The Promise of Autonomous Vehicles: Traffic reduction in lane merging and the concertina effect

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ABSTRACT

This paper investigates the potential of autonomous vehicles in the reduction of road congestion. The assumption is that due to their lack of reaction time and ability to follow a fixed protocol the autonomous vehicles will be able to avoid creating a propagating disruptive traffic patterns. The paper investigates this potential through simulating two experiments on agent based models. The first experiment is a study of the concertina effect, the effect of over-braking propagating back and reducing the throughput of a single lane on a road. With a finite reaction time for humans, and a null reaction time for the autonomous vehicles, the simulation is run with different numbers of cars. The second experiment is a study of lane merging. The human simulation follows a protocol that attempts to accurately model the behavior of humans during lane merging, and the autonomous vehicles all perfectly follow a rigid protocol. Again the simulation is run with different numbers of cars. As was hoped, the results show that autonomous vehicles perform better in both experiments. They reduce the time taken to resolve the concertina effect by a factor of 1.62 and is faster in lane merging by a factor of 1.72. This gives us hope for the future of congestion reduction in road transport.

DEDICATION

This thesis is dedicated to my mother and father who have both provided substantial intellectual, finantial and emotional support throughout my years of learning.

ACKNOWLEDGMENTS

I must chiefly acknowledge Prof. Bastien Chopard for his generous and indispensable guidance throughout the writing of this thesis.

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1.1.1 Road Traffic

Personal freedoms are paramount in modern society, featuring in the legal codes of many countries as fundamental human rights. Amongst the freedoms can be found the right to freedom of movement, with the *European Convention on Human Rights* stating in Protocol No.4, Article 2:

> Everyone lawfully within the territory of a State shall, within that territory, have the right to liberty of movement and freedom to choose his residence. [Hum, p.37]

We citizens of modern societies therefore have a quasi unlimited opportunity to go where we want, when we want, and naturally we enjoy exercising this right. While public transport is an important enabling factor, with 3238 trains being registered in Switzerland in 2015 [OFS18, p.20], and a total of 198 000 000km being driven on the rail system in 2017 [OFS18, p.24], the ownership of private motor vehicles is ever increasing due to the extra degrees of freedom it promotes. In 2017, 6.2 million road vehicles were registered in Switzerland, 4.6 million of these being tourist vehicles [OFS18, p.17], and the distance driven per private motor vehicle has increase 42% from 1995 to 2017 [OFS18, p.21].

1.1.2 Congestion, a Consequential Issue

This significant increase in the usage of public road networks without an according increase in their capacity leads to congestion, with the Swiss population clocking in 25 900 hours of traffic jams in 2017 [OFS18, p.24]. Congested traffic greatly reduces the throughput of public road networks and can therefore significantly increase the time motor vehicles spend on the road.

This increased idle time prolongs drivers' exposure to harmful substances emitted by surrounding vehicles which can lead to a variety of health issues, and the increased time spent sitting immobile contributes to the excess time a modern human spends in a seated position, which is not conducive to maintaining a healthy body. Time spent in traffic can also be very stressful, and therefore is a contributing factor to the many mental health issues that are plaguing modern society.

Two notable causes of congested traffic on motorways are lane merging and the concertina effect.

1.1.3 The Concertina Effect

The concertina effect, otherwise known as the accordion effect, is the propagation of over-braking in a lane of traffic. When a driver brakes, the driver directly behind will often overreact and brake more than necessary, as it is difficult to gauge the amount of force the car in front is braking with. This effect will in continue propagating back, with each subsequent car braking harder and harder, thereby resulting in a reduced throughput and congestion, and often will end up bringing the line of cars to a stand still.

1.1.4 Lane Merging

Lane merging on motorways is the situation in which two distinct lanes join to form one through the action of the merging lane gradually reducing in width until it's border coincides with the merged lane, as illustrated in Figure 1.

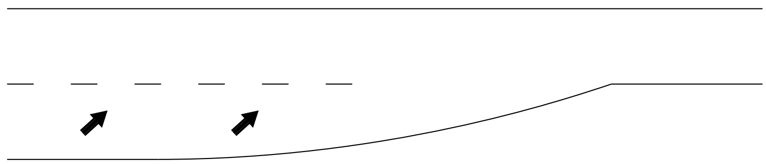

Figure 1: The merging of two lanes

The process of merging lanes is fraught with opportunities for drivers to make little mistakes. When merging lanes, the driver must ensure safety distance with the car in front, must check that there is space in the new lane, must indicate that he intends to change lanes, and must continue to maintain safety distance throughout the lane change.

Little mistakes can have large consequences in the throughput of the motorway, as testified by the concertina effect. A mild over braking caused by misjudging the distance to the car in front whilst changing lanes will propagate through the traffic behind.

Another significant problem with Lane Merging is that it inherently halves the potential throughput of a motorway due to the removal of one lane, and so will always pose a risk of congestion.

Another issue lies in the fact that humans will not all follow the same protocol during the merging of lanes. This will amplify the potential of little mistakes due to contradictory actions arising from different interpretations of a given situation.

These issues all acting at once result in a significant source of congestion on motorways, as lane merging is common on motorways, for example when joining a motorway a lane merge must be performed, as when exiting, and lane changing is a simplified case of lane merging but with the same potential issues.

1.1.5 Trivial, but Unavoidable

These omnipresent causes of congestion on motorways are so simple to understand and so trivial in nature that they give the impression of being entirely avoidable. One would assume that if all drivers on the road followed a well defined protocol, such issues would never arise or at least could be minimised. The unfortunate fact is that it would be exceedingly difficult to get all human drivers to adhere to such a protocol, with a single driver not adhering having the potential to completely negate the efforts of all the others.

