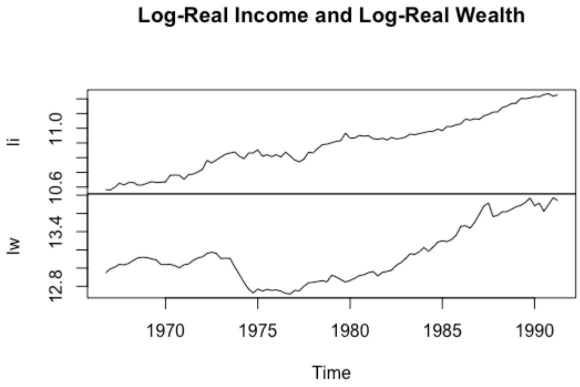
Time Series Econometrics Tutorial 6: Further Issues with Unit Roots – Spring 2019

Exercise 1:

Plotting the log of the income and wealth of the data shows that the processes are not stationary, although they could be trend-stationary.



After inspection, the income dataset appears to have a linear trend whilst wealth appears not have a defined trend. This uncertainty will lead us to testing all three models of Augmented Dickey-Fuller (ADF) tests (none, drift and trend), as well as utilising the Phillips-Persson test to confirm or deny the ADF results.

Log-Income

Augmented Dickey-Fuller Test:

Model: None

$$H_0: \pi = 0$$

$$H_1: \pi < 0$$

Critical Value (5%)		Test-Statistic
Tau1	-1.95	4.0885

The test statistic is greater than the critical value, therefore we fail to reject the null. i.e. The process has a unit root.

Model: Drift

$$H_0: \pi = 0, \qquad \pi = \beta_1 = 0$$

$$H_1: \pi < 0, \qquad \pi \neq \beta_1 \neq 0$$

Critical Value (5%)		Test-Statistic
Tau2	-2.89	-0.1562
Phi1	4.71	8.3001

The test-statistic is greater than the critical value Tau2, therefore we fail to reject the null that the process contains a unit root. However, the test-statistic is greater than Phi1. This means we can reject the joint null hypothesis that π and β_1 are equal 0. This means that the process has an intercept other than zero and the process does not contain a unit root. However, the result of the second null hypothesis Is not consistent with the first. The process must be differenced to yield consistent results.

Model: Trend

$$\mathrm{H}_0: \pi = 0, \quad \pi = \beta_1 = \beta_2 = 0, \quad \pi = \beta_2 = 0$$

$$\mathrm{H}_1: \pi < 0, \quad \pi \neq \beta_1 \neq \beta_2 \neq 0, \quad \pi \neq \beta_2 \neq 0$$

Critical Value (5%)		Test-Statistic
Tau3	-3.45	-2.0159
Phi2	4.88	7.0818
Phi3	6.49	2.0616

The Tau3 test statistic is greater than the Tau3 critical value, so we fail to reject the null hypothesis of a unit root. The Phi2 test statistic is greater than the critical value, so we reject the joint null hypothesis that the series' intercept, trend as zero and reject the null that the process has a unit root. The Phi3 test statistic is less than the critical value so we fail to reject the joint null hypothesis that the trend is zero a unit root.

After initial ADF testing of a non-differenced, it has been discovered that the process has a non-zero intercept coefficient and no trend, however these results are not consistent with the null hypotheses in each model type. To gain consistent results of the trend and intercept models, the process will be differenced. Before differencing, due to occasional type 1 errors and the low power of the ADF, Phillips-Perron (PP) test will also be used with both intercept and trend types to determine if a unit root is existent in the process.

<u>Phillips-Perron Test</u>

We use the Phillips-Perron test 'double-check' the results given by the ADF test.

Model: Intercept

$$\mathrm{H}_0: \alpha = 1$$
$$\mathrm{H}_1: \alpha < 0$$

Critical Value (5%)		Test-Statistic
$Z(\tau_{(\alpha)})$	-2.8909	-0.1024

Model: Intercept and Trend

$$\mathrm{H}_0: \alpha = 1$$
$$\mathrm{H}_1: \alpha < 0$$

Critical Value (5%)		Test-Statistic
$Z(\tau_{(\alpha)})$	-3.4557	-2.6412

After concluding that the process of log-income is a unit root, we need to determine the integration order (if its more than I(1)) due to conflicting results in the ADF types. After initial investigation, the integration order is at least 1. However, we will determine its actual integration order next by differencing the process.

