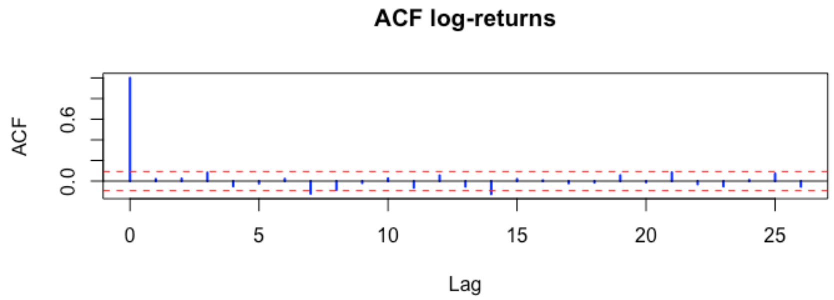
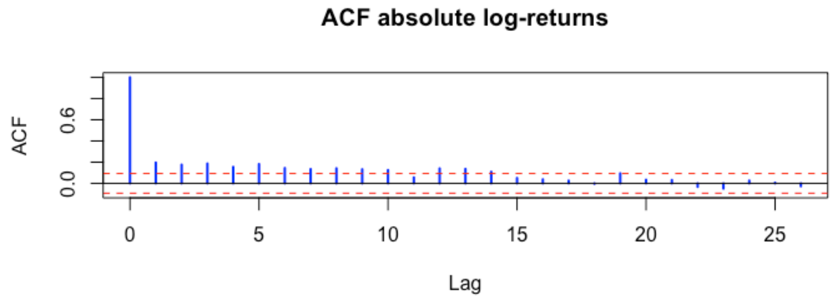
Time Series Econometrics Tutorial 8: Volatility Modelling with ARCH Models

Exercise 1:





The ACF of log-returns shows no significant serial correlations, except at lags 7 & 14. These are either minor correlations or simply just noise. However, the ACF of the absolute log-returns shows significant serial correlations.

Exercise 2:

$$H_0: \textit{No serial correlation}$$
$$H_1: \textit{Serial correlation}$$

Ljung – Box test results for Log-Returns:

X-squared test statistic	18.676
X-squared critical value	21.02607
df	12
p-value	0.09665

The X^2 test statistic is less than the critical value, therefore we fail to reject the null hypothesis. To confirm this, the p-value is greater than 0.05, showing that the log-return of Intel have no serial correlation.

Ljung – Box test results for Absolute Log-Returns:

X-squared test statistic	124.91
X-squared critical value	21.02607
df	12
p-value	<2.2e-16

The X^2 test statistic is greater than the critical value, therefore we reject the null hypothesis. To confirm this, the p-value significantly less than 0.05, showing that the absolute log-return of Intel has serial correlation. Therefore, the absolute log-returns exhibit significant levels of conditional heteroskedasticity.

Exercise 3:

$$H_0: \mu = 0$$
$$H_1: \mu \neq 0$$

t-test statistic	t-test critical value	p-value
2.3788	1.648301	0.01779

The test statistic is greater than the critical value, therefore the null hypothesis can be rejected, this is confirmed with the p-value taking on a value less than 0.05.

The coefficient estimate for μ is:

$$\hat{\mu} = 0.0143173$$

Exercise 4:

$$\boldsymbol{H_0: No\ serial\ correlation}$$
$$\boldsymbol{H_1: Serial\ correlation}$$

X-squared test statistic	92.939
X-squared critical value	21.02607
df	12
p-value	1.332e-14

The X^2 test statistic is greater than the critical value, therefore we reject the null hypothesis. To confirm this, the p-value significantly less than 0.05, showing that the residuals-squared of Intel has serial correlation. Therefore, the residual-squared returns indicate the presence of conditional heteroskedasticity.

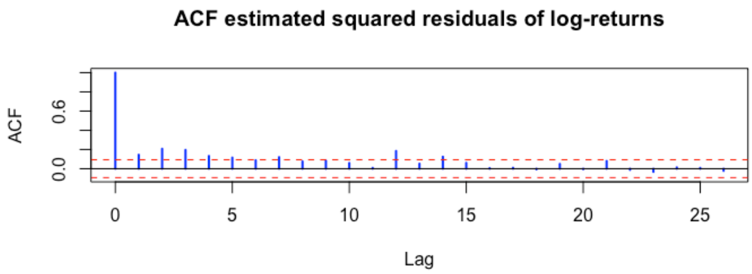
Exercise 5:

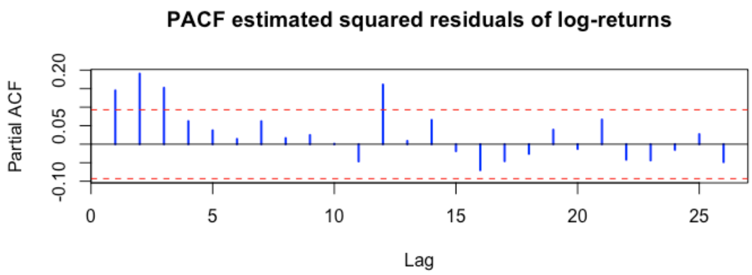
$$H_0: No\ serial\ correlation$$
$$H_1: Serial\ correlation$$

F test statistic	4.978
F critical value	1.775309
Df1	12
Df2	419
p-value	9.742e-08

The F test statistic is greater than the critical value, therefore we reject the null hypothesis. To confirm this, the p-value significantly less than 0.05, showing that the residual returns of Intel has serial correlation. Therefore, the residual returns indicate the presence of conditional heteroskedasticity.

Exercise 6:





The autocorrelations are significant for lags 1,2 and 3 as shown in the PACF. 11 is an outlier so we can assume it is either a minor serial correlation, or just noise.

Exercise 7:

An ARCH(3) model has been chosen based on the significant lags 1,2 and 3 in the PACF. The modelling equation therefore must include 3 lags, as they provide some insight into the time 0 residual value due to the serial correlation found in the PACF. The coefficients are also required to exhibit some regularity, so it is assumed that the residual white noise (ε_t) satisfies the widely adopted assumption of $\mathcal{N}\ (0,1)$.

The ARCH(3) model can be estimated as follows:

$$r_t = 0.012567 + a_t$$

and

$$\sigma_t^2 = 0.010421 + 0.232889a_{t-1}^2 + 0.075069a_{t-2}^2 + 0.051994a_{t-3}^2$$

	T test statistic	T critical value	p-value
Mu	2.279	1.648301	0.0227
Omega	8.418	1.648301	<2e-16
Alpha1	2.088	1.648301	0.0368
Alpha2	1.587	1.648301	0.1125
Alpha3	1.152	1.648301	0.2494

Mu, Omega and Alpha 1 are all statistically significant parameters because the t-value's are greater than the t-critical value, and the p-values are less than 0.05. However, Alpha 2 and Alpha 3 are not statistically significant.

Exercise 8:

Based on the analysis of the above, the model can be improved by removing the two statistically insignificant estimated coefficients. It is now an ARCH(1) model.

The model is now characterised by these equations:

$$r_t = 0.013130 + a_t$$

and

$$\sigma_t^2 = 0.011046 + 0.374976a_{t-1}^2$$

The error analysis is below:

	T test statistic	T critical value	p-value
Mu	2.469	1.648301	0.01355
Omega	9.238	1.648301	<2e-16
Alpha1	3.330	1.648301	0.00087

All coefficients are statistically significant at the 5% level.