Active vs Passive: Price Discovery in Automated Market Makers

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Stage 2 Presentation

24 May 2024

Thesis Background

Thesis title: **The Quality of Decentralised Markets**

- Decentralised market integrity
- The role decentralised markets play in price formation
- Liquidity in decentralised markets

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  - ◇ Prices set by market makers (Active market making)

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- Quotes drive price discovery in LOB markets (Brogaard, Hendershott, and Riordan, 2019)
  - Prices set by market makers (Active market making)
- **Can AMMs contribute to price discovery?**
  - AMM $\neq$ LOB

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- Compare monthly price discovery shares between Uniswap and Binance
  - Uniswap on Ethereum (CPMM, Concentrated Liquidity)
  - Binance (LOB)

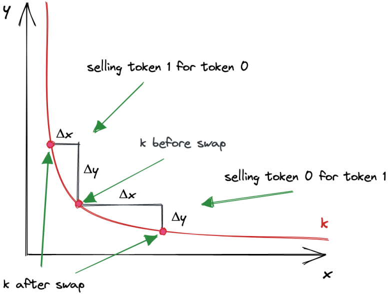
Uniswap

Constant Product Market Maker

Pricing function (Invariant):

$$\sqrt{xy} = L$$

where x and y are the quantities of the assets in the pool



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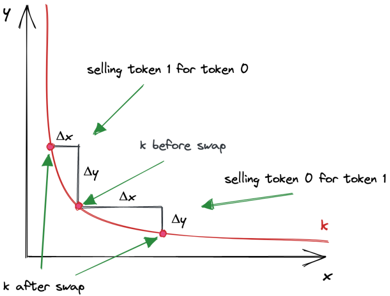
Holding L constant can be used to calculate the trade:

$$\sqrt{(x + \Delta x)(y + \Delta y)} = L$$



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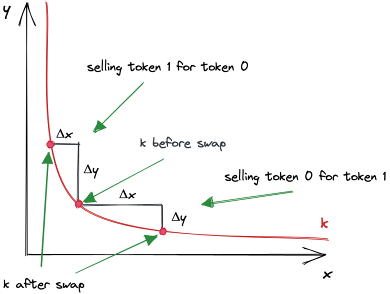
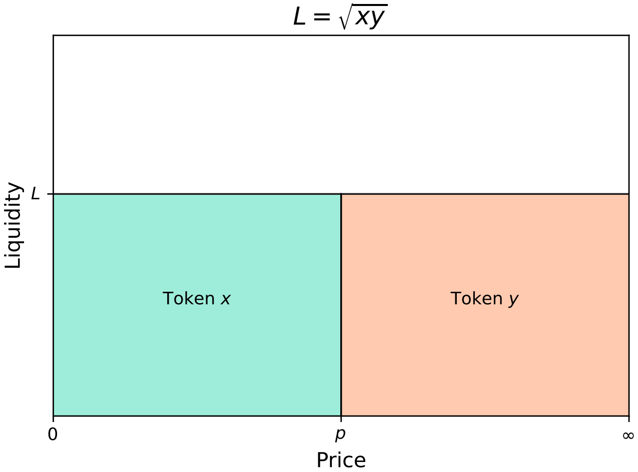


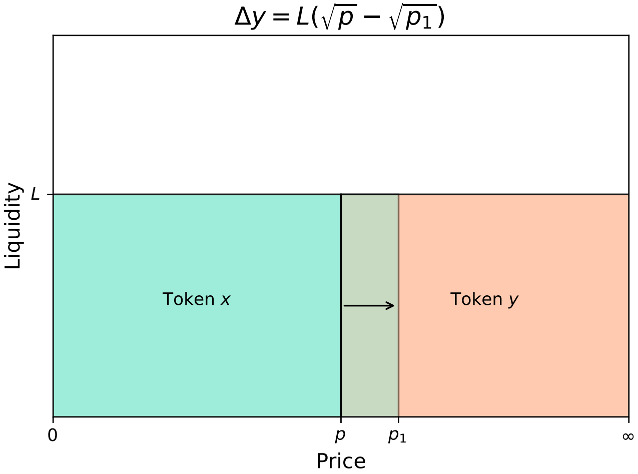
Concentrated liquidity:

