Millikan Oil Drop

Luke DeFreitas

February 28, 2025

Abstract

The charge of an electron can be experimentally measured using Millikan's classical method. By recording fall and rise velocities of small charged drops subject to gravity and a known external E field, one can find a multiple of the electron charge. In performing this experiment, we found the charge to be $e = 1.357 \times 10^{-19} \pm 3.85 \times 10^{-20}$ C which differs from the accepted value of $e = 1.602\,176\,634 \times 10^{-19}$ C by 15.3%.

1 Introduction

The electron was first discovered at the turn of the twentieth century by J.J. Thomson while researching cathode rays. Thomson measured how these rays bent under a magnetic field and how much energy they carried to find the charge to mass ratio, e/m, of an electron [4]. The first accurate measurement for the charge of an electron was Millikan's oil drop experiment in 1909. Millikan found that droplets of oil from a spray bottle carried a net charge of a few electrons. He built an apparatus in which droplets fall between two capacitor plates and observed their motion. A voltage applied between the two plates generate an electric field driving the drops up against gravity. By recording the motion of a droplet in response to gravity, the electric field, and viscous drag through air, Millikan was able to calculate the charge of a drop. He found that charges on drops occurred in integer multiples of the fundamental charge and gave the value for this fundamental charge to an accuracy of 1 part in 1000 [5]. This marked an important milestone in physics as it demonstrated that charge is quantized. Millikan's efforts awarded him a Nobel prize and motivated the development of quantum mechanics. In this paper we aim to experimentally determine the charge of an electron and show the quantization of charge by recreating Millikan's famous oil drop experiment with modern tools.

2 Theory

Consider the free body diagram for rising and falling drops below.

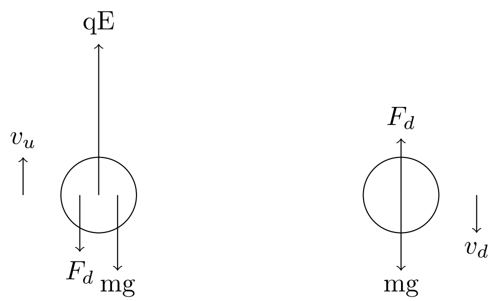

Figure 1: Two free body diagrams depicting oil drops. One is subject to an external electric field driving it up (left) and the other is freely falling under gravity (right). [5]

From figure 1 and the fact that these drops reach terminal velocity almost instantaneously ($ma = 0$) you get

$$F = qE - mg - F_d = ma = 0 \tag{1}$$

where F_d is the drag force, q is the charge of the droplet, $E = V/d$ is the electric field, m is the mass of the drop and g is the acceleration due to gravity. F_d can be accurately described by Stokes's Law,

$$F_d = 6\pi\eta rv \tag{2}$$

where η is the viscosity of air, r is the radius of the drop and v is the speed of the drop. Consider the case where the droplet is rising, plugging in for (1) and rearranging yields

$$qE = mg + 6\pi\eta rv_u \tag{3}$$

leaving only the mass of the drop m and radius r to be found. Assuming the drops are spheres, the mass can be written in terms of the density ρ and radius r as $m = \rho 4\pi r^3/3$. To find this radius r we can use Stokes's law again but this time for the case for the drop falling under gravity with no external field.

$$mg = 6\pi\eta rv_d = \rho 4\pi r^3 g/3 \tag{4}$$

Solving for r,

$$r = \sqrt{\frac{9\eta v_d}{2\rho g}} \tag{5}$$

now plug (4) and (5) into (3) to solve for q.

$$qE = 6\pi\eta\sqrt{\frac{9\eta v_d}{2\rho g}}v_d + 6\pi\eta\sqrt{\frac{9\eta v_d}{2\rho g}}v_u \tag{6}$$

To get an even more accurate measurement, a correction can be made to Stokes's law using a modified viscosity for air. This correction is necessary since the radii of the drops are so small that they are comparable to the mean free path of the air molecules (violating the assumption of a spherical object moving through a continuous fluid in Stokes's law). This modified viscosity is written as

$$\eta = \eta_0[1 + \frac{5.908 \times 10^{-5}}{rP}]^{-1} \tag{7}$$

where η_0 is the "ordinary" viscosity of air, P is the pressure in torr, and r is the drop radius in meters. Near room temperature, η_0 can be shown as

$$\eta_0 = 1.8228 \times 10^{-5} + (4.790 \times 10^{-8})(T - 21°\text{C}) \tag{8}$$

with T in °C. Once η_0 is calculated you can plug it into (5) to get a value for r. Then plug this r value into (7) to get your modified viscosity of air. Use this new value of η and plug back into (5) to get a more accurate r and repeat until both have converged.

