Applied Numerical Methods
(MATH 151A)
Homework 2

Due day: 3:50 p.m., Friday, Feb 8th.

1. Given that each of the following sequences $\{p_n\}_{n=0}^{\infty}$ converges to p^*, show that it converges linearly:

   (a) The sequence is $\begin{cases} p_{n+1} = \frac{1}{2}\ln(p_n + 1) \\ p_0 = 1 \end{cases}$ and the limit is $p^* = 0$;

   (b) The sequence is $\begin{cases} p_n = 1 + 2^{1-n} + \frac{1}{(n+2)^n} \\ p_0 = 4 \end{cases}$ and the limit is $p^* = 1$;

2. Show that the following sequences $\{p_n := 10^{(-2^n)}\}_{n=0}^{\infty}$ converges to p^*, show that it converges quadratically.

3. (a) Use the Lagrange interpolation method to find a polynomial f such that

   $$f(1) = 2, \quad f(2) = 1, \quad f(3) = 4, \quad f(4) = 3.$$

   (b) Use the Neville's Method instead to find the same polynomial f.

4. Programming problem: Consider the following function $f : [-1, 1] \to \mathbb{R}$

   $$f(x) = |x|$$

   (a) Plot the graph of the function f.

   (b) Given $n \in \mathbb{N}\backslash\{0\}$, define $x_n^k = -1 + \frac{2k}{n}$ for $0 \le k \le n$.
   Let $g_n(x)$ be the unique polynomial of degree n which results by interpolating the $n + 1$ data $\{(x_n^k, f(x_n^k))\}_{0 \le k \le n}$, i.e. $g_n(x_n^k) = f(x_n^k)$ for all $0 \le k \le n$. Plot the functions f, g_2, g_3, g_4 and g_5 on the same graph.

   (c) Plot the sequence $\{g_n(0.3)\}_{1 \le n \le 20}$.