Adaptive Systems

Lecture 3.2: Negative feedback & control

University of Sussex

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Spring 2026

Contacting me

Email

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Canvas discussions

- Everyone can see my answers
- I will normally check these at least twice a week

My office hours

- TBD

Lecture learning outcomes

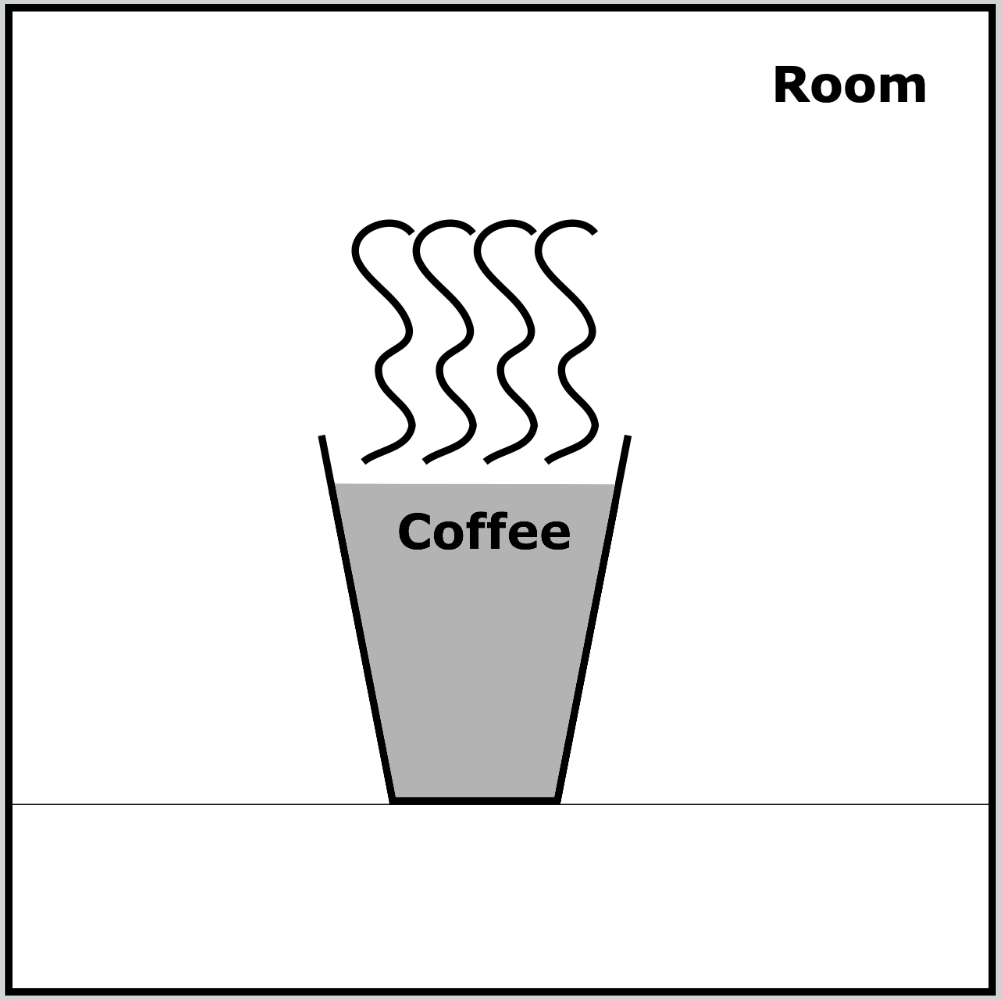
Main points:

- The basic principle of negative feedback control
- Negative feedback leads to stability
- Stability without control
- Delays in feedback loops

[1]

Lecture outline

1. Metaphors and analogies
2. Stable and unstable equilibria
3. A short introduction to negative feedback control
4. Negative feedback tends to lead to stability
5. Delays in feedback loops can cause problems
6. Some systems are stable without control
7. Some examples of systems which use negative feedback



Metaphors, analogies and models

Cybernetics: metaphors, or analogies?

metaphor

noun [C or U]

UK /ˈmet.ə.fɔːʳ/ **US** /ˈmet̬.ə.fɔːr/

Add to word list

C2

an expression, often found in literature, that describes a person or object by referring to something that is considered to have similar characteristics to that person or object:

- *"The mind is an ocean" and "the city is a jungle" are both metaphors.*
- *Metaphor and simile are the most commonly used figures of speech in everyday language.*

analogy

noun [C or U]

UK /əˈnæl.ə.dʒi/ **US** /əˈnæl.ə.dʒi/

Add to word list

C2

a comparison between things that have similar features, often used to help explain a principle or idea:

- *He **drew** an analogy **between** the brain and a vast computer.*
- *It is sometimes easier to illustrate an abstract concept **by** analogy **with** (= by comparing it with) something concrete.*

https://dictionary.cambridge.org/dictionary/english/

Cybernetics: metaphors, or analogies?

- Wiener called Cybernetics "Control and Communication in the Animal and the Machine"

- Mary Catherine Bateson put it more poetically:

Cybernetics makes poets out of us,

It teaches us to think in metaphors,

So that we discover how a city is like a lake,

How a university is like a forest,

And how a religion is like a body.

(Mary Catherine Bateson, quoted by her husband)

https://www.youtube.com/watch?v=jNCnkAPNiwA&t=3149s

Cybernetics: metaphors, or analogies?

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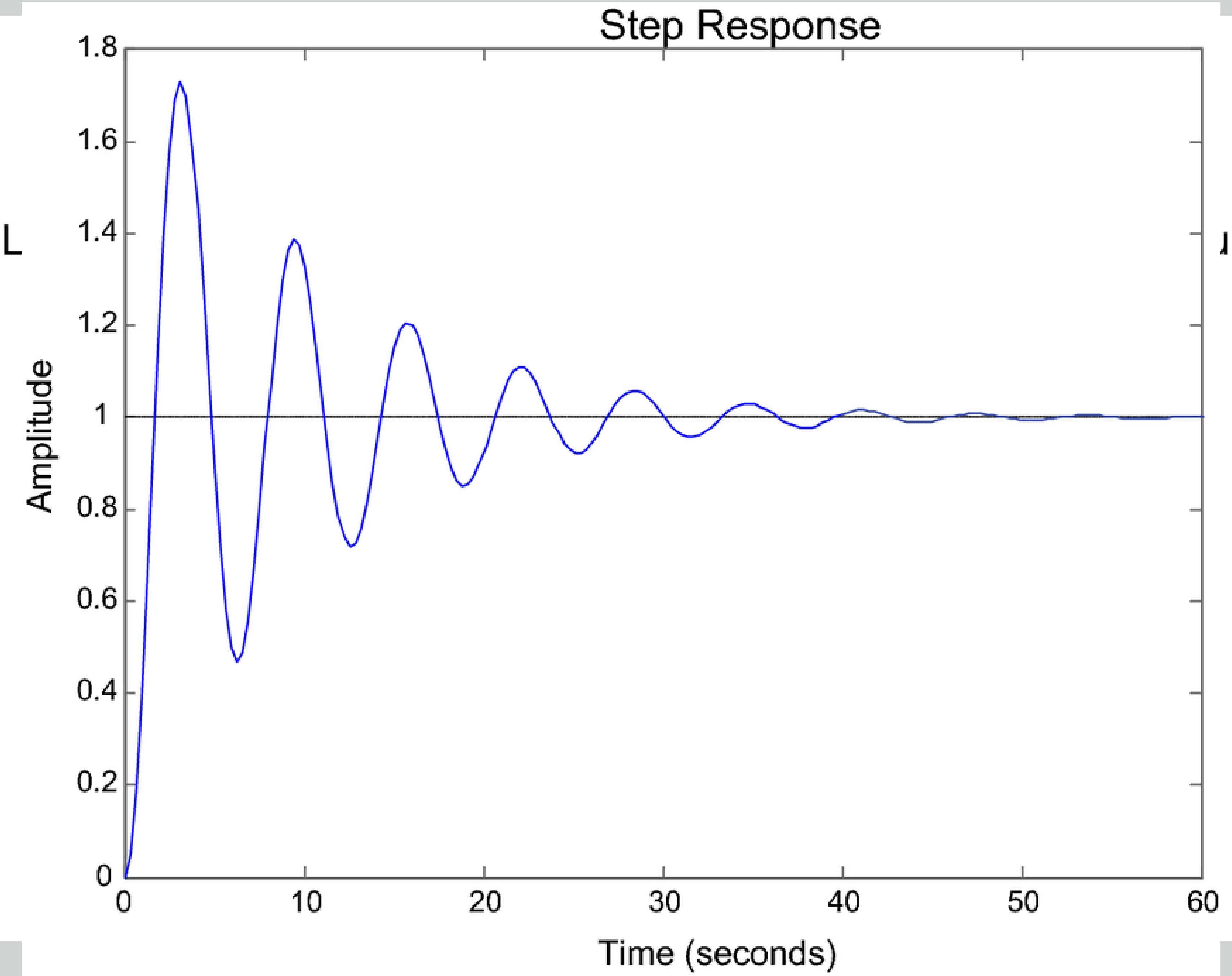

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[8]

$$m\ddot{x} = u - kx - d\dot{x}$$

$$L\ddot{q} = V - \frac{1}{C}q - R\dot{q}$$

- **These two systems are isomorphic**

Cybernetics: metaphors, or analogies?

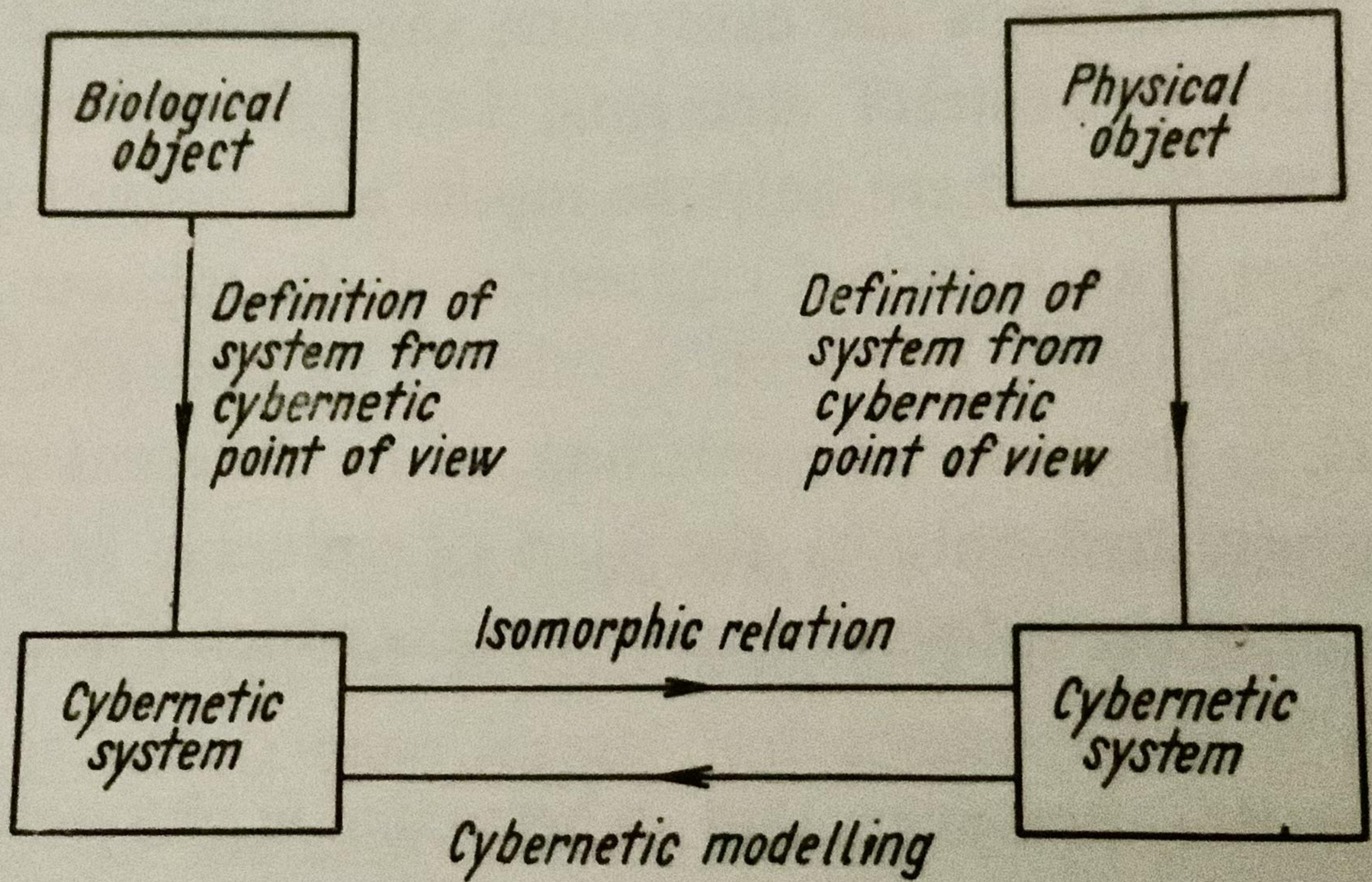
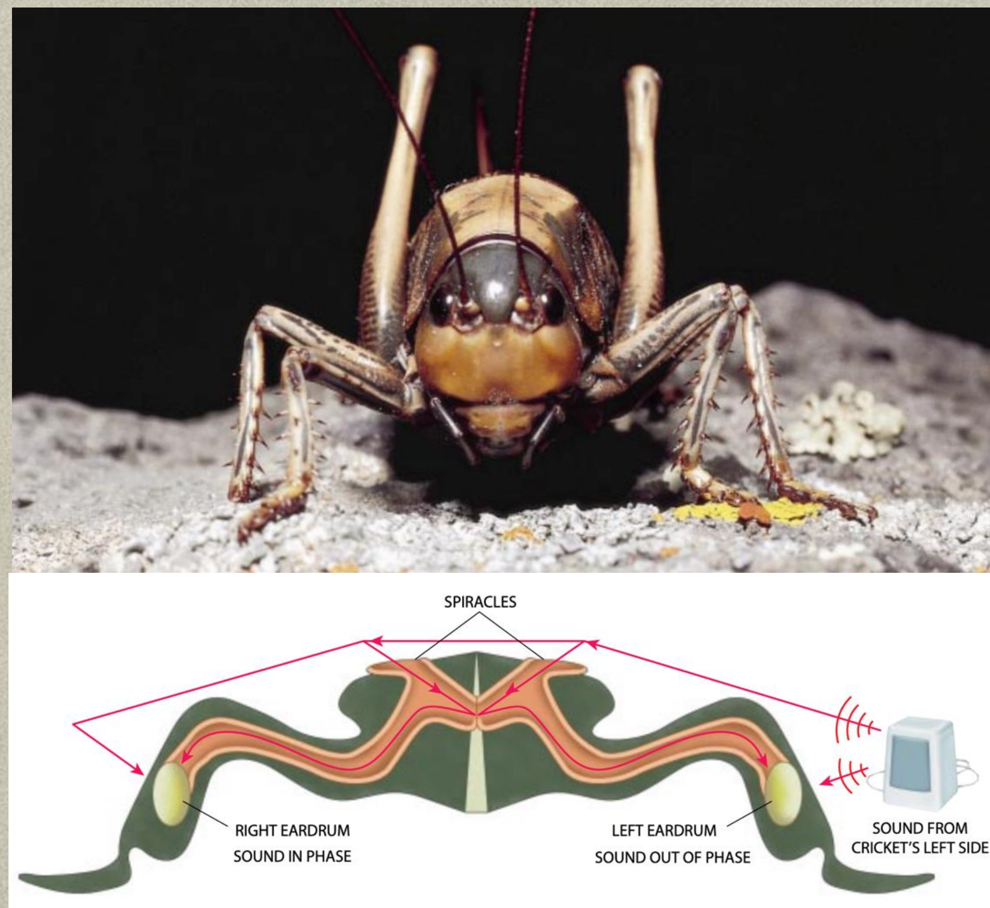


Fig. 5.7. Principles of cybernetic modelling

[9]

Cybernetics: metaphors, or analogies?



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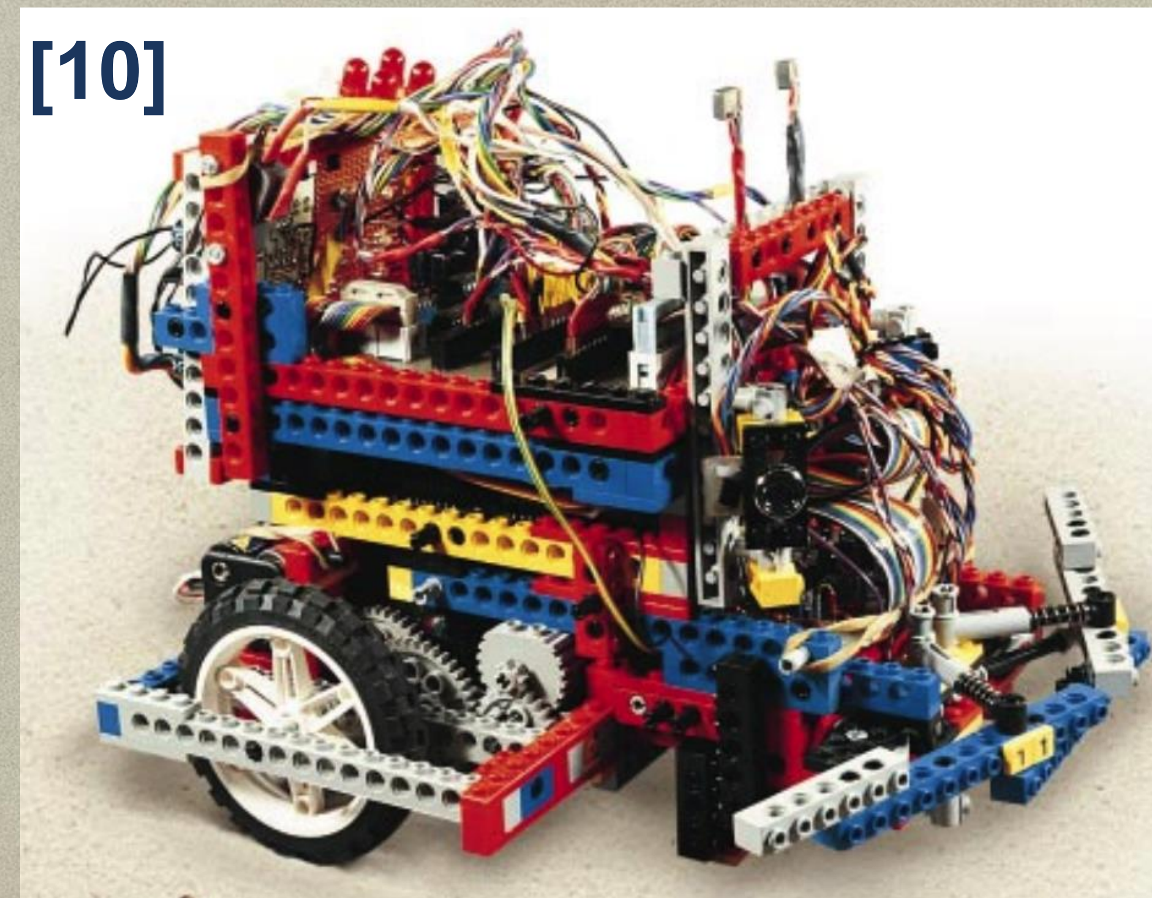

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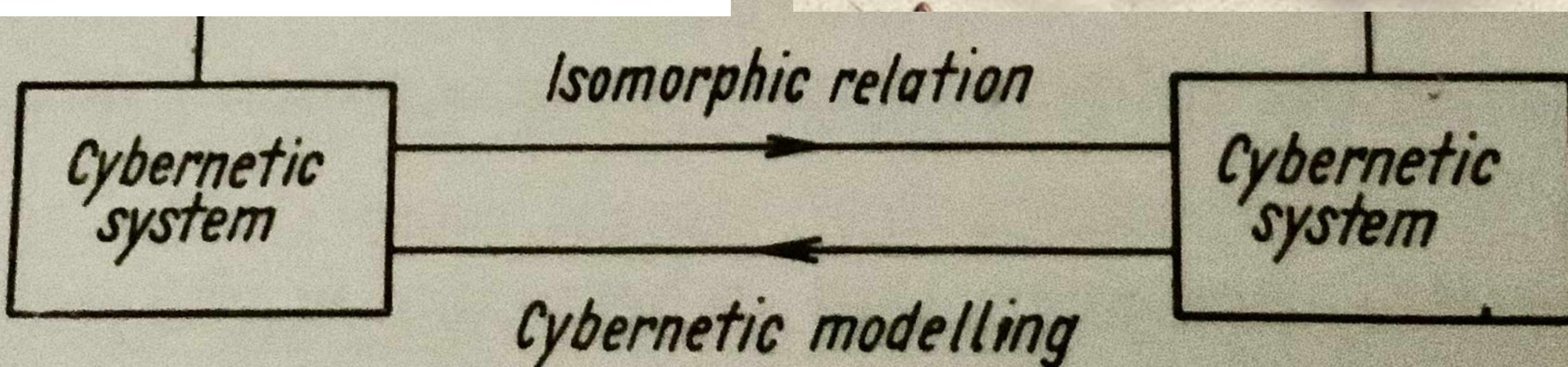
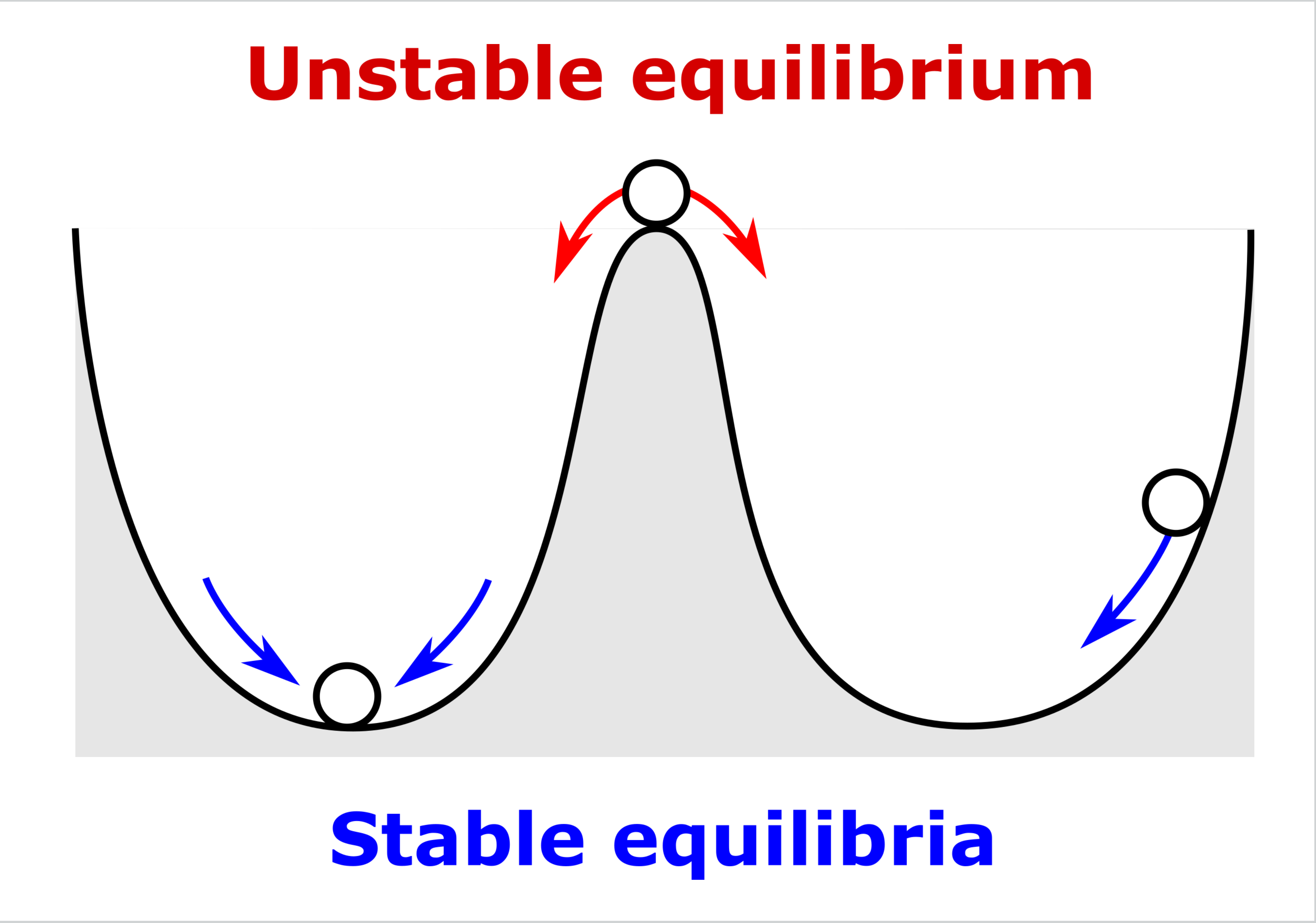


Fig. 5.7. Principles of cybernetic modelling

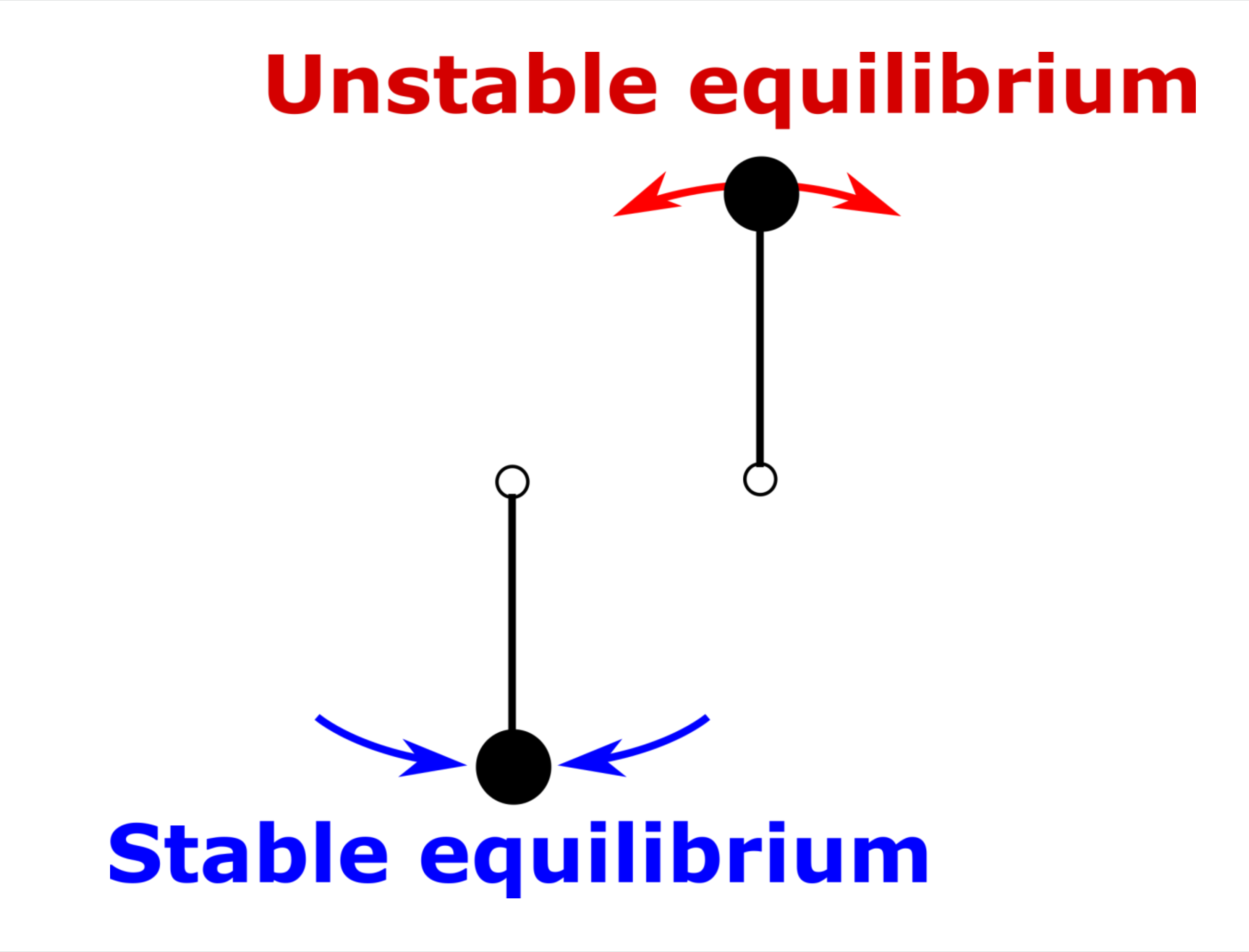
Stability and control

Stable and unstable equilibria



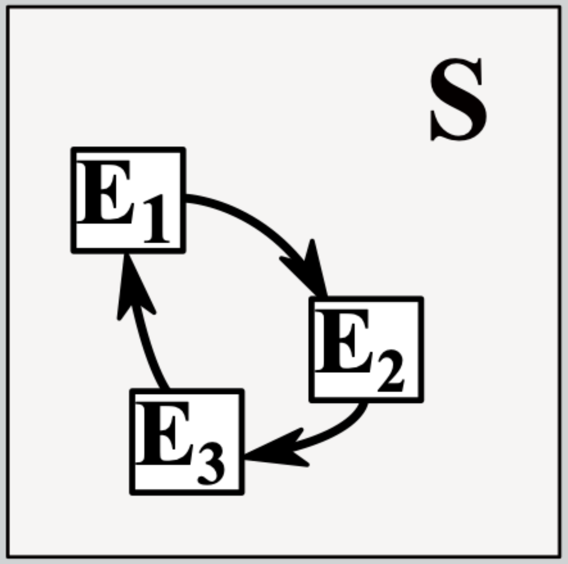
Equilibria - the pendulum

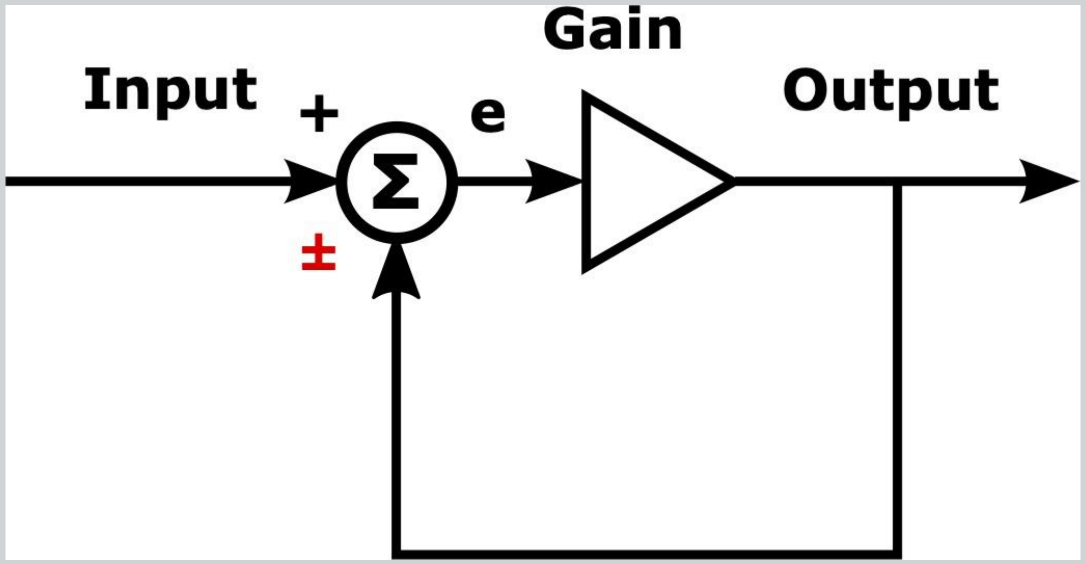
- A pendulum has a stable and an unstable equilibrium
- When the pendulum hangs straight down, it is at a stable equilibrium
- When it is perfectly balanced pointing upwards (or inverted), it is at an unstable equilibrium
- The unstable equilibrium of the inverted pendulum can be made stable by using negative feedback control (which is roughly how humans stand up straight)