3 Experimental Techniques

3.1 Setup

- Millikan Oil Drop Apparatus (TM-15)
- Oil Atomizer
- Watch Oil (Nye 140C)
- Kim Wipes
- Silicon Calibration Block
- Lamp
- Agilent Digital Multimeter
- Video Camera (WAT 902H2)
- GrabeeX+ Deluxe Video Adapter
- Computer and Software
  - Debut Video Capture
  - Igor Pro 6.3 and QuickTime
  - Find Particles.pxp
  - Excel/LibreOffice Calc

3.2 Experimental Procedures

The apparatus used to perform this experiment is a one-piece unit; the power supply, polarity reversing switch, light source, microscope, chamber, atomizer, and oil reservoir are all contained in one unit. A small video camera has been attached to the microscope to allow recording of videos of the drops' motion to the computer. Debut Video Capture can be used to make such recordings. The videos can then be analyzed using Windows Movie Maker and Igor Pro. Place the end of the atomizer tube in the reservoir. Close the chamber and turn on the unit. Set the voltage to zero. A gentle squeeze of the bulb should inject some droplets into the region between the plates. You will see a bright flash as the drops enter the chamber. You'll need to replace the plug into the inlet hole to keep air currents from interfering with the drops' motion. Note that the field of view of the telescope is somewhat limited, while you are viewing a single drop you will find it necessary to move the focus forward or backward to follow its motion. To find a drop to follow you will also have to move the focus back and forth; a squeeze of the atomizer will rarely inject a drop right in your field of view, but by looking at different depths you should be able to find one. Once you have found a drop, explore your ability to move it up and down using the electric field. Vary the voltage and polarity; you will see drops of opposite charges move in opposite directions. Drops with larger charges will move more rapidly. As noted above, the basic measurement that you will have to make on an individual drop is a measurement of the speed when the drop is driven upwards by an electric field, and then allowed to fall without a field. To find this speed, you need to calibrate how many video pixels correspond to 1 mm. For this purpose, a calibration block is used with two lines described accurately 1.000 ± 0.003 mm apart. As you are determining a strategy for taking your data, note the following: i) You need to concentrate on small drops that have only a few charges on them. You need drops with only a few charges on them so that you can distinguish between charge states. For instance, suppose your measurement error is 10%. Then it is easy to distinguish between $q = 2e$ and $q = 3e$, but $q = 10e$ and $q = 11e$ are much harder to tell apart. Small numbers of charges mean that the upward force qE will be small, so we also need to have small, light drops so that qE can be made larger than mg and the drops can be made to rise. To find small, lightly-charged drops to work with it can be helpful to first select droplets that fall rather slowly in zero field; typical fall times for good drops are in the range of 10 s to 30 s to fall about 2 mm. After you get a handle on how a small drop falls, you can check for ones that have only a few charges by applying a fairly large voltage ($200 - 500$ V) and seeing which droplets rise at speeds similar to those of the smaller falling droplets. Droplets that rise very rapidly probably have too much charge to be useful. ii) Repeated measurements on a single drop are very useful. There are two benefits: first, you will be able to estimate the error in your determinations of speed. Second, you will be able to get a better value for the charge on the drop by averaging the measurements that you obtain. Therefore, the best strategy is to work with a single drop for at least several (3-4) measurements. [5]

4 Data Analysis and Results

Table 1 below shows the constants used for this report. Note, that the voltage uncertainty is quite high since we didn't use the multi-meter for each run and noticed the voltage changing at times (according to the gauge).

V	$500 \pm 10\,\mathrm{V}$
d	$4.902 \pm 0.007\,\mathrm{mm}$
E	$1.020 \times 10^5 \pm 2.507 \times 10^3\,\mathrm{V/m}$
ρ	$861 \pm 1\,\frac{\mathrm{kg}}{\mathrm{m}^3}$
P	$753.3 \pm 0.01\,\mathrm{Torr}$
g	$9.8 \pm 0.01\,\frac{\mathrm{m}}{\mathrm{s}^2}$
T	$18.89 \pm 0.278°\mathrm{C}$

Table 1: Constants used with uncertainties

Table 2 below shows the average velocity for each drop either going up due to the external field or down due to gravity. Note that the uncertainty on some of these is quite high ($\approx 10\%$).

