Zeeman Effect

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Abstract

In this paper, we study the Zeeman effect in mercury. The Zeeman effect refers to the splitting of atomic energy levels in the presence of a strong magnetic field. This splitting allows for more energy level transitions to take place. These energy level transitions can be observed as the splitting of spectral lines in light emitted. Using a Fabry-Perot interferometer and ramping up an external magnetic field, we observe the interfered 579.066 nm spectral line from a mercury lamp source. Using the geometry of the Fabry-Perot interference pattern, we find the corresponding energy transition ΔE and use this to calculate the Bohr magneton μ_B since $\Delta E = \vec{\mu_B} \cdot \vec{B}$. Using this, we calculate the Bohr magneton as $(9.650 \pm 0.656)\,\frac{\mathrm{J}}{\mathrm{T}} \times 10^{-24}$ which is within error of the accepted value of μ_B.

1 Introduction

The Zeeman effect was first observed experimentally in 1896 by Dutch physicist Pieter Zeeman and refers to the splitting of spectral lines from an excited atom (which corresponds to energy level transitions) in the presence of a strong magnetic field [4].

The Zeeman effect can be explained quantum mechanically in terms of quantum numbers m, l, and g. When the Zeeman effect is present, energy levels split into $2l + 1$ separate levels. However, the allowed transitions of electrons follow selection rules restricting $\Delta m_j = 0, \pm 1$. The transitions we can observe in this experiment are from a mercury lamp and is shown by the figure below.

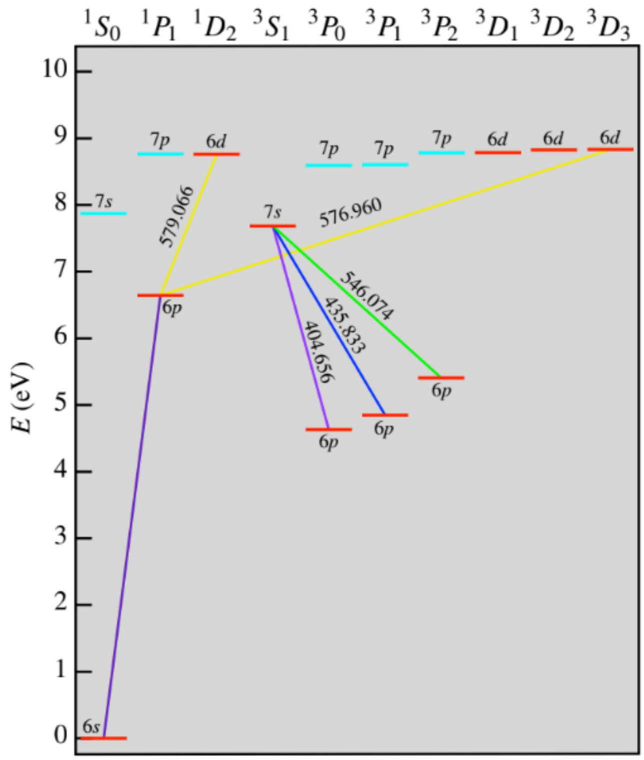

Figure 1: Hg spectral lines from electron transitions [2]

In this paper we focus on the $6d - 6p$ transition at 579.066 nm seen by the leftmost line in the yellow doublet. Under the presence of the Zeeman effect, there is a total of nine different transitions allowed between the two orbitals. However, we only see three split spectral lines since many of the transitions correspond to the same energy. This is known as the "normal" Zeeman effect and arises from the fact that the g values of each orbital are equal. The "anomalous" Zeeman effect on the other hand would correspond to nine split spectral lines with differing g values for each orbital.