1.1.6 The Promise of Autonomous Vehicles

We are on the eve of a mass adoption of autonomous vehicles, and can expect in the near future to live in a society in which they represent the vast majority of motor vehicles. Autonomous vehicles have the advantage over humans of being able to adhere perfectly to a defined protocol for congestion avoidance and have much faster reaction times, and so we might hope that as the usage of autonomous vehicles increases, congestion decreases accordingly.

It would be nice to show that this optimism is grounded in reality and that autonomous vehicles do in fact pose a real possibility for congestion limitation.

1.2 Thesis Goal

The goal of this thesis therefore is to explore the possibility of reducing congestion on motorways with autonomous vehicles, considering specifically the cases of the Concertina Effect, and of Lane

Merging. Efforts will therefore be made in accurately modelling humans and autonomous vehicles on a motorway and in developing a protocol for lane merging.

2 Modelling

2.1 Motivation

To be able to explore the potential of autonomous vehicles in reducing congested traffic, we must develop a computational model. We will use agent based modelling, a class of models that simulate individual autonomous agents separately in an attempt to understand their effects on a whole system. We must model single lane agents and lane changing agents and extend these models to the multi agent scenario. We must also introduce an extension to the model that factors in a human element, with the aim of comparing our congestion avoidance attempts to current situations found on the motorway.

2.2 Single Lane Agent

2.2.1 Goals, Parameters and Assumptions

The goal is to find a model of a car defined by a position and a velocity that are iteratively updated according to equations that we will derive. The the parameters for these equations, notably the rate of acceleration/braking, will be decided by the agent at each iteration based upon a protocol and in reaction to it's surroundings.

A simple model has been proposed in [Cho17] and is presented below.
A car (agent) is characterized by several parameters: mass m, length l, maximum velocity $v_{max} > 0$, maximum acceleration $a_{max} > 0$, and maximum deceleration $a_{min} < 0$. We will denote $x(t)$ the position of the car along a given road at time t, with $v(t)$ it's current velocity.
The equations of motion for the car are

$$\dot{x} = v, \qquad m\dot{v} = F \tag{1}$$

with F the force acting on the car at time t.
The assumption

$$F = \begin{cases} -m\gamma_- v & \text{if the car is braking} \\ +m\gamma_+(v_{max} - v) & \text{if the car is accelerating} \end{cases} \tag{2}$$

where $\gamma_\pm$ has a unit s^{-1} allows us to control the acceleration/deceleration through $\gamma_\pm$, which are measures of the force with which the driver brakes or accelerates, and are related to the maximum acceleration and deceleration.

2.2.2 Linear Motion

The linear motion of the car under no acceleration, i.e when

$$\gamma_\pm \to 0 \implies F \to 0 \tag{3}$$

is given by the following equations

$$\begin{aligned} v(t) &= v(t_0) \\ x(t) &= x(t_0) + v(t_0)(t - t_0) \end{aligned} \tag{4}$$

2.2.3 Braking

In the case of the car braking, the velocity is being reduced proportionally to the factor γ_- we introduced in (2), and so we find the equation of motion

$$\dot{v} = -\gamma_- v \tag{5}$$

whose solution is

$$v(t) = v(t_0)e^{(-\gamma_-(t-t_0))} \tag{6}$$

Integrating $v(t)$ we find the position $x(t)$

$$x(t) = x(t_0) + \frac{v(t_0)}{\gamma_-}(1 - e^{(-\gamma_-(t-t_0))}) \tag{7}$$

2.2.4 Accelerating

In the case of the car accelerating, we will make the assumption that the cars' ability to accelerate diminishes as it approaches its maximum velocity, and thus we find the equation of motion

$$\dot{v} = \gamma_+(v_{max} - v) \tag{8}$$

whose solution is

$$v(t) = v_{max} + (v(t_0) - v_{max})e^{(-\gamma_+(t-t_0))} \tag{9}$$

Integrating $v(t)$ we find the position $x(t)$

$$x(t) = x(t_0) + v_{max}(t - t_0) + \frac{v(t_0) - v_{max}}{\gamma_+}(1 - e^{(-\gamma_+(t-t_0))}) \tag{10}$$

2.2.5 $\gamma_\pm$ values

We can derive maximum values of $\gamma_\pm$ such that they satisfy known properties of a given car, for example stopping distance or time taken to accelerate to a given velocity. We can then vary their values within these bounds depending on the situation in which the autonomous vehicle finds itself.

Expanding (7) and substituting (6) we obtain

$$x(t) = x(t_0) + \frac{v(t_0)}{\gamma_-} - \frac{v(t_0)e^{(-\gamma_-(t-t_0))}}{\gamma_-} = x(t_0) + \frac{v(t_0) - v(t)}{\gamma_-} \tag{11}$$

We can rearrange (11) to find the maximum value γ_-^* of γ_- as derived from the knowledge of a cars minimum braking distance $x(t) - x(t_0)$ from a given velocity v_{ref}

$$\gamma_-^* = \frac{v_{ref}}{x(t) - x(t_0)} \tag{12}$$

Under the assumption that a car accelerates with as much force as it can from a standing start, $v(t_0) = 0$, to a given velocity v_{ref} we find

$$v_{ref} = v_{max} - v_{max}e^{(-\gamma_+(t-t_0))} \tag{13}$$

We can rearrange (13) to find the maximum value γ_+^* of γ_+ as derived from a given maximum and reference velocity, the start and end velocities of the braking period, and

$$\gamma_+^* = -\frac{1}{t - t_0}\ln(1 - \frac{v_{ref}}{v_{max}}) \tag{14}$$

Seeing as we are in the case of autonomous vehicles, we will consider the $\gamma_\pm^*$ values of a Tesla Model 3. The Model 3 stops in approximately $40m$ from $100km/h \approx 28m/s$ ($133ft$ from $60mph$)

