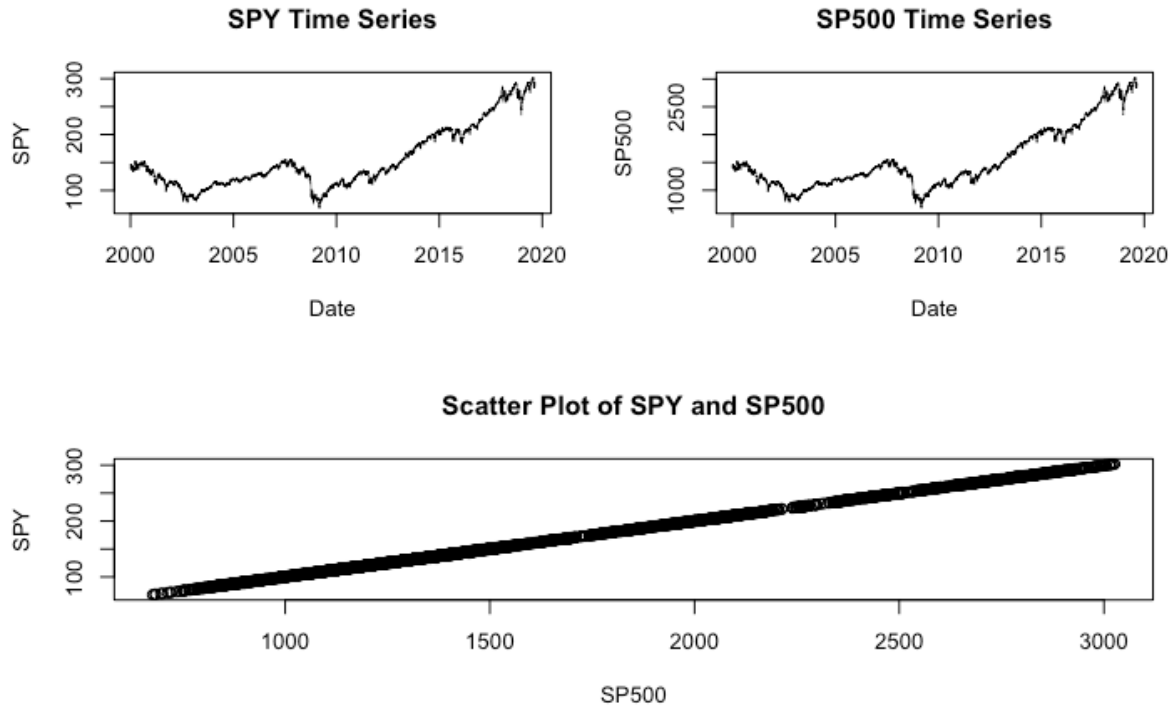
Time Series Econometrics Tutorial 4 Spring 2019

Exercise 1:



The graphs in row 1 support a possible cointegration relationship because they appear to be identical. The linearity of the scatter plot also implies cointegration.

An ETF holds a basket of assets which are divided into shares and operates with an arbitrage mechanism to keep it close to its net asset value, and thus mirroring the index. Financial institutions can purchase large blocks of ETF shares to 'redeem' and sell some of the assets in the basket, allowing for arbitrage. This arbitrage mechanic is what keeps the ETF close to the index value.

For example, if there is increased demand for an ETF, the share price will rise above the net value of the basket per share, incentivising purchasing of underlying assets of the ETF and selling off the individual shares, bringing the ETF share price back to equilibrium (the index value). If the opposite occurs, and the ETF share price is below the net value of the assets, arbitrageurs will buy the smaller shares and sell the underlying assets.

Exercise 2:

A Dickey-Fuller (DF) test was performed on both the index (S&P 500) and the ETF (SPY). The results yielded are as follows:

	Test-Statistic	Critical Value (α = 0.05)
Index	1.2279	-1.95
ETF	1.2175	-1.95

The DF test stipulates the hypothesis test of:

H_0: $\pi = 0$

H_1 : $\pi < 0$

Where $\Delta z_t = (\phi - 1)_{zt-1} + \epsilon_t =: \pi z_{t-1} + \epsilon_t$ is the equation of which π is found.

The Index test-statistic is not greater than the critical value, meaning that the test-statistic is not significantly different from the null, therefore the null cannot be rejected.

The ETF test-statistic is not greater than the critical value, meaning that the test-statistic is not significantly different from the null, therefore the null cannot be rejected.

By rejecting the null, this gives the high possibility (we are not certain because of type I errors) that both processes contain a unit root, meaning they are non-stationary. The DF test is necessary for establishing a cointegrating relationship because the result of a possible unit root in both processes allows for the further investigate the cointegrating coefficient.

Exercise 3:

The value of the estimated slope coefficient, $\hat{\theta}$, is 0.09972.
Key statistical results supporting the significant of $\hat{\theta}$ are a 1.96*standard error < the t-value because the SE is 1.509×10^{-5} and a t-value of 6613.19. An incredibly low and extremely high t-value indicate statistical significance of the slope coefficient. The results are so significant, the regression is indicating a linear relationship between the two processes.

Exercise 4:

A DF test was performed on the estimated residuals of the regression from Exercise 3.

	Test-statistic	Critical Value
$\widehat{z_t}$ Estimated Residuals	-35.7744	-1.95

The hypothesis test is as follows:

$$H_0: \pi = 0$$
$$H_1 : \pi < 0$$

The test-statistic is greater than the critical value. The test-statistic is significantly different from the null hypothesis; thus, we reject the null.