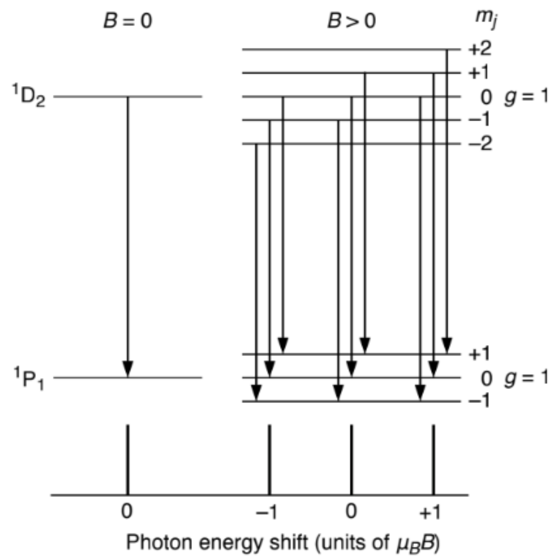

Figure 2: Energy level diagram illustrating normal Zeeman effect [2]

2 Theory

The Zeeman effect can be explained by the interaction of an atom's magnetic moment, $\vec{\mu}$, and an external magnetic field, $\vec{B} = B(t)\hat{z}$. This interaction is defined as H^1 and can be treated as a small perturbation to the total energy of the system.

$$H^1 = -\vec{\mu} \cdot \vec{B} \tag{1}$$

We can express μ for an atom using angular momentum operator $\hat{l}, \hat{s}$ corresponding to orbital and spin angular momentum for an electron i.

$$\vec{\mu} = -\frac{1}{2}\frac{e}{m}\sum_i \hat{l}_i - \frac{e}{m}\sum_i \hat{s}_i \tag{2}$$

With e and m being the charge and mass of an electron. If we define the total orbital angular momentum as $\sum_i \hat{l}_i = \vec{L}$, the total spin angular momentum as $\sum_i \hat{s}_i = \vec{S}$ and the sum of the two as $\vec{J} = \vec{S} + \vec{L}$ we can rewrite (2).

$$\vec{\mu} = -\frac{1}{2}\frac{e}{m}(\vec{L} + 2\vec{S}) = -\frac{1}{2}\frac{e}{m}(\vec{J} + \vec{S}) \tag{3}$$

Plugging into (1) yields

$$H^1 = \frac{1}{2}\frac{e}{m}(\vec{J} + \vec{S}) \cdot \vec{B} \tag{4}$$

we can approximate $\vec{J} + \vec{S}$ by replacing $\vec{S}$ with $\vec{S_{avg}}$ which is the projection of $\vec{S}$ along $\vec{J}$.

$$\vec{S_{avg}} = \frac{\vec{J}^2 + \vec{S}^2 - \vec{L}^2}{2\vec{J}^2}\vec{J} \tag{5}$$

Plugging $\vec{S_{avg}}$ in for $\vec{S}$ in (4) and using $\vec{J} \cdot \vec{B} = J_z B$ gives

$$H^1 = \frac{eB}{2m}(1 + \frac{\vec{J}^2 + \vec{S}^2 - \vec{L}^2}{2\vec{J}^2})J_z \tag{6}$$

To find the energy shifts of atomic levels we take the expectation value of H^1. This is simple since we know the expectation value of our angular momentum operators ($< \vec{J}^2 >= j(j+1)\hbar^2$, $< \vec{S}^2 >= s(s+1)\hbar^2$, $< \vec{L}^2 >= l(l+1)\hbar^2$, $< J_z >= m_j\hbar$).

$$< H^1 >= E_j = g(\frac{e\hbar}{2m})Bm_j \tag{7}$$

With $\frac{e\hbar}{2m}$ defined as the Bohr magneton and g given below.

$$g = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)} \tag{8}$$

Setting E_j equal to $h\Delta\nu$ from the Fabry-Perot interference pattern allows us to solve for the Bohr magneton.

$$\Delta\nu = \frac{c}{2L}\Delta m \tag{9}$$

Where Δm is an integer and $\Delta m = m' - m$ where m' and m are the orders of the diffracted beams [1].

