HW3

Q1 (a) True. B' do not correspond to a feasible solution, that is, new X_j is 0. while the origin X_i is not, which ~~must~~ won't make equilition hold

(b) False. While at least 1 solution will be BFS, there can be no basic optmal sol, when there is degeneracy or optimal solution lies in line segment or face of feasible region

(c) True. LP is unbounded according to simplex method & Bland's rule, we keep finding a new x', making value of funcion smaller. But due to unbounded, we can alway find basic direction $\vec{d}$. such that $C^T d < 0$, $d \geq 0$

Q2 (a) ~~minimize $y_1 + y_2 + y_3$~~

s. t.

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Q2

(a) introduce slack var & artificial var.

minimize $y_1 + y_2$

s.t. $x_1 - x_2 + 2x_3 - s_1 + y_1 = 2$

$x_2 - x_3 + 2x_4 + s_2 = 4$

$2x_1 + 3x_3 - x_4 + y_2 = 2$

$x_1, x_2, x_3, x_4, s_1, s_2, y_1, y_2 \geq 0$

(b) basic $\{x_1, x_4, y_2\}$, So $x_2 = x_3 = s_1 = s_2 = y_1 = 0$

that is, solving the sol, BFS $(2, 0, 0, 2, 0, 0, 0, 0)$

It's not optimal objective value of $y_1 + y_2 = 0$

but reduced cost of y_1 is -1 since it's not in the basis

(c) By Calculation, reduced cost is $-2.5 \quad 1.5 \quad -2 \quad -0.5 \quad 3$.

direction is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0 \\ -2 & 0.5 & 1 \end{bmatrix}$

x_2 is next to enter

~~B x' z' s~~

Q3 That is minimize $-x_1-2x_2-3x_3-8x_4$

s.t. $x_1-x_2+x_3+s_1=2$

$x_3-x_4+s_2=1$

$2x_2+3x_3+4x_4+s_3=8$

$x_1,x_2,x_3,x_4,s_1,s_2,s_3\geq 0$.

B	-1	-2	-3	-8	0	0	0	0	
5	1	-1	1	0	1	0	0	2	basic set {5,6,7}.
6	0	0	1	-1	0	1	0	1	basic solution (0,0,0,0,2,1,8)
7	0	2	3	4	0	0	1	8	objective function value = 0.
	0	-3	-2	-8	0	0	0	2	
1	1	-1	1	0	1	0	0	2	basic set {1,6,7}
6	0	0	1	-1	0	1	0	1	basic sol (2,0,0,0,0,1,8)
7	0	2	3	4	0	0	1	8	objective function value = 2
	0	0	$\frac{5}{2}$	-2	0	0	$\frac{3}{2}$	14	
1	1	0	$\frac{5}{2}$	2	1	0	$\frac{1}{2}$	6	basic set {1,2,6}
6	0	0	1	-1	0	1	0	1	basic sol (6,4,0,0,0,4,0)
2	0	1	$\frac{3}{2}$	2	0	0	$\frac{1}{2}$	4	objective function value = 14
	0	1	4	0	0	0	2	18	
1	1	0	1	0	1	0	0	2	basic set {1,4,6}
6	0	$\frac{1}{2}$	$\frac{7}{4}$	0	0	1	$\frac{1}{4}$	3	basic sol. (2,0,0,2,0,3,0)
4	0	$\frac{1}{2}$	$\frac{3}{4}$	1	0	0	$\frac{1}{4}$	2	objective function value = 18.

optimal sol (2,0,0,2)

opt. value 18

Q4

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convey it to auxiliary problem minimize y_1

$$\text{s.t. } 2x_1 - x_2 + 2x_3 + s_1 = -1$$
$$x_1 - x_2 - x_3 + s_2 = 4$$
$$x_2 - x_4 + y_1 = 0$$
$$x_1, x_2, x_3, x_4, s_1, s_2, y_1 \geq 0$$

$$\bar{c}_N = -\begin{bmatrix}0\\0\\1\end{bmatrix}^T \begin{bmatrix} 2 & -1 & 2 & 0 & 1 & 0 & 0 \\ 1 & -1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} = [0\ -1\ 0\ 1\ 0\ 0]$$

