HW9

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1(a) True. If LP is infeasible, then we add a new ~~constraints~~ constraints to make it IP, feasible region of IP is subset of LP, must be infeasible.

(b) False. Max $x+y$
s.t. $0 \le x < 2$ $0 \le y < 2$

For LP ~~sol~~ unbounded
IP sol $(1,1)$ value $= 2$.

(c) True. LP provides best possible objective value if it corresponds to IP when a new constraints is added, no better integer solution exist

(d) False. maximize $8x_1 + 5x_2$
s.t. $9x_1 + 5x_2 \le 43$
$x_1 + x_2 \le 6$
$x_1 \ge 4$
$x_1, x_2 \in \mathbb{Z}$

S1: $x_2 \le 1$, opt. val $= 40.56$, fractional
S2: $x_2 \ge 2$, infeasible
    S3: $x_1 = 4, x_2 \le 1$, opt-val $= 37$
    S4: $x_1 \ge 5$, $x_2 \le 1$, opt val 40

Meet the requirement
But Branch TWICE.

~~3~~ 2. (a) $\underline{z} =$ ~~5~~ 61.5 It must be no larger than the current opt. val 61.5

(b) $\bar{z} =$ ~~5~~ 64.5 It must be no smaller than the current possible opt. val 64.5.

(c) ~~FG~~ H I

For FG, current ~~lower~~ upper bound is smaller than solved optimal value, no need to explore:

H: It's infeasible, adding new constraints will still be infeasible. I: already ~~find~~ solve IP in this case.

(d) For [illegible] $x_4 = 0$ For E. $x_4 \ge 1$

3. Optimal solution on ~~original~~ relaxed LP is $(x, y) = (\frac{5}{3}, \frac{40}{3})$ opt. val $= \frac{205}{3}$.

So opti val for IP ≤ 68. Branch on $x = 1.66$; S1: $x_1 \le 1$ S2: $x \ge 2$.

For S1, sol on relaxed LP is $(x, y) = (1, 4)$ opt. val $= 65$

For S2, sol on relaxed LP is $(x, y) = (2, \frac{31}{14})$ opt. val $= 65$

In S2. opt. val for IP ≤ 65, Branch on y. S3: $y \le 2$ S4: $y \ge 3$.

For S3. sol on relaxed LP is $(2.6, 2)$ opti. val $= 68.2$.

For S4, infeasible.

Branch on $x = 2.6$ S5 $x \le 2$ S6 $x \ge 3$

For S5, sol $= (2, 2)$ opt. val $= 58 < 65$.

For S6 sol $= (3, \frac{10}{7})$ opt. sol $= \frac{477}{7}$

Branch on $y = \frac{10}{7}$ S7: $y \le 1$ S8: $y \ge 2$

For S7 sol $= (3.3, 1)$ opt. val $= 68.1$

For S8, infeasible.

Branch on $x = 3.3$ S9 $x \le 3$ S10 $x \ge 4$

For S9 sol $= (3, 1)$ opt. val $= 63 < 65$

For S10 sol $= (4, 0)$ opt. val $= 68 > 65$.