[Rep20], accelerates to $100km/h \approx 28m/s$ in $3.4s$, and has a maximum velocity of $261km/h \approx 72.5$ [Tes20]. With these values, we find

$$\begin{aligned} \gamma_-^* &= \frac{28}{40} \approx 0.69 \ s^{-1} \\ \gamma_+^* &= -\frac{1}{3.4} \ln(1 - \frac{28}{72.5}) \approx 0.14 \ s^{-1} \end{aligned} \tag{15}$$

2.2.6 Model

The final model is therefore initialised with these following parameters:

- $t = t_0$: initial time
- $v = v_0$: initial velocity
- v_{max}: maximum velocity
- $x = x_0$: initial x position
- $\gamma = \gamma_0$: initial value for the "acceleration" parameter
- $\gamma_+ = 0.14$: the maximum value of $-\gamma$ for a Tesla Model 3
- $\gamma_- = 0.69$: the maximum value of γ for a Tesla Model 3

and we impose the restriction $-\gamma_-^* < \gamma < \gamma_+^*$.
We update the cars' velocity and position using the following equations depending on the value of γ:

- $\gamma = 0$: (4)
- $\gamma > 0$: (6) and (7)
- $\gamma < 0$: (9) and (10)

Note that the position of the car will be taken to describe the position of its back wheels, and that the parameter γ is the only parameter modified by the agent when reacting to its surroundings.

2.3 Lane Changing Agent

2.3.1 Goals, Parameters and Assumptions

The goal is to find a model of a car defined by a position and a velocity that are iteratively updated according to equations that we will derive. The the parameters for these equations, notably the rate of acceleration/braking and the turning angle of the wheel, will be decided by the agent at each iteration based upon a protocol and in reaction to it's surroundings.

The Lane Changing Agent will simply be an extension of the model in 2.2 through the addition of a capacity to change lanes. We will base this extension on the work in [Cho18].

The description of the trajectory of a turning car in [Cho18] is presented below.
The car is approximated by a 1d line and is described in Figure 2.
We would like to calculate the new position $(x(t+dt), y(t+dt))$ and new angle $\theta(t+dt) = \theta + d\theta$.

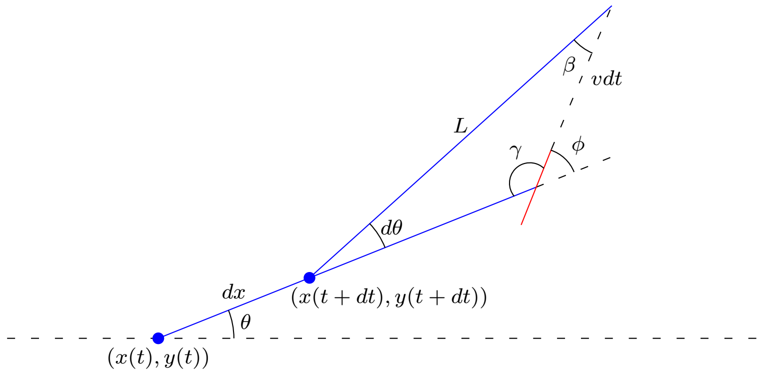

Figure 2: The car is of length L, with a position described by the (x, y) coordinates of the back wheels at time t. The car is at an angle θ to the road, with it's wheels at an angle ϕ to it's chassis. After a small time interval dt the back wheels have moved along the original path by a distance dx, and the front wheels a distance vdt along the path defined by ϕ.

2.3.2 Deriving $\theta(t + dt)$

To find a formula for $\theta(t + dt)$ we will begin by deriving a formula for $d\theta$.
From Figure 2, and with the law of sines, we find that

$$\sin(d\theta) = \frac{vdt}{L}\sin(\gamma) = \frac{vdt}{L}\sin(\phi) \tag{16}$$

where $\sin(\gamma) = \sin(\pi - \phi) = \sin(\phi)$ is obtained by expanding $\sin(\pi - \phi)$ using a trigonometric identity).
Rearranging we find

$$d\theta = \arcsin\left(\frac{vdt}{L}\sin(\phi)\right) \xrightarrow[dt\to 0]{} \frac{vdt}{L}\sin(\phi) \tag{17}$$

and thus

$$\frac{d\theta}{dt} = \frac{v}{L}\sin\phi \tag{18}$$

Integrating $\frac{d\theta}{dt}$ with respect to dt on the interval $[t, t + dt]$ we obtain the angle swept in the time interval dt thereby leading to the following formula for the new angle $\theta(t + dt)$

$$\theta(t + dt) = \theta(t) + \frac{1}{L}\int_t^{t+dt} v(\tau)\sin\phi(\tau)d\tau \tag{19}$$

Under the assumption that during the time interval dt the velocity and wheel angle are constant

$$(\tau) = v(t), \qquad \phi(\tau) = \phi(t) \tag{20}$$

we can obtain the following formula

$$\theta(t + dt) = \theta(t) + \frac{v(t)dt}{L}\sin(\phi(t)) \tag{21}$$

Henceforth we shall denote $\phi(t) = \phi$ and $v(t) = v$.