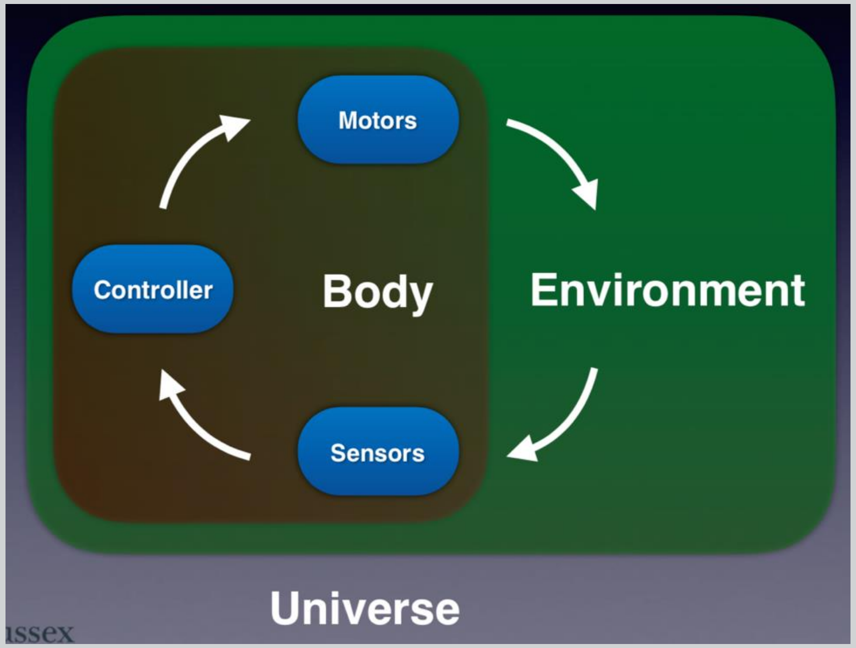


Cybernetic loops

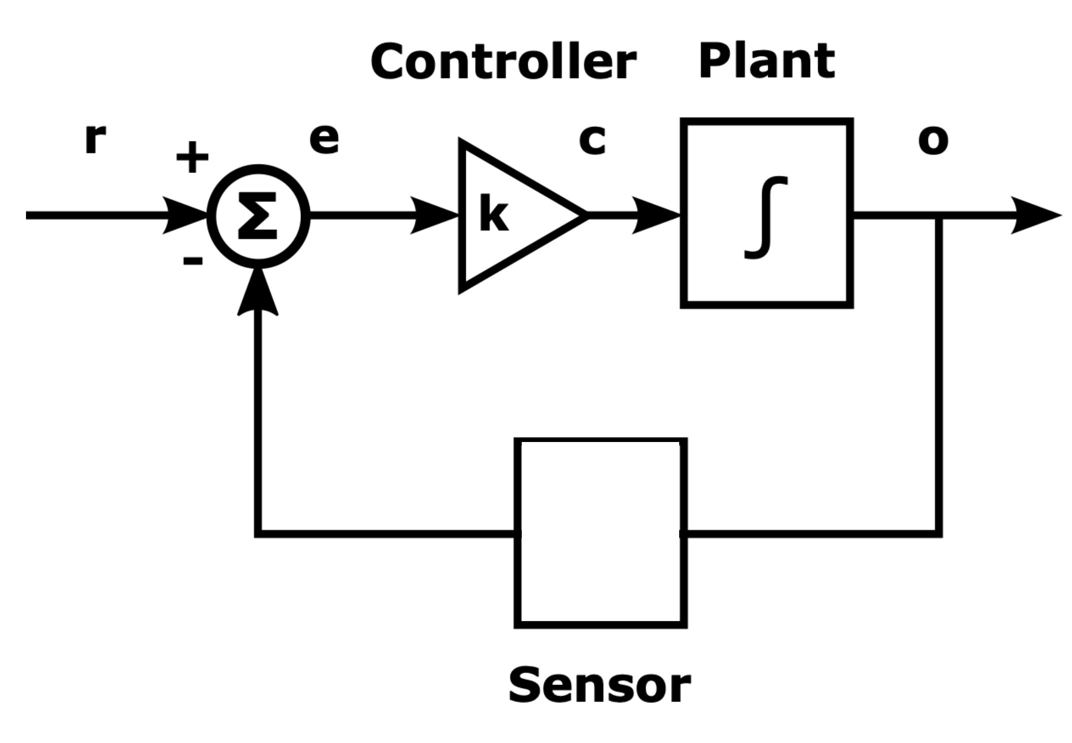
- Even before cybernetics had a name, Wiener and his collaborators had pointed out the importance of feedback
  - In systems with feedback, elements are connected in a loop
  - The cyberneticians referred to this as circular causality







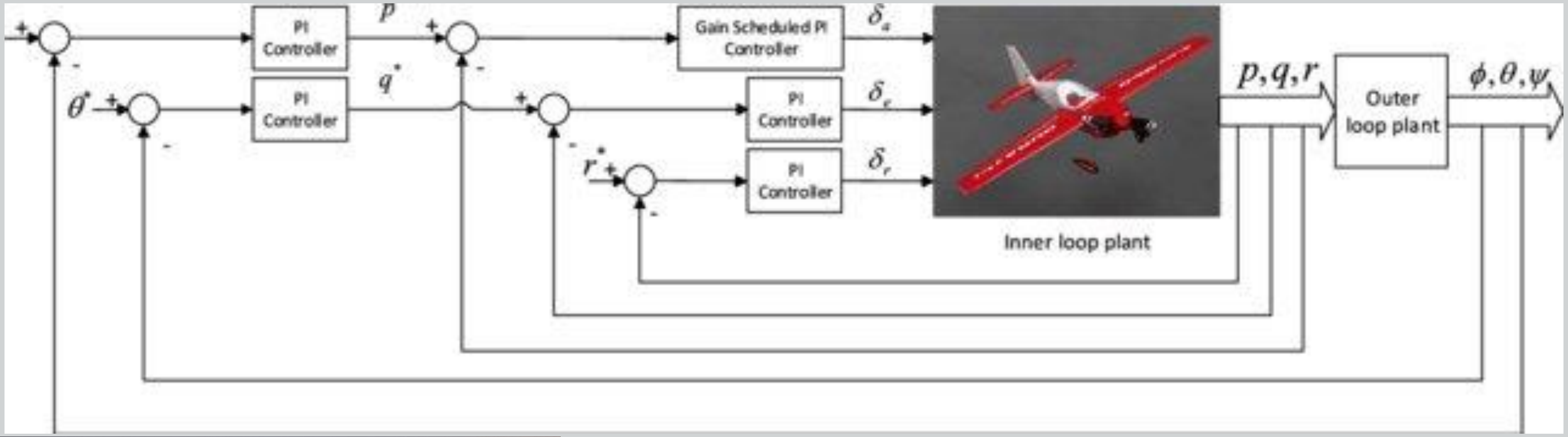
Negative feedback control



- The controlled system or process is often referred to as the "plant"
- The output from the plant is the controlled variable - this could be many things, e.g. the position of a robot arm, the production rate of a factory, the heading of a ship
- The reference signal, r, specifies the desired plant output, o (i.e. the goal of the system is to make o equal to r)
  - r can be a time-varying signal
- The actual output of the plant is *fed back* to the controller, so that the controller acts on the error, e, which is the difference between the desired and actual outputs

Negative feedback control

These are just a few examples of the many systems which require negative feedback loops to function correctly:



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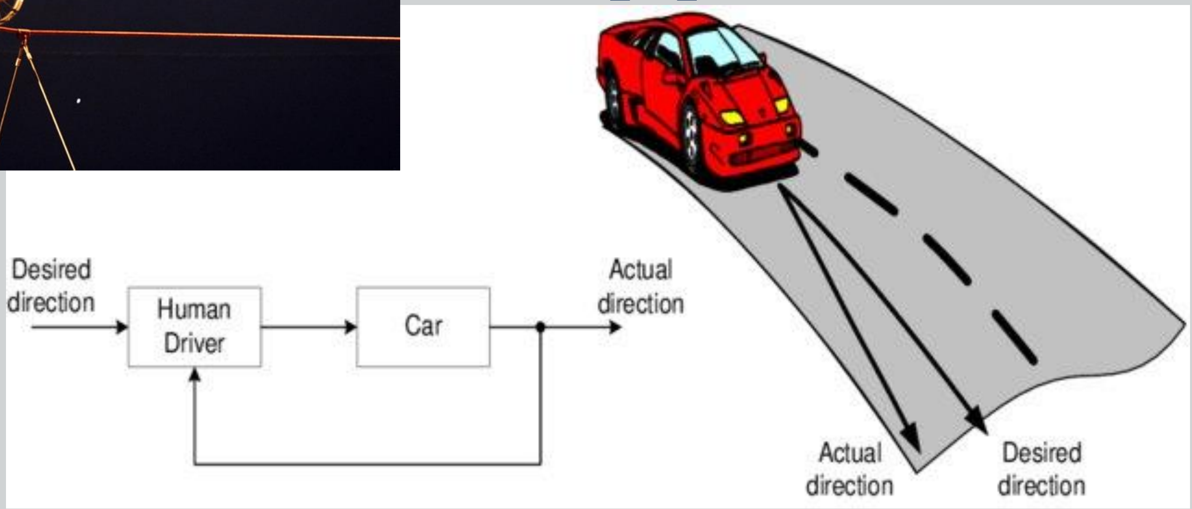

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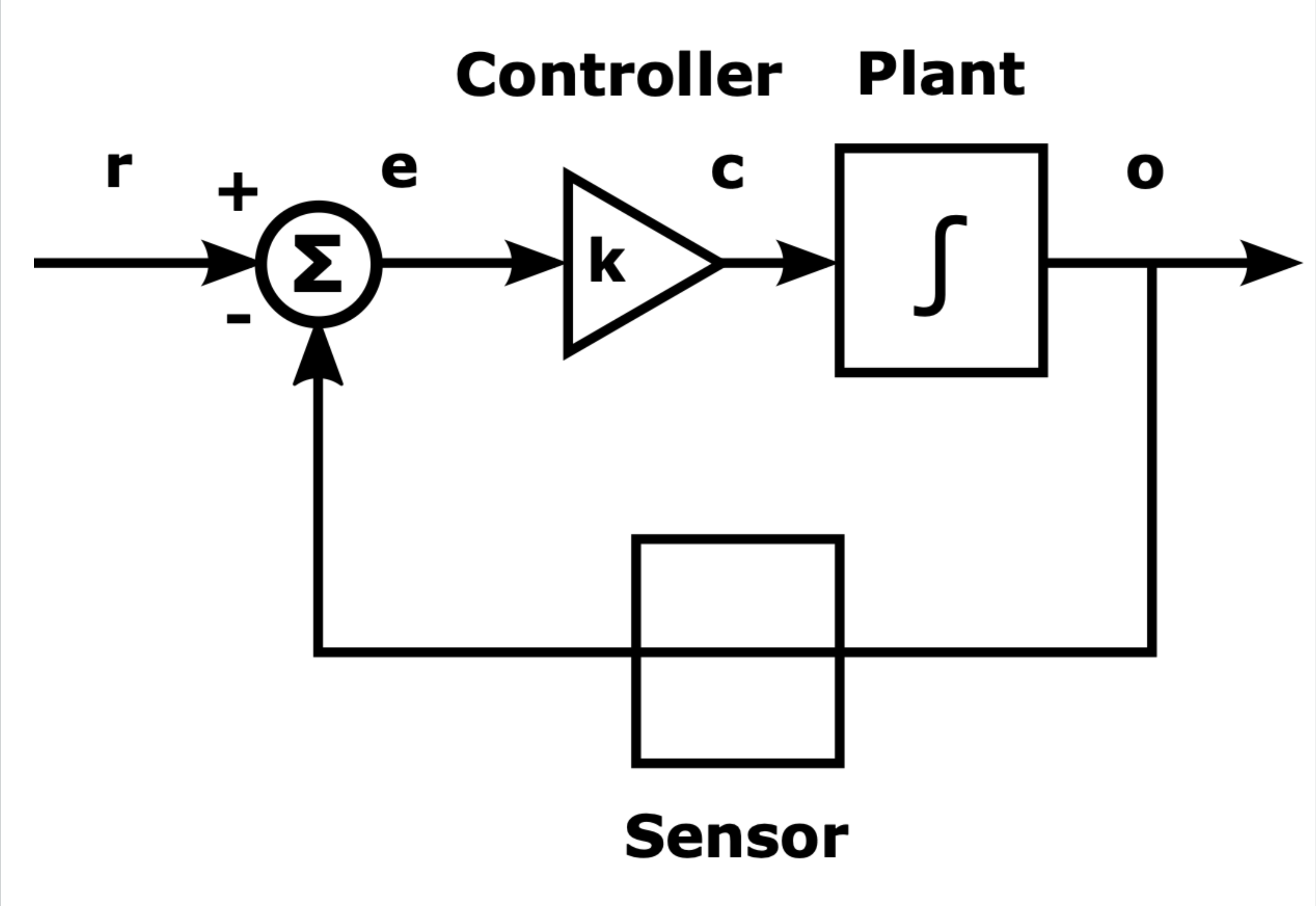
A simple feedback control example

An “on-paper” example:

- The plant (controlled system) is an *integrator*
  - i.e. the more we input to it, the more its output moves



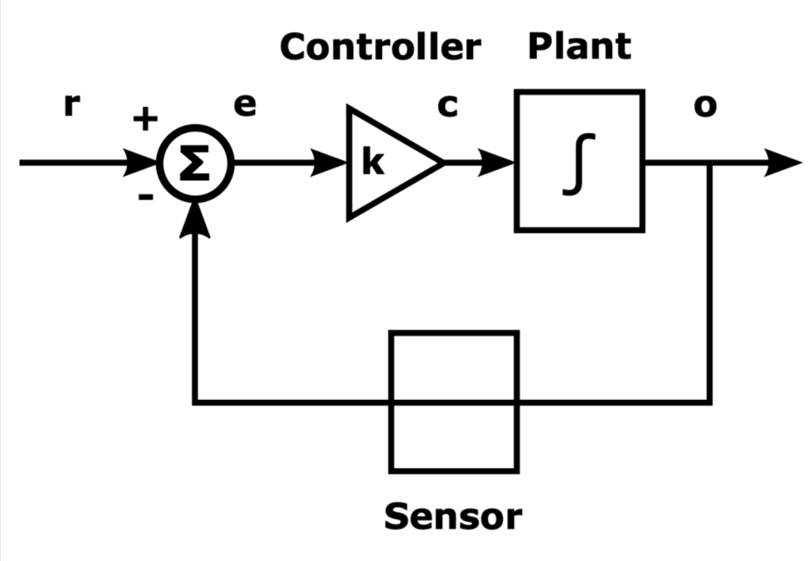
    - e.g. the plant could be a robot joint driven by a motor - the longer we supply energy (torque) to the motor, the more it turns - therefore its position is related to the integral of torque input

- The controller is a simple gain or amplifier, which we can call a *proportional* controller - it amplifies (or attenuates) the error signal

A simple feedback control example

An "on-paper" example:

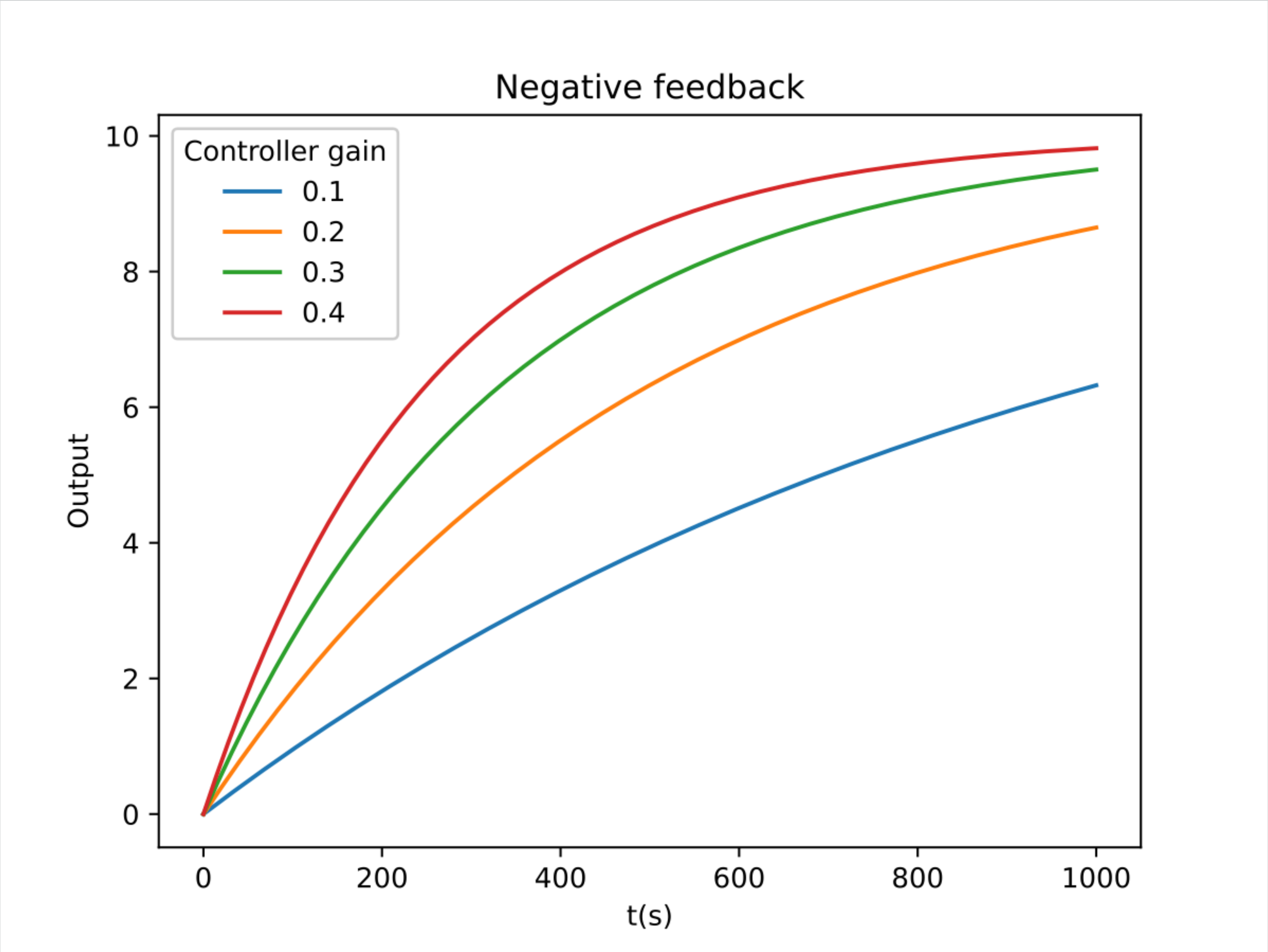
- We will assume a constant input (reference signal), $r = 10$, and a controller gain of $k = 0.1$
- At time $t = 0$, the output $o = 0$
- In every time step, we calculate:
  - $e = r - o$
  - $c = ke$
  - $o_{t+1} = o_t + c_t$



- After some n number of steps, (possibly large)
  - $o \approx r$
  - $e \approx c \approx 0$

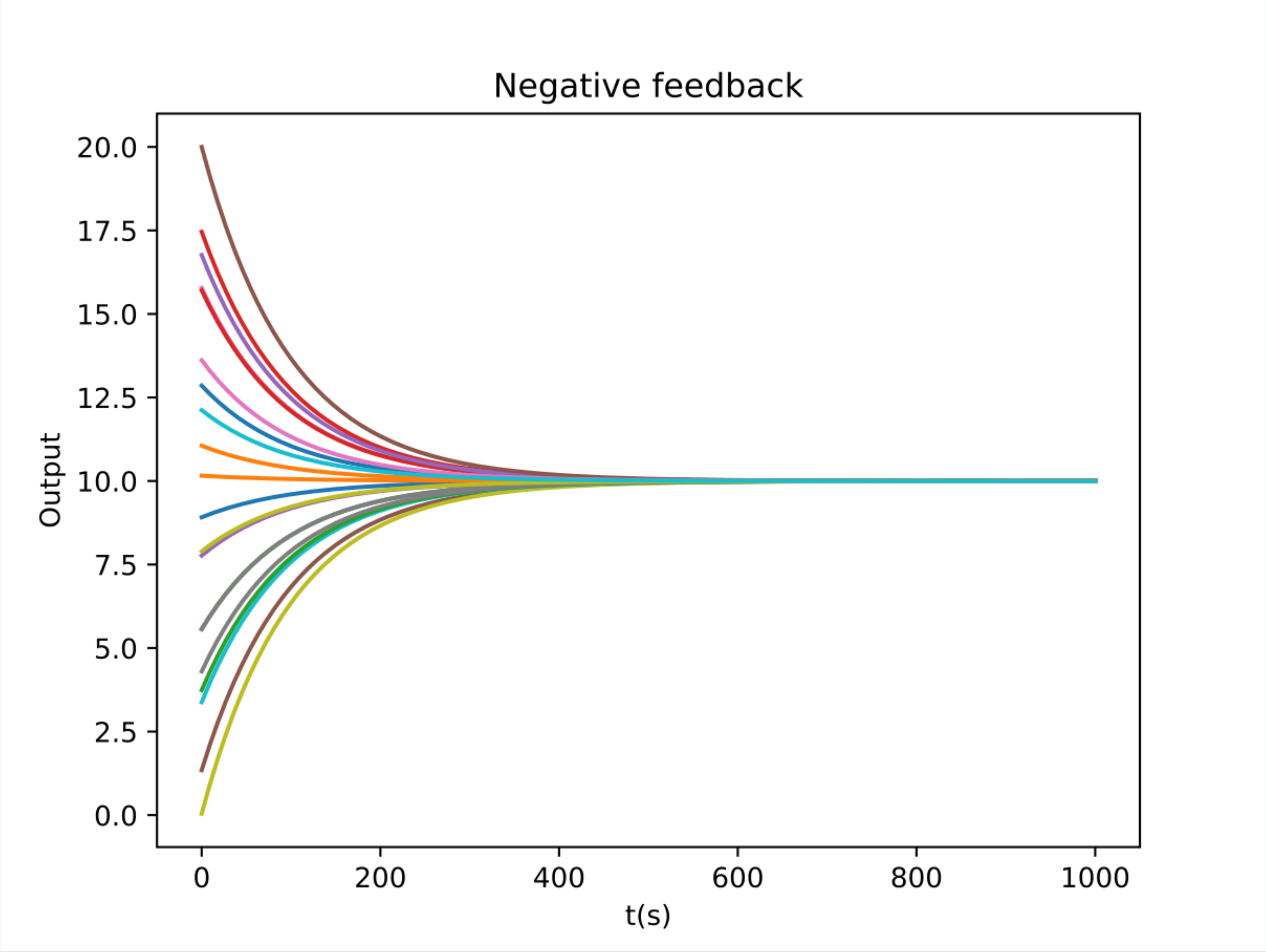
t	o	e	c
0	0	10	1
1	1	9	0.9
2	1.9	8.1	0.81
3	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
n	~10	~0	~0

