ASE 324L

Lab #11: High Cycle Fatigue and Fatigue Crack Growth

Background: Fatigue failure is one of the most common modes of failure of structural components. It occurs in components that are subjected to repeated or cyclic loading and can be broadly classified into three categories:

(a) *High cycle fatigue* – nominal stress levels are below the yield strength of the material and a large number (greater than 10^4) of cycles are required to cause failure. Rotating parts, pressurized fuselages, etc., fall into this category.

(b) *Low cycle fatigue* – nominal stress levels are above the yield strength of the material. The number of cycles to cause failure is relatively low (less than 10^3). Overloads are the most common cause of low cycle fatigue.

(c) *Fatigue crack growth* – pre-existing cracks may grow at $K < K_C$ under fatigue loading.

Although eventual failure in all three categories is due to fast crack growth, most of the lifetime of components in categories (a) and (b) is spent initiating, rather than propagating, cracks. As a result, fatigue resistance and strength is described in terms of nominal stresses or strains, rather than fracture parameters, which are used for fatigue failure in category (c).

High Cycle Fatigue

High cycle fatigue is often analyzed based on so-called ***S-N curves***. The name arises from the fact that stress amplitude,

$$\sigma_a = \left(\sigma_{\max} - \sigma_{\min}\right)/2 \tag{1}$$

is plotted (Fig. 1) against the number of cycles to failure, N. The stresses are usually applied in a sinusoidal manner at constant stress amplitude and frequency.

The curves, which essentially represent a material's resistance to high cycle fatigue, indicate that fatigue life (number of cycles to failure) decreases with increasing stress amplitude. There is usually a lower limit, σ_{fat}, known as the ***fatigue endurance limit*** of a material, below which fatigue failure does not occur. Any structural component that is designed to operate below this stress level can enjoy the prospect of an infinite life. If higher stresses are encountered then life predictions can be made based on a cumulative damage law, such as **Miner’s Law**, where the amount of damage, D, that occurs after n cycles at a stress amplitude that would lead to failure after N cycles is given by

$$D = n/N \, . \tag{2}$$

In other words, the amount of damage is linearly proportional to the number of cycles applied and failure occurs when $D = 1$. If a number of blocks of constant amplitude loading are applied, then the total damage is added so that

$$D_T = \Sigma \; n_i / N_i \tag{3}$$

where n_i and N_i are, respectively, the number of cycles applied and the lifetime associated with the ith loading block. Failure occurs when $D_T = 1$.

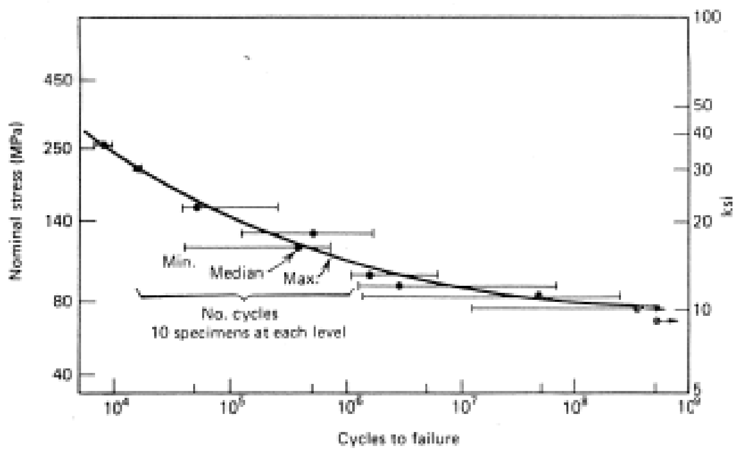


Figure 1. *S-N* Curve

The stress amplitude however is not the only parameter that controls high cycle fatigue failure. The mean stress level is also important, although its effect has only been determined empirically. Basically, increasing the mean stress level decreases the lifetime at a given stress amplitude (Fig. 2). One way of accounting for this is to use Goodman’s rule which indicates how the stress amplitude, σ_{am} at a nonzero mean stress must be decreased relative to σ_a, the stress amplitude at zero mean stress, to yield the same lifetime. Goodman’s rule can be written as

