Modeling of permanent magnets in three-dimensional space using edge finite elements

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A surface integration method for modeling of permanent magnets in three-dimensional space using edge finite elements is presented, where the value of the coercive magnetic force $\mathbf{H}_c$ is integrated only over the entire surface area of the permanent magnet. This allows direct computation of the equivalent source current values, which are afterwards assigned to the surface edges of the permanent magnet. The main advantages of the proposed method are: accurate results with less computational effort and improved convergence rate of the iterative solver. Verification of this surface integration method is carried by comparing the results with those obtained by the Biot–Savart law and the traditional current sheet method. Finally, a comparison between the numerical results obtained through the surface integration method and the measured results of a complex electromagnetic device with permanent magnets is given.
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I. INTRODUCTION

In recent years new permanent magnetic materials such as ceramic or rare earth magnets have been introduced and become commonly available at a relatively low price. The use of permanent magnets (PMs) in various electrical devices, therefore, has experienced a sizable increase making it very important to be able to accurately compute the magnetic field phenomena inside electrical devices with PMs.

The main reason why numerical modeling of PMs is not very common can be located in the existence of nonexplicit field sources such as excitation currents or known voltage sources. Therefore, several specific methods for the numerical modeling of PMs have been already investigated.[1,2] To model a PM system means to substitute the existing PM system with a non-PM system so that both systems are magnetically equivalent. The easiest and most widely employed method for modeling PM devices is the current sheet method which, however, has a major disadvantage: it is applicable only to simple geometrical PM shapes. Improvements of this method, with the aim of extending its applicability to more complicated PM shapes have been investigated, almost all of which assuming rather rigorous approximations and much effort.[3]

Owing mainly to its computational advantages, the finite element method (FEM) based on edge finite elements has recently become widely employed. However, as a result of nonexact satisfaction of the solenoidal character of the source current which usually occurs if edge finite elements are used in connection with the current sheet method (this being especially true for complicated shapes of PMs and ultra thin sheet conductors) poor convergence rate of the iteration process and long computation time can be experienced.

In this article, a surface integration method for three-dimensional modeling of PMs using edge finite elements is described. The proposed method is based on integration of the coercive force $\mathbf{H}_c$ along the entire surface area of PM. First, a short outline is given and a mathematical basis of the method emphasizing its application to edge based FEA is established. Next, verification is carried out using two models: a simple test model and an application model with rather complex geometry. The numerical results obtained using the surface integration method are compared with those obtained by the Biot–Savart law and the current sheet method for the test model, and with measured results for the application model. It is shown that the surface integration method is applicable to any PM shape and exhibits improvement of the convergence rate of the iterative solver attaining at the same time highly accurate results.

II. OUTLINE OF THE PROPOSED METHOD

A. Mathematical background

In edge FEA, the unknown variables, boundary conditions, and source vectors have to be assigned directly to the edges of the mesh, not to the nodes. This statement is also true for the source current values which, in edge FEA, can be assigned directly to mesh edges as source current intensity values, or indirectly by means of the current vector potential.[4] Using the current sheet method, only simple PM shapes can be modeled. Another problem is satisfaction of the solenoidal character of the source current which can be strongly emphasized in the cases of ultrathin conductors and complicated PM shapes. Use of ultrathin conductors in the current sheet method is imperative, however, if an accurate analysis is needed. Decreasing the thickness of the current sheet towards zero results in increasing the accuracy of the results. Thus, theoretically, for conductor thickness close to zero, the approximation will give best results. However, as the current carrying area decreases toward zero it is progressively more difficult to define the equivalent current density values. As obvious from the analysis presented next, using the proposed surface integration method over a mesh of edge based finite elements solves this problem elegantly.

Typical working demagnetization curve of PM can be expressed with the following equation

$$\mathbf{H}=\nu(\mathbf{B})\mathbf{B}-\mathbf{H}_c, \tag{1}$$

where ν is the reluctivity coefficient, and $\mathbf{H}_c$ is the coercivity. Substituting Eq. (1) in Ampere's law we obtain the following equation

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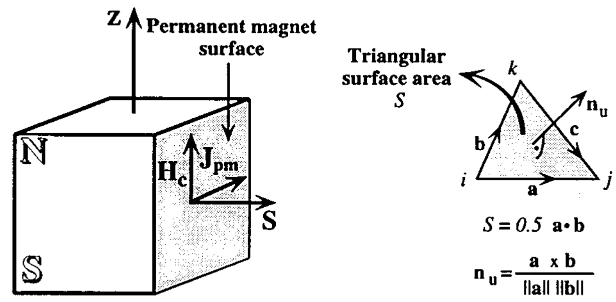

FIG. 1. Simple test model with definition of the equivalent source current $\mathbf{J}_{pm}$, normal vector $\mathbf{n}_u$, and triangular surface area S.

