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```
clear
clc
```

```
%
```

2b

```
syms x y z u ae J2 J3

U = -(3*u*ae*ae*J2*z*z)/(2*((x^2+y^2+z^2)^(5/2)))...
    + (u*ae*ae*J2)/(2*((x^2+y^2+z^2)^(3/2)))...
    - (5*u*ae*ae*ae*J3*z*z*z)/(2*((x^2+y^2+z^2)^(7/2)))...
    + (3*u*ae*ae*ae*J3*z)/(2*((x^2+y^2+z^2)^(5/2)));

dudx = diff(U,x);

dudy = diff(U,y);

dudz = diff(U,z);

acc = [dudx; dudy; dudz];

dadx = simplify(diff(acc,x));
dady = simplify(diff(acc,y));
```

```

dadz = simplify(diff(acc,z));
dadu = simplify(diff(acc,u));
dadJ2 = simplify(diff(acc,J2));
dadJ3 = simplify(diff(acc,J3));

fprintf('dadx \nx-hat: %s\ ny-hat: %s\ nz-hat: %s\n
\n',dadx(1),dadx(2),dadx(3))
fprintf('dady \nx-hat: %s\ ny-hat: %s\ nz-hat: %s\n
\n',dady(1),dady(2),dady(3))
fprintf('dadz \nx-hat: %s\ ny-hat: %s\ nz-hat: %s\n
\n',dadz(1),dadz(2),dadz(3))
fprintf('dadu \nx-hat: %s\ ny-hat: %s\ nz-hat: %s\n
\n',dadu(1),dadu(2),dadu(3))
fprintf('dadJ2 \nx-hat: %s\ ny-hat: %s\ nz-hat: %s\n
\n',dadJ2(1),dadJ2(2),dadJ2(3))
fprintf('dadJ3 \nx-hat: %s\ ny-hat: %s\ nz-hat: %s\n
\n',dadJ3(1),dadJ3(2),dadJ3(3))

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

close all
addpath(' ../bin')

%
```

Getting J2 & u accelerations

```

t1 = zeros(3,3);
tm = eye(3,3);
tr = zeros(3,3);
ml = [dadx(1), dady(1), dadz(1)
      dadx(2), dady(2), dadz(2)
      dadx(3), dady(3), dadz(3)];
mm = zeros(3,3);
mr = [simplify(diff(dudx,u)), simplify(diff(dudx,J2)),
      simplify(diff(dudx,ae))
      simplify(diff(dudy,u)), simplify(diff(dudy,J2)),
      simplify(diff(dudy,ae))
      simplify(diff(dudz,u)), simplify(diff(dudz,J2)),
      simplify(diff(dudz,ae))];
bl = zeros(3,3);
bm = zeros(3,3);
br = zeros(3,3);

stm = [t1 tm tr; ml mm mr; bl bm br];
stm2 = [t1 tm; ml mm]
```

%

Getting J2 & u accelerations

```
a1 = dudx + (15*J3*ae^3*u*x*z)/(2*(x^2 + y^2 + z^2)^(7/2)) -  
    (35*J3*ae^3*u*x*z^3)/(2*(x^2 + y^2 + z^2)^(9/2));  
a2 = dudy + (15*J3*ae^3*u*y*z)/(2*(x^2 + y^2 + z^2)^(7/2)) -  
    (35*J3*ae^3*u*y*z^3)/(2*(x^2 + y^2 + z^2)^(9/2));  
a3 = dudz - (3*J3*ae^3*u)/(2*(x^2 + y^2 + z^2)^(5/2)) +  
    (15*J3*ae^3*u*z^2)/(x^2 + y^2 + z^2)^(7/2) - (35*J3*ae^3*u*z^4)/  
    (2*(x^2 + y^2 + z^2)^(9/2));  
a_3 = [a1; a2; a3]
```

%

Givens

Orbital Elements

```
a = 7000; % km  
e = 0.001;  
i = 30*pi/180; % rad  
RAAN = 80*pi/180; % rad  
w = 40*pi/180; % rad  
ta = 0; % rad
```

State deviation vector

```
dx = [1; 0; 0; 0; .01; 0]; % km, km/s
```

Other

```
uE = 398600.4415; % km^3/s^2  
J2 = 0.0010826269;  
g = 9.81;  
Re = 6378.1;
```

%

Finding Initial Position and Setting State

Converting OE to ECI state

```
[r0, v0] = OE2ECI(a, e, i, RAAN, w, ta, uE); % km, km/s
```

Setting initial state vectors (6x1)

```
X0 = [r0; v0]
X02 = X0 + dx;

%
```

Propagating the State with Numerical Integration

Setting time frame

```
ti = 0; % sec
tf = 24*3600; % sec
dt = 1; % sec
time = ti:dt:tf; % sec
```

Setting integrator accuracy

```
tol = 1E-9;
options = odeset('RelTol',tol,'AbsTol',tol);
```

Propagating the State

```
[Times,States] = ode45(@StatOD_Hw1_Int,time,X0,options,uE,Re,J2);

plot3(States(:,1),States(:,2),States(:,3))
PlotBoi3('X, km', 'Y, km', 'Z, km', 14)
view(-60,15)
```

Propagating the State (X + dx)

```
[Times,States_pdx] = ode45(@StatOD_Hw1_Int,time,X02,options,uE,Re,J2);

figure
subplot(3,2,1)
plot(Times./3600,States(:,1)-States_pdx(:,1))
PlotBoi2('','X Error, km',14)
subplot(3,2,3)
plot(Times./3600,States(:,2)-States_pdx(:,2))
```

