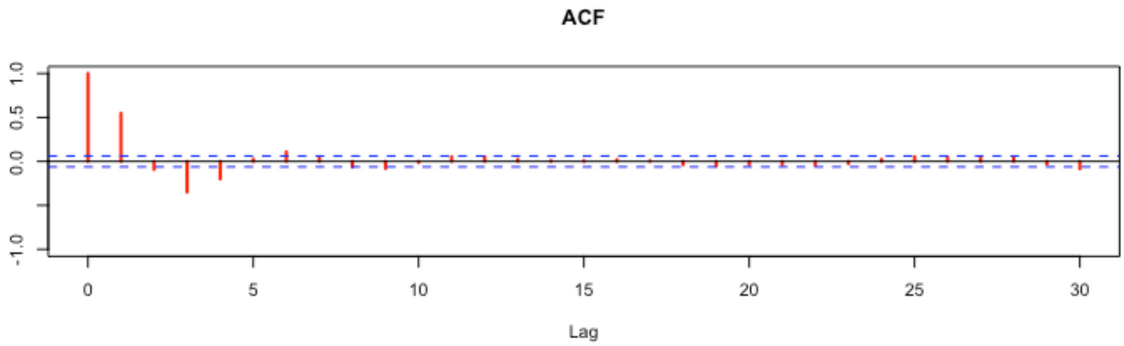
Spring 2019 Tutorial 2

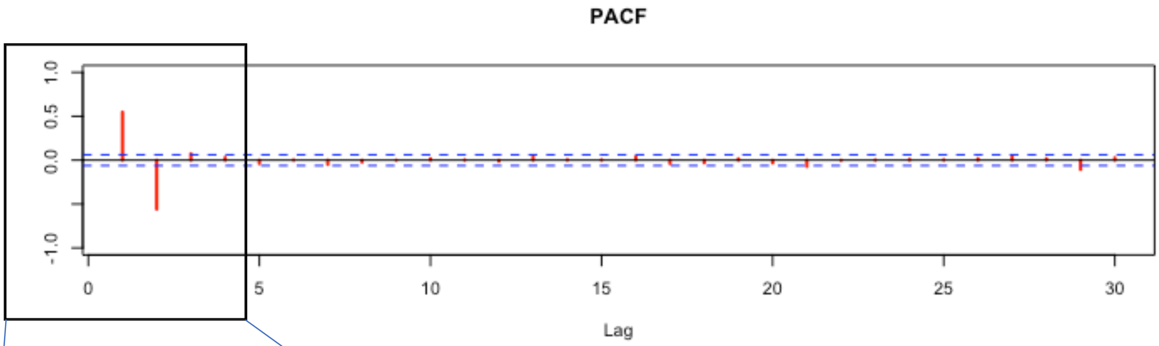
Exercise 1:

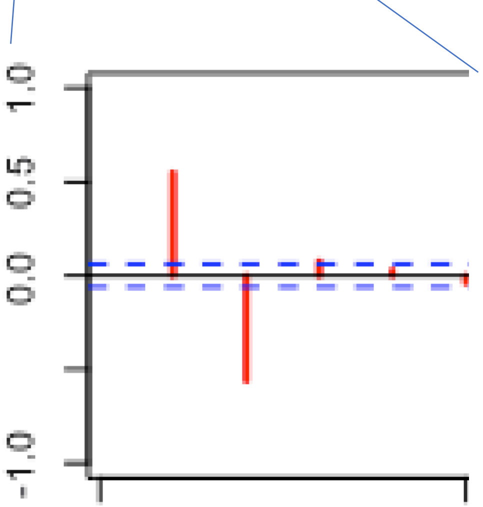
Code:

```
#Exercise 1
x <- ts(data = (read.xlsx("Tutorial 2.xlsx")), sheetName = "Exercise 
1", colIndex = 2
        ,start=1, end=1000, frequency=1)

layout(matrix(c(1,1,2,2), ncol = 1))
acf3 <- acf(x, main="ACF", ylab="", ylim=c(-1, 1), lwd = 2, col="red", 
ci.col="blue")
pacf3 <- pacf(x, main="PACF", ylab="", ylim=c(-1, 1), lwd = 2, 
col="red", ci.col="blue")
# Because the ACF is decaying, the process is at least an AR(?), the 
PACF determines the order.
# It appears to be of order 3
```







```
#Estimating the candidate AR(3) model, and check its stationarity and the significance of
#its coefficients
arma30 <- arima(x, order=c(3,0,0))
arma30
rootdf <- data.frame(Re(polyroot(c(1, -arma30$coef[1], -arma30$coef[2], -arma30$coef[3]))),
                     Im(polyroot(c(1, -arma30$coef[1], -arma30$coef[2], -arma30$coef[3]))))
colnames(rootdf)[1] = "real"
colnames(rootdf)[2] = "imag"
modulus <- sqrt(rootdf$real^2+rootdf$imag^2)
rootdf$modulus <- modulus
format(round(rootdf, 4), nsmall =4)

#The roots are > 1 therefore it is a stationary process.

#Testing the residuals of the candidate AR(3) model fro uncorrelatedness and normality
res30 <- residuals(arma30)
Box.test(res30, lag = 20, type = "Ljung-Box")
shapiro.test(res30)
#The results for both tests are not less than 0.05 (which is the alpha). So the null cannot be rejected.
#There is no correlation between the residuals and the residuals are normally distributed.

#Estimating an AR(4) model, and checking its stationarity and the significance of its coefficients.
arma40 <- arima(x, order=c(4,0,0))
arma40
rootdf <- data.frame(Re(polyroot(c(1, -arma40$coef[1], -arma40$coef[2], -arma40$coef[3], -arma40$coef[4]))),
                     Im(polyroot(c(1, -arma40$coef[1], -arma40$coef[2], -arma40$coef[3], -arma40$coef[4]))))
colnames(rootdf)[1] = "real"
colnames(rootdf)[2] = "imag"
modulus <- sqrt(rootdf$real^2+rootdf$imag^2)
rootdf$modulus <- modulus
format(round(rootdf, 4), nsmall =4)

#AR(4) Model, the coefficients 3 & 4 are insignificant according to the t student statistic

#Estimating an AR(2) model. and checking its stationarity and the significantce of its coefficients.
arma20 <- arima(x, order=c(2,0,0))
arma20
rootdf <- data.frame(Re(polyroot(c(1, -arma20$coef[1], -arma20$coef[2]))),
                     Im(polyroot(c(1, -arma20$coef[1], -arma20$coef[2]))))
colnames(rootdf)[1] = "real"
colnames(rootdf)[2] = "imag"
modulus <- sqrt(rootdf$real^2+rootdf$imag^2)
rootdf$modulus <- modulus
format(round(rootdf, 4), nsmall =4)
```

```
#AR(2) model, both the coefficients are significant according to the t student statistic.

#Testing the residuals of the AR(2) model
res20 <- residuals(arma20)
Box.test(res30, lag = 20, type = "Ljung-Box")
shapiro.test(res20)
#The residuals pass the test

#Comparing the AR3 and AR2 AIC, AR3 has (2814.81-2811.49) lower AIC. We choose the lower AIC value, the AR3.
#However we will now test ARMA Models

#Estimate an ARMA(2,1)
arma21 <- arima(x, order=c(2,0,1))
arma21
rootdf <- data.frame(Re(polyroot(c(1, -arma21$coef[1], -arma21$coef[2]))),
                     Im(polyroot(c(1, -arma21$coef[1], -arma21$coef[2]))))
colnames(rootdf)[1] = "real"
colnames(rootdf)[2] = "imag"
modulus <- sqrt(rootdf$real^2+rootdf$imag^2)
rootdf$modulus <- modulus
format(round(rootdf, 4), nsmall = 4)
#Doesn't have a lower AIC than the AR(3)


#Estimate ARMA(3,1)
arma31 <- arima(x, order=c(3,0,1))
arma31
rootdf <- data.frame(Re(polyroot(c(1, -arma31$coef[1], -arma31$coef[2], -arma31$coef[3]))),
                     Im(polyroot(c(1, -arma31$coef[1], -arma31$coef[2], -arma31$coef[3]))))
colnames(rootdf)[1] = "real"
colnames(rootdf)[2] = "imag"
modulus <- sqrt(rootdf$real^2+rootdf$imag^2)
rootdf$modulus <- modulus
format(round(rootdf, 4), nsmall = 4)
#Doesnt have a lower AIC than the AR(3)

#Estimate ARMA(3,2)
arma32 <- arima(x, order=c(3,0,2))
arma32
rootdf <- data.frame(Re(polyroot(c(1, -arma32$coef[1], -arma32$coef[2], -arma32$coef[3]))),
                     Im(polyroot(c(1, -arma32$coef[1], -arma32$coef[2], -arma32$coef[3]))))
colnames(rootdf)[1] = "real"
colnames(rootdf)[2] = "imag"
modulus <- sqrt(rootdf$real^2+rootdf$imag^2)
rootdf$modulus <- modulus
format(round(rootdf, 4), nsmall = 4)
#Not a lower AIC than the AR3

#Tested for robustness, AR3 still produces the lowest AIC, which is 2811.49
```