$$(x + \frac{L}{\sqrt{p_b}})(y + L\sqrt{p_a}) = L^2$$

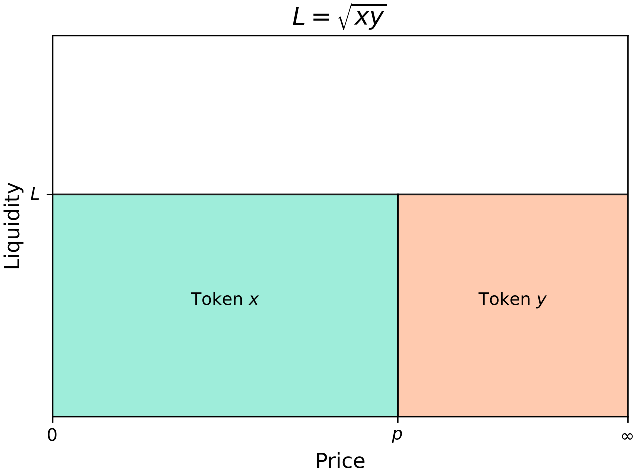
Constant Product Market Maker Example



Constant Product Market Maker Example



Constant Product Market Maker Example



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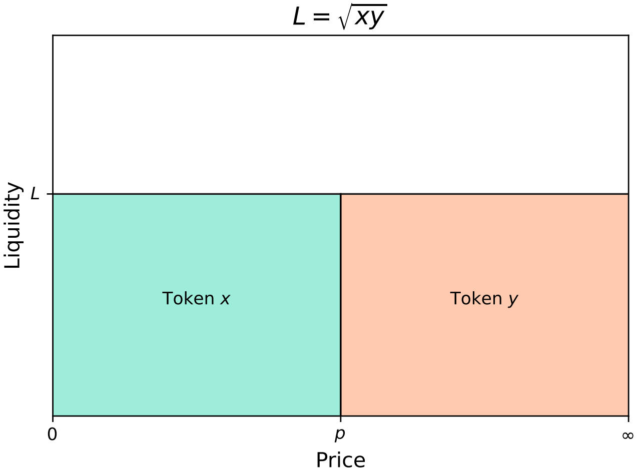
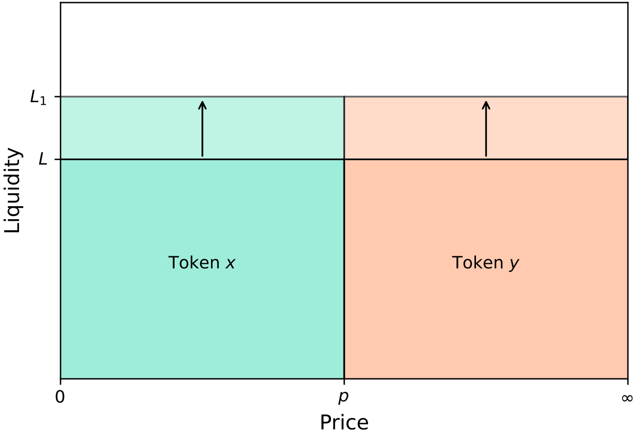


Constant Product Market Maker Example

$$L = \sqrt{(x+\Delta x)(y+\Delta y)}$$



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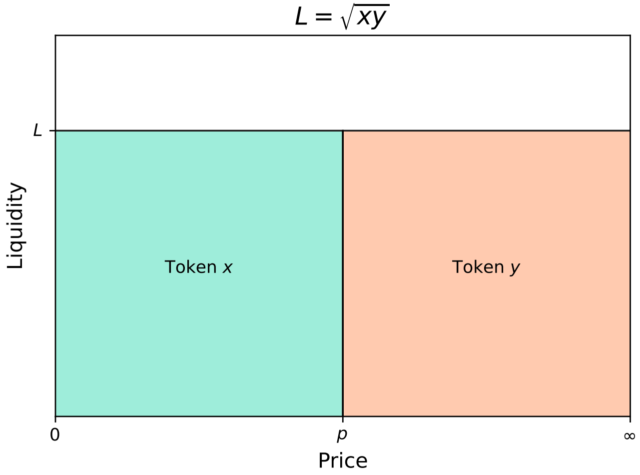

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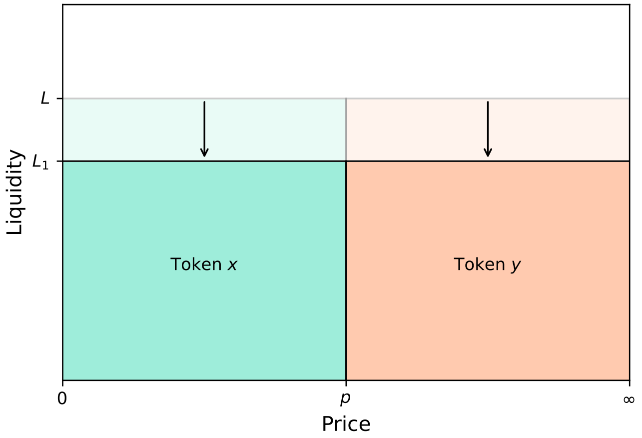

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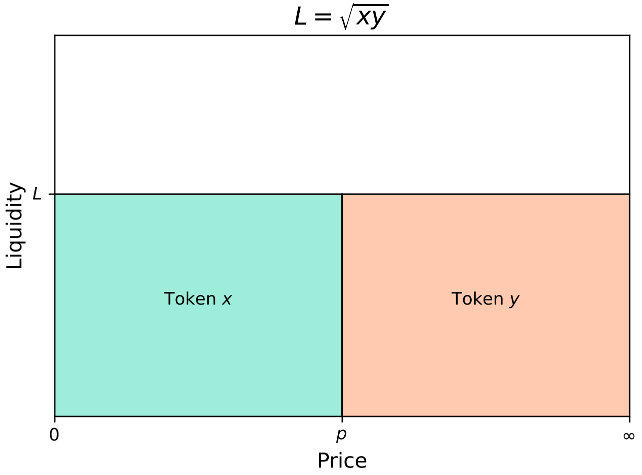

Concentrated Liquidity Example

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- Who decides the order of the transactions in the block?
    - Maximal extractable value (MEV)

Economics of Ethereum

John et al. (2024)

- Block builders (miners) control transactions

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- Proposer Builder Separation (Proof of stake)
  - ◇ Searcher, Builder, Relay, Proposer

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John et al. (2024)

- Block builders (miners) control transactions
- Proposer Builder Separation (Proof of stake)
  - Searcher, Builder, Relay, Proposer
- Searchers need to sufficiently incentivise the builder

Informed traders and AMMs

- Why would an informed trader trade on the AMM?
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  - Guarantee inclusion (For a price)
  - Max profit: settle discrete time market, then the continuous market

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  - (Capponi, Jia, and Yu 2023)
- Trade off between liquidity and cost of inclusion

Testable hypothesis

Informed traders can bid for priority inclusion to maximise profit

- Cost of inclusion
- Liquidity in AMM

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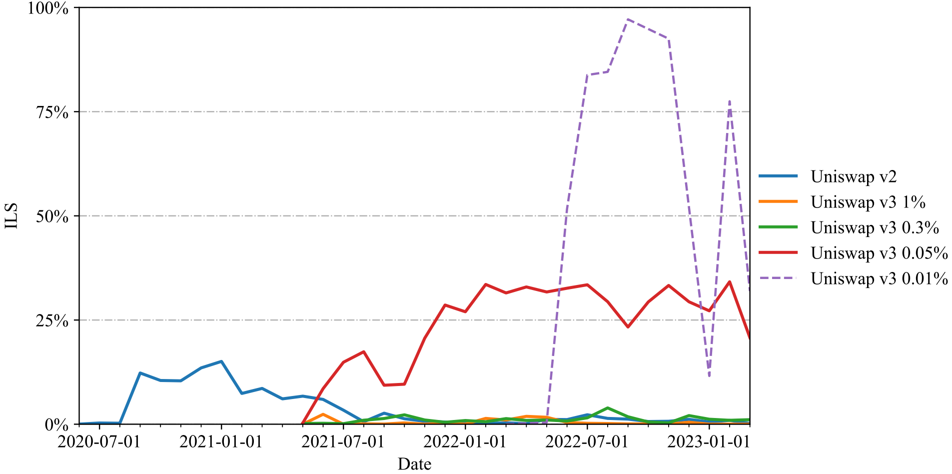
1. *The AMM will lead the price discovery process when it is more liquid then the LOB*
2. *The AMM will lead the price discovery process when priority inclusion is cheap*
3. *The AMM will lead the price discovery process when asset volatility is higher*