```

PlotBoi2('','Y Error, km',14)
subplot(3,2,5)
plot(Times./3600,States(:,3)-States_pdx(:,3))
PlotBoi2('Time, hr','Z Error, km',14)
subplot(3,2,2)
plot(Times./3600,States(:,4)-States_pdx(:,4))
PlotBoi2('','X-Dot Error, km',14)
subplot(3,2,4)
plot(Times./3600,States(:,5)-States_pdx(:,5))
PlotBoi2('','Y-Dot Error, km',14)
subplot(3,2,6)
plot(Times./3600,States(:,6)-States_pdx(:,6))
PlotBoi2('Time, hr','Z-Dot Error, km',14)

```

Propagating the State (STM)

```

[Times,States_dx] =
ode45(@StatOD_Hw1_STMInt,time,dx,options,uE,Re,J2);

figure
subplot(3,2,1)
plot(Times./3600,States_dx(:,1))
PlotBoi2('','X Error, km',14)
subplot(3,2,3)
plot(Times./3600,States_dx(:,2))
PlotBoi2('','Y Error, km',14)
subplot(3,2,5)
plot(Times./3600,States_dx(:,3))
PlotBoi2('Time, hr','Z Error, km',14)
subplot(3,2,2)
plot(Times./3600,States_dx(:,4))
PlotBoi2('','X-Dot Error, km',14)
subplot(3,2,4)
plot(Times./3600,States_dx(:,5))
PlotBoi2('','Y-Dot Error, km',14)
subplot(3,2,6)
plot(Times./3600,States_dx(:,6))
PlotBoi2('Time, hr','Z-Dot Error, km',14)
%

```

Numerical Integrators

```

function [ dY ] = StatOD_Hw1_Int(t,Y,u,ae,J2)
dY = zeros(6,1);

```

Unpack the state vector (ECI)

```

x = Y(1);
y = Y(2);

```

```

z = Y(3);
dy = Y(4:6); % Satellite Velocity, km/s

```

Output the derivative of the state

```

dY(1:3) = dy; % km/s
dY(4) = (15*J2*ae^2*u*x*z^2)/(2*(x^2 + y^2 + z^2)^(7/2)) -
    (3*J2*ae^2*u*x)/(2*(x^2 + y^2 + z^2)^(5/2)) - u*x/(norm(Y(1:3))^3); %
    km/s^2
dY(5) = (15*J2*ae^2*u*y*z^2)/(2*(x^2 + y^2 + z^2)^(7/2)) -
    (3*J2*ae^2*u*y)/(2*(x^2 + y^2 + z^2)^(5/2)) - u*y/(norm(Y(1:3))^3); %
    km/s^2
dY(6) = (15*J2*ae^2*u*z^3)/(2*(x^2 + y^2 + z^2)^(7/2)) -
    (9*J2*ae^2*u*z)/(2*(x^2 + y^2 + z^2)^(5/2)) - u*z/(norm(Y(1:3))^3); %
    km/s^2
end

```

```

function [ dY ] = StatOD_Hwl_STMInt(t,Y,u,ae,J2)
dY = zeros(6,1);
x = Y(1);
y = Y(2);
z = Y(3);
J3 = 0;

% stm = [
                                0,
                                0,
                                0, 1, 0, 0,
                                0,
                                0,
                                0
                                ]

```

```

0,

0, 0, 1, 0,

0,
0,

0

%

0,

0,

0, 0, 0, 1,

0,

0,

0

% -(3*J2*ae^2*u*(x^2 + y^2 + z^2)^3 - 15*J2*ae^2*u*z^2*(x^2 + y^2
+ z^2)^2 + 315*J3*ae^3*u*x^2*z^3 + 15*J3*ae^3*u*z*(x^2 + y^2 +
z^2)^2 - 35*J3*ae^3*u*z^3*(x^2 + y^2 + z^2) - 15*J2*ae^2*u*x^2*(x^2
+ y^2 + z^2)^2 - 105*J3*ae^3*u*x^2*z*(x^2 + y^2 + z^2) +
105*J2*ae^2*u*x^2*z^2*(x^2 + y^2 + z^2))/(2*(x^2 + y^2 +
z^2)^(11/2)),

