Question 7

Solve the following questions from the Discrete Math zyBook:

Exercise 3.1.1, sections a-g

a. True
b. False
c. True
d. False
e. True
f. False
g. False

Exercise 3.1.2, sections a-e

a. False
b. True
c. True
d. True
e. False

Exercise 3.1.5, sections b, d

b. $\{x \in \mathbb{Z}^{+}: \ x \ is \ a \ multiple \ of \ 3\}$

This set is an infinite set

d. $\{x \in \mathbb{N}: 0 \leq x \leq 1000 \ and \ x \ is \ a \ multiple \ of \ 10\}$

The cardinality of this set would be 101

Exercise 3.2.1, sections a-k

a. True
b. True
c. False
d. False
e. True
f. True
g. True
h. False
i. False

j. False
k. False

Question 8

Solve Exercise 3.2.4, section b from the Discrete Math zyBook.

Let A = {1, 2, 3}. What is $\{X \in P(A)\ :\ 2 \in X\}$?

$X\ =\ \{\{2\},\ \{1, 2\},\ \{2, 3\}\,\{1, 2, 3\}\}$

Question 9

Solve the following questions from the Discrete Math zyBook:

Exercise 3.3.1, sections c-e

c. $A \cap C = \{-3, 1, 17\}$

d. $A \cup (B \cap C) = \{-5, -3, 0, 1, 4, 17\}$

e. $A \cap B \cap C = \{1\}$

Exercise 3.3.3, sections a, b, e, f

$A_i = \{i^0, i^1, i^2\}$

$C_i = \{x \in \mathbb{R} : -1/i \leq x \leq 1/i\}$

a: $\bigcap_{i=2}^{5} A_i$

This is $A_2 \cap A_3 \cap A_4 \cap A_5$

$A_2 = \{1, 2, 4\}$

$A_3 = \{1, 3, 9\}$

$A_4 = \{1, 4, 16\}$

$A_5 = \{1, 5, 25\}$

$\bigcap_{i=2}^{5} Ai = \{1\}$

b. $\bigcup_{i=2}^{5} A_i$

This is $A_2 \cup A_3 \cup A_4 \cup A_5$

$\bigcup_{i=2}^{5} A_i = \{1, 2, 3, 4, 5, 9, 16, 25\}$

e. $\bigcap_{i=1}^{100} C_i$

$C_1 = \{x \in \mathbb{R} : \ -1/1 \ \le x \le 1/1\}$

$C_2 = \{x \in \mathbb{R} : \ -1/2 \ \le x \le 1/2\}$

$C_3 = \{x \in \mathbb{R} : \ -1/3 \le x \le 1/3\}$

...

$C_{100} = \{x \in \mathbb{R} : \ -1/100 \le x \le 1/100\}$

Each subsequent iteration gets smaller and smaller, so the intersection of this equation would be the smallest iteration.

$\bigcap_{i=1}^{100} C_i = \{x \in \mathbb{R} : \ -1/100 \le x \le 1/100\}$

f. $\bigcup_{i=1}^{100} C_i$

It is the union of all of these elements, and since each element gets smaller and smaller, the union would be the largest iteration.

$\bigcup_{i=1}^{100} C_i = \{x \in \mathbb{R} : \ -1/1 \ \le x \le 1/1\}$

Exercise 3.3.4, sections b, d

$A = \{a, b\}$

$B = \{b, c\}$

b. $P(A \cup B)$

$(A \cup B) = \{a, b, c\}$

$P(A \cup B) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

d. $P(A) \cup P(B)$

$P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
$P(B) = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$

$P(A) \cup P(B) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{c\}, \{b, c\}\}$

Question 10

Solve the following questions from the Discrete Math zyBook:

Exercise 3.5.1, sections b, c

$A = \{tall,\ grande,\ venti\}$
$B = \{foam,\ no - foam\}$
$C = \{non - fat, whole\}$

b: Write an element from the set $A \times B \times C$

$(tall, foam, non - fat)$

c: Write the set $B \times C$ using roster notation

$B \times C =$
$\{(foam,\ non - fat),\ (no - foam,\ non - fat),\ (foam,\ whole),\ (no - foam,\ whole)\}$

Exercise 3.5.3, sections b, c, e

b: $\mathbb{Z}^2 \subseteq \mathbb{R}^2$

This is true, because $\mathbb{Z}^2$ is the ordered pair of all integers, and $\mathbb{R}^2$ is the ordered pair of all real numbers, and $\mathbb{Z} \subseteq \mathbb{R}$

c: $\mathbb{Z}^2 \cap \mathbb{R}^3 = \emptyset$

True, because the intersection of $\mathbb{Z}^2$, which is the ordered pair of all integers, and the set of $\mathbb{R}^3$, the ordered triple of all real numbers, would be an empty set.

e: For any three sets, A, B, and C, if $A \subseteq B,\ then\ A \times C \subseteq B \times C$

This is True, because all the elements of A are in B, so it follows that all the elements of A x C would be in B x C.