Price Discovery Shares

- Yan-Zivot-Putnins Information Leadership Share (*ILS*)
- Significant difference in noise between markets
  - Information/Component Share both measure a relative avoidance of noise
- VECM with 300 lags ($\sim$1 hour)
  - Mid-quote prices for all Uniswap markets and Binance
  - Ethereum blocktime ($\sim$12 seconds)

Information Leadership Share



What Determines Price Discovery?

$$ILS_{i,t} = \alpha_{i,t} + Spread_{i,t} + Depth_{i,t} + Gas_t + Volatility_{i,t} + Controls_{i,t} + \epsilon_{i,t}$$

Spread

- Half Quoted Spread (Fees)
- Effective Spread

Depth

- 0-1%
- 1-2%
- 2-3%

Volatility

- Realized Volatility

Gas

- Gas price

Controls

- Volume

Determinants of price discovery

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dependent variable:	*ILS*	*ILS*	*ILS*	*ILS*	*ILS*	*ILS*	*ILS*	*ILS*
QuotedSpread	−0.099***	−0.096***	−0.096***	−0.097***	−0.082***	−0.085***	−0.085***	−0.088***
	(−8.84)	(−8.61)	(−8.65)	(−8.72)	(−8.56)	(−8.42)	(−8.40)	(−8.50)
EffectiveSpread	0.001				0.005			
	(0.12)				(1.13)			
$Depth_{[0-1\%]}$	−0.006	−0.007***			0.008	−0.003		
	(−1.15)	(−3.68)			(1.40)	(−1.34)		
$Depth_{[1-2\%]}$	−0.017**		−0.006***		−0.029***		−0.003	
	(−2.48)		(−3.09)		(−3.64)		(−1.36)	
$Depth_{[2-3\%]}$	0.020***			−0.002	0.022***			0.002
	(5.00)			(−1.06)	(5.25)			(1.32)
RealizedVolatility		−0.009***	−0.008***	−0.006***		−0.010***	−0.010***	−0.008***
		(−4.11)	(−3.77)	(−2.61)		(−4.08)	(−4.09)	(−3.42)
Volume	0.015***	0.012***	0.011***	0.008***	0.017***	0.011***	0.011***	0.007***
	(4.50)	(4.29)	(3.90)	(3.09)	(5.07)	(4.31)	(4.65)	(3.69)
Gas	−0.015***	−0.010***	−0.009***	−0.010***				
	(−4.16)	(−2.69)	(−2.61)	(−2.61)				
Pair Effects	N	N	N	N	Y	Y	Y	Y
Time Effects	N	N	N	N	Y	Y	Y	Y
Adjusted R^2	24.6%	23.7%	23.3%	22.6%	43.3%	42.3%	42.3%	42.2%

