Quiz 1

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Math 257

1 Problem 1

I am given:

$$2x_1 + x_2 - 2x_3 = 4$$

$$4x_1 + 2x_3 = 10$$

$$-6x_1 + 4x_2 - 15x_3 = -19$$

Transforming this into an augmented matrix, we have:

$$\begin{bmatrix} 2 & 1 & -2 & 4 \\ 4 & 0 & 2 & 10 \\ -6 & 4 & -15 & -19 \end{bmatrix}$$

To work towards a leading 1 in the first column, I will do the following elementary row operations:

$$3R_1 + R_3 \implies R_3$$

$$\begin{bmatrix} 2 & 1 & -2 & 4 \\ 4 & 0 & 2 & 10 \\ 0 & 7 & -21 & -7 \end{bmatrix}$$

$$2R_1 - R_2 \implies R_2$$

$$\begin{bmatrix} 2 & 1 & -2 & 4 \\ 0 & 2 & -6 & -2 \\ 0 & 7 & -21 & -7 \end{bmatrix}$$

$$\frac{1}{2}R_1 \implies R_1$$

$$\begin{bmatrix} 1 & \frac{1}{2} & -1 & 2 \\ 0 & 2 & -6 & -2 \\ 0 & 7 & -21 & -7 \end{bmatrix}$$

Now, I will work to get a leading one in the second column using the following elementary row operations:

$$\frac{1}{2}R_2 \implies R_2$$

$$\begin{bmatrix} 1 & \frac{1}{2} & -1 & 2 \\ 0 & 1 & -3 & -1 \\ 0 & 7 & -21 & -7 \end{bmatrix}$$

$$\frac{1}{7}R_3 \implies R_3$$

$$\begin{bmatrix} 1 & \frac{1}{2} & -1 & 2 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{bmatrix}$$

$$R_2 - R_3 \implies R_2$$

$$\begin{bmatrix} 1 & \frac{1}{2} & -1 & 2 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Using this matrix in row-echelon form, I can reconstruct the system of equations:

$$x_1 + \frac{1}{2}x_2 - x_3 = 2$$

$$x_2 - 3x_3 = -1$$

$$0x_1 + 0x_2 + 0x_3 = 0 \implies 0 = 0$$

As per the instructions, if I let $x_3 = t$, this yields:

$$x_1 + \frac{1}{2}x_2 - t = 2$$

$$x_2 - 3t = -1$$

Solving for x_2 with the parameter t, and then using backsubstitution, I get:

$$\boxed{x_2 = 3t - 1}$$

$$x_1 + \frac{1}{2}(3t - 1) - t = 2$$

$$x_1 + \frac{3}{2}t - t - \frac{1}{2} = 2$$

$$x_1 + \frac{1}{2}t = \frac{5}{2}$$

$$\boxed{x_1 = \frac{5}{2} - \frac{1}{2}t}$$

Therefore, the infinite set of solutions is:

$$x_1 = \frac{5}{2} - \frac{1}{2}t$$

$$x_2 = 3t - 1$$

$$x_3 = t$$

2 Problem 2

I am given:

$$x - y - z = 0$$

$$x + 2y - z = 6$$

$$2x - z = 5$$

Transforming this into an augmented matrix, I get:

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 1 & 2 & -1 & 6 \\ 2 & 0 & -1 & 5 \end{bmatrix}$$

To get a leading 1 in the first column, I will do the following elementary row operations:

$$R_1 - R_2 \implies R_2$$

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & -3 & 0 & -6 \\ 2 & 0 & -1 & 5 \end{bmatrix}$$

$$2R_1 - R_3 \implies R_3$$

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & -3 & 0 & -6 \\ 0 & -2 & -1 & -5 \end{bmatrix}$$

To get a leading one in the second column, I will do the following row operations:

$$-\frac{1}{3}R_2 \implies R_2$$

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & -2 & -1 & -5 \end{bmatrix}$$

$$2R_2 + R_3 \implies R_3$$

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

$$-R_3 \implies R_3$$

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Reconstructing a system of linear equations from the matrix in row echelon form, I get:

$$x - y - z = 0$$

$$\boxed{y = 2}$$

$$\boxed{z = 1}$$

Using backsubstitution with the values of y and z in the first equation, I have:

$$x - 2 - 1 = 0$$

$$\boxed{x = 3}$$

Therefore, the solution to the system of linear equations is:

$$(x, y, z) = \boxed{(3, 2, 1)}$$