$$\sigma_{am} = \sigma_a \left[1 - |\sigma_m| / \sigma_{UTS} \right] \tag{4}$$

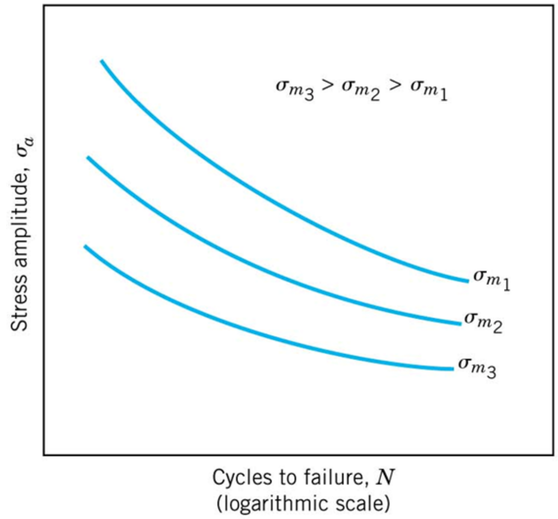


Figure 2. Effect of Mean Stress on high cycle fatigue

Fatigue Crack Growth

We saw in Lab 9 that fast fracture occurs under monotonic loading when $K = K_C$, the toughness of the material. It turns out that under cyclic loading cracks can grow slowly in a subcritical fashion at stress intensity factor levels that are much lower than K_C. This is analogous to the damage that is caused during high cycle fatigue at nominal stress levels that are below the yield strength of the material. Similar phenomena of subcritical slow crack growth ($K < K_C$) have been discussed in Lab 10 as environmentally assisted crack growth, where the loading condition is static rather than cyclic.

The resistance to fatigue crack growth in a material is expressed in a correlation between the crack growth rate, or crack extension per cycle, *da/dN*, and the change in stress intensity factor over one cycle, ΔK. Such a correlation is shown in Figure 3, where three regimes of fatigue crack growth are depicted.

(i) *Threshold region* – fatigue cracks will not grow for $K \leq K_{th}$, the threshold value.

(ii) *Power law region* – the linearity of the double logarithmic plot in this region suggests that

$$\frac{da}{dN} = A(\Delta K)^n \, . \tag{5}$$

which is known as the Paris Law. The parameters A and n are material properties that represent the material's resistance to fatigue crack growth.

(iii) *Fast fracture region* – the maximum value of the stress intensity factor in a cycle approaches K_C and crack growth accelerates.

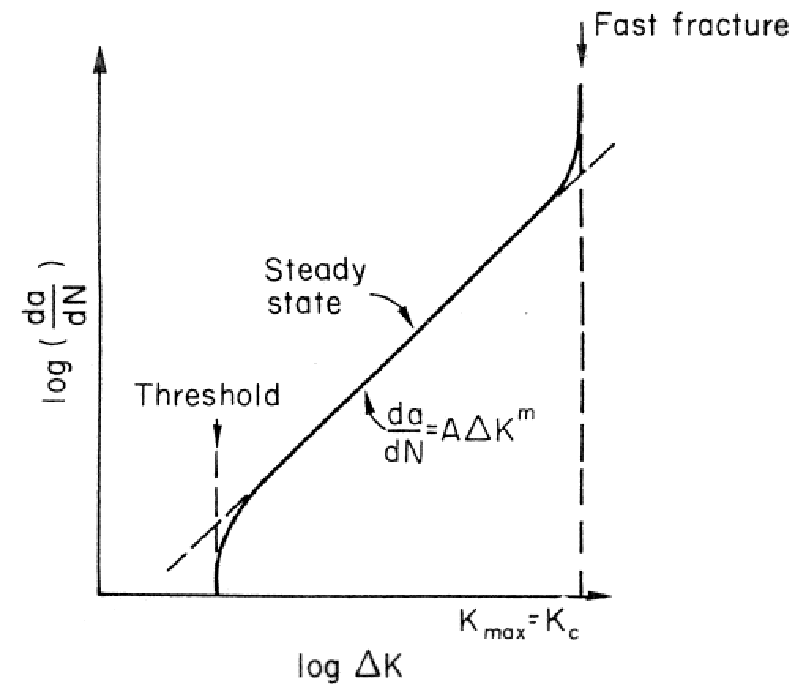


Figure 3: Crack Growth Rate Correlation

The amount of time spent in the power law region usually dominates, which means that once A and n in equation (5) have been determined, it is possible to predict fatigue crack growth in cracked structural components. By integrating (5) we see that the number of cycles, N, required to produce a crack length, a, in the component are given by