Phase I

B	0	-1	0	1	0	0	0	0
y_1	0	1	0	-1	0	0	1	0
→ B	0	0	0	0	0	0	1	0
y_1	0	1	0	-1	0	0	1	0

$$\bar{c} = (1, -1, 2, 0, 0, 0) - (0, 1, 0, -1, 0, 0, 1)$$
$$= (1, -2, 2, 1, 0, 0, -1)$$

Phase II

B	1	-2	2	1	0	0	0
s_1	2	-1	2	0	1	0	-1
s_2	1	-1	-1	0	0	1	4
x_2	0	1	0	-1	0	0	0

When we have negative reduced cost, the coloum entries is either negative or has the same basis. Thus, the question is unbounded

B

Q5

auxiliary problem minimize $y_1+y_2+y_3$

$$\text{s.t.}\quad \begin{aligned} &x_1+2x_2+4x_4+x_5+y_1=2\\ &x_1+2x_2-2x_4+x_5+y_2=2\\ &-x_1-4x_2+3x_3+y_3=1\\ &x_1,x_2,x_3,x_4,x_5,\ y_1,y_2,y_3\geq 0. \end{aligned}$$

$B=\{y_1,y_2,y_3\}$ $\quad A_B=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ $\quad x_B=\begin{bmatrix}2\\2\\1\end{bmatrix}$

$\bar{c}_N=c_N-c_B^TA_B^{-1}A_N=-(1,1,1)\cdot\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&2&0&4&1\\1&2&0&-2&1\\-1&4&3&0&0\end{bmatrix}$

$=(-1,0,-3,-2,-2)$

Phase I

B									
	-1	0	-3	-2	-2	0	0	0	-5
y_1	1	2	0	4	1	1	0	0	2
y_2	1	2	0	-2	1	0	1	0	2
y_3	-1	-4	3	0	0	0	0	1	1

basis $\{y_1,y_2,y_3\}$
sol $(0,0,0,0,0,2,2,1)$
value = 5

B									
	0	2	-3	2	-1	1	0	0	-3
x_1	1	2	0	4	1	1	0	0	2
y_2	0	0	0	-6	0	-1	1	0	0
y_3	0	-2	3	4	1	-1	0	1	3

basis $\{x_1,y_2,y_3\}$
sol $(2,0,0,0,0,0,0,3)$
value = -3

B									
	0	0	0	6	0	2	0	1	0
x_1	1	2	0	4	1	1	0	0	2
y_2	0	0	0	-6	0	-1	1	0	0
x_3	0	$-\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{1}{3}$	1

basis $\{x_1,x_3,y_2\}$
sol $(2,0,0,0,0,0,0,0)$
value = 0

B									
	0	0	0	0	0	1	1	1	0
x_1	1	2	0	0	1	$\frac{1}{3}$	$\frac{2}{3}$	0	2
x_4	0	0	0	1	0	$\frac{1}{6}$	$-\frac{1}{6}$	0	0
x_3	0	$-\frac{2}{3}$	1	0	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{3}$	1

basis $\{x_1,x_3,x_4\}$
sol $(2,0,1,0,0,0,0,0)$
value = 0

opt. value is 0. has BFS.

→ Phase II $\bar{c}=c^T-c_N^TA_B^{-1}A$

$=(1,3,0,1,-2)-(1,0,1)\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}^{-1}\begin{bmatrix}1&2&0&0&1\\0&0&0&1&0\\0&-\frac{2}{3}&1&0&\frac{1}{3}\end{bmatrix}=(0,1,0,0,-3)$

B						
	0	1	0	0	-3	-2
x_1	1	2	0	0	1	2
x_4	0	0	0	1	0	0
x_3	0	$-\frac{2}{3}$	1	0	$\frac{1}{3}$	1

basis $\{x_1,x_3,x_4\}$
sol $(2,0,1,0,0)$
value = 2.

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B	3	7	0	0	0	4
x_5	1	2	0	0	1	2
x_4	0	0	0	1	0	0
x_3	$-\frac{1}{3}$	$-\frac{4}{3}$	1	0	0	$\frac{1}{3}$

basis = $\{x_3, x_4, x_5\}$

sol = $\{0, 0, \frac{1}{3}, 0, 2\}$

value = -4

So sol is $(0, 0, \frac{1}{3}, 0, 2)$

objective function opt. value is -4.