Homework 4

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Problem 1

(a)

$$
\begin{aligned}
L_1(x) &= \frac{(x-0)(x-1)}{(-1-0)(-1-1)} = \frac{x^2-x}{2} \\
L_2(x) &= \frac{[x-(-1)](x-1)}{[0-(-1)](0-1)} = 1-x^2 \\
L_3(x) &= \frac{[x-(-1)](x-0)}{[1-(-1)](1-0)} = \frac{x^2+x}{2} \\
h(x) &= P(x) = \sum_{k=0}^{3} f(x_k)L_k(x) \\
&= \frac{x^2-x}{2}f(-1) + (1-x^2)f(0) + \frac{x^2+x}{2}f(1)
\end{aligned}
$$

(b)

$$
E(x) = \frac{f^{(3)}(\xi(x))}{3!}\prod_{i=0}^{2}(x-x_i)
$$

(c)

$$
\begin{aligned}
\int_{-1}^{1} h(x)\ dx &= \int_{-1}^{1} \frac{x^2-x}{2}f(-1) + (1-x^2)f(0) + \frac{x^2+x}{2}f(1)\ dx \\
&= \frac{1}{2}\int_{-1}^{1} (x^2-x)f(-1) + (2-2x^2)f(0) + (x^2+x)f(1)\ dx \\
&= \frac{1}{2}f(-1)\int_{-1}^{1} x^2 - x\ dx + f(0)\int_{-1}^{1} 1-x^2\ dx + \frac{1}{2}f(1)\int_{-1}^{1} x^2+x\ dx \\
&= \frac{1}{2}f(-1)[\frac{x^3}{3} - \frac{x^2}{2}]_{-1}^{1} + f(0)[x-\frac{x^3}{3}]_{-1}^{1} + \frac{1}{2}f(1)[\frac{x^3}{3}+\frac{x^2}{2}]_{-1}^{1} \\
&= \frac{1}{3}f(-1) + \frac{4}{3}f(0) + \frac{1}{3}f(1)
\end{aligned}
$$

(d) Yes. This is because when f is a polynomial of degree less than or equal to 2, the error term stated in part (b) is 0 as $f^{(3)}(\xi(x))$ is 0. This makes $\int_{-1}^{1} h(x)\ dx = \int_{-1}^{1} f(x)\ dx$.

(e)

$$
|\int_{-1}^{1} f(x)\ dx - \int_{-1}^{1} h(x)\ dx| \leq \frac{1}{6}\int_{-1}^{1}(x-x_0)(x-x_1)(x-x_2)f^{(3)}(\xi(x))\ dx
$$

Problem 2

(a)

$$\begin{aligned}\int_0^4 f(x)\ dx &= \frac{h}{3}[f(x_0) + 4f(x_1) + f(x_2)] - \frac{h^5}{90} f^{(4)}(\xi) \\ &\approx \frac{2}{3}[f(0) + 4f(2) + f(4)] \\ &= \frac{2}{3}[1 + 4 + 1] \\ &= 4\end{aligned}$$

(b)

$$\begin{aligned}\int_1^4 f(x)\ dx &= \frac{h}{3}[f(a) + 2\sum_{j=1}^{(n/2)-1} f(x_{2j}) + 4\sum_{j=1}^{(n/2)} f(x_{2j-1}) + f(b)] - \frac{b-a}{180} h^4 f^{(4)}(\mu) \\ &\approx \frac{1}{3}[f(0) + 2f(2) + 4(f(1) + f(3)) + f(4)] \\ &= \frac{1}{3}[1 + 2 * 1 + 4 * (2 + 2) + 1] \\ &= \frac{20}{3}\end{aligned}$$