$$N = \int_{a_o}^{a} \frac{da}{A(\Delta K)^n}. \tag{6}$$

In general ΔK will be a function of the applied stress level and crack length in the component. The variation with crack length is determined by stress analysis of the most likely path the fatigue crack will follow. The number of cycles, N_f, required to achieve fast fracture occur when the crack length in the upper limit of the integral becomes a_c, the critical crack length at the maximum stress level in the cycle. That is, when

$$\sqrt{\pi a_c}\; Q(a_c) = \frac{K_C}{\sigma_{\max}} \tag{7}$$

where $Q(a_c)$ is the configuration factor determined from the stress analysis of the cracked component.

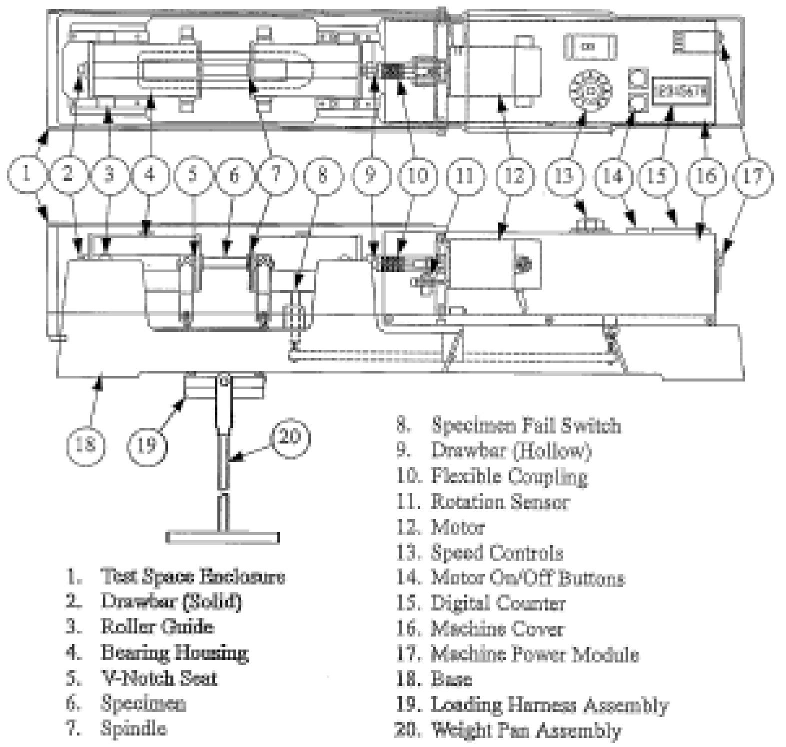


Fig. 4: Schematic of an R. R. Moore rotating bend device.

Procedure: The test in this week's lab is to determine the high-cycle fatigue behavior (S-N curve) of a 2024 T4 Aluminum alloy using the R.R. Moore High-speed Rotating Beam Testing Machine (Fig. 4). A specimen, whose circular cross section is a minimum at the center, is subjected to four-point bending while it rotates. This means that, over one cycle, any point on the surface experiences the maximum (tensile) and minimum (compressive) stresses provided by the device. From beam theory, $\sigma_{\max} = \frac{16WL}{\pi d_0^3} = -\sigma_{\min}$, where the overall load is W, L (4 in) is the distance between an outer support and the corresponding inner load point and d_0 is the diameter at the center of the specimen. As a result, the stress amplitude $\sigma_a = \frac{16WL}{\pi d_0^3}$ and the mean stress is zero in this test.

Each laboratory section will run the fatigue tests on two specimens, one with a standard lathe finish and the other with a sandblasted finish. The applied load (W) varies from section to section. The data will be pooled with existing data and those generated by other sections to obtain the S-N curves for the two types of specimens.

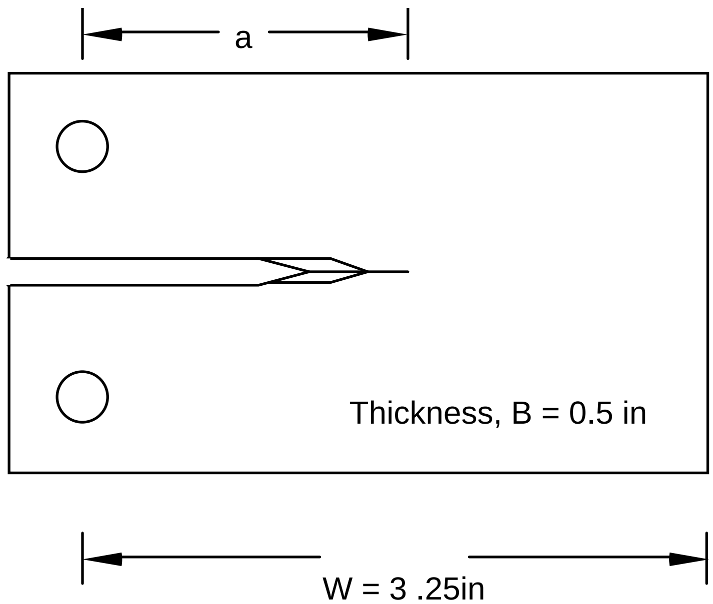


Figure 5: Fatigue Crack Growth Specimen

Next, the large compact tension specimen shown in Figure 5 is used to determine the fatigue resistance of a material. The specimen is loaded in a servohydraulic loading device which applies a cyclic load and automatically counts the number of cycles. The crack length is measured optically to obtain a plot of crack length *vs.* number of cycles. Crack growth rates (da/dN) are then calculated at various crack lengths. The stress intensity factor, K, at any of these crack lengths is given by

$$K = \frac{P}{B\sqrt{\pi W}} \frac{(2 + a/W)}{(1 - a/W)^{3/2}} Q\left(\frac{a}{W}\right) \tag{10}$$

where $Q(a/W)$ is tabulated below and a, W, B are shown in the figure. Thus the change in stress intensity factor, ΔK, over one cycle is given by

$$\Delta K = \frac{\Delta P}{B\sqrt{\pi W}} \frac{(2 + a/W)}{(1 - a/W)^{3/2}} Q\left(\frac{a}{W}\right) \tag{11}$$

where $\Delta P = P_{max} - P_{min} = (1 - R)\, P_{max}$ is the change in load P over one cycle. Note that the Δa over one cycle has a small effect and does not enter Eq. (11).

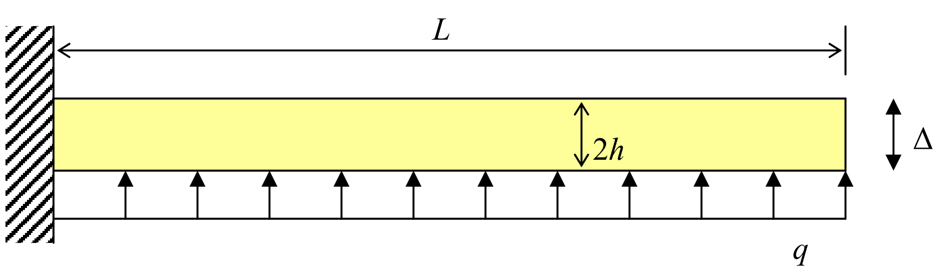
a/W	***Q(a/W)***
.2	2.070
.3	2.090
.4	1.836
.5	1.634
.6	1.460
.7	1.351
.8	1.314

Correlations of the type shown in Figure 3 are then made by plotting log (da/dN) *vs*. log (ΔK). The constants A and n are then extracted from the intercept and slope of the linear region.

Assignment: No formal lab report this week, but answer the following questions:

1. The data obtained from the rotating beam tests will be provided by the TA's from two sets of aluminum specimens. One set had the standard lathe finish while the other was sandblasted.
   a) Determine the stress amplitude from the load applied to each specimen.
   b) Plot the S-N curves for the two types of specimens.
   c) Fit the S-N curves to $\sigma_a = \sigma_{fat} + \frac{b}{N_f^c}$, and determine the properties σ_{fat}, b and c, for the two types of specimens.
   d) What is the mean stress during this test?

