

Time Series Econometrics Tutorial 9: Volatility Modelling with GARCH Models

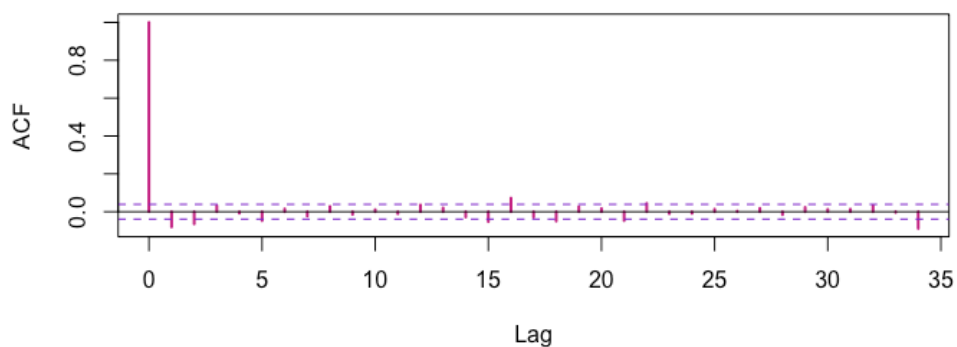
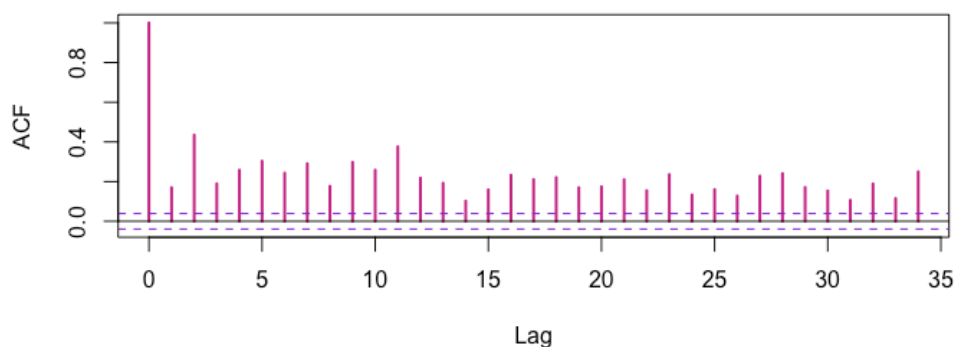
Exercise 1:

$$H_0: \mu = 0$$

$$H_1: \mu \neq 0$$

μ	T test statistic	T critical value (5% significance level)
7.24541e-05	0.26515	1.645455

The test statistic is less than the critical value therefore we fail to reject the null hypothesis that the mean is 0. The t-test returns estimated that the mean is around 0.0000724541.

Exercise 2:**ACF of log-returns****ACF of squared log-returns**

The ACF of the log-returns fails to show significant conditional heteroskedasticity after lag 2. However, when the log-returns are squared, it can be seen that there is significant evidence of conditional heteroskedasticity in the log-returns

Exercise 3:

$$H_0: \text{No serial correlation}$$

$$H_1: \text{Serial correlation}$$

Ljung – Box test results for squared Log>Returns:

X-squared test statistic	3088.8
X-squared critical value	31.41043
df	20
p-value	<2.2e-16

The X^2 test statistic is greater than the critical value, therefore we reject the null hypothesis. To confirm this, the p-value is significantly less than 0.05, showing that the squared log-return of the SPY has serial correlation. Therefore, the squared log-returns exhibit significant levels of conditional heteroskedasticity.

Exercise 4:

The fitted GARCH(2,1) model equations are as follows:

$$r_t = 0.00057243 + a_t$$

$$\text{where } a_t = \sigma_t \varepsilon_t \text{ and}$$

$$\sigma_t^2 = 0.0000023226 + 0.0019895a_{t-1}^2 + 0.1165a_{t-2}^2 + 0.87049\sigma_{t-1}^2$$

Exercise 5:

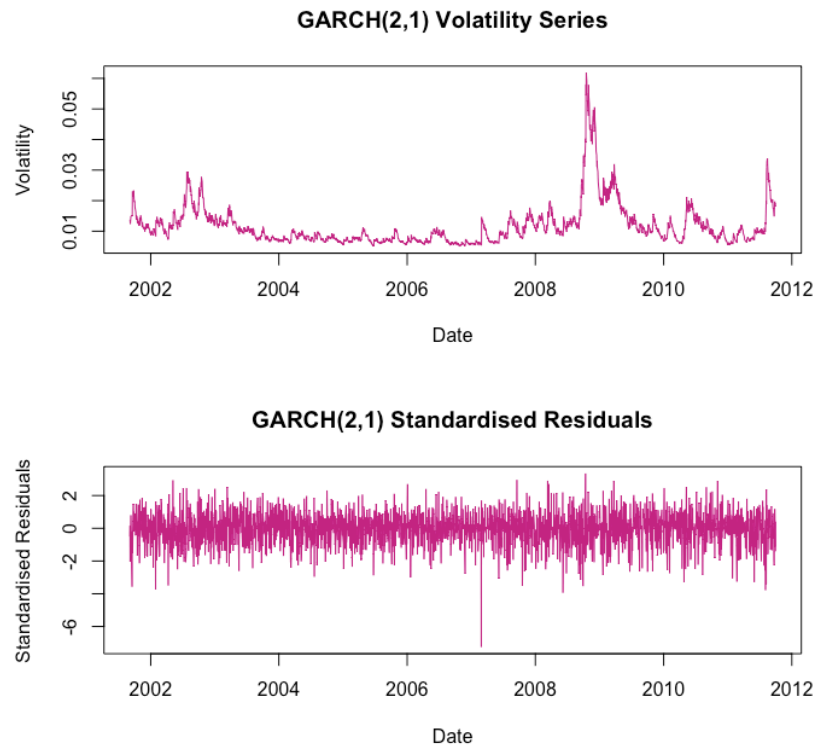
$$H_0: \text{No serial correlation}$$

$$H_1: \text{Serial correlation}$$

Ljung – Box test results for squared Log>Returns:

X-squared test statistic	20.47
X-squared critical value	31.41043
df	20
p-value	0.4289

The X^2 test statistic is less than the critical value, therefore we fail to reject the null hypothesis. To confirm this, the p-value is not less than 0.05, showing that the residuals from the fitted GARCH(2,1) model do not have serial correlation. Therefore, the residuals do not exhibit significant levels of conditional heteroskedasticity.

Exercise 6:

Observations show that there is high volatility for SPY around 2009, this is explained by the GFC. The average volatility lies around 0.01 for the SPY.

The Standardised Residuals shows one spike, which can be ignored. Around the time of the GFC, there is somewhat higher (negative) values of the expected residuals (greater than ± 2). The standardised residuals appear to be white noise, which is in line with model assumptions.

Exercise 7: