Homework #1: Fundamentals

Question I (10 points)

Suppose that A and B are nonempty sets and $A \times B \subseteq B \times A$. Show that $A = B$. Suggestion: show that $A \subseteq B$ and $B \subseteq A$, using proof by contradiction in each case.

Question II (10 points each)

Each case below give a relation on the set of all nonempty subsets of $\mathcal{N}$. In each case, say whether the relation is reflexive, whether it is symmetric, and whether it is transitive.

a. R is defined by: ARB if and only if $A \subseteq B$.

b. R is defined by: ARB if and only if $A \cap B \neq \emptyset$.

c. R is defined by: ARB if and only if $1 \in A \cap B$.

Question III (10 points)

Prove using mathematical induction that for every nonnegative integer n,

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}.$$

Question IV (5 points each)

In each case below, construct a truth table for the statement and find another statement with at most one operator ($\vee, \wedge, \neg$, or $\rightarrow$) that is logically equivalent.

a. $(p \rightarrow q) \wedge (p \rightarrow \neg q)$

b. $p \wedge (p \rightarrow q)$

c. $(p \rightarrow q) \wedge (\neg p \rightarrow q)$

d. $p \leftrightarrow (p \leftrightarrow q)$

e. $q \wedge (p \rightarrow q)$

QUESTION V (2 points each)

Let $P = \emptyset$, $Q = \{\epsilon\}$, $R = \{aba, bcb, cac\}$, and $S = \{aba, abc, abb\}$.

1. Is $P \subset Q$ correct?
2. What are the members of $R \cap S$?
3. What are the members of $R \triangle S$?
4. What is the cardinality of R?

QUESTION VI (3 points each)

Let $P = (x \rightarrow y) \wedge (y \rightarrow z)$ where x, y, and z are boolean variables.

1. What is the value of P when $x = z =$ true and $y =$ false?
2. What is the value of P when $x = z =$ false and $y =$ true?

QUESTION VII

1. (5 points) Design a 4-state finite automaton that accepts the words over $\{a, b\}$ that end with aba.
2. (6 points) Design a 6-state finite automaton that accepts the words over $\{0, 1\}$ that contain either 000 or 111.