Hw6

Q1 (a) $\bar{X}_A = \frac{32+30+33+32+29+34+32}{7} = \frac{222}{7}$ $S_A^2 = \frac{1}{6}\Sigma(X_{A_i}-\bar{X}_A)^2 = \frac{61}{21}$

$\bar{X}_B = \frac{33+35+36+37+35}{5} = \frac{176}{5}$ $S_B^2 = \frac{1}{4}\Sigma(X_B-\bar{X}_B)^2 = \frac{107}{50}$

$$S_E^2 = \frac{S_A^2}{7} + \frac{S_B^2}{5} = 0.843.$$

$$df = \frac{0.843^2}{\frac{(\frac{61}{147})^2}{6} + \frac{(\frac{107}{250})^2}{4}} = 9.54$$

$\bar{X}_A - \bar{X}_B = -3.49$ $t_{crit} = t_{0.025, 9.54} \approx 2.262$

$CI = (-3.49 \pm 2.262 \times 0.843)$

That is $CI = (-5.397, -1.583)$

(b) $t_{0.05, 9.54} \approx 1.833$

Upper $CI = (-3.49) + 1.833 \times 0.843 = -1.945$

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Q2. $\hat{P}_1 = \frac{840}{2100} = \frac{2}{5}$ $\hat{P}_2 = \frac{323}{1900} = \frac{17}{100}$.

$SE = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} \approx 0.0137$.

CI is $(\hat{p}_1 - \hat{p}_2) \pm Z_{0.05} SE$

That is $(0.23 - 1.645 \times 0.0137,\ 0.23 + 1.645 \times 0.0137)$

or $(0.2075,\ 0.2525)$

Q3. $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$

i $Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$ where μ_0 is 0, that is $Z = \frac{\bar{X}}{\sigma/\sqrt{n}}$ under H_0

For a two-tailed test with $\alpha = 0.05$, critical values for Z is ± 1.96 (Z_α) we reject H_0 if $|Z| > 1.96$.

ii $P(Z > Z_{\frac{\alpha}{2}} \text{ or } Z > Z_{1-\frac{\alpha}{2}}) = 2P(Z < Z_{\alpha/2}) = 2 \times \frac{\alpha}{2} = \alpha$

iii $\beta = P(\text{Fail to reject } H_0 \mid H_1 \text{ is true})$ $Z \sim N(\frac{1}{\sigma/\sqrt{n}}, 1)$

So $\beta = P(Z \le Z_\alpha \mid H_1: \mu = 1)$ P

$= P(\frac{\bar{X} - 1}{\sigma/\sqrt{n}} \le Z_\alpha)$

$= \Phi(Z_\alpha - \frac{1}{\sigma/\sqrt{n}})$

as $n \to \infty$ $\sigma/\sqrt{n} \to 0$ so $Z_\alpha - \frac{1}{\sigma/\sqrt{n}} \to -\infty$ sin Z_α is constant

so $\beta \to 0$ as $n \to \infty$, type-II error tend to 0.

iv $\bar{X} = \frac{1}{15}\sum_{i=1}^{15} X_i = -\frac{227}{375}$

$Z = \frac{\bar{X}}{1/\sqrt{15}} = -2.344 < -1.96$ which reject null hypothesis H_0 and we prefer H_1