2.3.3 Deriving $(x(t+dt), y(t+dt))$

The position $(x(t+dt), y(t+dt))$ can be described by the following equations

$$\begin{aligned} x(t+dt) &= x(t) + dx\cos(\theta) \\ y(t+dt) &= y(t) + dx\sin(\theta) \end{aligned} \tag{22}$$

We would now like to derive a formula for dx in (22).
From Figure 2, and with the law of sines, we find that

$$L - dx = L\frac{\sin(\beta)}{\sin(\gamma)} \implies dx = L\left(1 - \frac{sin(\beta)}{\sin(\gamma)}\right) = L\left(1 - \frac{sin(\phi - d\theta)}{sin(\phi)}\right) \tag{23}$$

where $\beta = \pi - d\theta - \gamma = \phi - d\theta$ with $\gamma = \pi - \phi$.
With the small angle approximation we find that $sin(\phi - d\theta) = sin(\phi) - d\theta\cos(\phi)$, and thus

$$dx = L\left(1 - \frac{sin(\phi - d\theta)}{sin(\phi)}\right) \xrightarrow[d\theta \to 0]{} L\left(1 - \frac{sin(\phi) - d\theta\cos(\theta))}{sin(\phi)}\right) = \frac{Ld\theta}{\tan(\phi)} \tag{24}$$

With (21) we have $d\theta = \frac{vdt}{L}\sin(\phi)$ and so

$$dx = \frac{vdt}{\tan(\phi)}\sin(\phi) = vdt\cos(\phi) \tag{25}$$

We substitute (25) into (22) to obtain

$$\begin{aligned} x(t+dt) &= x(t) + vdt\cos(\phi)\cos(\theta) \\ y(t+dt) &= y(t) + vdt\cos(\phi)\sin(\theta) \end{aligned} \tag{26}$$

2.3.4 Changing lanes

(21) and (26) now describe the new position $(x(t+dt), y(t+dt))$ and angle with respect to the road $\theta(t+dt)$ of a car after a small time interval dt in which the car has a constant velocity v and turning angle ϕ. Note that the velocity v can be varied between time steps dt and so the car can be accelerating or decelerating whilst turning.
We can extend this description to that of a car changing lanes by making the assumption that the turning angle is adjusted after each time interval dt in such a manner that the front wheels point to a desired target, being lane to change to. The desired target is therefore chosen to be

$$(x(t) + h, y_{\text{lane}}) \tag{27}$$

with h the horizon distance of the driver, and y_{lane} the y-coordinate of the lane to change to. The horizon distance of the driver is the distance ahead of the car at which he is aiming.
The new turning angle can be described thusly

$$\psi = \arctan\left(\frac{y_{\text{lane}} - y(t)}{h - L\cos(\theta)}\right) - \theta \tag{28}$$

So as to ensure a smooth lane change without drastic variations in turning angle, we can bound the difference of the turning angle between two time steps by a given maximum value, Δ_ϕ. The final formula for updating the angle ϕ between time steps is therefore

$$\phi(t+dt) = \begin{cases} \phi(t) - \Delta_\phi & \text{if } \psi - \phi(t) < -\Delta_\phi \\ \psi & \text{if } |\psi - \phi(t)| < \Delta_\phi \\ \phi(t) + \Delta_\phi & \text{if } \psi - \phi(t) > \Delta_\phi \end{cases} \tag{29}$$

The choice of $\Delta_\phi = 1$ results in a smooth lane change [Cho18], and the choice of $h = 50m$ is natural from personal experience, as that is approximately the distance ahead of me on which I fixate when changing lanes whilst driving.

2.3.5 Model

The final model is based on the model presented in Section 2.2.6 with the extended capability of changing lanes.
The model is therefore initialised with the following additional parameters

- $L = 4.69$: the length of the car (Tesla Model 3)
- $y = y_0$: the initial y position of the car
- y_{target}: the target of the lane change
- $\theta = \theta_0$: the initial angle to the road
- $\phi = \phi_0$: the initial turning angle
- $\Delta_\phi = 1$: the maximum variation in turning angle
- $h = 50$: the horizon distance of the driver

The lane change is described as a change in car angle with respect to the road, position and turning angle between small time steps dt. These changes are described respectively by (21), (26) and (29).

2.4 Multi Agent

2.4.1 Agent Interaction

In a multi agent environment the agents must be able to communicate with their surroundings. Each agent should:

- know the distance to the agent in front of them in the current lane,
- know the distance to the agents behind them and in front of them and in the adjacent lane
- know if the agent ahead of them has begun/finished merging

To aid the implementation of these functions we will assign a unique id to each agent.

2.4.2 Model

We must therefore extend the model presented in Section 2.3.5 to include this additional information.
The model is therefore initialised with the following additional parameters

- id: the id of the agent
- merging=false: flag to indicate the agent has begun merging
- merged=false: flag to indicate the agent has finished merging

2.5 Full Model

Full model is therefore initialised with the following parameters

- $t = t_0$: initial time
- $v = v_0$: initial velocity
- v_{max}: maximum velocity
- $x = x_0$: initial x position
- $\gamma = \gamma_0$: initial value for the "acceleration" parameter
- $\gamma_+ = 0.14$: the maximum value of $-\gamma$ for a Tesla Model 3
- $\gamma_- = 0.69$: the maximum value of γ for a Tesla Model 3
- $L = 4.69$: the length of the car (Tesla Model 3)
- $y = y_0$: the initial y position of the car
- y_{target}: the target of the lane change
- $\theta = \theta_0$: the initial angle to the road
- $\phi = \phi_0$: the initial turning angle
- $\Delta_\phi = 1$: the maximum variation in turning angle
- $h = 50$: the horizon distance of the driver
- id: the id of the agent
- merging=false: flag to indicate the agent has begun merging
- merged=false: flag to indicate the agent has finished merging

We update the cars' velocity, position, angle to the road and turning angle using the following equations depending on the value of γ and ϕ:

If $\phi == 0$
- $\gamma = 0$: (4)
- $\gamma < 0$: (6) and (7)
- $\gamma > 0$: (9) and (10)

If $\phi ! = 0$
- $\gamma = 0$: (26), (21) and (29)
- $\gamma < 0$: (6), then (26), (21) and (29)
- $\gamma > 0$: (9), then (26), (21) and (29)

Note that the parameters γ and ϕ are the only parameters modified by the agent when reacting to its surroundings.