Differencing the log-income process **once** yields the following results:

<u>Augmented Dickey-Fuller</u>
Model: None

$$\mathrm{H}_0: \pi = 0$$
$$\mathrm{H}_1: \pi < 0$$

Critical Value (5%)		Test-Statistic
Tau1	-1.95	-5.806

The test statistic is less than the critical value, therefore we reject the null. i.e. The process does not unit root.

Model: Drift

$$\mathrm{H}_0: \pi = 0, \qquad \pi = \beta_1 = 0$$

$$\mathrm{H}_1: \pi < 0, \qquad \pi \neq \beta_1 \neq 0$$

Critical Value (5%)		Test-Statistic
Tau2	-2.89	-7.0434
Phi1	4.71	24.8104

The test-statistic is less than the critical value Tau2, therefore we reject the null that the process contains a unit root i.e. the process is stationary. The Phi1 test-statistic is also greater than the critical value. This means we can reject the joint null hypothesis that π and β_1 are equal 0. This means that the process has an intercept other than zero and the process does not contain a unit root at the 5% level of significance.

Model: Trend

$$\mathrm{H}_0: \pi = 0, \qquad \pi = \beta_1 = \beta_2 = 0, \qquad \pi = \beta_2 = 0$$

$$\mathrm{H}_1: \pi < 0, \qquad \pi \neq \beta_1 \neq \beta_2 \neq 0, \qquad \pi \neq \beta_2 \neq 0$$

Critical Value (5%)		Test-Statistic
Tau3	-3.45	-7.0165
Phi2	4.88	16.4152
Phi3	6.49	24.6171

The Tau3 test statistic is less than the Tau3 critical value, so we reject the null hypothesis of a unit root. The Phi2 test statistic is greater than the critical value, so we reject the joint null hypothesis that the series' intercept, trend as zero and reject the null that the process has a unit root. The Phi3 test statistic is greater than the critical value so we reject the joint null hypothesis that the trend coefficient is zero and the process has a unit root.

The test results are consistent with each other, unlike the results yielded from a non-differenced process. It can be concluded using the ADF tests that the process is I(2). However, we will use the PP test to confirm this result.

Phillips-Perron Test

Model: Intercept

$$H_0: \alpha = 1$$
$$H_1: \alpha < 0$$

Critical Value (5%)		Test-Statistic
$Z(\tau_{(\alpha)})$	-2.8912	-11.7906

Model: Intercept and Trend

$$H_0: \alpha = 1$$
$$H_1: \alpha < 0$$

Critical Value (5%)		Test-Statistic
$Z(\tau_{(\alpha)})$	-3.456	-11.7495

The PP test results are consistent to the ADF results.

The process is I(2).

Log-Wealth

Augmented Dickey-Fuller Test:

Model: None

$$H_0: \pi = 0$$

$$H_1: \pi < 0$$

Critical Value (5%)		Test-Statistic
Tau1	-1.95	1.4209

The test statistic is greater than the critical value, therefore we fail to reject the null. i.e. The process has a unit root.

Model: Drift

$$H_0: \pi = 0, \qquad \pi = \beta_1 = 0$$

$$H_1: \pi < 0, \qquad \pi \neq \beta_1 \neq 0$$

Critical Value (5%)		Test-Statistic
Tau2	-2.89	0.272
Phi1	4.71	1.0282

The test-statistic is greater than the critical value Tau2, therefore we fail to reject the null that the process contains a unit root. However, the test-statistic is less than Phi1. This means we fail to reject the joint null hypothesis that π and β_1 are equal 0. This means that the process has an intercept other than zero and the process does not contain a unit root. However, the result of the second null hypothesis Is not consistent with the first. The process must be differenced to yield consistent results.

Model: Trend

$$H_0: \pi = 0, \qquad \pi = \beta_1 = \beta_2 = 0, \qquad \pi = \beta_2 = 0$$

$$H_1: \pi < 0, \qquad \pi \neq \beta_1 \neq \beta_2 \neq 0, \qquad \pi \neq \beta_2 \neq 0$$

Critical Value (5%)		Test-Statistic
Tau3	-3.45	-1.0153
Phi2	4.88	1.8719
Phi3	6.49	1.7902

The Tau3 test statistic is greater than the Tau3 critical value, so we fail to reject the null hypothesis of a unit root. The Phi2 test statistic is greater than the critical value, so we reject the joint null hypothesis that the series' intercept, trend as zero and reject the null that the process has a unit root. The Phi3 test statistic is less than the critical value so we fail to reject the joint null hypothesis that the trend is zero a unit root.

