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Homework 1

Question 1

A. Convert the following numbers to their decimal representation. Show your work.

1.A.1:

$10011011_2 = 155_{10}$

$1 * 2^0 + 1 * 2^1 + 0 * 2^2 + 1 * 2^3 + 1 * 2^4 + 0 * 2^5 + 0 * 2^6 + 1 * 2^7$
1 + 2 + 0 + 8 + 16 + 0 + 0 + 128
1 + 2 + 8 + 16 + 128 = 155

1.A.2:

$456_7 = 237_{10}$
$6 * 7^0 + 5 * 7^1 + 4 * 7^2$
6 + 35 + 196 = 237

1.A.3:

$38A_{16} = 906_{10}$
$A_{16} = 10_{10}$

$10 * 16^0 + 8 * 16^1 + 3 * 16^2$
10 + 128 + 768 = 906

1.A.4:

$2214_5 = 309_{10}$
$4 * 5^0 + 1 * 5^1 + 2 * 5^2 + 2 * 5^3$
4 + 5 + 50 + 250 = 309

B: Convert the following numbers to their binary representation:

1.B.1:

$69_{10} = 1000101_2$
Divide the decimal number by 2 and gather the remainder to represent the number in binary form
69 ÷ 2 = 34 R 1
34 ÷ 2 = 17 R 0
17 ÷ 2 = 8 R 1
8 ÷ 2 = 4 R 0
4 ÷ 2 = 2 R 0
2 ÷ 2 = 1 R 0
1 ÷ 2 = 0 R 1

1.B.2:

485_{10} = 111100101_2

485 ÷ 2 = 242 R 1
242 ÷ 2 = 121 R 0
121 ÷ 2 = 60 R 1
60 ÷2 = 30 R 0
30 ÷ 2 = 15 R 0
15 ÷ 2 = 7 R 1
7 ÷ 2 = 3 R 1
3 ÷ 2 = 1 R 1
1 ÷ 2 = 0 R 1

1.B.3:

$6D1A_{16}$ = 0110110100011010_2
$A_{16} = 10_{10} = 1010_2$
$1_{10} = 0001_2$
$D_{16} = 13_{10} = 1101_2$
$6_{10} = 0110_2$

C: Convert the following numbers to their hexadecimal representation:

1.C.1:

1101011_2 = $6B_{16}$
$1011_2 = 11_{10} = B_{16}$
$0110_2 = 6_{10} = 6_{16}$

1.C.2:

895_{10} = $37F_{16}$
Divide by 16 and take the remainder
895 ÷16 = 55 R 15
$15_{10} = F_{16}$
55 ÷16 = 3 R 7
3 ÷16 = 0 R 3

Question 2

Solve the following, do all calculation in the given base. Show your work.

2.1:

$7566_8 + 4515_8 = 14303_8$

$$\begin{array}{r} {\scriptstyle 1\,1\,1\;\;} \\ 7566 \\ +\ 4515 \\ 14303 \end{array}$$

2.2:

$10110011_2 + 1101_2 = 11000000_2$

$$\begin{array}{r} {\scriptstyle 1\,1\,1\,1\,1\,1\;\;} \\ 10110011 \\ +\quad 1101 \\ 11000000 \end{array}$$

2.3:

$7A66_{16} + 45C5_{16} = C02B_{16}$

$$\begin{array}{r} {\scriptstyle 1\,1\;\;\;\;} \\ 7A66 \\ +\ 45C5 \\ C02B \end{array}$$

2.4:

$3022_5 - 2433_5 = 34_5$

$$\begin{array}{r} {\scriptstyle 2\;4\;1\;\;} \\ \cancel{302}2 \\ -2433 \\ 34 \end{array}$$

Question 3

A. Convert the following numbers to their 8-bits two's complement representation. Show your work.

3.A.1:

$124_{10} = 01111100_{\text{8-bit 2 's comp}}$

Since it starts with a 0, I know it is positive. Divide by two, take the remainder.