Significance: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

Relative determinants of price discovery

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dependent variable:	*ILS*	*ILS*	*ILS*	*ILS*	*ILS*	*ILS*	*ILS*	*ILS*
QuotedSpreadRatio	−0.037***	−0.031***	−0.032***	−0.032***	−0.038***	−0.038***	−0.038***	−0.040***
	(−7.41)	(−6.87)	(−7.03)	(−7.15)	(−7.08)	(−7.03)	(−7.00)	(−7.05)
EffectiveSpreadRatio	0.007				0.008*			
	(1.48)				(1.88)			
$Depth_{[0-1\%]}Ratio$	−0.023***	−0.004*			0.003	−0.005**		
	(−4.46)	(−1.85)			(0.48)	(−2.24)		
$Depth_{[1-2\%]}Ratio$	0.031***		−0.001		−0.020***		−0.005**	
	(5.48)		(−0.51)		(−2.67)		(−2.35)	
$Depth_{[2-3\%]}Ratio$	−0.011***			−0.003*	0.014***			0.000
	(−4.12)			(−1.65)	(3.69)			(−0.22)
RealizedVolatilityRatio		−0.016***	−0.014***	−0.015***		0.000	0.000	0.001
		(−6.27)	(−5.91)	(−6.16)		(−0.02)	(−0.15)	(0.34)
VolumeRatio	0.015***	0.015***	0.013***	0.014***	0.028***	0.024***	0.024***	0.019***
	(6.41)	(6.52)	(6.40)	(7.52)	(7.36)	(7.82)	(8.23)	(8.70)
Gas	−0.014***	−0.014***	−0.015***	−0.015***				
	(−3.34)	(−3.42)	(−3.51)	(−3.61)				
Pair Effects	N	N	N	N	Y	Y	Y	Y
Time Effects	N	N	N	N	Y	Y	Y	Y
Adjusted R^2	16.2%	15.1%	14.7%	14.9%	40.2%	39.3%	39.3%	38.9%

Significance: * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

AMM Price Leadership

- Informed traders exploit trading costs
  - AMM fee negatively related with ILS
  - Gas fee negatively related with ILS
- Informed traders in AMM reduce liquidity (depth)
  - LPs adjust for adverse selection costs

Noise trader migration

Aoyagi and Ito (2021)

Coexisting exchanges: AMM and LOB

Noise traders outpace informed traders

- Noise trader tend to low cost market $\rightarrow$ AMM liquidity equilibrium $\uparrow$

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- $\downarrow$ LOB spread $\rightarrow$ LOB Noise traders $\uparrow$

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- $\downarrow$ AMM liquidity equilibrium $\rightarrow$ Liquidity providers step away

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- Informed traders follow liquidity $\rightarrow$ Adverse Selection LOB $\uparrow$, AMM $\downarrow$
- $\uparrow$ LOB spread $\rightarrow$ Noise trader tend to low cost market

Conclusion

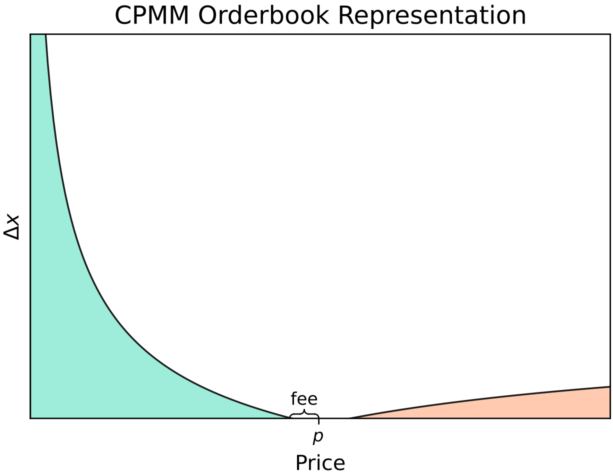
- AMM can play an important informational role in crypto currency markets
- Uniswap v3 0.05% pools have an average ILS of 24.1%
- Uniswap v3 0.01% pools have an average ILS of 61.1%
- Provide evidence of the noise trader feedback loop
  - Informed traders negatively impact AMM liquidity

Future Work

Progress Update

- AMM Liquidity Measurement
  - To revise (June 24)
- DEX Manipulation Index
  - For completion (November 24)
- Finalise Submission
  - Stage 3 (March 25)
  - Submission (April 25)

Constant Product Market Maker Orderbook Example



Price discovery estimation

For each pair-month, we estimate a reduced form VECM of the log price series ($p_{1,t}$ to $p_{n,t}$) with 300 lags (prices are sampled based on the Ethereum block time where trading is continuous in the AMM and the LOB).

$$\Delta p_t = aZ_{t-1} + \sum_{i=1}^{300} b_i \Delta p_{t-i} + \epsilon_t \tag{1}$$

where Δp_t is the $n \times 1$ midquote return vector, α is the $n \times (n-1)$ matrix of error correction coefficients, Z_{t-1} is the $n \times 1$ co-integrating vector, b_i is the $n \times n$ coefficient matrix for lag i and ϵ_t is the $n \times 1$ vector of residuals.

