Dataset

The dataset we use describes the Microsoft stock, including the price (Open, High, Low and Close) and the volume of sales[1]. In this project we will only focus on the closing price. The time series has 1511 samples collected daily and spans from 1 April 2015 to 31 May 2021. The graph of the time series can be seen in Figure 1.

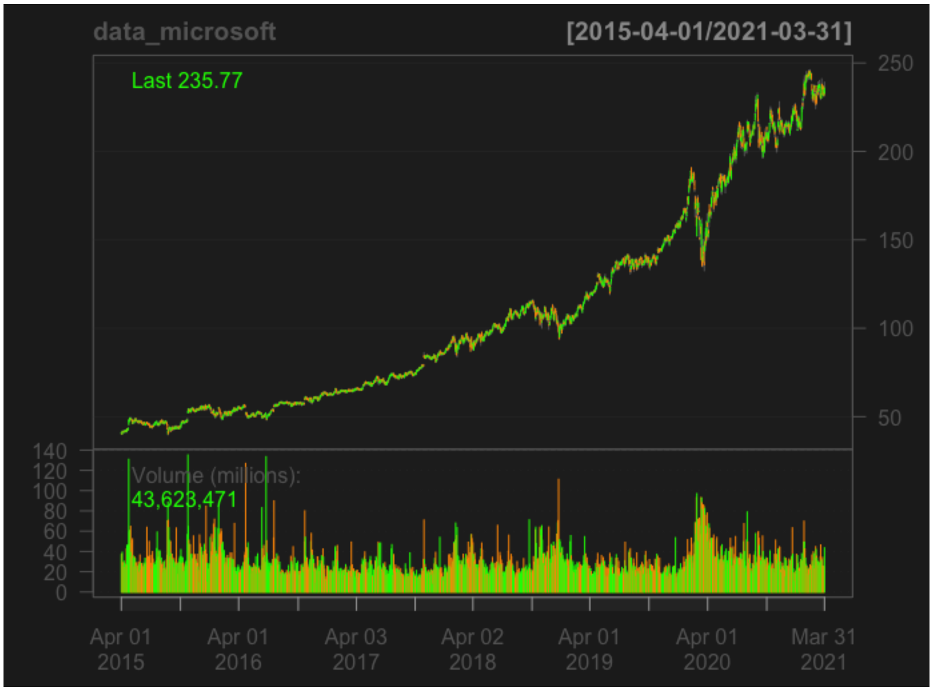

Figure 1: Time Series of Microsoft Stock including Price and Volume

Based on this chart, we can see that the price increased over time from around \$40 to \$235.77. Notable periods include the significant growth in 2017-2018 and the high volatility during the COVID-19 pandemic in 2020, when the price initially fell but quickly recovered. Trading volume spikes, particularly during market-wide events, indicate increased investor activity and confidence in the stock.

1.3 Data Processing and Analysis

Rather than analysing and modelling the price itself, the literature usually examines its variability over time. In particular, ARCH and GARCH models are used for this purpose. Data transformation usually includes the ratio of the price on a given day in comparisons, e.g. using a so-called log-transformation. Its advantage is that it helps in stabilizing the variance and normalizing the distribution of the returns, which are crucial for accurate financial analysis and forecasting.

$$\text{log\_return}_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

where P_t is the closing price at time t, and P_{t-1} is the closing price at time $t-1$.

By performing such an operation on the closing price of Microsoft data, we obtain the following time series.

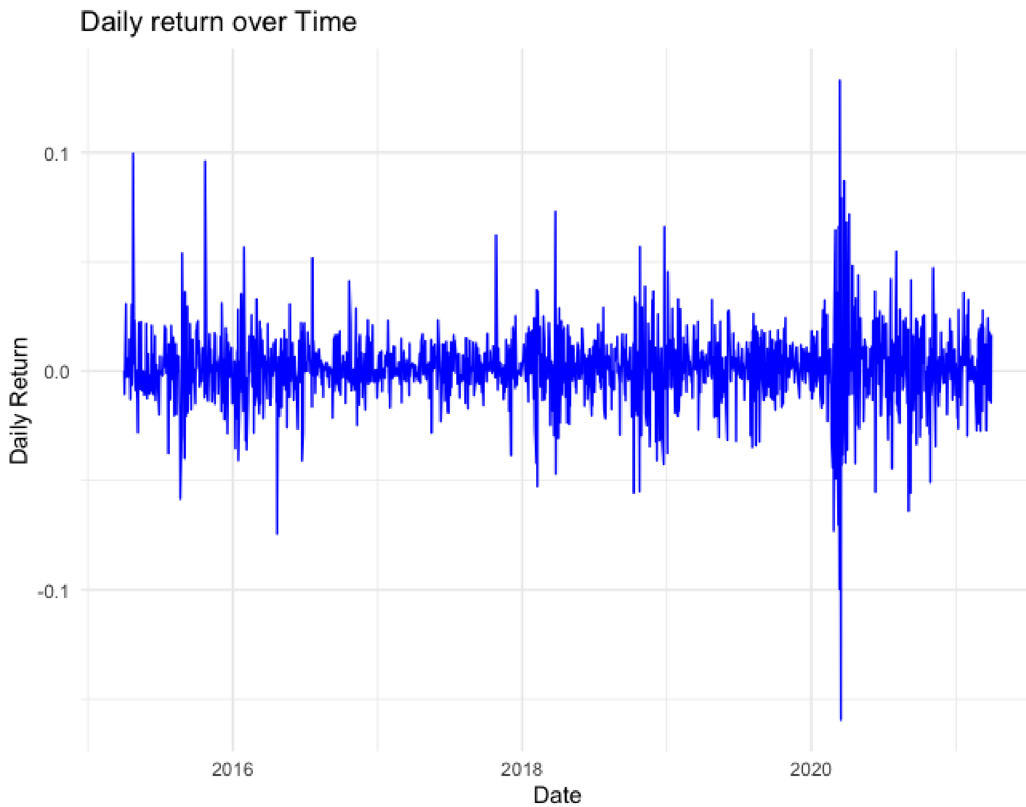

Figure 2: Time Series of Microsoft Stock closing price after log-transformation (log-returns).

From this perspective, it seems natural that we will analyse the volatility in the market, exactly how it has changed from day to day. Especially several key observations can be made:

- **Volatility Clustering**: The plot exhibits periods of high volatility followed by periods of low volatility, a common characteristic in financial time series known as volatility clustering.
- **Impact of Market Events**: Notable spikes in the returns can be observed around significant market events. For instance, the sharp increase and subsequent drop in returns around early 2020 correlate with the onset of the COVID-19 pandemic, which caused substantial market turmoil.
- **Mean Reversion**: Despite the volatility, the returns tend to revert to a mean value over time, indicating the mean-reverting nature of stock returns.

Another important feature is that for GARCH models we need to assume a distribution of the analysed stock data (e.g. normal or t-student distribution). For this reason, let us look at the histogram and the estimated density of the transformed closing price of Microsoft stock.

