Applied Numerical Methods
(MATH 151A)
Assignment 4

Note:

- **Due day**: 3:50 p.m., March 8th (Friday).

1. Let $f(x)$ be a function defined on the interval $[-1, 1]$, and $f \in C^4[-1, 1]$.

   (a) Let $h(x)$ be the Lagrange interpolation polynomial of $f(x)$ at the nodes $x = -1, 0, 1$. Write down the expression of $h(x)$.

   (b) Write down the error term $E(x) := f(x) - h(x)$ in terms of the derivatives of $f(x)$. (Recall the theorem about the error between the interpolation formula h and the exact function f.)

   (c) Compute the integral
   $$\int_{-1}^{1} h(x)dx$$
   exactly in terms of the values of $f(x)$ at points $x = -1, 0, 1$.

   (d) If we approximate the integral $\int_{-1}^{1} f(x)dx$ by $\int_{-1}^{1} h(x)dx$, is it true that the above approximation is exact if f is a polynomial of degree less than or equal to 2 ? Why ?

   (e) Write down an error bound of this approximation rule suggested in (d) directly based on the result in (b).

2. A function f has the values shown as below:

x	0	1	2	3	4
$f(x)$	1	2	1	2	1

   (a) Use Simpson's Rule and only the function values at $x = 0, 2, 4$ to approximate the integral $\int_0^4 f(x)dx$.

   (b) Use composite Simpson's Rule and the functions values at $x = 0, 1, 2, 3, 4$ to approximate the same integral $\int_0^4 f(x)dx$.

3. (Programming problem) Consider the integral:
   $$\int_0^{\pi} \cos x dx$$

(a) Write a program to use the composite trapezoidal to approximate the above integral by dividing $[0, \pi]$ to N equal spaces.

(b) Write a program to use the composite Simpson's approximate the above integral by dividing $[0, \pi]$ to N equal spaces.