MAT 3007 HW1.

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1(a) let x_1 be type 1's production, x_2 be type 2's.

$$\max_{x_1, x_2} \; (9-1.2)x_1 + (8-0.9)x_2$$

that is

$$\max_{x_1, x_2} \; 7.8x_1 + 7.1x_2$$

s.t.

$$\frac{1}{8}x_1 + \frac{1}{4}x_2 \le 90$$

$$\frac{1}{2}x_1 + \frac{1}{6}x_2 \le 80$$

$$x_1, x_2 \geq 0.$$

(b)

$$\min_{x_1, x_2} \; -7.8x_1 - 7.1x_2$$

$$\frac{1}{8}x_1 + \frac{1}{4}x_2 + S_1 = 90$$

$$\frac{1}{2}x_1 + \frac{1}{6}x_2 + S_2 = 80$$

$$x_1, x_2, S_1, S_2 \geq 0.$$

(c) let x_3 be overtime hours.

$$\max_{x_1, x_2, x_3} \; 7.8x_1 + 7.1x_2 - 8x_3$$

s.t.

$$\frac{1}{8}x_1 + \frac{1}{4}x_2 \le 90 + x_3$$

$$\frac{1}{2}x_1 + \frac{1}{6}x_2 \le 80$$

$$x_3 \le 40$$

$$x_1, x_2, x_3 \geq 0$$

(d) optimal solutions are $x_1 = 48$ $x_2 = 336$

optimal objective value 2760.

2. let $y_1 = |x_1 - x_3|$. $y_2 = |x_1 + 2|$ $y_3 = |x_2|$

That is

$$\min_{x_2, y_1} \; 2x_2 + y_1$$

s.t.

$$y_2 + y_3 \le 5$$

$$x_3 \le 1$$

$$x_3 \geq -1$$

$$y_1 \geq x_1 - x_3$$

$$y_1 \geq -x_1 + x_3$$

$$y_2 \geq x_1 + 2$$

$$y_2 \geq -x_1 - 2.$$

$$y_3 \geq x_2$$

$$y_3 \geq -x_2$$

→ standard

$$\min_{x_2, y_1} \; 2x_2 + y_1$$

s.t.

$$y_2 + y_3 + S_1 = 5$$

$$x_3 + S_2 = 1$$

$$-x_3 + S_3 = 1$$

$$x_1 - x_3 - y_1 + S_4 = 0$$

$$-x_1 + x_3 - y_1 + S_5 = 0.$$

$$x_1 + 2 - y_2 + S_6 = 0$$

$$-x_1 - y_2 + S_7 = 2.$$

$$x_2 - y_3 + S_8 = 0$$

$$-x_2 - y_3 + S_9 = 0$$

$$S_i \geq 0 \;\; \forall i$$

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3. (a) let X_{ij} be the number of cars moving from i to j. C_{ij} be the ~~distance~~ cost between i and j

$$\underset{X_{ij}}{\text{minimize}} \quad \sum_{j=1}^{5}\sum_{i=1}^{5} C_{ij}X_{ij}$$

s.t.;

$\sum_{i\neq 1} X_{i1} - \sum_{j\neq 1} X_{1j} \geq \cancel{200}\ 150 - 110$. that is $\sum_{i\neq 1} X_{i1} - \sum_{j\neq 1} X_{1j} \geq 40$

$\sum_{i\neq 2} X_{i2} - \sum_{j\neq 2} X_{2j} \geq 200 - 335$ $\qquad \sum_{i\neq 2} X_{i2} - \sum_{j\neq 2} X_{2j} \geq -135$

$\sum_{i\neq 3} X_{i3} - \sum_{j\neq 3} X_{3j} \geq 400 - \cancel{400}$ $\qquad \sum_{i\neq 3} X_{i3} - \sum_{j\neq 3} X_{3j} \geq +200$

$\sum_{i\neq 4} X_{i4} - \sum_{j\neq 4} X_{4j} \geq -420 + 200$ $\qquad \sum_{i\neq 4} X_{i4} - \sum_{j\neq 4} X_{4j} \geq -220$

$\sum_{i\neq 5} X_{i5} - \sum_{j\neq 5} X_{5j} \geq -610 + 390$ $\qquad \sum_{i\neq 5} X_{i5} - \sum_{j\neq 5} X_{5j} \geq 0 - 220$

$X_{ij} \geq 0 \ \forall i, j$ $\qquad X_{ij} \geq 0 \ \forall i, j$

(b) optimal solution ~~$X_{24}=135$ $X_{31}=40$ $X_{34}=85$ $X_{35}=75$~~

$X_{41}=40$ $\quad X_{43}=200$ $\quad X_{24}=20$, $X_{ij}=0$ otherwize

optimal value. ~~4390~~ 2400

4 Optimal path is $S \to 3 \to 5 \to 7 \to T$

YES. Due to the problem's structure, the relaxed problem tend to have integer values because for a give node, flow either goes along an edge ($X_{ij}=1$) or does not ($X_{ij}=0$)