1(a)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Smoke | Fire | (Smoke ⇒ Fire) | (¬Smoke ⇒ ¬Fire) | (Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire) |
| T | T | T | T | T |
| T | F | F | T | T |
| F | T | T | T | T |
| F | F | T | F | F |

Thus, the sentence is neither valid nor unsatisfiable (only unsatisfiable in case 4).

(b)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Smoke | Fire | Heat | (Smoke ⇒ Fire) | (Smoke ∨ Heat) | ((Smoke ∨ Heat) ⇒ Fire) | (Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire) |
| T | T | T | T | T | T | T |
| T | T | F | T | T | T | T |
| T | F | T | F | T | T | T |
| T | F | F | F | T | F | T |
| F | T | T | T | T | T | T |
| F | T | F | T | F | T | T |
| F | F | T | T | T | F | F |
| F | F | F | T | F | T | T |

Thus, the sentence is neither valid nor unsatisfiable (only unsatisfiable in case 7).

(c)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Smoke | Fire | Heat | ((Smoke ∧ Heat) ⇒ Fire) | (Smoke ⇒ Fire) | (Heat ⇒ Fire) | ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire)) |
| T | T | T | T | T | T | T |
| T | T | F | T | T | T | T |
| T | F | T | F | F | F | T |
| T | F | F | T | F | T | T |
| F | T | T | T | T | T | T |
| F | T | F | T | T | T | T |
| F | F | T | T | T | F | T |
| F | F | F | T | T | T | T |

Thus, the sentence is valid and not unsatisfiable.

2(a)

Mythical => Immortal

¬Mythical => (¬Immortal ∧ Mammal)

(Immortal ∨ Mammal) => Horned

Horned => Magical

(b)

(¬Mythical∨Immortal) ∧ (Mythical ∨¬Immortal) ∧ (Mythical ∨ Mammal) ∧ (¬Immortal∨ Horned) ∧ (¬Mammal ∨Horned) ∧ ( ¬Horned ∨Magical)

(c)

Let

(¬Mythical∨Immortal) 1

(Mythical ∨¬Immortal) 2

(Mythical ∨ Mammal) 3

(¬Immortal ∨ Horned) 4

(¬Mammal ∨Horned) 5

( ¬Horned ∨Magical) 6

Mythical:

Assume ¬Mythical 9

2,9: ¬Immortal 10