2.6 Implementation

2.6.1 Choice of Technology

I have chosen to use the C++ programming language based on its speed and its object oriented capabilities. This object oriented aspect will help simplify the implementation, as the components of the experiment can easily be modelled through classes.

2.6.2 Car Class

The Car class will be the model of the agent. Its purpose will be to contain the necessary attributes to describe it's position, speed, and the result of it's decision such as the acceleration parameter and turning angle. The Car class will also contain the methods necessary to update the position and the decision of the car.

Car
- id: int - velocity: float - max_velocity: float - x_pos: float - y_pos: float - accel_param: float - max_accel_param: float - min_accel_param: float - length: float - y_target: float - horizon: float - road_angle: float - turning_angle: float - max_delta_turning_angle: float
+ update_decision(Road): void + update_position(float time_step, int road_delim_x, int refill_protocol): void +get_x_pos(): float +get_y_pos(): float +get_length(): float

Figure 3: Car Class Diagram

Note that in the implementation the sign of γ_- has been flipped in the equations describing acceleration and braking due to the fact that the attribute γ (accel_param) in the model can take negative values as it represents both γ_- and γ_+, where as in the description of the model γ_- and γ_+ were separated and both took positive values.
Also note that this class will later be extended with additional attributes and methods that will aid the implementation of the protocols.

2.6.3 Road Class

The Road object will contain describtive attributes about the road, such as its length and where the lane merging begins, and will also contain the list of cars on the road. It will also have methods that will update the decisions and positions of all the cars on the road.
Note that the separate lanes of the road will be given integer values for there y positions. During the investigation of the concertina effect, we will ignore the y values, and when investigating lane merging, we will only use the two lanes 0 and 1, with the cars attempting to merge into lane 1.
Also note that the road length is fixed, unless set to 0. If the road length is set to 0 then the experiment will continue until the stopping condition is met and the length of the road will equal the distance travelled by the front car.

Road
- road_length: int - speed_limit: int - n_lanes: int - cars: std::vector<Car> - begin_lane_merge: float
+ update_car_decisions(): void + update_car_positions(): void + add_car(Car): void + get_cars(): std::vector<Car> + get_road_length(): int + get_speed_limit(): int + get_begin_lane_merge(): int

Figure 4: Road Class Diagram

2.6.4 Experiment Class

The main purpose of the Experiment class is to initialise the experiment and the road with the values described in the structure 'init_experiment' in Figure 6, and to contain the function that will loop for the desired number of iterations, updating the Cars' decisions and positions through calls to the Road.

Experiment
- road: Road - n_cars_per_lane: int - car_spacing: int - max_it: int - time_step: float
+ main_loop(): void

Figure 5: Experiment Class Diagram

The Experiment initialisation structure contains all the information necessary to create a new experiment. The main program takes these values as input and therefore new experiments can easily be run. For convenience of changing the experiment values the programme is called from a bash script.

Init_Experiment Structure
- velocity: float - max_velocity: float - accel_param: float - max_accel_param: float - min_accel_param: float - car_length: float - horizon: float - max_delta_turning_angle: float - road_length: int - n_lanes: int - begin_lane_merge: float - inlet_protocol: float - n_cars_per_lane: int - car_spacing: int - max_it: int - time_step: float

Figure 6: Experiment Class Diagram

2.6.5 Programs

The Concertina and Lane Merging experiment programs are implemented in two separate files, in each file the 'update_decision' method of the Car is implemented.

2.6.6 Visualisations

I have written a separate header file that contains two methods for visualisaing the data, 'write_space_time_Pbm' which draws a Space-Time diagram to a PBM file from the result of the Concertina experiment (which is converted after in the bash script to a png), and 'write_lane_changing_gif' which creates a Gif of the cars merging lanes from the result of the Lane Merging experiment.
The main purpose of these visualisations is to better understand the protocols that I have implemented and to ensure their validity.

3 Concertina Effect

3.1 Goal

The aim of this study of the concertina effect is to show that autonomous vehicles produce less of a concertina effect than humans.

3.2 Reaction times

3.2.1 Human v AV

We will investigate the simplest difference between a human and an autonomous vehicle, being their reaction time. A human has a non-zero reaction time, on average 0.25s TODO CITATION, whereas an autonomous vehicles reaction time is significantly lower, on average TODO CITATION, and therefore can be neglected. We will run the same experiment with a non-zero reaction time for the humans whilst maintaining 0 reaction times for the autonomous vehicles, varying the number of cars, and observe the difference.

3.2.2 Experiment Description

We assume the cars to be in the ideal motorway configuration at the beginning of the experiment, that is to say that the they are all travelling at the same speed and with the perfect safety distance between them.
The experiment begins when the first car brakes with maximum force for a short period, initiating a concertina. The first car will then accelerate back up to the speed limit, and the resultant time until all cars have returned to the speed limit will be measured. A greater concertina effect will result in the cars taking a longer time to return to the speed limit.
We must therefore develop strategies for the acceleration and braking of the cars.

3.2.3 Braking

Firstly, we will consider the braking of the vehicles. Braking will occur when a car gets too close to the car in front, i.e when the distance between the two cars is less that the required safety distance. The safety distance between two cars is defined as 2s in Switzerland TODO CITATION, and so can be calculated by multiplying the velocity [m/s] by 2. If the safety distance is violated then the car will brake according to the following strategy.

$$\gamma = \begin{cases} -\dfrac{v}{d} & \text{if } -\dfrac{v}{d} > \gamma_- \\ \gamma_-, & \text{otherwise} \end{cases} \tag{30}$$

where d is the distance to the car in front, and γ_- is the minimum values the acceleration parameter can take.
This rule ensures that the more a car violates the safety distance, the harder it brakes.