Exercise 2:

$$\begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} \begin{bmatrix} x_{1,t-1} \\ x_{2,t-1} \end{bmatrix} + \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} \begin{bmatrix} x_{1,t-2} \\ x_{2,t-2} \end{bmatrix} + \begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix}$$

1 2 3 4 5 6

Exercise 2

Demonstrate that the companion form representation can be expressed as a VAR(2) form.

$$\begin{bmatrix} x_{1,t} \\ x_{2,t} \\ x_{1,t-1} \\ x_{2,t-1} \end{bmatrix} = \begin{bmatrix} a^1_{1,1} & a^1_{1,2} & a^2_{1,1} & a^2_{1,2} \\ a^1_{2,1} & a^1_{2,2} & a^2_{2,1} & a^2_{2,2} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{1,t-1} \\ x_{2,t-1} \\ x_{1,t-2} \\ x_{2,t-2} \end{bmatrix} + \begin{bmatrix} u_{1,t} \\ u_{2,t} \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} a_{1,1}x_3 + a_{1,2}x_4 + a_{1,3}x_5 + a_{1,4}x_6 \\ a_{2,1}x_3 + a_{1,2}x_4 + a_{1,3}x_5 + a_{1,4}x_6 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ \cancel{x_3} \\ \cancel{x_4} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -x_3 \\ -x_4 \end{bmatrix} = \begin{bmatrix} a_{1,1}x_3 + a_{1,2}x_4 + a_{1,3}x_5 + a_{1,4}x_6 \\ a_{2,1}x_3 + a_{2,2}x_4 + a_{2,3}x_5 + a_{2,4}x_6 \\ \cancel{x_3} \\ \cancel{x_4} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -x_3 \\ \cancel{-x_4} \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_{1,1}x_3 + a_{1,2}x_4 \\ a_{2,1}x_3 + a_{1,2}x_4 \end{bmatrix} + \begin{bmatrix} a_{1,3}x_5 + a_{1,4}x_6 \\ a_{2,3}x_5 + a_{2,4}x_6 \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_{1,1} + a_{1,2} \\ a_{2,1} + a_{1,2} \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} a_{1,3} + a_{1,4} \\ a_{2,3} + a_{2,4} \end{bmatrix} \begin{bmatrix} x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix} = \begin{bmatrix} a^1_{1,1} & a^1_{1,2} \\ a^1_{2,1} & a^1_{2,2} \end{bmatrix} \begin{bmatrix} x_{1,t-1} \\ x_{2,t-1} \end{bmatrix} + \begin{bmatrix} a^2_{1,1} & a^2_{1,2} \\ a^2_{2,1} & a^2_{2,2} \end{bmatrix} \begin{bmatrix} x_{1,t-2} \\ x_{2,t-2} \end{bmatrix} + \begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix}$$

A VAR(2) model.

