Quiz 2

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Math 257

1 Problem 1

To solve this problem, I will create a system of equations representing the flow in and out of each node as a singular equation. Since there are 4 nodes, I will have 4 equations. For the top left node, I have:

$$x_1 + x_2 = 300$$

For the top right node:

$$x_1 + x_3 = x_4 + 150 \implies x_1 + x_3 - x_4 = 150$$

For the bottom left node:

$$200 + x_2 = x_3 + x_5 \implies -x_2 + x_3 + x_5 = 200$$

For the bottom right node:

$$x_4 + x_5 = 350$$

Answer to Part A: Taken together, these equations create the system:

$$x_1 + x_2 = 300$$

$$x_1 + x_3 - x_4 = 150$$

$$-x_2 + x_3 + x_5 = 200$$

$$x_4 + x_5 = 350$$

Answer to Part B: Now that I have 4 distinct equations that relate the variables to each other, I will represent the system as an augmented matrix:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 300 \\ 1 & 0 & 1 & -1 & 0 & 150 \\ 0 & -1 & 1 & 0 & 1 & 200 \\ 0 & 0 & 0 & 1 & 1 & 350 \end{bmatrix}$$

I immediately notice that there are less equations than there are variables, so the system must have infinitely many solutions or none. I will put the augmented matrix in reduced row echelon form using a calculator. This yields the new matrix:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 500 \\ 0 & 1 & -1 & 0 & -1 & -200 \\ 0 & 0 & 0 & 1 & 1 & 350 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

From this augmented matrix in reduced row-echelon form, I will rewrite the system of linear equations:

$$x_1 + x_3 + x_5 = 500$$

$$x_2 - x_3 - x_5 = -200$$

$$x_4 + x_5 = 350$$

Per the instructions, I will set $x_3 = t$ and $x_5 = r$ and solve using these parameters:

$$x_1 + t + r = 500 \implies x_1 = 500 - t - r$$

$$x_2 = t + r - 200$$

$$x_4 = 350 - r$$

From these equations, I have an infinite number of solutions given by any value of the parameter t and r:

$$(x_1, x_2, x_3, x_4, x_5) = \boxed{(500 - t - r, t + r - 200, t, 350 - r, r)}$$

Answer to Part C: Substituting $x_3 = t = 170$ and $x_2 = 35$ into the solution above, I have:

$$x_2 = \boxed{35} = 170 + r - 200 \implies r = x_5 = \boxed{65}$$

$$x_1 = 500 - 170 - 65 = \boxed{265}$$

$$x_4 = 350 - 65 = \boxed{285}$$

Putting all of these together, I have the solution:

$$(x_1, x_2, x_3, x_4, x_5) = \boxed{(265, 35, 170, 285, 65)}$$

2 Question 2

I will start by rearranging the equation:

$$3X + 2A = B \implies X = \frac{1}{3}(B - 2A)$$

I will then find $2A$:

$$2A = 2\begin{bmatrix} 1 & -5 \\ 2 & -3 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -10 \\ 4 & -6 \\ -8 & 0 \end{bmatrix}$$

Subtracting $2A$ from B, I have:

$$\begin{bmatrix} 2 & -2 \\ 4 & 1 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -10 \\ 4 & -6 \\ -8 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 8 \\ 0 & 7 \\ 12 & 1 \end{bmatrix}$$

Finally, multiply by a scalar of $\frac{1}{3}$, I have:

$$\frac{1}{3} \cdot \begin{bmatrix} 0 & 8 \\ 0 & 7 \\ 12 & 1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{8}{3} \\ 0 & \frac{7}{3} \\ 4 & \frac{1}{3} \end{bmatrix}$$