(15*ae^2*u*x*y*(J2*x^4 +
2*J2*x^2*y^2 - 5*J2*x^2*z^2 + 7*J3*ae*x^2*z + J2*y^4 - 5*J2*y^2*z^2
+ 7*J3*ae*y^2*z - 6*J2*z^4 - 14*J3*ae*z^3))/(2*(x^2 + y^2 +
z^2)^(11/2)),
-(315*J3*ae^3*u*x*z^4
+ 15*J3*ae^3*u*x*(x^2 + y^2 + z^2)^2 - 45*J2*ae^2*u*x*z*(x^2
+ y^2 + z^2)^2 + 105*J2*ae^2*u*x*z^3*(x^2 + y^2 + z^2) -
210*J3*ae^3*u*x*z^2*(x^2 + y^2 + z^2))/(2*(x^2 + y^2 + z^2)^(11/2)),
0, 0, 0,
-(ae^2*x*(3*J2*x^4 + 6*J2*x^2*y^2
- 9*J2*x^2*z^2 + 15*J3*ae*x^2*z + 3*J2*y^4 - 9*J2*y^2*z^2 +
15*J3*ae*y^2*z - 12*J2*z^4 - 20*J3*ae*z^3))/(2*(x^2 + y^2 +
z^2)^(9/2)),
-(3*ae^2*u*x*(x^2 + y^2 - 4*z^2))/(2*(x^2 + y^2 +
z^2)^(7/2)),
-
(3*ae*u*x*(2*J2*x^4 + 4*J2*x^2*y^2 - 6*J2*x^2*z^2 + 15*J3*ae*x^2*z +
2*J2*y^4 - 6*J2*y^2*z^2 + 15*J3*ae*y^2*z - 8*J2*z^4 - 20*J3*ae*z^3))/
(2*(x^2 + y^2 + z^2)^(9/2))

```

%

$$\begin{aligned} & (15*ae^2*u*x*y*(J2*x^4 + 2*J2*x^2*y^2 - \\ & 5*J2*x^2*z^2 + 7*J3*ae*x^2*z + J2*y^4 - 5*J2*y^2*z^2 + 7*J3*ae*y^2*z \\ & - 6*J2*z^4 - 14*J3*ae*z^3))/(2*(x^2 + y^2 + z^2)^(11/2)), - \\ & (3*J2*ae^2*u*(x^2 + y^2 + z^2)^3 - 15*J2*ae^2*u*z^2*(x^2 + y^2 \\ & + z^2)^2 + 315*J3*ae^3*u*y^2*z^3 + 15*J3*ae^3*u*z*(x^2 + y^2 + \\ & z^2)^2 - 35*J3*ae^3*u*z^3*(x^2 + y^2 + z^2) - 15*J2*ae^2*u*y^2*(x^2 \\ & + y^2 + z^2)^2 - 105*J3*ae^3*u*y^2*z*(x^2 + y^2 + z^2) + \\ & 105*J2*ae^2*u*y^2*z^2*(x^2 + y^2 + z^2))/(2*(x^2 + y^2 + \\ & z^2)^(11/2)), - (315*J3*ae^3*u*y*z^4 \\ & + 15*J3*ae^3*u*y*(x^2 + y^2 + z^2)^2 - 45*J2*ae^2*u*y*z*(x^2 \\ & + y^2 + z^2)^2 + 105*J2*ae^2*u*y*z^3*(x^2 + y^2 + z^2) - \\ & 210*J3*ae^3*u*y*z^2*(x^2 + y^2 + z^2))/(2*(x^2 + y^2 + z^2)^(11/2)), \\ & 0, 0, 0, - (ae^2*y*(3*J2*x^4 + 6*J2*x^2*y^2 \\ & - 9*J2*x^2*z^2 + 15*J3*ae*x^2*z + 3*J2*y^4 - 9*J2*y^2*z^2 + \\ & 15*J3*ae*y^2*z - 12*J2*z^4 - 20*J3*ae*z^3))/(2*(x^2 + y^2 + \\ & z^2)^(9/2)), - (3*ae^2*u*y*(x^2 + y^2 - 4*z^2))/(2*(x^2 + y^2 + \\ & z^2)^(7/2)), - \\ & (3*ae*u*y*(2*J2*x^4 + 4*J2*x^2*y^2 - 6*J2*x^2*z^2 + 15*J3*ae*x^2*z + \\ & 2*J2*y^4 - 6*J2*y^2*z^2 + 15*J3*ae*y^2*z - 8*J2*z^4 - 20*J3*ae*z^3))/ \\ & (2*(x^2 + y^2 + z^2)^(9/2)) \end{aligned}$$

%

$$\begin{aligned} & - \\ & (315*J3*ae^3*u*x*z^4 + 15*J3*ae^3*u*x*(x^2 + y^2 + z^2)^2 - \\ & 45*J2*ae^2*u*x*z*(x^2 + y^2 + z^2)^2 + 105*J2*ae^2*u*x*z^3*(x^2 \\ & + y^2 + z^2) - 210*J3*ae^3*u*x*z^2*(x^2 + y^2 + z^2))/(2*(x^2 + \\ & y^2 + z^2)^(11/2)), \\ & - (315*J3*ae^3*u*y*z^4 + 15*J3*ae^3*u*y*(x^2 + y^2 + z^2)^2 - \\ & 45*J2*ae^2*u*y*z*(x^2 + y^2 + z^2)^2 + 105*J2*ae^2*u*y*z^3*(x^2 + \\ & y^2 + z^2) - 210*J3*ae^3*u*y*z^2*(x^2 + y^2 + z^2))/(2*(x^2 + y^2 + \\ & z^2)^(11/2)), - (9*J2*ae^2*u*(x^2 + y^2 + z^2)^3 + 315*J3*ae^3*u*z^5 \\ & - 90*J2*ae^2*u*z^2*(x^2 + y^2 + z^2)^2 + 105*J2*ae^2*u*z^4*(x^2 + y^2 \\ & + z^2) + 75*J3*ae^3*u*z*(x^2 + y^2 + z^2)^2 - 350*J3*ae^3*u*z^3*(x^2 \\ & + y^2 + z^2))/(2*(x^2 + y^2 + z^2)^(11/2)), 0, 0, 0, (35*J3*ae^3*z^4 \\ & + 3*J3*ae^3*(x^2 + y^2 + z^2)^2 - 9*J2*ae^2*z*(x^2 + y^2 + z^2)^2 + \\ & 15*J2*ae^2*z^3*(x^2 + y^2 + z^2) - 30*J3*ae^3*z^2*(x^2 + y^2 + z^2))/ \\ & (2*(x^2 + y^2 + z^2)^(9/2)), - (3*ae^2*u*z*(3*x^2 + 3*y^2 - 2*z^2))/ \\ & (2*(x^2 + y^2 + z^2)^(7/2)), - (3*ae*u*(6*J2*x^4*z - 3*J3*ae*x^4 + \\ & 12*J2*x^2*y^2*z - 6*J3*ae*x^2*y^2 + 2*J2*x^2*z^3 + 24*J3*ae*x^2*z^2 + \\ & 6*J2*y^4*z - 3*J3*ae*y^4 + 2*J2*y^2*z^3 + 24*J3*ae*y^2*z^2 - 4*J2*z^5 \\ & - 8*J3*ae*z^4))/(2*(x^2 + y^2 + z^2)^(9/2)) \end{aligned}$$

%

0,

0,