2. An aircraft wing is subjected to two sources of loading. The first comes from the lift, which may be assumed to be a uniform spanwise loading of intensity q = 52 lb/in. The second comes from flutter, which is by cyclic end deflection with amplitude $\Delta = \pm 8.64$ in, equally upwards and downwards. Consider the wing as a cantilever beam of unit thickness (1 in) with the span L = 20 ft and depth $2h$ = 20 in (see figure), made of an aluminum alloy ($E = 10^7$ psi and σ_y = 360 ksi). There is an average of 453 flutter cycles per flight, and the stress due to the lift acts as a mean stress for the cyclic loading. The fatigue *S*-*N* curve of the aluminum can be described by a function, $\sigma_a = 180 - 20\log_{10}(N)$, where the stress is in the unit of ksi, and the fatigue strength is determined to be 40 ksi (below which the fatigue life become infinite). The mean stress effect is accounted for through Goodman's rule, $\sigma_a = \sigma_{a0}(1 - \sigma_m / \sigma_y)$.



   a) Determine the location where the fatigue failure most likely initiates.
   b) Determine how much fatigue damage (in percentage) is caused by 1,000 flights at the previously determined location.
   c) If, after 1,000 flights as described above, the aircraft changes to a different flight with q = 65 lb/in for the lift, $\Delta = \pm 10.4$ in for the flutter, and 900 flutter cycles per flight, how many flights would it takes for the aircraft to fail?

3. The fatigue crack growth history for an aluminum specimen is tabulated in Table II. The geometry and dimensions of the specimen are given in the Lab Manual (Figure 5 of Lab 11).

The maximum applied load is 2222 lbs and the ratio between the minimum and maximum loads is 0.1. The cyclic load frequency is 5 Hz.

Table II

Number of Cycles	**Crack Length (inches)**
0	1.250
1540	1.375
2740	1.500
3440	1.625
3900	1.750
4190	1.875
4300	2.000
4330	2.125

a) Make a plot of crack length (a) vs number of cycles (N).

b) Determine the crack growth rate $\left(\frac{da}{dN}\right)$ as a function of the crack length (a).

c) Determine the change of stress intensity factor, ΔK, as a function of crack length. Use Equation (11) and the table in the Lab Manual. For a/W not listed in the table, use linear interpolation to find Q.

d) Plot a curve correlating the crack growth rate with ΔK.

e) Fit the steady state part of the crack growth curve with Paris law, $\frac{da}{dN} = C\left(\Delta K\right)^m$, and determine the constants m and C.

4. Now use a crack growth law obtained in 3e) for a "damage tolerance" application: A large edge-cracked aluminum plate is subjected to a constant amplitude cyclic stress of σ_{max} = 10 ksi and stress ratio R = 0.2. If the fracture toughness of the 2024-T3 aluminum is K_{IC} = 30 ksi $\sqrt{\text{in}}$ and the smallest crack that can be detected by NDE methods is 0.01 in., determine the remaining life of the plate once the crack has been detected. Assume that the plate dimensions are much larger than the critical crack size.

5. A hydraulic actuator that is used for extending and retracting a landing gear consists of a piston in a cylinder. The cylinder is only pressurized when the landing gear is extended. After many flights, the cylinder failed due to the growth of a radial crack in its wall (see figure). The stress intensity factor of this configuration is: $K = \dfrac{5pR_o^2\sqrt{\pi a}}{2(R_o^2 - R_i^2)}$, where p is the internal pressure, R_o and R_i are the outer and inner radii of the cylinder (3 in and 2.5 in,

respectively). The plane strain toughness of the material is 40 ksi√in, and the steady state fatigue crack growth is described by Paris law with constants $C = 10^{-20}$ and $m = 4$.

a) For a crack length $a = 0.1$ in, 11 striations were observed over a distance of 0.0036 inch near the end of the crack surface (in other words, the crack growth rate is about 0.0036/11 inch per cycle for the current crack length). Determine the operating pressure of the actuator.
b) Based on Paris law, how many flights can be made before the crack runs across the wall of the cylinder, if the initial crack length is 0.05 in.