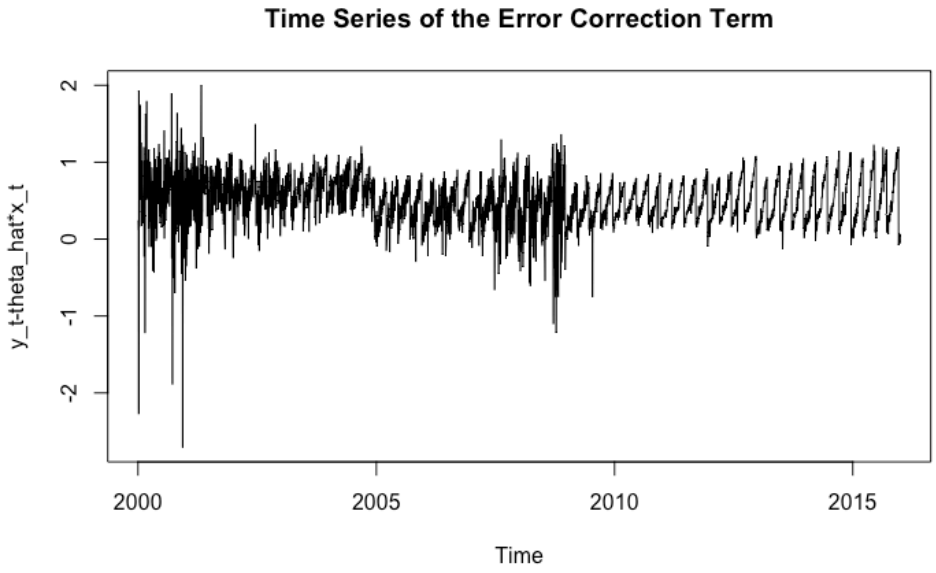
The outcome of this test shows that the residuals are a stationary process, i.e. the roots are not equal to one or less than one. This is consistent with the existence of a cointegrating relationship because the estimated residuals, a linear combination, should be of I(0) when the two processes are I(0). Linear combination says that the residuals should also be of I(1) if one of the processes is I(1), but since the residuals are of I(0), a cointegrating relationship exists.

Exercise 5:

$$\hat{\gamma} = -0.348852$$

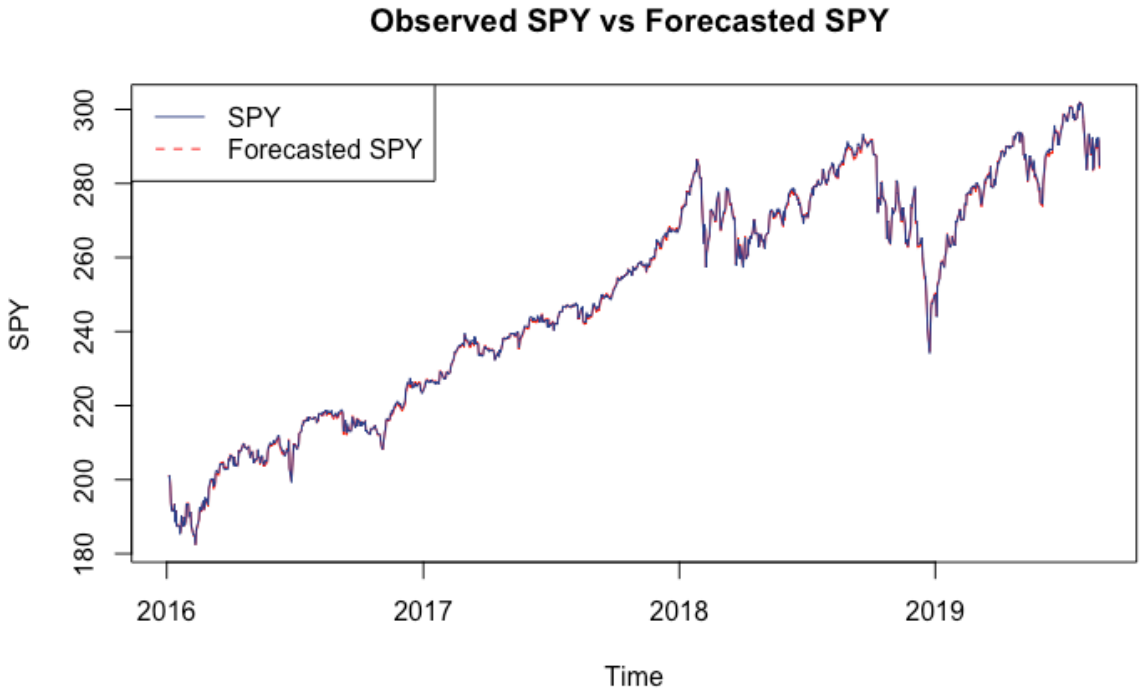
The z_{t-1} coefficient is significant because the p-value is extremely small (8.77×10^{-5}), indicating that the value of gamma suggests cointegration. The value of $\hat{\gamma}$ is negative, this allows the cointegration relationship to begin to 'correct' itself with each time step, approaching to the long-term mean. This supports the fact that the residuals are a stationary, or noise processes, adding to the case of a cointegration relationship between the processes.

Exercise 6:



Conclusion: The residuals look like a stationary process or I(0). This supports the claim that the two processes are cointegrated.

Exercise 7:



How useful is the index for forecasting the ETF? Extremely useful. The blue line (observed value of the ETF) is almost always the same as the forecasted ETF values). There is almost no red showing.