Varying parameters - gain



- **Every one of these curves is exponential**
- **They all eventually converge on the reference,** $r = 10$
- **Changing the gain only changes the time to stabilise**

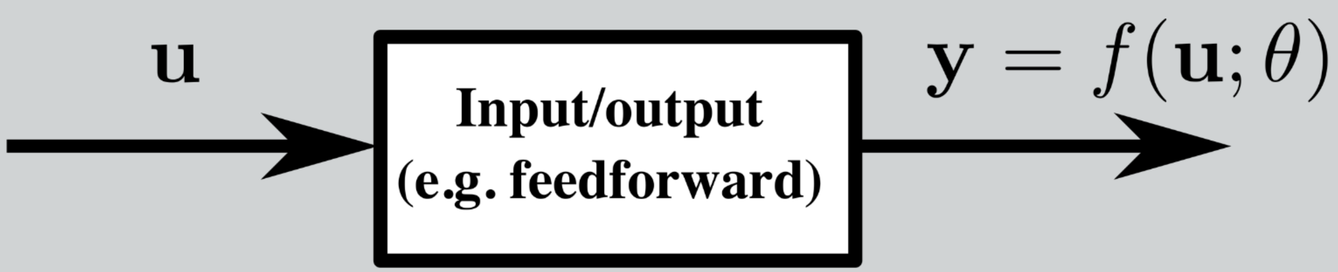
Varying initial conditions

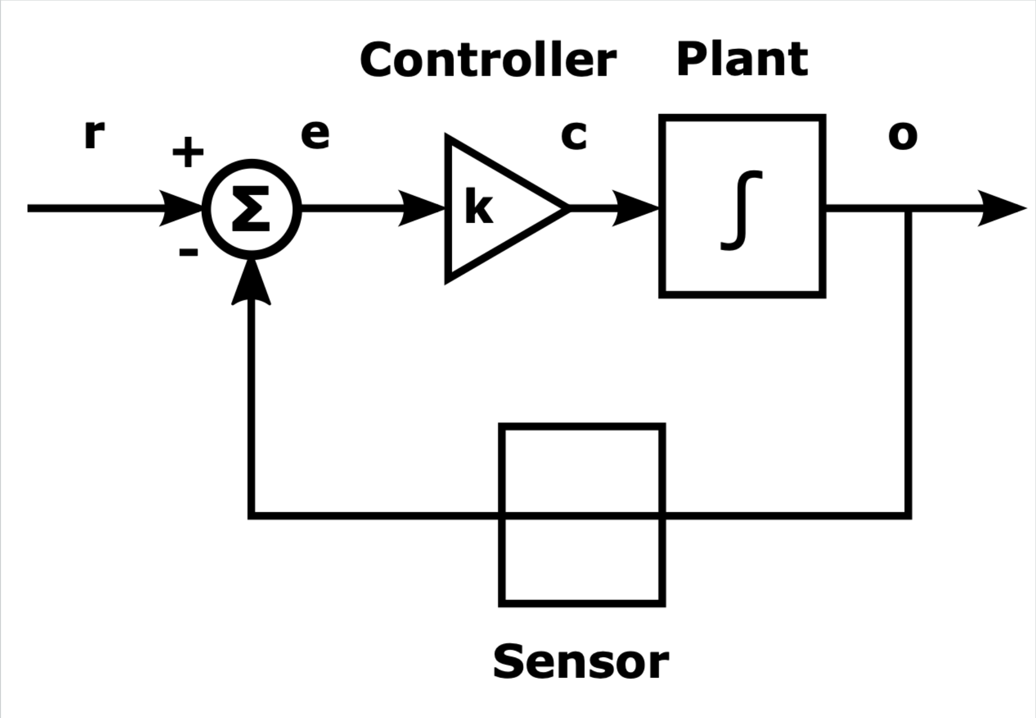


- **Every one of these curves is exponential**
- **They all eventually converge on the reference, $r = 10$**
- **Changing the output at time $t = 0$ only changes the time to stabilise**

Some comments on feedback control

- The very simple feedback control system that we have just seen is capable of controlling or regulating variables
    - But it is reactive *not* self-adaptive
        - It does not monitor its own performance
        - If a control system's parameters are not set correctly, it may perform very badly

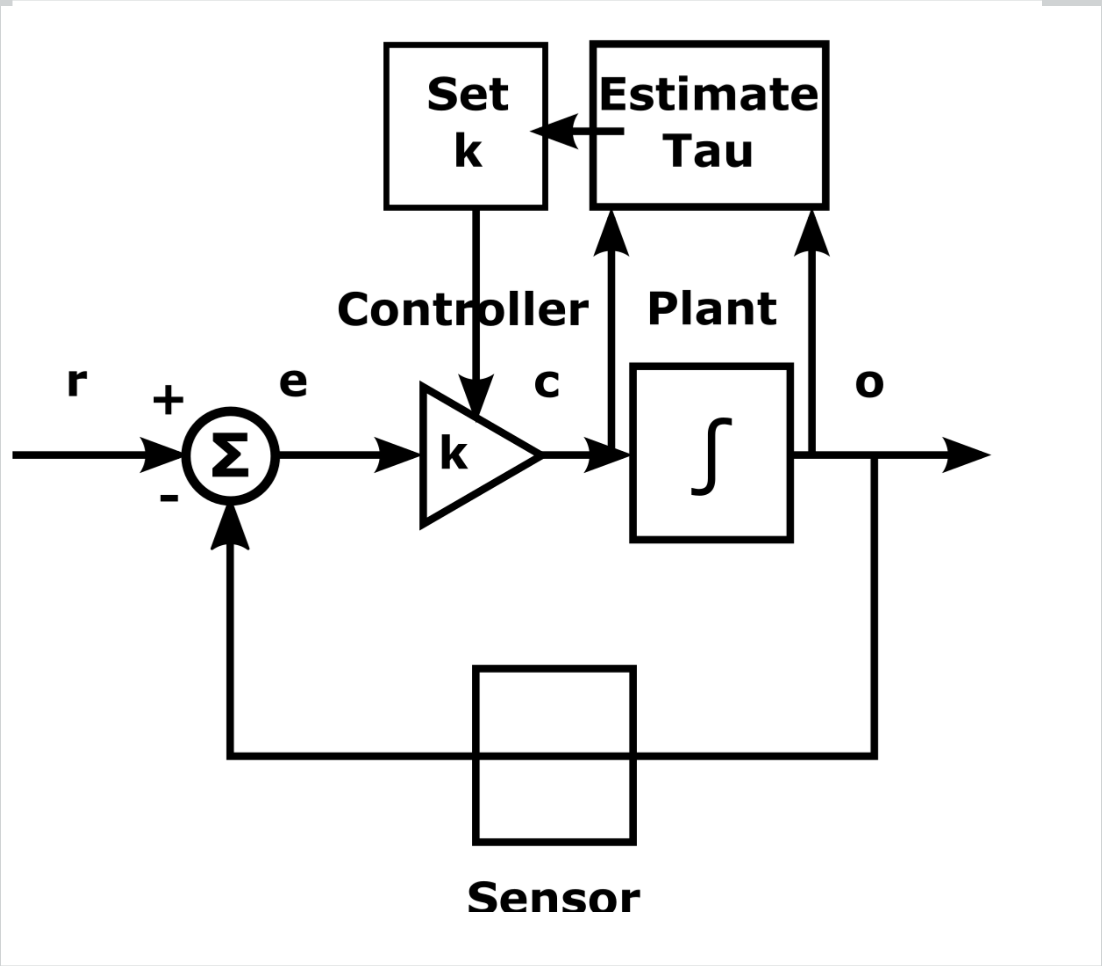




Brief look forwards

Briefly looking forward to later material:

- A self-adaptive feedback control system monitors its own performance
  - If the performance is not optimal, then the system may make changes to its parameters

