

# ML - HW3

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November 2023

## 1 Paper Problems

### 1.1

#### 1.1.1 a

We know that a hyperplane cannot have a margin if the function is not linearly separable. However, it is clear that the function is linearly separable. A perceptron is guaranteed to converge if the data is linearly separable. On desmos graph this is made clear with purple dots representing positive labels and green representing negative labels ((Fig. 1). To find the margin of the dataset, one must find the minimum distance any point in the dataset and the hyperplane. To do this, we can use the equation described in class of

$$\gamma = \min \frac{y_i(w^T x_i + b)}{\|w\|}$$

written in terms of the distance from a single data point to the hyperplane, this equation is

$$\frac{|b + w_1 x_1 + w_2 x_2|}{\sqrt{w_1^2 + w_2^2}}$$

When we find the distance for each data point, we find the minimum of these distances to be roughly  $Margin = 0.28$ . (From the point 1,1).

#### 1.1.2 b

We first must inspect that the dataset 2 is linearly separable. It appears when I graph the data that the last data point contradicts the other signs of the surrounding datapoints. This means the dataset is not linearly separable by the defined hyperplane. Therefore, the margin DNE. A hyperplane margin cannot exist on a dataset that is not linearly separable by the defined hyperplane.

## 1.2

### 1.2.1 a

### 1.2.2 b

We first must inspect that the dataset 2 is linearly separable. It appears when I graph the data that the last data point contradicts the other signs of the surrounding datapoints. This means the dataset is not linearly separable by any hyperplane. There is no 2d straight line that can separate the colors of these dots. Therefore, the margin DNE. A hyperplane margin cannot exist on a dataset that is not linearly separable.

## 1.3

It stands to reason that the upperbound of the number of mistakes made by the perceptron algorithm is still  $\frac{R^2}{\gamma^2}$ . The original assumption states that there exists a unit vector  $u$  such that for some margin, we have  $y_i(u^T x_i) \geq \text{margin}$  for all  $i$ . The modified assumption only states that there exists a vector  $u$  such that for some margin, we have  $y_i(u^T x_i) \geq \text{margin}$  for all  $i$ .

The modified assumption is weaker than the original assumption because the original assumption implies the modified assumption. This is because if  $u$  is a unit vector, then  $u^T x_i$  is equivalent to  $|u^T x_i| = 0$ . Therefore, if the original assumption holds, then the modified assumption must also hold.

Since the modified assumption is weaker than the original assumption, it follows that the upper bound.

## 1.4

A perceptron will have a number of mistakes less than or equal to  $\frac{R^2}{\gamma^2}$ . This is true when the data has a margin  $\gamma$  and every point inside a circle of radius  $R$ . Here, the radius of the ball that contains all of the training data is  $\sqrt{2n}$  because that is the max distance between any two datapoints, and the margin of the hyperplane is at least 1. Therefore, the upperbound for the number of mistakes of the perceptron is  $\frac{\sqrt{2n}^2}{1^2}$  which simplifies to  $2n$ .

## 1.5

A linear classifier that exists in a plane can shatter any 3 distinct points but not any four distinct points. The linear classifier can only divide the plane into two half spaces. There are 16 total ways to label 4 distinct points, so there must be two labelings that cannot be achieved by any linear classifier.

Proof: Let there exist a linear classifier that can shatter any 4 distinct points on a plane. This means there must be a line that can divide the plane into two half spaces such that a labeling of the four points can be made by choosing the half-space that contains the points labeled positive and excluding the half-space with negative labels. There are 16 ways to label 4 distinct points. For each

labeling for these points, we construct a line that divides the plane into two half-spaces so each labeling can occur. There are only 8 ways to divide the plane into two half spaces with a line, however. Therefore there are at least two labels that are not achieved by any possible line. Therefore, there is no linear classifier that can shatter 4 distinct points.

## 1.6

The VC dimension of a hypothesis space is the largest number of points that can be shattered by  $H$ . When rectangles are in a plane, the VC dimension is four because there are four points that can be shattered by rectangles. A set of points is only shattered by a hypothesis space when there is a hypothesis in that space which classifies each point as positive or negative. There are many examples of sets of four points where rectangles at similar vertices can classify points as positive and negative, and can be shattered.

## 2 Practice

### 2.1

I updated my machine learning library, added a README.md, and added a new folder titled perceptron.

### 2.2

#### 2.2.1

After implimenting the standard perceptron in python with  $T = 10$  epochs, the learned weight vector was [4102.605, 11291.479 8154.935, -5277.336]. The average prediction error for the test dataset was 0.724, which is obviously a lot of error.

#### 2.2.2

After implimenting the voted perceptron in python with  $T = 10$  epochs, the learned weight vectors and their counts is Weight vector: [-18.8312856 -14.068138 -17.351032 -11.6960836], Count: 5418.0. The average prediction error for the test dataset was 0.658, which is an improvement but still too much error.

#### 2.2.3

My learned weight vector for the average perceptron is [-15811.70503946 -11401.3072864 -7556.1951926 -12690.15358546]. The test error is only 0.074. Comparing with the previous weight vectors it is obvious that this weight vector just used averages to get to its final iteration.

#### **2.2.4**

Ultimately, the average prediction error for the averaged perceptron was the best method because it is the most efficient, simplest, and yielded the least errors. The standard perceptron was not the best because it only updated one weight vector and did not keep track of others. The voted vector was inefficient as well.