Drop & Dir.	Velocity ($\frac{\mathrm{m}}{\mathrm{s}}$)
1U	$1.67 \times 10^{-4} \pm 1.60 \times 10^{-6}\,\frac{\mathrm{m}}{\mathrm{s}}$
1D	$9.37 \times 10^{-5} \pm 1.40 \times 10^{-6}\,\frac{\mathrm{m}}{\mathrm{s}}$
2U	$3.76 \times 10^{-4} \pm 6.11 \times 10^{-6}\,\frac{\mathrm{m}}{\mathrm{s}}$
2D	$1.68 \times 10^{-4} \pm 1.84 \times 10^{-5}\,\frac{\mathrm{m}}{\mathrm{s}}$
3U	$3.60 \times 10^{-4} \pm 1.30 \times 10^{-5}\,\frac{\mathrm{m}}{\mathrm{s}}$
3D	$1.70 \times 10^{-4} \pm 2.06 \times 10^{-6}\,\frac{\mathrm{m}}{\mathrm{s}}$
4U	$1.45 \times 10^{-4} \pm 2.68 \times 10^{-6}\,\frac{\mathrm{m}}{\mathrm{s}}$
4D	$8.76 \times 10^{-5} \pm 1.24 \times 10^{-6}\,\frac{\mathrm{m}}{\mathrm{s}}$
5U	$2.10 \times 10^{-4} \pm 6.04 \times 10^{-6}\,\frac{\mathrm{m}}{\mathrm{s}}$
5D	$6.23 \times 10^{-5} \pm 1.64 \times 10^{-6}\,\frac{\mathrm{m}}{\mathrm{s}}$
6U	$2.64 \times 10^{-4} \pm 9.03 \times 10^{-6}\,\frac{\mathrm{m}}{\mathrm{s}}$
6D	$1.22 \times 10^{-4} \pm 1.32 \times 10^{-6}\,\frac{\mathrm{m}}{\mathrm{s}}$
7U	$4.01 \times 10^{-4} \pm 7.79 \times 10^{-6}\,\frac{\mathrm{m}}{\mathrm{s}}$
7D	$1.73 \times 10^{-4} \pm 1.06 \times 10^{-6}\,\frac{\mathrm{m}}{\mathrm{s}}$
8U	$5.67 \times 10^{-4} \pm 1.65 \times 10^{-5}\,\frac{\mathrm{m}}{\mathrm{s}}$
8D	$2.02 \times 10^{-4} \pm 2.40 \times 10^{-6}\,\frac{\mathrm{m}}{\mathrm{s}}$
9U	$2.06 \times 10^{-4} \pm 2.21 \times 10^{-6}\,\frac{\mathrm{m}}{\mathrm{s}}$
9D	$1.33 \times 10^{-4} \pm 9.17 \times 10^{-7}\,\frac{\mathrm{m}}{\mathrm{s}}$
10U	$4.095 \times 10^{-4} \pm 3.01 \times 10^{-5}\,\frac{\mathrm{m}}{\mathrm{s}}$
10D	$5.95 \times 10^{-5} \pm 1.07 \times 10^{-6}\,\frac{\mathrm{m}}{\mathrm{s}}$

Table 2: Average velocities for 10 drops

Table 3 below shows the calculated charge for each drop. Note that some drops are quite high in charge (such as drop 8) resulting in a high uncertainty ($\approx 1e$).

Drop	Charge (q)
1	$7.345 \times 10^{-19} \pm 2.25 \times 10^{-20}$ C
2	$2.118 \times 10^{-18} \pm 1.62 \times 10^{-19}$ C
3	$2.074 \times 10^{-18} \pm 8.06 \times 10^{-20}$ C
4	$6.296 \times 10^{-19} \pm 2.05 \times 10^{-20}$ C
5	$6.083 \times 10^{-19} \pm 2.56 \times 10^{-20}$ C
6	$1.260 \times 10^{-18} \pm 4.85 \times 10^{-20}$ C
7	$2.272 \times 10^{-18} \pm 7.06 \times 10^{-20}$ C
8	$3.312 \times 10^{-18} \pm 1.20 \times 10^{-19}$ C
9	$1.158 \times 10^{-18} \pm 3.32 \times 10^{-20}$ C
10	$1.020 \times 10^{-18} \pm 8.25 \times 10^{-20}$ C

Table 3: Drop number and Charge

Now we can graph the charges on a discrete plot with uncertainties. I will set the ticks in multiples of the accepted e value ($e = 1.602 \times 10^{-19}$) to make it easy to tell the charge on each drop and to see if our guesses are reasonable.