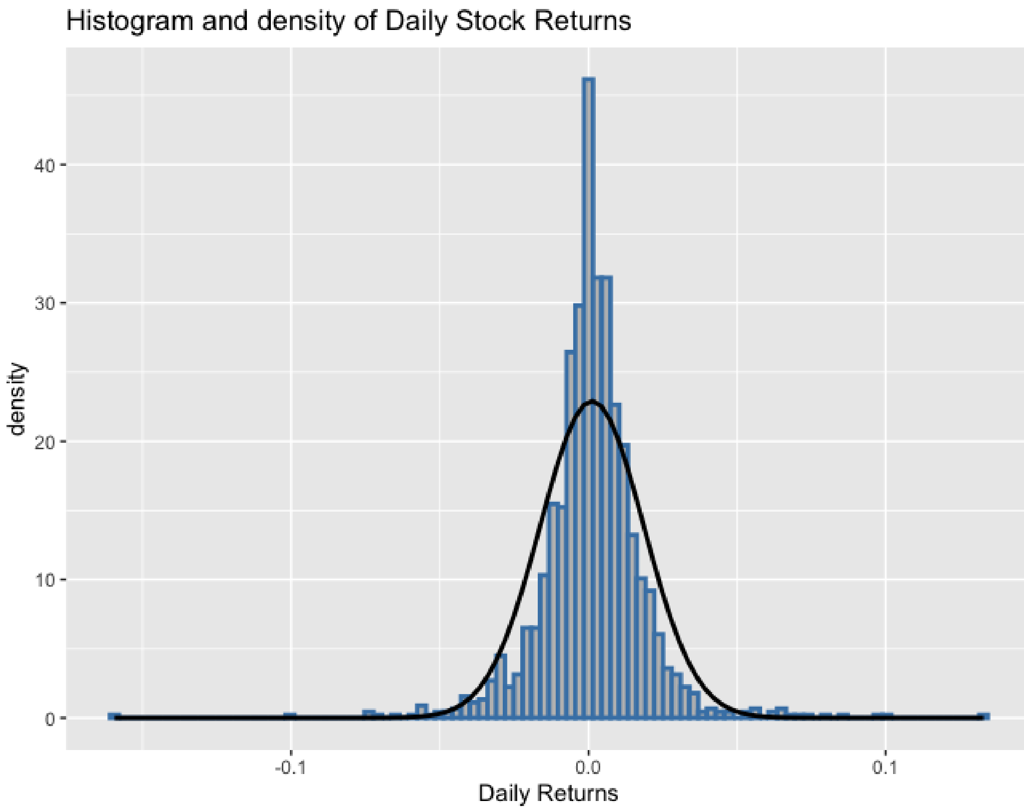

Figure 3: Histogram of the Time Series of Microsoft Stock closing price after log-transformation

The chart shows that most daily returns are close to zero, with occasional extreme positive or negative returns, suggesting the presence of volatility clustering and fat tails in the return distribution.

2 Methodology proposal

In this section, we will describe the methodology used to model the time series under consideration, and in particular focus on the mathematical theory behind the parameter estimation and subsequent prediction. We will also describe some of the assumptions that must be met and how to study the properties of a set of observations measured over time.

2.1 Analysis of trend in data

Before moving on to a factual analysis of the series, we will briefly describe what a trend in the data is, how to detect it, and its relevance to our problem.

A trend in time series data represents a long-term movement or direction in the data over time. It indicates a general tendency for the values to increase, decrease, or remain stable. Trends can be linear or non-linear and can provide significant insights into the underlying patterns of the data.

Trends can be detected using various statistical and graphical methods. Common techniques include:

- **Visual Inspection**: Plotting the data and visually identifying any upward or downward movements.
- **Moving Averages**: Applying moving averages to smooth out short-term fluctuations and highlight longer-term trends.
- **Decomposition Methods**: Decomposing the time series into trend, seasonal, and residual components.
- **Statistical Tests**: Using tests like the Mann-Kendall test to statistically confirm the presence of a trend.

If we find a trend in the series we are analysing, we need to remove it (e.g. by using the ARIMA model) before we can use the GARCH model.

The simplest method is to try to find a model of the linear relationship between time and the values of a given time series. Let's try to do this for our data and then analyse the results.

	Estimate	**Std. Error**	**t value**	**p-value**
(Intercept)	-1.510×10^{-3}	1.249×10^{-2}	-0.121	0.904
Date	1.517×10^{-7}	7.081×10^{-7}	0.214	0.830

Table 1: Summary of Linear Regression Coefficients

Table 1 shows that there is no statistically significant dependency between time and daily returns from the stock. These conclusion is even more visible on the chart of the trend.

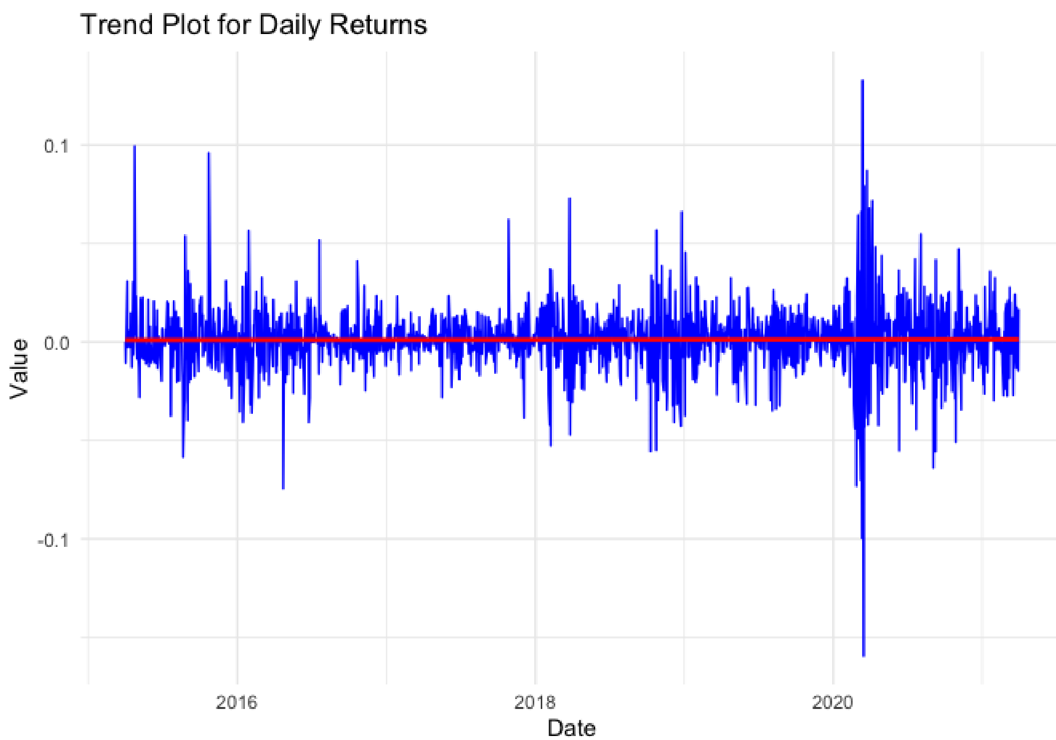

Figure 4: Daily returns over time with line of trend

As we can see in the figure 4, there is no trend pattern to be extracted from our data. Since there is no significant trend in the daily returns of Microsoft stock, we can proceed directly with the GARCH model, as there is no need to remove the trend from the raw data.

2.2 Stationarity analysis

To test the stationarity of the process, we have chosen to run both the Dickey-Fuller and the Kwiatkowski-Phillips-Schmidt-Shin tests. If the p-value of the Dickey-Fuller test is above the threshold for rejecting the null hypothesis ($p = 0.05$) [2], this indicates that the process is stationary. On the other hand, we can only assume that the process is stationary through the Kwiatkowski-Phillips-Schmidt-Shin test if the p-value of the test is below the null hypothesis rejection threshold ($p = 0.05$) [3]. In this case, if both tests return a p-value above $p = 0.05$ or both are below this value, we cannot assume that the process is stationary.

In order to be sure that the process is stationary, we should determine the differential time series and run both stationarity tests again until the returned p-values follow the above pattern. Only then can we proceed with the analysis and modelling.

