HW2

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Q1 $f(x) = \frac{3^x}{x!} \cdot e^{-3}$ let $Y = \sum_{i=1}^{50} X_i \sim N(\cancel{\frac{3}{\cdot}, \frac{3}{50}})$ $(3 \times 50 = 150, 3 \times 50 = 150)$

$P(1 \le Y \le 10) = P(\frac{1-150}{\sqrt{150}} \le \frac{Y-150}{\sqrt{150}} \le \frac{10-150}{\sqrt{150}})$

$= P(11.431 \le \frac{Y-150}{\sqrt{150}} \le 12.166)$

$\approx 1.69 \times 10^{-29} \approx 0$

Q2. let $Y = \sum_{i=1}^{n} X_i$. where $X_1 \cdots X_n$ has a random sample size of n from the identical distribution $\chi^2(1)$ So $X_1 \cdots X_n$ are iid.

$\mu_Y = n \cdot 1 = n$ $\sigma^2_Y = n \cdot \sigma_X^2 = 2n$.

Condition of Central limit Theorem satisfied.

$W = \frac{Y-n}{\sqrt{2n}} = \frac{Y-\mu_Y}{\sigma_Y}$ has limiting distribution $N(0,1)$

Q3 i $f(A) = \frac{\lambda_1^x}{x!} e^{-\lambda_1}$ $f(B) = \frac{\lambda_2^x}{x!} e^{-\lambda_2}$

$P(A+B=k) = \sum_{i=0}^{k} P(A=i, B=k-i) = \sum_{i=0}^{k} \frac{\lambda_1^i e^{-\lambda_1}}{i!} \cdot \frac{\lambda_2^{k-i} e^{-\lambda_2}}{(k-i)!}$

$= e^{-(\lambda_1+\lambda_2)} \sum_{i=0}^{k} \frac{\lambda_1^i \lambda_2^{k-i}}{i!(k-i)!}$

$\overset{\text{binomal espansion}}{=} e^{-(\lambda_1+\lambda_2)} \cdot \frac{(\lambda_1+\lambda_2)^k}{k!}$

So $P(A+B) \sim$ Possion $(\lambda_1+\lambda_2)$

ii when $n=1$ it is absolutely True. $z_1 \sim$ Possion(λ)

when $n=k$ suppose $\sum_{i=1}^{k} z_i \sim$ Possion$(k\lambda)$ stands True

now consider $n=k+1$ $\sum_{i=1}^{k+1} z_i = (\sum_{i=1}^{k} z_i) + z_{k+1}$

$\sum_{i=1}^{k} z_i \sim$ Possion$(k\lambda)$ $z_{k+1} \sim$ Possion(λ), From i, we know $\sum_{i=1}^{k+1} z_i \sim$ Possion $(k\lambda+\lambda)$ or Possion$((k+1)\lambda)$

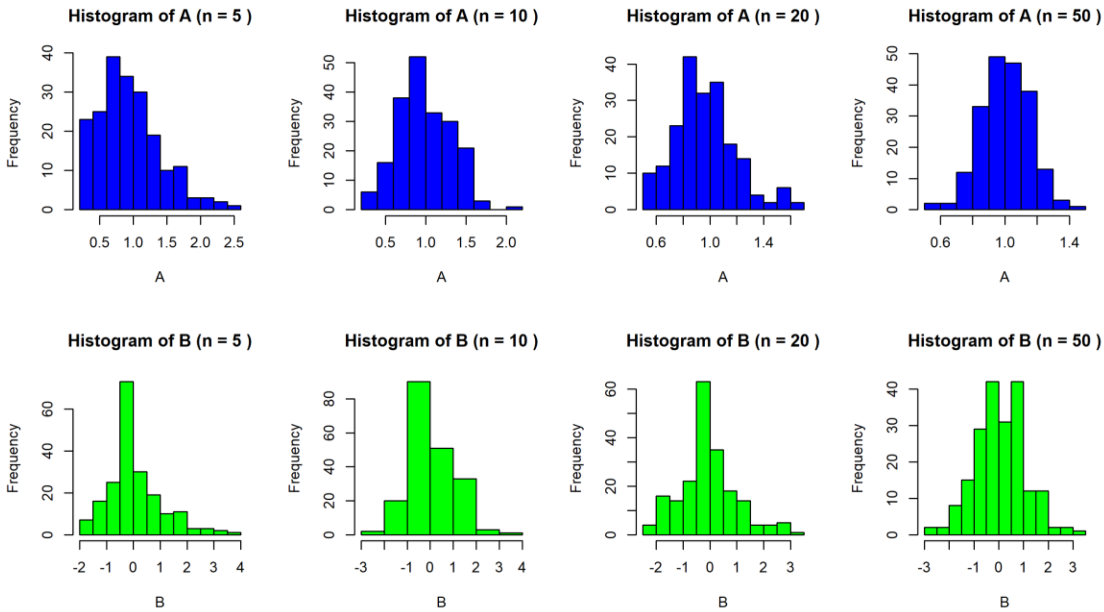
iii let $X = \frac{1}{n} \sum_{i=1}^{n}$ then $\mu_X = \lambda$ $\sigma^2_X = \frac{\lambda}{n}$.

From Central limit theorem. $\cancel{\sqrt{n}(\frac{1}{n}\sum_{i=1}^{n} z_i - \lambda) = \frac{X-\lambda}{\frac{\sqrt{\lambda}}{\sqrt{n}}}} = \frac{X-\mu_X}{\sigma_X \cdot \sqrt{\lambda}}$

$U_1 = \frac{\sum_{i=1}^{n} z_i - n\lambda}{\sqrt{n\lambda}} \sim N(0,1)$ $U_2 = \sqrt{n}(\frac{1}{n}\sum_{i=1}^{n} z_i - \lambda)$ $\cancel{\rightarrow N(0,\lambda)}$

$= \sqrt{\lambda} U_1 \sim N(0,\lambda)$

```r
# Load necessary library and initialize the parameter
library(ggplot2)

sample_sizes <- c(5, 10, 20, 50)
A_list <- list()
B_list <- list()

# Loop through each sample size
for (n in sample_sizes) {
  A_vals <- numeric(200)
  B_vals <- numeric(200)

  # Run 200 simulations for each sample size
  for (sim in 1:200) {
    Z <- rpois(n, 1)  # Generate Poisson distribution with lambda = 1
    A <- mean(Z)  # Calculate A
    B <- sqrt(n) * (A - 1)  # Calculate B

    A_vals[sim] <- A
    B_vals[sim] <- B
  }

  # Store results
  A_list[[paste0("n_", n)]] <- A_vals
  B_list[[paste0("n_", n)]] <- B_vals
}

# Plot histograms
par(mfrow = c(2, 4))  # 2 rows, 4 columns for the 8 histograms

# Plot histograms for A
for (n in sample_sizes) {
  hist(A_list[[paste0("n_", n)]], main = paste("Histogram of A (n =", n, ")"), xlab = "A", col =
"blue")
}

# Plot histograms for B
for (n in sample_sizes) {
  hist(B_list[[paste0("n_", n)]], main = paste("Histogram of B (n =", n, ")"), xlab = "B", col =
"green")
}
```



Histogram of A (n = 5)
Histogram of A (n = 10)
Histogram of A (n = 20)
Histogram of A (n = 50)
Histogram of B (n = 5)
Histogram of B (n = 10)
Histogram of B (n = 20)
Histogram of B (n = 50)
Frequency
A
B