124 ÷ 2 = 62 R 0
62 ÷ 2 = 31 R 0
31 ÷ 2 = 15 R 1
15 ÷ 2 = 7 R 1
7 ÷ 2 = 3 R 1
3 ÷ 2 = 1 R 1
1 ÷ 2 = 0 R 1

Add another 0 to the beginning to make it 8 bit two's complement

3.A.2:

$-124_{10} = 10000100_{\text{8-bit 2 's comp}}$

I already know positive 124 is 01111100 in 8 bit 2's complement.
Step 1: flip the bits
10000011

Step 2: add 1

$$\begin{array}{l} {\scriptstyle 1\,1} \\ 10000011 \\ + \qquad\;\; 1 \\ 10000100 \end{array}$$

To check my work, the additive inverse of both binary numbers should add up to 1 followed by 8 zeroes.

$$\begin{array}{rl} {\scriptstyle 1\,1\,1\,1\,1} & \\ 01111100 & = 24 \\ +\ 10000100 & = -24 \\ 100000000 & \end{array}$$

Thus, $-124_{10} = 10000100_{\text{8-bit 2's complement}}$

3.A.3:

$109_{10} = 01101101_{\text{8-bit 2's comp}}$

Since it starts with a 0, I know it is positive. Divide by two, take the remainder.

109 ÷ 2 = 54 R 1
54 ÷ 2 = 37 R 0
27 ÷ 2 = 13 R 1
13 ÷ 2 = 6 R 1
6 ÷ 2 = 3 R 0
3 ÷ 2 = 1 R 0
1 ÷ 2 = 0 R 1

Add a 0 at the beginning to make it 8-bit 2's complement

3.A.4:

$-79_{10} = 10110001_{\text{8-bit 2's comp}}$

First Step is to figure out positive 79 in 8 bit 2's complement. I do that by dividing by 2 and taking the remainder, and adding zeroes at the beginning if needed.

79 ÷ 2 = 39 R 1
39 ÷ 2 = 19 R 1
19 ÷ 2 = 9 R 1
9 ÷ 2 = 4 R 1
4 ÷ 2 = 2 R 0
2 ÷ 2 = 1 R 0
1 ÷ 2 = 0 R 1

$79_{10} = 01001111_{\text{8-bit 2's comp}}$

The second step is to flip the bits: 10110000, and then add 1 bit, to find the 8-bit two's complement

$$\begin{array}{r} 10110000 \\ +\quad\quad 1 \\ 10110001 \end{array}$$

To check my work, both numbers should add up to 1 followed by 8 zeroes.

$$\begin{array}{rl} {\scriptstyle 1\,1\,1\,1\,1\,1\,1} & \\ 01001111 & = 79 \\ +\ 10110001 & = -79 \\ 100000000 & \end{array}$$

Thus, $-79_{10} = 10110001_{\text{8-bit 2's comp}}$

B: Convert the following numbers (represented as 8-bit two's complement) to their decimal representation. Show your work.

3.B.1:

$00011110_{\text{8-bit 2 's comp}} = 30_{10}$

Since it starts with a 0, I know that it is positive. I just have to convert it to decimal.

$0 * 2^0 + 1 * 2^1 + 1 * 2^2 + 1 * 2^3 + 1 * 2^4$

2 + 4 + 8 + 16 = 30

3.B.2:

$11100110_{\text{8-bit 2 's comp}} = -26_{10}$

Since it starts with 1, I know it is a negative number. The additive inverse of the number will add up to 1 followed by 8 zeroes.

$$\begin{array}{r} {\scriptstyle 1111111} \\ 11100110 \\ +\ 00011010 \\ 100000000 \end{array}$$

Converting 00011010 to decimal:

$0 * 2^0 + 1 * 2^1 + 0 + 2^2 + 1 * 2^3 + 1 * 2^4 = 26$

Thus, it follows that the inverse, $11100110_{\text{8-bit 2 's comp}} = -26_{10}$

3.B.3:

$00101101_{\text{8-bit 2 's comp}} = 45_{10}$

Since it starts with 0, I know that it is positive. Converting to decimal:

1 ∗ 2 0 + 0 ∗ 2 1 + 1 ∗ 2 3 + 0 ∗ 2 4 + 1 ∗ 2 5 1 + 4 + 8 + 32 = 45

3.B.4:

$10011110_{\text{8-bit 2 's comp}} = -98_{10}$

Since it starts with 1, I know it is negative. Adding the inverse will add up to 1 followed by 8 zeroes.

$$\begin{array}{r} {\scriptstyle 111111} \\ 10011110 \\ +\ 01100010 \\ 100000000 \end{array}$$

Converting 01100010 to binary:

$0 * 2^0 + 1 * 2^1 + 0 * 2^2 + 0 * 2^3 + 0 * 2^4 + 1 * 2^5 + 1 * 2^6$

2 + 32 + 64 = 98

It follows that the inverse, $10011110_{\text{8-bit 2 's comp}} = -98_{10}$

Question 4

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.4, sections b, c

B:

$\neg(p \lor q)$

p	q	$p \lor q$	$\neg(p \lor q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

C:

$r \lor (p \land \neg q)$

p	q	r	$p \land \neg q$	$r \lor (p \land \neg q)$
T	T	T	F	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

2. Exercise 1.3.4, sections b, d

Give a truth table for each expression.

B:

$$(p \to q) \to (q \to p)$$

p	q	$(p \to q)$	$(q \to p)$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

D:

$$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$$

p	q	$(p \leftrightarrow q)$	$(p \leftrightarrow \neg q)$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	T	F	T

Question 5

Solve the following questions from the Discrete Math zyBook:

1: Exercise 1.2.7, sections b, c

B: $(B \land D) \lor (B \land M) \lor (D \land M)$

C: $B \land (D \lor M)$

2: Exercise 1.3.7, sections b – e:

B: $(s \lor y) \rightarrow p$

C: $p \rightarrow y$

D: $p \leftrightarrow (s \land y)$

E: $p \rightarrow (s \lor y)$

3. Exercise 1.3.9, sections c, d

C: $c \rightarrow p$

D: $c \rightarrow p$

Question 6

Solve the following questions from the Discrete Math zyBook:

1: Exercise 1.3.6, sections b - d

B: If Joe is eligible for the honors program, then he must maintain a B average.

C: If Rajiv can go on the roller coaster, then he is at least 4 feet tall.

D: If Rajiv is at least 4 feet tall, then he can go on the roller coaster.

2. Exercise 1.3.10, sections c– f

p: True

q: False

r: unknown

C: $(p \lor r) \leftrightarrow (q \lor r)$

$T \leftrightarrow (F \lor r)$

This statement is false, because $F \lor r$ will evaluate to false regardless of r’s truth value

D: $(p \land r) \leftrightarrow (q \land r)$

$(T \land r) \leftrightarrow F$

This statement is unknown, because the truth value of $T \land r$ depends on r’s truth value, and the truth value of the entire statement depends on whether $T \land r$ is true or false.

E: $p \rightarrow (r \lor q)$

$T \rightarrow (r\, v\, F)$

This statement is unknown, because the truth value of this statement is dependant on whether $r \lor q$ evaluates to true or false

F: $(p \land q) \rightarrow r$

$(T \land F) \rightarrow r$

$F \rightarrow r$

This statement is true, because it would only be false if $p \land q$ evaluated to true and r evaluated to false

Question 7

Solve Exercise 1.4.5, sections b – d, from the Discrete Math zyBook:

j: Sally got the job.

l: Sally was late for her interview

r: Sally updated her resume.

B:

If Sally did not get the job, then she was late for her interview or did not update her resume.

If Sally updated her resume and was not late for her interview, then she got the job.

$\neg j \rightarrow (l \lor \neg r)$

$(r \land \neg l) \rightarrow j$

The two statements are logically equivalent

j	**l**	**r**	$(l \lor \neg r)$	$(r \land \neg l)$	$\neg j$	$\neg j \rightarrow (l \lor \neg r)$	$(r \land \neg l) \rightarrow j$
T	T	T	T	F	F	T	T
T	T	F	T	F	F	T	T
T	F	T	F	T	F	T	T
T	F	F	T	F	F	T	T
F	T	T	T	F	T	T	T
F	T	F	T	F	T	T	T
F	F	T	F	T	T	F	F
F	F	F	T	F	T	T	T

C:

If Sally got the job then she was not late for her interview.
If Sally did not get the job, then she was late for her interview.

$j \to \neg l$
$\neg j \to l$

The two statements are not logically equivalent

j	l	$\neg j$	$\neg l$	$j \to \neg l$	$\neg j \to l$
T	T	F	F	F	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	F

D:
If Sally updated her resume or she was not late for her interview, then she got the job.
If Sally got the job, then she updated her resume and was not late for her interview.

$(r \lor \neg l) \to j$
$j \to (r \land \neg l)$

The two statements are not logically equivalent

j	l	r	$(r \lor \neg l)$	$(r \land \neg l)$	$(r \lor \neg l) \to j$	$j \to (r \land \neg l)$
T	T	T	T	F	T	F
T	T	F	F	F	T	F
T	F	T	T	T	T	T
T	F	F	T	F	T	F
F	T	T	T	F	F	T
F	T	F	F	F	T	T
F	F	T	T	T	F	T
F	F	F	T	F	F	T

Question 8

Solve the following questions from the Discrete Math zyBook:

1: Exercise 1.5.2, sections c, f, i

C: $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

$(p \to q) \wedge (p \to r)$	
$(\neg p \vee q) \wedge (\neg p \vee r)$	Conditional identities
$\neg p \vee (q \wedge r)$	Distributive Laws
$p \to (q \wedge r)$	Conditional Identities

F: $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

$\neg(p \vee (\neg p \wedge q))$	
$\neg((p \vee \neg p) \wedge (p \vee q)$	Distributive laws
$\neg(T \wedge (p \vee q)$	Complement Laws
$\neg(p \vee q)$	Identity Laws
$\neg p \wedge \neg q$	DeMorgan's Laws

I: $(p \wedge q) \to r \equiv (p \wedge \neg r) \to q$

$(p \wedge q) \to r$	
$\neg(p \wedge q) \vee r$	Conditional Identities
$(\neg p \vee \neg q) \vee r$	DeMorgan's Laws
$(\neg q \vee \neg p) \vee r$	Commutative laws
$\neg q \vee (\neg p \vee r)$	Associative
$(\neg p \vee r) \vee \neg q$	Commutative
$\neg(\neg p \vee r) \to \neg q$	Conditional Identities
$(\neg\neg p \wedge r) \to \neg q$	DeMorgan's Laws
$(p \wedge \neg r) \to \neg q$	Double Negation

2.Exercise 1.5.3, sections c, d

C: $\neg r \vee (\neg r \to p)$

$\neg r \vee (\neg\neg r \vee p)$	Conditional Identities
$\neg r \vee (r \vee p)$	Double Negation
$(\neg r \vee r) \vee (\neg r \vee p)$	Distributive
$T \vee (\neg r \vee p)$	Complement Laws
$T \vee (\neg r \vee p) \equiv T$	Domination Laws

D: $\neg(p \to q) \to \neg q$

$\neg(\neg p \vee q) \to \neg q$	Conditional Identities
$(\neg\neg p \wedge \neg q) \to \neg q$	DeMorgan's Laws
$(p \wedge \neg q) \to \neg q$	Double Negation
$\neg(p \vee \neg q) \vee \neg q$	Conditional Identities
$(\neg p \vee \neg\neg q) \vee \neg q$	DeMorgan's Laws
$(\neg p \vee q) \vee \neg q$	Double Negation
$\neg p \vee (q \vee \neg q)$	Associative
$\neg p \vee T$	Complement
$\neg p \vee T \equiv T$	Domination Laws

Question 9

Solve the following questions from the Discrete Math zyBook:

1.Exercise 1.6.3, sections c, d

C: $\exists x(x = x^2)$

D: $\forall x(x \leq x^2 + 1)$

2. Exercise 1.7.4, sections b - d

B: $\forall x(\neg S(x) \land W(x))$

C: $\forall x(S(x) \rightarrow \neg W(x))$

D: $\exists x(S(x) \land W(x))$

Question 10

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.7.9, sections c - i
   - C: $\exists x((x = c) \rightarrow P(x)) \equiv T$
   - D: $\exists x(Q(x) \land R(x)) \equiv T$
   - E: $Q(a) \land P(d) \equiv T$
   - F: $\forall x((x \neq b) \rightarrow Q(x)) \equiv T$
   - G:$\forall x(P(x) \lor R(x)) \equiv F$
   - H:$\forall x(Q(x) \rightarrow P(x)) \equiv T$
   - I: $\exists x(Q(x) \lor R(x)) \equiv T$
2. Exercise 1.9.2, sections b - i
   - B: $\exists x \forall y Q(x, y) \equiv T$
   - C:$\exists y \forall x P(x, y) \equiv T$
   - D:$\exists x \exists y S(x, y) \equiv F$
   - E:$\forall x \exists y Q(x, y) \equiv F$
   - F:$\forall x \exists y P(x, y) \equiv T$
   - G:$\forall x \forall y P(x, y) \equiv F$
   - H:$\exists x \exists y Q(x, y) \equiv T$
   - I: $\forall x \forall y \neg S(x, y) \equiv T$