2.3 Autocorrelation and partial autocorrelation functions

Autocorrelation (ACF) and partial autocorrelation (PACF) functions are statistical measures used to understand the relationship between an observation and its previous observations at different lags. Autocorrelation measures the linear relationship between an observation and its previous observations across various lags, indicating how much the current observation is influenced by past observations. On the other hand, partial autocorrelation focuses on the direct linear relationship between an observation and its previous observations at a specific lag, excluding the influences from intermediate lags. This distinction allows the identification of the order of autoregressive (AR) processes or moving average (MA) models. Table 2 indicates how the shape of both ACF and PACF graphs determine the time of process the time series follows.

Table 2: Relationship between ACF, PACF values and the autoregressive and moving average processes

	ACF	**PACF**
AR(p)	Slow decay to 0	Non-null until the lag p
MA(p)	Non-null until the lag p	Slow decay to 0

2.4 AR, MA, ARMA, and ARIMA processes

An auto-regressive (AR) process uses a linear combination of past values to predict the next value in the series. For instance, an AR(1) model predicts the current value based on the immediately preceding value, while an AR(2) model incorporates both the immediate past and the value two steps back. An AR(0) model represents white noise, indicating no dependency between terms [4]. Equation 1 presents the general expression of an AR process.

$$AR(p) : X_t = a_1 X_{t-1} + ... + a_p X_{t-p} + e_t \tag{1}$$

A moving average (MA) process is a type of time series process that includes moving average terms to capture patterns in the data. An MA(q) model, where q represents the order of the moving average, incorporates the effects of past errors (multiplied by coefficients) to predict future values [5]. Equation 2 presents the general expression of a MA process.

$$MA(q) : X_t = e_t + b_1 e_{t-1} + ... + b_q e_{t-q} \tag{2}$$

The auto-regressive moving average (ARMA) process combines the AR and MA models. It predicts future values based on past observations and errors, as shown in Equation 3.

$$ARMA(p, q) : X_t = a_1 X_{t-1} + ... + a_p X_{t-p} + b_1 e_{t-1} + ... b_q e_{t-q} + e_t \tag{3}$$

The integrated auto-regressive moving average (ARIMA) process follows the same structure of an ARMA process, however it is applied to the integrated time series. An ARIMA(p,d,q) assumes $\Delta^d X_t = ARIMA(p,d,q)$ as stationary, and $\phi(B)$ and $\theta(B)$ represent the AR and MA polynomials, respectively [4]. Equation 4 represents the general expression of ARIMA processes.

$$ARIMA(p,d,q) : \phi(B)\Delta^d X_t = \theta(B)e_t \tag{4}$$

2.5 GARCH modeling

Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models are a significant advancement in the field of finance and econometrics, providing a framework for understanding and predicting the volatility of financial markets. The core idea behind GARCH models is that the variability of financial returns is not constant over time but is instead autocorrelated, meaning that periods of high volatility tend to cluster together. This phenomenon is observable in stock returns, where periods of increased volatility in returns often occur consecutively 4. GARCH models aim to capture this volatility clustering effect, offering a more nuanced approach to understanding and forecasting financial market dynamics compared to traditional methods that assume constant volatility [6].

There are several variations of GARCH models, each designed to address specific characteristics of financial data and market behavior. Notable among these are. The first one is Nonlinear GARCH (NGARCH), which addresses correlation and observed "volatility clustering" of returns. The second variant is Integrated GARCH (IGARCH), functioning by restricting the volatility parameter, aiming to model the long-term persistence of volatility. The third type of GARCH is fGARCH, also known as Family GARCH, which nests a variety of other popular symmetric and asymmetric GARCH models, including APARCH, GJR, AVGARCH, NGARCH [7], [8].

GARCH models do not use model directly the stock time series. In return, they model the logarithmic returns of the original time series, has presented in Equation 5. Typically, the logarithmic returns (r_t), which are represented in Equation 5, present periods of high volatility clustered together, a marginal distribution with kurtosis and no serial correlation, however they have a strong serial correlation in r_t^2 [9].

$$r_t = \frac{x_t - x_{t-1}}{x_{t-1}} = \sigma_t \epsilon_t \tag{5}$$

In this type of modelling and considering GARCH(1,1), the variance of instant t is a linear combination of the squared value of the logarithmic returns and variance of instant $t-1$, as presented in Equation 6. Being that said, the residuals follow a normal distribution, as shown in Equation 7 [9]. The general equation of a GARCH(p,q) model takes into consideration the number of lags the series r and the variance of the process, as shown in Equation 8 [10].

$$\sigma_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{6}$$

$$\epsilon \sim N(0,1), \alpha_1 + \beta < 1 \tag{7}$$

$$GARCH(p,q): \sigma_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + ... + \alpha_1 r_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + ... + \beta_q \sigma_{t-q}^2 \tag{8}$$

In the just mentioned equations, it is assumed that $\alpha_0 > 0$ and that both α_1 and β_1 are equal to or greater than 0. Also, it for GARCH(1,1), $\alpha_1 + \beta_1 < 1$, which confers that the model is stationary.

2.6 GARCH models evaluation

To assess quality of fit for each model we have developed, we have used four criteria. These criteria are Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Hannan-Quinn Information Criterion (HQ), and Shibata's Information Criterion. Each criterion balances the goodness-of-fit of a model against its complexity, measured by the number of parameters used. The AIC value is given by Equation 9, where $\hat{\sigma}$ is the estimated model error variance and k the number of free parameters of the model. AIC measures the relative amount of information lost by a given model: a lower AIC score indicates a model that fits the data better [11].

$$AIC = -2log(\hat{\sigma}^2) + 2k - 1 - \log(2\pi) \tag{9}$$

The BIC value is given by Equation 10, where n represents the number of observations [11]. Similar to AIC, BIC also aims to balance model complexity and goodness of fit but places a stronger penalty on the number of parameters, making it more conservative in choosing complex models.

$$BIC = -2log(\hat{\sigma}^2) + k\log(n) - 1 - \log(2\pi) \tag{10}$$

The HQ value is given by Equation 11 and adjusts the penalty for the number of parameters in a model, providing a compromise between AIC and BIC [11].

$$HQ = -2log(\hat{\sigma}^2) + 2k\log(\log(n)) - 1 - log(2\pi) \tag{11}$$

The Shibata's criterion does not have a universally accepted single formula due to its focus on minimizing the mean integrated square error (MISE) across different sample sizes.

2.7 Volatility Forecasting

The forecast of future volatility is based on the recursive application of the volatility equation. For a forecast horizon h, the forecast of the conditional variance σ_{t+h}^2 is derived iteratively using past returns and variances. The one-step-ahead forecast of the conditional variance is given by:

$$\hat{\sigma}_{t+1}^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i+1}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j+1}^2 \tag{12}$$

For multi-period forecasting, this process can be extended. The two-step-ahead forecast is:

$$\hat{\sigma}_{t+2}^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \hat{\epsilon}_{t+2-i}^2 + \sum_{j=1}^{p} \beta_j \hat{\sigma}_{t+2-j}^2 \tag{13}$$

where $\hat{\epsilon}_{t+1}$ and $\hat{\sigma}_{t+1}^2$ are the one-step-ahead forecasts. This iterative method continues for the desired forecast horizon h.

2.8 Daily Return Forecasting

Once the future volatility is forecasted, the future returns can be predicted by incorporating the mean return and the expected innovation term. The forecasted return for horizon h is:

$$\hat{r}_{t+h} = \mu + \hat{\epsilon}_{t+h} \tag{14}$$

Assuming that the innovation term ϵ_t follows a normal distribution with zero mean and forecasted variance σ_{t+h}^2, the future return can be estimated. The innovation term can be expressed as:

$$\hat{\epsilon}_{t+h} \sim N(0, \hat{\sigma}_{t+h}^2) \tag{15}$$

Thus, the forecasted return is:

$$\hat{r}_{t+h} = \mu + \hat{\sigma}_{t+h} z_{t+h} \tag{16}$$

where $z_{t+h}$is a standard normal random variable.