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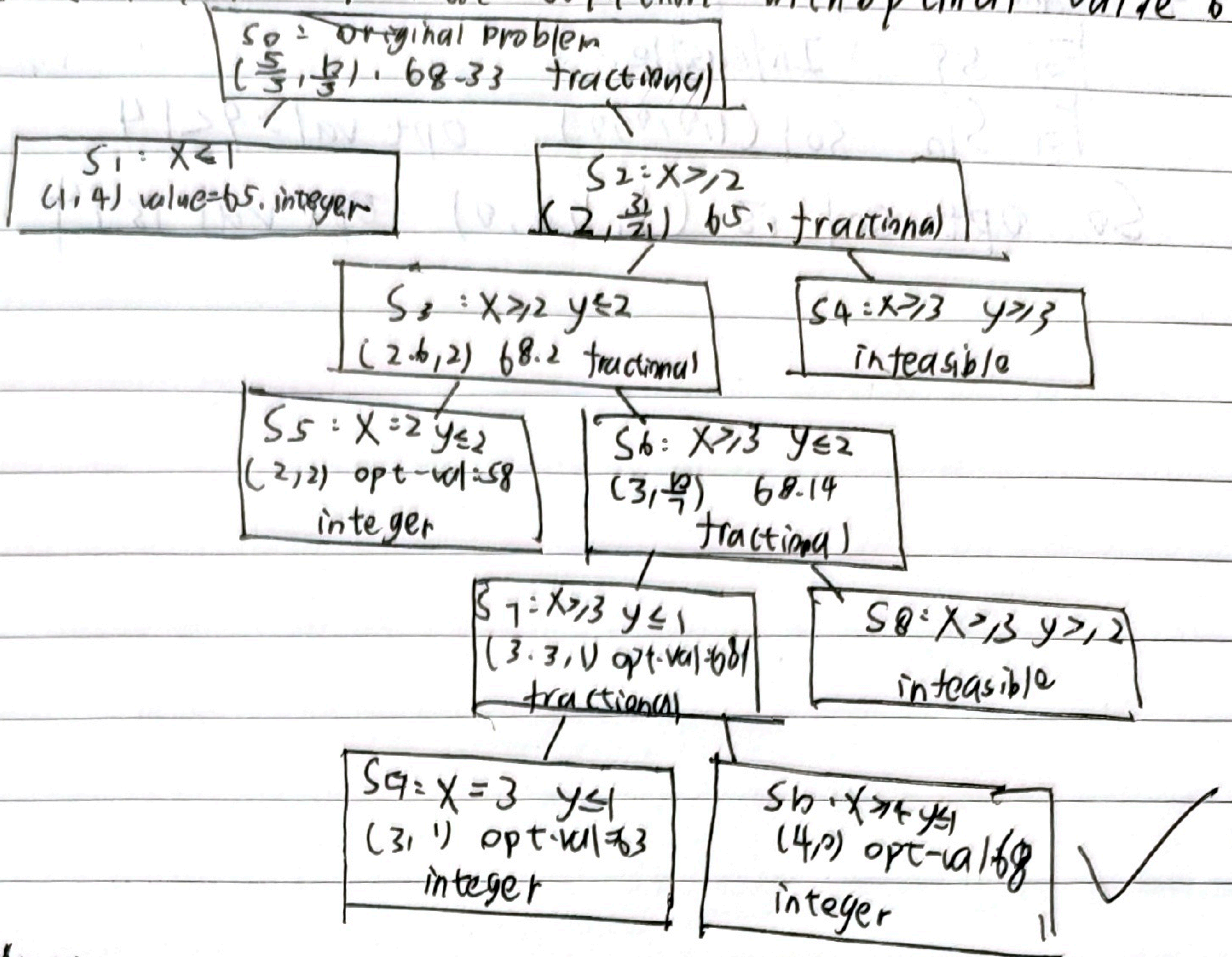
So (4, 0) is optimal solution with optmal value 68.



$X_3 + X_4 \le 1$:

4. (a) The company can at most build one warehouse.

$X_3 - X_1 \le 0$. If the company plan to build warehouse X_3, factory X_1 must be built. $X_4 - X_2 \le 0$ If the company plan to build warehouse X_4, factory X_2 must be built.

(b) optimal soluton on relaxed LP is (0.833, 1, 0, 1) opt.val = 16.5

Branch on X_1. S_1: $X_1 = 1$ S_2: $X_1 = 0$

For S_1 sol (1, 0.8, 0, 0.8) opt.val 16.2

For S_2 sol (0, 1, 0, 1) opt.val 9 integer

Branch on X_2 S_3: $X_2 = 1$ S_4: $X_2 = 0$

For S_3 sol (1, 1, 0, 0.5) opt.val = 16

For S_4 sol (1, 0, 0.8, 0) opt.val = 13.8.

For S_3 Brach on X_4 S_5: $X_4 = 1$ S_6 $X_4 = 0$

For S_5 ~~sol~~ Infeasible

For S_6 sol (1, 1, 0.2, 0) opt.val 15.2

Branch on X_3 S_7: $X_3 = 1$ S_8: $X_4 = 0$

For S_7 Infeasible 14

For S_8 (1, 1, 0, 0) opt.val 14 integer 14 > 9.