3.2.4 Acceleration

Secondly, we will consider the acceleration of the vehicles. A car will accelerate to the speed limit whenever it is not violating the safety distance between it and the car in front. We will adopt the rule given in [Cho17], adding a condition to set the acceleration parameter to 0 if the velocity is close enough to the target velocity. This condition is added to keep the time taken for the acceleration parameter to converge to 0 reasonable, and to avoid issues arising in the implementation when $\gamma \to 0 (\implies v \to v_{target})$ in which the value of $1 - e^{(-\gamma_+(t-t_0))}$ in (10)

becomes 0 due to the finite precision of the machine and results in the position being updated as if the car were travelling at maximum velocity.
The acceleration parameter is thus updated as follows when accelerating.

$$\gamma = \begin{cases} \left(1 - \frac{v}{v_{target}}\right) v_+^*, & \text{if abs}(v - v_t arget) < 0.1 \\ 0, & \text{otherwise} \end{cases} \tag{31}$$

where 0.1 chosen as an arbitrary small enough bound. This results in a smooth acceleration up to the speed limit as the car accelerates less and less strongly as it approaches the target velocity.

3.2.5 Reaction Time

The reaction time will be implemented through inserting the acceleration parameter decisions into a buffer, the buffer being equal to the number of iterations of the experiment that correspond to the reaction time of the car. This means that a car will make a decision, and that the decision will only be taken into effect after the reaction time of the car has elapsed, as is the case in a real world scenario.

3.2.6 Validation

I ran a number of experiments to ensure that the strategies were valid. Shown in Figure 3.2.6 is an example space time diagram of an experiment with 20 cars and no reaction time.

Figure 7: Example Space-Time Diagram

The concertina effect is clearly visible in the diagram.

3.2.7 Results

To begin with, I modelled the humans as all having a fixed reaction time of the average, being 0.25, and varied the number of cars. I plotted this against the model of autonomous vehicles where the reaction time was 0. The results can be seen in Figure 3.2.7.

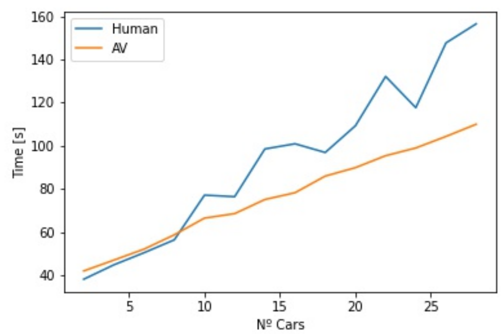

Figure 8: Concertina: Human Reaction Time = 0.25

The slope of the Human times is 4.55 and the slop of the AV times is 2.61. This indicates that the autonomous vehicles are in this case on average 1.74 times faster than the humans, which represents a significant improvement. We can also see that the effect of the reaction times is more noticeable when there are more cars involved, as the concertina can propagate backwards further and have more effect.
The human model was however not perfect as it did not take into account the random variation in the reaction times of humans. I therefore redid the experiment but initialised the reaction times to be randomly chosen from the interval $[1.5, 3.5]$, and for each number of cars I repeated the experiment 5 times and took an average of the times to average out the random initialisation of the reaction times. The results of this experiment are shown in Figure 3.2.7.

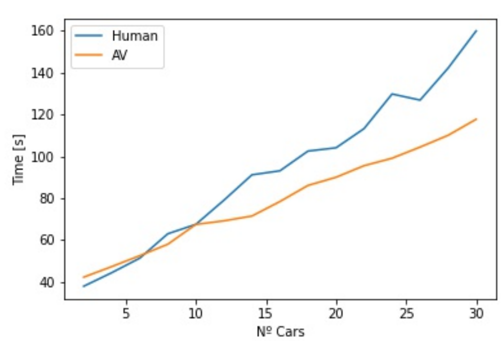

Figure 9: Concertina: Human Reaction Time in $[1.5, 3.5]$

In this case the slopes indicate that the autonomous vehicles are on average 1.62 times faster than the humans.

3.2.8 Conclusions

We have been able to show in this very simple experiment that autonomous vehicles produce less concertina effect than humans under the simple assumption that they have a reaction time of 0. In actuality, the autonomous vehicles may be even better than this simple experiment has shown, as the braking strategy used here for the humans is in all probability far more optimal that an actual braking strategy of a human, and does not take into account the differences between the humans' braking strategies. An interesting extension would therefore be to repeat the same experiment assuming 0 reaction time but with two different braking strategies, and repeat the experiment a third time combining the sub-optimal braking strategy with the non zero reaction times in the human model.

3.2.9 Issues and Improvements

An issue with the experiment above arises from my choice of reaction times when varying the reaction times for the humans. I should sampled taken reaction times following a normal distribution rather than a uniform and should have taken the mean and standard deviation of this gaussian from a study of human reaction times. It however is safe to assume that the results would not be significantly different.

4 Lane Merging

4.1 Goal

The aim of this study of lane merging is to show that autonomous vehicles have the potential to merge lanes more efficiently than humans do.

4.2 Protocols

4.2.1 Experiment Description

We assume the cars to be in the ideal motorway configuration at the beginning of the experiment, that is to say that the they are all travelling at the same speed and with the perfect safety distance between them.
The experiment begins once the first car crosses the boundary at which lane merging may begin. The time taken until all cars are in lane 1 and are travelling at the speed limit will be measured.

4.2.2 Human v AV

The difference between the humans and the autonomous vehicles will be their merging protocol, and their reaction times. The humans merging protocol will attempt to model a real world situation, and will therefore include a certain randomness and difference between the decisions taken by the humans, and will also include a non-zero reaction time, as humans have an average reaction time of 0.25s TODO CITATION. The autonomous vehicles protocol will be more rigid and will have a reaction time of 0 as the compute time is negligible when compared to the reaction time of humans. We will run the same experiment with the two different protocols, for different numbers of cars, and observe the difference.