Table 3: P-values of the Augmented Dickey-Fuller Test and KPSS Test for assessing stationarity

	ADF Test	**KPSS Test**
Close	0.7388	<0.01
log returns	<0.01	>0.1
Squared log returns	<0.01	0.01796

3 Results

In this section we will look at the application of the methodology described to our data. We will start with an analysis of the ACF and PACF, move on to the selection of the most optimal model, and end with an analysis of the residuals of the model to justify its goodness of fit.

3.1 Stationarity of time series

To assess the stationarity of the time series, we have performed the Augmented Dickey-Fuller (ADF) test and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test. The results of the rejection of the null hyposthesis (expressed by the respective p-values) are presented in Table 3.

Analysing the ADF results, the *Close* series shows a p-value of 0.7388, indicating that there is insufficient evidence to reject the null hypothesis of a unit root, suggesting that the *Close* may be non-stationary. The *log returns* series has a p-value less than 0.01, which strongly rejects the null hypothesis of a unit root, implying that the log returns series is likely stationary. The *squared log returns* series also has a p-value less than 0.01, again rejecting the null hypothesis of a unit root, suggesting that this series is stationary.

When it comes to the KPSS results, *Close* series has a p-value less than 0.01, which is significant evidence against the null hypothesis of stationarity, indicating that this series is likely non-stationary. The *log returns* series has a p-value greater than 0.1, failing to provide strong evidence against the null hypothesis of stationarity, thus suggesting that this series might be stationary. The *squared log returns* series has a p-value of 0.01796, which is below the conventional significance level of 0.05 but still suggests that there is a possibility of the series being non-stationary, although the evidence is weaker compared to the other tests.

In summary, the ADF tests suggest that the *log returns* and *squared log returns* series are stationary, while the KPSS tests indicate that only the *log returns* series might be stationary. Therefore, we con only be confident of the stationarity of *log returns*.

3.2 ACF and PACF of time series

Firstly, it is worth observing the squared values of the data, specifically their ACF analysis. If we detect that the observations are highly correlated with each other, it is a sign that it is worth using the GARCH modeling to understand their variability (daily returns). As seen in Figure 5, the ACF of the squared raw time series decays very slowly to zero. It corroborates the fact that this time series has long memory and observations are highly correlated.

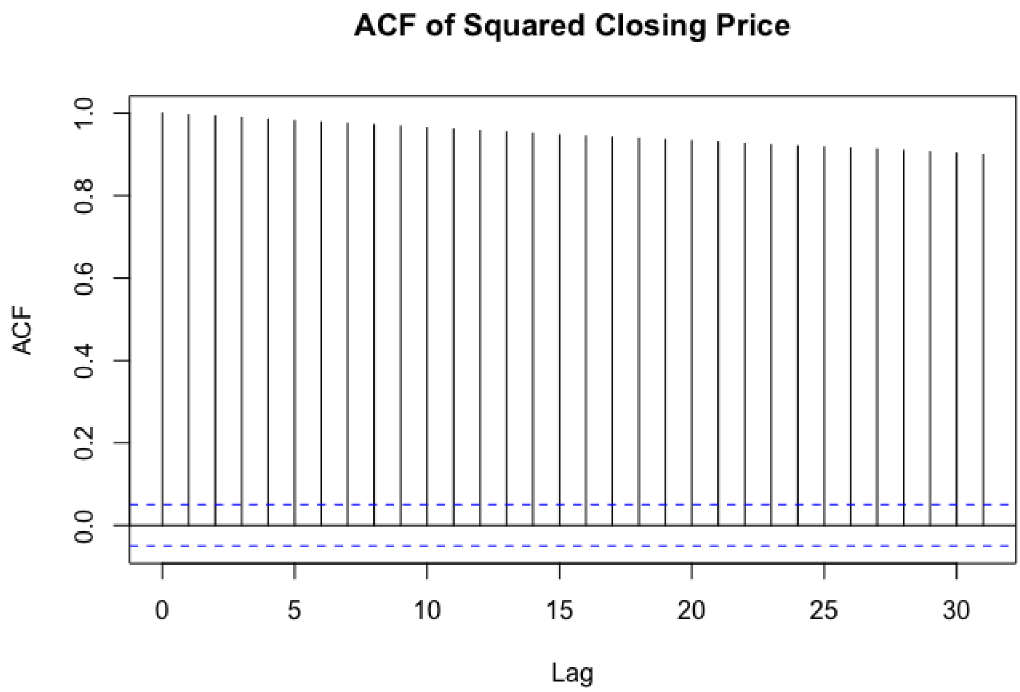

Figure 5: Autocorrelation function of Squared Closing Price

In order to further apply GARCH modelling, the logarithmic transformation was applied to the original *Close* time series, which is also referred to as *log returns*. Let us now look at the ACF and PACF graphs of log returns. The values of both ACF and PACF to do not decrease continuously to zero, as seen in Figure 6, showing considerable volatility and odd auto-correlation of the time series. This strongly suggests that the time series should be modeled through GARCH [12]. When we observe the ACF and PACF values over the lags in Figure 7, both functions decay slower to zero, especially ACF. This was observed by Hossain et al. [13], when the ACF values the log returns of Japanese stock exchange quickly decayed to zero and the ACF values of the squared log returns of the same time series dacayed slowly to zero. With this information, they indicated that such behaviour indicated that log returns time series presented the ARCH or GARCH effect, which corroborates the approach taken in this project.

