Outline - APPM 5460 Proposal Outline

Luke Bury & Don Kuettel

March 21, 2018

0.1 Needs

- 2-4 pages
- Lit review
- dynamical system modeled by ordinary differential equations
- material from class

0.2 Outline

- Intro (don)
  - In this proposal, we will be investigating homoclinic orbits in the Circular Restricted Three-Body Problem
  - (tie to competition history)
  - Homoclinic orbits are ...
  - They are important because ...
- History w/ Historical Lit Review (luke)
  - As documented by Andersson and Barrow-Green [1, 2], in 1885, Acta Mathematica announced a mathematics competition to the world. This competition, sponsored by King Oscar II of Sweden, encouraged interested parties to make an attempt at solving one of four selected problems.
  - Henri Poincaré, a prominant mathematician of the time (who was largely favored to win the competition) chose to attempt the first problem, which essentially asked for a solution to the n-body problem. However, instead of the n-body problem, Poincaré decided to first attempt the three-body problem, since it was the first order of the problem that remained unsolved. To further simplify his initial effort, he restricted the three-body system in a manner known today as the Circularly Restricted Three Body Problem (CR3BP).

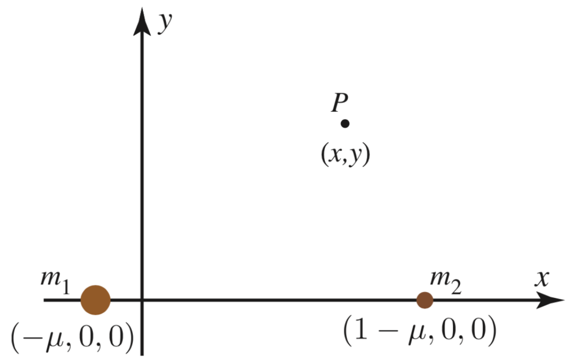

Figure 1: Layout of Circular Restricted Three Body Problem [4]

- In the CR3BP, the origin is set at the barycenter of two bodies of interest (eg, Earth & Moon), and the frame rotates so these bodies remain stationary on the x-axis. The bodies are assumed to move in perfectly circular orbits, and act as point masses from a gravitational perspective. The system is normalized so that the masses of the two bodies sum to 1 ($m_1 = \mu,\ m_2 = 1-\mu$), the distance between the bodies is 1, and the gravitational constant G is equal to 1.
- Under these conditions, the equations of motion for the CR3BP are:

$$\ddot{x} = 2\dot{y} + x - (1-\mu)\left(\frac{x+\mu}{R_1^3}\right) - \mu\left(\frac{x-1+\mu}{R_2^3}\right) \tag{1}$$

$$\ddot{y} = -2\dot{x} + y\left(-\frac{1-\mu}{R_1^3} - \frac{\mu}{R_2^3} + 1\right) \tag{2}$$

$$\ddot{z} = z\left(-\frac{1-\mu}{R_1^3} - \frac{\mu}{R_2^3}\right) \tag{3}$$

- Poincaré submitted his work on the CR3BP, won the competition, and collected the prize. However, around the time of some of the first printings of his work, a discussion with Lars Edvard Phragmén led to the discovery of an error within Poincaré's submissiion that held significant ramifications.
- The error was rooted in Poincaré's failing "to take proper account of the exact geometric nature of a particular curve" [3]. In Theorem III of the paper's first, and flawed, edition, Poincaré claimed that a particular invariant curve was closed (Figure 2(a,b)). He failed to consider that the curve could be self-intersecting (Figure 2(c)) - a mistake whose discovery cost him the fortune he had won, but also cemented his place among legendary mathematicians for his resulting discovery of *doubly asymptotic*, or *homoclinic*, points in dynamical systems.

• Application w/ Modern Lit Review

- Much research has been conducted in this field since (references, references, references)

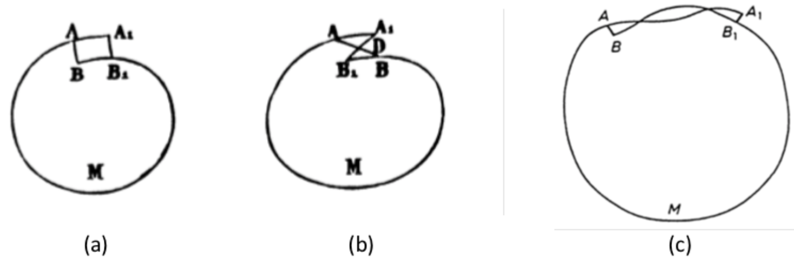

Figure 2: Invariant curve with self intersection from Poincaré's corrected work [3]

- (pg 72+ of book)
- we will look at / recreate Poincaré's work
- we will create homoclinic orbits in the CR3BP using intersections of stable and unstable orbits in Poincaré plots

References

[1] K. G. Andersson. Poincaré's discovery of homoclinic points. *Archive for History of Exact Sciences*, 48(2):133–147, 1994.

[2] J. Barrow-Green. Oscar ii's prize competition and the error in poincaré's memoir on the three body problem. *Archive for History of Exact Sciences*, 48(2):107–131, 1994.

[3] J. Barrow-Green. *Poincaré and the Three Body Problem.* American Mathematical Society, 1997.

[4] Lo M. Marsden J. Ross S. Sang Koon, W. *Dynamical Systems, The Three-Body Problem, and Space Mission Design.* Marsden Books, 2011.