3 Experimental Techniques

3.1 Setup

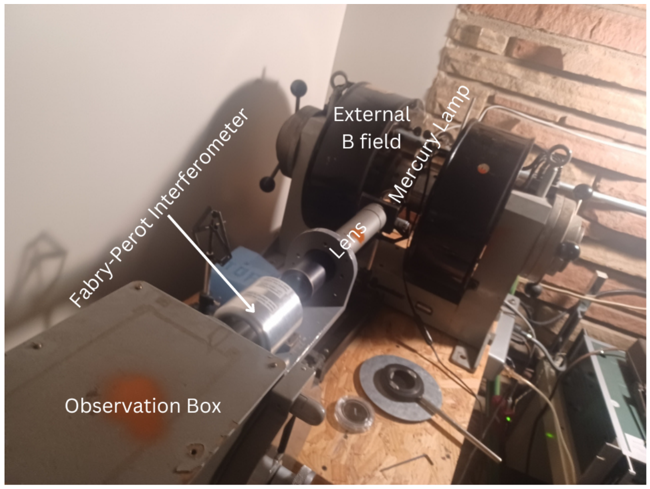

Figure 3: Lab setup for Zeeman effect

3.2 Parts and Equipment [3]

- Mercury (Hg) Light Source and Power Supply
- Electromagnet (Varian V-4004)
- Magnet Power Supply (PAD 160-7L)
- Digital Multimeter (Keithley 197)
- Grating Spectrometer and Optics
  - Etalon/Fabry-Perot Interferometer
  - Eyepiece
  - Condenser Lens
  - Polarizers
  - Neutral-Density Filter
- CCD Camera (Meade DSI)
- Gaussmeter
- LabJack (U3)
- Computer and Software
  - LabVIEW
  - LabJack UD Driver Package
  - Autostar Envisage
  - Igor Pro 6.3 and QuickTime
  - ZA ZeemanAnalysis.ipf

3.3 Procedure [3]

The procedure for this lab can be broken down into six steps.

1. Finding a Spectral Line
   - Turn on the mercury lamp located in-between the poles of the magnet.
   - Insert the eyepiece into the observation box.
   - Ensure the Etalon is not in place yet, adjust the condenser lens so a bright image from the lamp appears.
   - Begin to turn the knob on the observation box with white numbered labels to change the wavelength of the spectrometer until the yellow doublet is visible.
   - Adjust the lens slit width until the doublet is almost overlapping.
2. Saving a Picture
   - With the doublet in view, replace the eyepiece with the camera then connect the camera to the computer.
   - Open Autostar Envisage from the desktop. Increase the exposure time in Autostar to increase the brightness of the image.
   - Set the save directory for photos in the settings menu.
   - Save the image and check to make sure it saved in the specified directory.
3. Adding the Etalon
   - Replace the camera with the eyepiece.
   - Open up the lens slit a bit and insert the Etalon between the condenser lens and the observation box.
   - Adjust the slit width, and the etalon pitch/yaw until about five horizontal interference lines are visible in each doublet line.
4. Applying a Magnetic Field (Manually and by Computer)
   - Turn on the multi-meter.
   - Turn the current and voltage knobs all the way down on the power supply then turn it on. We will control the current via a control voltage from the LabJack which is controlled by the computer.
   - Open Set Voltage.vi on the computer, set the input voltage to 3 V and run the vi.

- Set the current knob to maximum so the supply is limited by the voltage knob.
- Slowly increase the voltage knob until you reach the maximum voltage, write this number down.
- Turn down the voltage knob back to zero and set the input voltage on the vi to zero then run it again. At this point the power supply current and the voltage should be zero.

5. Test Run
    - Ensure the yellow doublet is in view with the Etalon in place. The doublet should almost overlap and there should be about five interference lines in each doublet line.
    - With the voltage knob set to zero on the power supply, set Voltage.vi to 3 V to set a high current limit.
    - Take a picture using Autostar Envisage.
    - Slightly increase the voltage and take another picture.
    - Repeat increasing voltage and taking picture until max voltage is reached.
    - Once the max voltage has been reached, capture a final photo. Then turn the power supply's voltage knob back to zero.
    - Open the ZeemanAnalysis.ipf using Igor Pro.
    - Select Zeeman Analysis in the menu bar, then select Slicer Window.
    - Follow the instructions in Slicer Window. This should result with a picture that shows the evolution of the pictured spectral line as the voltage/magnetic field increase.