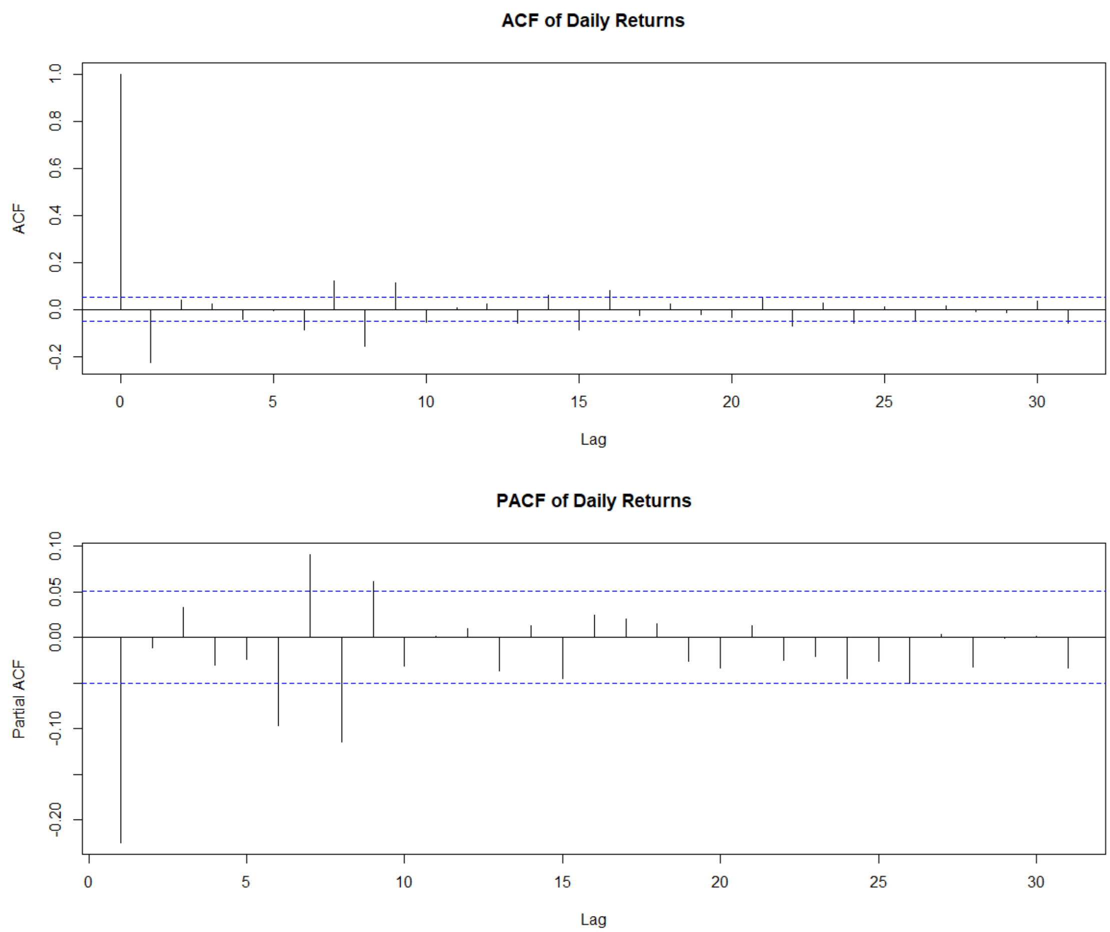

Figure 6: Autocorrelation and partial autocorrelation functions of daily log returns

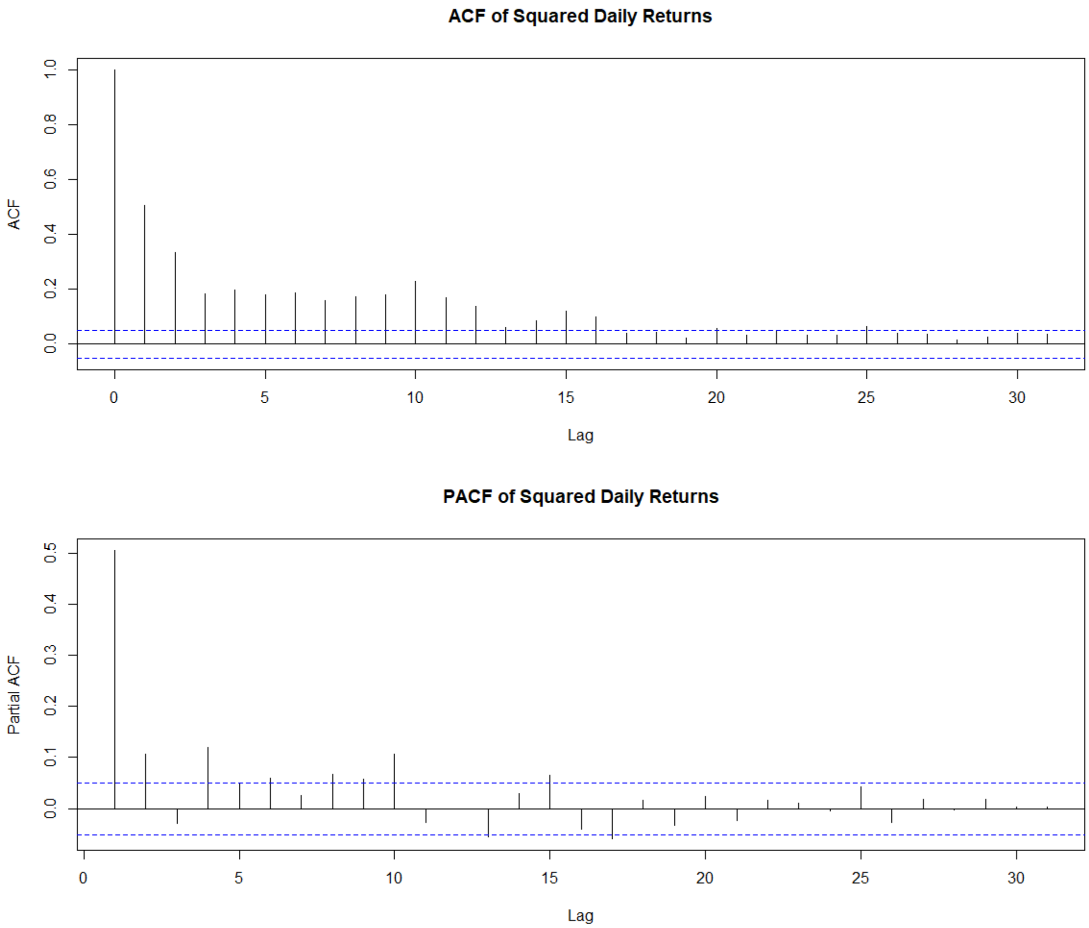

Figure 7: Autocorrelation and partial autocorrelation functions of squared daily log returns

3.3 ARIMA/GARCH modelling

After an initial analysis of the ACF and PACF of the raw and reprocessed data, it is appropriate to model the stock price volatility over time, which allows to explain this phenomenon. For this reason, it is worth focusing on the parameters to be selected. As we described in the theoretical introduction on GARCH, these will be the parameters p and q. In addition, we can verify if adding the ARMA component increases the fitting-goodness of the model to any extent. Tables 4 and 5 show the results of the so-called grid search, containing all the possible values for both p and q in the GARCH and ARMA components. As we can see, the best fit was the GARCH(1, 1) x ARMA(1, 2) model, since the AIC, BIC, Shibata criterion and HQ values were the lowest, as mentioned in the introductory part.

p	q	AR(p)	MA(q)	AIC	BIC	Shibata	HQ
1	0	0	0	-5.471359	-5.460796	-5.471367	-5.467425
1	0	1	0	-5.471752	-5.457667	-5.471766	-5.466506
1	0	0	1	-5.471943	-5.457858	-5.471957	-5.466697
1	0	1	1	-2.393483	-2.375877	-2.393504	-2.386926
1	0	1	2	-5.505505	-5.484378	-5.505536	-5.497637
1	0	2	2	-1.517039	-1.492391	-1.517082	-1.507860
2	0	0	0	-5.521259	-5.507174	-5.521273	-5.516014
2	0	1	0	-5.524582	-5.506976	-5.524604	-5.518025
2	0	2	0	-5.524655	-5.503528	-5.524687	-5.516788
2	0	2	2	-5.541687	-5.513518	-5.541743	-5.531197
0	1	0	0	-5.264105	-5.253541	-5.264113	-5.260171
0	1	1	0	-5.309866	-5.295781	-5.309880	-5.304621
0	1	2	0	-5.311352	-5.293746	-5.311374	-5.304796
0	1	0	1	-5.307998	-5.293913	-5.308012	-5.302753
0	1	2	1	-5.320809	-5.299682	-5.320840	-5.312941
1	1	0	0	-5.574924	-5.560839	-5.574938	-5.569679
1	1	1	0	-5.580044	-5.562438	-5.580066	-5.573488
1	1	2	0	-5.579674	-5.558547	-5.579706	-5.571807
1	1	0	1	-5.580476	-5.562870	-5.580498	-5.573920
1	1	1	1	-5.584520	-5.563393	-5.584551	-5.576652
1	1	2	1	-5.587538	-5.562890	-5.587581	-5.578359
1	1	0	2	-5.579735	-5.558608	-5.579766	-5.571867
1	**1**	**1**	**2**	**-5.587584**	**-5.562936**	**-5.587627**	**-5.578405**
1	1	2	2	-5.586389	-5.558219	-5.586445	-5.575899
2	1	0	0	-5.573678	-5.556072	-5.573700	-5.567122
2	1	1	0	-5.578772	-5.557645	-5.578804	-5.570904
2	1	2	0	-5.578351	-5.553702	-5.578393	-5.569172
2	1	0	1	-5.579201	-5.558074	-5.579233	-5.571334
2	1	1	1	-5.583250	-5.558602	-5.583293	-5.574071
2	1	2	1	-5.586214	-5.558045	-5.586270	-5.575724