```
0, 0, 0, 0,
0, 0,
0
%
0,
0, 0, 0, 0,
0, 0,
0
%
0,
0, 0, 0, 0,
0, 0,
0];
stm = [
0,
```

0,

0, 1, 0, 0

0,

0,

0, 0, 1, 0

0,

0,

0, 0, 0, 1

$$-(3*J2*ae^2*u*(x^2 + y^2 + z^2)^3 - 15*J2*ae^2*u*z^2*(x^2 + y^2 + z^2)^2 + 315*J3*ae^3*u*x^2*z^3 + 15*J3*ae^3*u*z*(x^2 + y^2 + z^2)^2 - 35*J3*ae^3*u*z^3*(x^2 + y^2 + z^2) - 15*J2*ae^2*u*x^2*(x^2 + y^2 + z^2)^2 - 105*J3*ae^3*u*x^2*z*(x^2 + y^2 + z^2) + 105*J2*ae^2*u*x^2*z^2*(x^2 + y^2 + z^2))/(2*(x^2 + y^2 + z^2)^{(11/2)}),$$

$$(15*ae^2*u*x*y*(J2*x^4 + 2*J2*x^2*y^2 - 5*J2*x^2*z^2 + 7*J3*ae*x^2*z + J2*y^4 - 5*J2*y^2*z^2 + 7*J3*ae*y^2*z - 6*J2*z^4 - 14*J3*ae*z^3))/(2*(x^2 + y^2 + z^2)^{(11/2)}),$$

$$-(315*J3*ae^3*u*x*z^4 + 15*J3*ae^3*u*x*(x^2 + y^2 + z^2)^2 - 45*J2*ae^2*u*x*z*(x^2 + y^2 + z^2)^2 + 105*J2*ae^2*u*x*z^3*(x^2 + y^2 + z^2) - 210*J3*ae^3*u*x*z^2*(x^2 + y^2 + z^2))/(2*(x^2 + y^2 + z^2)^{(11/2)}),$$

0, 0, 0

$$(15*ae^2*u*x*y*(J2*x^4 + 2*J2*x^2*y^2 - 5*J2*x^2*z^2 + 7*J3*ae*x^2*z + J2*y^4 - 5*J2*y^2*z^2 + 7*J3*ae*y^2*z - 6*J2*z^4 - 14*J3*ae*z^3))/(2*(x^2 + y^2 + z^2)^{(11/2)}), -$$

```

(3*J2*ae^2*u*(x^2 + y^2 + z^2)^3 - 15*J2*ae^2*u*z^2*(x^2 + y^2
+ z^2)^2 + 315*J3*ae^3*u*y^2*z^3 + 15*J3*ae^3*u*z*(x^2 + y^2 +
z^2)^2 - 35*J3*ae^3*u*z^3*(x^2 + y^2 + z^2) - 15*J2*ae^2*u*y^2*(x^2
+ y^2 + z^2)^2 - 105*J3*ae^3*u*y^2*z*(x^2 + y^2 + z^2) +
105*J2*ae^2*u*y^2*z^2*(x^2 + y^2 + z^2))/(2*(x^2 + y^2 +
z^2)^(11/2)),
-(315*J3*ae^3*u*y*z^4
+ 15*J3*ae^3*u*y*(x^2 + y^2 + z^2)^2 - 45*J2*ae^2*u*y*z*(x^2
+ y^2 + z^2)^2 + 105*J2*ae^2*u*y*z^3*(x^2 + y^2 + z^2) -
210*J3*ae^3*u*y*z^2*(x^2 + y^2 + z^2))/(2*(x^2 + y^2 + z^2)^(11/2)),
0, 0, 0

```

```

-
(315*J3*ae^3*u*x*z^4 + 15*J3*ae^3*u*x*(x^2 + y^2 + z^2)^2 -
45*J2*ae^2*u*x*z*(x^2 + y^2 + z^2)^2 + 105*J2*ae^2*u*x*z^3*(x^2
+ y^2 + z^2) - 210*J3*ae^3*u*x*z^2*(x^2 + y^2 + z^2))/(2*(x^2 +
y^2 + z^2)^(11/2)),
-(315*J3*ae^3*u*y*z^4 + 15*J3*ae^3*u*y*(x^2 + y^2 + z^2)^2 -
45*J2*ae^2*u*y*z*(x^2 + y^2 + z^2)^2 + 105*J2*ae^2*u*y*z^3*(x^2 +
y^2 + z^2) - 210*J3*ae^3*u*y*z^2*(x^2 + y^2 + z^2))/(2*(x^2 + y^2 +
z^2)^(11/2)), -(9*J2*ae^2*u*(x^2 + y^2 + z^2)^3 + 315*J3*ae^3*u*z^5 -
90*J2*ae^2*u*z^2*(x^2 + y^2 + z^2)^2 + 105*J2*ae^2*u*z^4*(x^2 + y^2 +
z^2) + 75*J3*ae^3*u*z*(x^2 + y^2 + z^2)^2 - 350*J3*ae^3*u*z^3*(x^2 +
y^2 + z^2))/(2*(x^2 + y^2 + z^2)^(11/2)), 0, 0, 0];
dY = stm * Y;

```

end

dadx