After initial ADF testing of a non-differenced, it has been discovered that the process has a non-zero intercept coefficient and no trend, however these results are not consistent with the null hypotheses in each model type. To gain consistent results of the trend and intercept models, the process will be differenced. Before differencing however, due to occasional type 1 errors and the low power of the ADF, Phillips-Perron (PP) test will also be used with both intercept and trend types to determine if a unit root is existent in the process.

Phillips-Perron Test

We use the Phillips-Perron test 'double-check' the results given by the ADF test.

Model: Intercept

$$\mathrm{H}_0: \alpha = 1$$
$$\mathrm{H}_1: \alpha < 0$$

Critical Value (5%)		Test-Statistic
$Z(\tau_{(\alpha)})$	-2.8909	0.0881

We fail to reject the null because the test-statistic is greater than the critical value at 5% significance.

Model: Intercept and Trend

$$\mathrm{H}_0: \alpha = 1$$
$$\mathrm{H}_1: \alpha < 0$$

Critical Value (5%)		Test-Statistic
$Z(\tau_{(\alpha)})$	-3.4557	-0.9743

We fail to reject the null because the test-statistic is greater than the critical value at 5% significance.

After concluding that the process of log-income is a unit root, we need to determine the integration order (if its more than I(1)) due to conflicting results in the ADF types. After initial investigation, the integration order is at least 1. However, we will determine its actual integration order next by differencing the process.

Differencing the log-income process **once** yields the following results:

Augmented Dickey-Fuller
Model: None

$$H_0: \pi = 0$$
$$H_1: \pi < 0$$

Critical Value (5%)		Test-Statistic
Tau1	-1.95	-5.8101

The test statistic is less than the critical value, therefore we reject the null that the process has a unit root. i.e. The process does not unit root.

Model: Drift

$$H_0: \pi = 0, \qquad \pi = \beta_1 = 0$$

$$H_1: \pi < 0, \qquad \pi \neq \beta_1 \neq 0$$

Critical Value (5%)		Test-Statistic
Tau2	-2.89	-5.983
Phi1	4.71	17.9028

The test-statistic is less than the critical value Tau2, therefore we reject the null that the process contains a unit root. The Phi1 test-statistic is also greater than the critical value. This means we can reject the joint null hypothesis that π and β_1 are equal 0. This means that the process has an intercept other than zero and the process does not contain a unit root at the 5% level of significance.

Model: Trend

$$H_0: \pi = 0, \qquad \pi = \beta_1 = \beta_2 = 0, \qquad \pi = \beta_2 = 0$$

$$H_1: \pi < 0, \qquad \pi \neq \beta_1 \neq \beta_2 \neq 0, \qquad \pi \neq \beta_2 \neq 0$$

Critical Value (5%)		Test-Statistic
Tau3	-3.45	-7.0165
Phi2	4.88	16.4152
Phi3	6.49	24.6171

The Tau3 test statistic is less than the Tau3 critical value, so we reject the null hypothesis of a unit root. The Phi2 test statistic is greater than the critical value, so we reject the joint null hypothesis that the series' intercept, trend as zero and reject the null that the process has a unit root. The Phi3 test statistic is greater than the critical value so we reject the joint null hypothesis that the trend coefficient is zero and the process has a unit root.

The test results are consistent with each other, unlike the results yielded from a non-differenced process. It can be concluded using the ADF tests that the process is I(2). However, we will use the PP test to confirm this result.

Phillips-Perron Test

Model: Intercept

$$H_0: \alpha = 1$$
$$H_1: \alpha < 0$$

Critical Value (5%)		Test-Statistic
$Z(\tau_{(\alpha)})$	-2.8912	-11.7906

We fail to reject the null because the test-statistic is greater than the critical value at 5% significance.

Model: Intercept and Trend

$$H_0: \alpha = 1$$
$$H_1: \alpha < 0$$

Critical Value (5%)		Test-Statistic
$Z(\tau_{(\alpha)})$	-3.456	-11.7495

We fail to reject the null because the test-statistic is greater than the critical value at 5% significance.

The PP tests confirm the ADF tests.

The process is I(2).

Exercise 2:

The table below summarises the outcomes of the various unit root tests. Where; ADF (Augmented Dickey-Fuller), PP (Phillips-Perron), ERS (Elliot-Rothenberg-Stock), SP (Schmidt-Phillip) and KPSS (Kwiatkowski-Phillips-Schmidt-Shin) tests.

