Question 7

Solve the following questions from the Discrete Math zyBook:

Exercise 6.1.5, sections b-d

A 5-card hand is dealt from a perfectly shuffled deck of playing cards. What is the probability of each of the following events?

b. $\frac{C(13,1) * C(4,3) * C(12,2) * C(4,1) * C(4,1)}{C(52,5)} = \frac{54912}{2{,}598{,}960} = \frac{88}{4165}$

c. $\frac{C(4,1) * C(13,5)}{C(52,5)} = \frac{4 * 1287}{C(52,5)} = \frac{5148}{2598960} = \frac{33}{16660}$

d. $\frac{C(13,1) * C(4,2) * C(12,3) * C(4,1) * C(4,1) * C(4,1)}{C(52,5)} = \frac{1098240}{2598960} = \frac{352}{833}$

Exercise 6.2.4, sections a-d

A 5-card hand is dealt from a perfectly shuffled deck of playing cards. What is the probability of each of the following events?

a. $1 - \frac{C(39,5)}{C(52,5)} = \frac{575757}{2598960} = \frac{2109}{9520}$

b. $1 - \frac{C(13,5) * 4^5}{C(52,5)}$

$\frac{C(13,5) * 4^5}{C(52,5)} = \frac{1317888}{2598960}$

$1 - \frac{1317888}{2598960} = \frac{2112}{4165}$

c. $\frac{C(13,1)*C(39,4)}{C(52,5)} + \frac{C(13,1)*C(39,4)}{C(52,5)} - \frac{C(13,1) * C(13,1) * C(26,3)}{C(52,5)}$

$= \frac{65351}{99960}$

d. $1 - \frac{C(26,5)}{C(52,5)}$

$\frac{C(26,5)}{C(52,5)} = \frac{65780}{2598960} = \frac{253}{9996}$

$1 - \frac{253}{9996} = \frac{9743}{9996}$

Question 8

Solve the following questions from the Discrete Math zyBook:

Exercise 6.3.2, sections a-e

a. $p(A) = \frac{6!}{7!} = \frac{1}{7}$

$p(B) = \frac{7!}{2!} = \frac{1}{2}$

$p(C) = \frac{5!}{7!} = \frac{1}{42}$

b. $p(A|C) = \frac{|A \cap C|}{|C|} = \frac{2! * 3!}{5!} = \frac{1}{10}$

c. $p(B|C) = \frac{|B \cap C|}{|C|} = \frac{\frac{5!}{2}}{5!} = \frac{1}{2}$

d. $p(A|B) = \frac{|A \cap B|}{|B|} = \frac{3*5!}{\frac{7!}{2!}} = \frac{1}{7}$

e. A and C are not independent, because $P(A|C) \neq P(A)$

B and C are independent, because $P(B|C) = P(B) = \frac{1}{2}$

A and B are independent, because $P(A|B) = P(A) = \frac{1}{7}$

Exercise 6.3.6, sections b, c

b. First 5 heads = $\frac{1}{3} * \frac{1}{3} * \frac{1}{3} * \frac{1}{3} * \frac{1}{3} = (\frac{1}{3})^5$

Last 5 Tails = $\frac{2}{3} * \frac{2}{3} * \frac{2}{3} * \frac{2}{3} \frac{2}{3} = (\frac{2}{3})^5$

$(\frac{2}{3})^5 * (\frac{1}{3})^5 = \frac{32}{59049}$

c. $\frac{1}{3} * (\frac{2}{3})^9 = \frac{512}{59049}$

Exercise 6.4.2, section a

a. $A\ =\ The\ dice\ chosen\ is\ fair$

$\overline{A}\ =\ The\ biased\ dice\ is\ chosen$

$P(A)\ =\ P(\overline{A})\ =\ \frac{1}{2}$

$B\ =\ the\ dice\ came\ up\ 4,\ 3,\ 6,\ 6,\ 5,\ 5$

$P(B|\overline{A})\ =\ (\frac{3}{20})^4\ *\ (\frac{1}{4})^2$

$P(B|A)\ =\ (\frac{1}{6})^6$

$$P(A|B)\ =\ \frac{P(B|A)P(A)}{P(B|A)P(A)+P(B|\overline{A})P(\overline{A})}$$

$$=\ \frac{(\frac{1}{6})^6\frac{1}{2}}{(\frac{1}{6})^6*\frac{1}{2}*(\frac{3}{20})^4(\frac{1}{4})^2(\frac{1}{2})}\ =\ \frac{40000}{99049}\ =\ 0.404$$

Question 9:

Solve the following questions from the Discrete Math zyBook:

Exercise 6.5.2, sections a, b

a. The range of *A* is {0,1,2,34}

b.