Exercise 3:

Exercise 3:

· Formulate a 3-dimensional VAR(2) model where 3 does not Granger cause the other

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \\ y_{3,t} \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 0.2 & -0.7 & 0 \\ 0.4 & -0.5 & 0.6 \\ 0.6 & -0.3 & 0.7 \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \\ y_{3,t-1} \end{bmatrix} +$$

$$\begin{bmatrix} 0.3 & -0.1 & 0 \\ 0.6 & -0.2 & -0.6 \\ 0.9 & 0.4 & 0.8 \end{bmatrix} \begin{bmatrix} y_{1,t-2} \\ y_{2,t-2} \\ y_{3,t-2} \end{bmatrix} + \begin{bmatrix} u_{1,t} \\ u_{2,t} \\ u_{3,t} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -0.2 & +0.7 & 0 & -0.3 & +0.1 & 0 \\ 0 & 1 & 0 & -0.4 & +0.5 & -0.6 & -0.6 & +0.2 & +0.6 \\ 0 & 0 & 1 & -0.6 & +0.3 & -0.7 & -0.9 & -0.4 & -0.8 \end{bmatrix} \quad \text{Lag Polynomial}$$

The top right value must be 0 because that location is corresponding to variable 3 acting on variable 1.

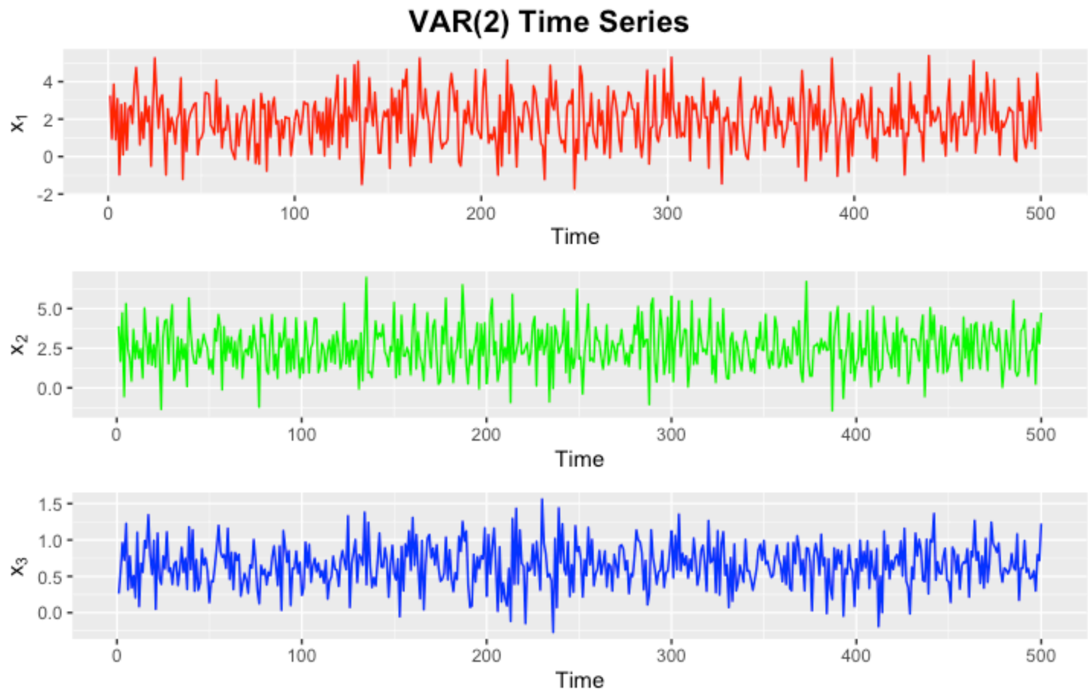
Exercise 4:

```
#Exercise 4
library(ggplot2)
library(latex2exp)
library(dse)
library(vars)
library(grid)
library(gridExtra)

const <- c(3,2,1)
lagpoly <- array(c(1, -0.2, -0.3, 0, -0.4, -0.6,
                   0, -0.6, -0.9, 0, 0.7, 0.1, 1, 0.5, 0.2, 0,
                   0.3, -0.4, 0, 0, 0, 0, -0.6, 0,6, 1, -0.7, -0.8),
                 c(3,3,3))
residcovmat <- diag(3)

var2 <- ARMA(A= lagpoly, B= residcovmat, TREND=const)
varsim <- simulate(var2, sampleT=500, noise=list(w=matrix(rnorm(1500),
nrow=500, ncol=3)),
                   rng=list(seed=c(123456)))

time <- c(1:500)
var2df <- data.frame(time, matrix(varsim$output, nrow=500, ncol=3))
p1 <- ggplot(var2df, aes(x=time, y=X1)) + geom_line(size=0.5,
colour="red") +
  labs(x="Time", y=TeX("x_1"))
p2 <- ggplot(var2df, aes(x=time, y=X2)) + geom_line(size=0.5, colour =
"green") +
  labs(x="Time", y=TeX("x_2"))
p3 <- ggplot(var2df, aes(x=time, y=X3)) + geom_line(size=0.5, colour =
"blue") +
  labs(x="Time", y=TeX("x_3"))
grid.arrange(p1, p2, p3, ncol=1, top=textGrob("VAR(2) Time Series",
gp=gpar(fontsize = 15, font = 2)))
arrangeGrob(p1, p2, p3, ncol=1, main=textGrob("VAR(2) Time Series",
gp=gpar(fontsize = 15, font = 2)))
```