Dataset	Variable	ADF	PP	ERS	SP	KPSS
Raotbl3	*lc*	I(1)	I(1)	I(1)	I(1)	I(1)
nporg	*log(gnp.r)*	I(1)	I(1)	I(1)	I(1)	I(0)
nporg	*log(gnp.n)*	I(1)	I(1)	I(1)	I(1)	I(0)
nporg	*bnd*	I(1)	I(1)	I(1)	I(1)	I(0)
nporg	*log(wg.n)*	I(1)	I(1)	I(1)	I(1)	I(0)

Raotbl3 lc:

ERS (DF_GLS)[Model: Trend]

Critical Value (5%)	Test-statistic
-3.03	-1.9108

ERS(P-Test)[Model: Trend]

Critical Value (5%)	Test-statistic
5.64	20.6103

In both ERS cases, the test-statistic is greater than the critical value at the 5% significance level, therefore we fail to reject the null of a unit root.

SP [Model: Tau], [Poly.deg =2]

Critical Value (5%)	Test-statistic
-3.65	-2.607

SP [Model: Rho], [Poly.deg =2]

Critical Value (5%)	Test-statistic
-23.7	-13.7177

In both SP cases, the test statistic is greater than the critical value at the 5% significance level, therefore we fail to reject the null of a unit root.

KPSS

Critical Value (5%)	Test-statistic
0.146	0.1508

$$\mathrm{H}_0: \pi < 0$$
$$\mathrm{H}_1: \pi = 0$$

Confirming the critical values on the one-sided test of the null being a stationary process.

	10pct	5pct	2.5pct	1pct
Critical values	0.119	0.146	0.176	0.216

The test statistic is greater than the critical value. Therefore, we reject the null hypothesis that the process is stationary.

Nporg Log(gnp.r):

ADF

Critical Value (5%)	Test-statistic
-1.95	2.1707

The test-statistic is greater than the critical value, therefore we fail to reject the null hypothesis of a unit root.

PP

Critical Value (5%)	Test-statistic
-3.4836	-1.9759

We fail to reject null of a unit root at the 5% significance, because the test statistic is greater than the critical value.

SP

Critical Value (5%)	Test-statistic
-3.65	-3.3168

Fail to reject the null of a unit root at the 5% significance, because the test statistic is greater than the critical value.

KPSS

Critical Value (5%)	Test-statistic
0.146	0.1336

We fail to reject the null that the process is stationary because the test statistic is less than the crucial value at the 5% significance.

Nporg log(gnp.n)

ADF

Critical Value (5%)	Test-statistic
-1.95	2.293

Fail to reject null of a unit root at the 5% significance level. The test-statistic is greater than the critical value.

PP

Critical Value (5%)	Test-statistic
-3.4836	-1.8033

Fail to reject null of a unit root at the 5% significance level. The test-statistic is greater than the critical value.

ERS

Critical Value (5%)	Test-statistic
-3.03	-1.5856

The test-statistic is greater than the critical value as the 5% significance level, therefore we fail to reject the null of a unit root.

KPSS

Critical Value (5%)	Test-statistic
0.146	0.1138

Because the test-statistic is less than the critical value, we fail to reject the null hypothesis that the process is stationary.

Nporg bnd

ADF

Critical Value (5%)	Test-statistic
-1.95	1.6779

The test-statistic is greater than the critical value as the 5% significance level, therefore we fail to reject the null of a unit root.

PP

Critical Value (5%)	Test-statistic
-3.4739	0.7142

The test-statistic is greater than the critical value as the 5% significance level, therefore we fail to reject the null of a unit root.

ERS

Critical Value (5%)	Test-statistic
-3.03	-0.7951

The test-statistic is greater than the critical value as the 5% significance level, therefore we fail to reject the null of a unit root.

SP

Critical Value (5%)	Test-statistic
-3.65	-0.8653

The test-statistic is greater than the critical value as the 5% significance level, therefore we fail to reject the null of a unit root.

Nporg log(wg.n)

ADF

Critical Value (5%)	Test-statistic
-1.95	2.4759

The test-statistic is greater than the critical value as the 5% significance level, therefore we fail to reject the null of a unit root.

PP

Critical Value (5%)	Test-statistic
-3.4739	-2.0632

The test-statistic is greater than the critical value as the 5% significance level, therefore we fail to reject the null of a unit root.

ERS

Critical Value (5%)	Test-statistic
-3.03	-2.0665

The test-statistic is greater than the critical value as the 5% significance level, therefore we fail to reject the null of a unit root.

SP

Critical Value (5%)	Test-statistic
-3.65	-3.3808

The test-statistic is greater than the critical value as the 5% significance level, therefore we fail to reject the null of a unit root.