HW8.

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1 (a) $\beta(\theta) = P(Y > c \mid \theta = \frac{1}{2})$ where $P(Y \le y) = P(\max(X_1 \cdots X_n) \le y)) = P(X_1 \le y)^n$

$= (\frac{y}{\theta})^n$

So $P(Y > c) = 1 - (\frac{c}{\theta})^n$

So $\beta\theta = P(Y > c \mid \theta) = \begin{cases} 1 - (\frac{c}{\theta})^n & c \le \theta \\ 0 & c > \theta \end{cases}$

(b) $P(Y > c \mid \theta = 0.5) = 0.05 \rightarrow 1 - (2c)^n = 0.05 \rightarrow c = \frac{0.95^{\frac{1}{n}}}{2}$.

(c) $P(Y > 0.48 \mid \theta = 0.5) = 1 - (\frac{0.48}{0.5})^{20} = 0.5580 > 0.05$

So we fail to reject H_0 at 5% significant level

(d) $P(Y > 0.52 \mid \theta = 0.5) = 1 - (\frac{0.52}{0.5})^{20} < 0 \rightarrow$ we reject H_0.

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2(a) let X be death before Holiday

Under H_0 $E(X) = 1919 \times \frac{1}{2} = 959.5$. $\sigma = \sqrt{n\theta(1-\theta)} \approx 21.9$

$T(X) = \frac{X - E(X)}{\sigma} \sim N(0,1)$ $T(\hat{X}) \approx -1.71$

$P = Prob(|T(X)| \geq |T(\hat{X})|) = 2\times(1-\Phi(1.71)) = 0.0872$.

So In null hypothesis, there would be a probability of 8.72%

that observing 922 or fewer deaths in 1919 total deaths observed

$P = 0.0872 > 0.05$ → fail to reject H_0.

Inter Classing

(b) $\hat{\theta} = \frac{997}{1919}$.

So CI $\hat{\theta} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n}}$ that is $(0.498, 0.522)$

3. (a) $P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$

$\log L(p) = \log \binom{n}{x} + x \log p + (n-x) \log(1-p)$

Likelihood ratio test is $\lambda = 2x(\log \hat{p} - \log p_0) + 2(n-x)(\log(1-\hat{p}) - \log(1-p_0)) = 2x \log\left(\frac{x}{np_0}\right) + 2(n-x) \log\left(\frac{1-\frac{x}{n}}{1-p_0}\right)$

We reject H_0 if $\lambda > \chi^2_{1,2}$

(b) Wald test statistic $W = \sqrt{n} \frac{\hat{p} - p_0}{\sqrt{\hat{p}(1-\hat{p})}} = \sqrt{n} \frac{\frac{x}{n} - p_0}{\sqrt{\frac{x}{n}(1-\frac{x}{n})}}$

$l(\theta_0) = l(\hat{\theta}) + (\hat{\theta} - \theta_0) l'(\hat{\theta}) + \frac{1}{2}(\hat{\theta} - \theta_0)^2 l''(\hat{\theta}) + O((\hat{\theta} - \theta_0)^3)$ where $l'(\hat{\theta}) = 0$

$\lambda = 2 \log\left(\frac{L(\hat{\theta})}{L(\theta_0)}\right) = -(\hat{\theta} - \theta_0)^2 l''(\hat{\theta}) + O((\hat{\theta} - \theta_0)^3)$

$W^2 = \frac{(\hat{\theta} - \theta_0)^2}{\hat{se}(\hat{\theta})^2} = nI(\hat{\theta})(\hat{\theta} - \theta_0)^2$

$\frac{\lambda}{W^2} = \frac{n^{-1} l''(\hat{\theta})}{-I(\hat{\theta})} + O(\hat{\theta} - \theta_0)$

Under H_0, $\frac{1}{I(\hat{\theta})} \to \frac{1}{I(\theta_0)} \to I(\theta_0) = E_{\theta_0}\left[\frac{\partial^2 \log f(X; \theta_0)}{\partial \theta^2}\right]$

$l''(\theta) = \sum \frac{\partial^2 \log f(X; \theta)}{\partial \theta^2}$.

$n^{-1} l''(\hat{\theta}) \to I(\theta_n)$

$\to W^2/\lambda \to 1$. $\to W \xrightarrow{d} N(0,1) \to$ We reject H_0 if $|z| > Z_{\alpha/2}$

Programming Question : Likelihood Ratio Test statistic 0.2615767

p-value = 0.609039 "Fail to reject H_0"