Date. Page.

3. (a) let X_{ij} be the number of cars moving from i to j. C_{ij} be the distance between i and j

$$\min_{X_{ij}} \sum_{j=1}^{5} \sum_{i=1}^{5} C_{ij} X_{ij}$$

s.t.;

$\sum_{i\neq1} X_{i1} - \sum_{j\neq1} X_{1j} \geq \cancel{200}\ 150 - 110$. that is $\sum_{i\neq1} X_{i1} - \sum_{j\neq1} X_{1j} \geq 40$

$\sum_{i\neq2} X_{i2} - \sum_{j\neq2} X_{2j} \geq 200 - 335$ $\quad \sum_{i\neq2} X_{i2} - \sum_{j\neq2} X_{2j} \geq -135$

$\sum_{i\neq3} X_{i3} - \sum_{j\neq3} X_{3j} \geq 600 - 400$ $\quad \sum_{i\neq3} X_{i3} - \sum_{j\neq3} X_{3j} \geq +200$

$\sum_{i\neq4} X_{i4} - \sum_{j\neq4} X_{4j} \geq 420 + 200$ $\quad \sum_{i\neq4} X_{i4} - \sum_{j\neq4} X_{4j} \geq -220$

$\sum_{i\neq5} X_{i5} - \sum_{j\neq5} X_{5j} \geq -610\ 1390$ $\quad \sum_{i\neq5} X_{i5} - \sum_{j\neq5} X_{5j} \geq 0 - 220$

$X_{ij} \geq 0 \ \forall i, j$ $\quad X_{ij} \geq 0 \ \forall i, j$

(b) optimal solution $\cancel{X_{24} = 135}$ $\cancel{X_{31} = 40}$ $\cancel{X_{34} = 85}$ $\cancel{X_{35} = 75}$

$X_{41} = 40$ $\quad X_{43} = 200$ $\quad X_{24} = 20$, $X_{ij} = 0$ otherwise

optimal value. $\cancel{4390}$ 2400

4 Optimal path is $S \to 3 \to 5 \to 7 \to T$

YES. Due to the problem's structure, the relaxed problem tend to have integer values because for a give node, flow either goes along an edge ($X_{ij} = 1$) or does not ($X_{ij} = 0$)

MAT 3007 HW1.

1 (a) let x_1 be type 1's production, x_2 be type 2's.

$$\underset{x_1,\ x_2}{\text{maximize}}\ (9-1.2)x_1 + (8-0.9)x_2$$

that is

$$\underset{x_1,\ x_2}{\text{maximize}}\ 7.8x_1 + 7.1x_2$$

$$\text{s.t.}\quad \frac{1}{8}x_1 + \frac{1}{4}x_2 \le 90$$

$$\frac{1}{2}x_1 + \frac{1}{6}x_2 \le 80$$

$$x_1, x_2 \geq 0.$$

(b)

$$\underset{x_1\ x_2}{\text{minimize}}\ -7.8x_1 - 7.1x_2$$

$$\frac{1}{8}x_1 + \frac{1}{4}x_2 + s_1 = 90$$

$$\frac{1}{2}x_1 + \frac{1}{6}x_2 + s_2 = 80$$

$$x_1, x_2, s_1, s_2 \geq 0.$$

(c) let x_3 be overtime hours.

$$\underset{x_1\ x_2\ x_3}{\text{maximize}}\ 7.8x_1 + 7.1x_2 - 8x_3$$

$$\text{s.t.}\quad \frac{1}{8}x_1 + \frac{1}{4}x_2 \le 90 + x_3$$

$$\frac{1}{2}x_1 + \frac{1}{6}x_2 \le 80$$

$$x_3 \le 40$$

$$x_1, x_2, x_3 \geq 0$$

(d) optimal solutions are $x_1 = 48$ $x_2 = 336$

optimal objective value 2760.

2. let $y_1 = |x_1 - x_3|$. $y_2 = |x_1 + 2|$ $y_3 = |x_2|$

That is

$$\underset{x_2\ y_1}{\text{minimize}}\ 2x_2 + y_1$$

$$\text{s.t.}\quad y_2 + y_3 \le 5$$

$$x_3 \le 1$$

$$x_3 \geq -1$$

$$y_1 \geq x_1 - x_3$$

$$y_1 \geq -x_1 + x_3$$

$$y_2 \geq x_1 + 2$$

$$y_2 \geq -x_1 - 2.$$

$$y_3 \geq x_2$$

$$y_3 \geq -x_2$$

$\rightarrow$ standard

$$\underset{x_2\ y_1}{\text{minimize}}\ 2x_2 + y_1$$

$$\text{s.t.}\quad y_2 + y_3 + s_1 = 5$$

$$x_3 + s_2 = 1$$

$$-x_3 + s_3 = 1$$

$$x_1 - x_3 - y_1 + s_4 = 0$$

$$-x_1 + x_3 - y_1 + s_5 = 0.$$

$$x_1 + 2 - y_2 + s_6 = 0$$

$$-x_1 - y_2 + s_7 = 2.$$

$$x_2 - y_3 + s_8 = 0$$

$$-x_2 - y_3 + s_9 = 0$$

$$s_i \geq 0\ \forall i$$