Maths for Machine learning

Linear Algebra - Chapter 2

2 intro, 2.1 Sys of lin eq, 2.2 Matrices

types of vector objects:

- Geometric vectors. Arrows
- Polynomials.
- Elements of $\mathbb{R}^n$

Matrix

Matrix A is an $m \cdot n$ tuple of elements a_{ij}, $i = 1, \dots, m$ $j = 1, \dots, n$ ordered as m rows & n columns

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}, a_{ji} \in \mathbb{R}$$

by convention:
- $(1, n)$ = rows
- $(m, 1)$ = Cols

Also called row column vectors

Matrix Addition & Multiplication

Addition is performed elementwise:
$A \in \mathbb{R}^{m \times n}$, $B \in \mathbb{R}^{m \times n}$

$$A + B := \begin{bmatrix} a_{11} + b_{11} & \cdots & a_{1n} + b_{1n} \\ \vdots & \vdots & \vdots \\ a_{m1} + b_{m1} & \cdots & a_{mn} + b_{mn} \end{bmatrix}$$

Multiplication is permormed as a dot product

$$\underbrace{A}_{n \times k} \underbrace{B}_{k \times m} = \underbrace{C}_{n \times m}$$

A rows multiplie by B cols

the neighbours (k) must match

Properties of matricies:

Associativity: group numbers in different orders for the same outcome

$\forall A \in \mathbb{R}^{m \times n}, B \in \mathbb{R}^{n \times p}, C \in \mathbb{R}^{p \times q}$:

$(AB)C = A(BC)$

Distributivity:

- Describes how an operation interacts with another operation
- Matrix multiplication is distributive over addition

$(A + B)C = AC + BC$

$A(C + D) = AC + AD$

Multiplication w/ indentity

$\forall A \in \mathbb{R}^{m \times n}$:

$I_m A = A I_n = A$

Note order LHS$_m$ or RHS$_n$ matters

LHS for row match identity

RHS for col match identity

As neighbour must match

2.2.2 inverse & transpose

Inverse

$AB = I_n = BA$

B is the inverse of A & denoted as A^{-1}

Not every matrix posses an inverse

if one does exist the matrix A is called regular/invertible/non-singular

otherwise ~~sig~~ singular or non-invertible

when an inverse exists it is unique

Transpose

A^T

write columns as rows