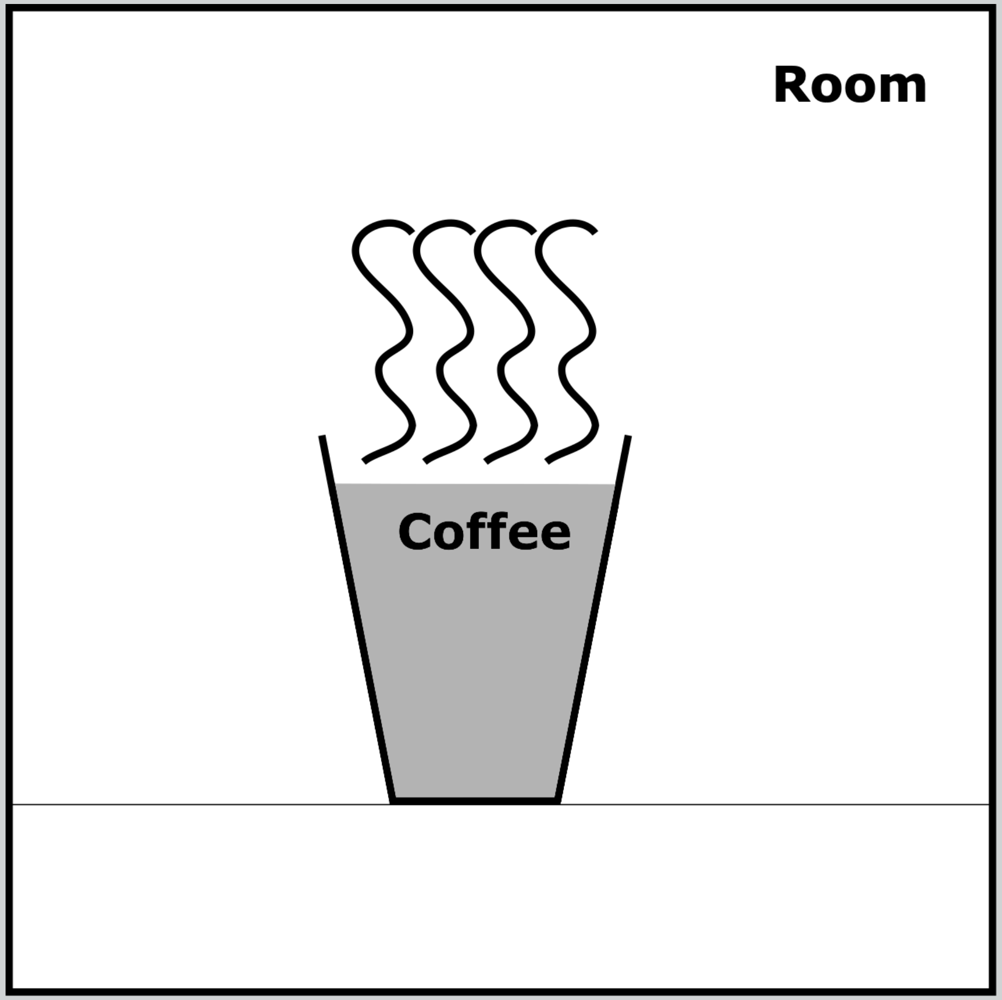


Problems with delays…

- When there is no error, the feedback controller does nothing
- This might seem fine - every time an error appears, the controller will act against it
  - But what if there is delay in the feedback path?
    - In fact, there will always be *some* delay - the only question is whether or not it is large enough to be a problem
  - A delay in the feedback path can cause at least two problems:
    1. Every time an error appears, there will be some period of time where the process is in error before the controller does anything
    2. A delay in the feedback path means that the controller is always working with out of date information, and negative feedback may even turn into positive feedback!

Stability without control

- Many systems tend towards stability without control
  - An example, a cup of coffee:
  - T_c = Temperature of coffee
  - T_r = Temperature of room
  - According to Newton's law (model) of cooling:
    - $\dot{T}_c = -k(T_c - T_r)$
      - where k is a rate parameter, which lumps various factors together
      - and we assume evaporation is negligible

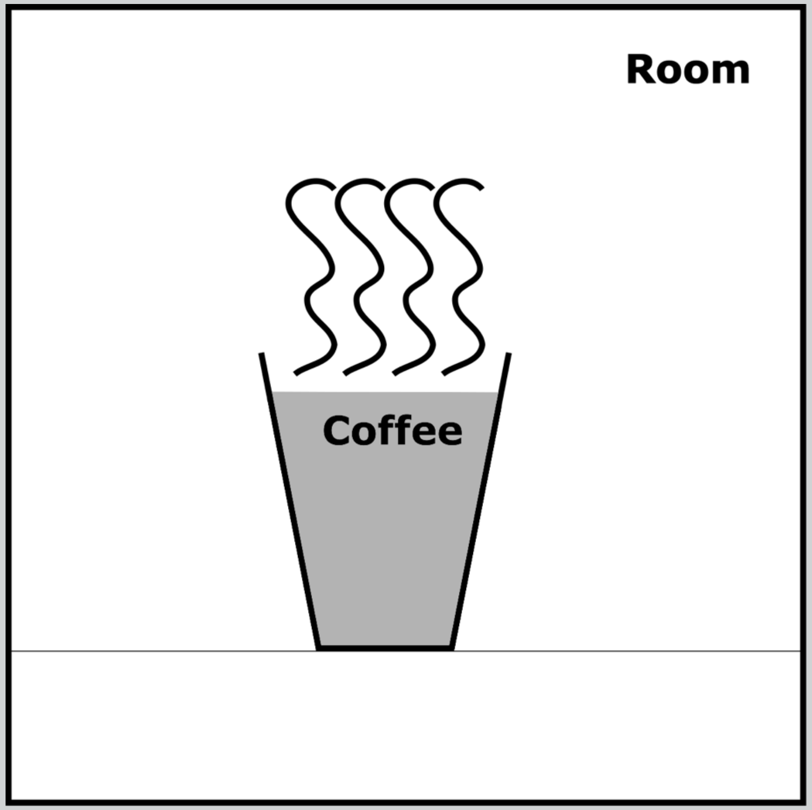


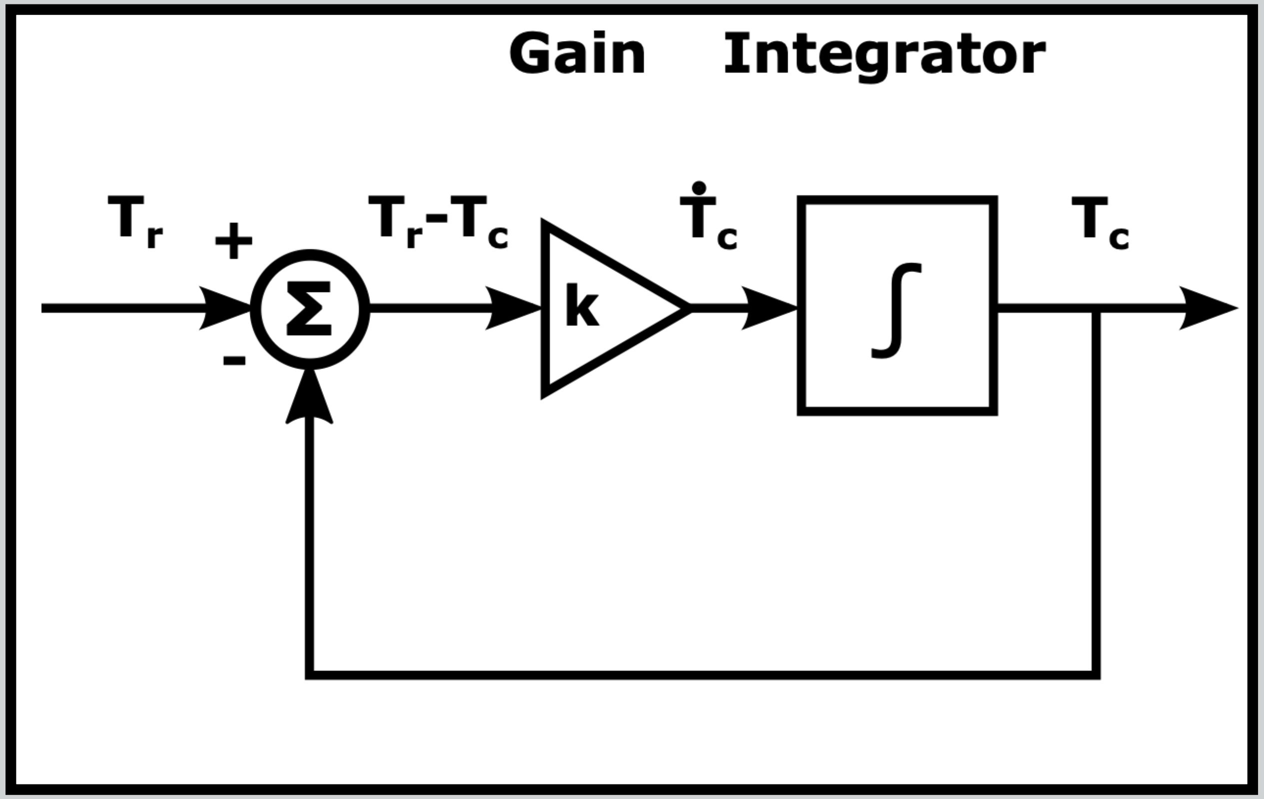
(Example from Meadows [6])

Stability without control

$\dot{T}_c = -k(T_c - T_r)$

- This equation describes a system with negative feedback!



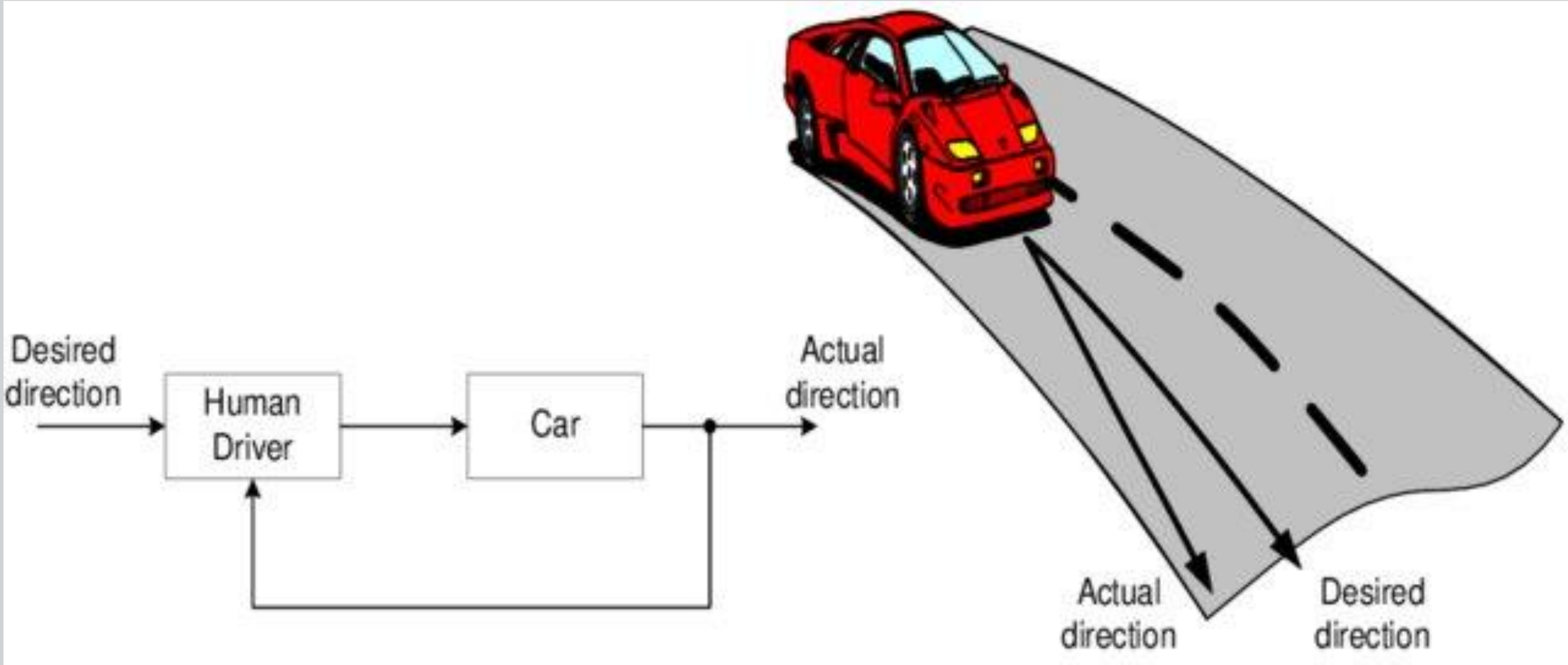


- In control theoretic terms, the room temperature is analogous to the reference signal, and the temperature of the cup is analogous to the controlled variable