4.2.3 AV Protocol

The idea for the autonomous vehicles protocol is for all the vehicles to slow down to a slower speed, with the throughput of the road with both lanes equal when at the slower speed to the throughput of the single lane when at the speed limit. The cars will then accelerate and merge from the front of the queue.
We can see this protocol in the Figure 4.2.3. At stage 2, once the vehicles have begun the merge (the vertical line represents the beginning of the lane merge), they slow down to the slower speed. In stage 3, the first car in the lane to merge to (lane 1) accelerates away, the car in lane 0 observes the distance to the car ahead, and begins merging once the safety distance is respected as we can see in 4. The cars behind repeat the same process as we can see in stages 4 and 5.

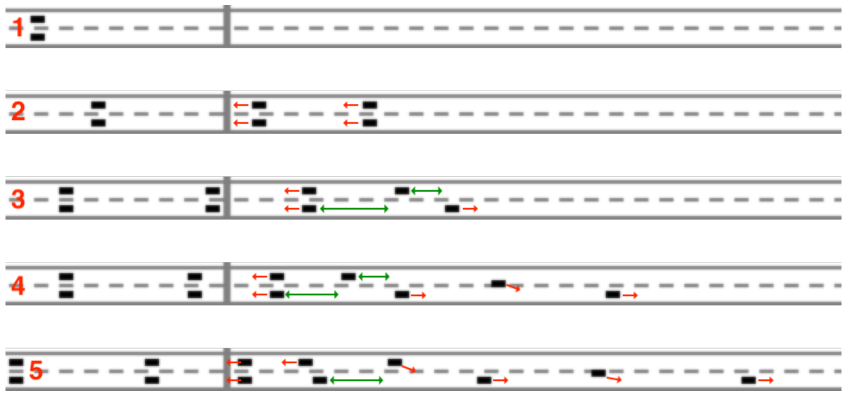

Figure 10: AV Lane Merging Protocol

This protocol can be more formally expressed in pseudo-code as shown in Algorithm 2.

Algorithm 1: AV Lane Merging Protocol Pseudo-Code

```
did_slow = false;
merging = false;
merged = false;
accelerating = false;
if not did_slow then
    slow();
    if at target slow velocity then
        did_slow = true;
else
    if starting lane == 0 then
        if not merged and not merging and safety distance
          respected then
            merging = true;
        if merging then
            merge();
            if in lane 1 then
                merged = true;
                merging = false;
        if merging or merged then
            accelerate();
    else if starting lane == 1 then
        if not accelerating and safety distance respected then
            accelerating = true;
        if accelerating then
            accelerate();
end
```

where slow() is a function that updates the acceleration parameter to slow the car down to the target slower velocity, merge() is a function that updates the turning angle, accelerate() is a

function that updates the acceleration parameter respecting the safety distance, and lane 0 and 1 are respectively the top and bottom lanes.
Note that this is the protocol respected once the cars have passed the beginning of the lane merge, before the beginning of the lane merge the cars simply attempt to keep to the speed limit whilst respecting the safety distance.

4.2.4 Human Protocol

The idea for the human protocol is for all the cars in lane 0 the lane to merge from to begin merging as soon as possible, i.e when there is space. It is up to the cars in lane 1 to allow the space. As in real life, the cars in lane 1 will be reluctant to allow space, and so will wait a random time from 1-4s before slowing down to allow the car next to them to merge. The cars in lane 1 will always allow the car next to them to merge, and the cars in lane 0 will always stay behind the car ahead in lane 1, for ease of implementation.
We can see this protocol in the Figure 4.2.4. At stage 2, once the car in lane 1 has waited it's random time, it will slow down to allow space for the car in lane 0. In stage 3 the car in lane 0 notices that there is space to merge and so merges, and the car that allowed it to merge begins accelerating again, maintaining the safety distance. In stage 4 the same process is repeated with the cars behind, as in stage 5.

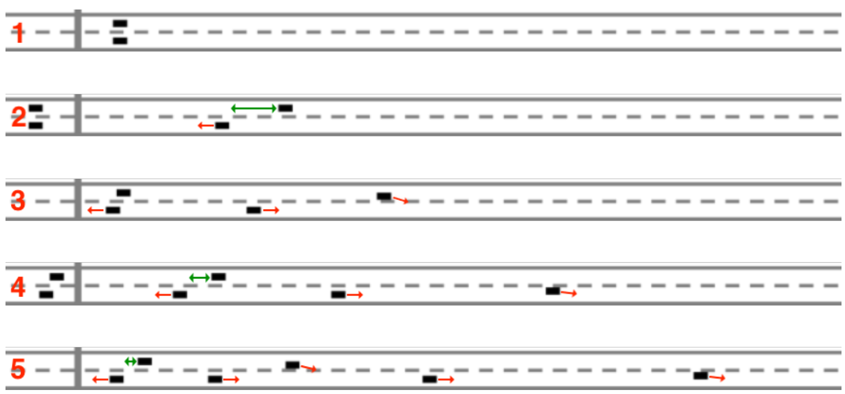

Figure 11: Human Lane Merging Protocol

Algorithm 2: AV Lane Merging Protocol Pseudo-Code

```
did_slow = false;
merging = false;
merged = false;
wait = 0;
waited = false;
if before begin merging then
    match the speed of the car ahead in lane 1;
else
    if starting lane == 0 then
        if not merged and not merging and space to merge to
         then
            merging = true;
        if merging then
            merge();
            if in lane 1 then
                merged = true;
                merging = false;
        if merging or merged then
            accelerate();
        else
            match the speed of the car ahead in lane 1;
        end
    else if starting lane == 1 then
        if car to allow in from lane 0 hasn't yet begun merging
         then
            if not wait == 0 then
                wait–;
                if wait == 0 then
                    waited = 0;
                accelerate();
            else if wait == 0 and not waited then
                wait = random(upper: 1, lower: 4); accelerate();
            else if wait == 0 and waited then
                slow();
        else
            accelerate();
        end
end
```

where slow() is a function that updates the acceleration parameter to slow the car down to allow space for the car beside to merge, merge() is a function that updates the turning angle, accelerate() is a function that updates the acceleration parameter respecting the safety distance, and lane 0 and 1 are respectively the top and bottom lanes.