```

x-hat: -(3*J2*ae^2*u*(x^2 + y^2 + z^2)^3 - 15*J2*ae^2*u*z^2*(x^2 +
y^2 + z^2)^2 + 315*J3*ae^3*u*x^2*z^3 + 15*J3*ae^3*u*z*(x^2 + y^2 +
z^2)^2 - 35*J3*ae^3*u*z^3*(x^2 + y^2 + z^2) - 15*J2*ae^2*u*x^2*(x^2
+ y^2 + z^2)^2 - 105*J3*ae^3*u*x^2*z*(x^2 + y^2 + z^2) +
105*J2*ae^2*u*x^2*z^2*(x^2 + y^2 + z^2))/(2*(x^2 + y^2 + z^2)^(11/2))
y-hat: (15*ae^2*u*x*y*(J2*x^4 + J2*y^4 - 6*J2*z^4 + 2*J2*x^2*y^2
- 5*J2*x^2*z^2 - 5*J2*y^2*z^2 - 14*J3*ae*z^3 + 7*J3*ae*x^2*z +
7*J3*ae*y^2*z))/(2*(x^2 + y^2 + z^2)^(11/2))
z-hat: -(315*J3*ae^3*u*x*z^4 + 15*J3*ae^3*u*x*(x^2 + y^2 + z^2)^2 -
45*J2*ae^2*u*x*z*(x^2 + y^2 + z^2)^2 + 105*J2*ae^2*u*x*z^3*(x^2 +
y^2 + z^2) - 210*J3*ae^3*u*x*z^2*(x^2 + y^2 + z^2))/(2*(x^2 + y^2 +
z^2)^(11/2))

```

dady

```

x-hat: (15*ae^2*u*x*y*(J2*x^4 + J2*y^4 - 6*J2*z^4 + 2*J2*x^2*y^2
- 5*J2*x^2*z^2 - 5*J2*y^2*z^2 - 14*J3*ae*z^3 + 7*J3*ae*x^2*z +
7*J3*ae*y^2*z))/(2*(x^2 + y^2 + z^2)^(11/2))
y-hat: -(3*J2*ae^2*u*(x^2 + y^2 + z^2)^3 - 15*J2*ae^2*u*z^2*(x^2 +
y^2 + z^2)^2 + 315*J3*ae^3*u*y^2*z^3 + 15*J3*ae^3*u*z*(x^2 + y^2 +
z^2)^2 - 35*J3*ae^3*u*z^3*(x^2 + y^2 + z^2) - 15*J2*ae^2*u*y^2*(x^2
+ y^2 + z^2)^2 - 105*J3*ae^3*u*y^2*z*(x^2 + y^2 + z^2) +
105*J2*ae^2*u*y^2*z^2*(x^2 + y^2 + z^2))/(2*(x^2 + y^2 + z^2)^(11/2))
z-hat: -(315*J3*ae^3*u*y*z^4 + 15*J3*ae^3*u*y*(x^2 + y^2 + z^2)^2 -
45*J2*ae^2*u*y*z*(x^2 + y^2 + z^2)^2 + 105*J2*ae^2*u*y*z^3*(x^2 +

```

$$y^2 + z^2) - 210*J3*ae^3*u*y*z^2*(x^2 + y^2 + z^2))/(2*(x^2 + y^2 + z^2)^(11/2))$$

dadz

$$x\text{-hat} : -(315*J3*ae^3*u*x*z^4 + 15*J3*ae^3*u*x*(x^2 + y^2 + z^2)^2 - 45*J2*ae^2*u*x*z*(x^2 + y^2 + z^2)^2 + 105*J2*ae^2*u*x*z^3*(x^2 + y^2 + z^2) - 210*J3*ae^3*u*x*z^2*(x^2 + y^2 + z^2))/(2*(x^2 + y^2 + z^2)^(11/2))$$

$$y\text{-hat} : -(315*J3*ae^3*u*y*z^4 + 15*J3*ae^3*u*y*(x^2 + y^2 + z^2)^2 - 45*J2*ae^2*u*y*z*(x^2 + y^2 + z^2)^2 + 105*J2*ae^2*u*y*z^3*(x^2 + y^2 + z^2) - 210*J3*ae^3*u*y*z^2*(x^2 + y^2 + z^2))/(2*(x^2 + y^2 + z^2)^(11/2))$$

$$z\text{-hat} : -(9*J2*ae^2*u*(x^2 + y^2 + z^2)^3 + 315*J3*ae^3*u*z^5 - 90*J2*ae^2*u*z^2*(x^2 + y^2 + z^2)^2 + 105*J2*ae^2*u*z^4*(x^2 + y^2 + z^2) + 75*J3*ae^3*u*z*(x^2 + y^2 + z^2)^2 - 350*J3*ae^3*u*z^3*(x^2 + y^2 + z^2))/(2*(x^2 + y^2 + z^2)^(11/2))$$

dadu

$$x\text{-hat} : -(ae^2*x*(3*J2*x^4 + 3*J2*y^4 - 12*J2*z^4 + 6*J2*x^2*y^2 - 9*J2*x^2*z^2 - 9*J2*y^2*z^2 - 20*J3*ae*z^3 + 15*J3*ae*x^2*z + 15*J3*ae*y^2*z))/(2*(x^2 + y^2 + z^2)^(9/2))$$

$$y\text{-hat} : -(ae^2*y*(3*J2*x^4 + 3*J2*y^4 - 12*J2*z^4 + 6*J2*x^2*y^2 - 9*J2*x^2*z^2 - 9*J2*y^2*z^2 - 20*J3*ae*z^3 + 15*J3*ae*x^2*z + 15*J3*ae*y^2*z))/(2*(x^2 + y^2 + z^2)^(9/2))$$