$(0, \frac{C(48,5)}{C(52,5)}), (1, \frac{C(4,1)*C(48,4)}{C(52,5)}), (2, \frac{C(4,2)*C(48,3)}{C(52,5)}), (3, \frac{C(4,3) * C(48, 2)}{C(52,5)}),$
$(4, \frac{C(4,4)*C(48,1)}{C(52,5)})$

Exercise 6.6.1, section a

a.

0, 1, or 2 potential choices

C(10,2) potential outcomes

0 = C(3,2)

1 = (7,1)

2 - (7,2)

$(0 * \frac{3}{45}) + (1 * \frac{21}{45}) + (2 * \frac{21}{45})$

$E\lfloor G\rfloor = 1.4$

Exercise 6.6.4, sections a, b

a. Range of $X = \{1, 4, 9, 16, 25, 36\}$

$E\lfloor X\rfloor = (1 * \frac{1}{6}) + (4 * \frac{1}{6}) + (9 * \frac{1}{6}) + (16 * \frac{1}{6}) + (25 * \frac{1}{6}) + (36 * \frac{1}{6})$

$E\lfloor X\rfloor$ = 15.166

b. Range of $Y = \{0, 1, 4, 9\}$

$E\lfloor Y\rfloor = (0 * \frac{1}{8}) + (1 * \frac{1}{8}) + (4 * \frac{1}{8}) + (9 * \frac{1}{8})$

$E\lfloor Y\rfloor = 3$

Exercise 6.7.4, section a

a. $P(child\ gets\ coat) = \frac{1}{10}$

$\frac{1}{10} * 10 = 1$

Question 10:

Solve the following questions from the Discrete Math zyBook:

Exercise 6.8.1, sections a-d

The probability that a circuit board produced by a particular manufacturer has a defect is 1%. You can assume that errors are independent, so the event that one circuit board has a defect is independent of whether a different circuit board has a defect.

a. $b(2; 100, \frac{1}{100}) = C(100, 2) * (\frac{1}{100})^2(\frac{99}{100})^2$ = 0.18

b. Find the probability with 0 defects and 1 defect and subtract it from 1

$$1 - (C(100, 0) * (\frac{1}{100})^0(\frac{99}{100})^{100} + C(100, 1) * (\frac{1}{100})^1(\frac{99}{100})^{99}$$

c. $100 * \frac{1}{100} = 1$

d.

Probability that 1 batch out of 50 (2 out of 100) has a defect:
Find the probability that none have defects:

$$b(0;\ 50,\ \frac{1}{100}) = C(50, 0)^0(\frac{99}{100})^{50}$$

Subtract it from 1:

$$1 - b(0;\ 50,\ \frac{1}{100}) = C(50, 0)^0(\frac{99}{100})^{50}$$

Expected numbers of circuit boards with defects out of 100:

$$2 * (50 * \frac{1}{100}) = 1$$

The probability that at least 2 have defects differ if they are made separately versus made in batches, but the expected number of circuit boards with defects are the same.

Exercise 6.8.3, section b

b. You reach an incorrect conclusion if the number of heads in the coin flips are more than 3.

$$
\begin{aligned}
P(0;\ 10,\ 0.3) &= C(10, 0)^{0}(0.3)(0.7)^{10} \\
P(1;\ 10,\ 0.3) &= C(10, 1)^{1}(0.3)(0.7)^{9} \\
P(2;\ 10,\ 0.3) &= C(10, 2)^{2}(0.3)(0.7)^{8} \\
P(3;\ 10,\ 0.3) &= C(10, 1)^{3}(0.3)(0.7)^{7}
\end{aligned}
$$

Add up the sum and subtract the sum from $1 = 0.35$