Exercise 5:

```
#Exercise 5
library(dse)
library(vars)

const <- c(3,2,1)
lagpoly <- array(c(1, -0.2, -0.3, 0, -0.4, -0.6,
                   0, -0.6, -0.9, 0, 0.7, 0.1, 1, 0.5, 0.2, 0,
                   0.3, -0.4, 0, 0, 0, 0, -0.6, 0,6, 1, -0.7, -0.8),
                 c(3,3,3))
residcovmat <- diag(3)

var2 <- ARMA(A = lagpoly, B=residcovmat, TREND=const)
var2sim <- simulate(var2, sampleT=500, noise=list(w=matrix(rnorm(1500),
nrow=500, ncol=3)),
                    rng=list(seed=c(123456)))
var2df <- data.frame(matrix(var2sim$output, nrow = 100, ncol=3))
colnames(var2df) <- c("x1", "x2", "x3")
VARselect(var2df, lag.max=5, type="const")

varsimest <- VAR(var2df, type ="const", lag.max=5, ic="AIC")
summary(varsimest, equation = "x1")
summary(varsimest, equation = "x2")
summary(varsimest, equation = "x3")

roots(varsimest)
```

According to the output of the `VARselection`, the optimal lag order for the time series is 1.

When matching the summary values to the model equation in Exercise 3, the model does not come close to the simulated values. This is probably due to the process not being stationary because all of the output roots are less than 1 (`0.7870349 0.7870349 0.7325394 0.7325394 0.6824037 0.6824037 0.5206097 0.5206097 0.3320007`)

```
#Exercise 6
serial.test(varsimest, lags.pt = 20, type="PT.asymptotic")
#Auto Regressive Heterscedasticity (ARCH) test
arch.test(varsimest, multivariate.only = TRUE)

normality.test(varsimest, multivariate.only = TRUE)
```

The output of the Portmanteau test has a p-value of 0.8959, an insignificant value. Therefore, there is no serial correlation among the residuals.
The output of the AutoRegressive Heteroscedasticity (ARCH) Linear Model test has a p-value of 0.6982, an insignificant value. Therefore, there is no heteroscedasticity among the residuals.
The output of the normality test has insignificant values of 0.33 (Jaques-Bera), 0.282 (Skewness) and 0.384 (Kurtosis). This leads to the conclusion that the residuals are normally distributed.

Exercise 7:

```
#Exercise 7
#Granger Causality Test
causalityTest <- causality(varsimest, cause = "x3")
causalityTest$Granger
format(round(qf(.95, df1=causalityTest$Granger$parameter[3],
                df2=causalityTest$Granger$parameter[1:2]), 4),
nsmall=4)
#p-value is not less than 0.05, i.e. not signficiantly different from
the null. Therefore we cannot reject the null
```

The p-value is not less than 0.05, i.e. not significantly different from the null. Therefore we cannot reject the null. According to the simulated model, the 3rd variable does Granger cause 1&2.