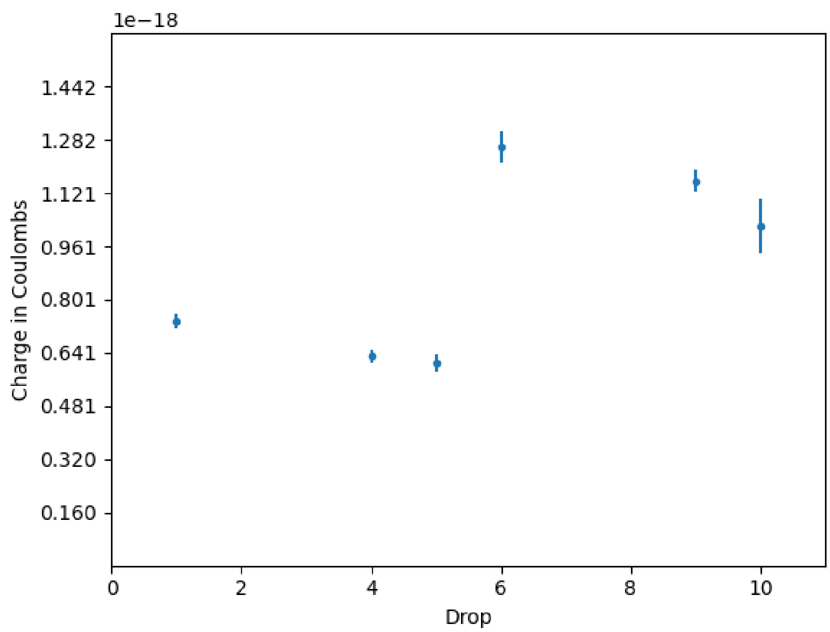

Figure 2: Drop number vs Charge graph with ticks spaced at $1e$. Y-axis from $1e - 10e$

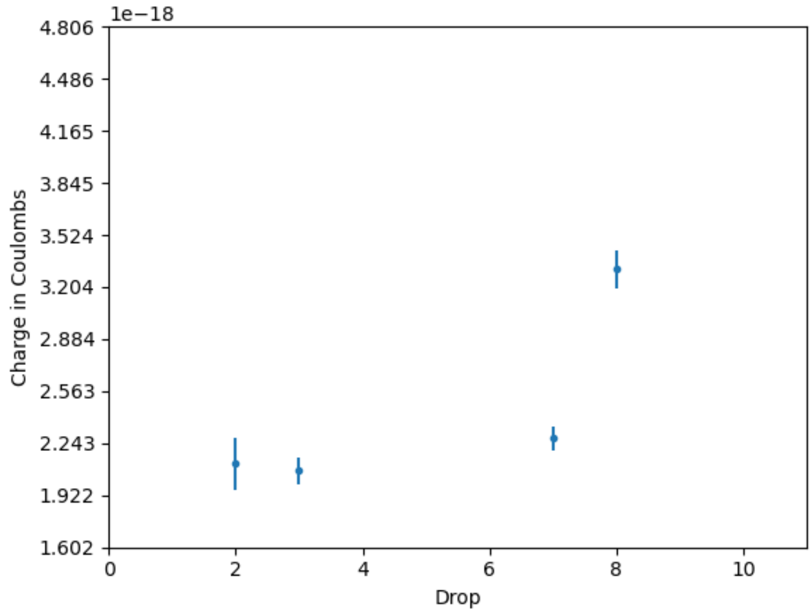

Figure 3: Drop number vs Charge graph with ticks spaced at $2e$. Y-axis from $10e-25e$

Based on figures 2 and 3 above, we can identify some candidates to find our value of e. For example, the difference between drops 4 and 1 should give us our calculated e value as this is the smallest step of difference in charge. Same for drops 9 and 6, 10 and 9, 3 and 7.