Table 4: GARCH/ARMA model parameters and statisticc

p	**q**	**AR(p)**	**MA(q)**	**AIC**	**BIC**	**Shibata**	**HQ**
2	1	0	2	-5.578411	-5.553763	-5.578454	-5.569232
2	1	1	2	-5.586261	-5.558091	-5.586317	-5.575771
2	1	2	2	-5.585065	-5.553375	-5.585136	-5.573264
0	2	0	0	-5.264911	-5.250826	-5.264925	-5.259666
0	2	1	0	-5.312720	-5.295114	-5.312741	-5.306163
0	2	2	0	-5.311527	-5.290399	-5.311558	-5.303659
0	2	0	1	-5.310992	-5.293386	-5.311014	-5.304436
0	2	1	1	-5.311487	-5.290360	-5.311518	-5.303619
0	2	2	1	-5.320890	-5.296242	-5.320933	-5.311711
0	2	0	2	-5.311853	-5.290726	-5.311884	-5.303985
0	2	1	2	-5.310545	-5.285897	-5.310588	-5.301366
0	2	2	2	-5.317433	-5.289263	-5.317488	-5.306942
1	2	0	0	-5.573937	-5.556331	-5.573959	-5.567381
1	2	1	0	-5.578899	-5.557772	-5.578930	-5.571031
1	2	2	0	-5.578434	-5.553786	-5.578477	-5.569255
1	2	0	1	-5.579317	-5.558190	-5.579348	-5.571449
1	2	1	1	-5.583335	-5.558686	-5.583378	-5.574156
1	2	2	1	-5.576642	-5.548473	-5.576698	-5.566152
1	2	0	2	-5.578500	-5.553851	-5.578542	-5.569321
1	2	1	2	-5.577005	-5.548835	-5.577061	-5.566515
2	2	0	0	-5.572613	-5.551486	-5.572645	-5.564746
2	2	1	0	-5.577575	-5.552927	-5.577618	-5.568396
2	2	2	0	-5.577111	-5.548941	-5.577166	-5.566620
2	2	0	1	-5.577993	-5.553345	-5.578036	-5.568814
2	2	1	1	-5.582011	-5.553842	-5.582067	-5.571521
2	2	2	1	-5.577242	-5.545551	-5.577312	-5.565440
2	2	0	2	-5.577176	-5.549007	-5.577232	-5.566686
2	2	1	2	-5.584944	-5.553254	-5.585015	-5.573143
2	2	2	2	-5.583748	-5.548536	-5.583835	-5.570635

Table 5: GARCH/ARMA model parameters and statisticc

3.4 Analysis of Model Residuals

In addition to the previous analysis, we need to carry out the analysis of the residuals of the model to fully assess its fit. First, we must verify that the residuals have a white noise distribution and that they are not correlated. This would mean that we have indeed extracted a random process from our observations and that the model can be used to predict future market volatility.

Let us first look at the time series (Figure 8) of the residuals extracted from the model and their squared values. We see several outliers, particularly at the beginning and in the middle. This is particularly evident in Figure 9, which shows the squared residuals. We have also observed present $\mu = 0$ and $\sigma = 1$, indicating they might follow the criteria of white noise, however, performing a

Shapiro-Walk test, we have observed that the p-value was inferior to 2.2×10^{-16}, indicating that the residuals do not follow a normal distribution.

