If x_1, x_2 are optimal sol, then any convex combination of x_1, x_2, is also optimal ($c_1x_1 + c_2x_2$, $c_1, c_2 \in \mathbb{R}$)

HW2

Q1

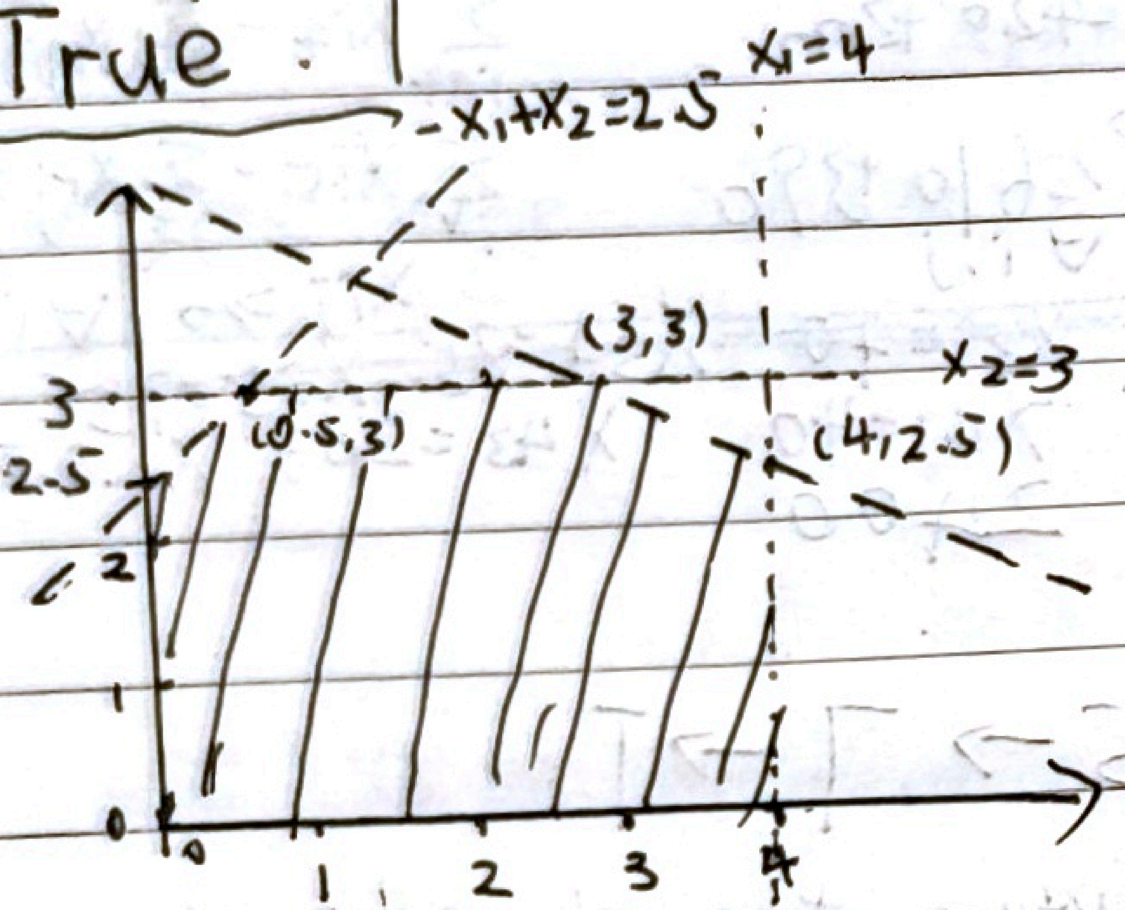
1 False ~~s.t.~~ e.g. minimize 2 s.t. $x \geq 0$ opitimal sol. is not bounded

2 False. e.g. same as 1 minimize 2 s.t. $x \geq 0$ any $x \geq 0$ is feasible, or infinite x.

3. True. → The LP is the same as

$$\begin{aligned} \underset{x_1, x_2}{\text{maximize}} \quad & x_1 + x_2 \\ \text{s.t.} \quad & -x_1 + x_2 \leq 2.5 \\ & x_1 + 2x_2 \leq 9 \\ & 0 \leq x_1 \leq 4 \quad 0 \leq x_2 \leq 3 \end{aligned}$$

Q2



we can find all feasible sol. from the graph: $(0,0)$, $(0,2.5)$, $(0.5,3)$, $(3,3)$, $(4,2.5)$, $(4,0)$

The optimal solution is $(4,2.5)$ and optimal value is 6.5

So active constraints $x_1 \leq 4$, $x_1 + 2x_2 \leq 9$.

Q3.

(a)

$$\begin{aligned} \text{minimize} \quad & -x_1 - 2x_2 - 4x_3. \\ \text{s.t.} \quad & x_1 + x_3 + s_1 = 8 \\ & x_2 + 2x_3 + s_2 = 14 \\ & x_1, x_2, x_3, s_1, s_2 \geq 0. \end{aligned}$$

(b) we have 2 constraints, at most 2 variables can be positive in any feasible solutions, which means there exist an optimal solution where at most 2 of x_1, x_2 or x_3 are positive, while the rest are zero

(c)

B	{1,2}	{1,3}	~~{1,4}~~	{1,5}	{2,3}	{2,4}	{3,4}	{3,5}	{4,5}
x_B	(8,14)	(1,7)		(8,14)	(-2,8)	(14,8)	(7,1)	(8,[illegible])	(8,14)
Obj. Value	-36	-29		-8	=	-28	-28	~~-32~~	-0
BFS	BFS	BFS		BFS	BS Not Feasible	BFS	BFS	BF Not Feasible	BFS

(d) Optimal solution is $(8,14,0,0,0)$

Q4 optimal solution is $x_a = x_b = x_c = x_d = x_e = x_f = x_g = x_h = x_i = x_j = 0$.

Optimal value is 5

But infact, the true value is 6. So we cannot move the integer constraints. in this case.

$\rightarrow (X^*, b-AX^*)$ must be a BFS of Q.

Q5 Suppose $(X^*, b-AX^*)$ is not basic feasible sol of Q

$\rightarrow$ we can find (a_1, b_1) (a_2, b_2) on polyhedron

Suppose $(X^*, b-Ax^*) = \lambda(a_1, b_1) + (1-\lambda)(a_2, b_2)$ where $\lambda \in [0, 1]$

if X^* is an extreme point,

$\forall j \in \{1, 2, \cdots m\}$, $A_j X^* = b_j$

also (a_1, b_1) $(a_2, b_2) \in Q \rightarrow A_j a_1 = b_j$ $a_j a_2 = b_j$

So $X^* = \lambda a_1 + (1-\lambda) a_2$

So X^* is not an extrom point of $P \rightarrow$ contradicts

$\rightarrow (X^*, b-AX^*)$ must be a BFS of Q