Homework 5

CSC 445-01: Theory of Computation

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2.15 Matthew

2.30 Matthew

We will prove L is not a context free language using a proof by contradiction and the Pumping Lemma.

$$L = \{t_1\#t_2\#...\#t_k | k \geq 2, t_i \in \{a, b\}^*, \exists(t_i = t_j, i \neq j)\}$$

Suppose L is a CFL. Then for a string $s \in L$ of length greater than p, there exists some decomposition of s, $s = uvxyz$ such that

1. $uv^ixy^iz \in L$ for $i \geq 0$
2. $|vy| > 0$
3. $|vxy| \leq p$

We let $s =$ because $|s| \geq p + 1$ and $s \in L$.

Ooooof

3.8

b

On input string w

1. Move the head to the front of the tape. Scan the tape and mark the first 1 that has not been marked. If none are found, move to stage 5.
2. Move the head to the front of the tape. Scan the tape and mark the first 0 that has not been marked. If no 0 is found, reject.
3. Scan the tape and mark the first 0 that has not been marked. If no 0 is found, reject.
4. Go to stage 1
5. Move the head to the front of the tape. Scan the tape to see if any unmarked 0s remain. If none remain, accept, else reject.

c

Run the machine from part b on input w. If it accepts, then reject. If it rejects, then accept. Alternatively, build a new TM as described below.

On input string w

1. Move the head to the front of the tape. Scan the tape and mark the first 1 that has not been marked. If none are found, move to stage 5.

2. Move the head to the front of the tape. Scan the tape and mark the first 0 that has not been marked. If no 0 is found, accept.
3. Scan the tape and mark the first 0 that has not been marked. If no 0 is found, accept.
4. Go to stage 1
5. Move the head to the front of the tape. Scan the tape to see if any unmarked 0s remain. If none remain, reject, else accept.

3.15

b Matthew

Suppose we have Turing Machines M_1 and M_2 that decide languages L_1 and L_2 respectively. We describe the Turing Machine M' that decides the concatenation of L_1 and L_2.

On input string w

c

Suppose we have a Turing Machine M that decides the language L. We describe the nondeterministic two-tape Turing Machine M' that decides the language L^*.

On input string w

1. Nondeterministically select an integer $k \leq |w|$
2. Nondeterministically partition w into k components, $w_1, w_2, ..., w_k$
3. Run M on each string $w_1, w_2, ..., w_k$
   - If M accepts all the strings $w_1, w_2, ..., w_k$ then accept

d

Suppose we have a Turing Machine M that decides the language L. We describe the Turing Machine M' that decides the language L^c.

On input string w

1. Run M on w.
   - If M accepts, then reject
   - Else accept

e

Having proved that decidable languages are closed under union and complement, DeMorgan's Law proves they are closed under intersection as well (as seen in other classes of languages).

Suppose we have Turing Machines M_1 and M_2 that decide languages L_1 and L_2 respectively. We describe the Turing Machine M' that decides the intersection of L_1 and L_2.

On input string w

1. Run M_1 on w
   - If M_1 accepts, continue to stage 2
   - Else reject
2. Run M_2 on w
   - If M_2 accepts, accept
   - Else reject