Homework 2

CSC 445-01: Theory of Computation

Matthew Mabrey, Luke Kurlandski

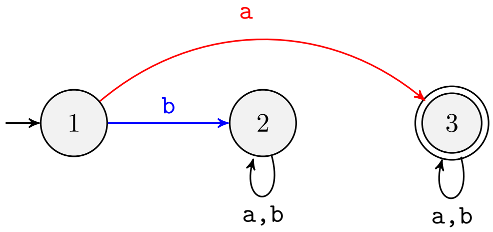
February 18, 2021

1.4

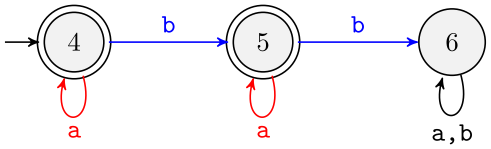
We use blue for 'b' and red for 'a' when space is limited

e

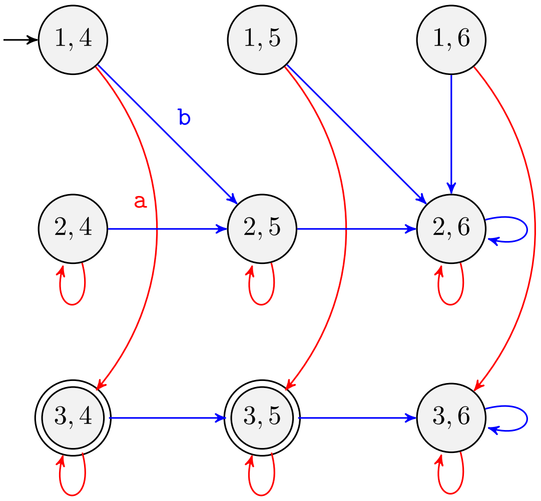
w starts with an 'a'



w has at most one 'b'

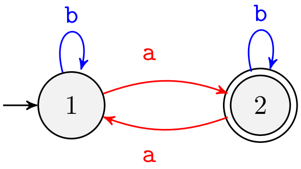


w starts with an 'a' and has at most one 'b'

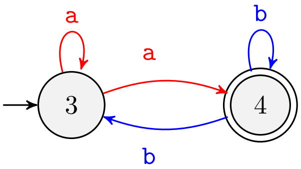


f

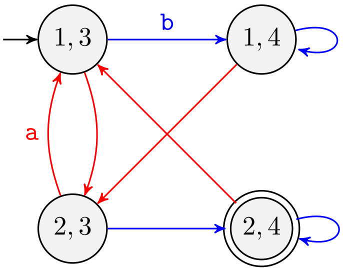
w has an odd number of ’a’



w ends in a ’b’

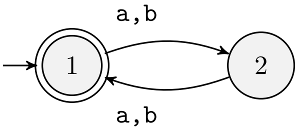


w has an odd number of ’a’ and ends in a ’b’

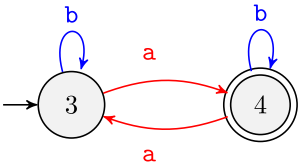


g

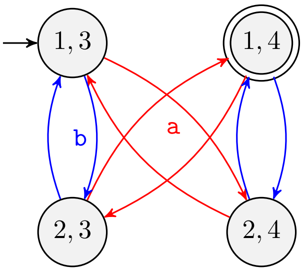
w has an even length



w has an odd number of ’a’



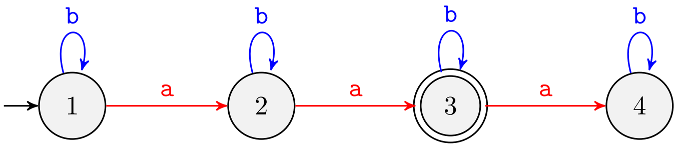
w has an even length and has an odd number of ’a’



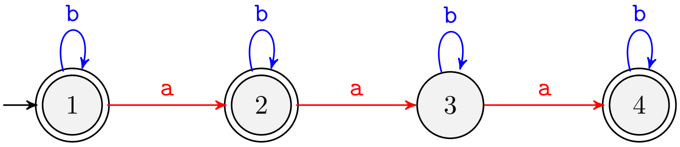
1.5

g

w is any string that contains exactly two 'a'



w is any string that does not contain exactly two 'a'



1.7

b

c

e

1.31

1.33

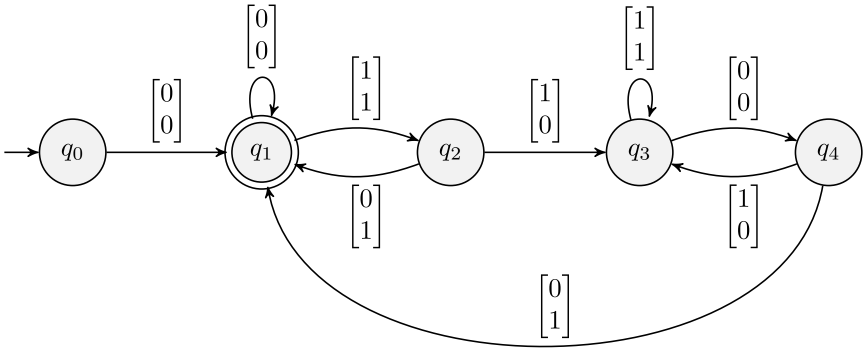
From problem 1.31, we know we are allowed to "wind the tape backwards" i.e., we will demonstrate that C^R is a regular language, thus prove that C itself is regular. To prove that C^R is regular, we will construct a nondeterministic finite automata that recognizes C^R.

For each symbol, we will read its top bit and judge whether or not the bottom bit is correct such that $3\text{row}_1 = \text{row}_2$. To do so, we track two conditions: 1) whether or not 3row_1 results in a carry-out in base two and 2) what the previous top symbol is. With these two pieces of information, given a top bit, we can check to see if the bottom bit is correct.

These two pieces of information result in an automata with 4 states to keep track of, but we had an additional start state.

- q_0 : the start state, to prevent the automata from accepting the empty string
- q_1 : the state where there is no carry-in from previous step and the previous top symbol is a 0
- q_2 : the state where there is a carry-in and the previous top value is a 1
- q_3 : the state where there is no carry-in and the previous top value is a 1
- q_4 : the state where there is a carry-in from previous step and the previous top value is a 0

Our transition function merely Recall that we read the string backwards, so the binary number is read from the 2^0th place first.


q_0
$\begin{bmatrix}0\\0\end{bmatrix}$
q_1
$\begin{bmatrix}0\\0\end{bmatrix}$
$\begin{bmatrix}1\\1\end{bmatrix}$
$\begin{bmatrix}0\\1\end{bmatrix}$
q_2
$\begin{bmatrix}1\\0\end{bmatrix}$
q_3
$\begin{bmatrix}1\\1\end{bmatrix}$
$\begin{bmatrix}0\\0\end{bmatrix}$
$\begin{bmatrix}1\\0\end{bmatrix}$
q_4
$\begin{bmatrix}0\\1\end{bmatrix}$

1.34