6. Measuring / Calibrating Magnetic Field
    - Connect the hall probe to the gaussmeter and turn the unit on.
    - Ensure that probe is selected on the unit.
    - Zero out the probe by putting the end of the probe in the zero flux chamber and selecting zero on the probe. Wait for the autozero to complete.
    - With the power supply knobs set to zero, put the probe in-between the magnets. The probe should be parallel to the faces magnet's poles. Record this background magnetic field.
    - Begin to calibrate the magnet by recording the magnetic field vs Set Voltage.vi. Take readings at $0.5V$ increments from $0\,\mathrm{V} - 3\,\mathrm{V}$.

- Fit a third order polynomial to your graph of Voltage(Tesla).
- Open the LabView VIRamp Field.vi and insert your fitting parameters. This program will then automatically ramp up your magnetic field at a constant rate.

We are now ready to take meaningful data. Begin ramping up the magnetic field with LabView (via Ramp Field.vi) while taking pictures. Make sure to allow for enough points while ramping up the magnetic field to get ≈ 300 pictures for a full run. The Autostar program takes a photo every $4 - 5$ seconds meaning we need ≈ 1500 points for the magnetic field ramping as one point corresponds to one second.

4 Data Analysis and Results

Table 1 below shows the calibration data used for the fitting parameters as described in the procedure.

V	B
0.0 ± 0.05	0.0433 ± 0.0001
0.5 ± 0.05	0.265 ± 0.0002
1.0 ± 0.05	0.585 ± 0.02
1.5 ± 0.05	0.776 ± 0.02
2.0 ± 0.05	0.988 ± 0.06
2.5 ± 0.05	1.085 ± 0.06
3.0 ± 0.05	1.15 ± 0.06

Table 1: Voltage (V) vs Magnetic Field in Tesla (T) used for calibration

Taking the calibration data and fitting a third order polynomial for $V(B)$ gives the parameters necessary for RampField.vi. The graph created for this report was made in excel and is shown below.

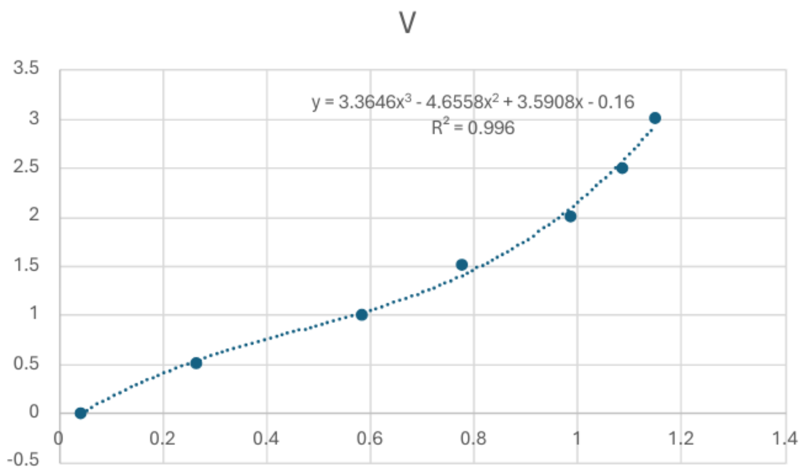

Figure 4: Fitting curve for RampField.vi

With the camera focused on the leftmost line of the yellow doublet produced by Hg ($\lambda = 579.066\,\text{nm}$), we ramp the field up from $0\,\text{T} \rightarrow 1.15\,\text{T}$ at a constant rate for 1500 points ($t_{total} = 25\,\text{min}$). Taking pictures every $4 - 5$ seconds and running the ZeemanAnalysis.vi produces the figure below.

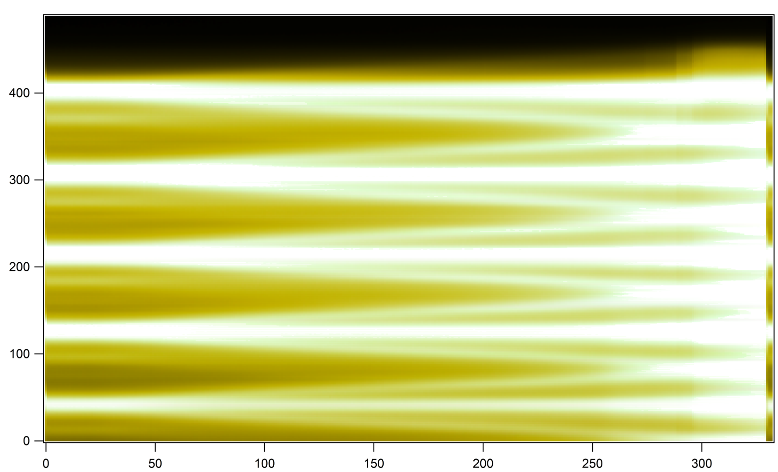

Figure 5: Time evolution illustrating Zeeman Effect in Hg, $\lambda = 579.066\,\text{nm}$

