

Outline - APPM 5460 Proposal Outline

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March 21, 2018

0.1 Needs

- 2-4 pages
- Lit review
- dynamical system modeled by ordinary differential equations
- material from class

0.2 Outline

- Intro (don)
 - In this proposal, we will be investigating homoclinic orbits in the Circular Restricted Three-Body Problem
 - (tie to competition history)
 - Homoclinic orbits are ...
 - They are important because ...
- History w/ Historical Lit Review (luke)
 - As documented by Andersson and Barrow-Green [1, 2], in 1885, Acta Mathematica announced a mathematics competition to the world. This competition, sponsored by King Oscar II of Sweden, encouraged interested parties to make an attempt at solving one of four selected problems.
 - Henri Poincaré, a prominent mathematician of the time (who was largely favored to win the competition) chose to attempt the first problem, which essentially asked for a solution to the n-body problem. However, instead of the n-body problem, Poincaré decided to first attempt the three-body problem, since it was the first order of the problem that remained unsolved. To further simplify his initial effort, he restricted the three-body system in a manner known today as the Circularly Restricted Three Body Problem (CR3BP).

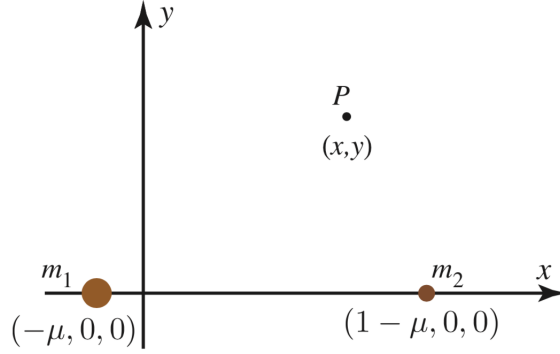


Figure 1: Layout of Circular Restricted Three Body Problem [4]

- In the CR3BP, the origin is set at the barycenter of two bodies of interest (eg, Earth & Moon), and the frame rotates so these bodies remain stationary on the x-axis. The bodies are assumed to move in perfectly circular orbits, and act as point masses from a gravitational perspective. The system is normalized so that the masses of the two bodies sum to 1 ($m_1 = \mu$, $m_2 = 1 - \mu$), the distance between the bodies is 1, and the gravitational constant G is equal to 1.
- Under these conditions, the equations of motion for the CR3BP are:

$$\ddot{x} = 2\dot{y} + x - (1 - \mu) \left(\frac{x + \mu}{R_1^3} \right) - \mu \left(\frac{x - 1 + \mu}{R_2^3} \right) \quad (1)$$

$$\ddot{y} = -2\dot{x} + y \left(-\frac{1 - \mu}{R_1^3} - \frac{\mu}{R_2^3} + 1 \right) \quad (2)$$

$$\ddot{z} = z \left(-\frac{1 - \mu}{R_1^3} - \frac{\mu}{R_2^3} \right) \quad (3)$$

- Poincaré submitted his work on the CR3BP, won the competition, and collected the prize. However, around the time of some of the first printings of his work, a discussion with Lars Edvard Phragmén led to the discovery of an error within Poincaré's submission that held significant ramifications.
- The error was rooted in Poincaré's failing "to take proper account of the exact geometric nature of a particular curve" [3]. In Theorem III of the paper's first, and flawed, edition, Poincaré claimed that a particular invariant curve was closed (Figure 2(a,b)). He failed to consider that the curve could be self-intersecting (Figure 2(c)) - a mistake whose discovery cost him the fortune he had won, but also cemented his place among legendary mathematicians for his resulting discovery of *doubly asymptotic*, or *homoclinic*, points in dynamical systems.
- Application w/ Modern Lit Review
 - Based on these famous results of Poincaré's work, our project will involve locating the intersection points of stable and unstable manifolds for our system of differential equa-

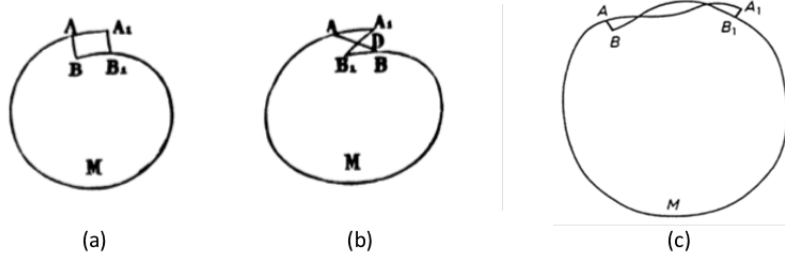


Figure 2: (a) Diagrams of incorrectly closed invariant curves from the first edition of Poincaré's memoir. (b) Invariant curve with self intersection from Poincaré's corrected work [3]

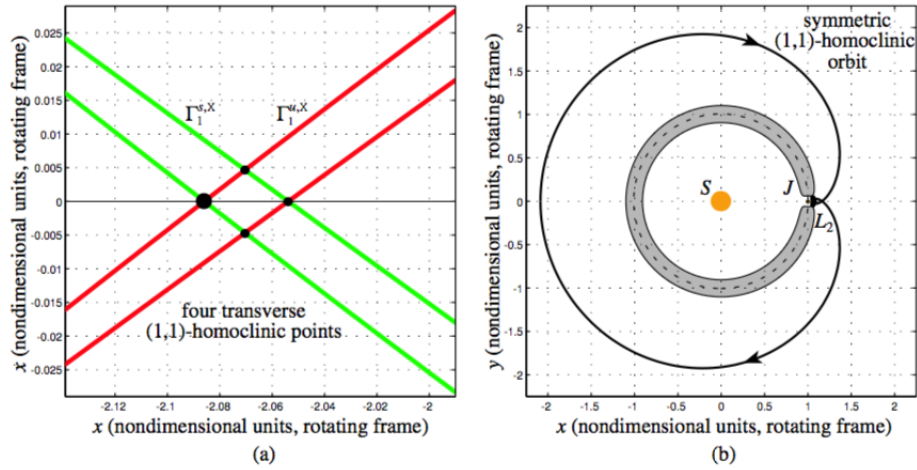


Figure 3: (a) A group of four homoclinic points lying at the intersection of stable (green) and unstable (red) manifolds for the Sun-Jupiter system shown in (b) [4]

tions in order to find homoclinic trajectories. Our first goal will be to use this method to recreate the trajectory shown in Figure 3. The second goal of our project will be to investigate how these manifolds and homoclinic trajectories change with small deviations in μ , the mass ratio for a 3-body system.

References

- [1] K. G. Andersson. Poincaré's discovery of homoclinic points. *Archive for History of Exact Sciences*, 48(2):133–147, 1994.
- [2] J. Barrow-Green. Oscar ii's prize competition and the error in poincaré's memoir on the three body problem. *Archive for History of Exact Sciences*, 48(2):107–131, 1994.

- [3] J. Barrow-Green. *Poincaré and the Three Body Problem*. American Mathematical Society, 1997.
- [4] Lo M. Marsden J. Ross S. Sang Koon, W. *Dynamical Systems, The Three-Body Problem, and Space Mission Design*. Marsden Books, 2011.