- When the rate of change of a variable is a function of that variable, then you have feedback

  - In this case, $\dot{T}_c$ is a function of T_c

Examples - steering



[3]

- A straightforward example of negative feedback
  - If the car goes to the left of its lane, it is steered back to the right, and vice versa
- This system is interesting because it combines human and machine

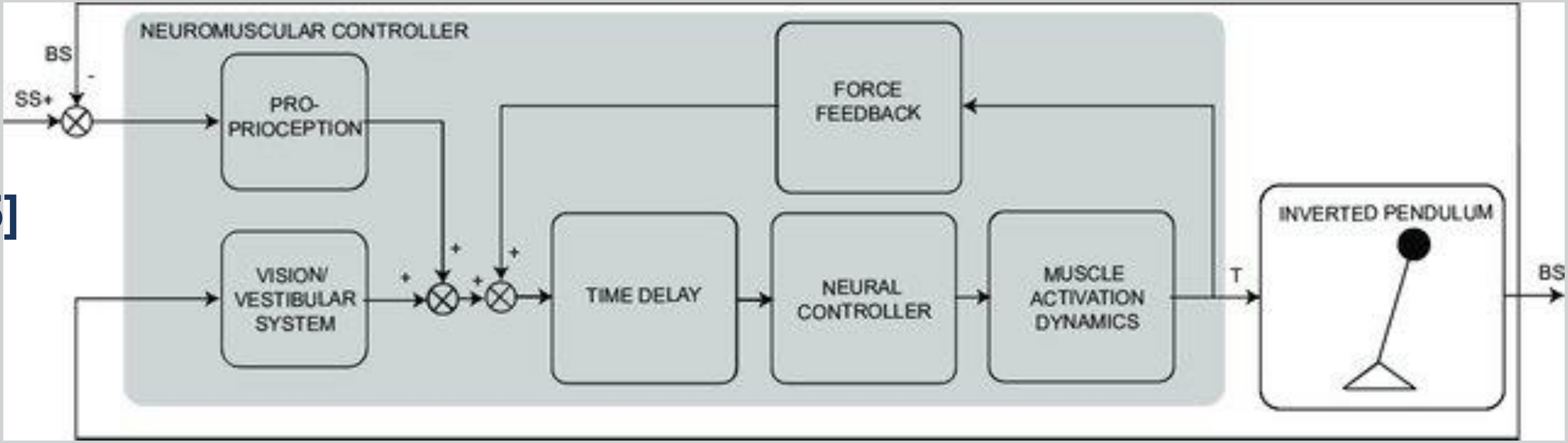
Examples - balancing

[2] 

[1]

- **Balancing is another problem that we solve using negative feedback**
  - **When we tilt one way, we lean towards the other**

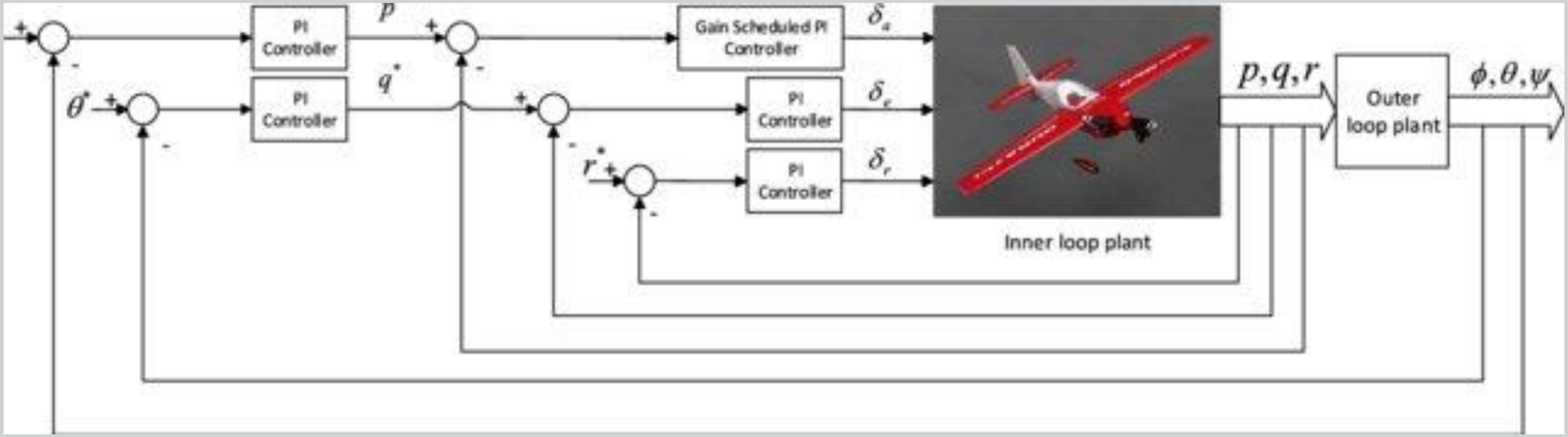
Examples - balancing



[5]

- A model of human balancing
  - The body is modelled as an inverted pendulum
  - There are three feedback loops here, using three separate sensory modalities

Examples - stabilising in air and sea



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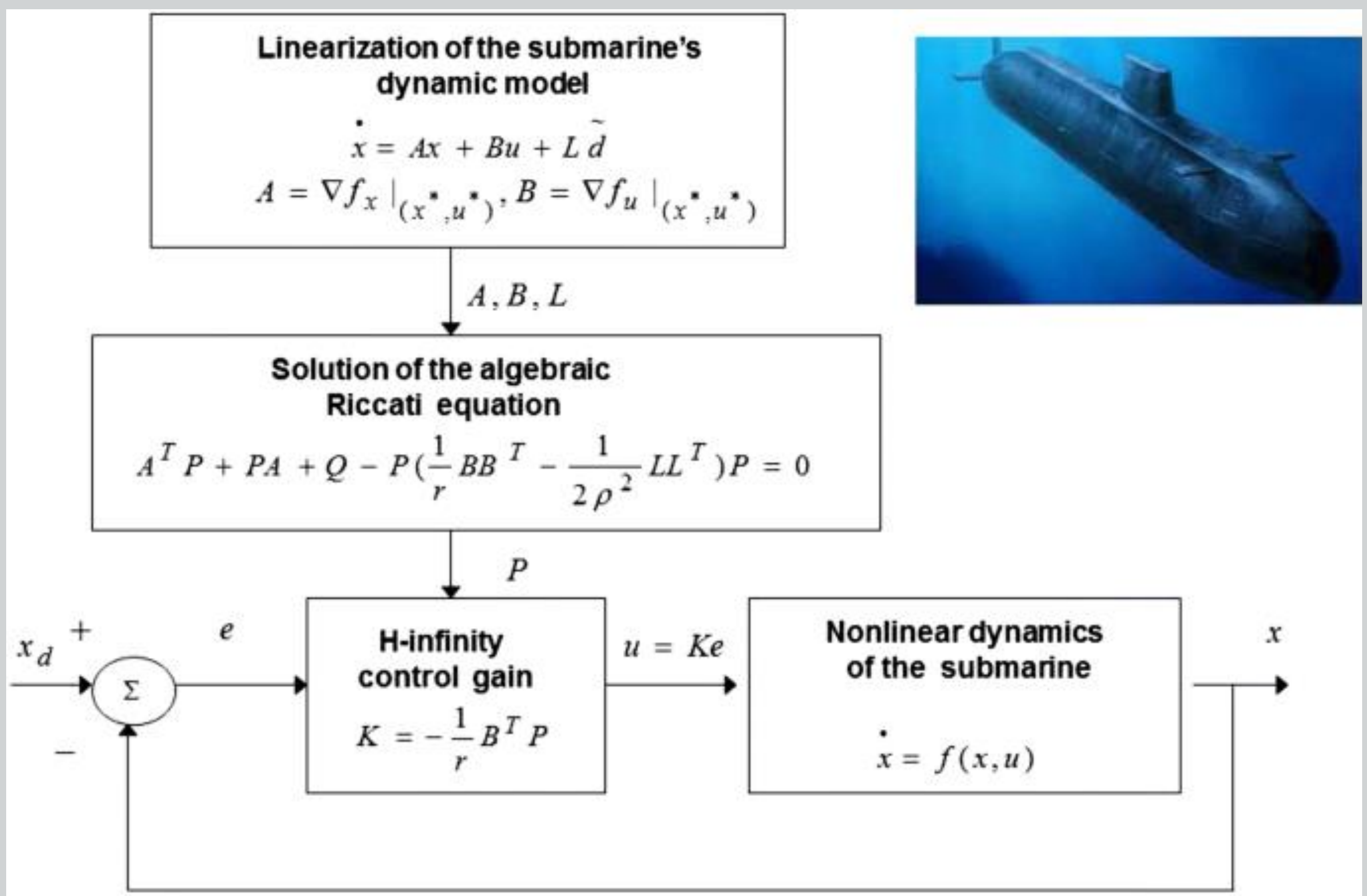

[7]

Summary

1. Systems with loops in their interconnections have feedback, or circular causality
2. Negative feedback tends to lead to stability
3. Some systems are naturally stable, or self-stable
4. In control, negative feedback can be used to turn unstable points in a system's dynamics into stable points
5. Although negative feedback tends to lead to stability, delays in feedback loops can cause problems
6. Systems which are self-stable may be mathematically equivalent to feedback-controlled systems! (or vice versa, e.g. see point 4)
7. Self-adaptive systems have multiple feedback loops

Bibliography

Recommended articles and books

[6] Donella H Meadows. Thinking in Systems: A primer. Chelsea Green Publishing, 2008.

[9] Klir, J., & Valach, M. (1967). Cybernetic modelling.

[10] Webb, B. (1996). A cricket robot. Scientific American, 275(6), 94-99.

Other cited articles and books

[7] Rigatos, G., Siano, P., Zouari, F. et al. Nonlinear optimal control of autonomous submarines' diving. Mar Syst Ocean Technol 15, 57–69 (2020). https://doi.org/10.1007/s40868-019-00070-3

Bibliography

Relevant websites

None this time.

Bibliography

Other websites (sources of pictures)

[1] https://github.com/topics/self-balancing-robot

[2] http://darkroom.baltimoresun.com/ph-ho-vw-circus-9/

[3] https://www.researchgate.net/publication/267039448_Human_and_Nature_Minding_Automation_An_Overview_of_Concepts_Methods_Tools_and_Applications/figures

[4] https://www.researchgate.net/publication/317040874_Gain_Scheduled_Attitude_Control_of_Fixed-Wing_UAV_With_Automatic_Controller_Tuning/figures

[5] https://www.researchgate.net/publication/282419509_Changes_in_sensory_reweighting_of_proprioceptive_information_during_standing_balance_with_age_and_disease/figures

[8] https://www.researchgate.net/figure/Step-response-of-the-nominal-mass-spring-damper-system_fig3_336601858