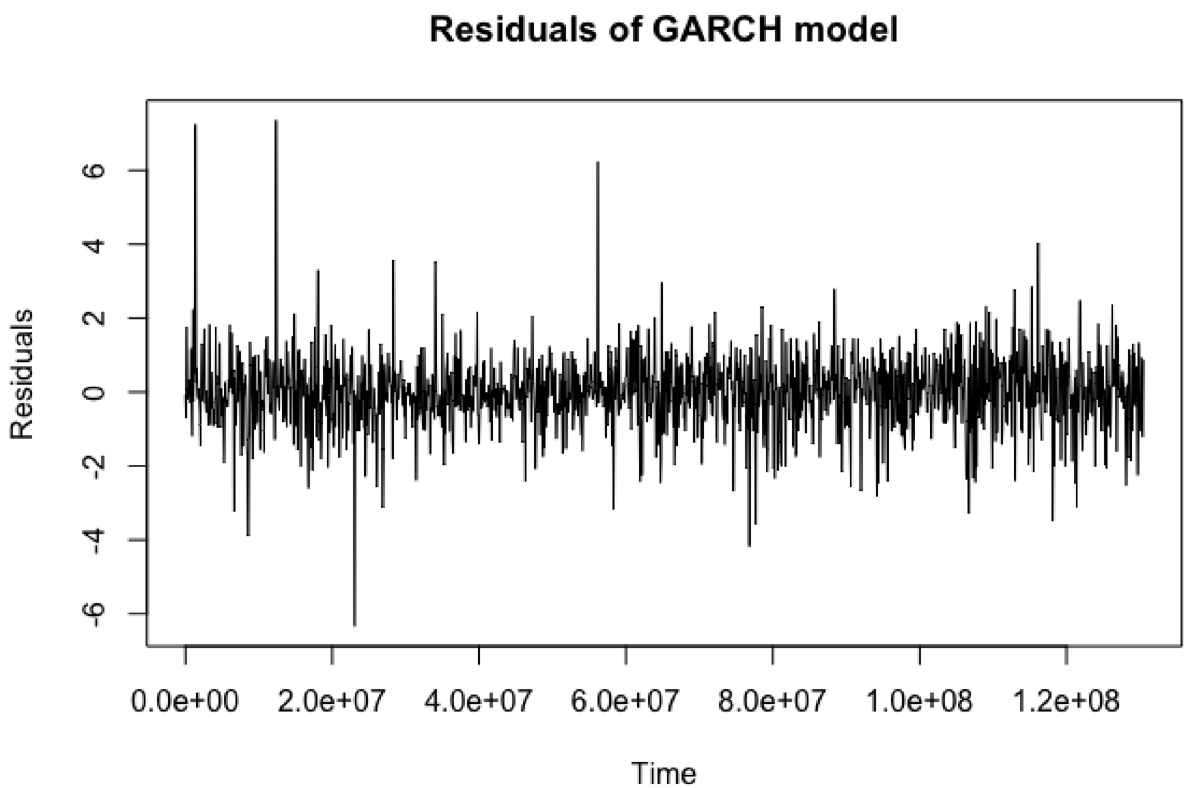

Figure 8: Model Residuals over Time

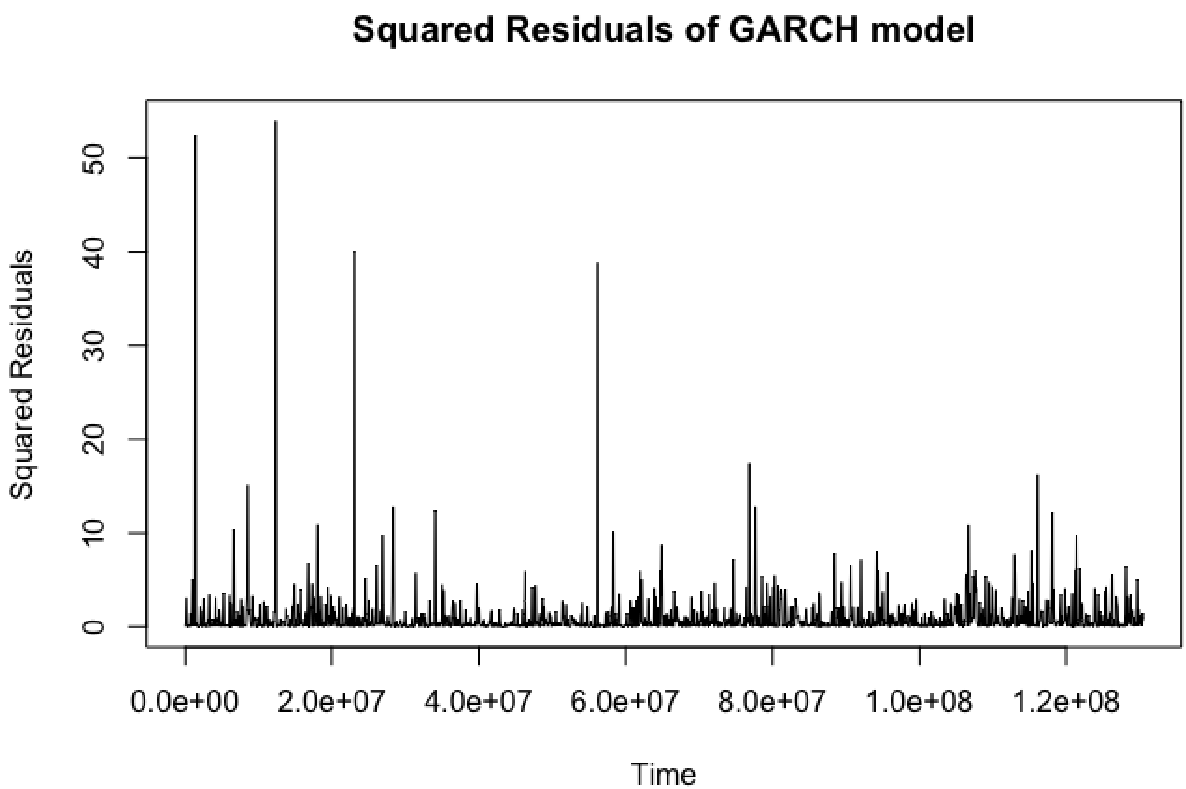

Figure 9: Squared Model Residuals over Time

However, we proceeded with the analysis of the residuals, as we were particularly interested in the ACF and PACF analysis of the standardised residuals. This is shown in Figure 10. As can be seen, all points are within the confidence interval, indicating that there is no statistical correlation between observations over time and indicating a good fit of the model.

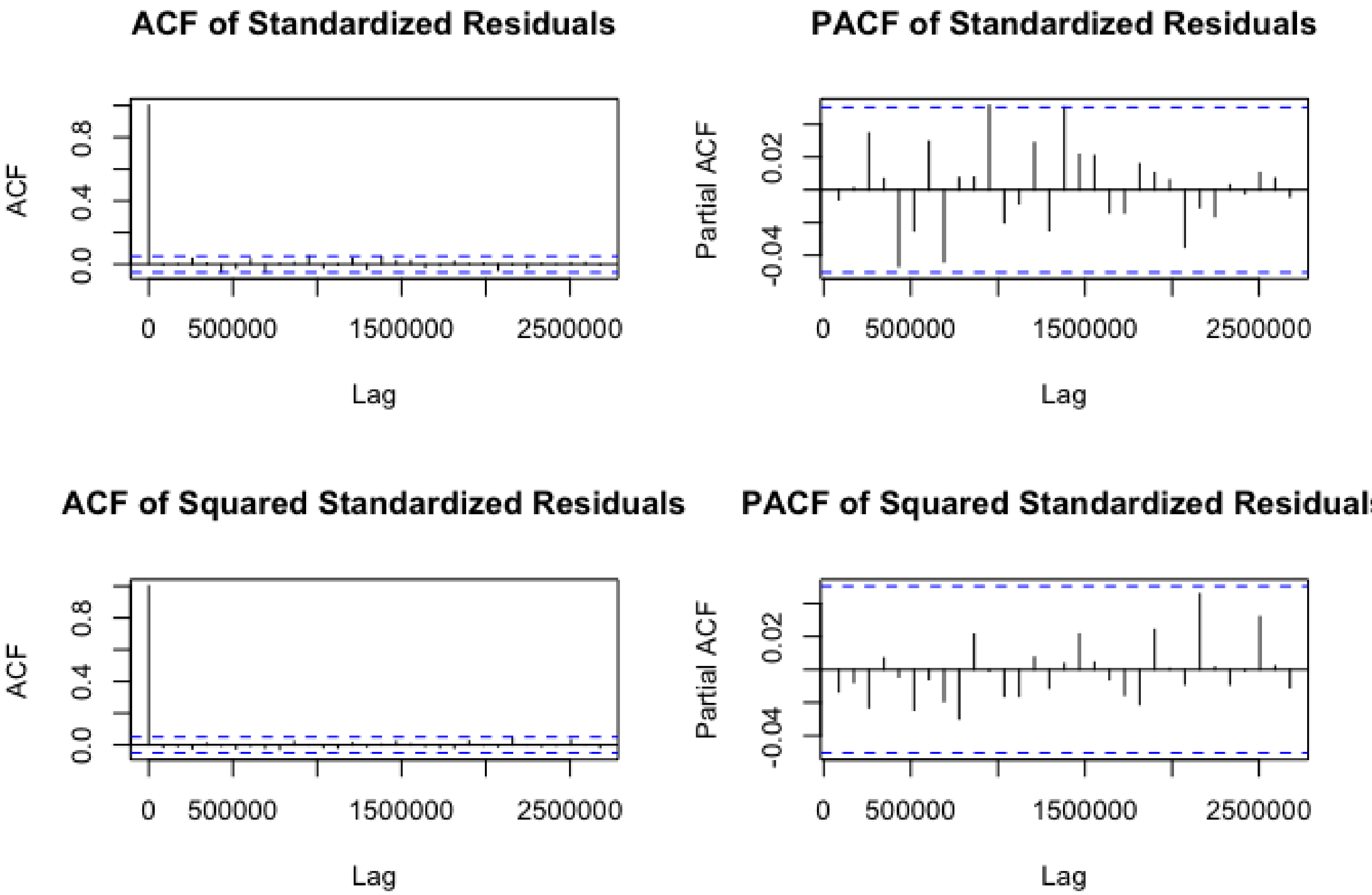

Figure 10: Autocorrelation function of Model Residuals

3.5 Forecasting of Daily Returns and Volatility

The process of modeling and forecasting daily returns and their volatility is crucial in financial econometrics for risk management, portfolio optimization, and financial decision-making. The GARCH model is widely used for this purpose because it effectively captures the time-varying volatility and volatility clustering commonly observed in financial time series data.

The dataset consists of daily log returns of a financial instrument. The data is split into training and test sets with an 80-20 split ratio. The training data is used to fit the GARCH model, and the test data is used to evaluate the model's forecasting performance. The GARCH(1,1) model, which includes one lag of the squared residuals (ARCH term) and one lag of the conditional variance (GARCH term), is specified along with an ARMA(1,2) model for the mean equation.

The first plot (Figure 11) illustrates the actual daily returns (in blue) along with the forecasted return (in red) and the 95% confidence intervals (shaded in red). The plot of daily returns shows that the GARCH model captures the general trend and variability of returns. The forecasted returns align well with the actual returns, and the confidence intervals provide a reasonable range around the forecasted values.