$$\nabla \times \nu(\mathbf{B})\mathbf{B} = \mathbf{J}_0 + \nabla \times \mathbf{H}_c . \tag{2}$$

In Eq. (2), the second term on the right side, $\nabla \times \mathbf{H}_c$, is the equivalent current density value of the permanent magnet $\mathbf{J}_{pm}$, whose numerical implementation is the main topic of this article and is being addressed in the next section. Here, we point out that in the case of a linear magnetic circuit, the reluctivity coefficient ν has to be computed only once as $\nu = \mathbf{H}_c/\mathbf{B}_r$, where $\mathbf{B}_r$ is the residual magnetization. For nonlinear magnetic circuits, however, the demagnetization curve must be shifted to the right for the amount of the coercivity $\mathbf{H}_c$, and the reluctivity coefficient must be recomputed at each nonlinear step as $\nu(\mathbf{B}) = (\mathbf{H} + \mathbf{H}_c)/\mathbf{B}$.

B. Numerical implementation of the surface integration method in edge based FEA

Using magnetic vector potential formulation, the governing equation for magnetostatic problems without source current is

$$\nabla \times (\nu \nabla \times \mathbf{A}) = \nabla \times \mathbf{H}_c , \tag{3}$$

where $\mathbf{A}$ is the magnetic vector potential. In Eq. (3) the existence of the current source $\mathbf{J}_0$ is neglected in order to simplify the problem and in no way aggravates the generalization of our approach. The right hand term $\nabla \times \mathbf{H}_c$ is the equivalent current density vector of the permanent magnet $\mathbf{J}_{pm}$. Applying the Curl theorem on the right hand side of Eq. (3) we obtain

$$\int_V \nabla \times \mathbf{H}_c \, dV = \int_S \mathbf{H}_c \times d\mathbf{S}, \tag{4}$$

where $d\mathbf{S}$ is the vectorial surface area with intensity equal to the area of each triangular surface and direction $\mathbf{n}_u$ normal to that surface (see Fig. 1). This procedure must be performed for each finite element ''inside'' the PM area. Since the outward normals of two adjoining surfaces always have opposite directions, the first and very important conclusion is that the integral Eq. (4) for each surface that lies inside PM is canceled. Therefore, the integration is reduced only to the entire PM surface area. However, from Eq. (4) another important conclusion emerges: since the integral Eq. (4) involves the cross-product between the coercive vector $\mathbf{H}_c$ and the outward normal of the PMs surface area $\mathbf{n}_u$, all surfaces which have an outward normal collinear with the direction of the coercivity vector $\mathbf{H}_c$ must also be excluded from the analysis, further reducing the size of the computational region.

Finally, using Eq. (4) we can compute the intensity and direction of the equivalent source current vector $\mathbf{I}_e$ using the following equation

$$\mathbf{I}_e = \frac{(\mathbf{H}_c \times \mathbf{n}_u) S}{l_e} . \tag{5}$$

In continuation, vector $\mathbf{I}_e$ must be assigned to surface edge e with length l_e. The computation of the triangular surface

TABLE I. Proposed method vs current sheet method and the Biot–Savart law.

			Bio–Savart law	Proposed method
Coil thickness (mm)	Current sheet method		0.2	1.0
B_{max} (T)	1.012	0.999	1.025	1.020
Number of iterations	48	32	⋯	22

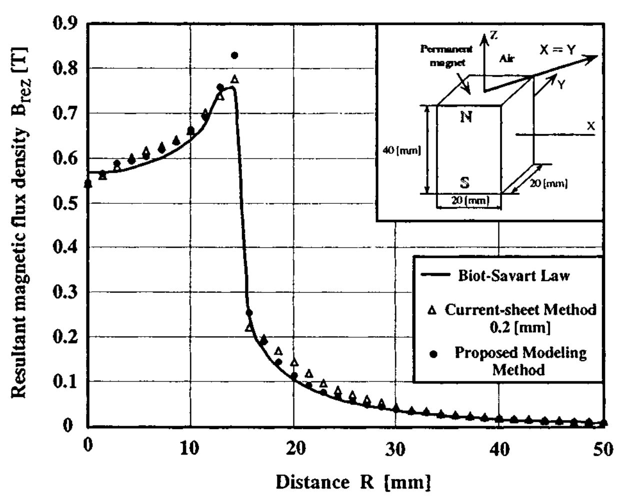

FIG. 2. Results comparison for resultant magnetic flux density along line $X=Y$ at $Z=20$ mm.