$$z\text{-hat} : (35*J3*ae^3*z^4 + 3*J3*ae^3*(x^2 + y^2 + z^2)^2 - 9*J2*ae^2*z*(x^2 + y^2 + z^2)^2 + 15*J2*ae^2*z^3*(x^2 + y^2 + z^2) - 30*J3*ae^3*z^2*(x^2 + y^2 + z^2))/(2*(x^2 + y^2 + z^2)^(9/2))$$

dadJ2

$$x\text{-hat} : -(3*ae^2*u*x*(x^2 + y^2 - 4*z^2))/(2*(x^2 + y^2 + z^2)^(7/2))$$

$$y\text{-hat} : -(3*ae^2*u*y*(x^2 + y^2 - 4*z^2))/(2*(x^2 + y^2 + z^2)^(7/2))$$

$$z\text{-hat} : -(3*ae^2*u*z*(3*x^2 + 3*y^2 - 2*z^2))/(2*(x^2 + y^2 + z^2)^(7/2))$$

dadJ3

$$x\text{-hat} : -(5*ae^3*u*x*z*(3*x^2 + 3*y^2 - 4*z^2))/(2*(x^2 + y^2 + z^2)^(9/2))$$

$$y\text{-hat} : -(5*ae^3*u*y*z*(3*x^2 + 3*y^2 - 4*z^2))/(2*(x^2 + y^2 + z^2)^(9/2))$$

$$z\text{-hat} : (ae^3*u*(6*x^2*y^2 - 24*x^2*z^2 - 24*y^2*z^2 + 3*x^4 + 3*y^4 + 8*z^4))/(2*(x^2 + y^2 + z^2)^(9/2))$$

stm2 =

[

0,

0,

0, 1, 0, 0]

[

0,

0,

0, 0, 1, 0]

[

0,

0,

0, 0, 0, 1]

[-(3*J2*ae^2*u*(x^2 + y^2 + z^2)^3 - 15*J2*ae^2*u*z^2*(x^2 + y^2 + z^2)^2 + 315*J3*ae^3*u*x^2*z^3 + 15*J3*ae^3*u*z*(x^2 + y^2 + z^2)^2 - 35*J3*ae^3*u*z^3*(x^2 + y^2 + z^2) - 15*J2*ae^2*u*x^2*(x^2 + y^2 + z^2)^2 - 105*J3*ae^3*u*x^2*z*(x^2 + y^2 + z^2) + 105*J2*ae^2*u*x^2*z^2*(x^2 + y^2 + z^2))/(2*(x^2 + y^2 + z^2)^(11/2)),

(15*ae^2*u*x*y*(J2*x^4 + 2*J2*x^2*y^2 - 5*J2*x^2*z^2 + 7*J3*ae*x^2*z + J2*y^4 - 5*J2*y^2*z^2 + 7*J3*ae*y^2*z - 6*J2*z^4 - 14*J3*ae*z^3))/(2*(x^2 + y^2 + z^2)^(11/2)),
-(315*J3*ae^3*u*x*z^4 + 15*J3*ae^3*u*x*(x^2 + y^2 + z^2)^2 - 45*J2*ae^2*u*x*z*(x^2 + y^2 + z^2)^2 + 105*J2*ae^2*u*x*z^3*(x^2 + y^2 + z^2) - 210*J3*ae^3*u*x*z^2*(x^2 + y^2 + z^2))/(2*(x^2 + y^2 + z^2)^(11/2)),
0, 0, 0]

[

(15*ae^2*u*x*y*(J2*x^4 + 2*J2*x^2*y^2 - 5*J2*x^2*z^2 + 7*J3*ae*x^2*z + J2*y^4 - 5*J2*y^2*z^2 + 7*J3*ae*y^2*z - 6*J2*z^4 - 14*J3*ae*z^3))/(2*(x^2 + y^2 + z^2)^(11/2)), -
(3*J2*ae^2*u*(x^2 + y^2 + z^2)^3 - 15*J2*ae^2*u*z^2*(x^2 + y^2 + z^2)^2 + 315*J3*ae^3*u*y^2*z^3 + 15*J3*ae^3*u*z*(x^2 + y^2 + z^2)^2 - 35*J3*ae^3*u*z^3*(x^2 + y^2 + z^2) - 15*J2*ae^2*u*y^2*(x^2 + y^2 + z^2)^2 - 105*J3*ae^3*u*y^2*z*(x^2 + y^2 + z^2) +

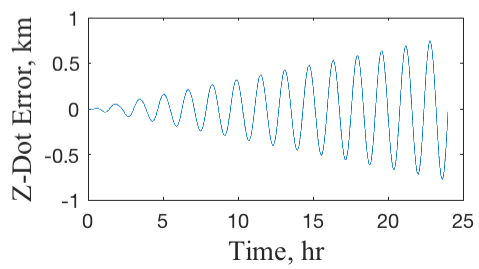