Drops	Difference
$q_1 - q_4 = e_1$	$1.049 \times 10^{-19} \pm 3.04 \times 10^{-20}$ C
$q_6 - q_9 = e_2$	$1.020 \times 10^{-19} \pm 5.88 \times 10^{-20}$ C
$q_9 - q_{10} = e_3$	$1.380 \times 10^{-19} \pm 8.89 \times 10^{-20}$ C
$q_7 - q_3 = e_4$	$1.980 \times 10^{-19} \pm 1.07 \times 10^{-19}$ C
e_{avg}	$1.357 \times 10^{-19} \pm 3.85 \times 10^{-20}$ C

Table 4: Shows difference for charges separated by $1e$

We can see that our calculated e_{avg} is reasonable as the error extends into the accepted value of e. However, it is not very precise as our value of e differs from the accepted by 15.3% and the associated error is about $\frac{24e}{100}$.

5 Discussion

The e value found in this paper is wildly imprecise. Of course, some error comes from randomness such as Brownian motion and air currents. However, a large proportion of it comes from how we took data. Looking at the error associated with v_{up} and v_{down} it is easy to see that we should've allowed the drops to fall and rise longer to get more precise measurements. This mistake stems from the fact that instead of focusing on finding good drops, we focused more on trackable drops. One difficulty when taking data was that the drops would be moving from side to side, in and out of frame, so we were limited to what drops to analyze. Another difficulty was trying to close the holes in the chamber, there were in total three holes (one for the light source, one to spritz through, and one for the camera). Of which we could only close one (the one for spritzing) since the other two were necessary. This makes the air currents in the chamber more frequent leading to worse data. Another consideration is the fact that the pressure in the chamber varies from outside it slightly. This source of error is much less than air currents and bad velocity measurements but should still be considered when looking at results. Nevertheless, we conclude that our calculated value of e agrees reasonably with the accepted value as it's on the same magnitude and is within error. If done carefully, one can find a much better e value following the steps in this paper and taking data more carefully.

References

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[2] Luke DeFreitas. Charge plot. https://drive.google.com/file/d/1X6eJ3Fw0MzkmZEgewXAJshDn7EXITCcU/view?usp=sharing, 2024.

[3] Luke DeFreitas. Drop data. https://drive.google.com/file/d/1fuoQpLZA8ajvWXlUbr4xfUY06ai28YfJ/view?usp=sharing, 2024.

[4] APS editing. October 1897: The discovery of the electron. *American Physical Society*, Octorber, 2000.

[5] Colorado State Univeristy. Millikan oil drop lab manual. January, 2019.

A Error Propagation

$$\delta E = E \times \sqrt{(\frac{\delta V}{V})^2 + (\frac{\delta d}{d})^2} \tag{9}$$

$$\delta T_c = \delta T_f \times \frac{5}{9}, \tag{10}$$

for converting Fahrenheit to Celsius.

$$\delta v = \frac{1}{N}\sqrt{(\delta v_1)^2 + (\delta v_2)^2 + (\delta v_3)^2 + ...} \tag{11}$$

for N velocities of v_{up} or v_{down}. Also used for q_{avg} with $v- > q$

$$\delta \eta_0 = \delta \eta_{max} = \delta T_c \times 4.790 \times 10^{-8} = 1.33 \times 10^{-8} \tag{12}$$

$$\delta r_0 = \delta_{max} r = \sqrt{(\frac{\partial r}{\partial \eta_0}\delta \eta_0)^2 + (\frac{\partial r}{\partial v_d}\delta v_d)^2 + (\frac{\partial r}{\partial \rho}\delta \rho)^2 + (\frac{\partial r}{\partial g}\delta g)^2} \tag{13}$$

$$\delta r_0 = \frac{1}{2}\sqrt{\frac{9 v_d}{2\rho g \eta_0}(\delta \eta_0)^2 + \frac{9\eta_0}{2\rho g v_d}(\delta v_d)^2 + \frac{9\eta_0 v_d}{2\rho^2 g}(\delta \rho)^2 + \frac{9\eta_0 v_d}{2\rho g^2}(\delta g)^2} \tag{14}$$

$$\delta_{max} q = \sqrt{[\frac{6\pi r_0(v_d + v_u)}{E}\delta \eta_0]^2 + [\frac{6\pi \eta_0(v_d + v_u)}{E}\delta r_0]^2 + [\frac{6\pi r_0 \eta_0}{E}\delta v_d]^2 + [\frac{6\pi r_0 \eta_0}{E}\delta v_u]^2 + [\frac{6\pi r_0 \eta_0(v_d + v_u)}{E^2}\delta E]^2} \tag{15}$$

$$\delta q_{diff} = \sqrt{(\delta q_i)^2 + (\delta q_j)^2} \tag{16}$$

For $q_i - q_j$ or $q_j - q_i$

B Excel

[3]

C Code

[1] [2]