

①

ML Week 6 - Model Valid 2

last week = wider picture of data problem
preprocessing, (collaboration)

today = More preprocessing & Evaluation

- Dimensionality Reduction
- Evaluation Metrics
- Evaluation strategies

Recap Eigenvalues & Eigenvectors

Eigens stretch or squish an arrow vector

for any square matrix $A \in \mathbb{R}^{B \times B}$

there exists a non-zero vector $v \in \mathbb{R}^B$ such that :

$$Av = \lambda v$$

λ = scalar

the scalar has the same effect as the matrix transformation

v is the eigenvector of A
 λ is the eigen value

Eigen vectors & values are useful for analysing a matrix in the context of linear mapping

(2)

One of their uses is Eigendecomposition

A square Matrix $A \in \mathbb{R}^{B \times B}$ can be decomposed as

$$A = P \cdot D \cdot P^{-1}$$

P = Matrix of eigenvectors of A

$$\begin{bmatrix} v_1 & \cdots & v_B \\ \vdots & \ddots & \vdots \\ v_1 & \cdots & v_B \end{bmatrix}$$

D = A diagonal matrix whose entries are the corresponding eigenvalues of A

$$\begin{bmatrix} \lambda^1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda^B \end{bmatrix}$$

this only true if the eigenvectors are lin independent

that is, none can be written as a linear combination of the others

Dimensionality Reduction

Input w/ 10k dims

memory & time concerns
Complexity

Model complexity = concern about overfitting

Solution

- Reduce dims of train data
- But in a way to not loose ~~valuable~~ info valuable for the MC task

Approaches:

- * Based on domain knowledge (Hand craft features)
 - Feature engineering
- * Exhaustive comparison of diff combs of features
 - Not efficient & often possible for large dims
 - Could be useful for dims that have already been pruned using domain knowledge
- * Representation / feature learning (week 9)
- * Other methods
 - PCA → today lecture
 - ↳ does not reduce itself
 - Maps data into other spaces

(4)

Principal Component Analysis (PCA)

maps input data $x \in \mathbb{R}^p$ to $\tilde{x} \in \mathbb{R}^p$

such that the new \tilde{x} (PC's) are ordered

1st PC is most informative

- captures largest variance in orig data

2nd PC next most informative

- captures largest var not captured by 1st PC

$O^{th}\text{ etc. . .}$

Dim Reduction goal is to:

- * make data dims more compact (reduced)
- * w/ minimal loss of information (variance)

reframe this as:

transform x into new space where a single feature \tilde{x}_1 captures the max variance in the original space

$$\tilde{x}_{1,n} = b_1^\top x_n \quad \forall n, n = 1, 2, \dots, N$$

and maximising

$$V\tilde{x}_1 = \frac{1}{N} \sum_{n=1}^N \tilde{x}_{1,n}^2$$

where $V\tilde{x}_1 = \text{var of } \tilde{x}_1$

$b_1 \in \mathbb{B}$ set of orthonormal basis vectors that map x_n to \tilde{x}_n

Orthogonal means orthogonal vector, each normalized to unit length

(5)

Stopped taking notes of Dimensional Reduction as reading will be better

Next section:

Evaluation metrics

Proportion of Accuracy \rightarrow Classification, correct classifications

$$= \frac{\sum_{n=1}^N \begin{cases} 1 & \text{if } \hat{y}_n = y_n \\ 0 & \text{otherwise} \end{cases}}{N} = 0 \leq \text{acc} \leq 1$$

0 = total wrong, 1 = total now

How to judge accuracy?

- * Random guess = lower Baseline $\rightarrow \frac{1}{N}$
- * Compare to human performance / Predictions

lots of metrics in weeks 3/4 labs

Confusion Matrix

A repres of classif Performance

		Pred	
		0	1
Act	0	TN	FP
	1	FN	TP

$$\text{Acc} = \frac{TP + TN}{TN + FP + FN + TP}$$

Can be extended to Multiclass

6

Problem w/ accuracy

- Report accuracy but never on its own
- is a Biased metric → hides errors & frequencies
- confusion matrix are most transparent

↳ but sometimes you need a single metric

Options → Precision, recall, f1 score

Precision: prop of predicted class that are correct
 $= \text{TP} / (\text{TP} + \text{FP})$

Recall: prop of true class that are correct
 $= \text{TP} / (\text{TP} + \text{FN})$

these two are conflicting metrics, but aim of to opt both

f1 Score captures the balance between them

$$f1 = 2 \times \frac{\text{Prec} \times \text{Recall}}{\text{Prec} + \text{Recall}} = \frac{2\text{TP}}{2\text{TP} + \text{FP} + \text{FN}}$$

Can/should compute f1 for each class

Other Options

- ROC Curve = True pos rate vs false pos rate

~~AUC =~~
 Area under
 Curve
 ; since
 more

Plots the different outcomes for diff decision bounds

→ is probabilistic bounds from model

(7)

Other metrics

- * Correlation metrics
- * Specialist metrics
- * Common regression metrics

Next section:

Evaluation Strategies

Divide data into train + test

for eval

How to split?

Hold out validation ← week 2

80/20

Problem? Bias Splits

k-fold cross valid

• $k = \text{split}/\text{fold}$

• each time we use one fold as test as iterate through folds

• less biases

$$k = N$$

• higher level of k-cross = leave one out valid
 → max size of train
 → slow & costly

$k=10$ = typical

(8)

Alternatives to k-cross?

Splits data by groups, i.e. people

i.e. leave out = all data for partic group

then can also take a k-cross appear for group

basically as hand selected k-cross

How to report k-cross valid?

Opt 1 - • Report mean & SD of folds
 • can't use confusion matrix

Opt 2 - • Combine all test labels & preds across folds
 • then, compute metric

Validation Set - week 3

train → test → valid