Question 11

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.10.4, sections c - g

    C: $\exists x \exists y(x + y = xy)$

    D: $\forall x \forall y(((x > 0) \land (y > 0)) \to (x{:}\, y > 0))$

    E: $\forall x(((x > 0) \land (x < 1)) \to (1/x > 1))$

    F: $\forall x \exists y(y < x)$

    G: $\forall x \exists y((x \neq 0) \to (xy = 1))$

2. Exercise 1.10.7, sections c - f

    C: $\exists x((N(x) \land D(x))$

    D: $\exists x \forall y((x = Sam) \land ((D(y) \land P(x, y)))$

    E: $\exists x \forall y(N(x) \land P(x, y))$

    F: $\exists x(N(x) \land D(x))$

3. Exercise 1.10.10, sections c – f

    C: $\forall x \exists y((y \neq Math\,101) \land T(x, y))$

    D: $\exists x \forall y((y \neq Math101) \to (T(x, y))$

    E:$\forall x \exists y \exists z((x \neq Sam) \land (T(x, y) \land T(x, z))$

    F:$\exists x \exists y \exists z(T(Sam, x) \land T(Sam, y) \land \neg T(Sam, z))$

Question 12

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.8.2, sections b – e

B: Every patient was given the medication or the placebo or both.

$\forall x(D(x) \lor P(x))$	
$\neg\forall x(D(x) \lor P(x))$	Negation
$\exists x\neg(D(x) \lor P(x))$	
$\exists x(\neg D(x) \land \neg P(x))$	DeMorgan's Law

English:
There is a patient who was not given the medication and not given the placebo.

C: There is a patient who took the medication and had migraines.

$\exists x(D(x) \land M(x))$	
$\neg\exists x(D(x) \land M(x))$	Negation
$\forall x\neg(D(x) \land M(x))$	
$\forall x(\neg D(x) \lor \neg M(x))$	DeMorgan's Law

English:
Every patient either was not given medication or did not have migraines.

D: Every patient who took the placebo had migraines.

$\forall x(P(x) \to M(x))$	
$\neg\forall x(P(x) \to M(x))$	Negation
$\exists x\neg(P(x) \to M(x))$	
$\exists x\neg(\neg P(x) \lor M(x))$	Conditional identities
$\exists x(\neg\neg P(x) \land \neg M(x))$	DeMorgan's Laws
$\exists x(P(x) \land \neg M(x))$	Double negative

English:
Some patient took the placebo and did not have migraines

E: There is a patient who had migraines and was given the placebo.

$\exists x(M(x) \land P(x))$	
$\neg\exists x(M(x) \land P(x))$	Negation
$\forall x\neg(M(x) \land P(x))$	
$\forall x(\neg M(x) \lor \neg P(x))$	DeMorgan's Laws

English:
Every patient either did not have migraines or did not take the placebo.

2. Exercise 1.9.4, sections c - e

C:

$\exists x\forall y(P(x,y) \to Q(x,y))$	
$\neg\exists x\forall y(P(x,y) \to Q(x,y))$	
$\forall x\exists y\neg(P(x,y) \to Q(x,y))$	
$\forall x\exists y\neg(\neg P(x,y) \lor Q(x,y))$	Conditional Identities
$\forall x\exists y(\neg\neg P(x,y) \land \neg Q(x,y))$	DeMorgan's Laws
$\forall x\exists y(P(x,y) \land \neg Q(x,y))$	Double Negatives

D:

$\exists x \forall y (P(x,y) \leftrightarrow P(y,x))$
$\neg \exists x \forall y (P(x,y) \leftrightarrow P(y,x))$
$\forall x \exists y \neg (P(x,y) \leftrightarrow P(y,x))$
$\forall x \exists y \neg ((P(x,y) \ \rightarrow P(y,x)) \ \land \ (P(y,x) \rightarrow P(x,y)))$ Conditional Identities
$\forall x \exists y \neg ((\neg P(x,y) \ \lor \ P(y,x)) \ \land \ (\neg P(y,x) \ \lor \ P(x,y))$ Conditional Identities
$\forall x \exists y (\neg\neg P(x,y) \ \land \ \neg P(y,x)) \ \land \ (\neg\neg P(y,x) \ \land \ \neg P(x,y))$ DeMorgan's Laws
$\forall x \exists y (P(x,y) \ \land \ \neg P(y,x)) \ \land \ (P(y,x) \ \land \ \neg P(x,y))$ Double Negatives

E:

$\exists x \exists y P(x,y) \ \land \ \forall x \forall y Q(x,y)$
$\neg \exists x \exists y P(x,y) \ \land \ \neg \forall x \forall y Q(x,y)$
$\forall x \forall y \neg P(x,y) \ \land \ \exists x \exists y \neg Q(x,y)$