Exercise 3.5.6, sections d, e

d. $\{xy\colon \ where\ x \in \{0\} \cup \{0\}^2 \ and\ y \in \{1\} \ \cup \{1\}^2\}$

$\{01, 0011, 001, 011\}$

e. $\{xy\colon \ where\ x \in \{aa, ab\}\ and\ y \in \{a\} \ \cup \{a\}^2\}$

$\{aaa, aaaa, aba, abaa\}$

Exercise 3.5.7, sections c, f, g

A={a}

B={b,c}

C={a,b,d}

c. $(A \times B) \cap (A \times C)$

$(A \times B) = \{(a, b), (a, c)\}$

$(A \times C) = \{(a, a),\ (a, b),\ (a, d)\}$

$(A \times B) \cap (A \times C) = \{(a, b)\}$

f. $P(A \times B)$

$=\{(a, b), (a, c),\ \{(a, b), (a, c)\},\ \emptyset\}$

e. $P(A) \ \times P(B)$

$P(A) \ = \ \{\emptyset, \{a\}\}$

$P(B) \ = \{\emptyset,\ \{b\},\ \{c\},\ \{b, c\}\}$

$P(A) \ \times P(B) \ =$

$\{(\emptyset, \emptyset),\ (\emptyset, \{b\}),\ (\emptyset, \{c\}),\ (\emptyset, \{b, c\}),\ (\{a\},\ \emptyset),\ (\{a\}, \{b\}),\ (\{a\}, \{c\}),\ (\{a\}, \{b, c\})\}$

Question 11

Solve the following questions from the Discrete Math zyBook:

Exercise 3.6.2, sections b, c

Use the set identities given in the table to prove the following new identities. Label each step in your proof with the set identity used to establish that step.

b. $(B \cup A) \cap (\overline{B} \cup A) = A$

$(B \cup A) \cap (\overline{B} \cup A)$	beginning expression
$(A \cup B) \cap (A \cup \overline{B})$	Commutative Law
$A \cup (B \cap \overline{B})$	Distributive Law
$A \cup \emptyset$	Complement Law
A	Domination Law

c. $\overline{A \cap \overline{B}} = \overline{A} \cup B$

$\overline{A \cap \overline{B}} = \overline{A} \cup B$

$\overline{A \cap \overline{B}}$	Beginning Expression
$\overline{A} \cup \overline{\overline{B}}$	DeMorgan's Law
$\overline{A} \cup B$	Double Complement

Exercise 3.6.3, sections b, d

Show that each set equation given below is not a set identity.

b. $A - (B \cap A) = A$

Let $A = \{1, 2, 3, 4, 5\}$
Let B=$\{4, 5, 6, 7, 8\}$
$(B \cap A) = \{4, 5\}$
$A - (B \cap A) = \{1, 2, 3\} \neq A$

d. $(B - A) \cup A = A$

Let $A = \{1, 2, 3, 4, 5\}$
Let $B = \{6, 7, 8, 9, 10\}$
$B - A = \{6, 7, 8, 9, 10\}$
$(B - A) \cup A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \neq A$

Exercise 3.6.4, sections b, c

Use the set subtraction law as well as the other set identities given in the table to prove each of the following new identities. Label each step in your proof with the set identity used to establish that step.

b. $A \cap (B - A) = \emptyset$

$A \cap (B - A)$	Beginning Expression
$A \cap (B \cap \overline{A})$	Set subtraction law
$A \cap (\overline{A} \cap B)$	Commutative Law
$(A \cap \overline{A}) \cap B$	Associative Law
$\emptyset \cap B$	Complement Law
$B \cap \emptyset$	Commutative Law
$\emptyset$	Domination Law

c. $A \cup (B - A) = A \cup B$

$A \cup (B - A)$	Beginning Expression
$A \cup (B \cap \bar{A})$	Set Subtraction Law
$(A \cup B) \cap (A \cup \bar{A})$	Distributive Law
$(A \cup B) \cap U$	Complement Law
$A \cup B)$	Identity Law