Note that this is the protocol respected once the cars have passed the beginning of the lane merge, before the beginning of the lane merge the cars simply attempt to keep to the speed limit whilst respecting the safety distance.

Also note that accelerate() is called whilst waiting to ensure the safety distance with the car ahead before allowing space for the car beside to merge.

4.2.5 Results

To begin with, I modelled the humans as all having a fixed reaction time of the average, being 0.25, and varied the number of cars. The waiting time before a car allows the car next to it to merge is a random number between 1 and 4 seconds. I plotted this against the model of autonomous vehicles where the reaction time was 0. The results can be seen in Figure 4.2.5.

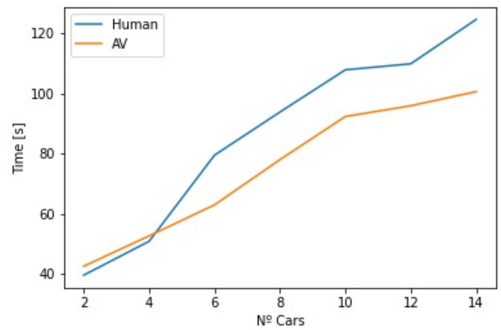

Figure 12: Lane Merge: Human Reaction Time = 0.25

The slope of the Human times is 7.07 and the slop of the AV times is 4.83. This indicates that the autonomous vehicles are in this case on average 1.46, a significant improvement when factoring in the amount of time this could potentially save.
To take into account the random variation in the reaction times of humans, I redid the experiment but initialised the reaction times to be randomly chosen from the interval $[1.5, 3.5]$, and for each number of cars I repeated the experiment 5 times and took an average of the times to average out the random initialisation of the reaction times. The results of this experiment are shown in Figure 4.2.5.

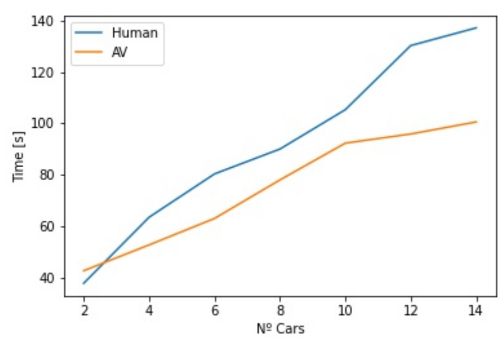

Figure 13: Lane Merge: Human Reaction Time in $[1.5, 3.5]$

In this case the slopes indicate that the autonomous vehicles are on average 1.72 times faster than the humans.

4.2.6 Conclusions

Again we have been able to show that the autonomous vehicles are better at lane merging than humans, under some simple assumptions about the protocol that humans follow. Again, as in the concertina experiment, the real world result might favour the autonomous vehicles even more as the protocol for the humans is in all probability too rigid, that is to say that in the real world humans would not be so precise in their decision, would often not let the car next to them in (I know this from experience), and there would be more random variation. This coupled with the fact that human braking is worse than what I modelled, and that I took a conservative estimate for the reaction times means that the actual case may be even worse for humans.
Considering however that I varied the protocol and the reaction times for the humans, this experiment has not made it clear which aspect had the largest effect on the time taken to lane merge. An interesting extension to this study would therefore to be to vary only one at a time and to determine whether it is simply the reaction times producing more concertina effect that reduces the time to merge or whether it is the protocol that the cars follow, or a combination of both.

4.2.7 Issues and Improvements

There are several improvements that may be interesting to study.
Firstly the protocol for the autonomous vehicles is sub-optimal. It would have been good to investigate in more detail the speed to which the autonomous vehicles slow down to and to optimise it with a mathematical argument showing that the throughput before and after slowing down remains constant with the optimal slow speed.
Secondly as discussed above, the human protocol is too optimal. Human's don't follow such a rigid protocol as was implemented, they aren't so courteous as in the protocol often waiting longer than what was implemented. I chose however to implement a reasonable set of humans to show that even in the case of humans driving sensibly, the autonomous vehicles will still provide a significant improvement.
Again as discussed above, another improvement would be to investigate in more detail the source of the reduced merging time.

5 Conclusion

In conclusion, we have managed to show that the hope of ability of autonomous vehicles to reduce congestion is not misplaced. With conservative models of humans, that is to say with human models that drive far more optimally than real humans would, we have still shown a significant decrease in the time taken to resolve the concertina effect and the time taken to merge two lanes. We have seen that the reaction time of humans is an important contributing factor to their sub-optimality, along with the protocol that they follow (or lack there-of).
We can hope therefore, that as autonomous vehicles become ever more present on our road systems, we will spend less and less time sitting in traffic, which is a very exciting prospect.

5.1 Possible Further Investigation

A possible area of further investigation would be, as discussed previously, to study the lane merging in more detail to attempt to determine the main source of the increased time to merge of the humans, if their is one. This would allow us to better understand the benefits of autonomous vehicles.
Another interesting and important area of investigation would be to perform the experiments of this thesis with both autonomous vehicles and humans drivers on the same road, in varying proportions. This would allow us to better understand the coming transition to autonomous vehicles, and to see whether this transition would produce early fruit in the form of congestion reduction when there are only a few autonomous vehicles on the road, or whether we must wait until the majority of cars on the road are autonomous vehicles before the traffic reduction becomes noticeable.

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