Maths for ML - C3.1 - Norm

A norm on a vector space V is a function

$$\|\cdot\| : V \to \mathbb{R}$$

A norm translates a vector space to a number

3 conditions of Norms:

$\lambda \in \mathbb{R} \quad x, y \in V$

Absolute homogeneous: $\|\lambda x\| = |\lambda| \|x\|$

if you multi a vector by a scalar the resulting scalar is the same as the absolute value of the scalar $|\lambda|$ times the norm of the original vector $\|x\|$

Triangle inequality: $\|x + y\| \leq \|x\| + \|y\|$

the norm of two vectors must be less or equal to the sum of the norms of indivial norms combined

Intuition is that $\|x + y\|$ represent the 3rd line of a triangle which cannot be longer than the sum of lengths of the other 2 $\|x\| + \|y\|$

Positive Definite: $\|x\| \geq 0$ & $\|x\| = 0 \leftrightarrow x = 0$

L_1 Norm $= \|x\|_1 := \sum_{i=1}^{n} |x_i|$

• where $|\cdot| =$ absolute value

L_2 Norm $=$ Euclidean

$\|x\|_2 = \sqrt{\sum x_i^2} = \sqrt{x^T x}$

- Computed as distance from origin