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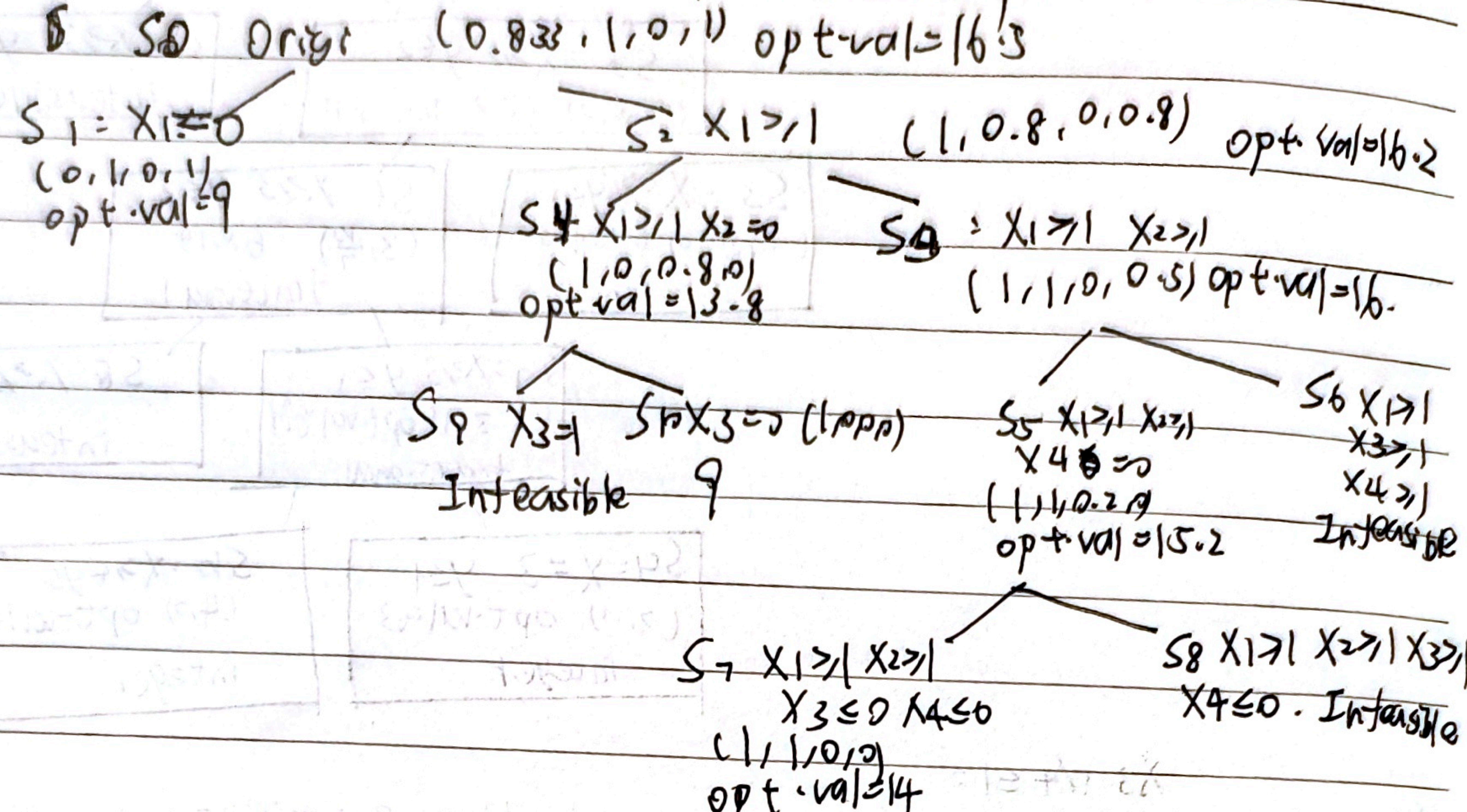
For S_4 Branch on X_3 S_9 $X_3=1$ S_{10} $X_3=0$

For S_9 Infeasible.

For S_{10} sol $(1,0,0,0)$ opt val $=9<14$

So opt. sol is $(1,1,0,0)$ opt val is 14



→ opt. sol $(1,1,0,0)$ opt. val $=14$