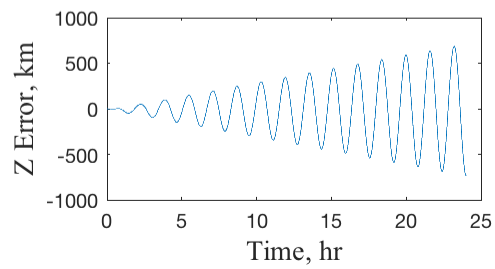
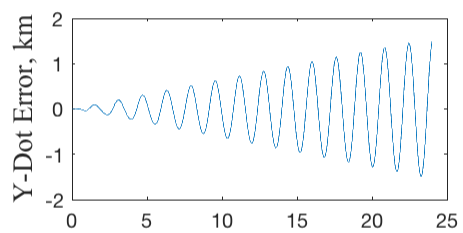
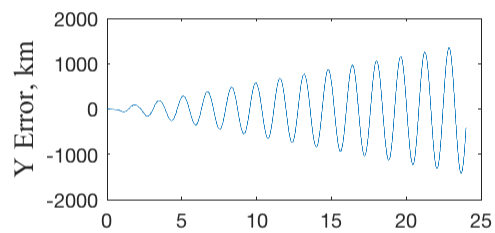
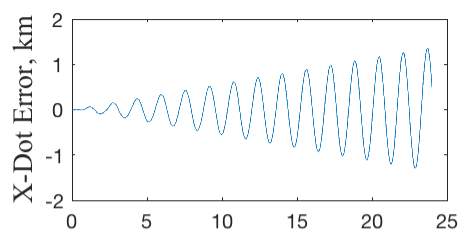
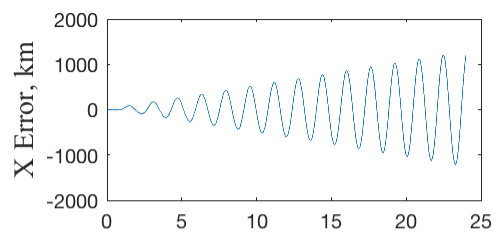
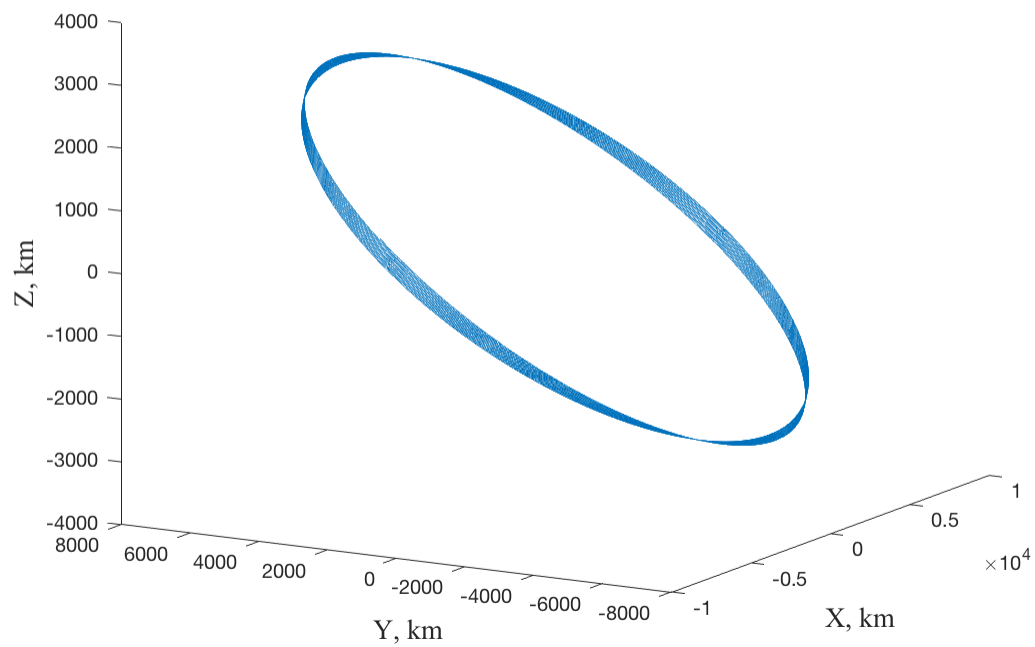
$$\begin{aligned}
& 105*J2*ae^2*u*Y^2*z^2*(x^2 + y^2 + z^2))/(2*(x^2 + y^2 + z^2)^{(11/2)}), \\
& -(315*J3*ae^3*u*Y*z^4 + 15*J3*ae^3*u*Y*(x^2 + y^2 + z^2)^2 - 45*J2*ae^2*u*Y*z^3*(x^2 + y^2 + z^2)^2 + 105*J2*ae^2*u*Y*z^3*(x^2 + y^2 + z^2) - 210*J3*ae^3*u*Y*z^2*(x^2 + y^2 + z^2))/(2*(x^2 + y^2 + z^2)^{(11/2)}), \\
& 0, 0, 0] \\
& [\\
& - \\
& (315*J3*ae^3*u*x*z^4 + 15*J3*ae^3*u*x*(x^2 + y^2 + z^2)^2 - 45*J2*ae^2*u*x*z^3*(x^2 + y^2 + z^2)^2 + 105*J2*ae^2*u*x*z^3*(x^2 + y^2 + z^2) - 210*J3*ae^3*u*x*z^2*(x^2 + y^2 + z^2))/(2*(x^2 + y^2 + z^2)^{(11/2)}), \\
& - \\
& -(315*J3*ae^3*u*Y*z^4 + 15*J3*ae^3*u*Y*(x^2 + y^2 + z^2)^2 - 45*J2*ae^2*u*Y*z^3*(x^2 + y^2 + z^2)^2 + 105*J2*ae^2*u*Y*z^3*(x^2 + y^2 + z^2) - 210*J3*ae^3*u*Y*z^2*(x^2 + y^2 + z^2))/(2*(x^2 + y^2 + z^2)^{(11/2)}), \\
& -(9*J2*ae^2*u*(x^2 + y^2 + z^2)^3 + 315*J3*ae^3*u*z^5 - 90*J2*ae^2*u*z^2*(x^2 + y^2 + z^2)^2 + 105*J2*ae^2*u*z^4*(x^2 + y^2 + z^2) + 75*J3*ae^3*u*z^3*(x^2 + y^2 + z^2)^2 - 350*J3*ae^3*u*z^3*(x^2 + y^2 + z^2))/(2*(x^2 + y^2 + z^2)^{(11/2)}), 0, 0, 0]
\end{aligned}$$

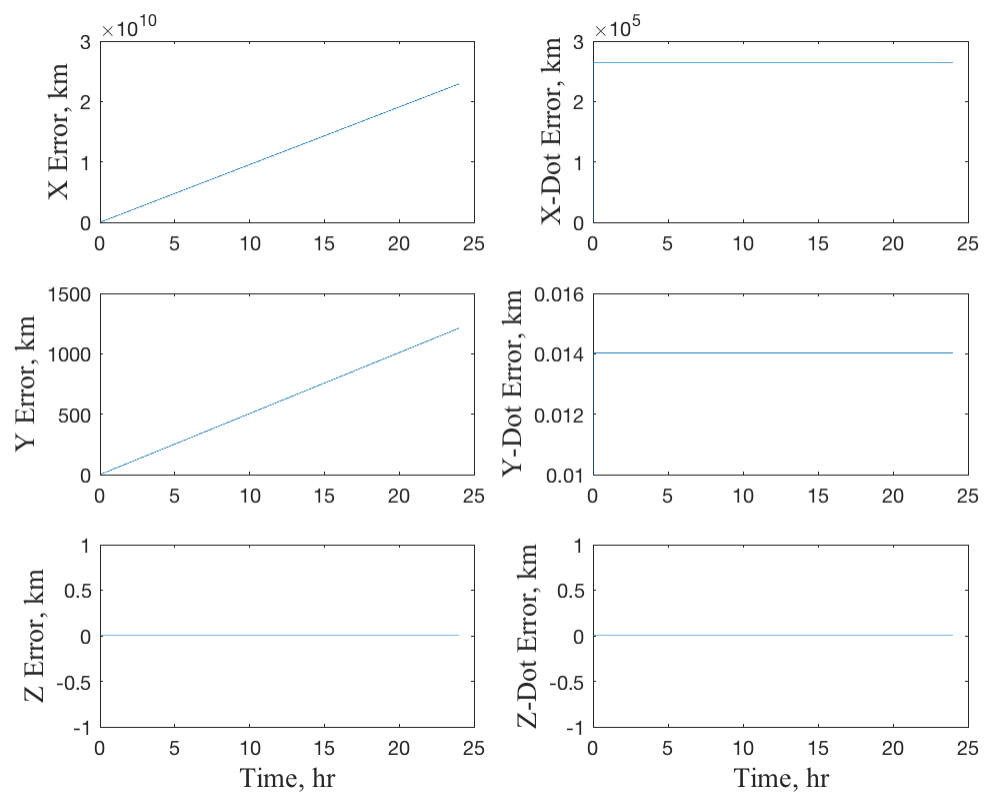
a_3 =

$$\begin{aligned}
& (15*J2*ae^2*u*x*z^2)/(2*(x^2 + y^2 + z^2)^{(7/2)}) - (3*J2*ae^2*u*x)/(2*(x^2 + y^2 + z^2)^{(5/2)}) \\
& (15*J2*ae^2*u*Y*z^2)/(2*(x^2 + y^2 + z^2)^{(7/2)}) - (3*J2*ae^2*u*Y)/(2*(x^2 + y^2 + z^2)^{(5/2)}) \\
& (15*J2*ae^2*u*z^3)/(2*(x^2 + y^2 + z^2)^{(7/2)}) - (9*J2*ae^2*u*z)/(2*(x^2 + y^2 + z^2)^{(5/2)})
\end{aligned}$$

X0 =

$$\begin{aligned}
& 1.0e+03 * \\
& -2.903431386573331 \\
& 5.951541650706099 \\
& 2.247506877268984 \\
& -0.005778159649183 \\
& -0.003911418840069 \\
& 0.002893197846789
\end{aligned}$$





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