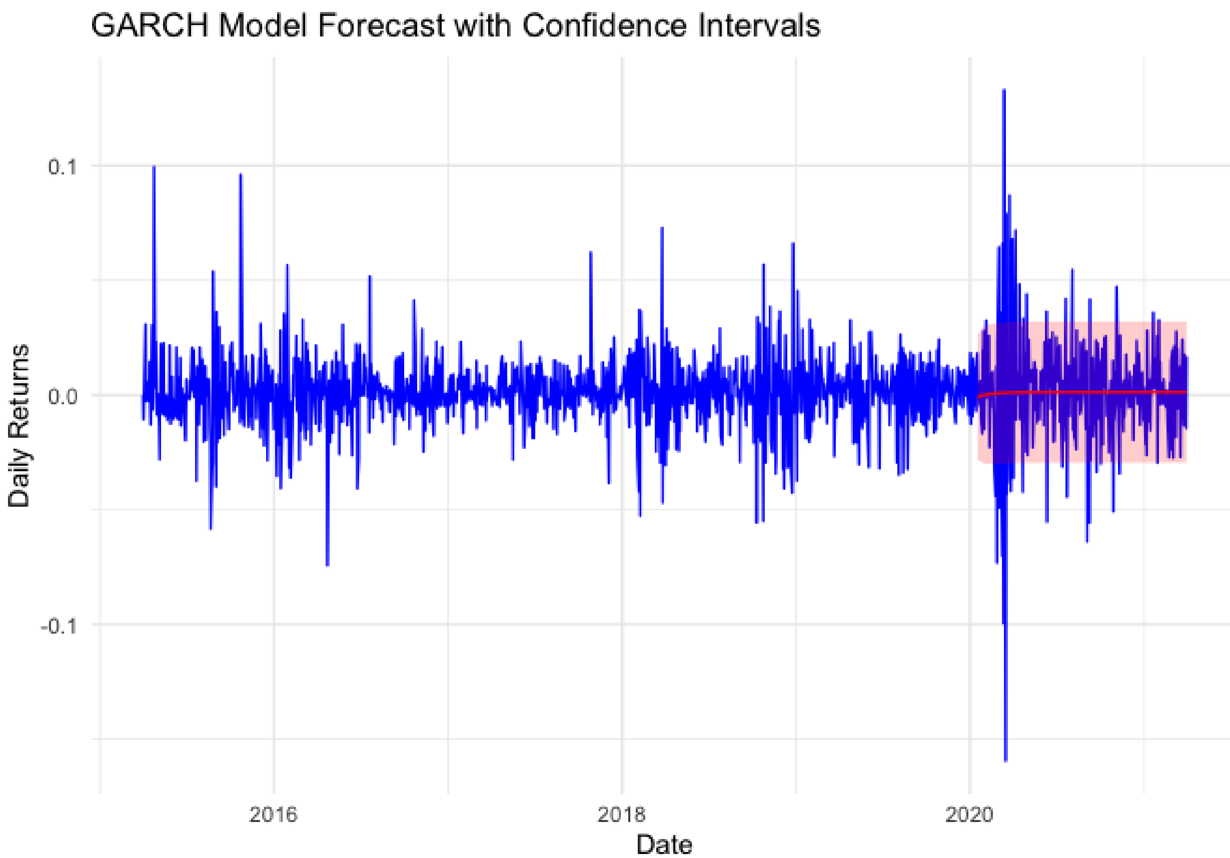

Figure 11: Confidence Intervals of the Forecast

For forecasting volatility, a one-step-ahead rolling forecast is performed. The model is refitted incrementally with expanding windows of the data, and the one-step-ahead forecasted volatility (conditional standard deviation) is recorded. The second plot (Figure 12) shows the actual volatility (in blue) and the predicted volatility (in red), demonstrating the model's performance in capturing the time-varying nature of volatility.

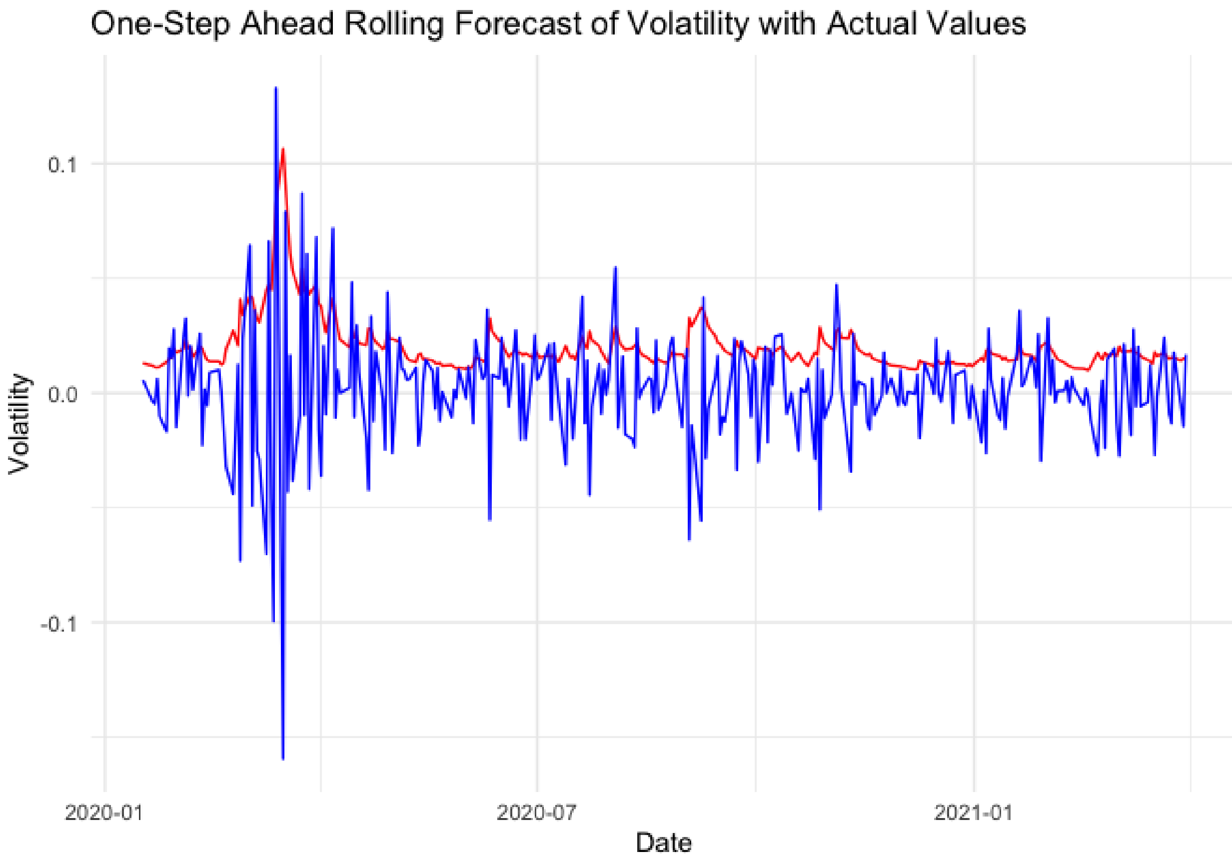

Figure 12: Forecast of the test data Volatility

The rolling forecast of volatility indicates that the GARCH model effectively captures the periods of high and have some trouble with the low volatility. In general the predicted volatility closely follows the actual volatility, demonstrating the model's ability to adapt to changing market conditions.

4 Conclusions and Future Work

In conclusion, the comprehensive analysis of the time series data, including the assessment of stationarity through ADF and KPSS tests, the examination of ACF and PACF properties, and the fitting of an ARIMA/GARCH model, provides a robust framework for understanding and predicting the dynamics of financial markets. The GARCH(1,1) x ARMA(1,2) model emerged as the most suitable model based on the criteria of AIC, BIC, Shibata, and HQ, offering a solid foundation for forecasting both daily returns and volatility. The analysis of residuals confirmed that the model is appropriate, displaying characteristics of white noise and no auto-correlation, further validating its predictive capabilities. This approach not only aids in risk management and portfolio optimization but also enhances financial decision-making processes by accurately capturing the volatile nature of financial markets.

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