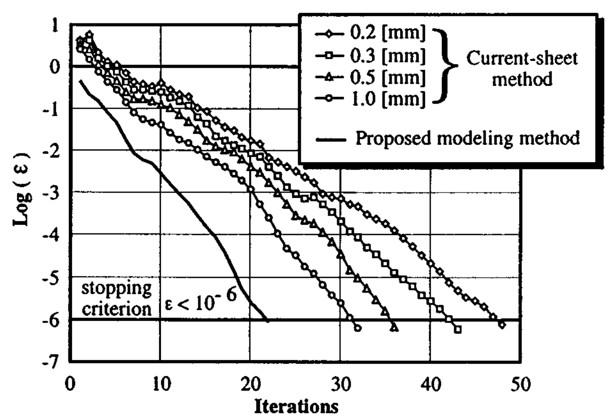

FIG. 3. Convergence rate.

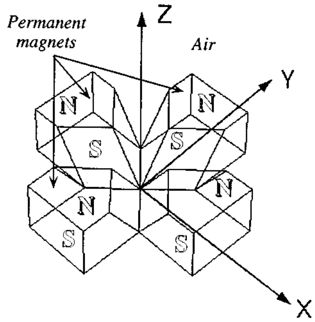

FIG. 4. Application model with complicated geometry.

area S and its outward normal vector $\mathbf{n}$ is computationally inexpensive and can be computed easily using two out of its three edges as shown in Fig. 1.

III. VERIFICATION OF THE SURFACE INTEGRATION METHOD

A. Simple test model

A rectangular PM as shown in Fig. 2, with relative permeability coefficient $\nu_r = 1.07$ and coercivity $\mathbf{H}_c = \{0, 0, H_{cz}\} = \{0, 0, 870000\}$ A/m was considered as a simple test model. This model was analyzed using the proposed surface integration method, the current sheet method, and the Biot–Savart Law. For the current sheet method, in order to evaluate the influence of the sheet thickness on the accuracy of the results, several current sheets with various thicknesses were developed. The obtained results for the resultant magnetic flux density $\mathbf{B}$ along line $X = Y$ at $Z = 20$ mm are presented in Fig. 2. The maximum relative errors along this line for the current sheet method and for the proposed surface integration method were 29.5 and 9.9%, respectively. It is apparent that the accuracy of the proposed method is greater than that of the current sheet method. In addition, the proposed method exhibits improvements in the convergence rate. This can be understood from the results given in Table I and Fig. 3. From Fig. 3 it is clear that the surface integration method requires fewer numbers of iterations for the same residual. In the case of the current sheet method, the number of iterations and the accuracy of the results are inversely proportional to the current sheet thickness.

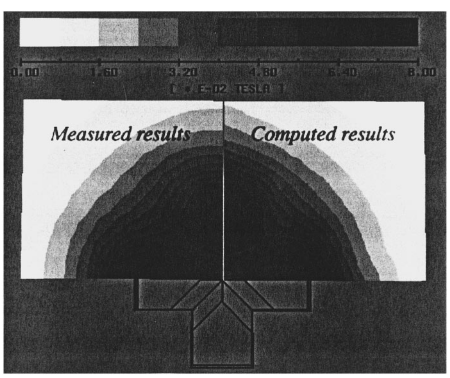

FIG. 5. Comparison between measured and computed results.

B. Application model

The surface integration method was also applied for analysis of the magnetic field distribution of the three-dimensional model with complex geometry presented in Fig. 4. This model comprises four symmetrical PMs with rather complicated shapes. Only 1/4 of the model was analyzed using the proposed modeling method. Due to the complexity of the model, the Biot–Savart law was not applicable for verification of the results. For this model, however, the measured results were available. In Fig. 5, a comparison between the three-dimensional magnetic flux density distribution obtained by the proposed surface integration method and that obtained from the measurements is presented. It is apparent that both results are almost identical.

IV. CONCLUSIONS

A new surface integration method for the modeling of PMs in three-dimensional space using edge finite elements is presented. The method is based on the integration of the coercive force vector over the entire PM surface area and exhibits several improvements over the conventional current sheet method, the most noteworthy improvements being: computational efficiency and accuracy, and easy modeling of arbitrary three-dimensional PM shapes. The results obtained with the surface integration method are in very good agreement with the measurements and with the results obtained by the Biot–Savart law.

[1] J. R. Brauer, L. A. Larkin, and V. D. Overbye, J. Appl. Phys. **55**, 2183 (1984).
[2] R. H. Vander Heiden, J. R. Brauer, J. J. Ruehl, and G. A. Zimmerlee, IEEE Trans. Magn. **24**, 2931 (1988).
[3] F. A. Fouad, T. W. Nehl, and N. A. Demerdash, IEEE Trans. Magn. **17**, 3002 (1981).
[4] V. Čingoski, K. Kaneda, and H. Yamashita, *Applied Electromagnetics in Materials*, edited by Y. Ishihara and E. Matsumoto, JSAEM Studies in Applied Electromagnetics (JSAEM, 1994), Vol. 3, pp. 357–367.