Price discovery estimation

From the reduced form VECM estimates in 1 we derive the corresponding infinite lag VMA representation in structural form assuming recursive contemporaneous causality running from the first through to the last price series.

$$
\begin{aligned}
\Delta p_{1,t} &= \sum_{l=0}^{\infty} A_{1,l}\varepsilon_{1,t-1} + \sum_{l=0}^{\infty} A_{2,l}\varepsilon_{2,t-1} + \cdots + \sum_{l=0}^{\infty} A_{n,l}\varepsilon_{n,t-1} \\
\Delta p_{2,t} &= \sum_{l=0}^{\infty} B_{1,l}\varepsilon_{1,t-1} + \sum_{l=0}^{\infty} B_{2,l}\varepsilon_{2,t-1} + \cdots + \sum_{l=0}^{\infty} B_{n,l}\varepsilon_{n,t-1} \\
&\vdots \qquad\qquad\qquad\qquad \vdots \qquad\qquad\qquad\qquad \vdots \\
\Delta p_{n,t} &= \sum_{l=0}^{\infty} N_{1,l}\varepsilon_{1,t-1} + \sum_{l=0}^{\infty} N_{2,l}\varepsilon_{2,t-1} + \cdots + \sum_{l=0}^{\infty} N_{n,l}\varepsilon_{n,t-1}
\end{aligned}
$$

Price discovery estimation

We obtain the structural VMA coefficients by computing the orthogonalized impulse response functions and the (contemporaneously uncorrelated) structural VMA errors ($\varepsilon_{1,t}$ to $\varepsilon_{n,t}$) by mapping their relation to the reduced form errors. Innovations in the permanent component (the efficient price, m_t) are given by

$$\Delta m_t = \theta_{\varepsilon 1}\varepsilon_{1,t} + \theta_{\varepsilon 2}\varepsilon_{2,t} + \cdots + \theta_{\varepsilon n}\varepsilon_{n,t}$$

The variance of the innovations in the efficient price is therefore:

$$\begin{aligned} Var(\Delta m_t) &= Var(\theta_{\varepsilon 1}\varepsilon_{1,t} + \theta_{\varepsilon 2}\varepsilon_{2,t} + \cdots + \theta_{\varepsilon n}\varepsilon_{n,t}) \\ &= \theta_{\varepsilon 1}^2 Var(\varepsilon_{1,t}) + \theta_{\varepsilon 2}^2 Var(\varepsilon_{2,t}) + \cdots + \theta_{\varepsilon n}^2 Var(\varepsilon_{n,t}) \end{aligned}$$

Price discovery estimation

Information shares (IS) are obtained as each priceâs contribution to the variance of the efficient price innovations

$$IS_n = \frac{\theta_{\varepsilon n}^2 Var(\varepsilon_{n,t})}{Var(\Delta m_t)}$$

Component shares (CS) are obtained by normalizing the permanent price impacts of each price series in the reduced form model.

$$CS_n = \frac{\theta_{\epsilon n}}{\sum_{i=1}^{n} \theta_{\epsilon i}}$$

Price discovery estimation

Finally, we calculate the information leadership share (ILS). In the two-price case, marketâs propensity to reflect new information (how much market *i*s price responds to an innovation in the efficient price) can be obtained from the ratio $\beta_i = \frac{IS_i}{CS_i}$, which when normalized gives the information leadership share

$$ILS_n = \frac{\beta_n^2}{\sum_{i=1}^{n} \beta_i^2}$$