This image is a compilation of 329 images sliced to demonstrate the Zeeman effect. At the end of the figure you can see six extra photos included after the magnetic field was ramped back down to zero. We can approximate when two adjacent lines intersect as $\approx 290 \pm 20$ images in. Then we can estimate the magnetic field at this time, assuming the field is increasing at a constant rate.

$$B = (1.15 \times \frac{290}{323} \pm 1.15 \times \frac{20}{323})\,\mathrm{T} = (1.03 \pm 0.07)\,\mathrm{T} \tag{10}$$

In figure 5 we see each line split into three separate levels suggesting we have the normal Zeeman effect. We can verify this knowing the spectral line imaged corresponds to the ${}^1D_2 \rightarrow^1 P_1$ transition, allowing for us to calculate the g values for each energy level as this notation is given as ${}^{2S+1}L_J$.

$$g_D = 1 + \frac{2(2+1) + 0(0+1) - 2(2+1)}{2 \times 2(2+1)} = 1 \tag{11}$$

$$g_P = 1 + \frac{1(1+1) + 0(0+1) - 1(1+1)}{2 \times 1(1+1)} = 1 \tag{12}$$

Since we are looking at the crossing point of two adjacent lines, $\Delta m = 1$ in (9) and we can solve for the Bohr magneton (μ_B). With $2L = (1.0000 \pm 0.00005)\,\mathrm{cm}$ given.

$$h\Delta v = \frac{hc}{2 \times (1.0000 \pm 0.00005)\,\mathrm{cm}} = \mu_B B = (9.939 \pm 0.00005)\,\mathrm{J} \times 10^{-24} \tag{13}$$

$$^{found}\mu_B = (9.650 \pm 0.656)\,\frac{\mathrm{J}}{\mathrm{T}} \times 10^{-24} \tag{14}$$

With $\delta h\Delta\nu = h\Delta\nu \times \frac{\delta(2L)}{2L}$, $\delta\mu_B = \mu_B\sqrt{(\frac{\delta(h\Delta\nu)}{h\Delta\nu})^2 + (\frac{\delta B}{B})^2}$. This experimentally calculated value is within error of the accepted value of the Bohr magneton, $^{true}\mu_B = 9.27 \times 10^{-24}\,\frac{\mathrm{J}}{\mathrm{T}}$.

5 Discussion

In this paper, we investigate the normal Zeeman effect in mercury by studying the $\lambda = 579.066\,\mathrm{nm}$ spectral line produced by Hg. The setup for this calculation consists of a mercury light source in-between two poles of strong magnets which shines through a condenser lens, into a spectrometer, where the diffraction was then observed. By the geometry of the spectrometer and the theory behind the Zeeman effect, we came up with an experimentally calculated value of the Bohr magneton. $\mu_B = (9.650 \pm 0.656)\,\frac{\mathrm{J}}{\mathrm{T}} \times 10^{-24}$ which is within error of the accepted value of μ_B.

One unconsidered source of error comes from the fitting parameters and the regime of RampField.vi. Further analysis could be done to associate some error to the RampField.vi procedure but is not included in this report.

References

[1] University of Puget Sound Greg Elliott. Optical spectroscopy and the zeeman effect. *Beyond the First Year workshop*, July 25-27 2012.

[2] Colorado State Univeristy J Harton. Zeeman effect lab manual. April, 2019.

[3] Colorado State Univeristy J Harton. Zeeman effect quick start. April, 2019.

[4] P. Zeeman. The Effect of Magnetisation on the Nature of Light Emitted by a Substance. , 55(1424):347, February 1897.